Two-Phase Behaviour of Financial Markets

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Buying and selling behaviour in complex financial markets are driven by demand, which can be quantified by the imbalance in the number of shares transacted by buyers and sellers over a time interval $\Delta t$. We analyze the probability distribution of demand, conditioned on its local noise intensity $\Sigma$, and find the surprising existence of a critical threshold $\Sigma_c$ separating two market phases (a) an equilibrium phase ($\Sigma < \Sigma_c$) in which neither buying nor selling behaviour dominates, and (b) an out-of-equilibrium phase ($\Sigma > \Sigma_c$) in which approximately half the time the market behaviour is mainly buying and half the time the market behaviour is mainly selling. For the equilibrium phase the most probable value of demand is approximately zero, while for the out-of-equilibrium phase two most probable values emerge that are symmetric around zero demand, corresponding to excess demand and excess supply [1].

To quantify the demand, we use the transactions and quotes database to analyze each and every transaction of the 116 most actively-traded stocks in the two-year period 1994-1995. Specifically, we calculate the volume imbalance $\Omega(t)$ for a huge sequence of short time intervals $\Delta t$ ($\approx 10^4$ intervals if $\Delta t = 15$ min). The volume imbalance is defined to be the difference between the number of shares $Q_B$ traded in buyer-initiated transactions and the number of shares $Q_S$ traded in seller-initiated transactions in $\Delta t$ [2,3],

$$\Omega(t) \equiv Q_B - Q_S = \sum_{i=1}^{N} q_i a_i, \quad (1)$$

where $i = 1, \ldots, N$ labels each of the $N$ transactions in the time interval $\Delta t$, $q_i$ denotes the number of shares traded in transaction $i$, and $a_i = \pm 1$ denotes buyer-initiated and seller-initiated trades respectively [2].

We also calculate, for the same sequence of intervals, the local noise intensity,

$$\Sigma(t) \equiv \langle |q_i a_i - \langle q_i a_i \rangle | \rangle, \quad (2)$$

where $\langle \ldots \rangle$ the denotes local expectation value, computed from all transactions of that stock in the time interval $\Delta t$.  

We find (Fig. 1a) that for small $\Sigma$, the conditional distribution $P(\Omega|\Sigma)$ is single-peaked, displaying a maximum at zero demand $\Omega = 0$. For $\Sigma$ larger than a critical threshold $\Sigma_c$, the behaviour of $P(\Omega|\Sigma)$ undergoes a qualitative change, becoming double-peaked with a pair of new maxima appearing at nonzero values of demand, $\Omega = \Omega_+$ and $\Omega = \Omega_-$, which are symmetric around $\Omega = 0$.

Our findings for the financial market problem are identical to what is known to occur in all phase transition phenomena, wherein the behaviour of a system undergoes a qualitative change at a critical threshold $K_c$ of some control parameter $K$. The change in behaviour at $K_c$ can be quantified by an order parameter $\Psi(K)$, where $\Psi(K) = 0$ for $K < K_c$ and $\Psi(K) \neq 0$ for $K > K_c$.

For the financial market problem, we find that the order parameter $\Psi = \Psi(\Sigma)$ is given by the values of the maxima $\Omega_{\pm}$ of $P(\Omega)$. Figure 1b shows that the change in $\Psi(\Sigma)$ as a function of $\Sigma$ is described by

$$
\Psi(\Sigma) = \begin{cases} 
0 & [\Sigma < \Sigma_c] \\
\Sigma - \Sigma_c & [\Sigma \gg \Sigma_c]
\end{cases}.
$$

(3)

We interpret these two market phases as corresponding to two distinct conditions of the financial market:

(a) The “$\Sigma < \Sigma_c$ market phase”, where the distribution of demand $\Omega$ is single peaked with the most probable value being zero, we interpret to be the market *equilibrium phase*, since the price of the stock is such that the probability of a transaction being buyer initiated is equal to the probability of a transaction being seller initiated [4]. Thus, in the equilibrium phase, there is statistically no *net* demand, and prices fluctuate around their “equilibrium” values, suggesting that most of the trading results from “noise traders” who trade from misperception of information or for idiosyncratic reasons [5–7].

(b) The “$\Sigma > \Sigma_c$ market phase”, where the distribution of demand is bimodal, we interpret to be the *out-of-equilibrium phase*, since the price of the stock is such that there is an excess of either buyers or of sellers and there is a non-zero net demand for the stock. Thus, in the out-of-equilibrium phase, the prevalent “equilibrium” price has very recently changed, so
the stock price is now being driven to the market’s new evaluation of a fair value, consistent with the possibility that most of the trading arises from “informed” traders who possess superior information [5–7].

The major interest of the above results is to identify statistical regularities in the reaction of prices on imbalances in demand. What they add to existing literature is the clear identification of a threshold for imbalances below which no clear tendencies of price change exist.

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REFERENCES


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FIGURES

(a) Conditional density $P(\Omega|\Sigma)$

(b) Order parameter $\Psi(\Sigma)$

Equilibrium phase, out-of-equilibrium phase

Volume imbalance $\Omega$

Local deviation $\Sigma$

$\Sigma_c$
FIG. 1. Empirical evidence supporting the existence of two distinct phases in a complex financial market. (a) Conditional density $P(\Omega|\Sigma)$ for varying $\Sigma$ computed using data for all stocks. For each stock, $\Omega$ and $\Sigma$ are normalized to zero mean and unit first centered moment. The distribution displays a single peak for $\Sigma < \Sigma_c$ (solid line). For $\Sigma \approx \Sigma_c$ (dotted line), the distribution flattens near the origin, and for $\Sigma > \Sigma_c$, $P(\Omega|\Sigma)$ displays two peaks (dashed line). (b) Order parameter, $\Psi$ (positions of the maxima of the distribution $P(\Omega|\Sigma)$), as a function of $\Sigma$. For small $\Sigma$, $P(\Omega|\Sigma)$ displays a single maximum whereas for large $\Sigma$ two maxima appear. To locate the extrema as accurately as possible, we compute all probability densities using the density estimator of [8]. Also shown, via shading, is a schematic phase diagram representing the two distinct market phases. Here $\Delta t = 15$ min; we have tested that our results hold for $\Delta t$ ranging from 15 min up to approximately 1/2 day, beyond which our statistics is insufficient.