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Hierarchy in social organization

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Abstract

We find that area and population distributions of nations follow an inverse power-law, as is known for cities, but with a different exponent. To interpret this result, we develop a growth model based on the geometrical properties of partitioning of the plane. The substantial agreement between the model and the actual nation distributions motivates the hypothesis that the distribution of aggregates of organisms is related to land partitioning. To test this hypothesis we follow the development of bacterial colonies of *Escherichia coli*, which, compared to humans, are on a completely different level of complexity. We find that the distributions of *E. coli* colonies follow an inverse power law with exponent similar to that of nations.

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1. Introduction

Universality in the behavior of complex systems often manifests itself in the form of scale-invariant distributions that are essentially independent of the details of the

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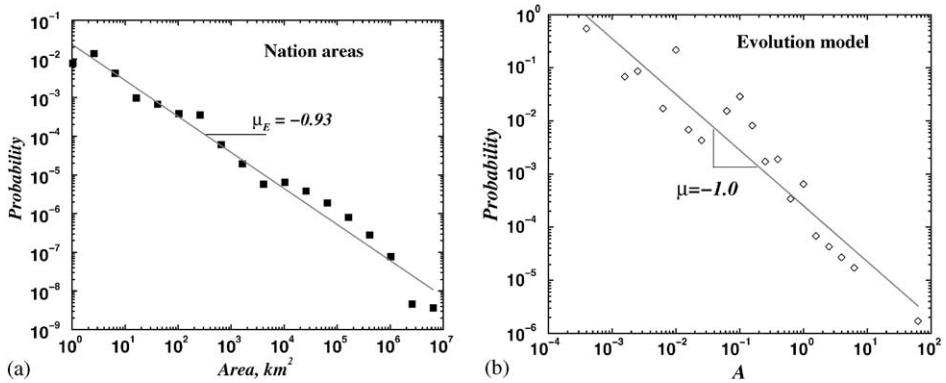


Fig. 1. (a) Double-logarithmic plot of the histogram of areas, A , of the 255 nations of the world in 1998; the linear regression coefficient is $\mu_N \approx 0.93$ (source: <http://www.stats.demon.nl>). (b) Double-logarithmic plot of the histogram of areas for the *evolution* model; the linear regression coefficient is $\mu \approx 1$.

microscopic dynamics. The interaction of individuals gives rise to a wide variety of collective phenomena such as demographic evolution, cultural and technological development, and economic activity that strongly differ from individual dynamics. A striking example of such cooperative phenomena is the formation of urban aggregates [1,2] and nations. The study of hierarchy in social organization has a rich history. Half a century ago, Zipf observed that the population distribution of cities follows a power-law behavior with exponent $\mu_C \approx 2$ [3–8]. The Zipf law has a remarkable “universal” character since it holds at the world level as well as within a single nation, and the exponent is essentially independent of the area of the nation and its socio-economical conditions. More recently, it has been observed [5] that the area distribution of satellite images of cities, towns and villages around large urban centers also obeys a power-law with exponent ≈ 2 . A population distribution close to that obeyed by cities was obtained through a model based on a master equation approach [9].

Individuals living in the same city are related to each other by a number of “links”, defining the very concept of city [10]. In fact the exponent 2 for the degree distribution is predicted by the preferential attachment model of a growing network [11]. People living within the same nation are also related to each other, e.g. they may share linguistic or cultural heritage. The interactions among the inhabitants of a nation may differ from those among people living in the same city, being presumably “weaker”. Thus we ask if the distribution of nations obeys the same law as that of cities. In order to answer this question we analyze the area distribution of the world’s nations. The log–log plot of the area distribution $f(A)$ is shown in Fig. 1a for all world nations. The distribution follows a power law, $f(A) \sim A^{-\mu_N}$, where $\mu_N = 0.93$. The population density of a nation depends only weakly on the area of a nation and we find that measures of size in terms of population or area lead to almost identical exponents. The fine structure of the nation area distribution due to small island nations and historical

peculiarities such as Andorra and Monaco is not discernible on a log–log plot and is ignored in the following models.

Although the social and historical processes that lead to distributions of nations are complex, the essence of the distribution can be captured in a geometric model which incorporates the features characterizing the land occupation process. There is no restriction to the land accessible to a nation except that of the total existing land: a sufficiently powerful nation could expand to absorb other nations. Cities, being the result of spontaneous aggregation of individuals around sites having attractive features, can form at any distance from existing ones [4,5]. The formation of cities separates the plane region into the overlapping land basins, and each new city attracts resources from a newly formed basin of a certain size.

The basins of attraction are usually organized in a hierarchical fashion: the capital of the nation draws (“attracts”) resources from the entire nation, capitals of provinces resources from the provinces, centers of the counties resources from their counties, and villages resources from the surrounding fields. If one assumes that the population of a city or village is proportional to the area of its “basin of attraction” and that the local hierarchy has many levels, each consisting of z subdivisions, it follows that the number of cities in the n th level of the hierarchy is z^n and the size of these cities is proportional to z^{-n} . The probability density of the cities of size $A = z^{-n}$ is thus proportional to $z^n / (z^{-n} - z^{-n-1}) = A^{-2} z / (z - 1)$, leading to a power-law distribution with exponent $\mu_C = 2$. Ideally, the resulting distribution is a step-function but, in reality, because z is not constant and the sizes of subdivisions are not equal, a log–log plot of the distribution yields an approximate straight line with a slope of -2 . Changing the economical or political situation may lead to a re-organization of the basins, the administrative capitals being not necessarily the largest cities, but the underlying hierarchical structure remains to govern the city size distribution.

The resulting size distribution is not modified when a city expands to absorb nearby cities and gives rise to a compact urban aggregate. In fact, unlike nations, cities usually do not lose their land to neighbors: small towns and villages retain their identities and usually become administrative districts of a larger aggregate that obeys, as shown in Refs. [4,5], the same area distribution that holds for separate cities. Thus, the way in which land is occupied by cities differs from that of nations: for cities the accessible land is fragmented into overlapping hierarchical basins while nations occupy all available land area.

The above considerations suggest that the land occupation processes of nations might be modeled in the framework of random partitioning of the plane [12–14]. One of the simplest ways of partitioning the plane is to divide it using straight lines that are randomly oriented and positioned. Each line divides the region into two portions, of which the smaller is selected and the larger is further partitioned. Because this partition model resembles the positioning of fences on open land, in the following we refer to it as the *fence* model.

This model can be solved analytically. If, after n partitions, the land available for further division is A_n , then

$$A_{n+1} = r_{n+1} A_n, \quad (1)$$

where r_n is a random factor uniformly distributed between $\frac{1}{2}$ and 1. The area of the n th portion S_n is

$$S_n = (1 - r_n)A_{n-1}, \quad (2)$$

and its logarithm can be written as

$$\ln S_n = \sum_{i=1}^{n-1} \ln r_i + \ln(1 - r_n) + \ln A_0. \quad (3)$$

If we plot on the x -axis the values of $\ln S_n$, we obtain random points the average distance between which is equal to $-\langle \ln r_i \rangle = 1 - \ln 2 \approx 0.3$. The largest nation corresponds to almost an entire continent A_0 , and the smallest one corresponds to $A_0 e^{-0.3n}$, where n is the number of nations on the continent. Thus, the distribution of the logarithm of a nation's area in each continent is a flat distribution between $\ln A_0 - 0.3n$ and $\ln A_0$ described by the probability density

$$P(\ln S) = \frac{1}{0.3n}. \quad (4)$$

Some continents, such as Australia and North America, have very few nations and some, such as Europe and Africa, have many. On the average, there are approximately $n = 50$ nations per continent, hence we expect the approximate distribution of the logarithms of the number of nations to be uniformly distributed between the average continent area $\sim 2 \times 10^7 \text{ km}^2$ and $2 \times 10^7 e^{-15} \text{ km}^2 \approx 6 \text{ km}^2$, with a spread similar to that observed in real nations distribution. The flat distribution of the logarithms $\text{const} d(\log S)$ corresponds to the distribution $(\text{const}/S) dS$ of the areas, which is close to what we observe in Fig. 1a.

The above model is oversimplified, lacking the dynamic evolution of nation areas. This concern can be alleviated by incorporating into the fence model the possibility that nations can evolve, growing or shrinking. Thus, we propose an extension of the fence model (called, in the following, evolution model) in which with probability $\frac{1}{2}$ a nation can grow or shrink by some random factor. This model is known as random multiplicative process [16]. The $P(A)$ histogram, remarkably, is not affected. To see this, first note that such change in area corresponds to the variable $\log(A)$ increasing or decreasing by a random number, i.e., a simple random walk of the variable $\log(A)$. In the absence of boundaries, the random walk probability density $P[\log(A)]$ converges to a Gaussian distribution [15], corresponding to a lognormal distribution of A . The probability density of the lognormal distribution with the large enough variance is known [16,17] to be well approximated around its central part by a power-law distribution $P(A) \sim 1/A$. Furthermore, in the presence of reflecting boundaries which confine the random walk, its distribution converges to a constant over the confined region. As mentioned above, $P(A) \sim 1/A$ is equivalent to $P[\log(A)] \sim \text{const}$ [18]. In the present case, there are indeed such reflecting boundaries, since $A_{\max} = A_0$ and A_{\min} is some minimal size of the nation. Thus, $P[\log(A)]$ converges to a constant and, hence, $P(A) \sim 1/A$ so the $P(A)$ distribution is not affected by the addition of evolutionary dynamics. The probability density of logarithms of nation areas obtained by numerical simulations

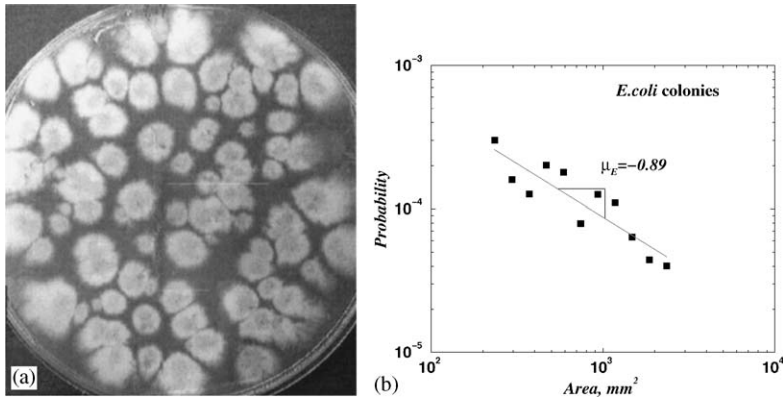


Fig. 2. (a) Bacteria colonies on a Petri dish after 6 days from inoculation. (b) Double-logarithmic plot of the histogram of areas, A , of the *E. coli* colonies; the linear regression correlation coefficient is $\mu_E \approx 0.89$.

of the evolution model is shown in Fig. 1b. The model distribution agrees well with the empirical data. This suggests the hypothesis that the distribution of social aggregates is a natural result of the partitioning of the plane.

Social aggregation occurs in all animal species, though the kind and depth of the relationships among individuals may vary enormously. At the opposite extreme of the complexity scale with respect to humans, there are unicellular organisms such as bacteria. Hence, analyzing aggregation processes of bacteria may provide a significant test of our hypothesis that plane partitioning underlies the distribution of aggregates of organisms. To this aim bacterial colonies of *Escherichia coli* were grown on a Petri dish. It is known that bacterial colonies grown on a semisolid agar substrate exhibit a rich variety of patterns under the influence of stress, chemical gradients, agar hardness, temperature and other environmental factors (see Refs. [19–25]). In order to elucidate basic geometrical features, in our study the colonies are grown under nutrient rich conditions, where the individual colony shapes are compact and do not exhibit tip splitting, branching or more complex structures [26,27]. The colonies are allowed to evolve for about 5 or 6 days after inoculation. Through a digital camera coupled to a microscope we analyze the structure of the colonies (Fig. 2a) and calculate their area distribution $P_E(A_E)$ (Fig. 2b). We find that the distribution can be fit by a power-law function, $P_E(A_E) \sim A_E^{\mu_E}$, with $\mu_E \approx 0.89$. The value of μ_E is close to the power-law exponent μ_N for land area distributions of nations.

In spite of the enormously different scales and degree of complexity of the social aggregates analyzed, nations and bacterial colonies, our results point to the possible identity of the power-law exponents of their respective area distributions. Although the slopes of the distributions for bacteria and nations are similar, the broadness of these distributions are dramatically different from each other, suggesting that bacterial colonies evolve according to the evolution model with comparable A_{\min} and A_{\max} rather than according to the fence model.

2. Methods

2.1. Pattern generation in bacterial colonies

Using a Monte Carlo Java Applet from the Boston University Center for Polymer Studies [28], we generate a random pattern of 50 spots within a circle of the same diameter as the 50 mm Petri dishes used. This pattern is transferred one spot at a time to a Petri dish containing nutrient rich agar (Bacto-Agar from Difco). 10 μ l aliquots from a solution of wild type *E. coli* bacteria, a gift from Prof. G. Jacobson, are spotted onto the agar with a platinum wire. The wire is flame-sterilized between inoculations. Bacterial growth occurs in an incubator at 37°C, in darkness. At periodic intervals, we take the plates briefly to a microscope station for recording images. Photomicrographs of the petri dishes are taken with a digital camera coupled to a microscope. Cultures are followed in this manner for about 6 days. Images are analyzed using Scion Image, which is a PC port of the popular NIH Image analysis package [29], in order to identify the individual colonies, and to determine the enclosed area of each. Colonies that were initially randomly spotted near each other grew until they apparently touched each other. These were counted as separate colonies.

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