QRE: Quick Robustness Estimation for large complex networks

Sebastian Wandelt\textsuperscript{a,b} \textsuperscript{*}, Xiaoqian Sun\textsuperscript{a,b,*}, Massimiliano Zanin\textsuperscript{c,d}, Shlomo Havlin\textsuperscript{e}

\textsuperscript{a} School of Electronic and Information Engineering, Beihang University, 100191 Beijing, China
\textsuperscript{b} Beijing Laboratory for General Aviation Technology, 100191 Beijing, China
\textsuperscript{c} The Innaxis Foundation and Research Institute, 28006 Madrid, Spain
\textsuperscript{d} Universidade Nova de Lisboa, 2829-516 Caparica, Portugal
\textsuperscript{e} Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Graphical abstract of the robustness estimation method.}
\end{figure}

\begin{table}
\centering
\begin{tabular}{|c|c|}
\hline
Method & Robustness Estimation Time \tabularnewline \hline
QRE & \textit{O}(N) \tabularnewline Betweenness Centrality & \textit{O}(N^2) \tabularnewline \hline
\end{tabular}
\caption{Comparison of robustness estimation methods.}
\end{table}

\section{Introduction}

During the last decades, empirical studies have characterized a plethora of real-world systems as complex networks \cite{1,2}, including air transportation networks \cite{3–5}, electrical power grids \cite{6,7}, Internet backbone \cite{8}, inter-bank networks \cite{9}, etc. Most networks present a well-recognized resistance against random failures \cite{10} but disintegrate rapidly under intentional attacks targeting relatively important nodes in the network first \cite{11,12}. Moreover, initial shocks can sometimes lead to cascading failures \cite{13}. Examples of recent extensive, wide-ranging network failures include European air traffic disruption caused by Icelandic volcano Eyjafjallajökull \cite{14}, large-scale power outages in the United States \cite{15}, computer virus spreading \cite{16}, cross-continental supply-chain shortages in the Japanese tsunami aftermath \cite{17}. Such disruptions cause high economic costs \cite{18}, making the analysis and improvement of resilience become one of the most critical challenges in applied network theory \cite{19,20}.

A first proposal for a metric assessing the network robustness was presented in \cite{21}. Given a network with \( N \) nodes, the robustness is defined as

\[
R = \frac{1}{N} \sum_{Q=1}^{N} s(Q),
\]

where \( s(Q) \) is the size of the giant component (GC size) after removing \( Q \) nodes. The value of \( R \) depends significantly on the underlying attacking strategy, i.e., the order in which node removals occur. Studies on network robustness usually select a single network metric, e.g., degree, betweenness, or collective influence \cite{22}, to rank nodes or links; such
2.1. Robustness estimation

Inspired on the well-known concept of percolation in statistical physics [30–34], the robustness of a network is usually defined as the critical fraction of nodes that causes a sudden disintegration [11], the latter being measured as the relative reduction in the size of the giant (largest connected) component. Note that the disintegration is higher when the GC size is smaller [35]. The intuition here is that the functionality of a network strongly depends on the number of connected nodes. In this paper we use the robustness measure \( R \), as defined in [21] and described in Section 1. Given a network with \( N \) nodes, the robustness is defined as \( R = \frac{1}{N} \sum_{Q=1}^{N} s(Q) \), where \( s(Q) \) is the GC size after removing \( Q \) nodes. The computation of \( R \) for a network requires a strategy to derive a node ranking, based on which the nodes are removed from the network. In Fig. 1, we show the robustness curves and their corresponding \( R \) values for three example networks, when nodes are removed according to their degree, i.e., highly connected nodes first (please refer to Section 3 for a description of the corresponding datasets). The football network (Fig. 1, left panel) can be considered robust: Only after attacking 50% of the nodes, the GC size is significantly reduced. The network netscience (Fig. 1, right panel), on the other hand, is very fragile: Attacking less than 5% of the nodes leads to an almost completely disintegrated network. As previously discussed, these results only provide a limited view on the network robustness, as they correspond to a specific attack strategy; one may then ask if a better node ranking criterion could be devised, and what would be the impact on the robustness estimation.

2.2. Single attack traces

In previous studies on network robustness, node rankings are usually defined by a single network metric. While the use of network metrics have been proposed in the past, we review five most significant ones:

1. **Degree**: The number of neighbors of a node, i.e., how many nodes can be reached within one hop. The degree of all nodes can be obtained in linear time regarding the number of nodes.
2. **Betweenness centrality**: \( B_i = \sum_{s \neq t \neq s} \frac{\sigma_{st}(i)}{\sigma_{st}} \), where \( \sigma_{st} \) is the number of shortest paths going from node \( s \) to node \( t \); \( \sigma_{st}(i) \) is the number of shortest paths going from node \( s \) to node \( t \) and passing through node \( i \) [36]. This metric essentially indicates the number of shortest paths going through a node. The betweenness of all nodes can be obtained in quadratic time regarding the number of nodes.
3. **Eigenvector centrality**: The eigenvector for the largest eigenvalue of the adjacency matrix. Nodes with high eigenvector centrality also connect to other nodes which have high eigenvector centrality.
4. **Katz centrality**: Computes the relative influence of a node within a network by measuring the number of the immediate neighbors (first degree nodes) and also all other nodes in the network that connect to the node under consideration through these immediate neighbors [31].
5. **PageRank**: A ranking of the nodes based on the structure of the incoming links, developed for assessing the importance of web pages [37].

Each of these metrics can be used to define an attack: Nodes are removed according to their ranking, from highly important (i.e. High metric values) to secondary (low metric values) ones. In general, two variants of a metric can be used: Static attack (which computes the metric only one time for the complete network) and interactive attack (which recomputes the metric after each removal of the top-ranked node) [38–40]. The choice of the metric is far from trivial. While the betweenness may be a good option,
Fig. 1. Robustness curves and computed R values for three networks: Football (rather robust with $R = 0.430$), adjnoun (slightly fragile with $R = 0.258$), netscience (very fragile with $R = 0.036$). We have used the interactive degree of nodes to attack the network.

Fig. 2. An example of a network attack. The process starts with a target network (a), where we want to attack two nodes. In (b) we show an optimal attack, which reduces the GC size from 13 to 5. A degree/betweenness-based attack, as shown in (c) and (d), reduces the GC size to 7; while attacking different node pairs. The combined knowledge from both metrics could be helpful to find the optimal attack, since node $d$ is identified as important by the degree-based ranking and node $a$ is identified as important by the betweenness-based ranking.

Fig. 3. Comparison of attacking strategies induced by different network metrics ($S =$ static, $I =$ interactive). Different network metrics induced significantly different robustness curves, and consequently, notably different R values. For instance, with interactive betweenness centrality as an attack strategy, football (left) becomes similar fragile as adjnoun (center). The difference in R values for football makes up 40% of the range of R (from minimum 0 to maximum 0.5).

due to its ability in describing between-communities connectivity, it understimates the importance of local connectivity patterns, which are best captured by metrics like the node degree. Therefore, the choice of the network metric is instrumental for obtaining a realistic R value. This issue is illustrated in Fig. 2 through a simple toy example; Fig. 3 further develops the idea, by showing how the GC size evolves as a function of the considered attack. Note that no exact bound on the time complexity is known for the metrics Eigenvector, Katz, and PageRank, yet experiments usually show a linear running time regarding the number of nodes and links.

An additional aspect often neglected by the physical community is the complexity cost associated with metric computation. While many metrics can be computed in linear time (regarding the number of nodes/links in the network), some need at least quadratic time complexity. If the difference is not relevant for small networks, it becomes important for large graphs, up to the point that the computation of a single betweenness-based attack is often intractable. To illustrate, the computation of a node-degree based attack requires only 2.7 s for a network with 16,706 nodes, while the betweenness computation for the same network takes more than one hour. The divergence increases tremendously with larger networks. Computing an R value is thus a trade-off between accuracy and time complexity, where spending more time often corresponds to higher accuracy. As we will show in our evaluation, the combination of few fast-to-compute network metrics can often outperform computationally expensive metrics.

2.3. Single partial attack traces

In the previous section we have discussed how to compute the R value of a network based on a node ranking. For large networks, the process of computing the R value itself, given an existing ranking, requires a significant amount of time: For each node in the ranking list, the node is removed and the GC size is computed. Since measuring the GC sizes takes at least $O(N)$ steps, the overall computation takes quadratic time. Therefore, computing the exact value of $R$ for a large network – even if a good ranking is provided – is still intractable. Below, we discuss how to obtain an approximate $R$ value based on sampling.

The intuition for sampling-based approximation is that, given the formulation of $R = \frac{1}{N} \sum_{Q=1}^{N} s(Q)$, the contribution of a single
s(Q) term is rather small for large networks: The maximum value of s(Q) is 1 and therefore the contribution of s(Q) cannot exceed \(\frac{1}{N}\). For large N, this value converges towards zero. Therefore, we propose to sample the measurements of s(Q) for a limited number of values in \(Q = \{1, \ldots, N\}\). Assuming that we sample the measurements at the interval points \([l_1, \ldots, l_m]\), how do we approximate the values of s(Q) for \(l_i < Q < l_{i+1}\)? In general, we do not just want any approximation, but we want to avoid underestimation of R, as this may be troublesome in real applications. Thus, we discard linear interpolation. We propose to set s(Q) = s(l) for \(l_i < Q < l_{i+1}\), i.e., all points inherit the GC size from the closest sample point to the left. Using this strategy, the robustness might be overestimated between two samples, but never underestimated, yielding an upper bound.

Given the upper bound approximation between sample points, the next question is how to distribute sample points. One solution is to perform an equal-width sampling, where all intervals have exactly the same size. This strategy is good for an initial robustness estimation without additional knowledge about the robustness trace. However, once we obtained an initial approximation of the robustness trace, it is often beneficial to adapt the sampling according to the currently best-known trace. Particularly, if the network is very fragile, results become more accurate, if samples points are preferably placed during the initial phase of the attack trace generation.

Instead of equal-width sampling, we propose a method inspired by equal-depth binning for the approximation of an attack trace. The idea is to split the area under the currently best-known curve into regions of approximately the same size. This strategy focuses on parts of the network with higher robustness. Once the network is almost destroyed, it does not make sense to collect further samples in this region anymore, since improvements will only slightly change the R value. For a comparison of equi-width and equi-depth sampling, please see Fig. 4.

### 2.4. Multiple attack traces, multiple iterations, and approximate betweenness

So far, we have discussed the case of using a single network metric for generating an attack trace by sampling points based on equi-depth sampling. This section deals with the use of several metrics and how to select the one mostly reducing the GC size. Our hypothesis, supported by a comprehensive evaluation in Section 3, is that combining several metrics which have a low time complexity, can outperform harder-to-compute metrics such as betweenness. Our evaluation on real-world networks below confirms this hypothesis.

Finally, in addition to the previously discussed network metrics, we propose to exploit approximate betweenness [41] for ranking

---

**Algorithm 1 Function:** QRE(G, metrics, iter, maxSampleCount)

**Input:** Graph G, set of metrics metrics, number of iterations iter, maximum number of samples maxSampleCount

**Output:** Estimated R

```plaintext
1: Let M = [1, \ldots, 1] be a list of length |G.nodes()|
2: Let initgcs = \[\frac{LC}{|G.nodes()|}\], where LC is the largest component of G
3: for i \in \{1, \ldots, iter\} do
4:   Let samples_s = getSampleIntervals(M, maxSampleCount)
5:   Let pat = getPartialAttackTraceDynamic(G, samples_s, metrics, initgcs)
6:   Let M = updateM(M, samples_s, pat)
7: end for
8: Let R = sum(M) / |M|
```

**Algorithm 2 Function:** getSampleIntervals(M, maxIntervalCount)

**Input:** Current minimum trace M, sample points samples_s, sample observations samples_g

**Output:** Updated M

```plaintext
1: Let R = sum(M) / |M|
2: Let result = []
3: Let cursum = 0
4: for i \in \{0, \ldots, |M| - 2\} do
5:   cursum = cursum + M[i]
6:   if cursum \geq \frac{\text{result} + R}{\text{maxIntervalCount}} then
7:     Append i to result
8: end if
9: end for
```

the importance of nodes. Instead of computing the exact betweenness of all nodes, an approximation is computed based on a given number of sample node pairs. Thanks to this metric, the time complexity is significantly reduced, while results are still comparable to the exact betweenness analysis.

### 2.5. Overall framework

In this section, we present our framework for scalable robustness estimation, merging all the insights discussed in Sections 2.1–2.4, we design an algorithm for estimating the robustness of a network. Our overall algorithm is presented in Algorithm 1. First, we initialize the list M, representing the aggregation of currently best known attack traces, with ones.

Firstly we define M as a vector of the size |N|, which will encode the aggregation of the best attack traces known at each iteration. Intuitively, M represents the best known attack trace, i.e., the relative GC size as a function of the number of nodes disabled. We initialized all elements in M with 1, indicating that we have no knowledge about any trace initially. Next, we execute the loop

Fig. 4. Sampling for the network adjnoun. The complete robustness curve for adjnoun (left); Equal-width sampling creates many measurements at network snapshots where the network is already disintegrated (center); Equi-depth sampling creates sampling points for network snapshots where the network is still robust (right).
Algorithm 3: Function: getPartialAttackTraceDynamic(G, samples, metrics, initgcs)

Input: Network G, sample points samples, set of metrics, initial size of giant component initgcs

Output: Sampled values samplesgeo

1: Let samplesgeo = []
2: Let G1 be a copy of G
3: Let gcs = G (SizeRelative(G1,G))
4: Append gcs to samplesgeo
5: for i ∈ {0, . . . , |samples| − 1} do
6:   Let end = |G1|nodes(i)
7:   if i + 1 < |samples| then
8:     end = samples[i + 1]
9:   end if
10:  Let minges = 1
11:  Let bestrank = []
12: for metric ∈ metrics do
13:   Let G2 be a copy of G1
14:   Let ranking be the ranking obtained from metric over G2
15:   for i ∈ {samples[i], . . . , end} do
16:     Remove node n from G2
17:   end for
18:   Let gcs = LC(sizes(G2) / |G|nodes(i)), where LC is the largest component of G2
19:   if gcs < minges then
20:     minges = gcs
21:     bestrank = ranking
22:   end if
23: end for
24: for n ∈ bestranking do
25:   Remove node n from G1
26: end for
27: Append minges to samplesgeo
28: end for

Algorithm 4: Function: updateM(M, samples, samplesgeo)

Input: Current minimum trace M, sample points samples, sample observations samplesgeo

Output: Updated M

1: for i ∈ {0, . . . , |samples|} do
2:   Let end = |M|
3:   if i + 1 < |samples| then
4:     end = samples[i + 1]
5:   end if
6:   for j ∈ {samples[i], . . . , end} do
7:     if samplesgeo[i] < |M| then
8:       Let M[i] = samplesgeo[i]
9:     end if
10: end for
11: end for

being attacked throughout the remainder. Second, we iterate over all sample points in samples. We determine the number of to-be-removed nodes first. Then we iterate over the provided network metrics in metrics. For each metric we create a copy of the currently attacked network, and then remove size of giant component top-ranked nodes from the network. We calculate the relative GC size and compare it to the previously best known attack. Throughout the loop, we keep track of the best attack and append the lowest recorded relative GC size to samplesgeo. Moreover, we remove the nodes with the best ranking from G1.

In Algorithm 4, we update the currently best known attack trace minimum by using the sample indices samplesgeo and their relative GC size measurements samplesgeo. Essentially, the algorithm iterates over all points in G and checks whether the current point is below the sampling-induced lowest relative GC size for that point. If yes, we update M accordingly. The time complexity of Algorithm 4 is $O(N^2)$, since we access each element in M exactly one time.

Fig. 5 presents an example of the whole process, for five iterations of the algorithm when applied to the adjnoun network; it depicts the change of M in each iteration for the network together with the selected sample points.

3. Results

3.1. Evaluation setup and data sources

We report the results of our evaluation on 12 real-world networks. All experiments were executed on a server with 32 cores and 386 GB RAM, running Fedora 24 (Linux 4.7.5–200.fc24.x86_64). The QRE framework was implemented in a single-threaded fashion, using the network analysis library NetworKit.\textsuperscript{1} All datasets are available to download from the UCI Network Data Repository.\textsuperscript{2} and have been studied extensively in the past.\textsuperscript{42–45} These networks cover a variety of network structures and network scales, see Table 1. We use six network metrics in our evaluation: Degree (deg), betweenness (betw), approximate betweenness (abettw), eigenvector (ev), pagerank (pr), and Katz centrality (katz).

3.2. Fine-tuning of parameters

First, we perform a sensitivity analysis of the approximate betweenness against the number of sample node pairs. For four networks of different sizes (dolphins, celegansneural, netscience, and power), we have computed the exact betweenness of nodes

---

\textsuperscript{1} Available at https://networkit.iti.kit.edu/.

\textsuperscript{2} Available at https://networkdata.ics.uci.edu/index.php.
Fig. 5. Visualization of \( M \) and ten sampling points with Algorithm 1 for the example network adjnoun and the strategy interactive degree. The red line in all charts corresponds to the robustness curve obtained without sampling, therefore representing the baseline. Initially, \( M \) is a vector filled with ones, as represented by the blue area in Iteration 0. After the first equi-depth sampling, which is equivalent to equi-width here, we obtain the \( M \) for Iteration 1. Subsequent iterations improve the approximation, with \( M \) converging to the actual robustness curve without sampling (red lines). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

### Table 1

List of twelve datasets used in our experiments, ranked by increasing number of nodes. We report standard network properties: Number of nodes \((N)\) and links \((L)\), density, number of connected components \((\text{Comp.})\), relative size of the giant component \((\text{Rel. GC})\), diameter, radius, and assortativity.

<table>
<thead>
<tr>
<th>Network</th>
<th>(N)</th>
<th>(L)</th>
<th>Density</th>
<th>Comp.</th>
<th>Rel. GC</th>
<th>Diameter</th>
<th>Radius</th>
<th>Assortativity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karate</td>
<td>34</td>
<td>78</td>
<td>13.9%</td>
<td>1</td>
<td>100.0%</td>
<td>5</td>
<td>3</td>
<td>–0.48</td>
</tr>
<tr>
<td>Dolphins</td>
<td>62</td>
<td>159</td>
<td>8.41%</td>
<td>1</td>
<td>100.0%</td>
<td>8</td>
<td>5</td>
<td>–0.04</td>
</tr>
<tr>
<td>Lesmis</td>
<td>77</td>
<td>254</td>
<td>8.68%</td>
<td>1</td>
<td>100.0%</td>
<td>5</td>
<td>3</td>
<td>–0.17</td>
</tr>
<tr>
<td>Adjnoun</td>
<td>112</td>
<td>425</td>
<td>6.84%</td>
<td>1</td>
<td>100.0%</td>
<td>5</td>
<td>3</td>
<td>–0.13</td>
</tr>
<tr>
<td>Football</td>
<td>115</td>
<td>613</td>
<td>9.35%</td>
<td>1</td>
<td>100.0%</td>
<td>4</td>
<td>3</td>
<td>0.16</td>
</tr>
<tr>
<td>Celegansneural</td>
<td>297</td>
<td>2148</td>
<td>4.89%</td>
<td>1</td>
<td>100.0%</td>
<td>5</td>
<td>3</td>
<td>–0.16</td>
</tr>
<tr>
<td>Polblogs</td>
<td>1490</td>
<td>16715</td>
<td>1.51%</td>
<td>268</td>
<td>82.01%</td>
<td>–</td>
<td>–</td>
<td>–0.22</td>
</tr>
<tr>
<td>Netscience</td>
<td>1589</td>
<td>2742</td>
<td>0.22%</td>
<td>396</td>
<td>23.85%</td>
<td>–</td>
<td>–</td>
<td>0.46</td>
</tr>
<tr>
<td>Power</td>
<td>4941</td>
<td>6594</td>
<td>0.05%</td>
<td>1</td>
<td>100.0%</td>
<td>46</td>
<td>23</td>
<td>0.00</td>
</tr>
<tr>
<td>hep-th</td>
<td>8361</td>
<td>15751</td>
<td>0.05%</td>
<td>1332</td>
<td>69.79%</td>
<td>–</td>
<td>–</td>
<td>0.29</td>
</tr>
<tr>
<td>astro-ph</td>
<td>16706</td>
<td>121251</td>
<td>0.09%</td>
<td>1029</td>
<td>88.86%</td>
<td>–</td>
<td>–</td>
<td>0.24</td>
</tr>
</tbody>
</table>

and compared it to the approximate betweenness scores as a function of the number of sample points \([41]\). The number of samples is a function of the number of nodes \(N\) in the network. We have evaluated three cases: \(\log(|N|)\), \(\sqrt{|N|}\), and \(|N|\) pairs of nodes. Note that for the exact betweenness computation one needs to consider a number of node pairs quadratic in the number of nodes. Fig. 6 reports the results of our experiments. We compare the ranks of nodes according to the exact betweenness computation (x-axis) with the rank of nodes for approximate betweenness (y-axis). The ranking differences between \(\log(|N|)\) and \(\sqrt{|N|}\) are insignificant, with the former being 4–8 times faster than the latter. The case with \(N\) samples obtains very good rankings, but is even slightly slower than the exact betweenness computation. We conclude that \(\log(|N|)\) is a good trade-off to perform a sampling-based approximation of betweenness, with a good initial ranking at low computation costs. However, if the approximate betweenness of a node is wrongly estimated, the importance of the node can be identified by one of the other metrics or be revealed during a later iteration with different sample points. Our results indicate that \(\log(|N|)\) sample pairs are often sufficient to find interesting sub-attacks.

In the following, we analyze the sensitivity of the computed \(R\) value given a number of sample points for partial attack traces. We evaluate four datasets and six static network metrics. The results are visualized in Fig. 7. After an initial phase of steep descent, a plateau is reached, after which increasing the number of sample points does not yield improved results. Our experimental results suggest, that 100 sample points are sufficient to accurately estimate the robustness. The non-determinism of approximate betweenness can be observed for the network netscience. In very few cases, the \(R\) value based on approximate betweenness can be rather high, if the number of samples is too small. For the remainder of our experiments we set the number of sample points to 100. We will show below that 100 samples points are sufficient,
(a) The number of sample points is set as \( \log(|N|) \).

(b) The number of sample points is set as \( \sqrt{|N|} \).

(c) The number of sample points is set as \( |N| \).

**Fig. 6.** Sensitivity analysis of the number of samples for the approximate betweenness computation, with different number of samples: \( \log(|N|) \), \( \sqrt{|N|} \), and \( |N| \), where \( |N| \) is the number of nodes in the network. The title of each subchart lists the dataset, as well as, the speed up factor, i.e., how many times faster the approximate betweenness calculation is, compared to the exact computation. For the network netscience, we observe that the approximate betweenness has problems to distinguish the betweenness centrality of several nodes, which is probably due to the larger number of components in netscience: With a few sample pairs, the approximate betweenness algorithm leaves several nodes undiscovered.

**Fig. 7.** Sensitivity analysis for the number of sample points for computing a partial attack trace. In the charts, we highlight the case of 100 sample points with a dashed vertical line.
The influence of the number of iterations on the estimated robustness of a network is evaluated next. Again, we evaluate four datasets and six static network metrics. Fig. 8 presents the results, with 100 sample points. For all network metrics, except from approximate betweenness, we can see that the robustness estimation is rather stable after a few number of iterations (horizontal line). Approximate betweenness, on the other hand, can still find lower $R$ values, with an increasing number of iterations. Overall, we propose that for all network metrics, a small number of iterations is sufficient. For the remainder of our experiments we fix the number of iterations to either 1 (fast) or 4 (stable) for all five static network metrics. The variation of approximate betweenness will be
addressed by using multiple partial attack traces, instead of single partial traces (see evaluation below).

### 3.3. Robustness estimations by QRE

Our sensitivity analysis in the previous section has led to the following parameters for Algorithm 1: Approximate betweenness is sampled by \( \log(|N|) \) random node pairs, we compute partial attack traces with 100 sample points, and iterate the main loop in Algorithm 1 for either one or four times.

In Fig. 9, we report the results of considering the network metric separately. In total we have 24 competitors, implemented in QRE: six metrics, static/interactive, and 1/4 iterations. We can see that the best metrics are usually approximate betweenness (abetw) and exact betweenness (betw). In a few cases, degree (deg) is in the Pareto front as well, with very short running times. Overall, we can conclude that for the majority of networks, only betweenness-based metrics can accurately estimate the robustness; where for larger networks, exact betweenness takes much longer to compute than the approximate variant. In Fig. 10, we visualize the distribution of \( R \) values of the 24 competitors for each dataset as a box-plot. It can be seen that the variation of \( R \) values is rather large; eventhemedianisratherfarawayfromtheminimumvalues. This highlights that one should not randomly select a network metric to compute the \( R \) values, because this often leads to a significant overestimation of network robustness.

In Fig. 11, we analyze the running times of the interactive QRE instances with 4 iterations; according to our evaluation above, these instances provide the best results. The running time of exact betweenness computation increases tremendously with the
number of nodes in the network. The execution times of the other
five metrics (including approximate betweenness) grow signifi-
cantly slower. The kink around 1500 nodes is caused by the dataset
polblogs, which has a significantly higher number of links than
the other datasets with similar number of nodes, e.g., netscience.
Therefore, the computations for polblogs always take longer.

In Fig. 12, we further evaluate QRE with multiple network met-
rics. For these experiments, we have enumerated all possible sub-
sets of \{betw = A, betw = B, deg = D, ev = E, katz = K, pagerank = P\} and four iterations, such that ADEI denotes
the sequential application of metrics. Again, we analyze the Pareto
front. We can conclude that for each network, only a few instances
of QRE belong to the best competitors. Particularly often, we find
that ADI (approximate betweenness, degree; interactive) is among
the best results, and the difference between ADI and the best result
per network is usually small, i.e., less than 0.005.

4. Discussion

In Table 2, we summarize the robustness estimations for five
standard (single) interactive network metrics using our methodol-
ogy, together with the cases of ADI (approximate betweenness,
degree; all interactive) and ADEKPI (approximate betweenness,
degree, eigenvector, katz, pagerank; all interactive). In all but one
case, the two ADI/ADEKPI identify the minimum \( R \) values for the
networks. It should be noted that the difference between ADI and
ADEKPI is usually less than 0.005 (except for polblogs). Among sin-
gle network metrics, interactive betweenness performs best as an
attacking strategy. The difference to other network metrics is sig-
nificant (up to 0.1 smaller \( R \) values). However, the major limitation
of interactive betweenness is its high computation time. In
Table 3, we compare the computation times for all approaches. We
can clearly see the computational requirements for interactive be-
tweenness. Moreover, the interactive degree is usually the fastest
method to attack the network, but the obtained \( R \) might not be ac-
curate. ADI and ADEKPI provide very good robustness estimations
at acceptable running times.

In synthesis, we have proposed a computationally efficient
framework, Quick Robustness Estimation (QRE), for estimating the
robustness of networks. We show that instances of QRE can be
used to obtain \( R \) values smaller than the state-of-the-art with
subquadratic computation times. The results of our study show that
efficient, yet accurate robustness estimation is possible even for
very large networks. We believe that this work contributes to a
better understanding of real-world network robustness in face
of big data. We envision that our technique can be extended to
analyze robustness of networks of networks [46–48]. Moreover,
recent progress on exploitation of artificial intelligence search
techniques for network resilience analysis promotes to go beyond
the use of network metrics only [49–51].

Acknowledgment

This study is supported by the National Natural Science
Foundation of China (Grant Nos. 61650110516 and 61601013).
Table 3

Comparison of robustness estimation running time (in s) by standard network metrics (betw, degl, evl, karzd, prl), and two selected QRE instances: ADI (approximate betweenness, degree, all interactive) and ADEPKI (approximate betweenness, degree, eigenvector, karz, pagerank; all interactive). Minimum values (=better) are highlighted in bold.

<table>
<thead>
<tr>
<th>Network</th>
<th>betw</th>
<th>degl</th>
<th>evl</th>
<th>karzd</th>
<th>prl</th>
<th>ADI</th>
<th>ADEPKI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karate</td>
<td>0.002</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
<td>0.008</td>
<td>0.022</td>
</tr>
<tr>
<td>Dolphins</td>
<td>0.012</td>
<td>0.002</td>
<td>0.003</td>
<td>0.003</td>
<td>0.005</td>
<td>0.021</td>
<td>0.062</td>
</tr>
<tr>
<td>Lesmis</td>
<td>0.024</td>
<td>0.01</td>
<td>0.005</td>
<td>0.015</td>
<td>0.006</td>
<td>0.041</td>
<td>0.081</td>
</tr>
<tr>
<td>Polbooks</td>
<td>0.067</td>
<td>0.004</td>
<td>0.007</td>
<td>0.008</td>
<td>0.019</td>
<td>0.049</td>
<td>0.15</td>
</tr>
<tr>
<td>Adjnoun</td>
<td>0.076</td>
<td>0.005</td>
<td>0.006</td>
<td>0.007</td>
<td>0.021</td>
<td>0.056</td>
<td>0.166</td>
</tr>
<tr>
<td>Football</td>
<td>0.136</td>
<td>0.004</td>
<td>0.008</td>
<td>0.008</td>
<td>0.023</td>
<td>0.071</td>
<td>0.202</td>
</tr>
<tr>
<td>Conceptual</td>
<td>0.672</td>
<td>0.012</td>
<td>0.032</td>
<td>0.046</td>
<td>0.234</td>
<td>0.743</td>
<td></td>
</tr>
<tr>
<td>Polblogs</td>
<td>9.741</td>
<td>0.065</td>
<td>0.102</td>
<td>0.102</td>
<td>0.222</td>
<td>1.402</td>
<td>5.364</td>
</tr>
<tr>
<td>Netsciences</td>
<td>0.764</td>
<td>0.043</td>
<td>0.064</td>
<td>0.066</td>
<td>0.076</td>
<td>0.454</td>
<td>2.061</td>
</tr>
<tr>
<td>Power</td>
<td>24.704</td>
<td>0.145</td>
<td>0.178</td>
<td>0.18</td>
<td>0.462</td>
<td>2.258</td>
<td>7.69</td>
</tr>
<tr>
<td>hep-th</td>
<td>64.099</td>
<td>0.233</td>
<td>0.324</td>
<td>0.327</td>
<td>0.703</td>
<td>4.561</td>
<td>15.331</td>
</tr>
<tr>
<td>astro-ph</td>
<td>997.342</td>
<td>0.559</td>
<td>0.968</td>
<td>0.988</td>
<td>2.777</td>
<td>18.065</td>
<td>56.356</td>
</tr>
</tbody>
</table>

References

Sebastian Wandelt is a professor at Beihang University. Before joining Beihang, he was a postdoc in the group Knowledge Management in Bioinformatics at Humboldt-University of Berlin. He obtained a Ph.D. degree on Semantic Web reasoning techniques at Hamburg University of Technology in 2011. His work is broadly in the intersection between intelligent transportation systems and computer science. In addition, he is interested in scalable techniques for storing and indexing data. His research is published in venues such as IEEE Transactions on Intelligent Transportation Systems, Transportation Research Part E, Transportmetrica A/R, PVLDB, IEEE TCBB, and SIGMOD Record.

Xiaoqian Sun is an associate professor with the School of Electronic and Information Engineering at Beihang University. She obtained her Ph.D. in Aerospace Engineering from German Aerospace Center/Hamburg University of Technology in 2012. From 2012 to 2015, she worked as a postdoc in German Aerospace Center, Hamburg. She received her M.S. and B.S. degrees in Aerospace Engineering from Northwestern Polytechnical University in Xi’an, China. Dr. Sun has several publications in Transportation Research Part E, IEEE Transactions on Intelligent Transportation Systems, Transportmetrica A/R, and Journal of Aircraft. Her research interests mainly include air transportation networks and multi-criteria assessment.

Massimiliano Zanin is a Principal Researcher at Innaxis, graduated in Aeronautical Management at the Universidad Autónoma de Madrid and got his Ph.D. from Universidade Nova de Lisboa. With more than 90 published peer-reviewed contributions in international conferences and journals, he has vast experience in complex systems and data mining research. His main topics of interest are Complex Networks, Data Science and their application to several real-world problems, including air transport and biomedicine. He is a member of the editorial team of Nature Scientific Reports, the European Journal of Social Behaviour, PeerJ and PeerJ Computer Science.

Shlomo Havlin graduated from Bar-Ilan and Tel-Aviv Universities with Highest Distinction. He obtained an academic position at Bar-Ilan University in 1972 where he became a full Professor in 1984. During 1978–1979 he was a Royal Society Visiting Fellow at the University of Edinburgh. In 1984 he became the Chair of the Physics Department at Bar-Ilan University until 1988. During 1983–1984 and 1989–1991, Professor Havlin was a Visiting Scientist at NIH where he collaborated much with Drs. George Weiss, Ralph Nossal and other members of NIH. During 1984–1985 and 1991–1992 he was a Visiting Professor at Boston University, where he collaborated with Professor H. Eugene Stanley. He served as President of the Israel Physical Society (1996–1999), Dean of Faculty of Exact Sciences (1999–2001), Chairman, Department of Physics (1984–1988), Professor Havlin had more than 100 graduate students and postdocs, and collaborated with more than 300 scientists around the globe. He published more than 600 articles and 11 books.