Current Flow in Random Resistor Networks:

The Role of Percolation in Weak and Strong Disorder

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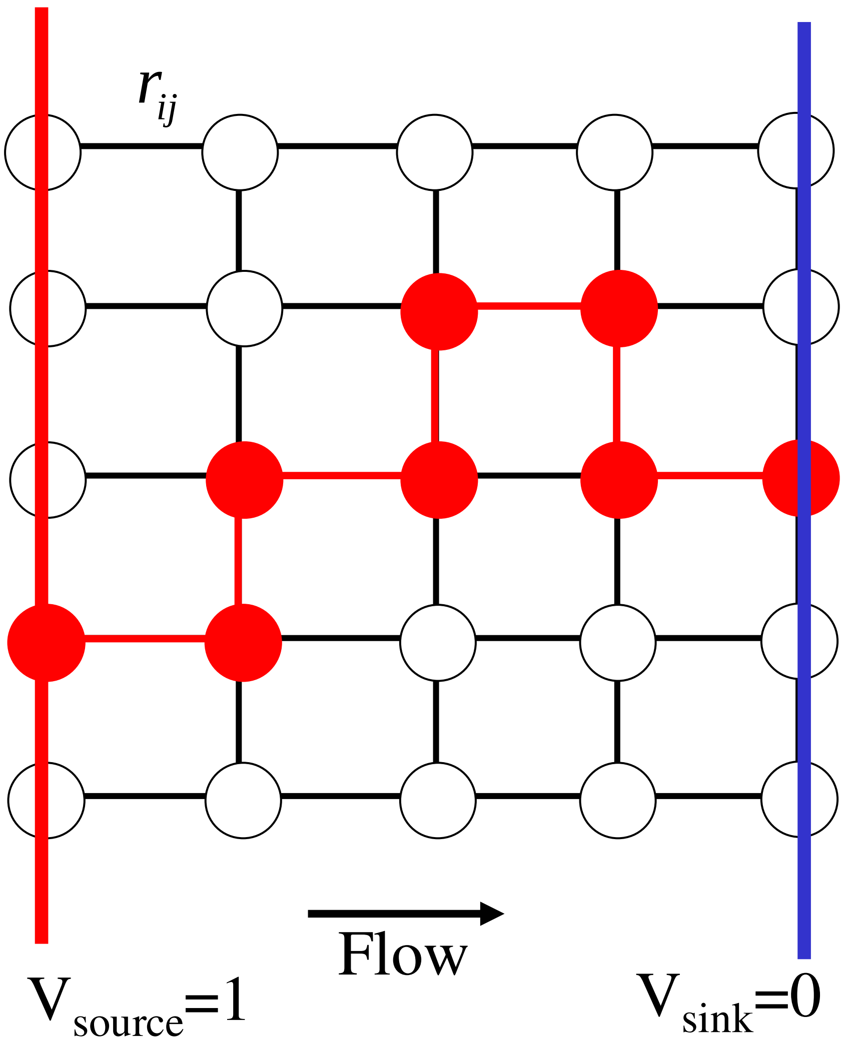
What are the questions?

- What are the properties of current flow on a disordered lattice?
- Is there any specific path which dominates current flow?
- How to characterize the dominant current flow path?

Tracer flow on bond disordered media



- Tracers (electrons, information packet…) flow in a *disordered* network with specific *structure*, according to the *rules* they obey.

- ex. *Resistor Lattice*, r_{ij} satisfies *some disorder distribution*.
- current flow obeys the *Kirchhoff law*.

●—● : Flow path

Disorder Distribution is Exponential

$$r_{ij} = e^{a x_{ij}}$$

$$x_{ij} \in [0,1]$$

$$\Rightarrow$$

$$P(r_{ij}) = \frac{1}{a r_{ij}}$$

$$r_{ij} \in [1, e^{a}]$$

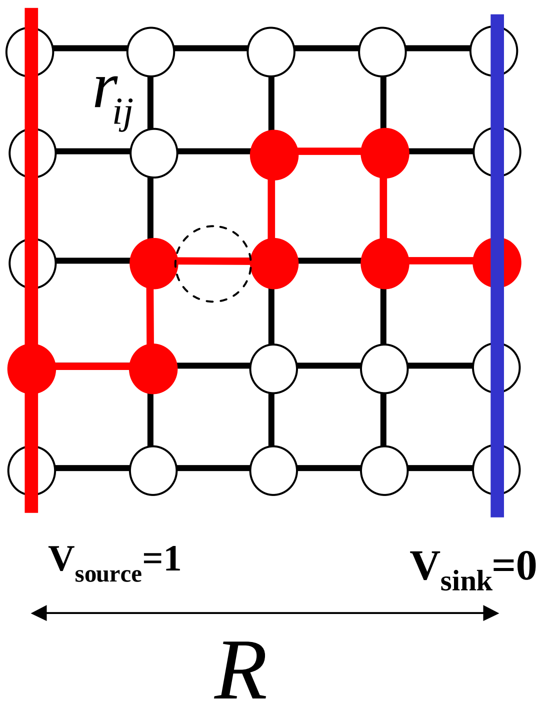
a : disorder strength, controls the broadness of the distribution.

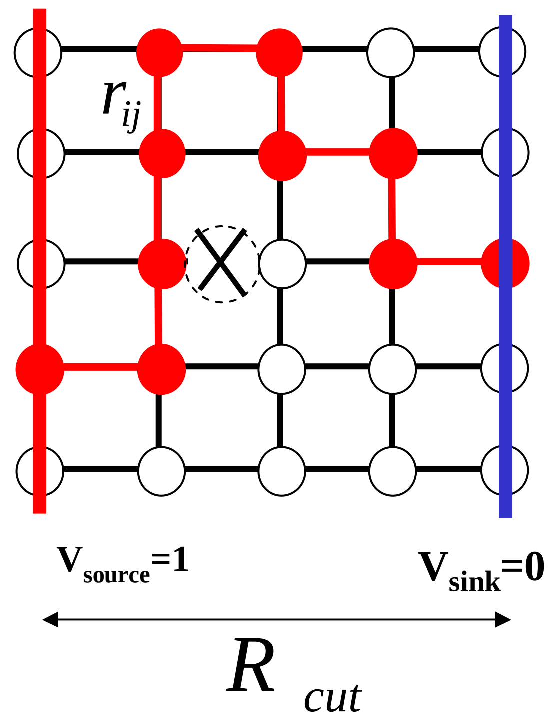
Experimental Realizations:

- Hopping conductivity of quenched condensed granular Ni thin films. [1]
- The temperature dependence of hopping conductivity in amorphous material, where $a \sim 1/\mathrm{T}$.[2]

[1]. Y. M. Strelniker et. al, Phys. Rev. E, **69**, 065105(R) (2004)
[2]. J. Bernasconi, Phys. Rev. B, **7**, 2252 (1973)

"One dominates all"[3]

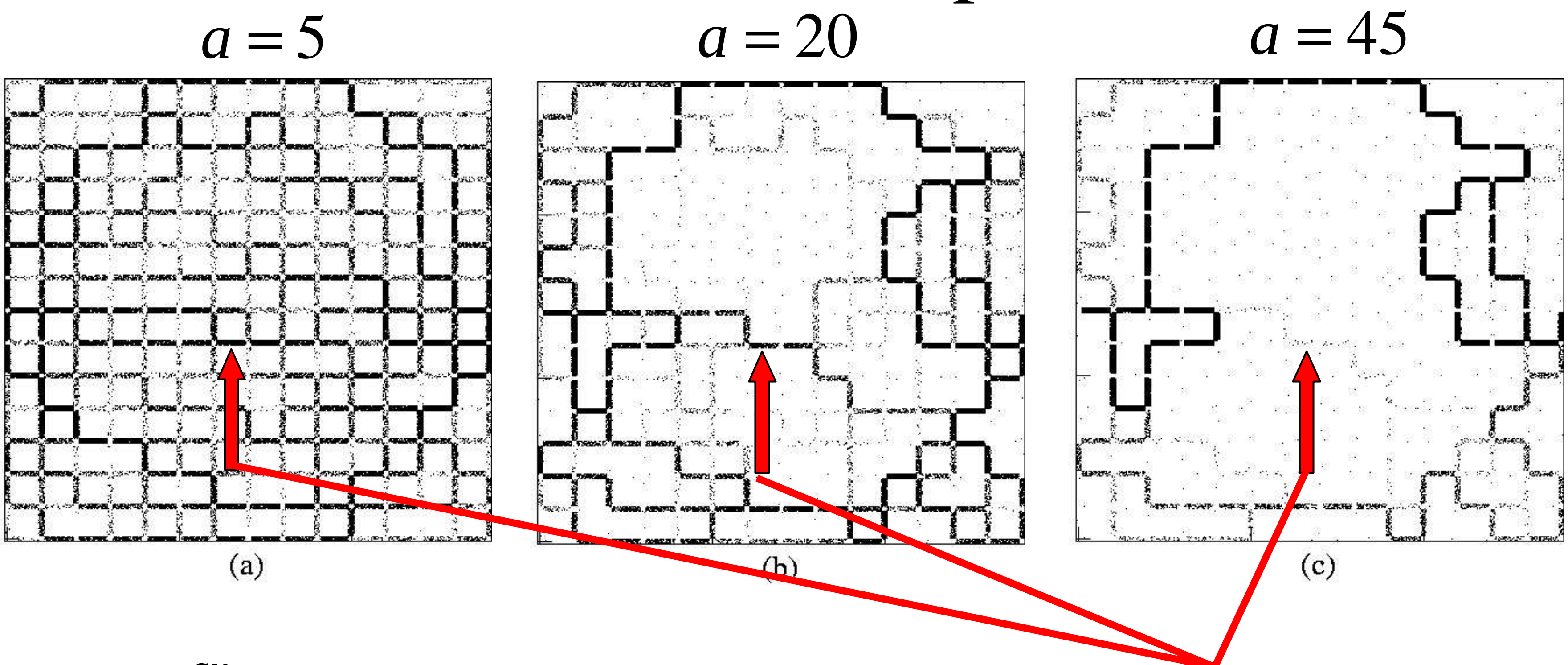




When one bond, which has the maximal current, was cut, the resistance of whole system increases dramatically.

[3]. Y. M. Strelniker et. al, Phys. Rev. E, **69**, 065105(R) (2004)

Current Map



$$r_{ij} = e^{ax_{ij}}, x_{ij} \in [0,1]$$

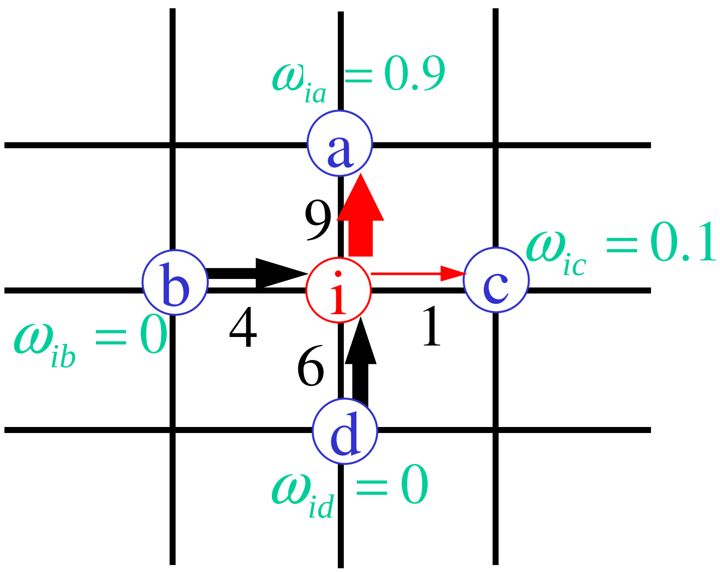
x_{ij}

As disorder strength a increase, the set of paths responsible for carrying current changes (more localized).

Goal: Find Current Paths ("tracer flow")

Solution: *Particle launching algorithm*: tracers flow[4]

- Step 1: calculate current I_{ij} (Solving Kirchhoff equation).
- Step 2: calculate flow probability: ω_{ij}



$$\omega_{ij} = \frac{J_{ij}}{\sum_j J_{ij}}$$

$$J_{ij} = \begin{cases} I_{ij} & I_{ij} \geq 0 \\ 0 & I_{ij} < 0 \end{cases}$$

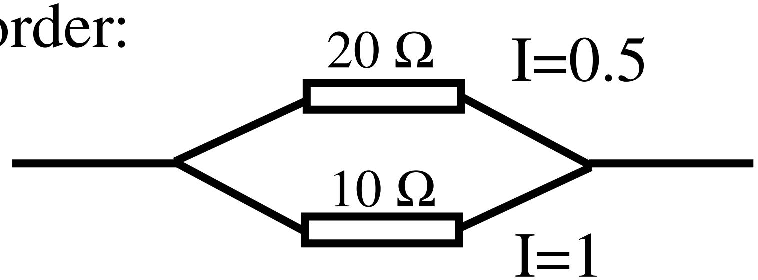
Convective tracers flow accurately according to the actual current.

[4]. E. Lopez et. al, Phys. Rev. E, 67, 056314 (2003)

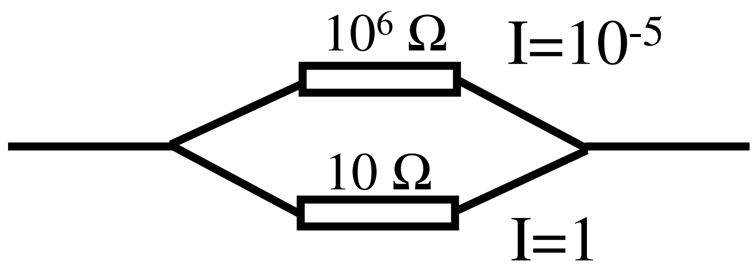
What decision tracers will make?

Two Categories of Disorder:

1) Weak disorder:



2) Strong disorder:

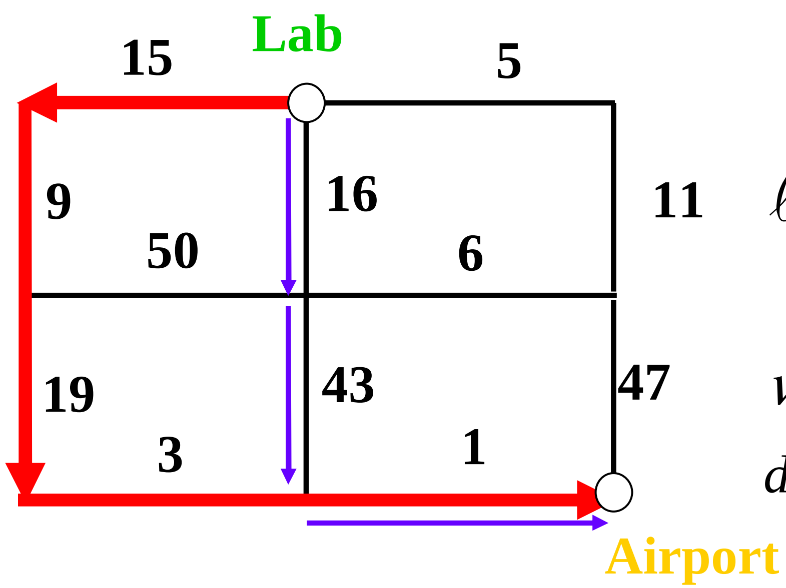


Tracers will ALWAYS choose the minimum total resistance path in the strong disorder limit ($a \to \infty$).

Motivates concept of *Optimal Path.*

Optimal Path

<u>Def</u>: Optimal path is the path that minimizes the total cost $(ex: \sum_i e^{ax_i})$ between any two points. [5-8]



$$\ell_{opt} \sim \begin{cases} L & L >> a^{\nu} \quad \textit{Weak disorder} \\ L^{d_{opt}} & L << a^{\nu} \quad \textit{Strong disorder} \end{cases}$$

(L: system size)

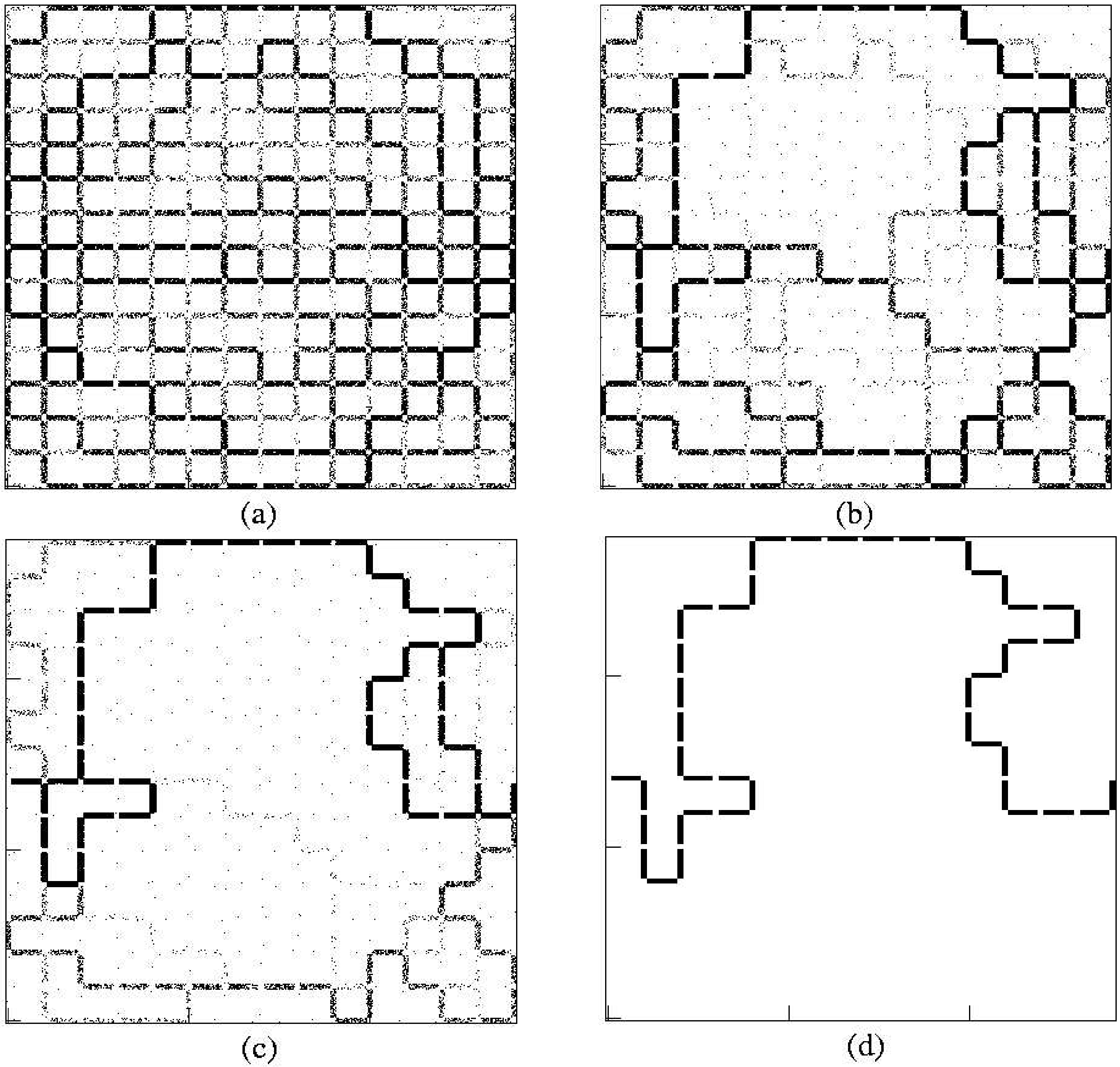
$\nu = 4/3$

$d_{opt} = 1.22 \pm 0.01$ in 2 dimensions

[5]. Cieplak, et al, Phys. Rev. Lett. **72**, 2320 (1004); **76**, 3754 (1996).
[6] Sameet, et al. Physica A, **346**, 174-182 (2005).
[7]. M. Porto, et. al, Phys. Rev. E **60**, 2448 (1999).
[8]. L. A. Braunstein, et al, Phys. Rev. Lett. **91**, 168701 (2003)

$a = 5$

$a = 20$

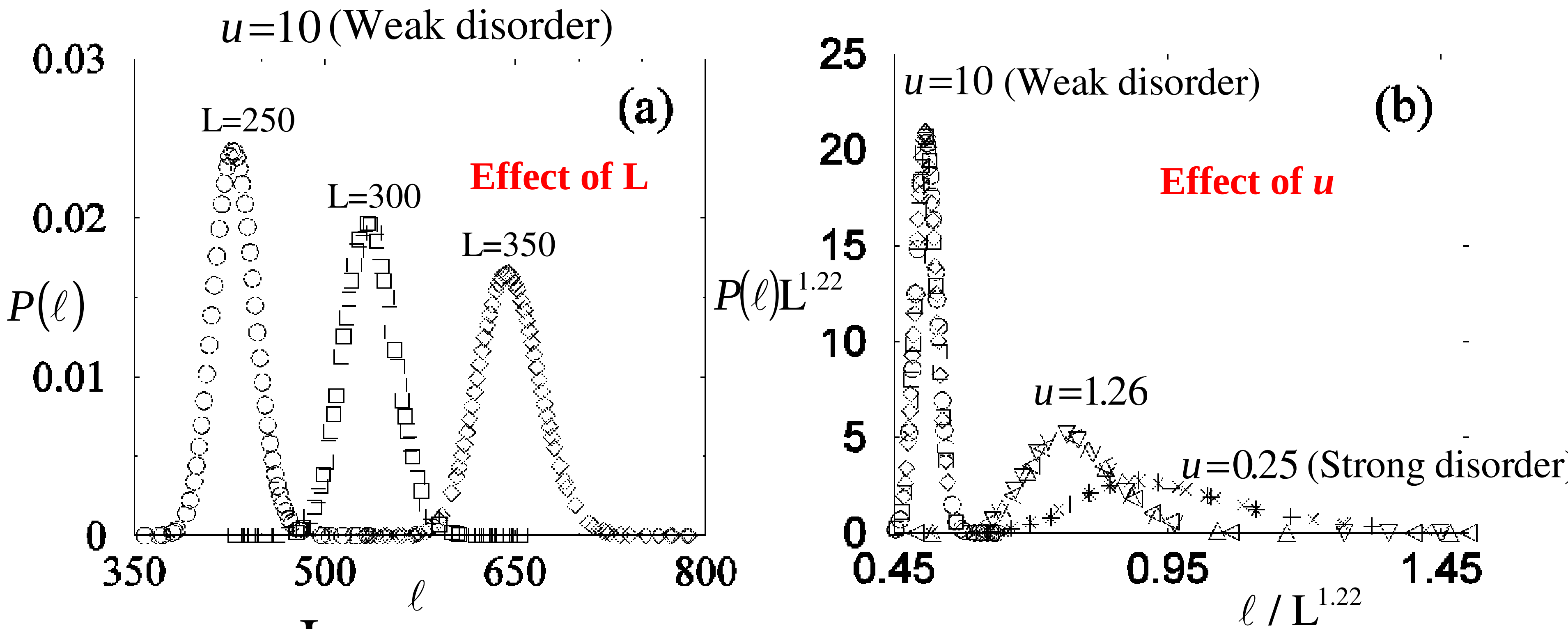


$a = 45$

Optimal path

Results

Distribution of the length of tracer paths



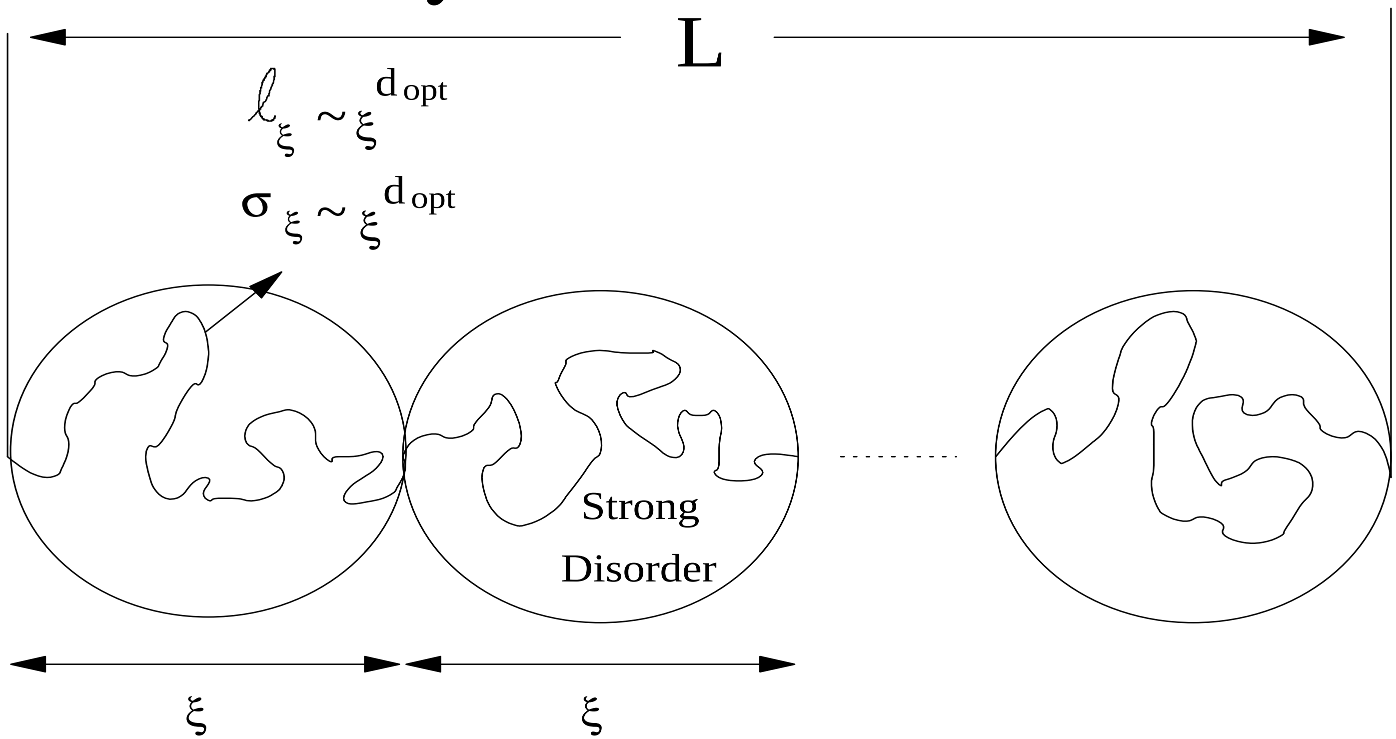
$$u \equiv \frac{L}{a^{\nu}}$$

$u >> 1$ weak disorder

$u << 1$ strong disorder

$$P(\ell \mid L, a) \sim \frac{1}{L^{d_{opt}}} f_u\left(\frac{\ell}{L^{d_{opt}}}\right)$$

Prediction: Physics of intermediate disorder



Strong disorder *connectedness length:* $\xi \sim a^{\nu}$

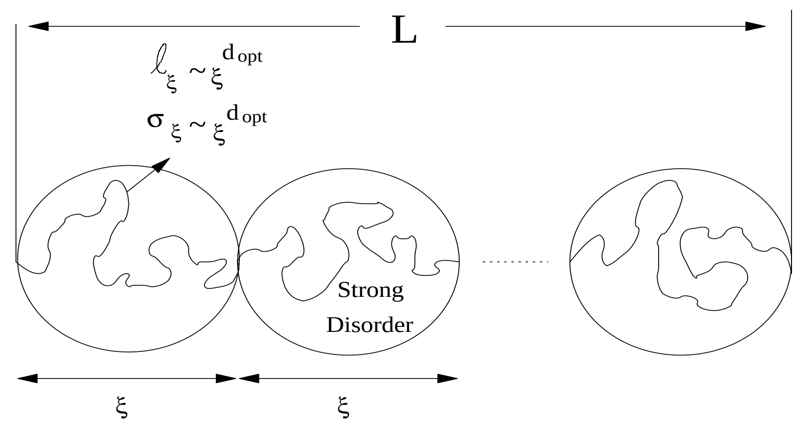
Weak disorder system with system size L is composed of:

$$u \equiv \frac{L}{a^{\nu}} \sim \frac{L}{\xi}$$ blobs of strong disorder subsystems.

Prediction: intermediate disorder

ℓ^*: the maximum of $P(\ell \mid L, a)$

σ : the standard deviation of ℓ



$$\ell^* \sim \overline{\ell} \sim u\xi^{d_{\text{opt}}} = L^{d_{\text{opt}}} u^{1-d_{\text{opt}}}$$

$$\sigma \sim \sqrt{u}\,\xi^{d_{\text{opt}}} = L^{d_{\text{opt}}} u^{1/2-d_{\text{opt}}}$$

$$u \equiv \frac{L}{a^{\nu}}$$

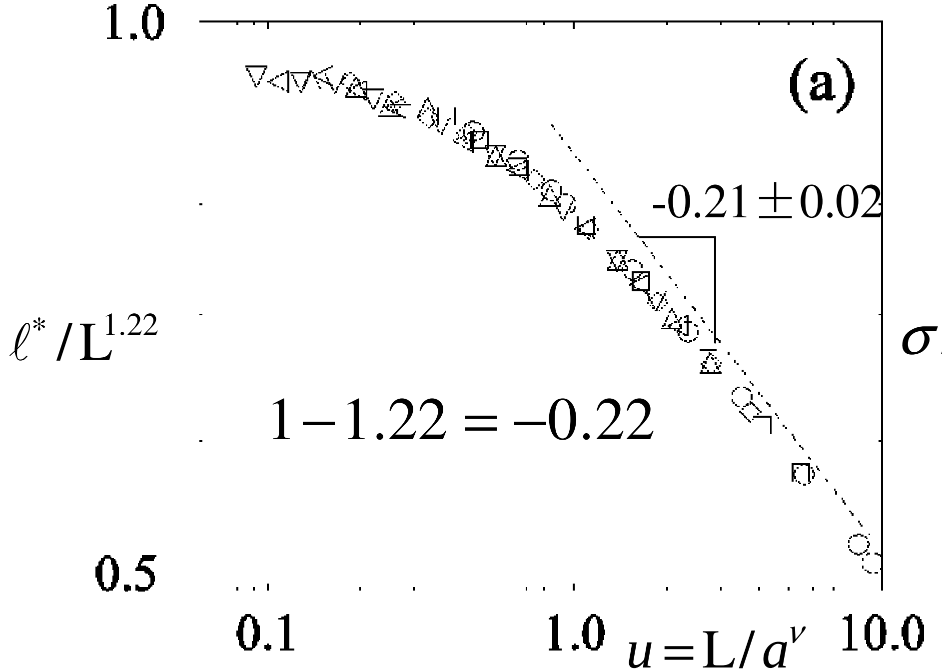
Results

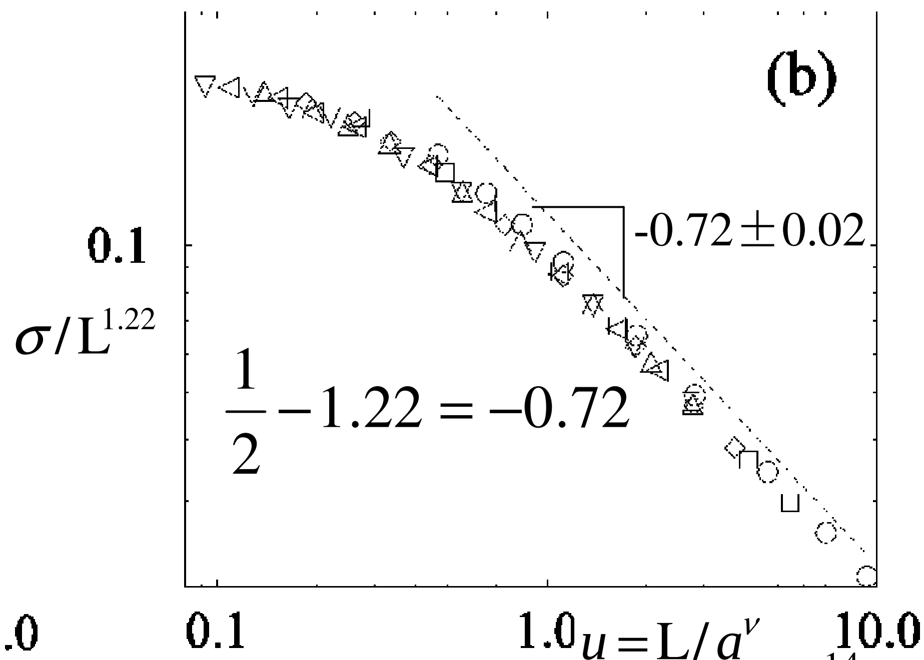
Measure ℓ^* and σ

$[d_{\text{opt}} = 1.22 \pm 0.01]$

$$\ell^* \sim \mathrm{L}^{d_{\text{opt}}} g_\ell(u) \qquad \sigma \sim \mathrm{L}^{d_{\text{opt}}} g_\sigma(u)$$

$$g_\ell(u) \sim \begin{cases} u^{1-d_{\text{opt}}} & u >> 1 \text{ weak} \\ 1 & u << 1 \text{ strong} \end{cases} \qquad g_\sigma(u) \sim \begin{cases} u^{1/2-d_{\text{opt}}} & u >> 1 \text{ weak} \\ 1 & u << 1 \text{ strong} \end{cases}$$





Summary

- Tracer path for *exponential disorder* satisfies similar *scaling* as the optimal path, with the same exponents.
- The ratio $u \equiv L/a^{\nu}$ fully determines the distribution of ℓ for all ranges of value u.
- In weak disorder, there exists a connectedness length $\xi \sim a^{\nu}$, where for length scale of the path below ξ , strong disorder and critical percolation exist.