RECOVERY PROCESSES AND DYNAMICS IN SINGLE AND INTERDEPENDENT NETWORKS

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Outline

1.1 Single networks: phase diagram
1.2 Phase flipping in single networks

2.1 Interacting networks: phase diagram
2.2 Problem of optimal repairing
2.3 Dynamics of interacting networks
1.1 Single networks: phase diagram
MOTIVATION

Fundamental processes:
- failures
- recoveries
- damage spread

Connected phenomena and problems:
- spontaneous recovery
- optimal repairing
Each node in a network can be **active** or **failed**.

We suppose there are **TWO** possible reasons for the nodes’ failures: **INTERNAL** and **EXTERNAL**.

1. **INTERNAL failure**: intrinsic reasons inside a node

2. **EXTERNAL failure**: damage “imported” from neighbors
   --> damage spread.

3. **RECOVERY**: A node can also **recover** from each kind of failure.

LET’S SPECIFY/MODEL THE RULES.
p- rate of internal failures (per unit time, for each node). During interval $dt$, there is probability $pdt$ that the node fails.

**Recovery**: A node recovers from an internal failure after a time period $\tau$. 
2. EXTERNAL FAILURES – if the neighborhood of a node is too damaged

IF: “CRITICALLY DAMAGED neighborhood”: less than or equal to \( m \) active neighbors, where \( m \) is a fixed threshold parameter.

THEN: There is a probability \( r \, dt \) that the node will experience externally-induced failure during \( dt \).

\( r \) - external failure rate

A node recovers from an external failure after time \( \tau' \).
<table>
<thead>
<tr>
<th>FAILURE TYPE</th>
<th>RULE</th>
<th>RECOVERY</th>
</tr>
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<tbody>
<tr>
<td>Internal failure</td>
<td>With rate $p$ on each node</td>
<td>After time $\tau$</td>
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<tr>
<td>External failure</td>
<td>IF($\leq m$ active neighbors) THEN</td>
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Out of these 5 parameters, we fix three of them: $m=4$, $\tau = 100$ and $\tau' = 1$.

We let $(p,r)$ to vary.

It turns out it is convenient to define $p^* = \exp(-p\tau)$.
So we use $(p^*, r)$ instead of $(p, r)$.

We measure activity $Z$ of the network as a function of $(p^*, r)$. 
Phase diagram (single network, random regular)

Green; High activity Z
Orange: Low activity Z

In the hysteresis region both phases exist, depending on the initial conditions or the memory/past of the system.

Blue line: critical line (spinodal) for the abrupt transition I → II

Red line: critical line (spinodal) for the abrupt transition II → I
Model simulation [Random regular networks]

\(<Z>\) - average fraction of active nodes (Z fluctuates)

For some values of \(r\) we have a hysteresis loop.
1.2 Phase flipping in single networks
Let’s pick point A, take a small system N=100, and run the simulation
Finite size effects

→ Sudden transitions

1. Why?

2. Is there any forewarning?

( Remember : \( Z = \) Fraction of active nodes )
It turns out it can be predicted.

Trajectory $\left(r_\lambda(t), p_\lambda^*(t)\right)$ in the phase diagram (white line, see below).

The trajectory crosses the spinodals (critical lines) interchangeably, and causes the phase flipping.

(Observing the network on the micro-level: counting internal and external failures)
Second finite size phenomenon: Flash crashes

An interesting (and unexpected) by-product of the model:

Sometimes the network rapidly crashes, and then quickly recovers (green circles).
Model predicts the existence of “flash crashes”.

Explanation: Unsuccessful transitions to a lower state.

Real stock markets also show a similar phenomenon.

Q: Possible relation?

“Flash Crash 2010”
2.1. Interacting networks

Network A

Network B
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<td>IF($\leq m$ active neighbors) THEN Extra failure rate $r$</td>
<td>After time $\tau'$</td>
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<td>Dependency failure</td>
<td>IF(companion node from the opposite network failed) THEN Extra failure rate $r_d$</td>
<td>After time $\tau''$</td>
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Elements of the phase diagram

• 2 critical points
• 4 triple points
• 10 allowed transitions
• 2 forbidden transitions

Diagram:

- Critical points: 11, 12, 21, 22
- Allowed transitions: Arrows connecting the points
- Forbidden transitions: Dashed lines connecting the points
TOTAL PHASE DIAGRAM
HYSTERESIS LOOPS
2.2 Problem of optimal repairing
PROBLEM

- Consider two interacting networks, both damaged (low activity).
- What is the most efficient strategy to repair such a system?
- We want minimal intervention (intervention might be expensive or invasive)
- Scenario common in medicine

Question: What is the minimum number of repairs to bring the system to the fully functional state, and how do we allocate them (repairs) between the two networks?
SOLUTION:
Explanation:
2.3 Dynamics of interacting networks
Activity z (simulation)
Activity z (real coupled networks)
Density plots for the activity of real networks.

Correlation between individual CDS signals.
Measuring and estimating model parameters

Building the network

Correlation between individual CDS signals.
Estimating internal failure parameters ($p^*_{EU}$ and $p^*_{LA}$): observing high activity states

Estimate:

$$p^*_{EU} = 0.07 \pm 0.01 \quad \text{and} \quad p^*_{LA} = 0.11 \pm 0.02$$
Alternative method for measuring internal failure and recovery rates: observing micro dynamics (dynamics of individual nodes)

- Time of recovery, $\tau_{EU}$ and $\tau_{LA}$
- Crude failure rates $p_{EU}$ and $p_{LA}$

\[
\begin{align*}
\tau_{EU} &= 13 \pm 2, \quad p_{EU} = (3.0 \pm 0.5) \times 10^{-3} \\
\tau_{LA} &= 9 \pm 1, \quad p_{LA} = (7.6 \pm 0.8) \times 10^{-3} \\
p_{EU}^* &= 0.07 \pm 0.01 \text{ (same as before)} \\
p_{LA}^* &= 0.11 \pm 0.02 \text{ (same as before)}
\end{align*}
\]
Estimating the external parameters $r_{EU}$ and $r_{LA}$: observing low activity states

- Contribution from internal failures known

- Contribution from interdependent failures temporary neglected, we introduce a small correction later

- Small influence of thresholds $m_{EU}$ and $m_{LA}$ on the average value of activities $z_{EU}$ and $z_{LA}$

By measuring $\langle z_i \rangle$ in the low states ($z_{EU} < 1/2$ & $z_{LA} < 1/2$) and already knowing $p_i^*$, we obtain estimates $r_{EU} = 0.81 \pm 0.03$ and $r_{LA} = 0.88 \pm 0.03$. 
Estimating the thresholds $m_{EU}$ and $m_{LA}$: visiting times

parameter $m_{frac}$ shape of the potential barrier between the wells.

simulating decoupled ($r_d = 0$) EU and LA networks

previously measured parameters $(p^*_EU, p^*_LA, \tau_{EU}, \tau_{LA}, r_{EU}, r_{LA})$

$m_{frac,EU} = 0.57 \pm 0.02$ and $m_{frac,LA} = 0.50 \pm 0.02$ for $\langle k \rangle = 3$, to $m_{frac,EU} = 0.59 \pm 0.02$ and $m_{frac,LA} = 0.50 \pm 0.02$ for $\langle k \rangle = 7$ ($\langle k \rangle$)
Estimating $r_d$ : correlation between networks EU and LA
Similarity in fluctuation size structure
• Thank you for your time.