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# RECOVERY PROCESSES AND DYNAMICS IN SINGLE AND INTERDEPENDENT NETWORKS


Antonio Majdandzic

**Boston University**

***Advisor*:**

H. E. Stanley


**PhD Committee:**

W. Skocpol

L. Sulak

P. Ivanov

S. Havlin

H.E. Stanley

***Collaborators:***

| | |
|---|---|
| B. Podobnik | L. Braunstein |
| S. Havlin | I. Vodenska |
| S. V. Buldyrev | C. Curme |
| D. Kenett | S. Levy-Carciente |
| T. Lipic | D. Horvatic |

# *Outline*

- 1.1 **Single networks:** phase diagram
  - 1.2 Phase flipping in single networks
- 2.1 **Interacting networks:** phase diagram
  - 2.2 Problem of optimal repairing
  - 2.3 Dynamics of interacting networks

## 1.1 **Single networks:** phase diagram

# MOTIVATION

Fundamental processes:
-failures
-recoveries
-damage spread

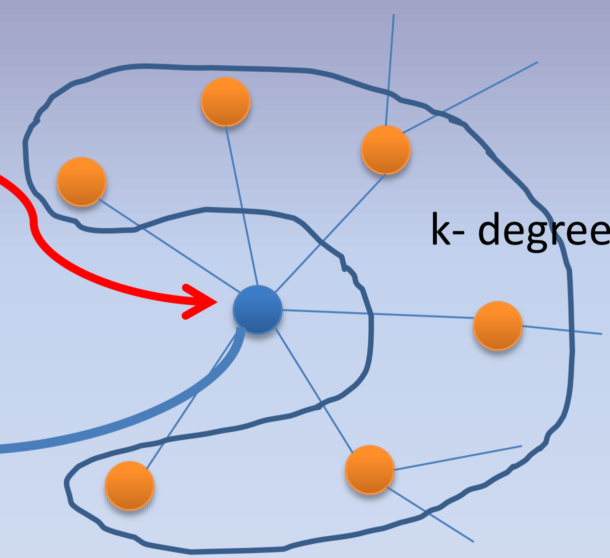

Connected phenomena and problems:
-spontaneous recovery
-optimal repairing

# SINGLE NETWORKS: FRS MODEL

• Each node in a network can be **active** or **failed**.

• We suppose there are TWO possible reasons for the nodes' failures: INTERNAL and EXTERNAL.

**1. INTERNAL failure**: intrinsic reasons inside a node



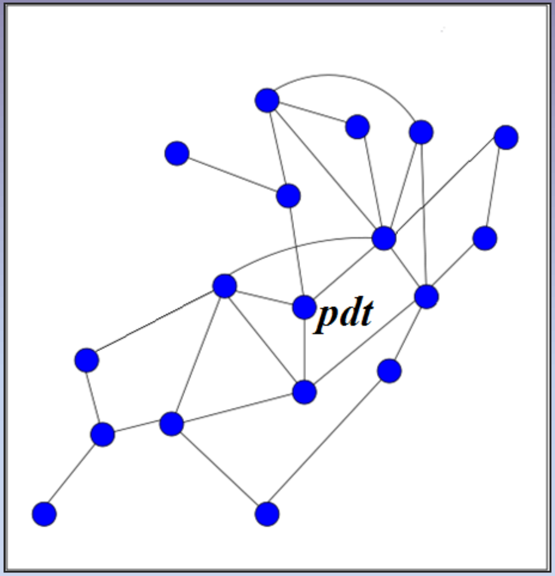

**2. EXTERNAL failure**: damage "imported" from neighbors
-->damage spread.

**3. RECOVERY**: A node can also **recover** from each kind of failure.

LET'S SPECIFY/MODEL THE RULES.

# 1. INTERNAL FAILURES

**p- rate of internal failures** (per unit time, for each node). During interval *dt*, there is probability **p***dt* that the node fails.




**Recovery**: A node *recovers from* an *internal failure after a time period* **τ** .

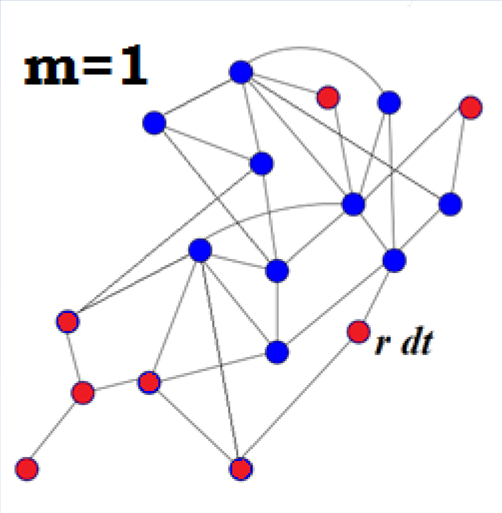
## 2. EXTERNAL FAILURES – if the neighborhood of a node is too damaged

IF: “CRITICALLY DAMAGED neghborhood”: **less than or equal to $m$ active neighbors, where $m$ is a fixed treshold parameter.**

THEN: There is a probability $r\,dt$ that the node will experience externally-induced failure during *dt*.

$r$ - external failure rate

*A node recovers from an external failure after time* $\tau'$.



| FAILURE TYPE | RULE | RECOVERY |
| --- | --- | --- |
| Internal failure | With rate ***p*** on each node | After time ***τ*** |
| External failure | IF(<= m active neighbors) THEN<br>Extra rate **r** on each node | After time ***τ’*** |

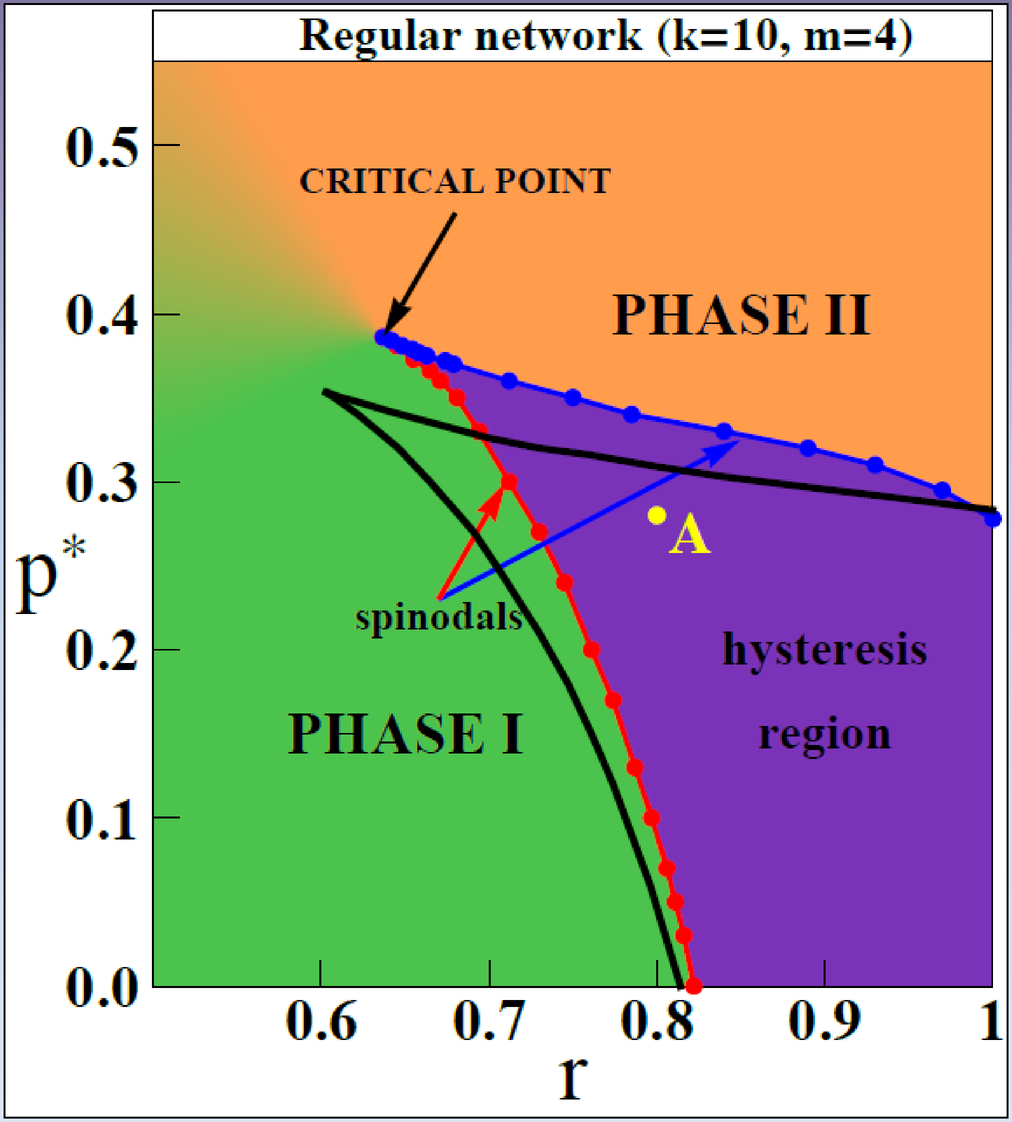
Out of these 5 parameters, we fix three of them:
m=4, $\tau=100$ *and* $\tau'=1$.

We let (p,r) to vary.

It turns out it is convenient to define $p^*=\exp(-p\tau)$.
So we use (p*,r) instead of (p,r).

We measure activity Z of the network as a function of (p*,r).

# Phase diagram (single network, random regular)




GREEN; High activity Z
ORANGE: Low activity Z

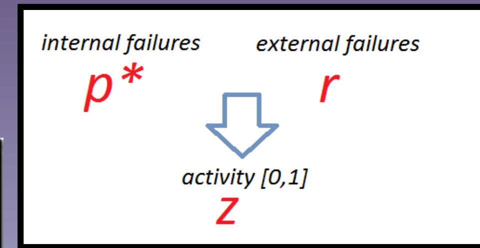
In the hysteresis region both phases exist, depending on the initial conditions or the memory/past of the system.

Blue line: critical line (spinodal) for the abrupt transition
I→ II

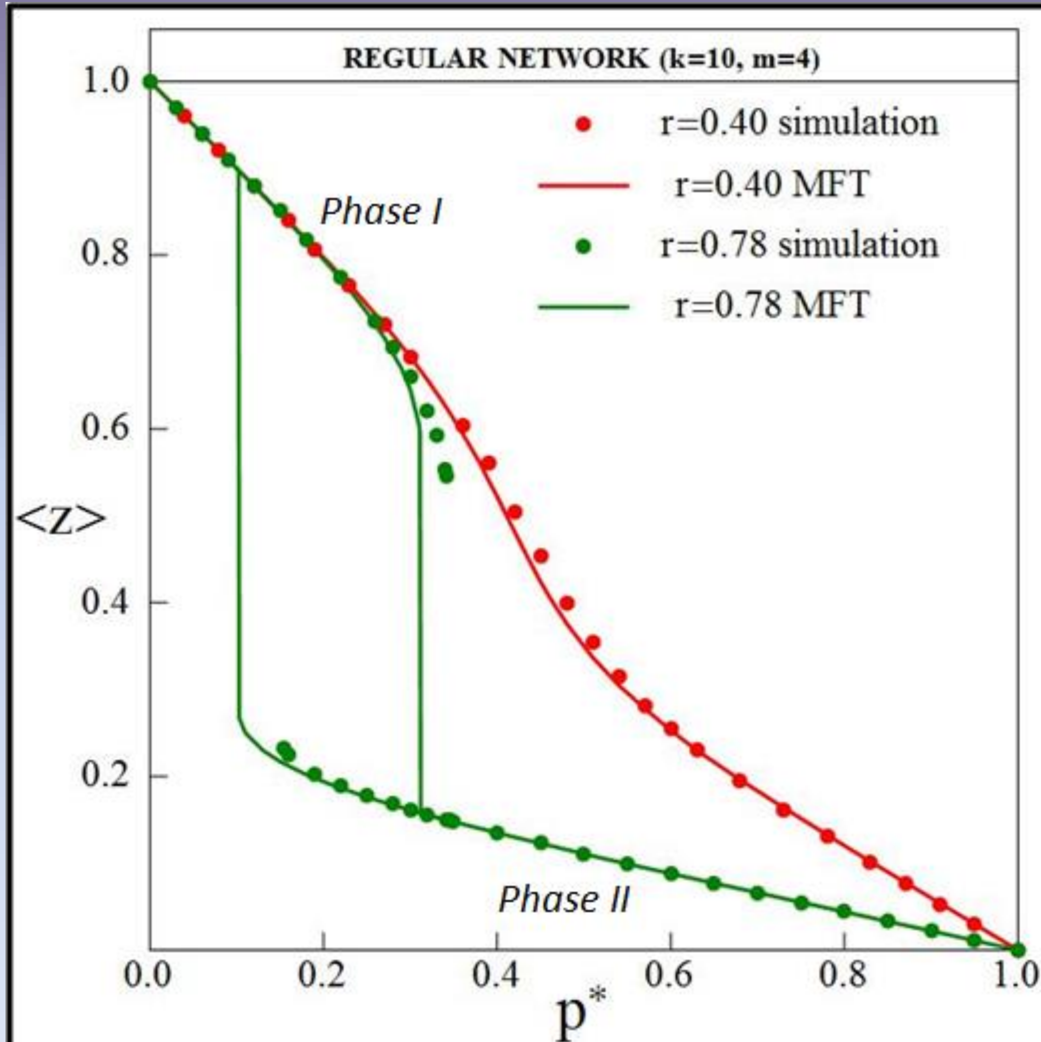
Red line: critical line (spinodal) for the abrupt transition
II→I

# Model simulation

[Random regular networks]



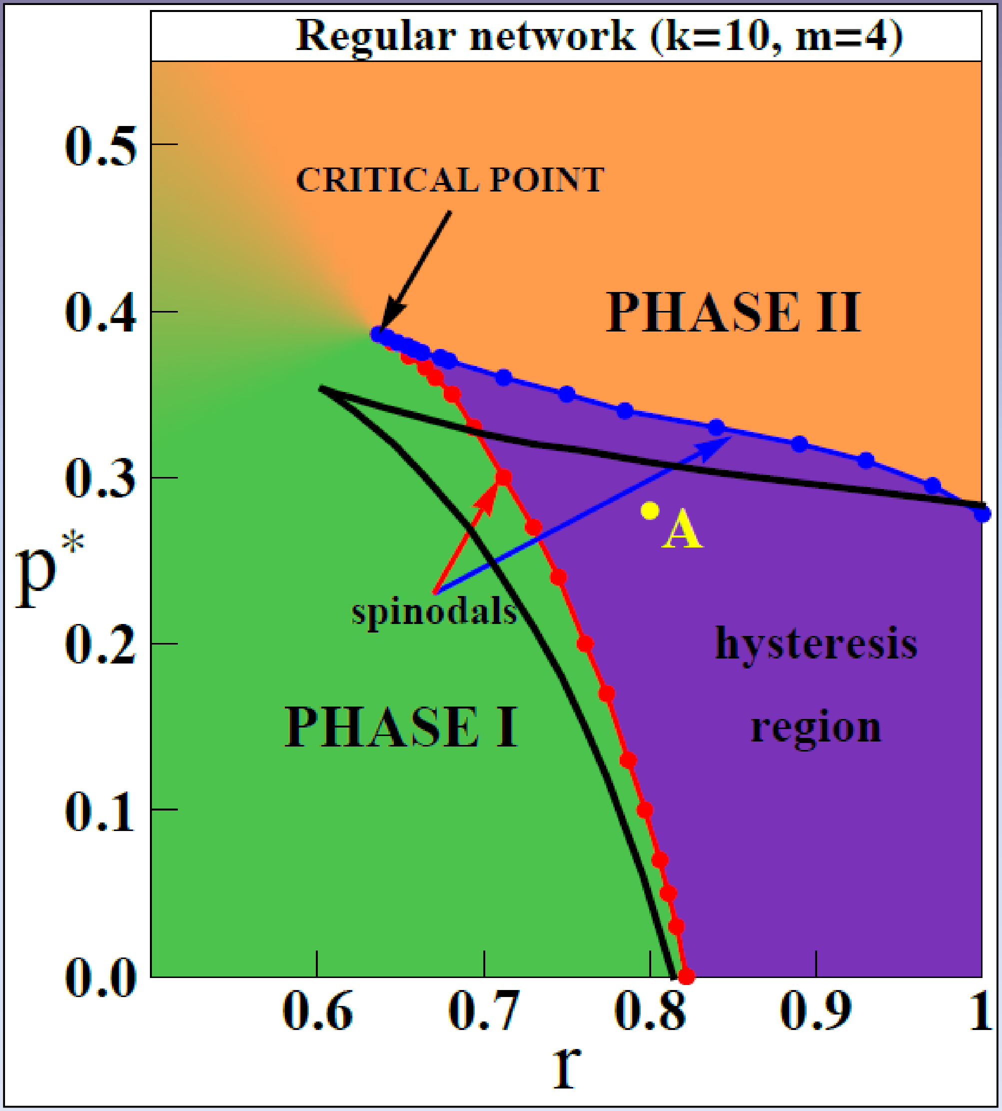

**<z>**- average fraction of active nodes (Z fluctuates)



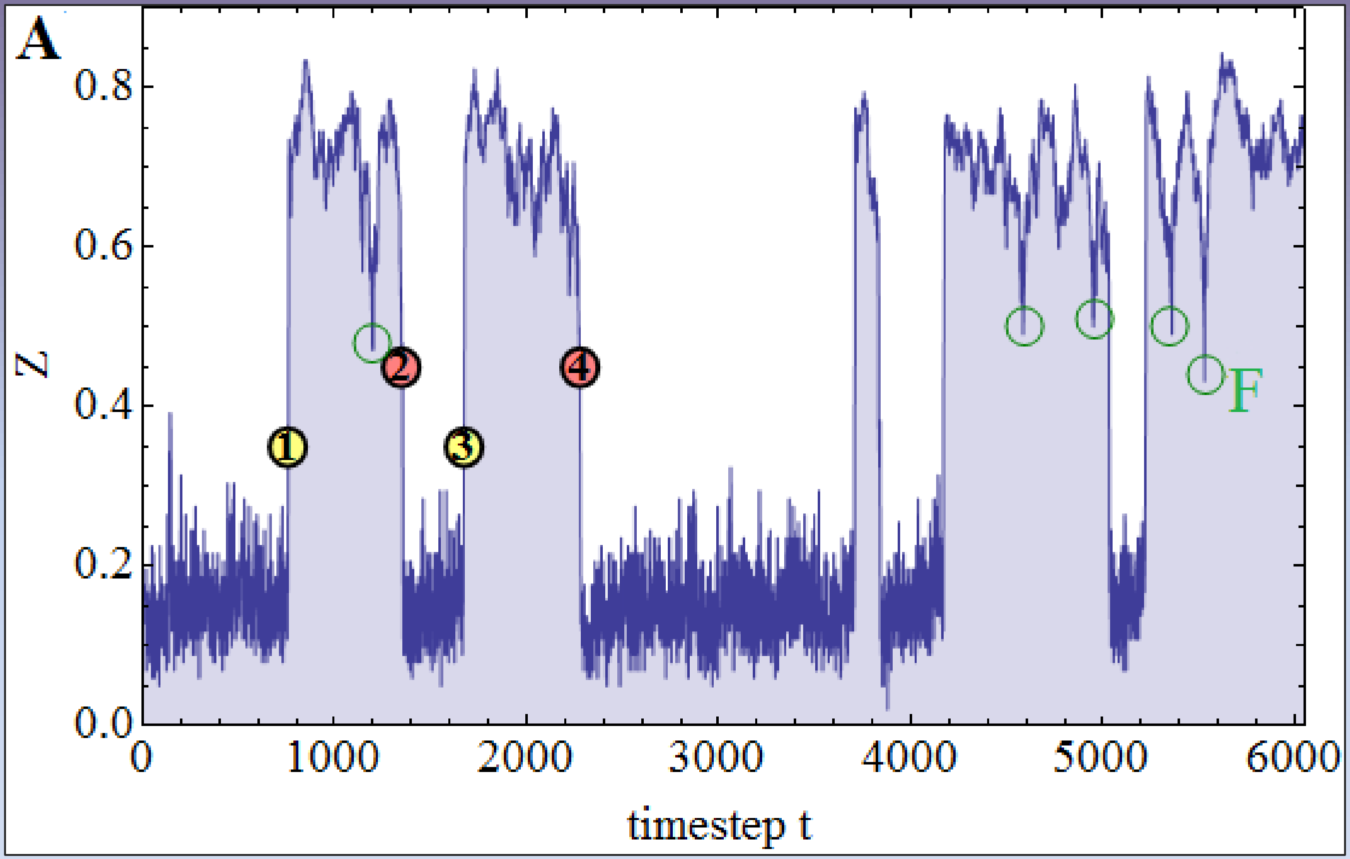

For some values of *r* we have a hysteresis loop.

# 1.2 Phase flipping in single networks

Let's pick point A, take a small system N=100, and run the simulation



# Finite size effects



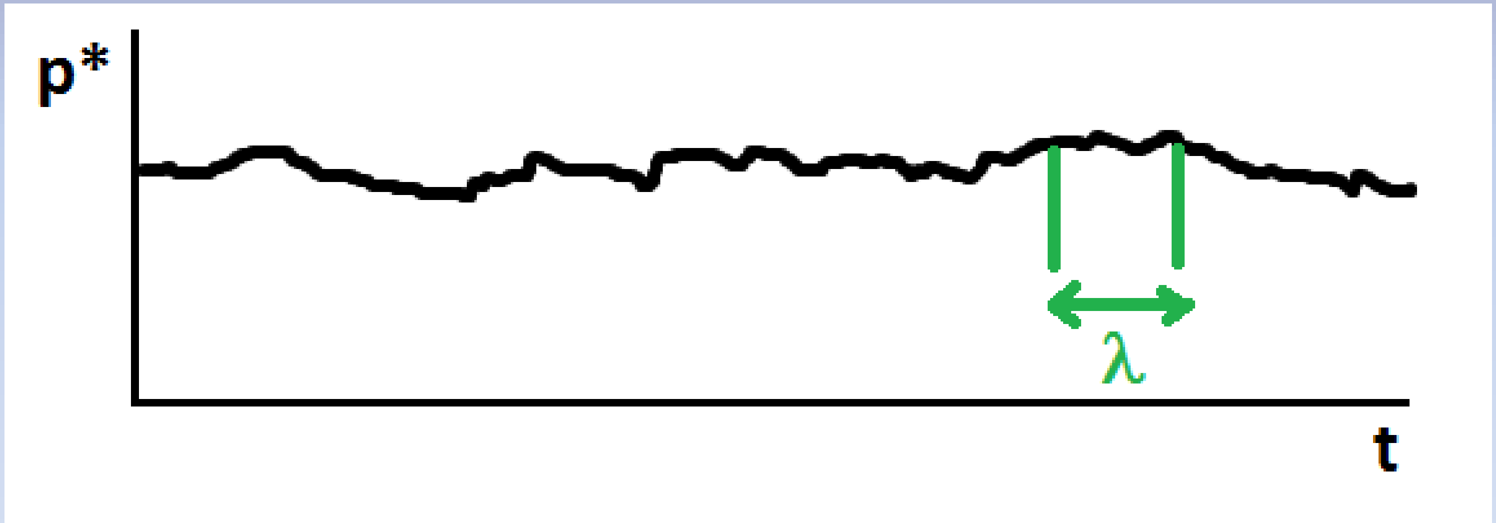

→ Sudden transitions

1. Why?

2. Is there any forewarning?

( Remember : Z = Fraction of active nodes )

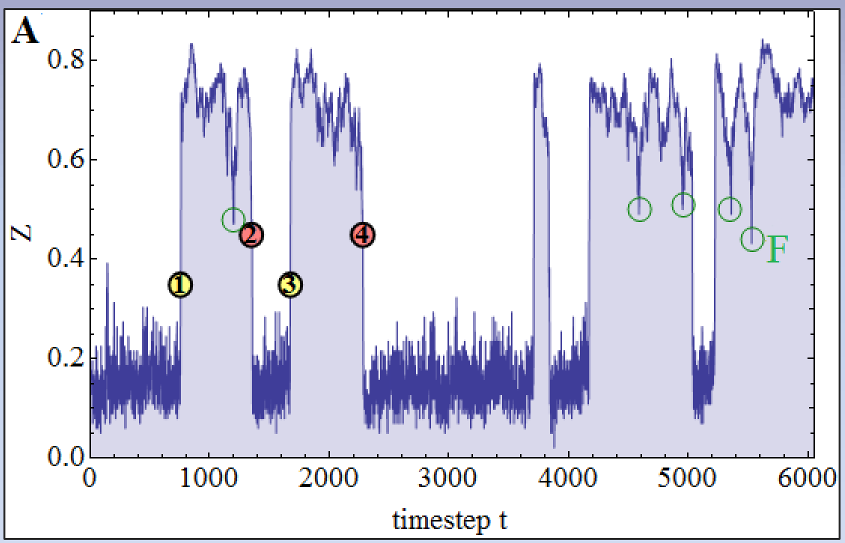

GEORGE WASHINGTON
1st PRESIDENT 1789-1797

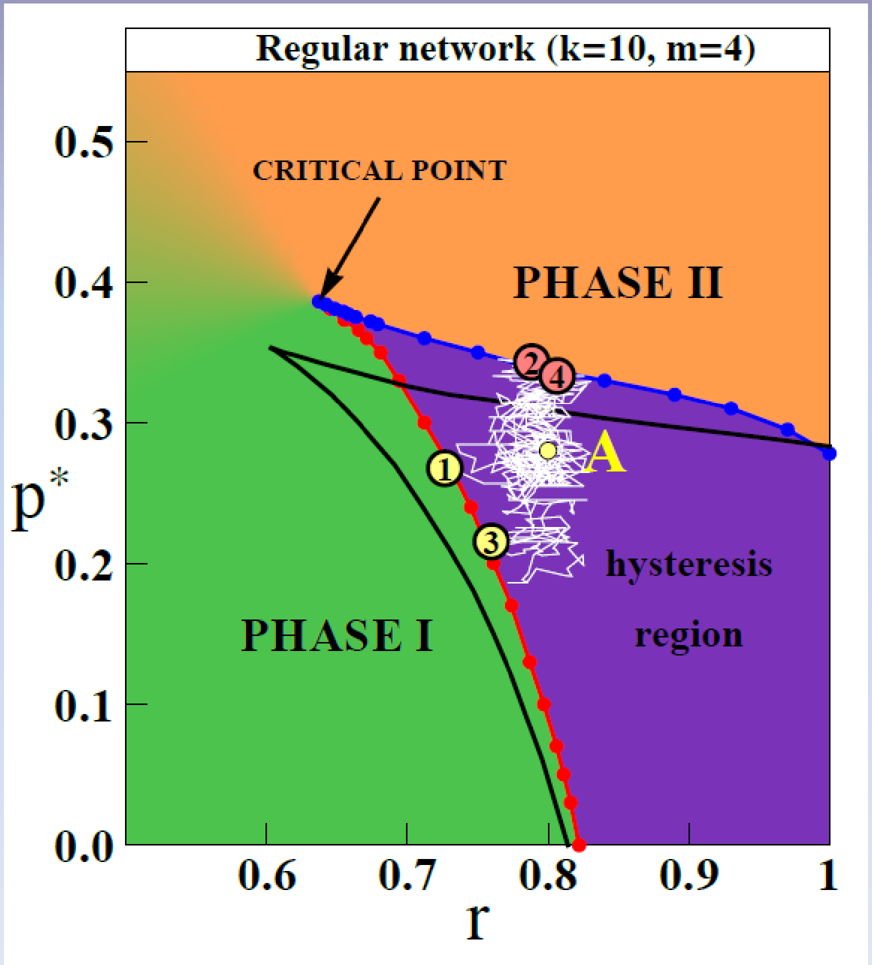

p*
λ
t

It turns out it can be predicted.

Trajectory $(r_\lambda(t),\ p_\lambda{}^*(t))$ in the phase diagram (white line, see below).

The trajectory crosses the spinodals (critical lines) interchangeably, and causes the phase flipping.

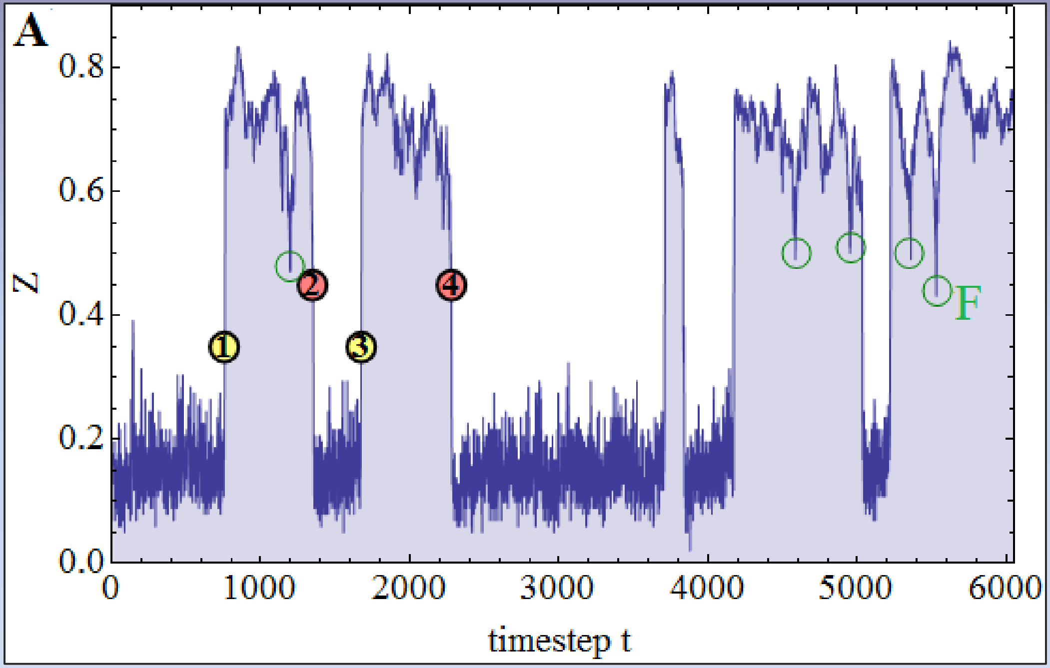



(Observing the network on the micro-level: counting internal and external failures)

# Second finite size phenomenon: Flash crashes

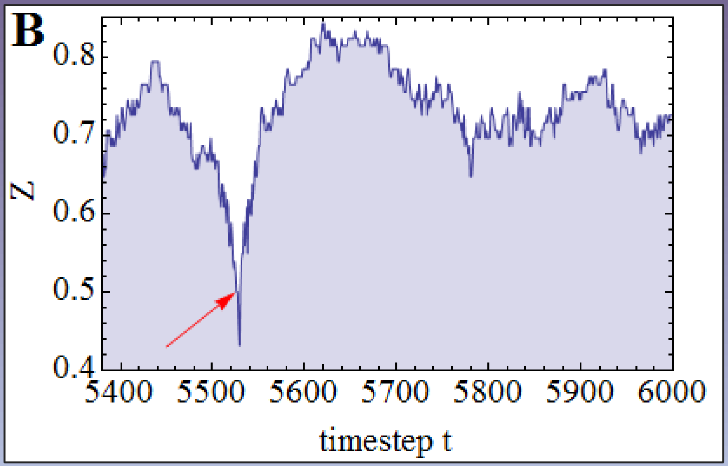
An interesting (and unexpected) by-product of the model:




Sometimes the network rapidly crashes, and then quickly recovers (green circles).

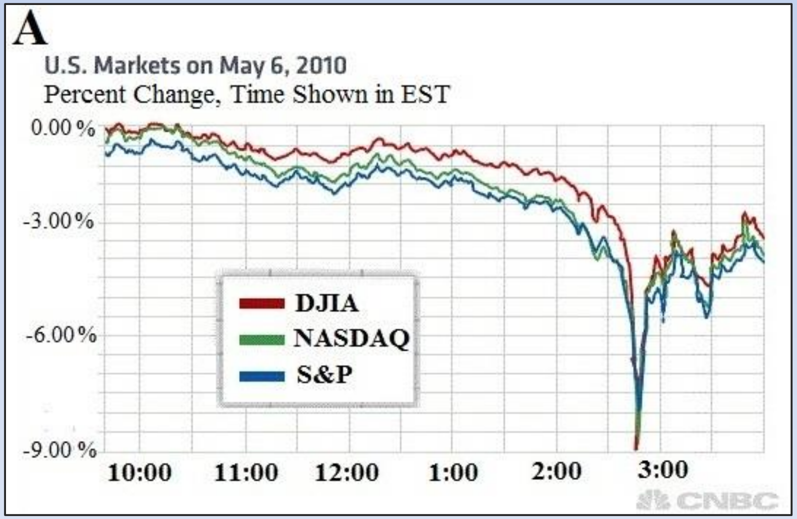
Model predicts the existance of "flash crashes".

Explanation: Unsuccessful transitions to a lower state.



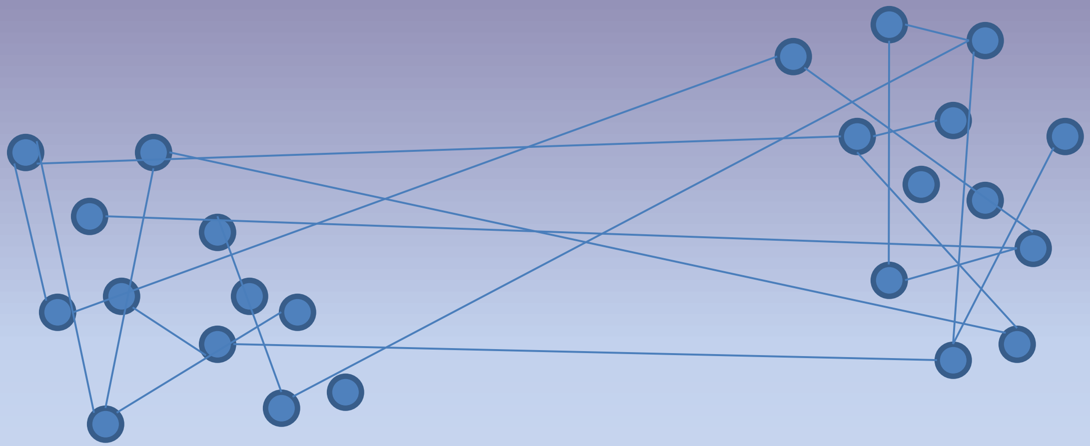

Real stock markets also show a similar phenomenon.

Q: Possible relation?



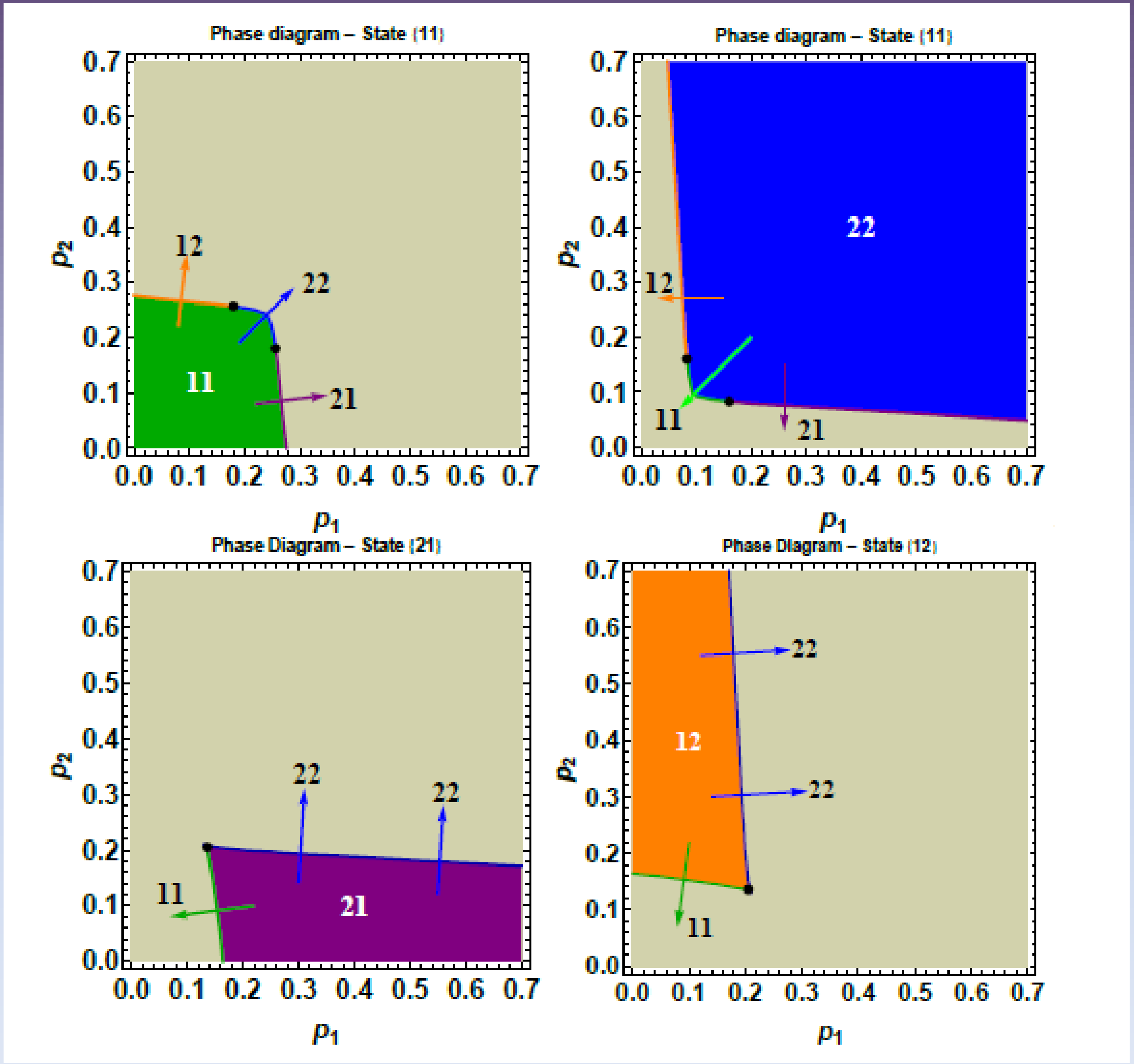

"Flash Crash 2010"

# 2.1. Interacting networks



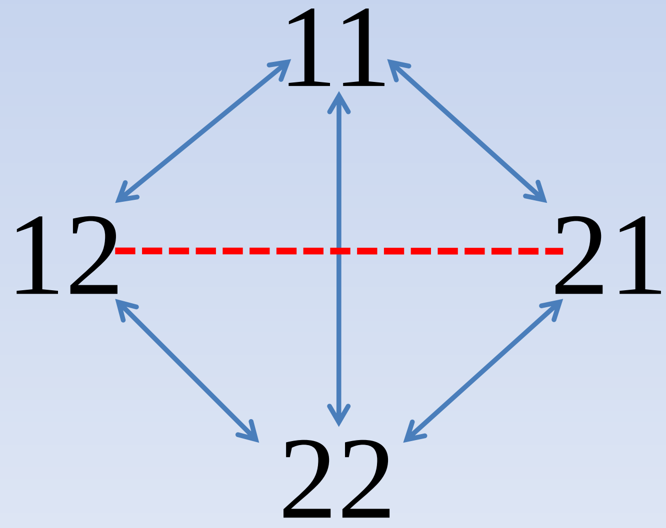
# MODEL

| FAILURE TYPE | RULE | RECOVERY |
|---|---|---|
| Internal failure | With rate ***p*** on each node | After time $\tau$ |
| External failure | IF(<= m active neighbors)<br>THEN<br>Extra failure rate **r** | After time $\tau'$ |
| Dependency failure | IF(companion node from the opposite network failed)<br>THEN<br>Extra failure rate $\mathbf{r_d}$ | After time $\tau''$ |

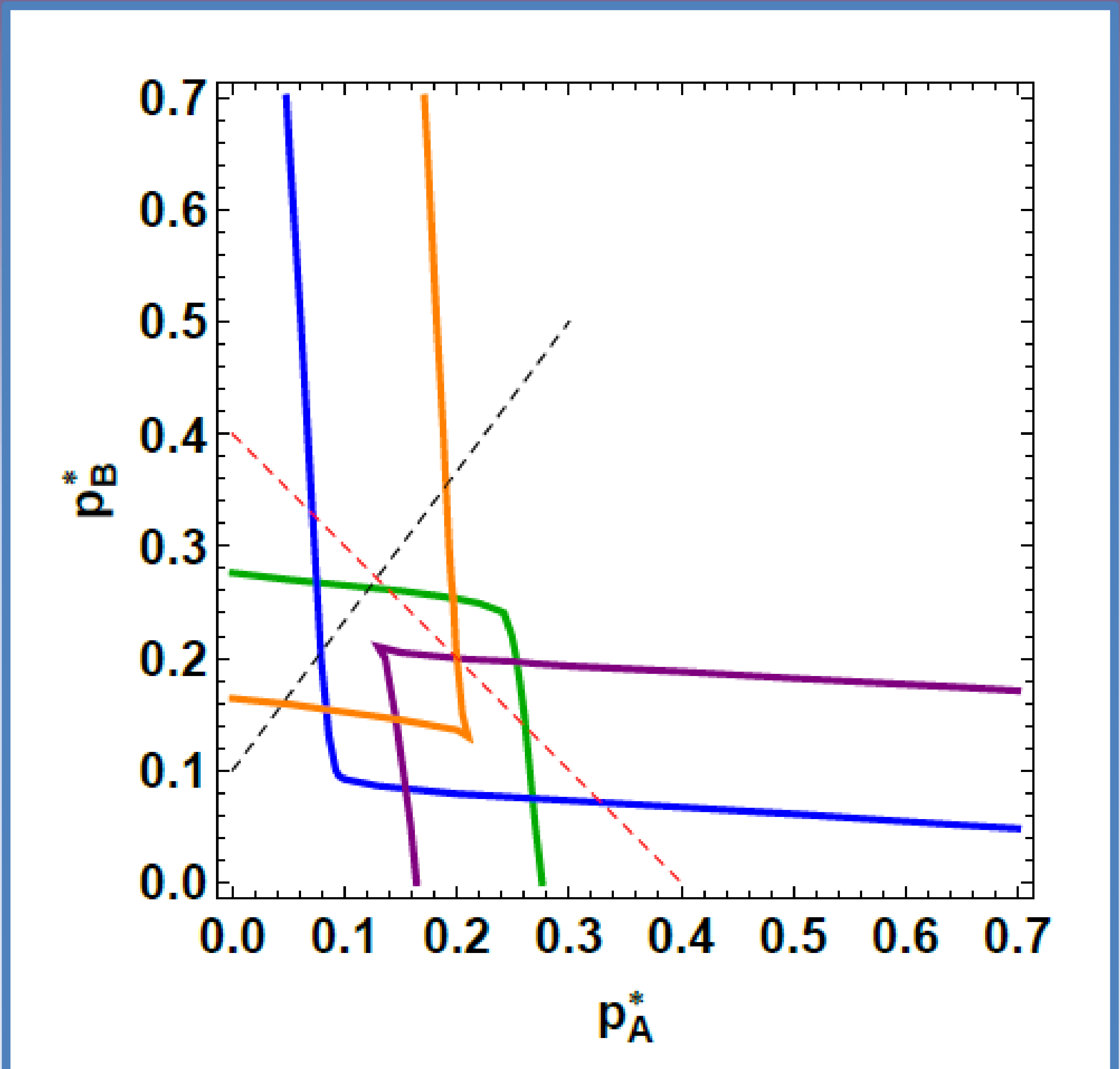

Phase diagram – State |11⟩
12
22
11
21
Phase diagram – State |11⟩
22
12
11
21
Phase Diagram – State |21⟩
22
22
11
21
Phase Diagram – State |12⟩
22
12
22
11
$p_1$
$p_2$
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7

# Elements of the phase diagram

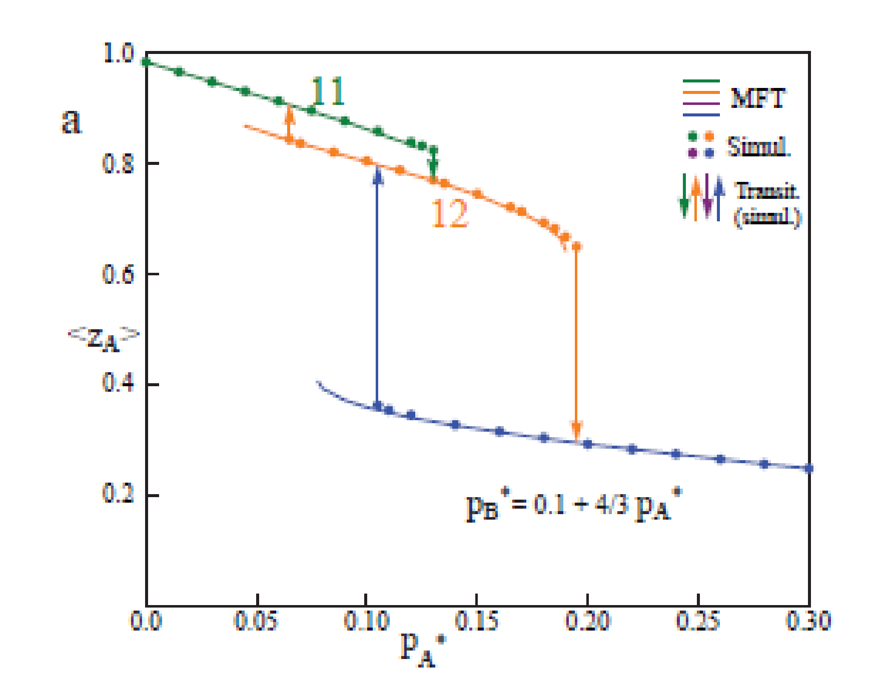
- 2 critical points
- 4 triple points
- 10 allowed transitions
- 2 forbidden transitions

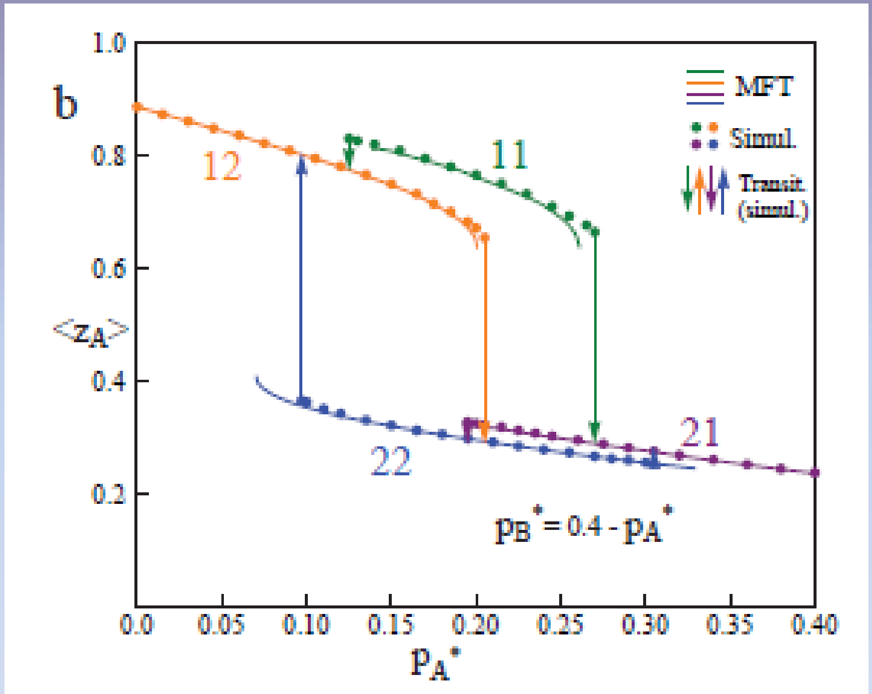


# TOTAL PHASE DIAGRAM



# HYSTERESIS LOOPS



# 2.2 Problem of optimal repairing

PROBLEM

- Consider two interacting networks, both damaged (low activity).

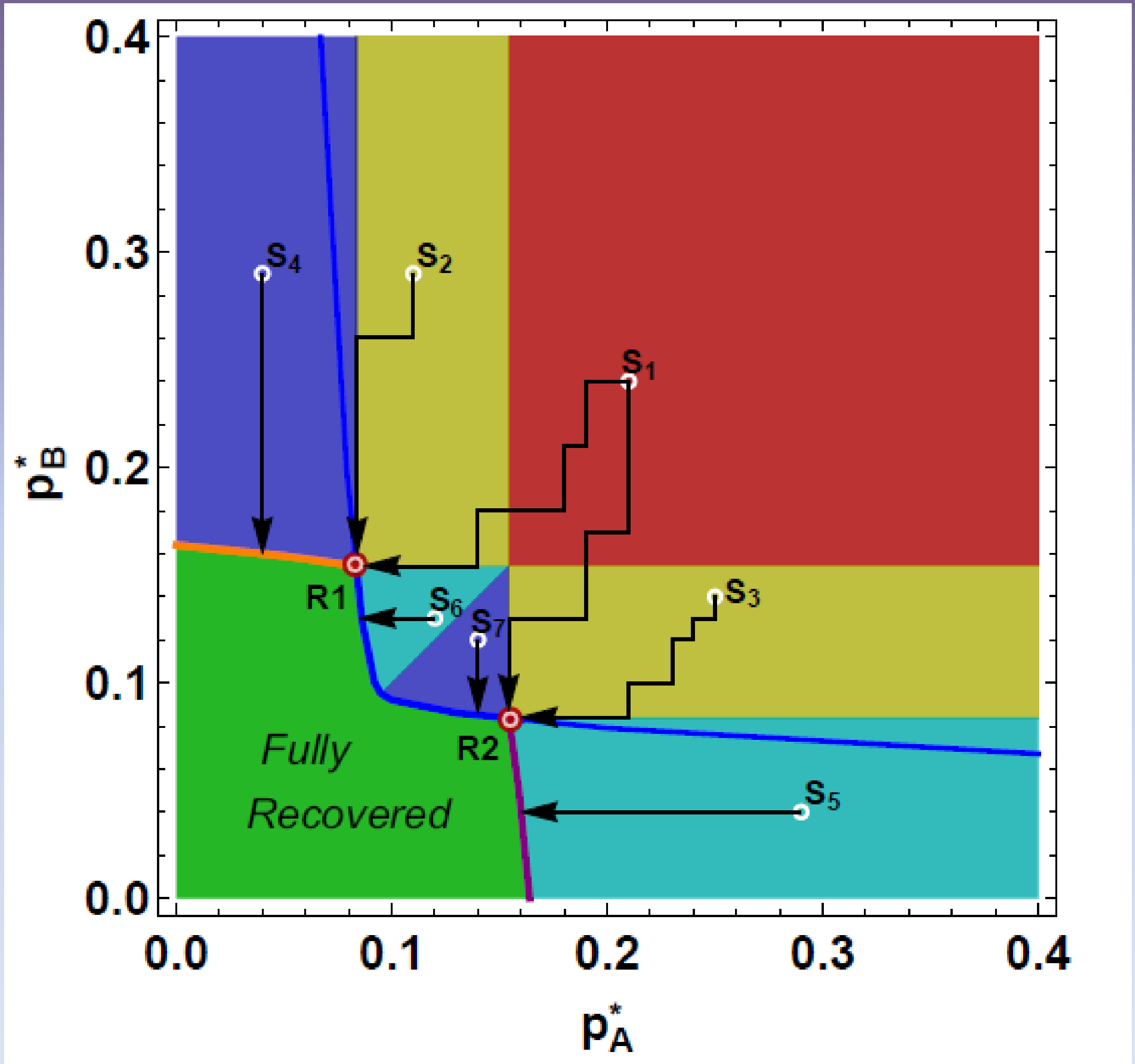
- What is the most efficient strategy to repair such a system?

- We want **minimal intervention** (intervention might be expensive or invasive)

- Scenario common in medicine

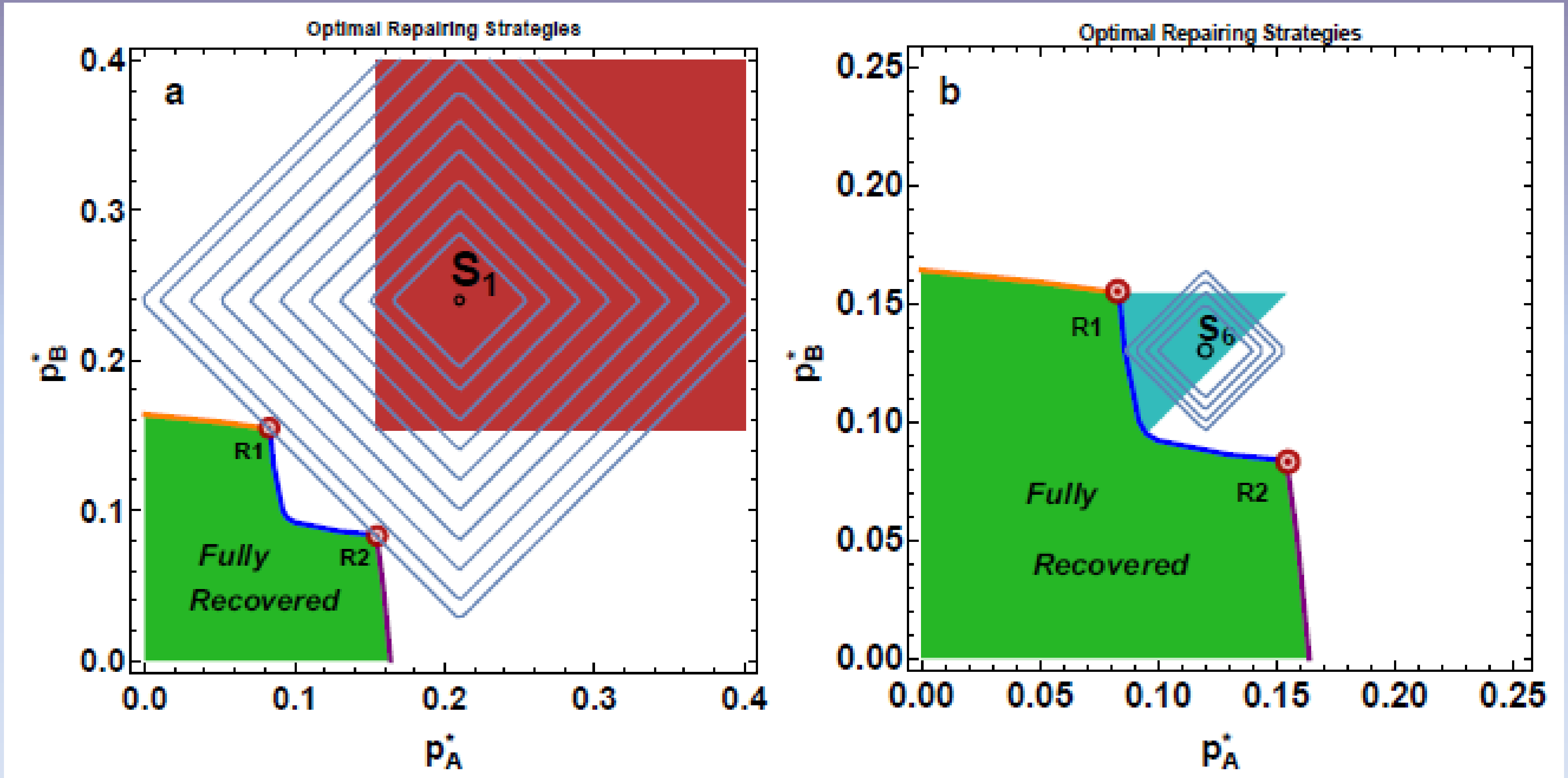
Question: What is the **minimum number of repairs** to bring the system to the fully functional state, and **how do we allocate them** (repairs) between the two networks?

SOLUTION:



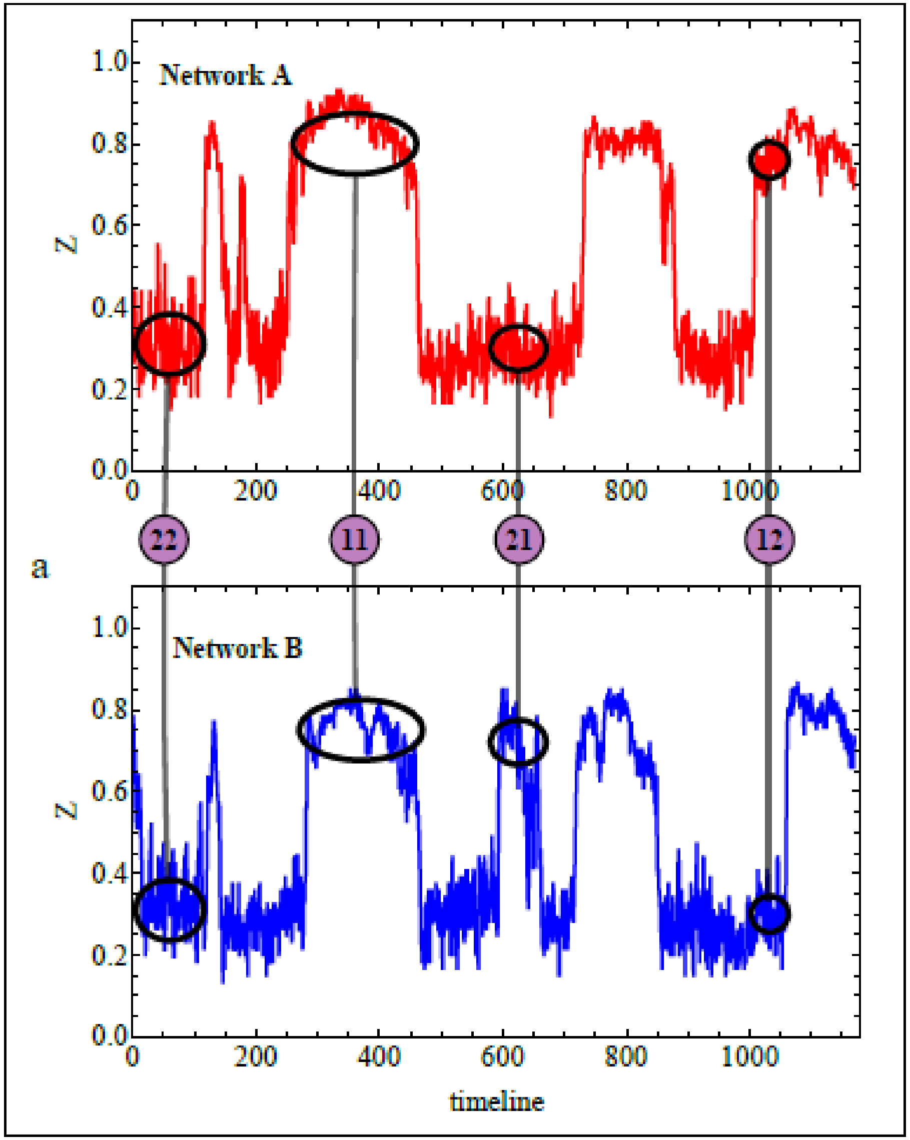
0.4
0.3
0.2
0.1
0.0
$p_B^*$
S4
S2
S1
S3
S6
S7
S5
R1
R2
Fully
Recovered
0.0
0.1
0.2
0.3
0.4
$p_A^*$

Explanation:



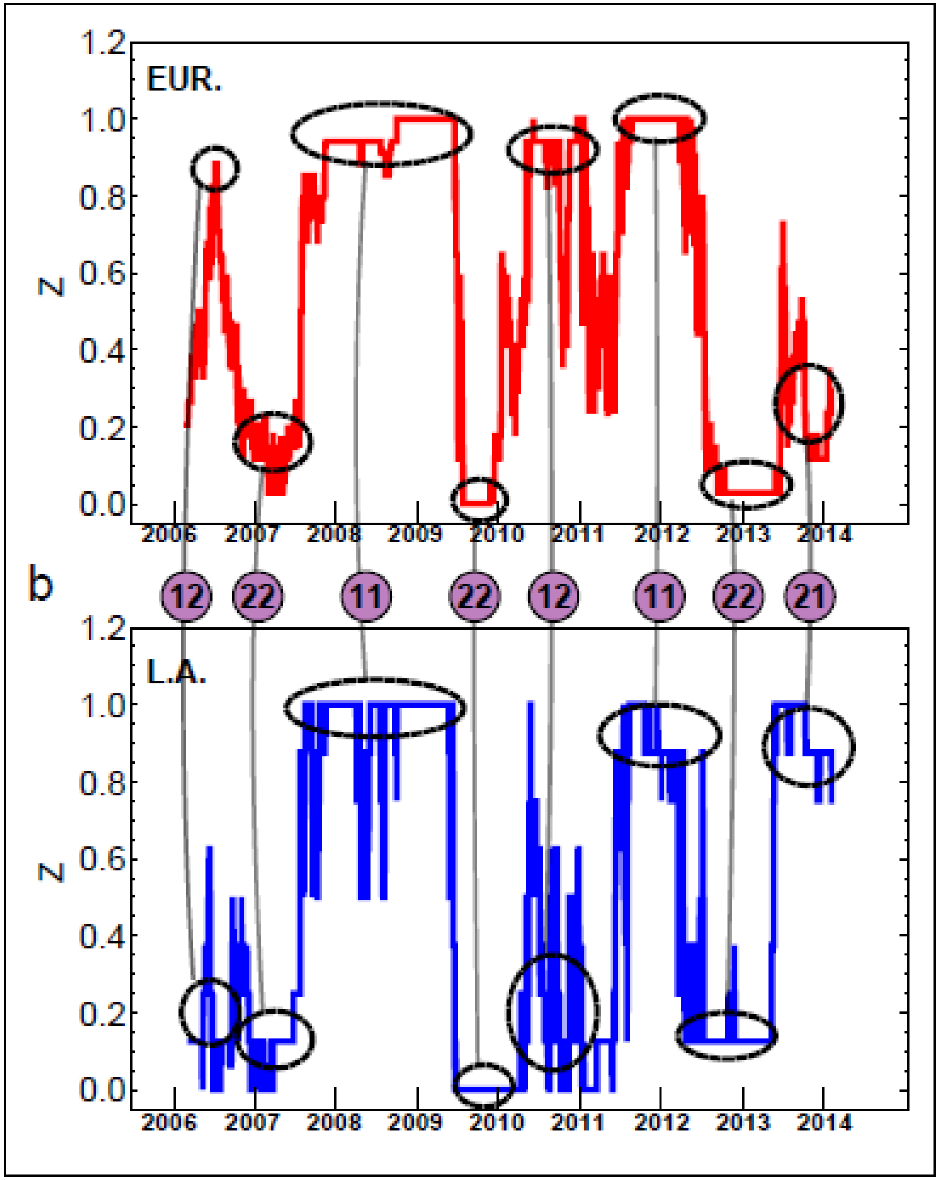
# 2.3 Dynamics of interacting networks

Activity z
(simulation)



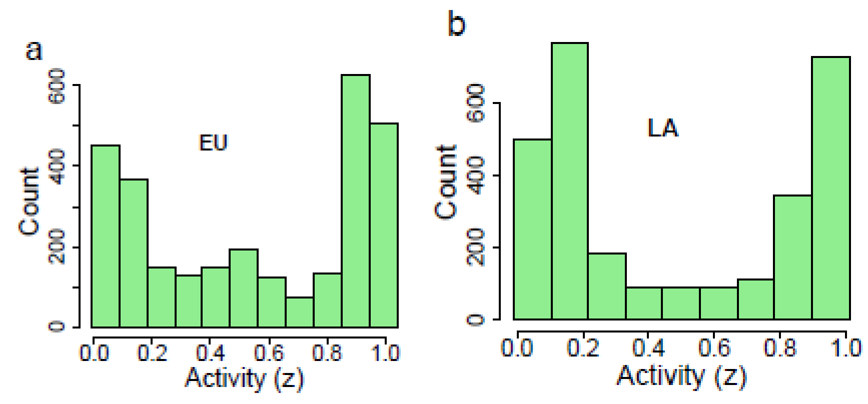

Collective dynamics in simulated interacting networks.

Activity z
(real coupled networks)



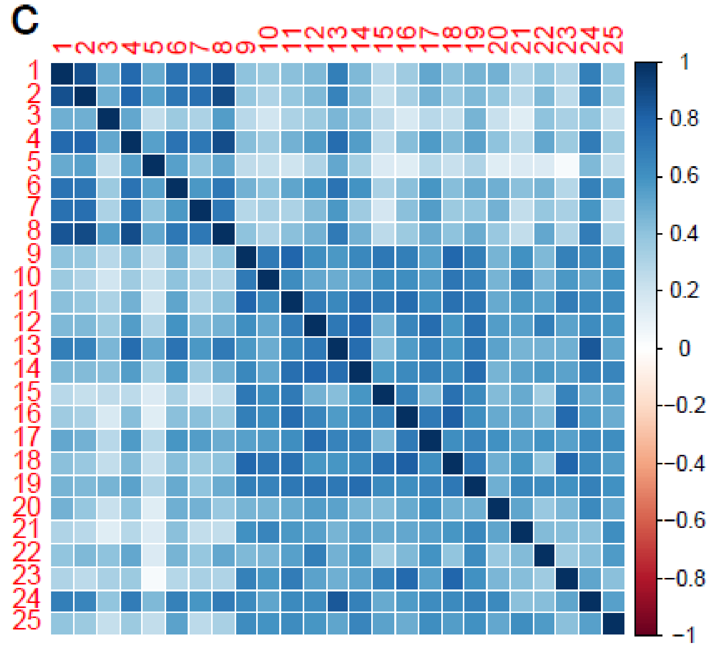

Collective dynamics in real interacting networks.

Density plots




Density plots for the activity of real networks.

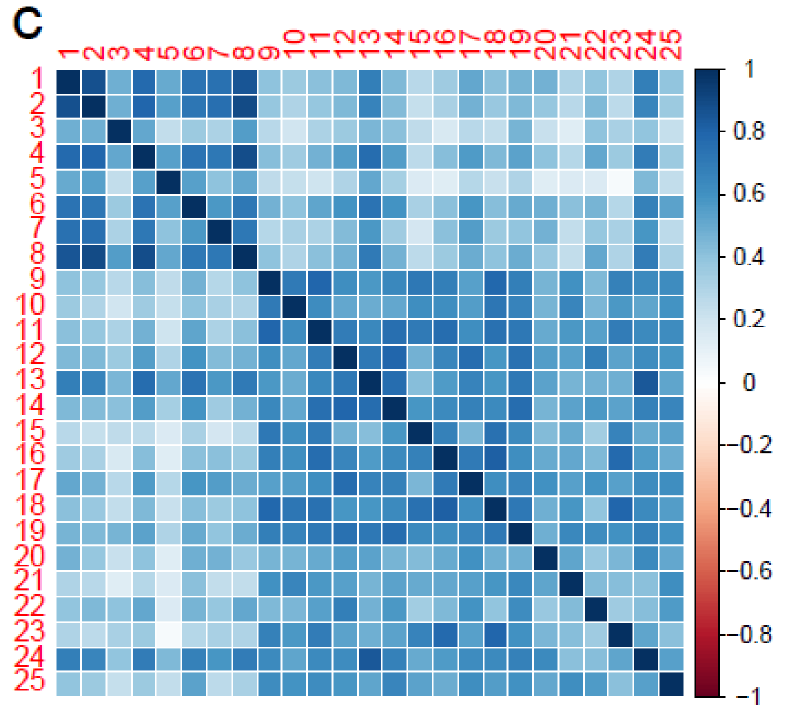
Corr. matrix




Correlation between individual CDS signals.

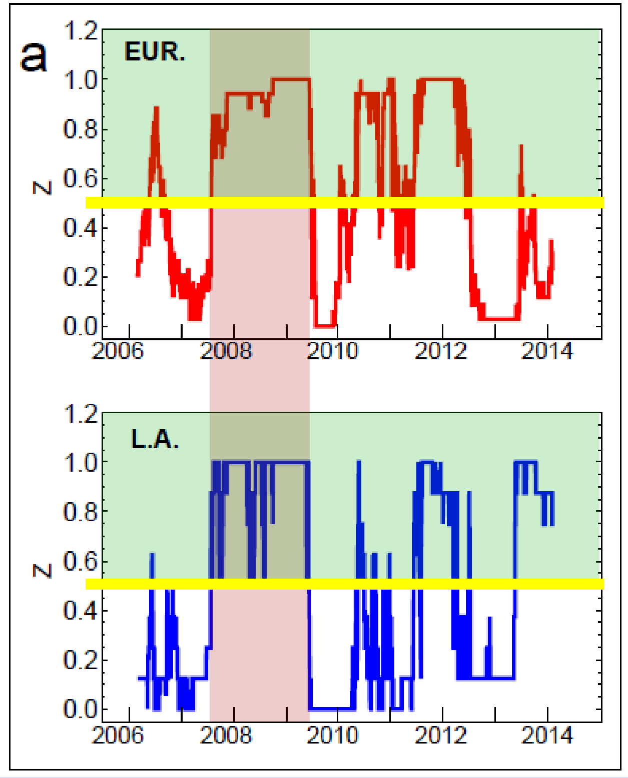
# Measuring and estimating model parameters

Building the network




Correlation between individual CDS signals.

Estimating internal failure parameters (p*_EU and p*_LA): observing high activity states



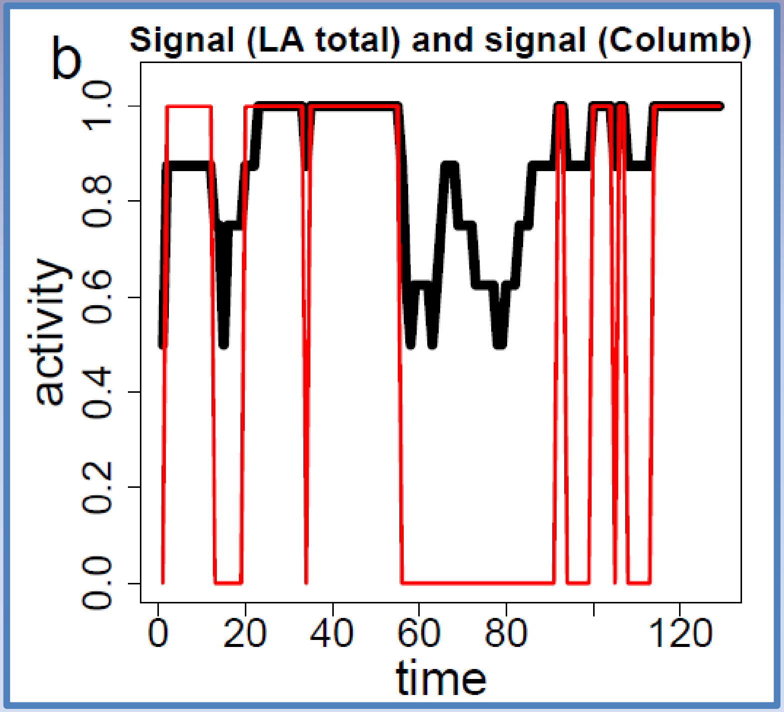

Estimate:

$$p^*_{\mathrm{EU}} = 0.07 \pm 0.01 \quad \text{and} \quad p^*_{\mathrm{LA}} = 0.11 \pm 0.02$$

Alternative method for measuring internal failure and recovery rates: observing micro dynamics (dynamics of individual nodes)

time of recovery, $\tau_{EU}$ and $\tau_{LA}$

crude failure rates $p_{EU}$ and $p_{LA}$




$\tau_{EU} = 13 \pm 2,\ p_{EU} = (3.0 \pm 0.5) * 10^{-3}$

$p^*_{EU} = 0.07 \pm 0.01$ (same as before)

$\tau_{LA} = 9 \pm 1,\ p_{LA} = (7.6 \pm 0.8) * 10^{-}3$

$p^*_{LA} = 0.11 \pm 0.02$ (same as before)

Estimating the external parameters r_EU and r_LA:
**observing low activity states**

- Contribution from internal failures known

- Contribution from interdependent failures temporary neglected, we introduce a small correction later

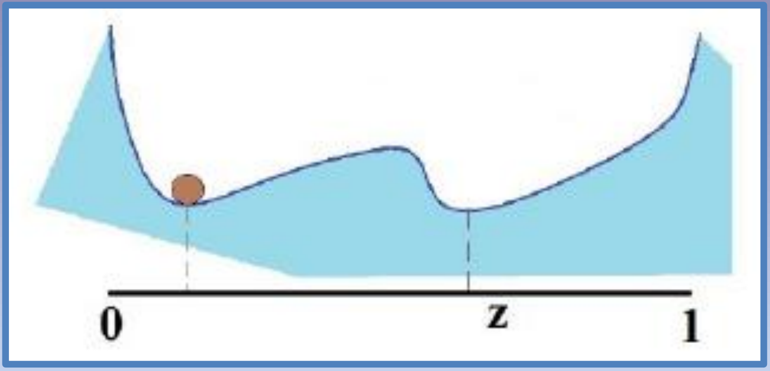
- Small influence of thresholds m_EU and m_LA on the average value of activities z_EU and z_LA

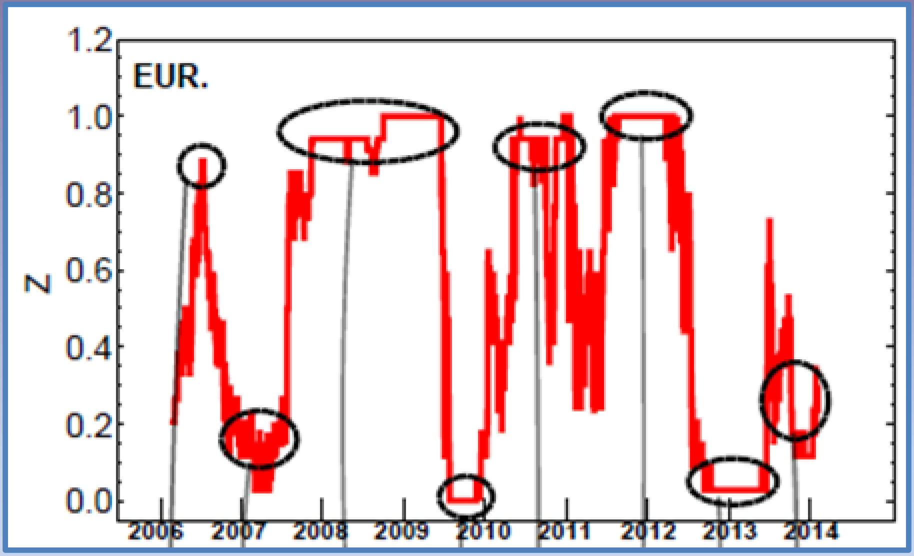
By measuring $\langle z_i \rangle$ in the low states ($z_{\mathrm{EU}} < 1/2$ & $z_{\mathrm{LA}} < 1/2$) and

already knowing $p_i{}^*$, we obtain estimates $r_{\mathrm{EU}} = 0.81 \pm 0.03$ and $r_{\mathrm{LA}} = 0.88 \pm 0.03$.

## Estimating the thresholds m_EU and m_LA: visiting times

double well,



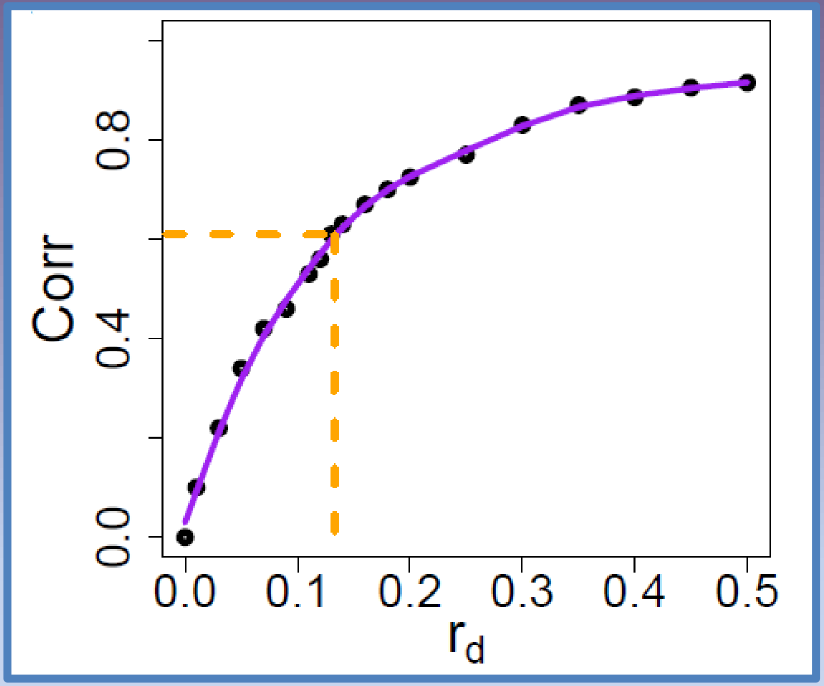

parameter $m_{frac}$ shape of the potential barrier between the wells.

simulating decoupled ($r_d = 0$) EU and LA networks

previously measured parameters ($p^*_{\rm EU}$, $p^*_{\rm LA}$, $\tau_{\rm EU}$, $\tau_{\rm LA}$, $r_{\rm EU}$, $r_{\rm LA}$)

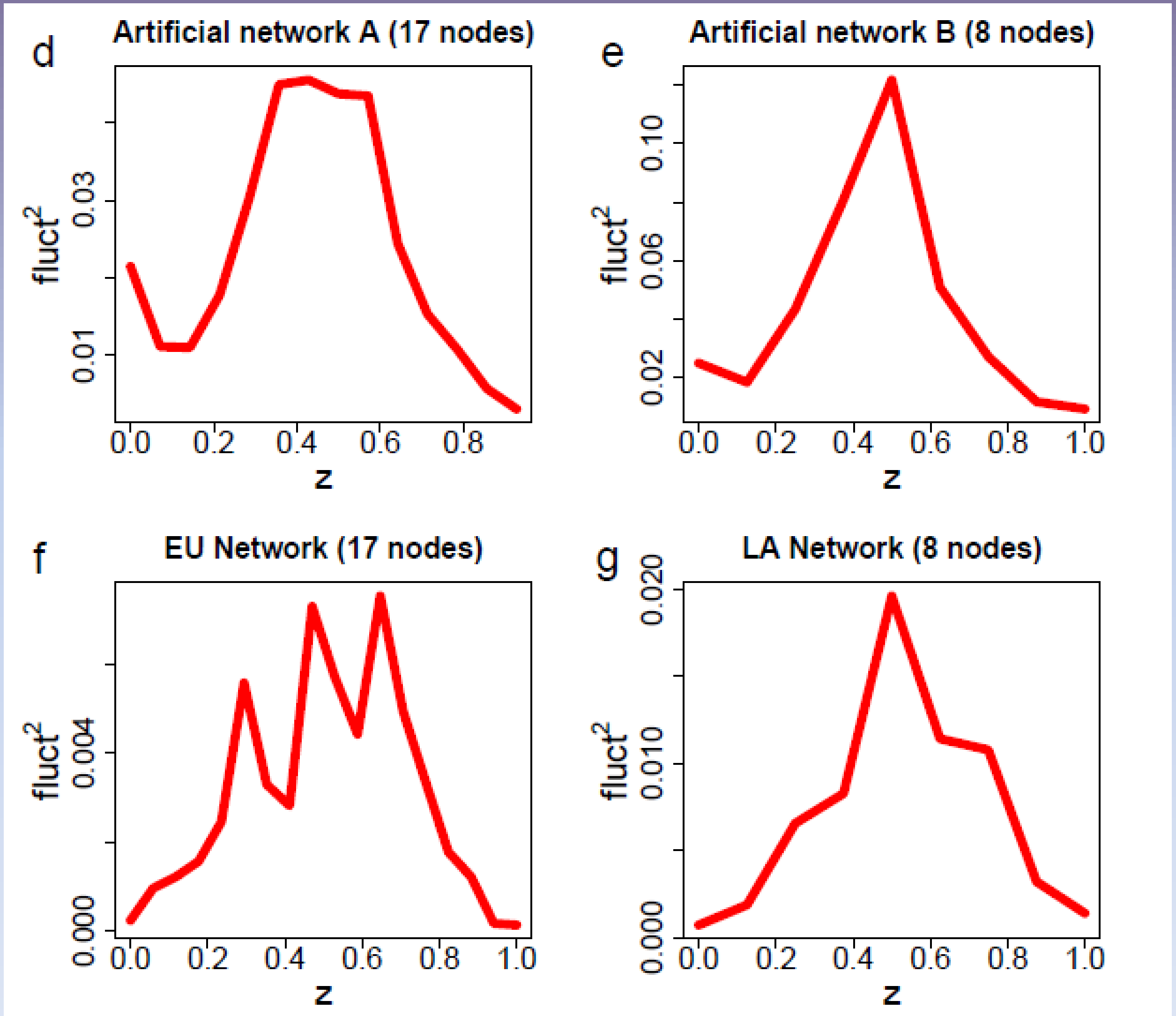
$m_{\rm frac,EU} = 0.57 \pm 0.02$ and $m_{\rm frac,LA} = 0.50 \pm 0.02$ for $\langle k \rangle = 3$, to $m_{\rm frac,EU} = 0.59 \pm 0.02$ and $m_{\rm frac,LA} = 0.50 \pm 0.02$ for $\langle k \rangle = 7$ ($\langle k \rangle$

Estimating r_d : correlation between networks EU and LA




| $\langle k \rangle$ | $m_{frac,EU}$ | $m_{frac,LA}$ | $r_EU$ | $r_LA$ | $r_d$ |
|---|---|---|---|---|---|
| 3 | 0.55 | 0.43-0.49 | 0.78 | 0.87 | 0.16 |
| 4 | 0.55 | 0.43-0.49 | 0.79 | 0.87 | 0.14 |
| 5 | 0.57 | 0.43-0.49 | 0.79 | 0.87 | 0.13 |
| 6 | 0.57 | 0.43-0.49 | 0.79 | 0.87 | 0.11 |
| 7 | 0.57 | 0.43-0.49 | 0.79 | 0.87 | 0.10 |

# Similarity in fluctuation size structure



- Thank you for your time.