# CASCADING FAILURES IN COMPLEX SYSTEMS

## k-core percolation in interdependent networks


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Collaborators

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# Outline

- Cascading failures in Complex networks
  - What are cascading failures ?
  - How does complex networks approach it ?
- First approach: k-core percolation
  - What is k-core percolation ?  Cascading failure
  - Integer vs. fractional k-cores ?
- Second approach: What are interdependent networks ?
  - What is interdependency ? How do we quantify it ?
- What happens when we combine both ? (My work)

# US Blackout 2003

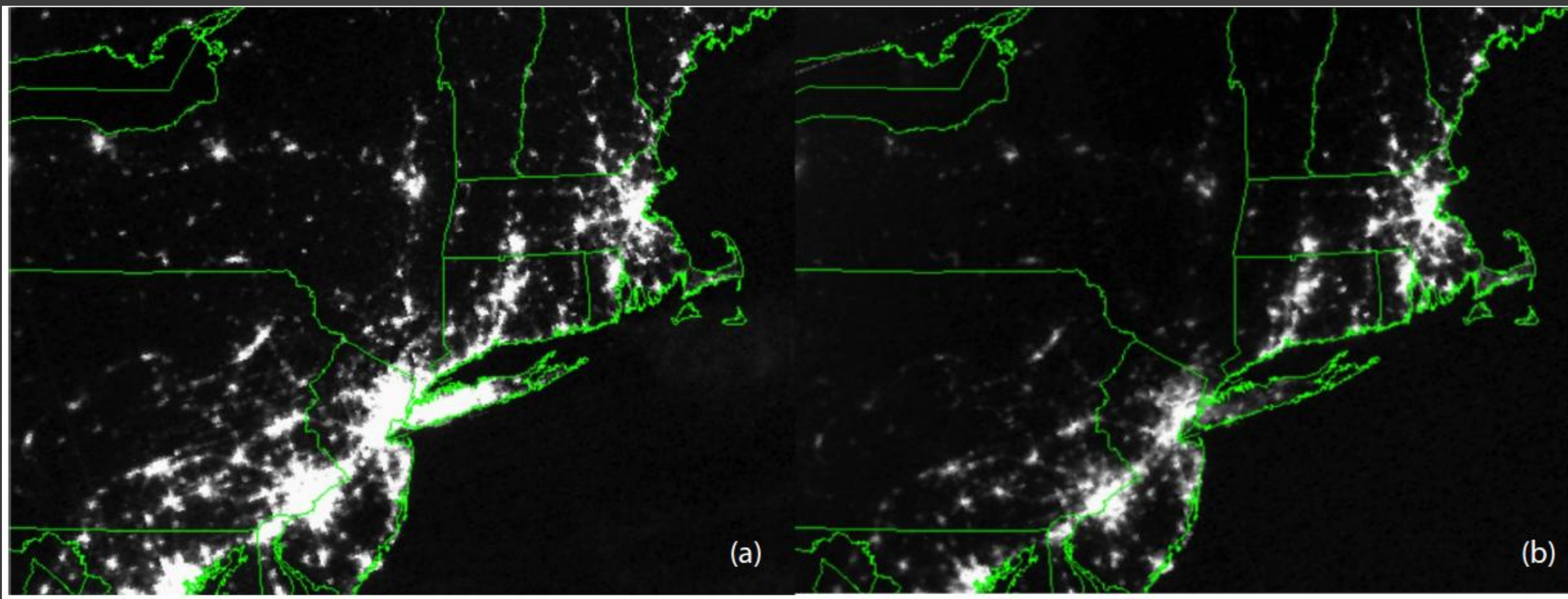
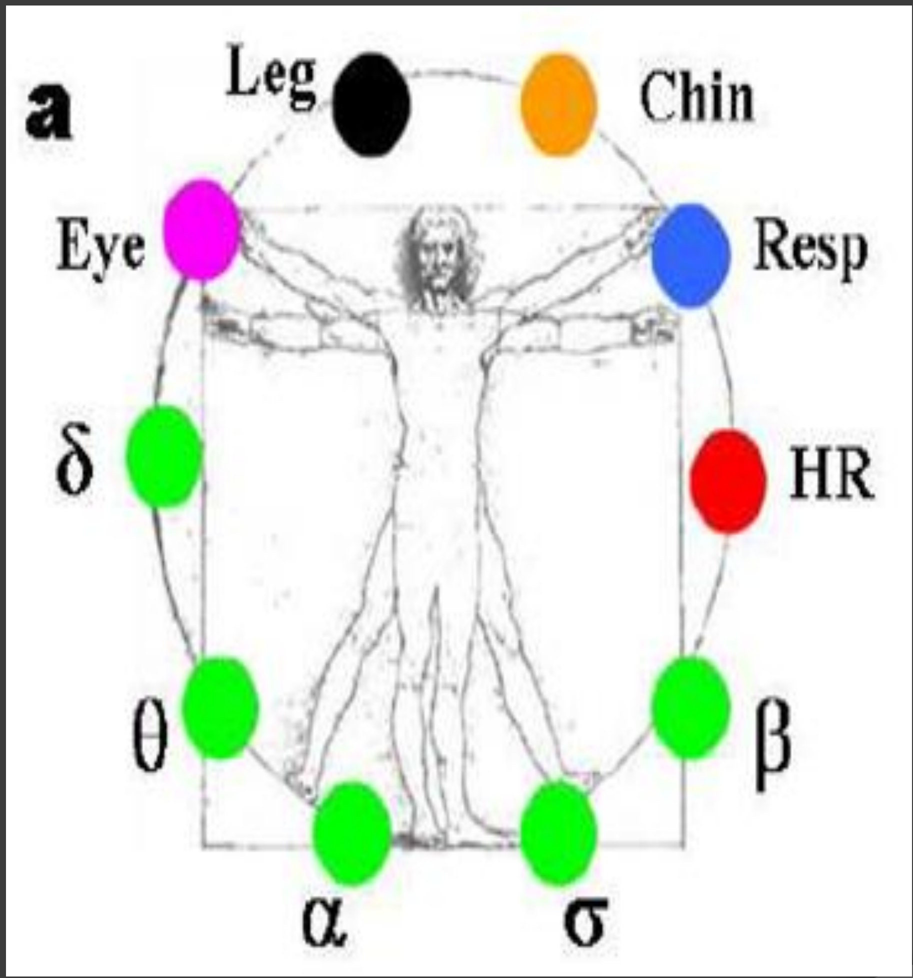



**Fig. 1.4** A satellite map of the US Northeastern blackout in 2003, (a) before blackout and (b) after blackout. The satellite map represents a real image of the US on August 14, 2003, the night of a major blackout that left an estimated 45 million people in eight U.S. states and another 10 million in Ontario without power.

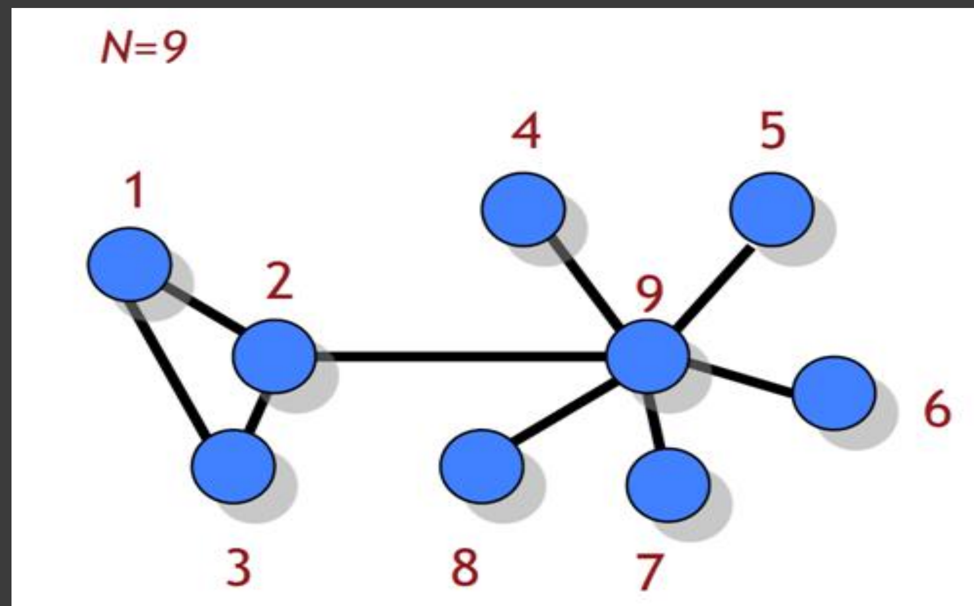
# Cascading failure in Human Physiology



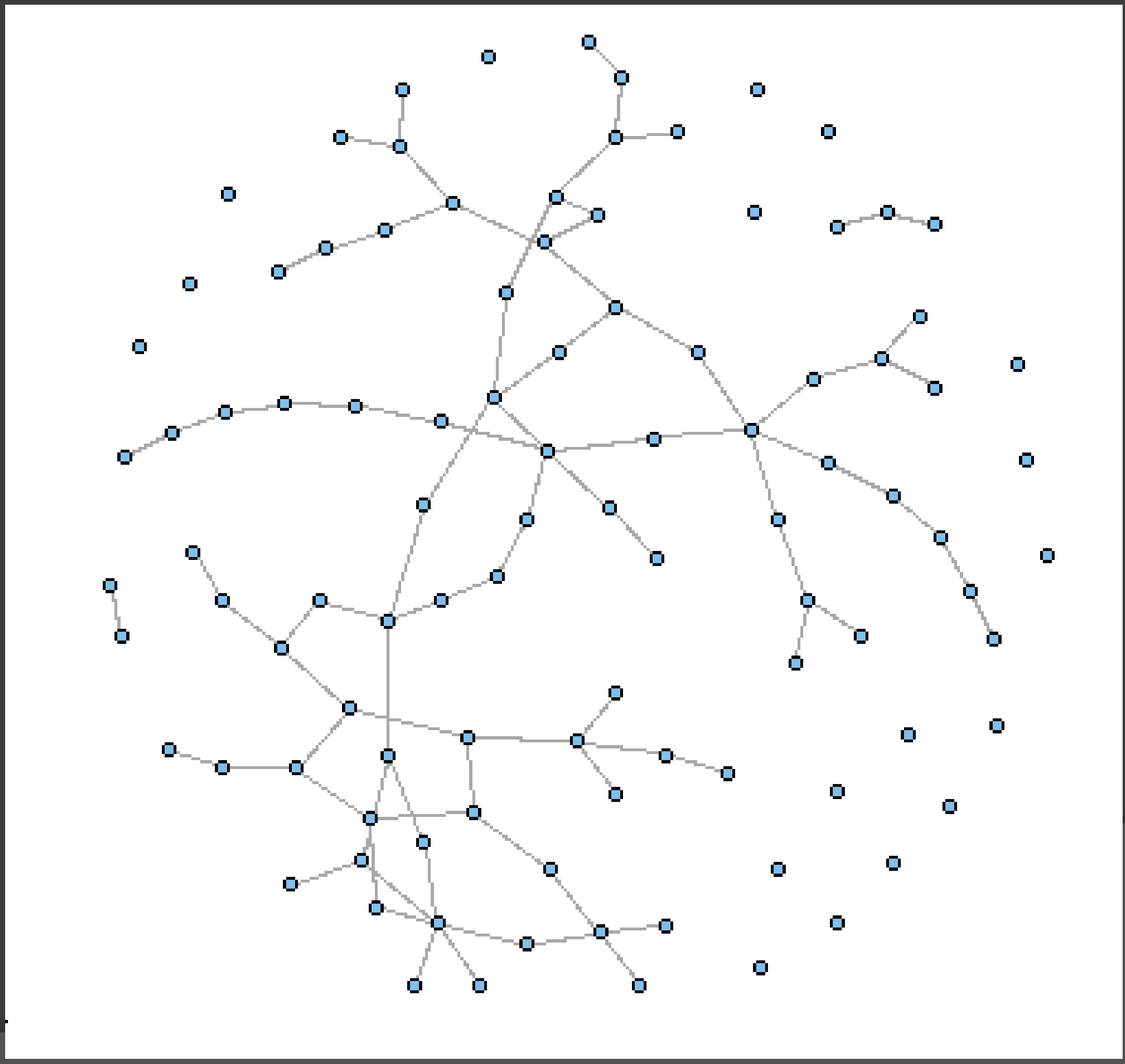

HUMAN BODY
Bashan et al Nat. Comm. (2012)

# Complex network elements

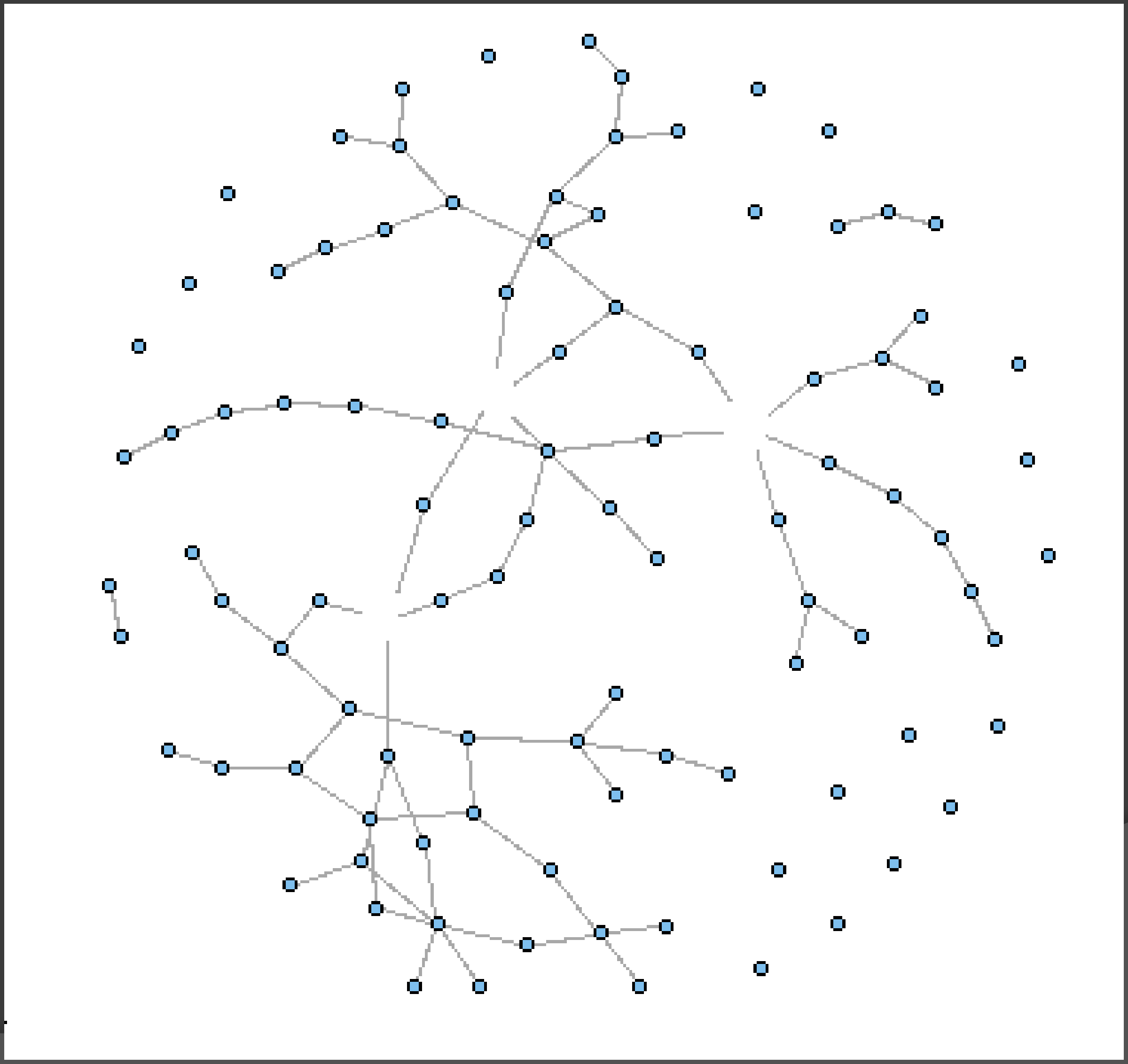
- Node
- Link
- Degree
- Degree distribution



# How do we define failure in complex network ?



# Random node failure leads to many network fragments

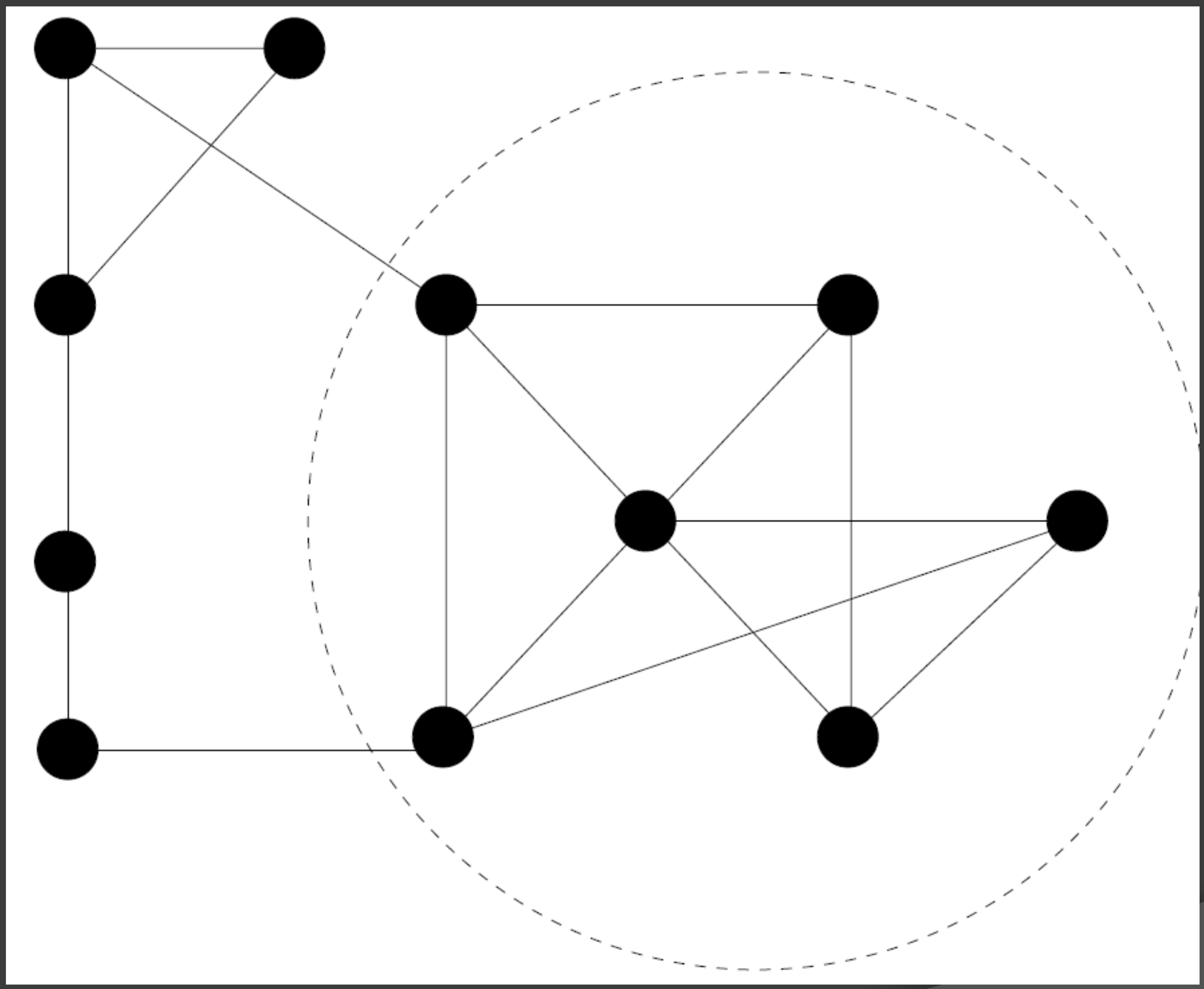


# k-core percolation

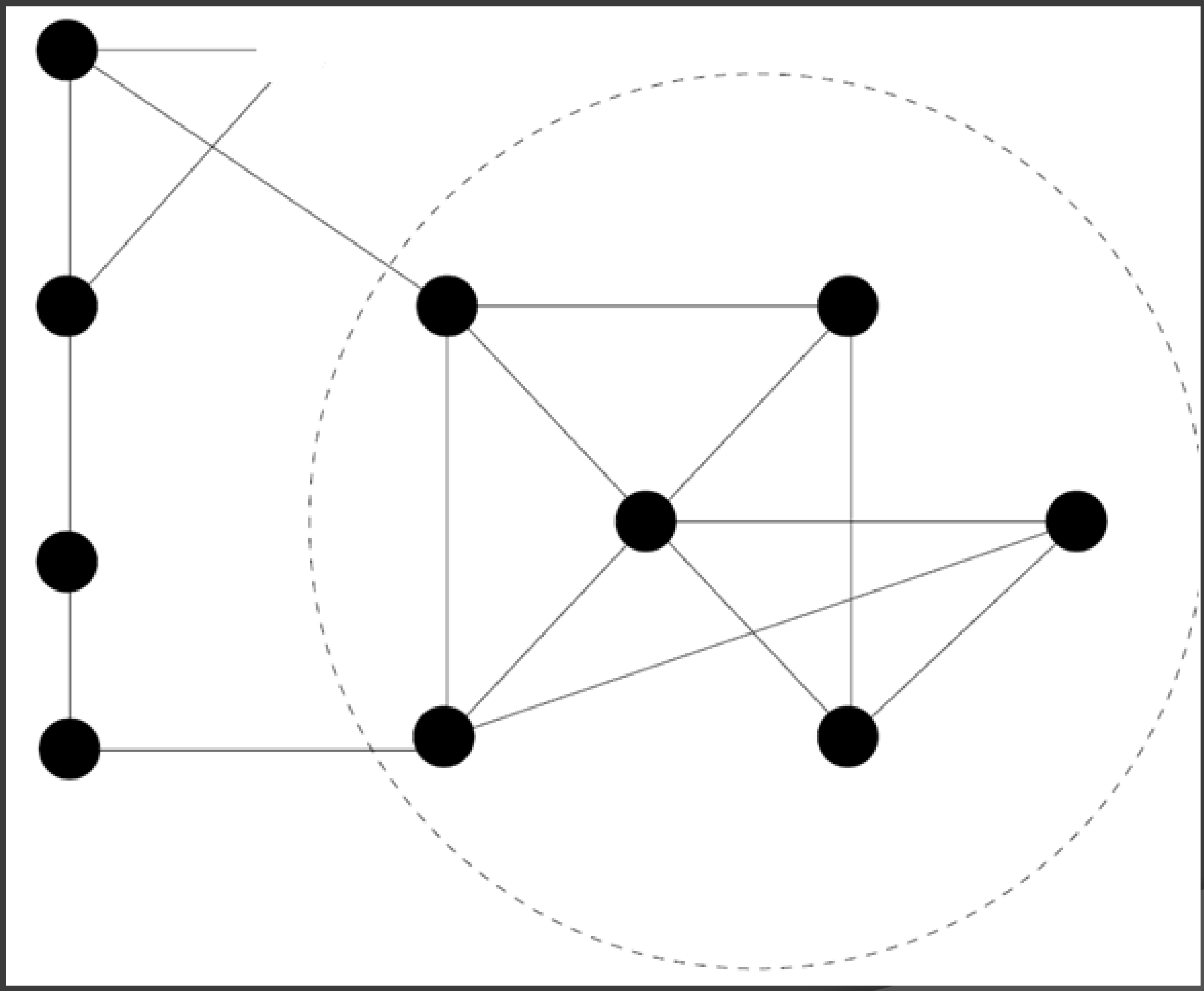
# What is k-core percolation ?

- After the initial damage, remove all nodes that do not have atleast ‘k’ neighbors:
- 3-core percolation means all nodes that have fewer than 3 neighbors are removed
- 3 is the local threshold

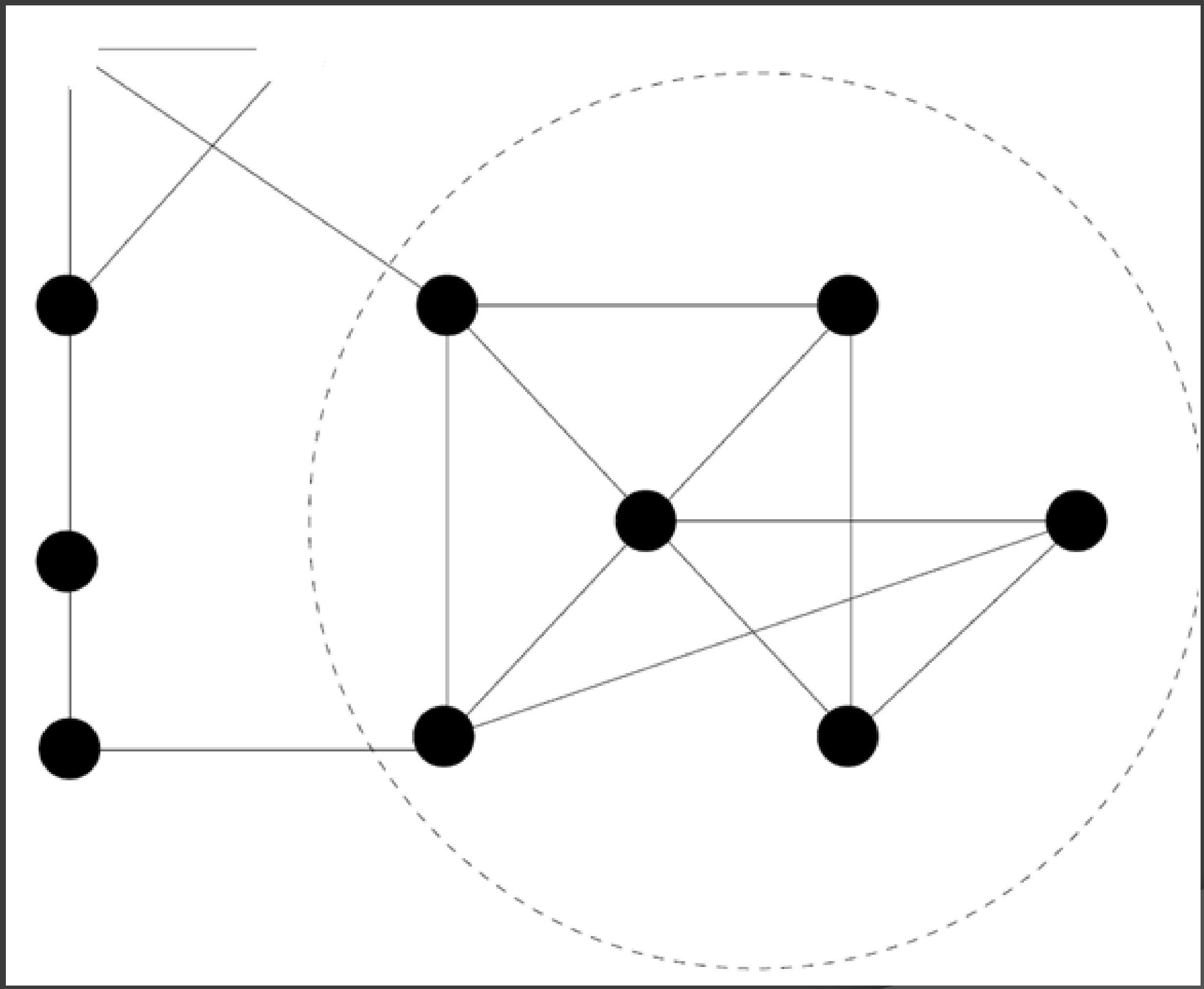
# Example: 3-core percolation



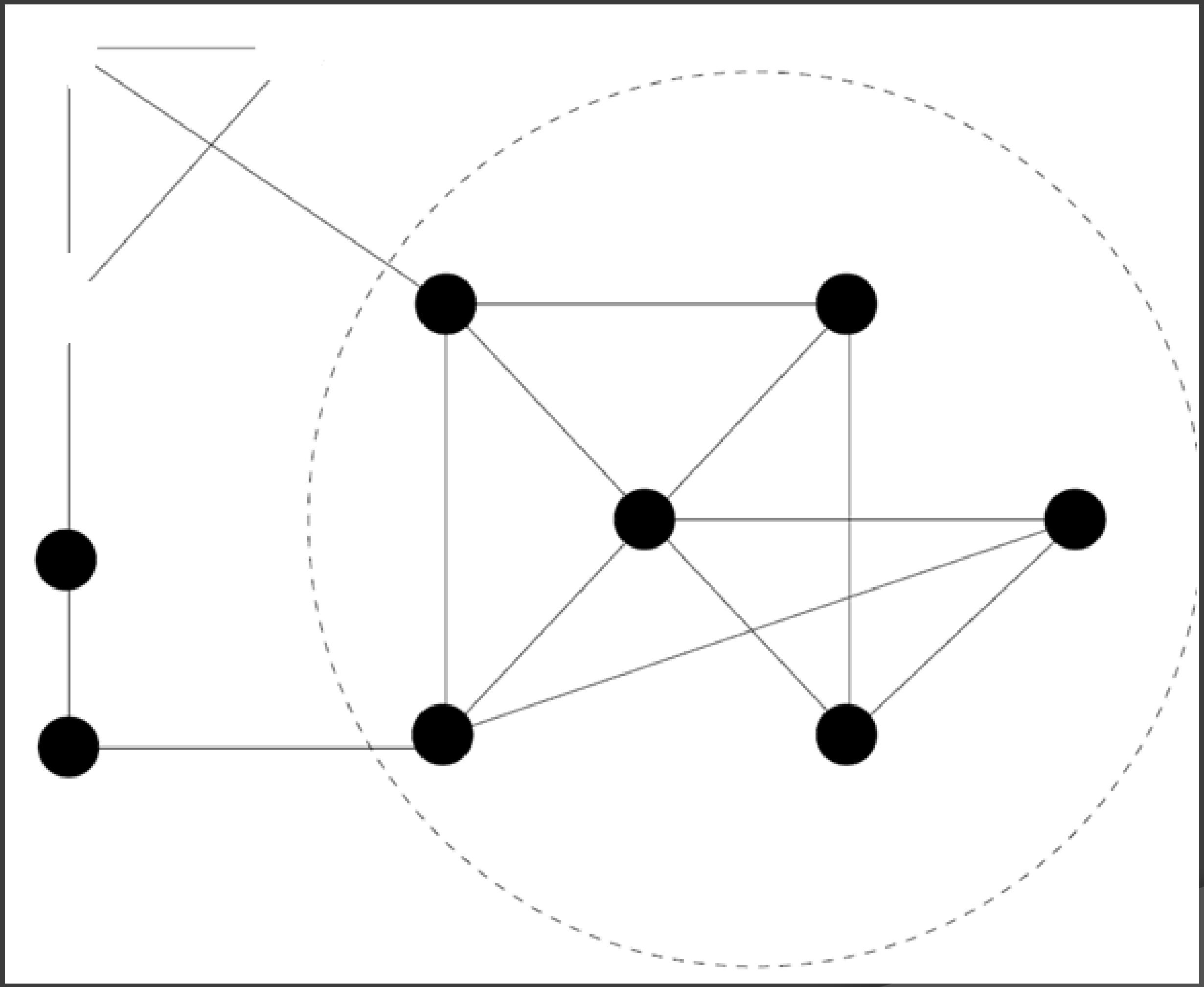
# Example: 3-core percolation



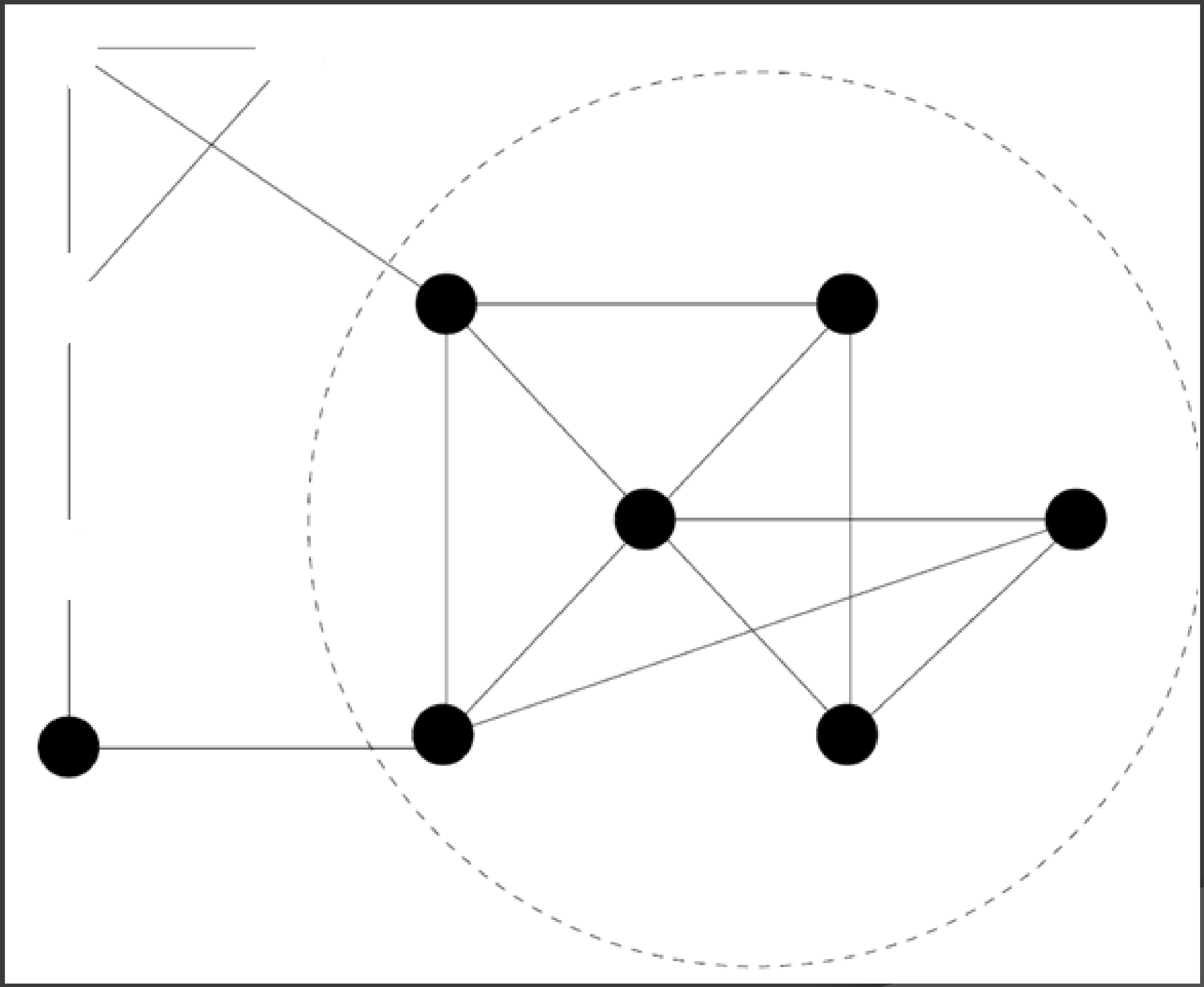
# Example: 3-core percolation



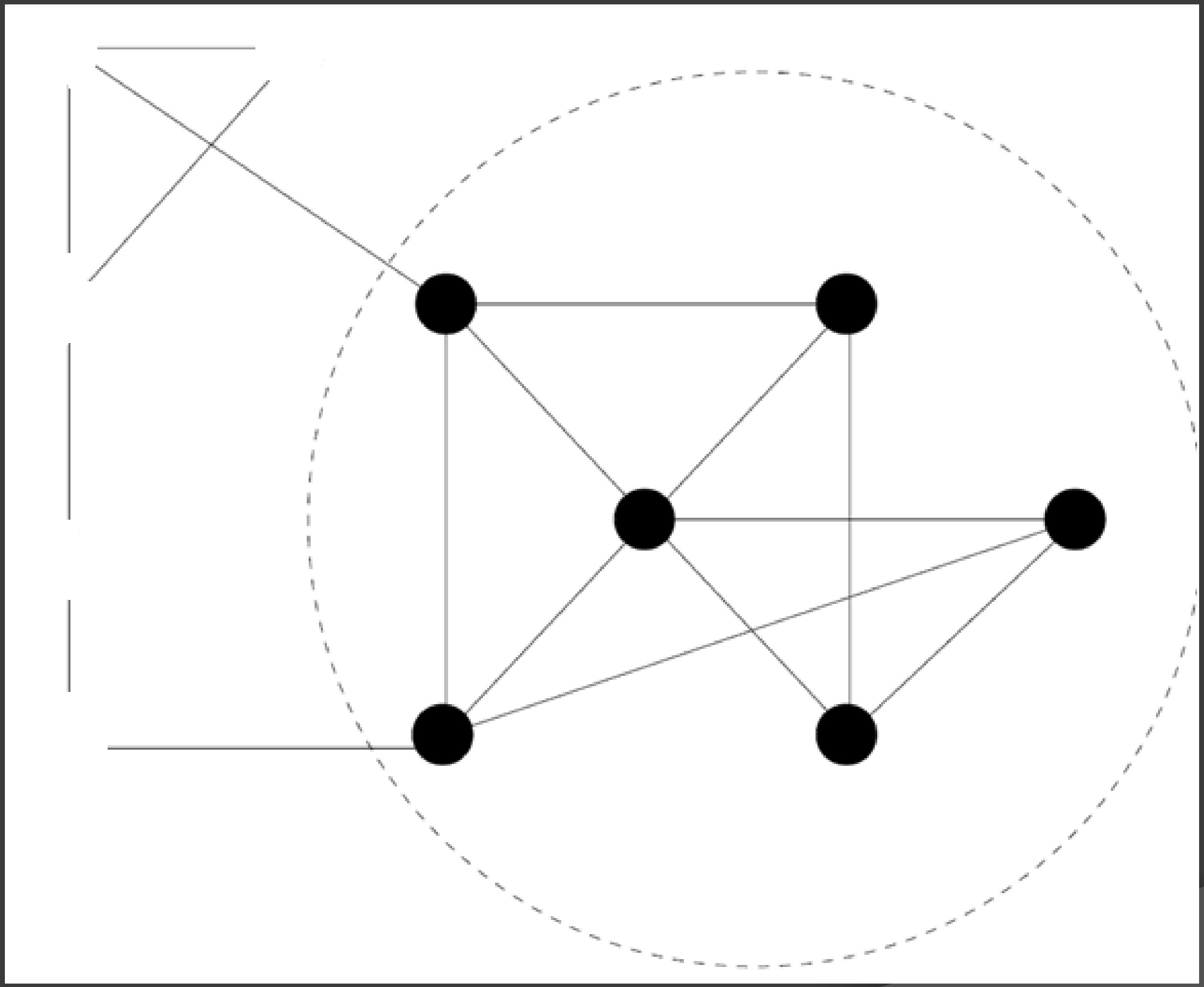
# Example: 3-core percolation



# Example: 3-core percolation



# Example: 3-core percolation



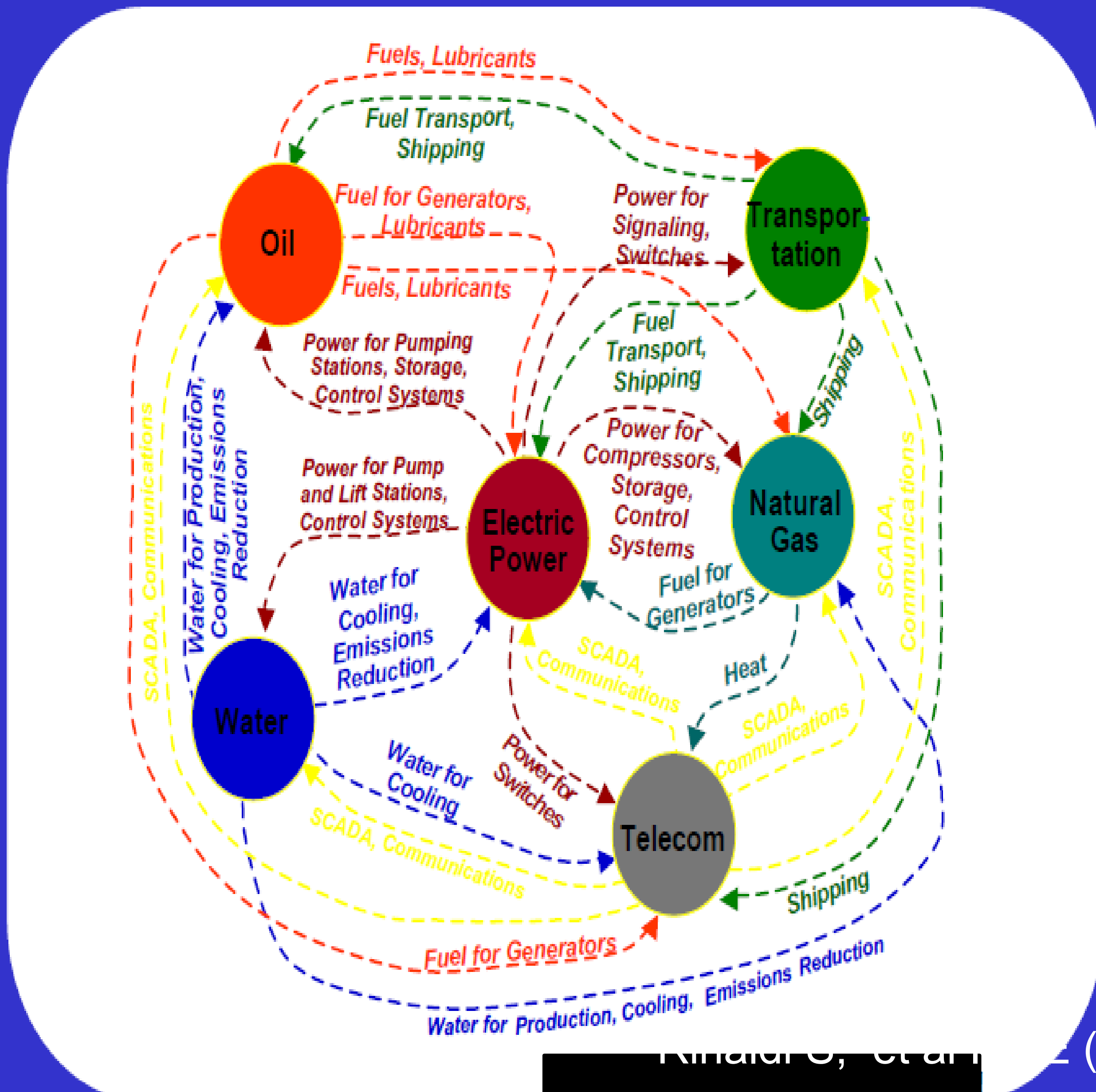
# What is k-core percolation ?

- Initial damage to nodes leads to cascading failures in the network
- The remaining nodes form the k-core

# Interdependent Networks

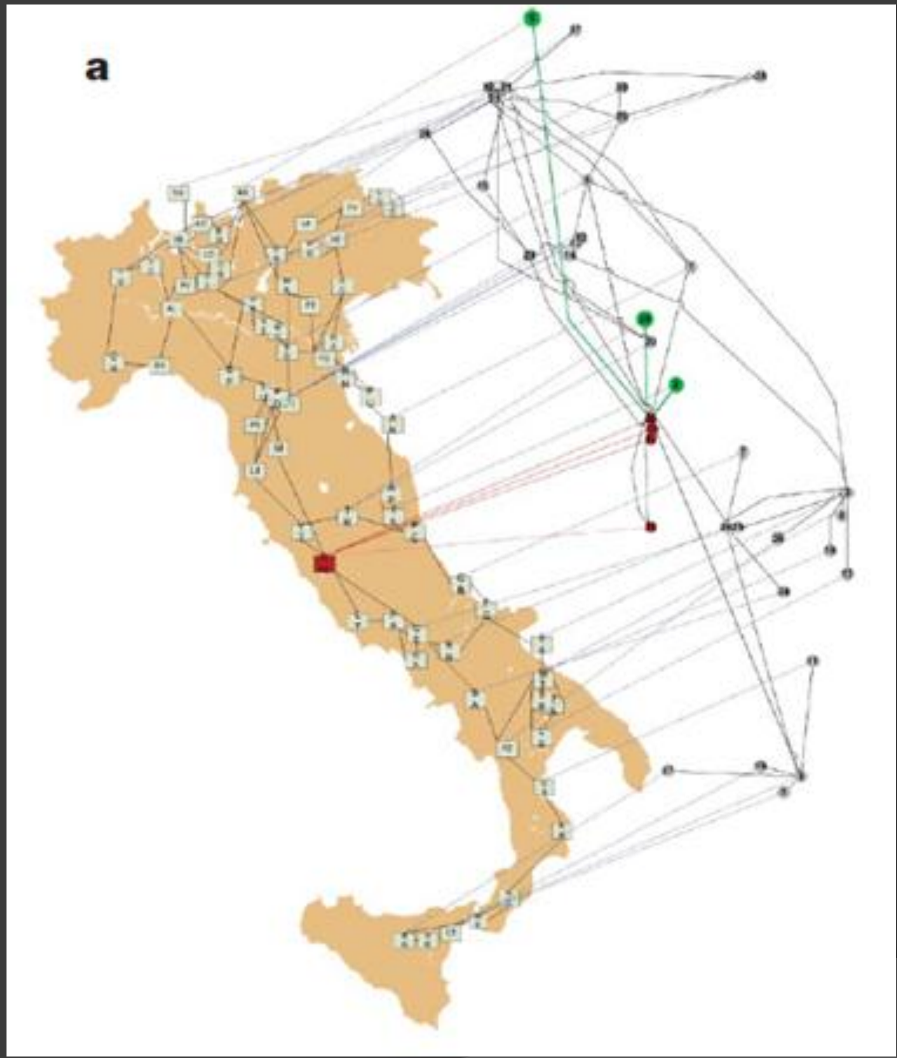
Second reason for cascading failure

# How interdependent are infrastructures?

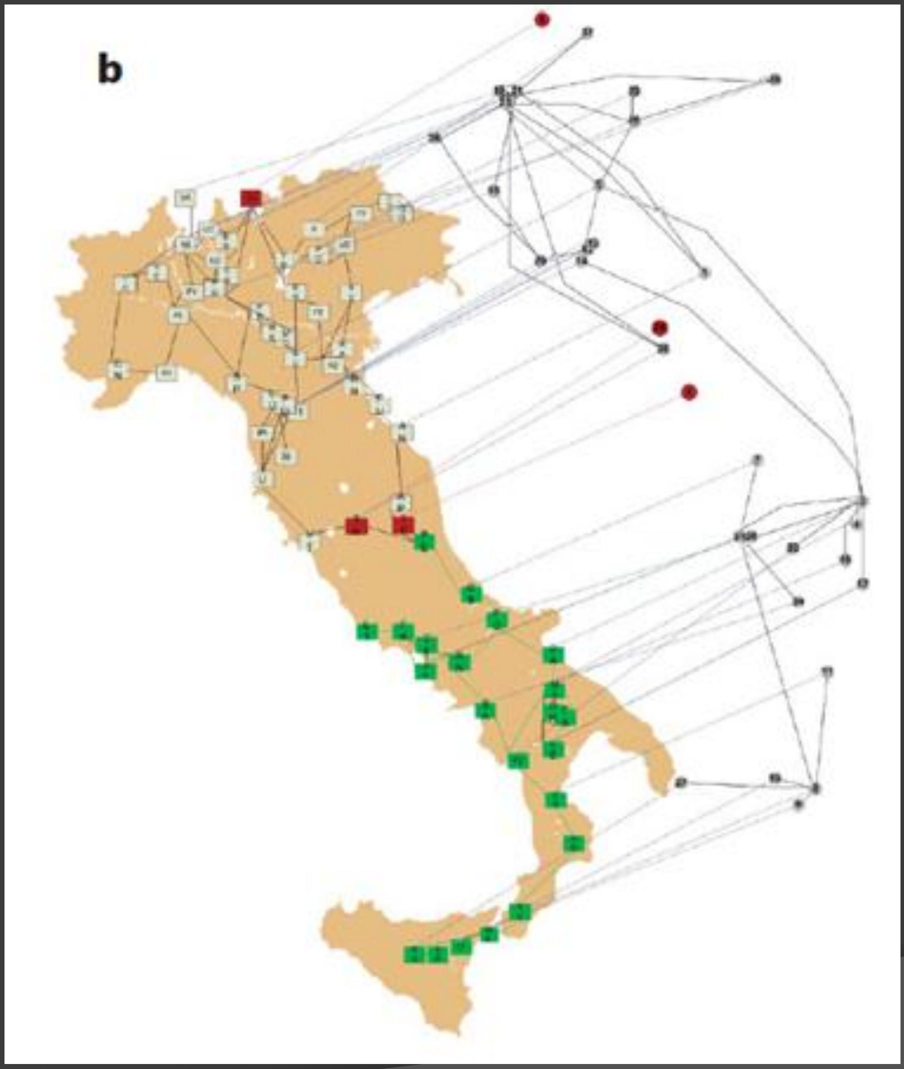



[redacted] (2001)

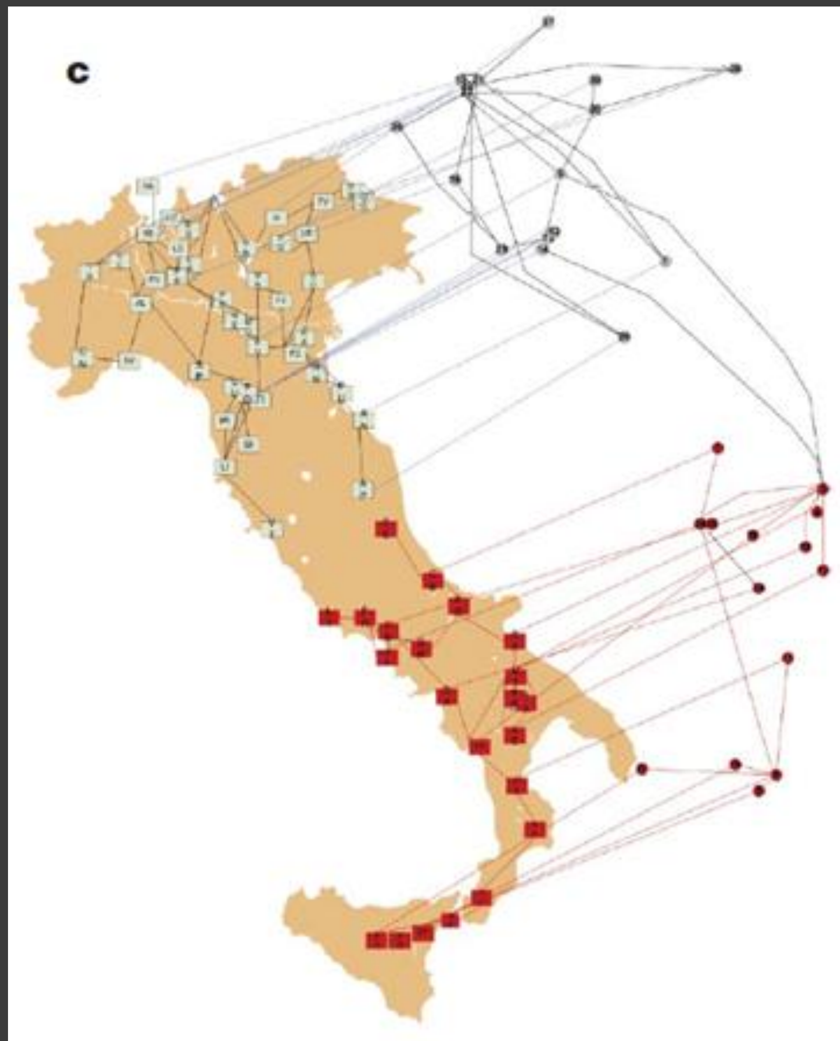
# How does interdependency cause cascade ?



# How does interdependency cause cascade ?



# How does interdependency cause cascade ?

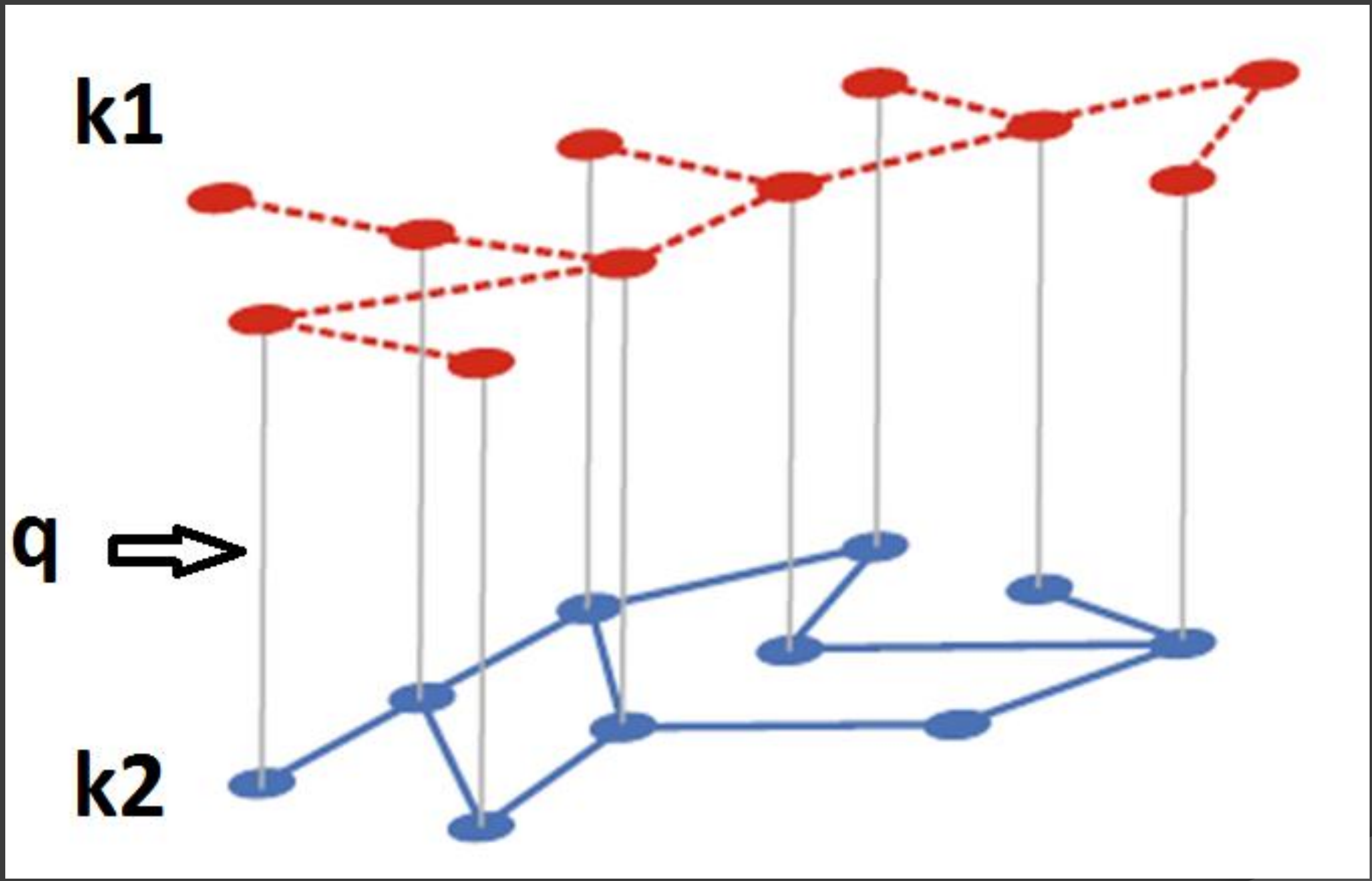


# How do we quantify coupling between networks ?

- Coupling q = fraction of nodes in one network depending on nodes in another network

# K-core Percolation in Interdependent Network

# K-core percolation in interdependent networks

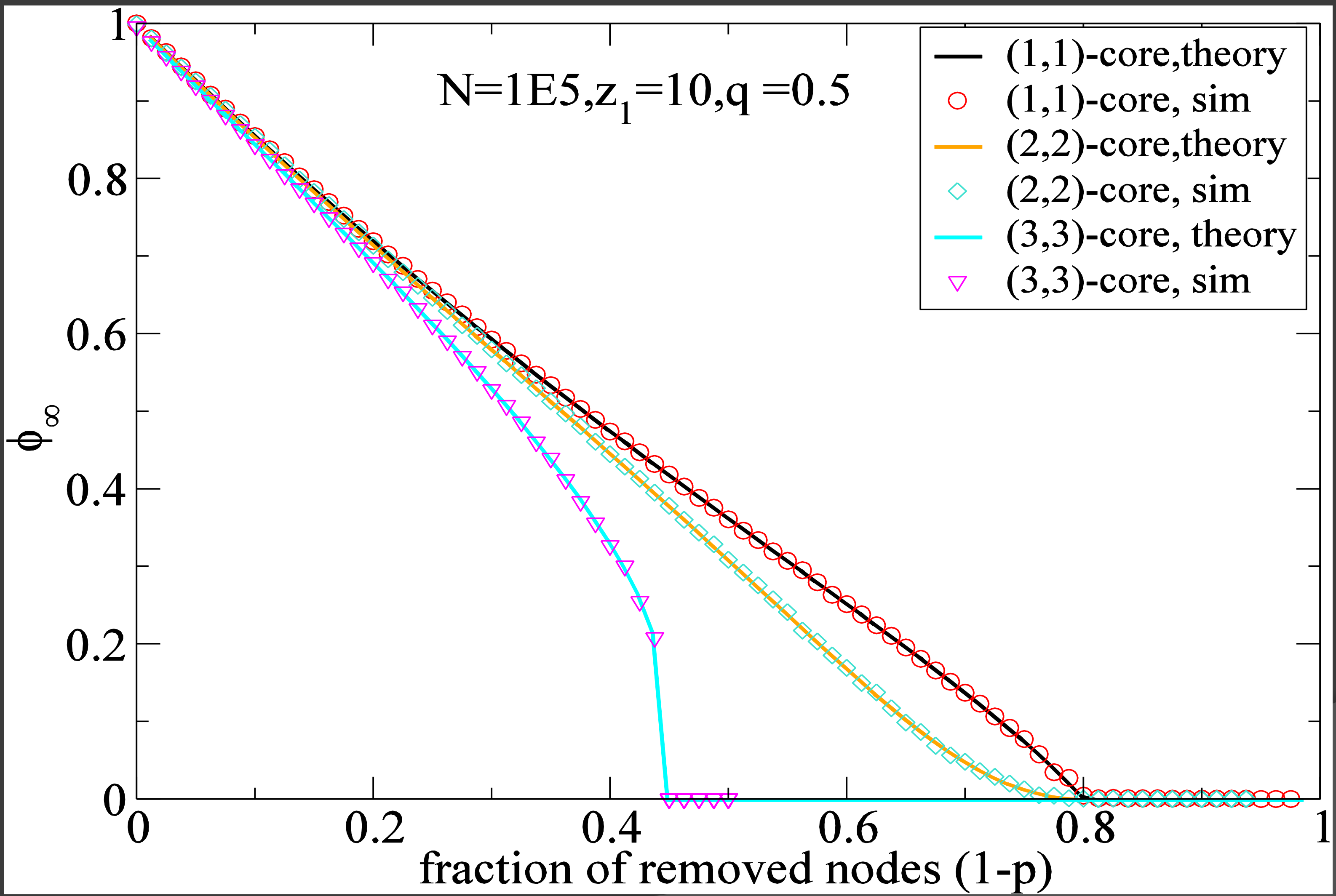


# (k,k)-core percolation

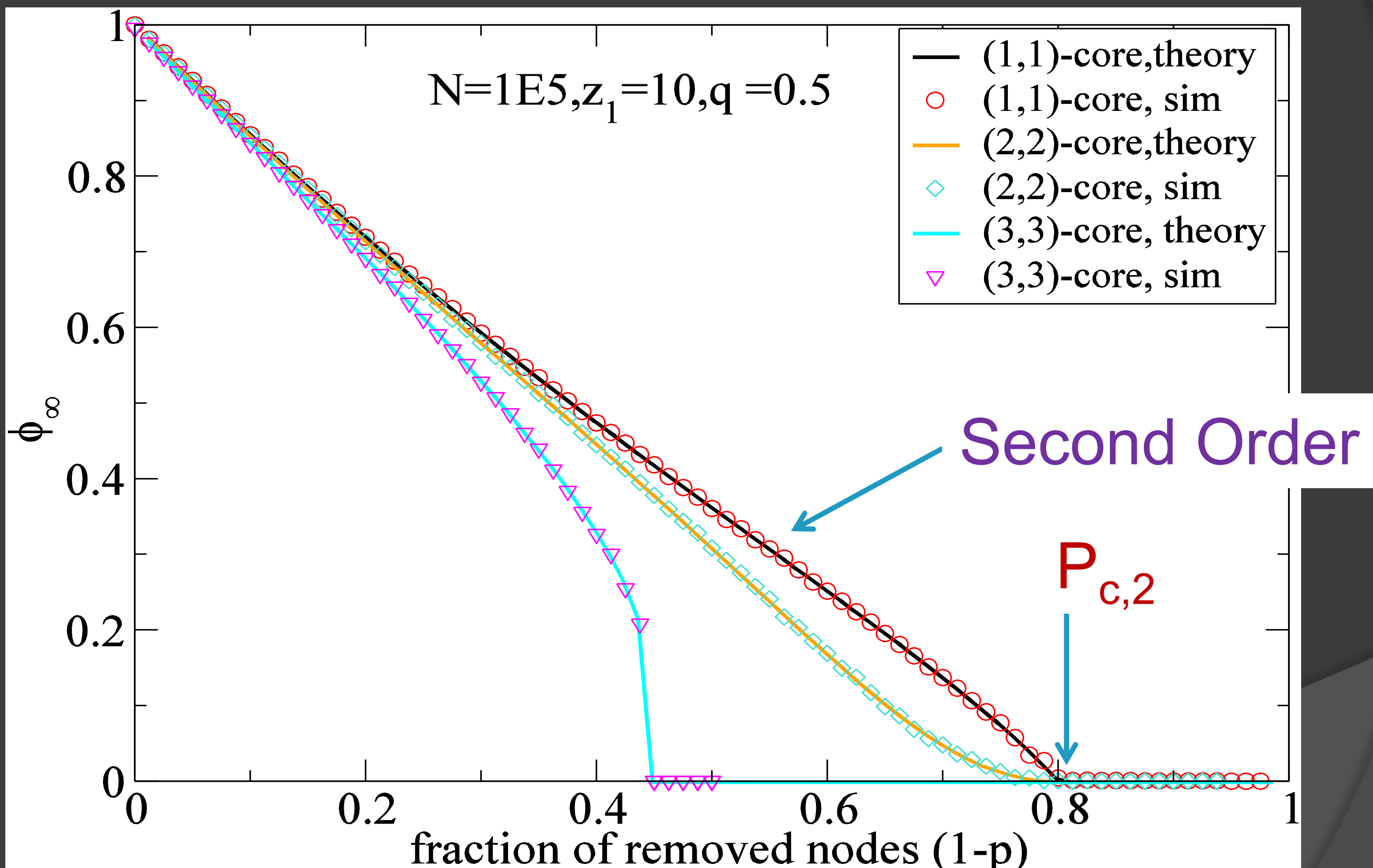
- In this talk, we take the average local threshold ‘k’ to be same for both networks

- Both networks are same as Erdos-Renyi

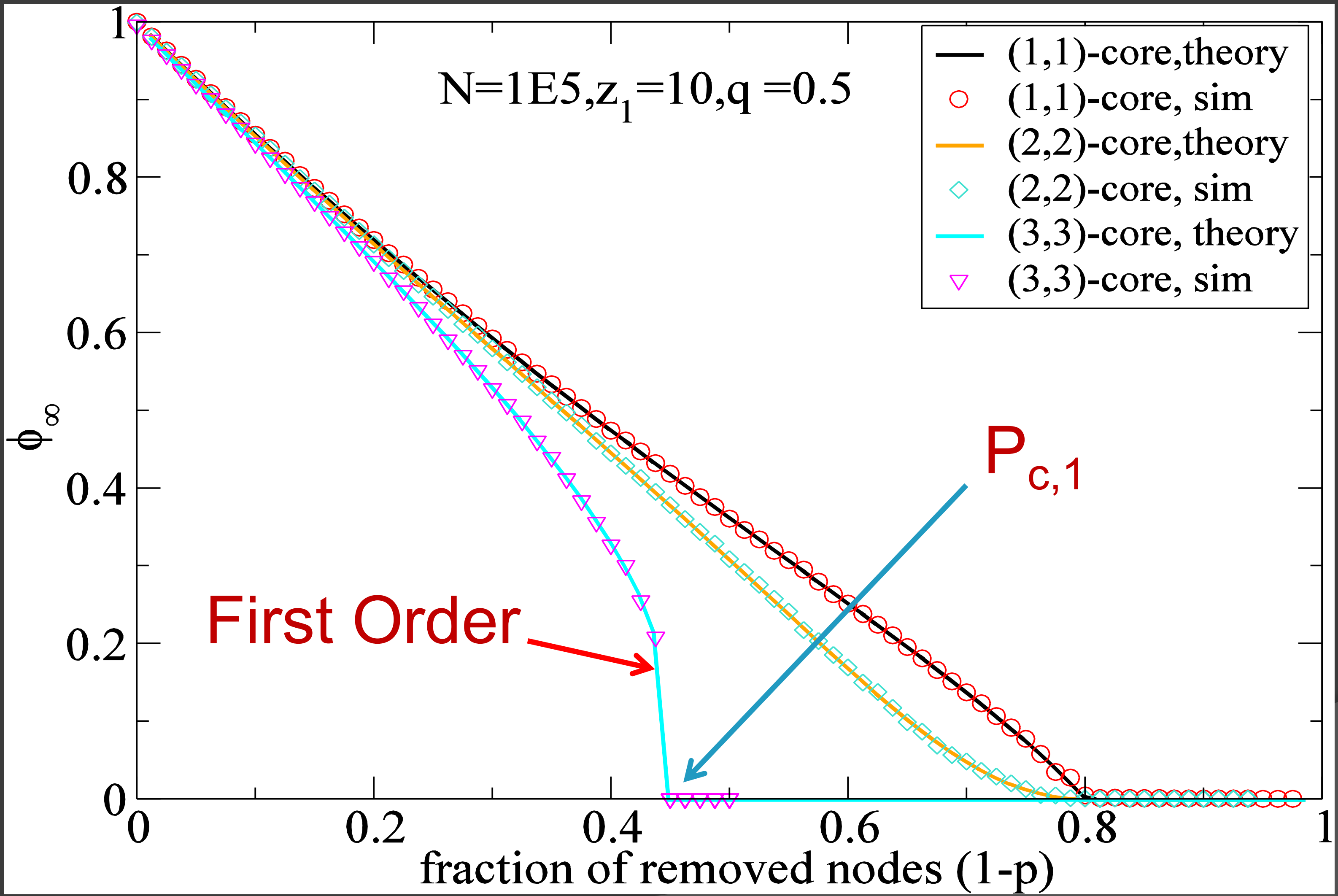
# Simulation vs theory



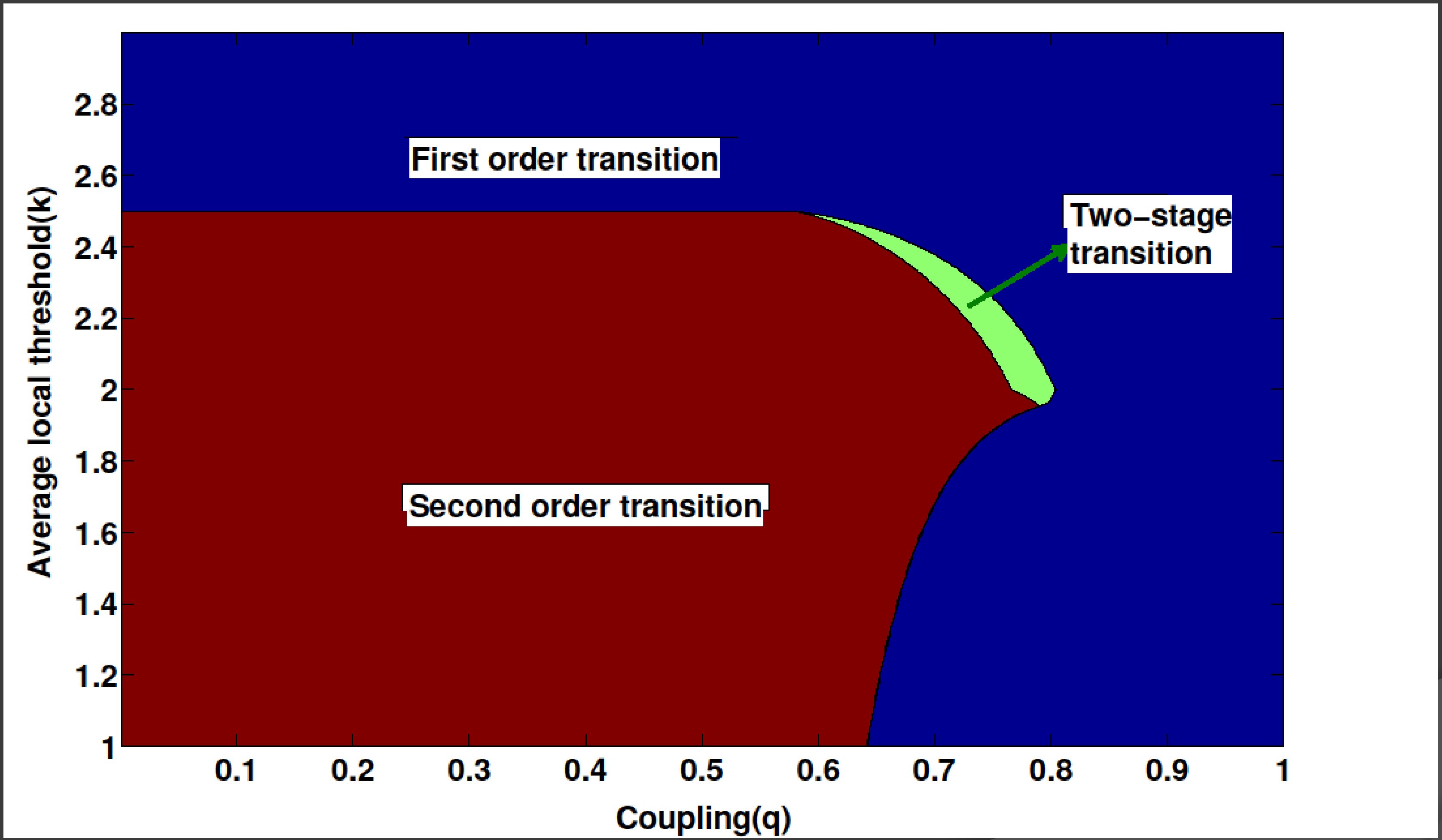
# Second order transition



# First order transition



# Complete Phase diagram



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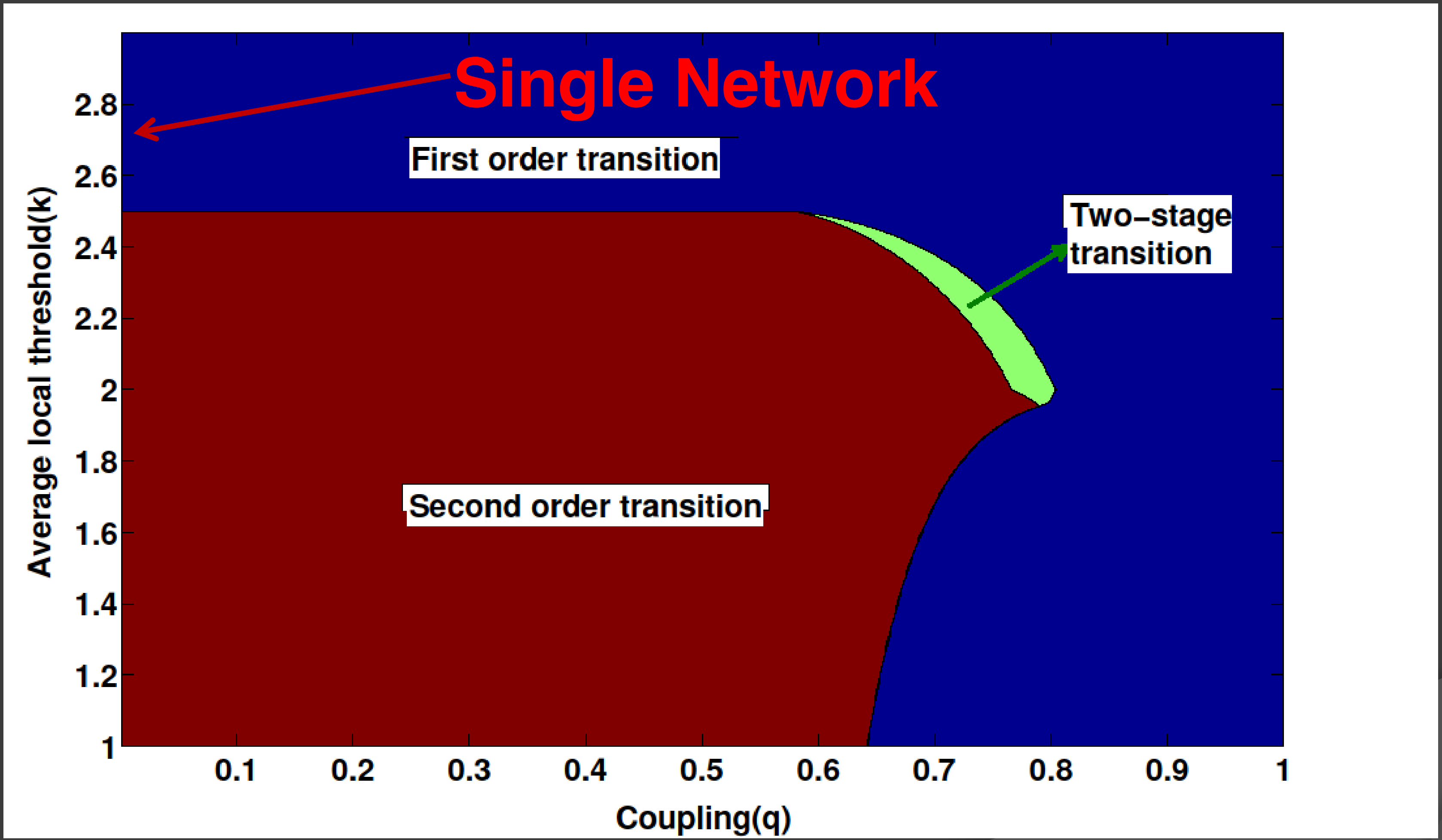
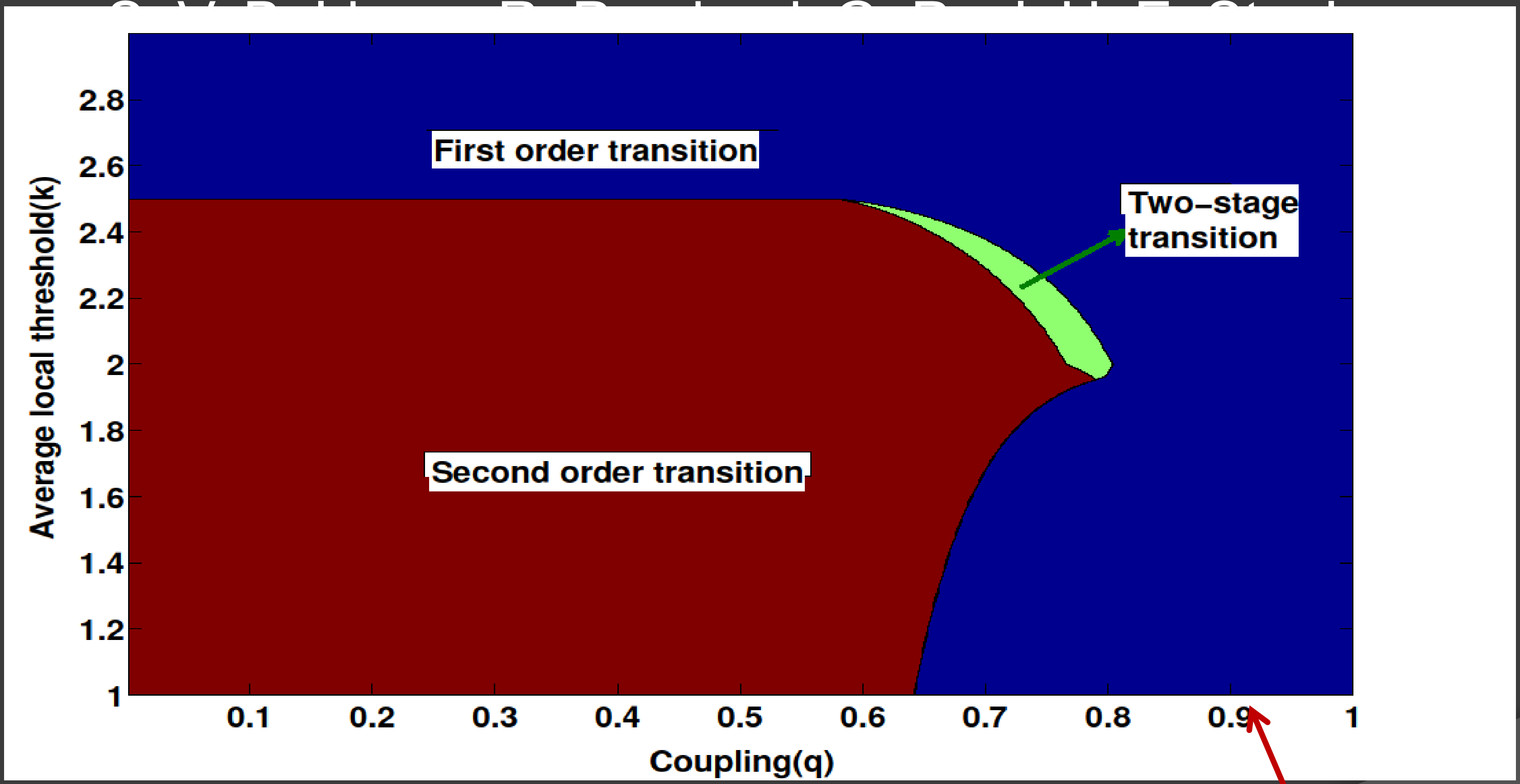
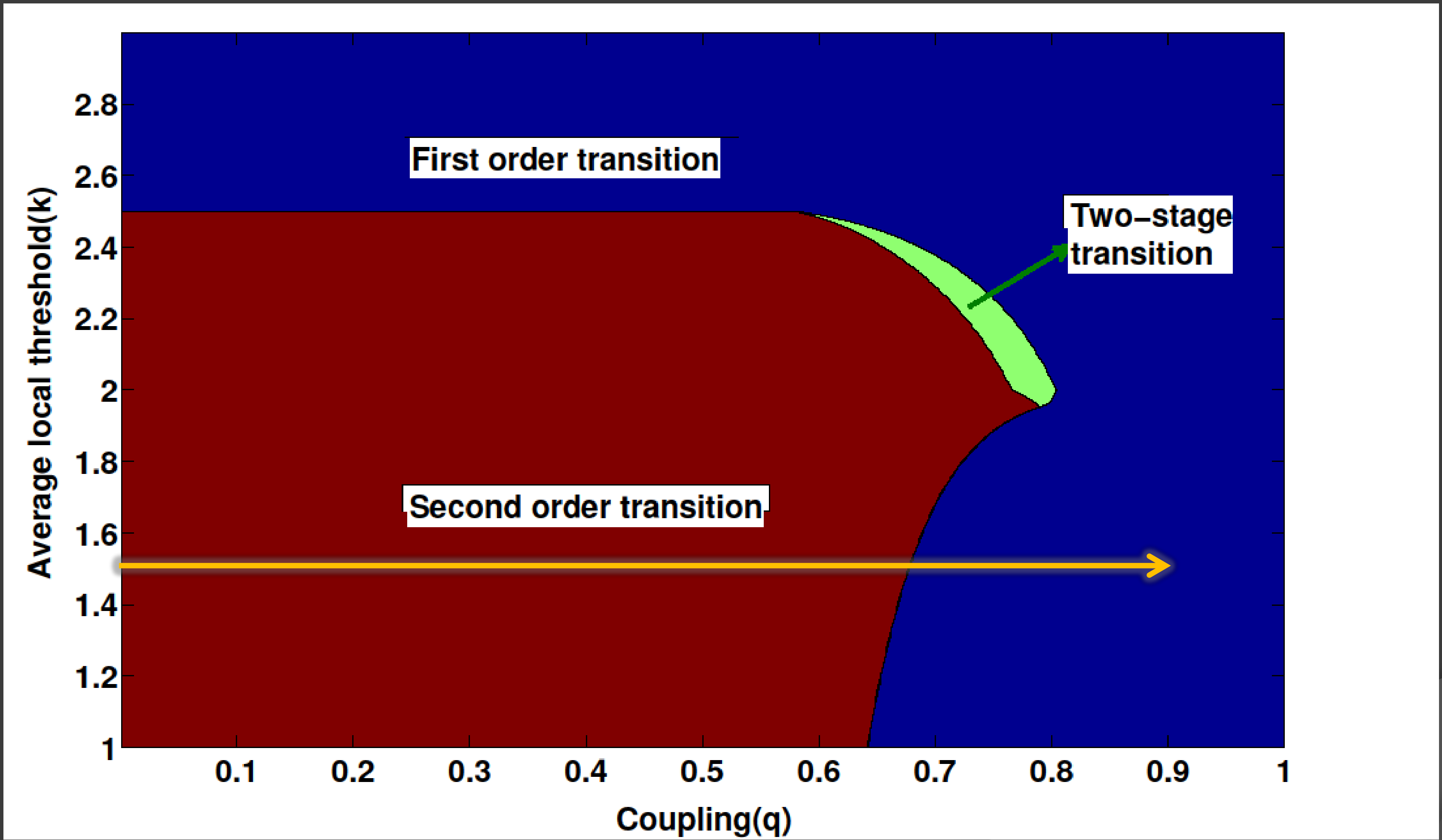


# k = 1 line -> Regular percolation

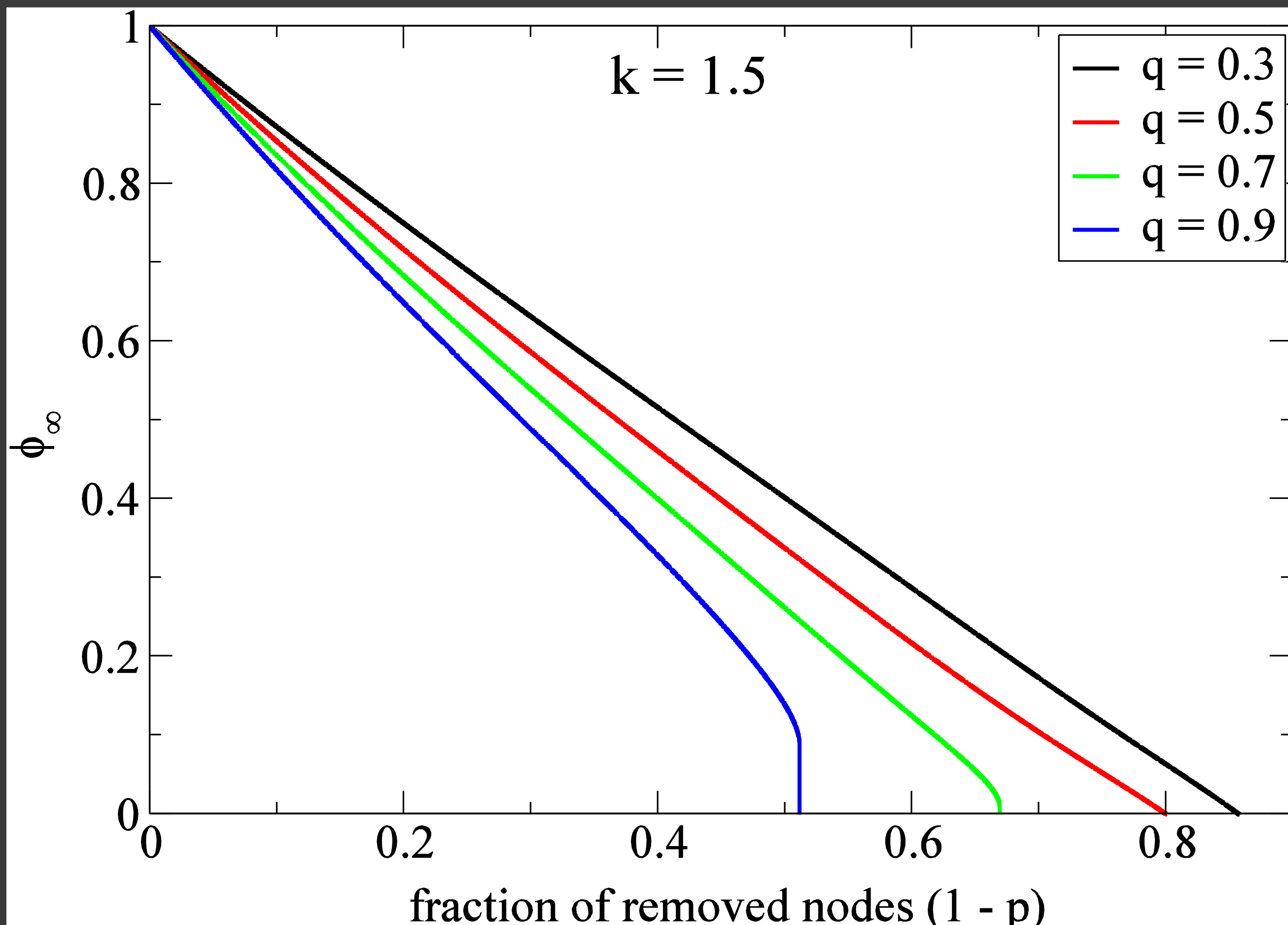


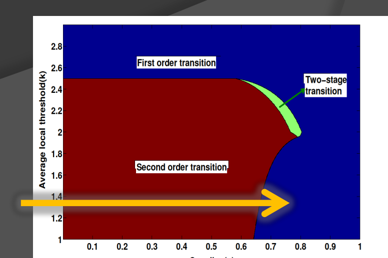

S. V. Buldyrev, R. Parshani, G. Paul, H. E. Stanley,S. Havlin, Nature 464, 1025 (2010)
R. Parshani, S. V. Buldyrev, and S. Havlin, Phys. Rev.Lett. 105, 048701 (2010)

# Complete Phase diagram

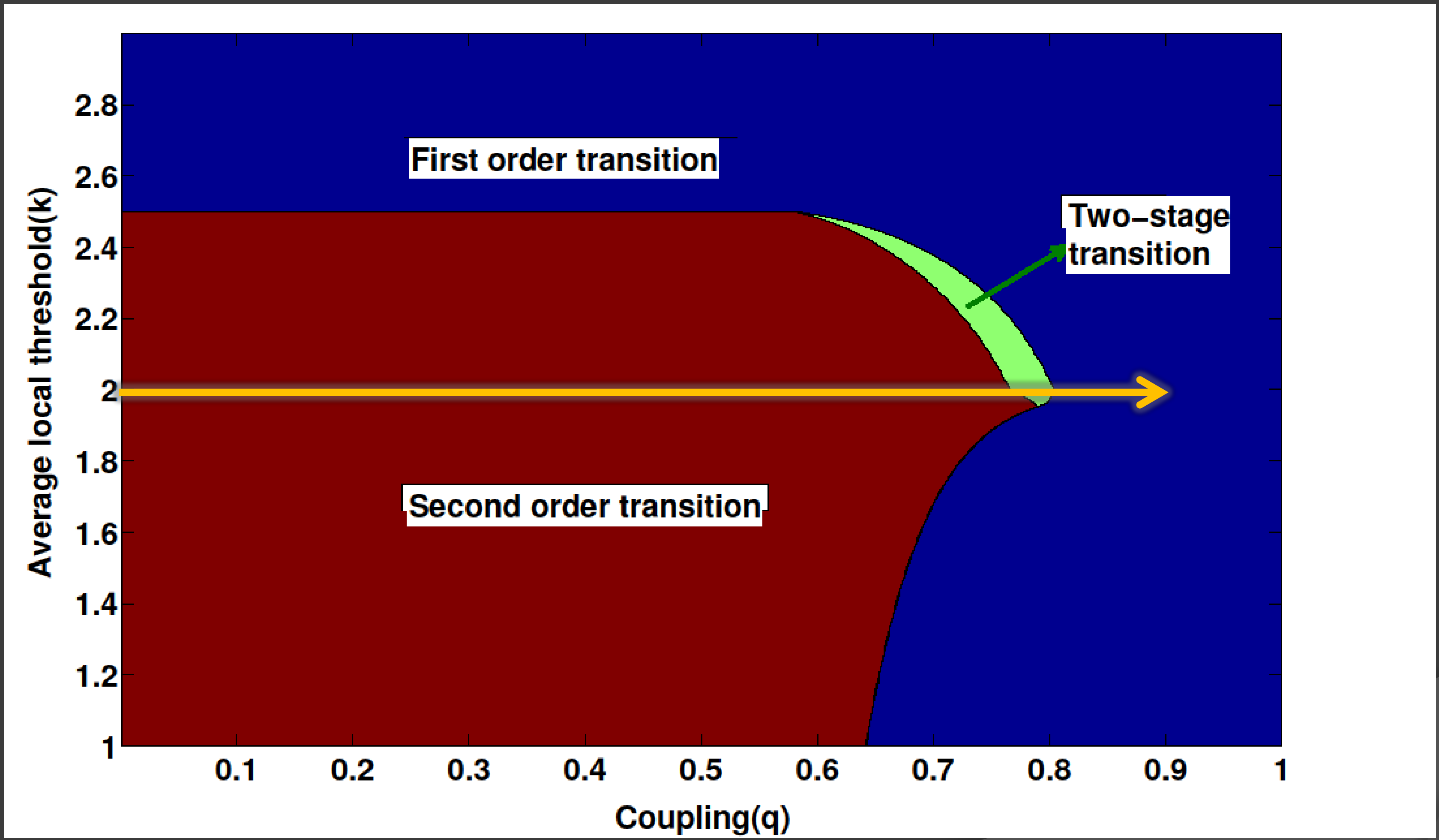


# Threshold k = 1.5, Tricritical point

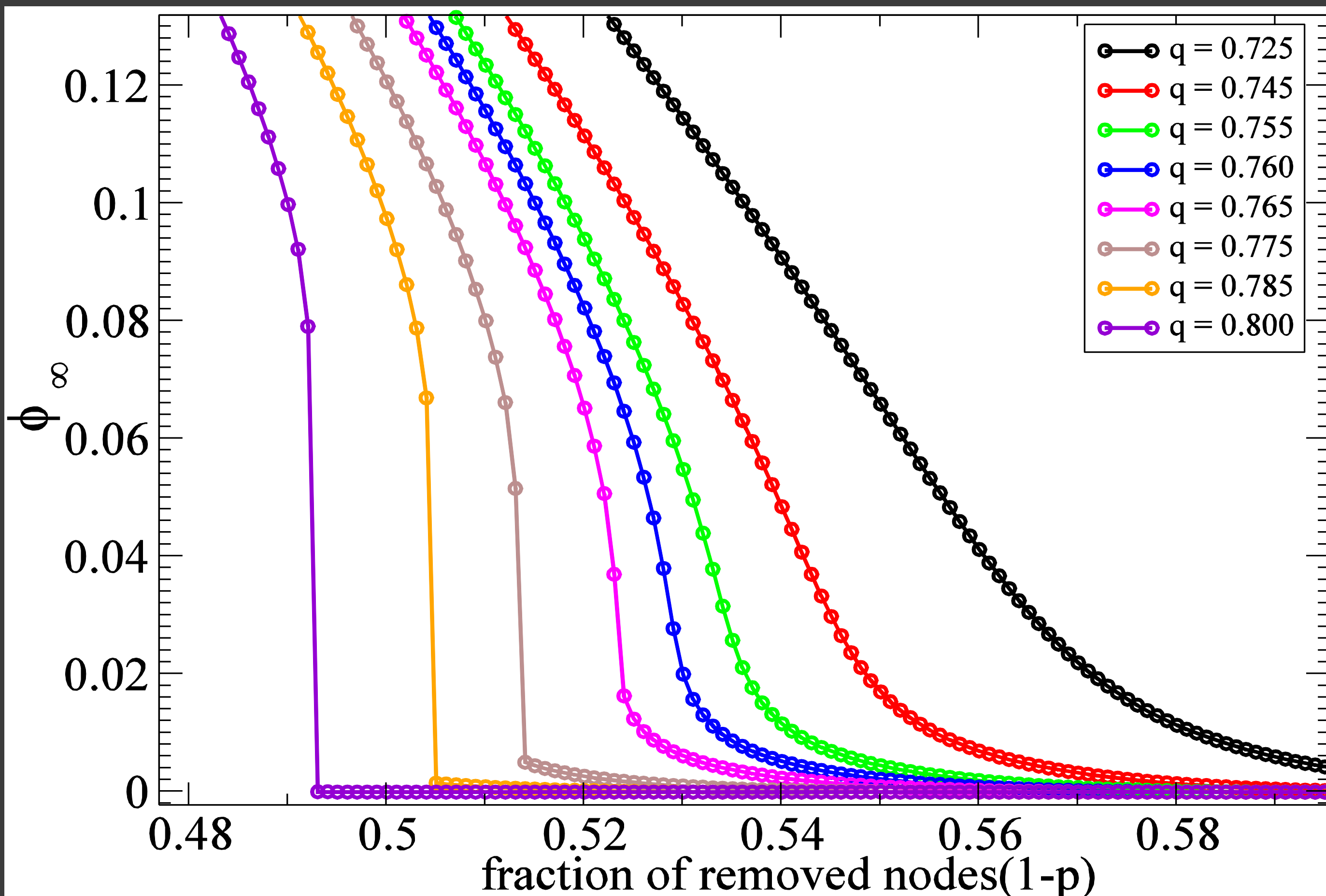


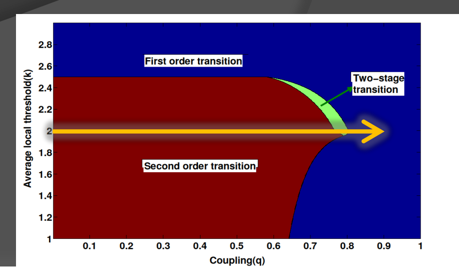



# Complete Phase diagram



# Threshold k = 2.0 (Two-stage)



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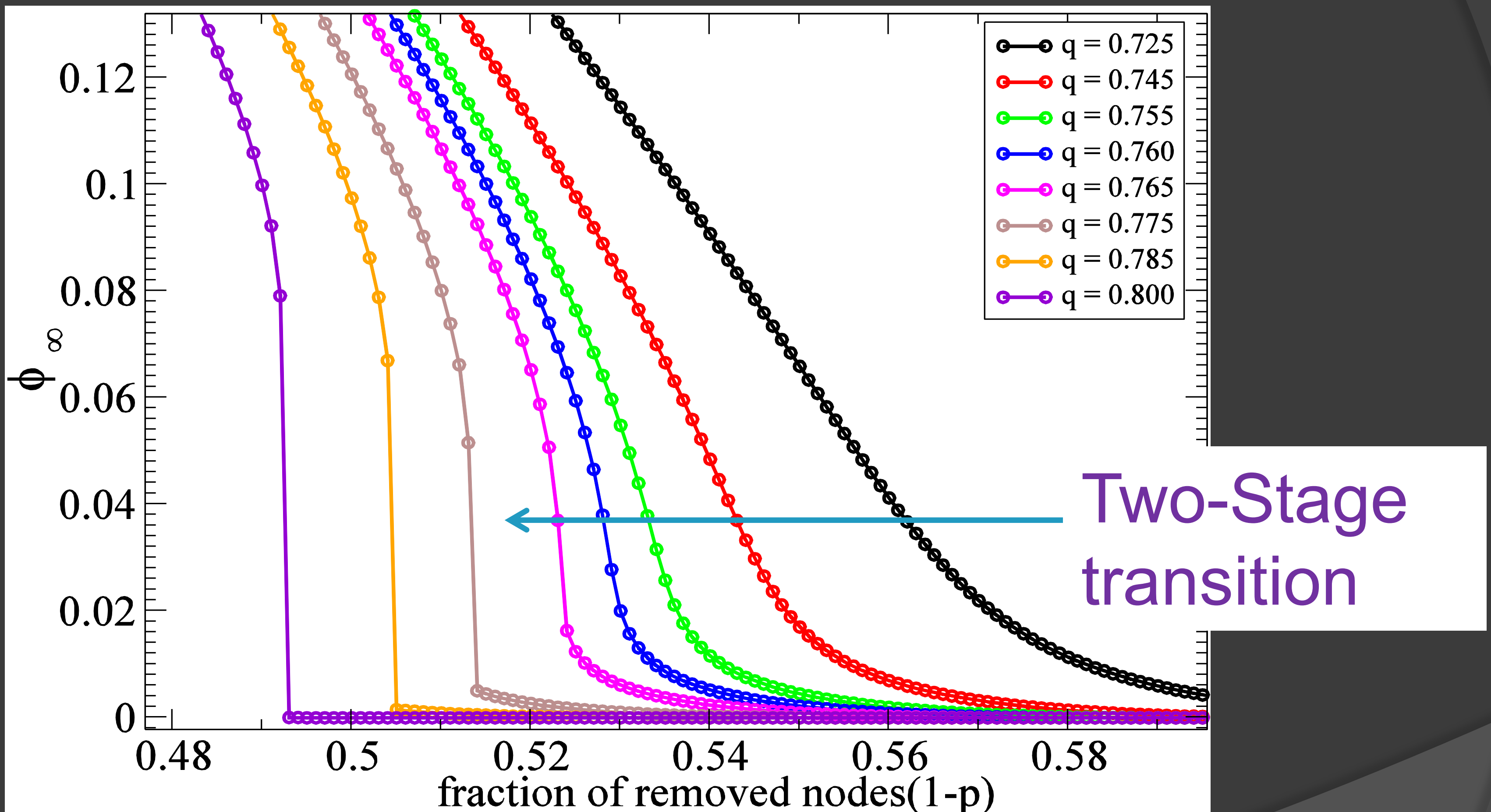
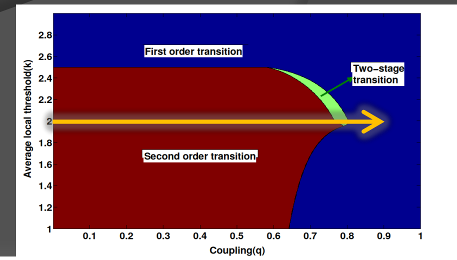
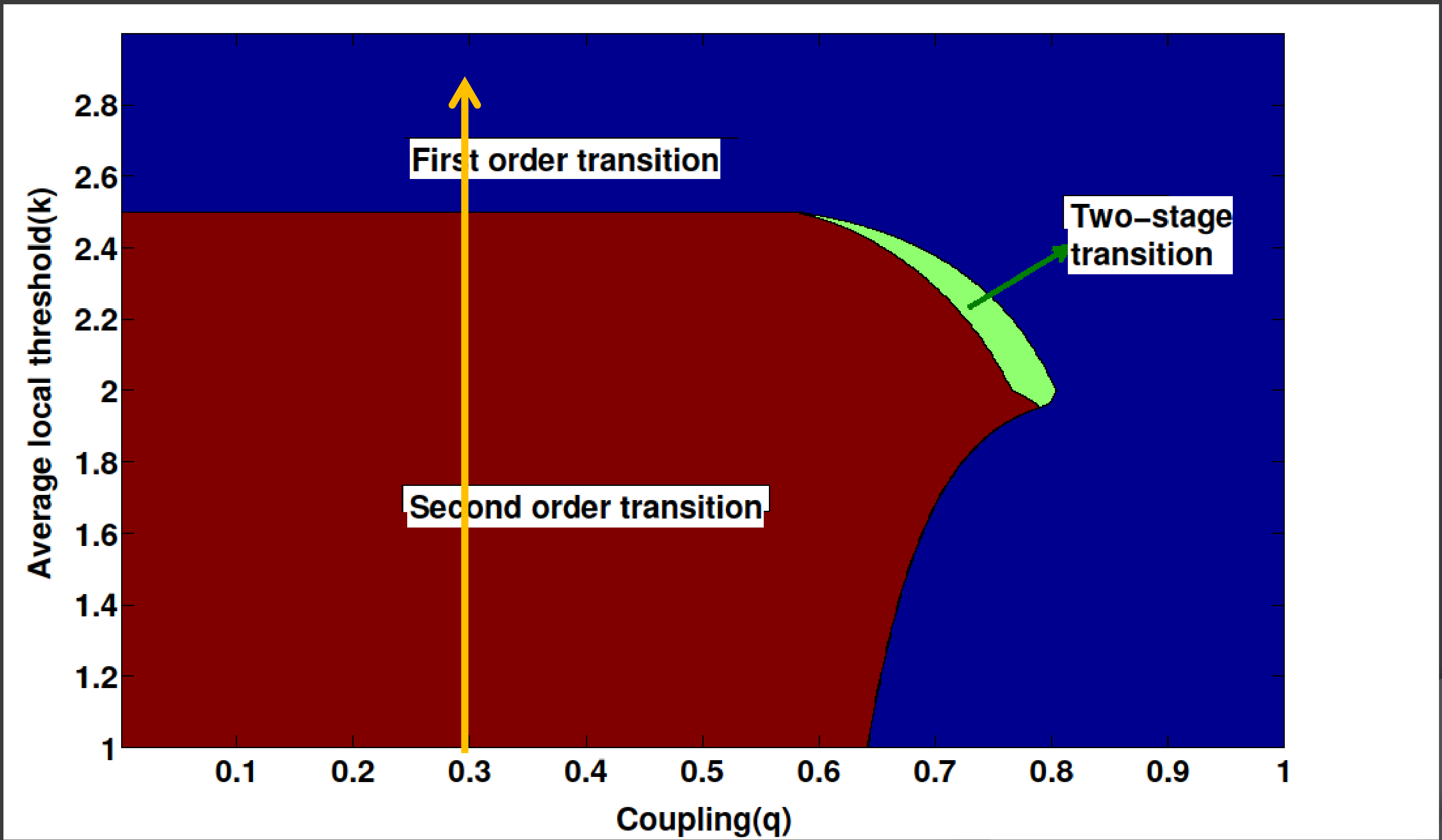
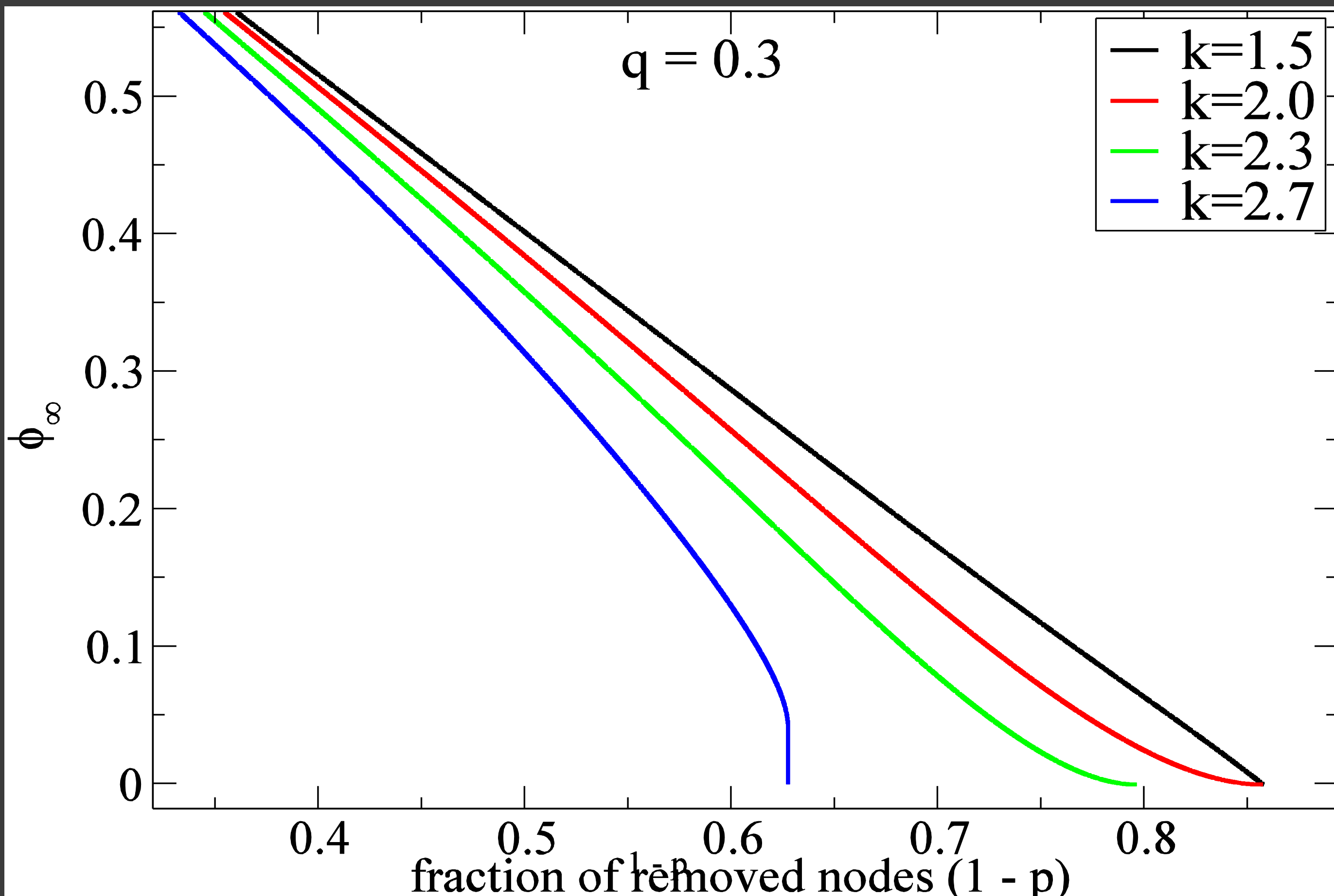


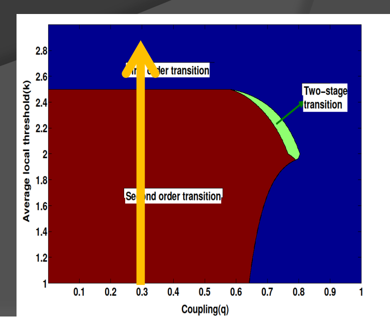



# Complete Phase diagram



# Coupling q = 0.3



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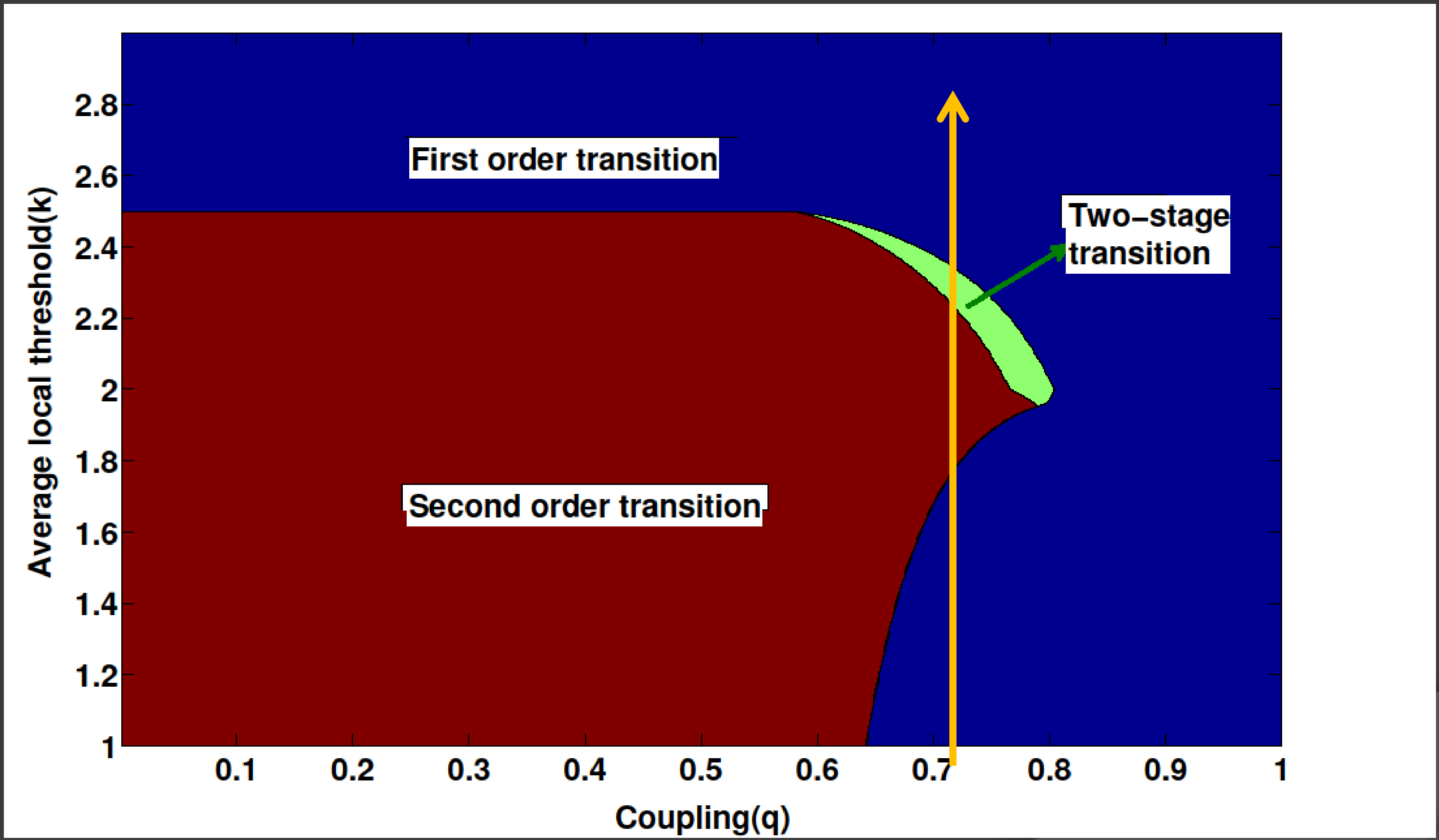
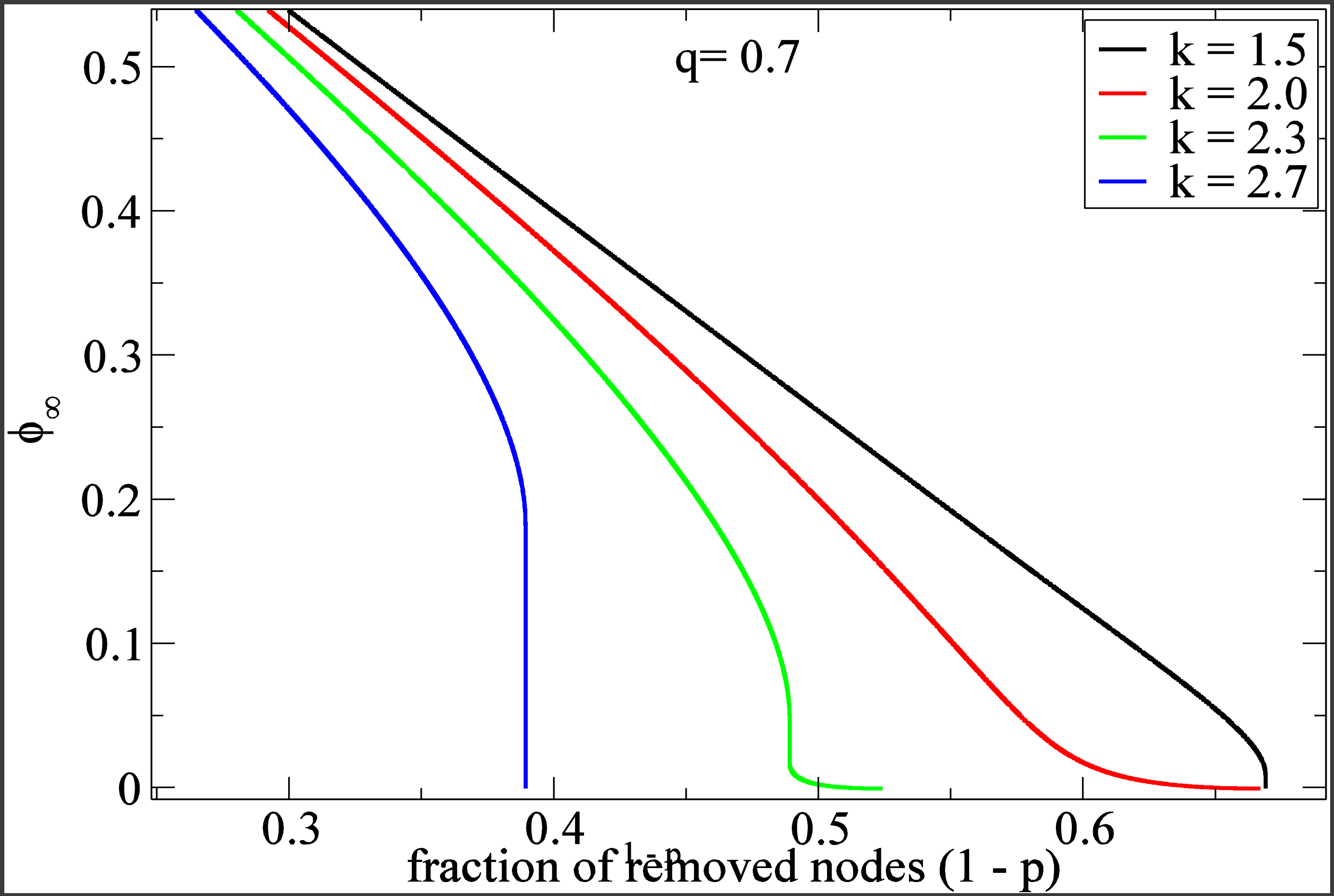
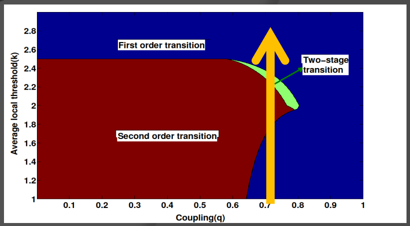
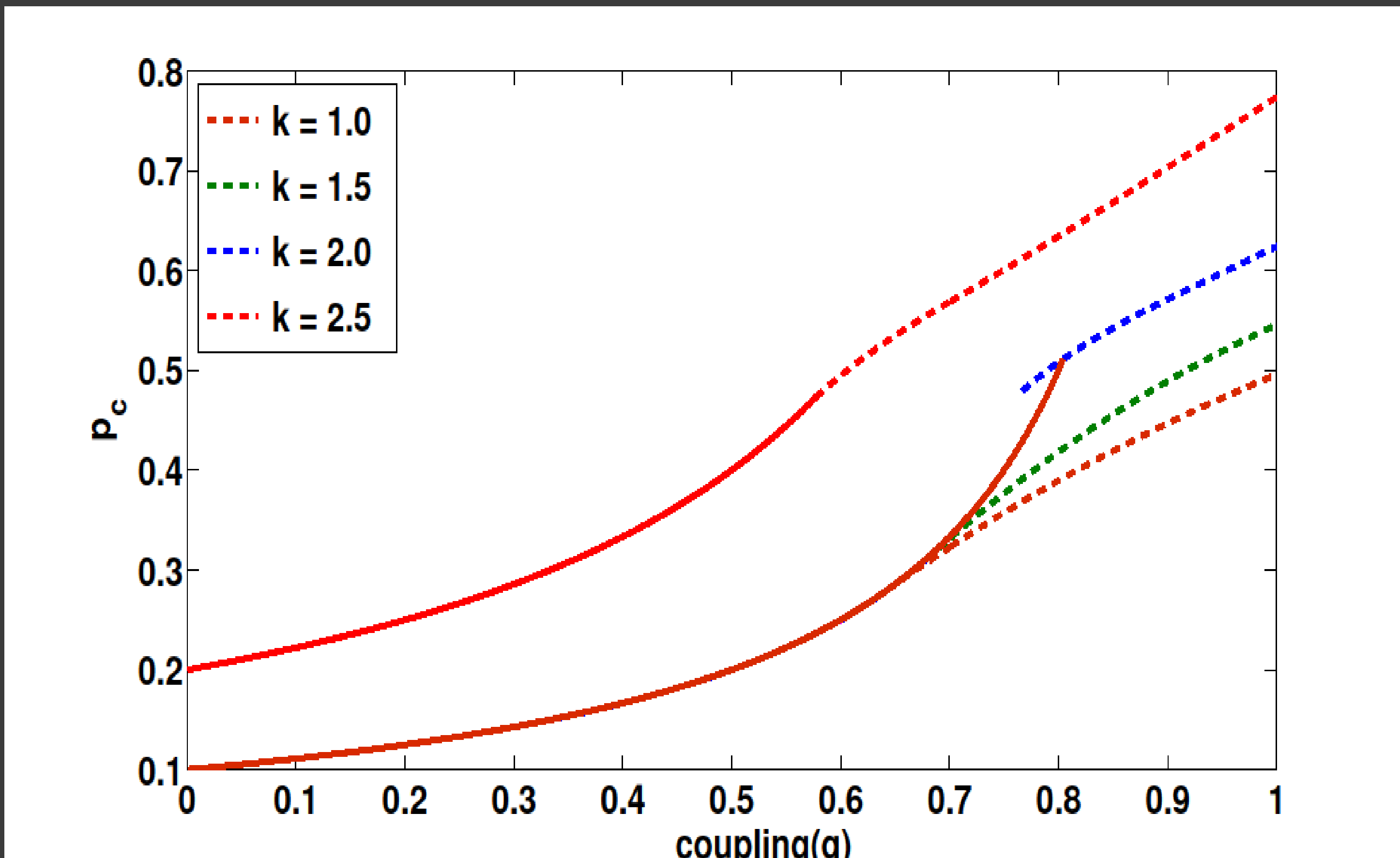


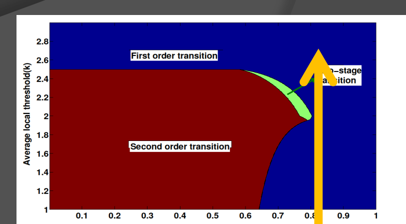
# Coupling q = 0.7



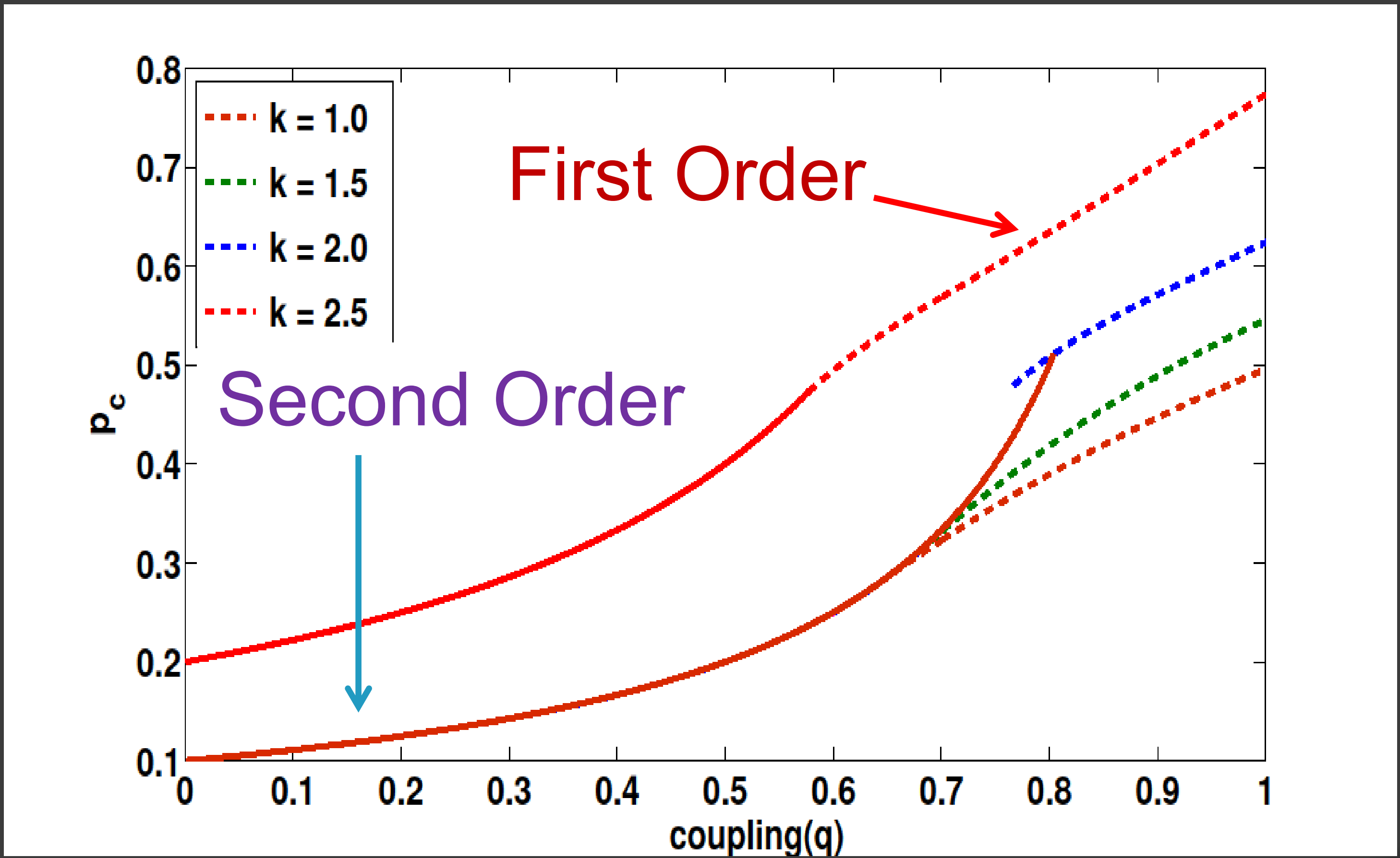



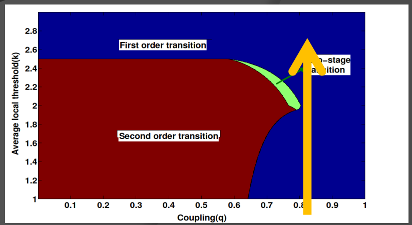
# $p_c$ vs coupling(q)



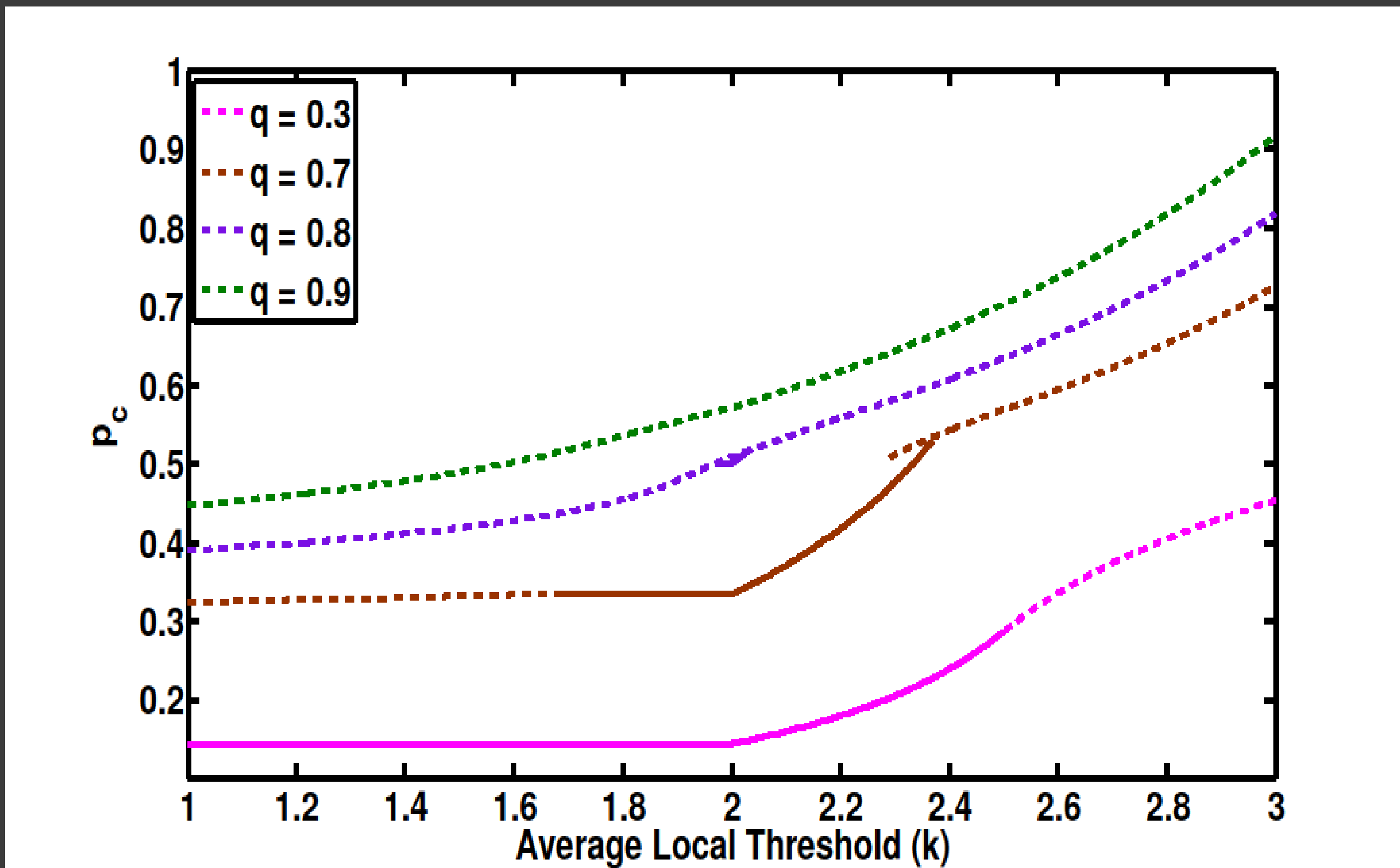



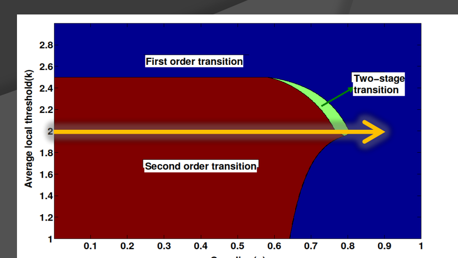
# $p_c$ vs coupling(q)



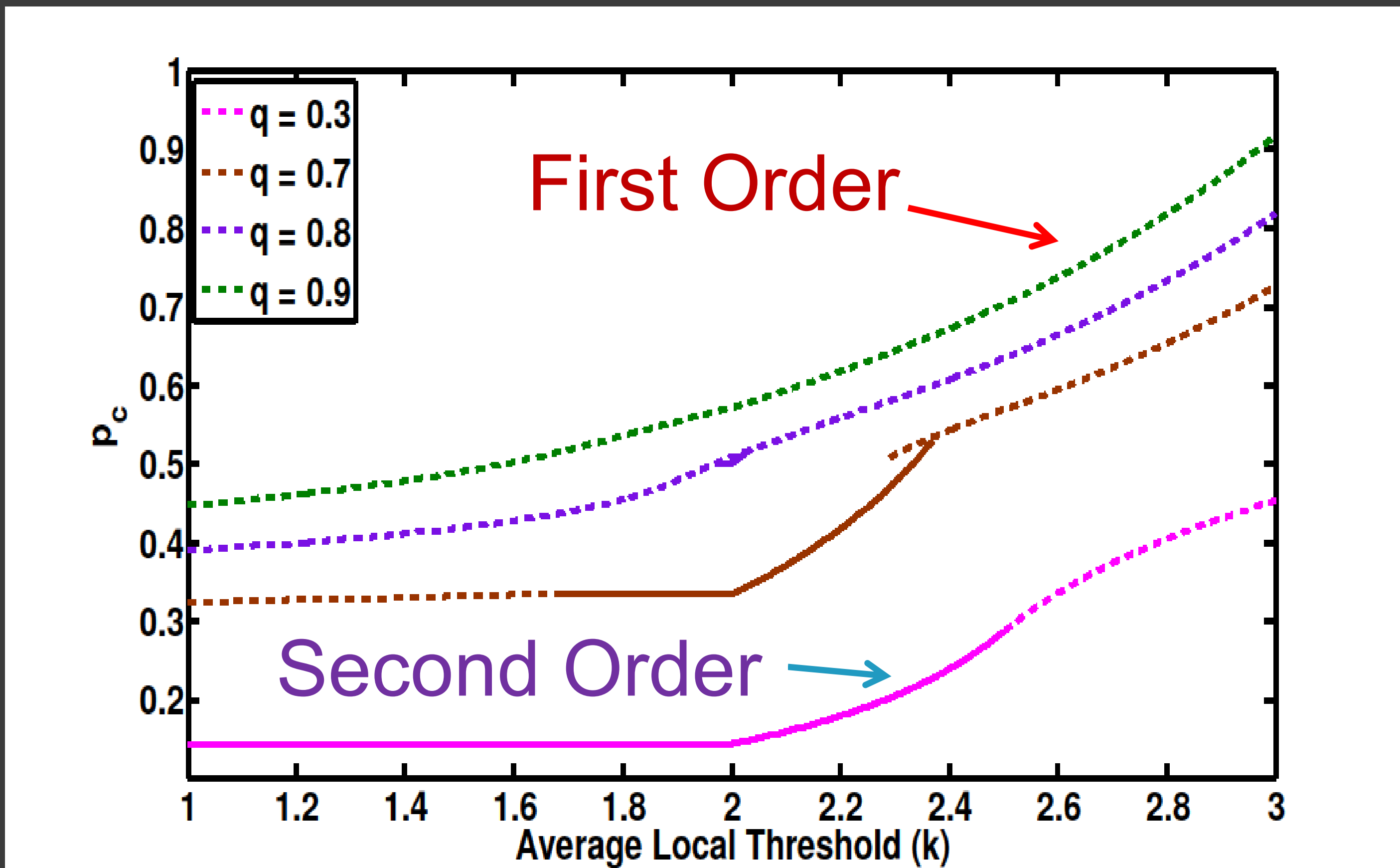



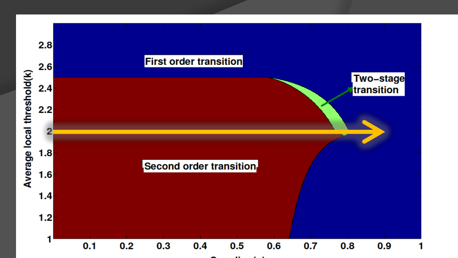
# $p_c$ vs threshold (k)



# $p_c$ vs threshold (k)



# Conclusions

- Combining two models leads to a richer cascading failure properties as shown in the phase diagram
- Understanding the combined effect of k-core percolation and interdependency leads to better design rules for infrastructure networks
- They also help to design a good recovery process to salvage the network in case of failure

# References: k-core in single networks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# References: Interdependent networks


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- [4] N. Azimi-Tafreshi, J. Gomez-Garde nes, and S. N. Dorogovtsev, Phys. Rev. E 90, 032816 (2014).


Thank you!

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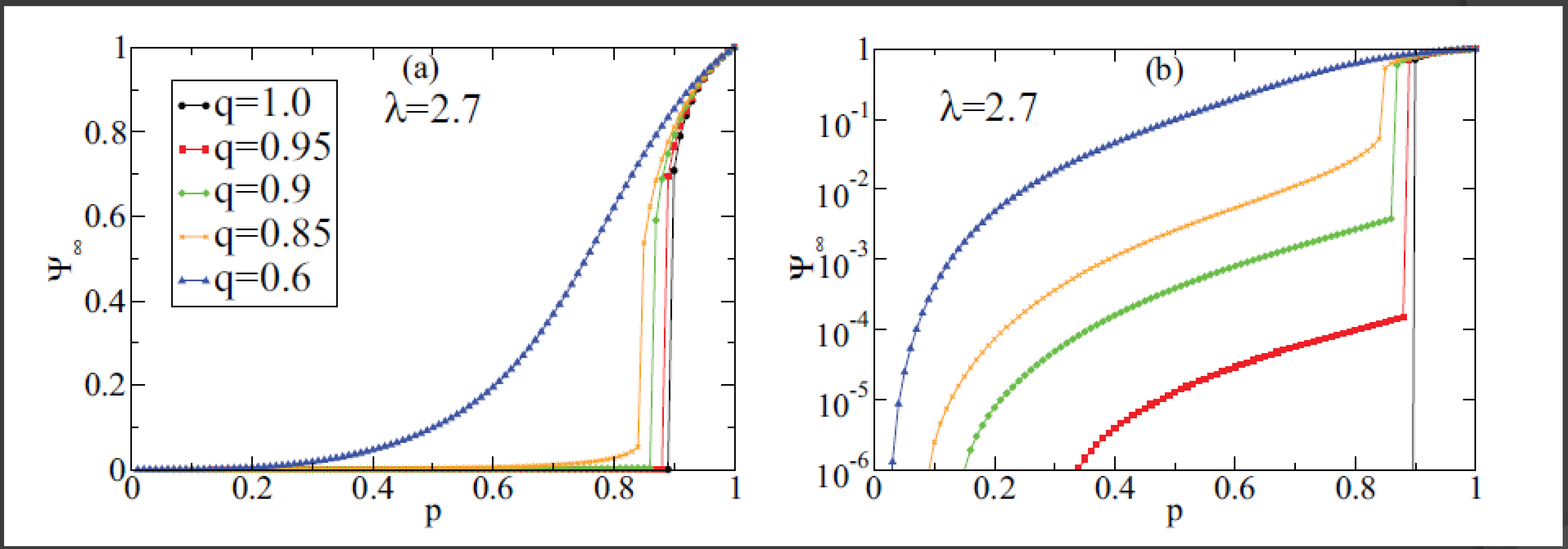
# More results

# SF networks



# Scale free network