CASCADING FAILURES IN COMPLEX SYSTEMS

k-core percolation in interdependent networks

Nagendra Panduranga

Collaborators
Shlomo Havlin
Jianxi Gao
Xin Yuan
Eugene Stanley
Outline

- Cascading failures in Complex networks
  - What are cascading failures?
  - How does complex networks approach it?
- First approach: k-core percolation
  - What is k-core percolation? Cascading failure
  - Integer vs. fractional k-cores?
- Second approach: What are interdependent networks?
  - What is interdependency? How do we quantify it?
- What happens when we combine both? (My work)
US Blackout 2003

Fig. 1.4 A satellite map of the US Northeastern blackout in 2003, (a) before blackout and (b) after blackout. The satellite map represents a real image of the US on August 14, 2003, the night of a major blackout that left an estimated 45 million people in eight U.S. states and another 10 million in Ontario without power.
Cascading failure in Human Physiology

HUMAN BODY
Complex network elements

- Node
- Link
- Degree
- Degree distribution
How do we define failure in complex network?
Random node failure leads to many network fragments
k-core percolation
What is k-core percolation?

- After the initial damage, remove all nodes that do not have at least ‘k’ neighbors:
- 3-core percolation means all nodes that have fewer than 3 neighbors are removed
- 3 is the local threshold
Example: 3-core percolation
Example: 3-core percolation
Example: 3-core percolation
Example: 3-core percolation
Example: 3-core percolation
Example: 3-core percolation
What is k-core percolation?

- Initial damage to nodes leads to cascading failures in the network.
- The remaining nodes form the k-core.
Interdependent Networks

Second reason for cascading failure
How interdependent are infrastructures?
How does interdependency cause cascade?
How does interdependency cause cascade?
How does interdependency cause cascade?
How do we quantify coupling between networks?

- Coupling $q = \text{fraction of nodes in one network depending on nodes in another network}$
K-core Percolation in Interdependent Network
K-core percolation in interdependent networks
(k,k)-core percolation

- In this talk, we take the average local threshold ‘k’ to be same for both networks
- Both networks are same as Erdos-Renyi
Simulation vs theory

N=1E5, z_1 = 10, q = 0.5

fraction of removed nodes (1-p)

\( \phi^8 \)
Second order transition

\[ N=10^5, z_1=10, q=0.5 \]

Fraction of removed nodes \( (1-p) \)

\( \Phi^8 \)

Second Order

\( P_{c,2} \)
First order transition

N=1E5, z₁=10, q = 0.5

First Order

fraction of removed nodes (1-p)

$\phi^8$
Complete Phase diagram

- First order transition
- Second order transition
- Two-stage transition
Complete Phase diagram

Single Network

First order transition

Second order transition

Two-stage transition
$k = 1$ line $\rightarrow$ Regular percolation

Complete Phase diagram

- First order transition
- Second order transition
- Two-stage transition
Threshold $k = 1.5$, Tricritical point
Complete Phase diagram

- **First order transition**
- **Second order transition**
- **Two-stage transition**
Threshold $k = 2.0$ (Two-stage)
Threshold $k = 2.0$ (Two-stage)

Two-Stage transition
Complete Phase diagram
Coupling $q = 0.3$
Complete Phase diagram

- First order transition
- Second order transition
- Two-stage transition
Coupling $q = 0.7$
$p_c$ vs coupling(q)
$p_c$ vs coupling($q$)
$p_c$ vs threshold ($k$)
$p_c$ vs threshold ($k$)

First Order

Second Order
Conclusions

- Combining two models leads to a richer cascading failure properties as shown in the phase diagram

- Understanding the combined effect of k-core percolation and interdependency leads to better design rules for infrastructure networks

- They also help to design a good recovery process to salvage the network in case of failure
References: k-core in single networks

References: Interdependent networks

Thank you!
Back up Slides

More results
SF networks
Scale free network

N=1E5, γ=2.5, q =0.5

- (1,1)-core, theory
- (1,1)-core, sim
- (2,2)-core, theory
- (2,2)-core, sim
- (3,3)-core, theory
- (3,3)-core, sim

fraction of removed nodes (1-p)