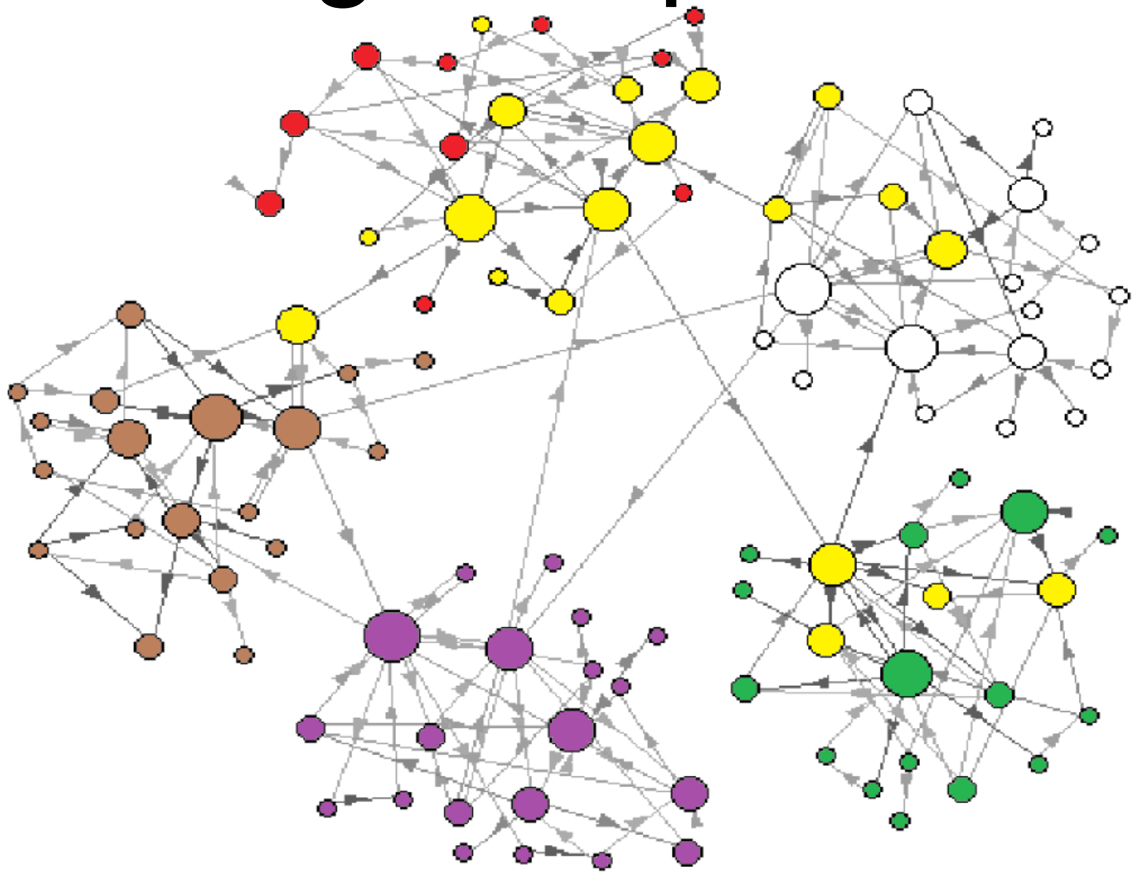
Epidemics in Dynamic and Interacting Complex Networks



Mark Dickison

April 9, 2012

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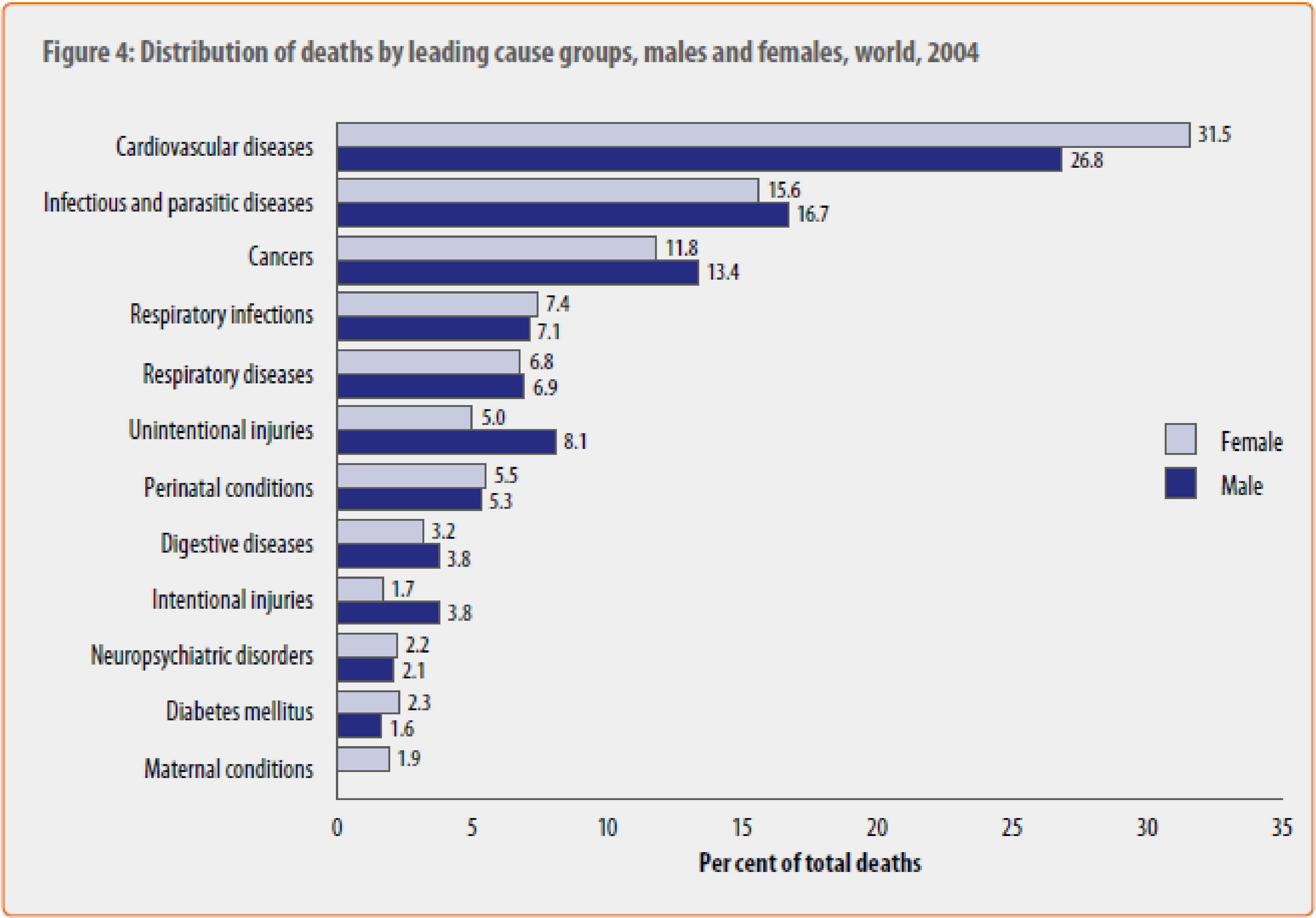
Dept. of Physics
Boston University

Outline

- Motivation
- Networks, our Disease Model and Epidemics
- Epidemics in Dynamically Quarantined Networks
- Epidemics in Interacting Networks
- Conclusions and Ongoing Work

Motivation



Source: WHO, Global Burden of Disease (2008)

Motivation

Table 5. Estimated Costs of Selected Virus and Worm Attacks, 1999-2003

(in billions of dollars)

Attack	Year	Mi2g
SoBig	2003	30.91
Slammer	2003	1.05
Klez	2002	14.89
BadTrans	2002	0.68
Bugbear	2002	2.70
Nimda	2001	0.68
Code Red	2001	2.62
Sir Cam	2001	2.27
Love Bug	2000	8.75
Melissa	1999	1.11

Source: CRS Report for Congress (2004)

Motivation

The Big Question(s)

How effective is quarantine
at preventing epidemics?

How does a network of networks
structure impact the spread of
epidemics?

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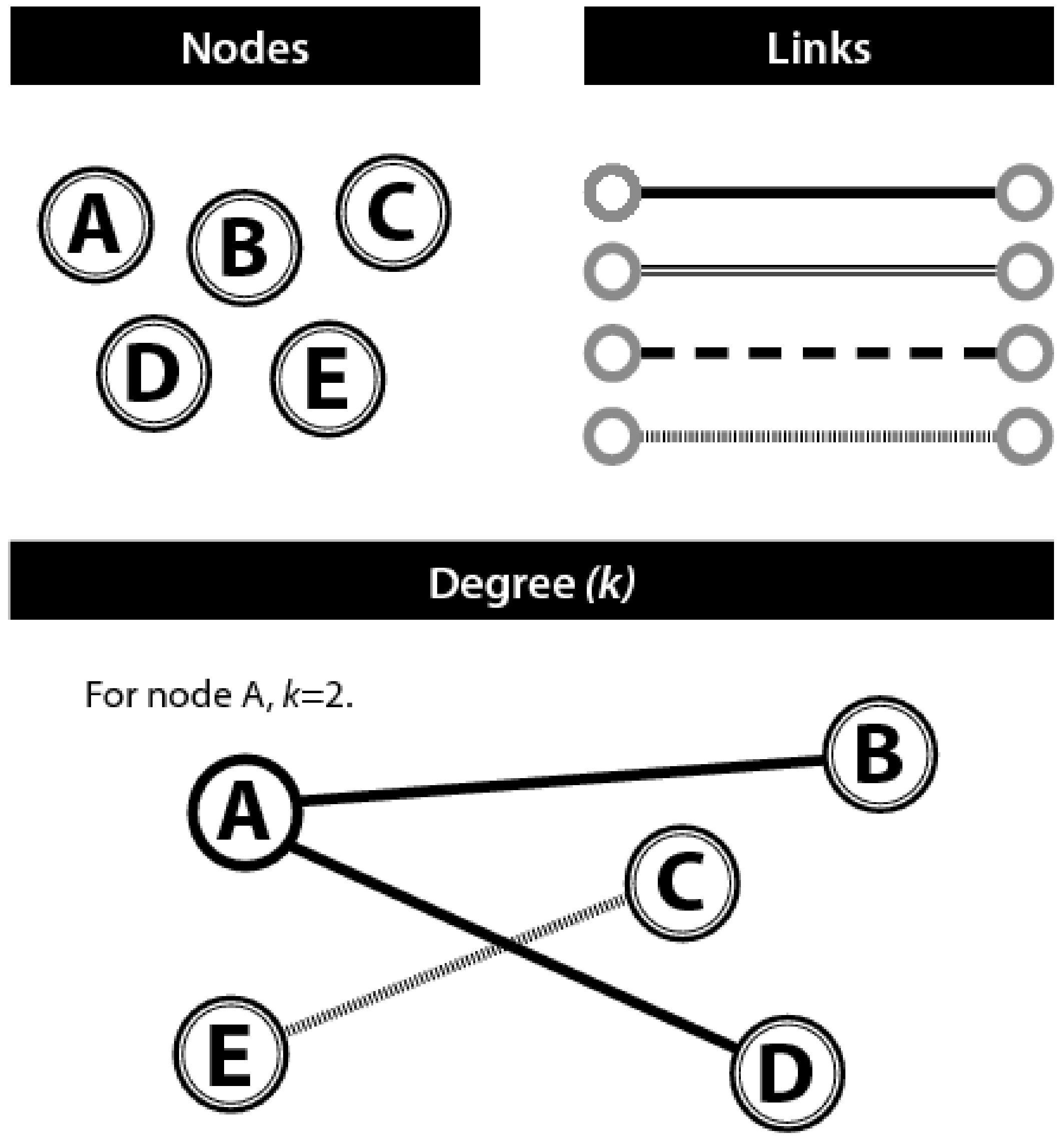
Network Basics

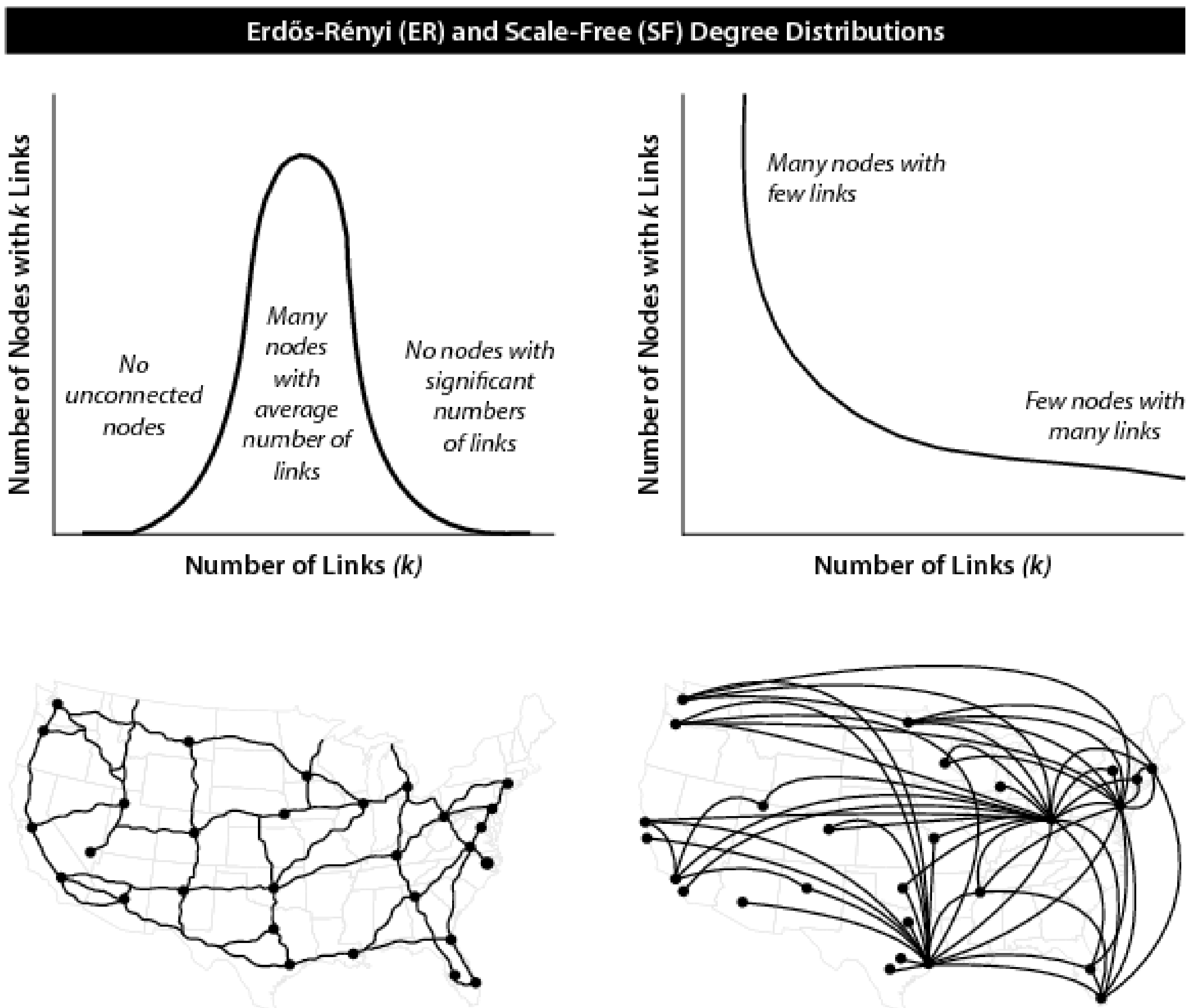


Types of Network

Networks are generally categorized by degree distributions
Two common types:

Random Networks

Scale-free Networks



EXAMPLE: US Highway System
SOURCE: US Department of Transportation

EXAMPLE: US Airline Routes
SOURCE: US Bureau of Transportation Statistics

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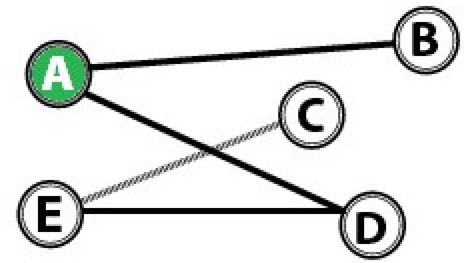
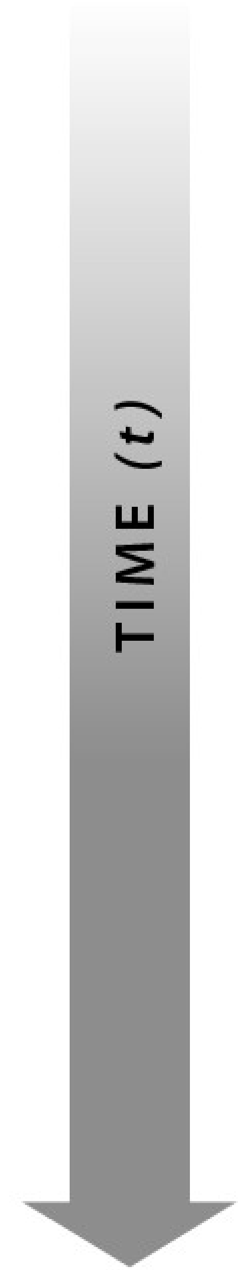
SIR Disease Model
(Susceptible Infected Recovered)

- Healthy nodes: white

  Infected nodes: **green**

  (Recovered nodes: **black**)





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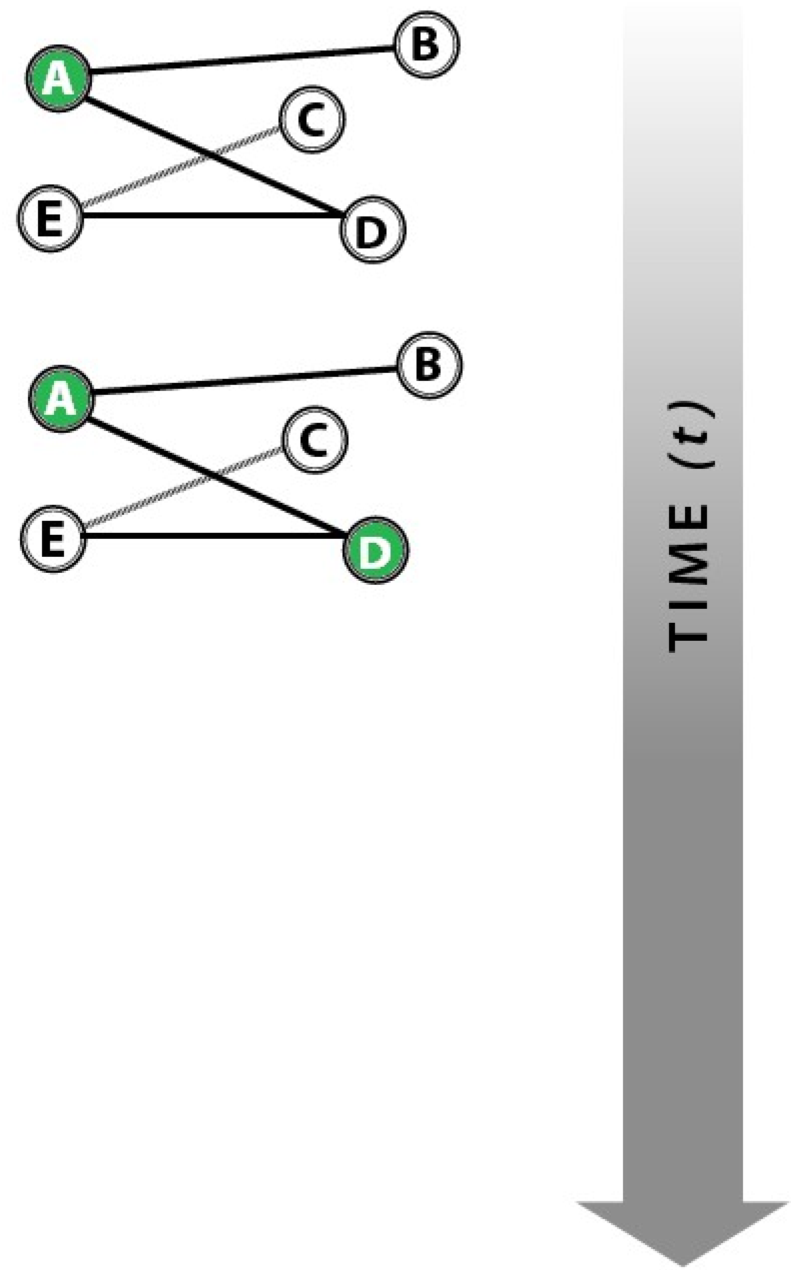

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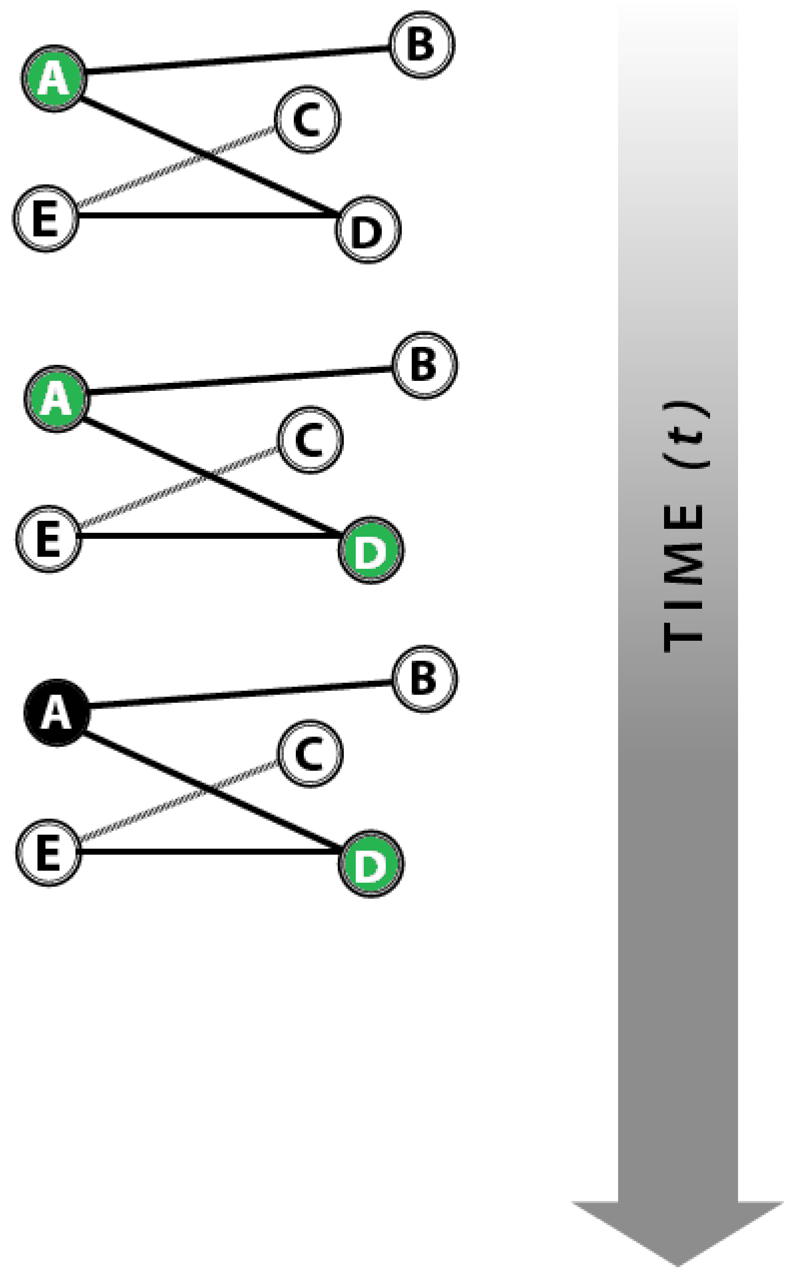
- t_{rec} = recovery time

  No reinfection



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- Healthy nodes: white

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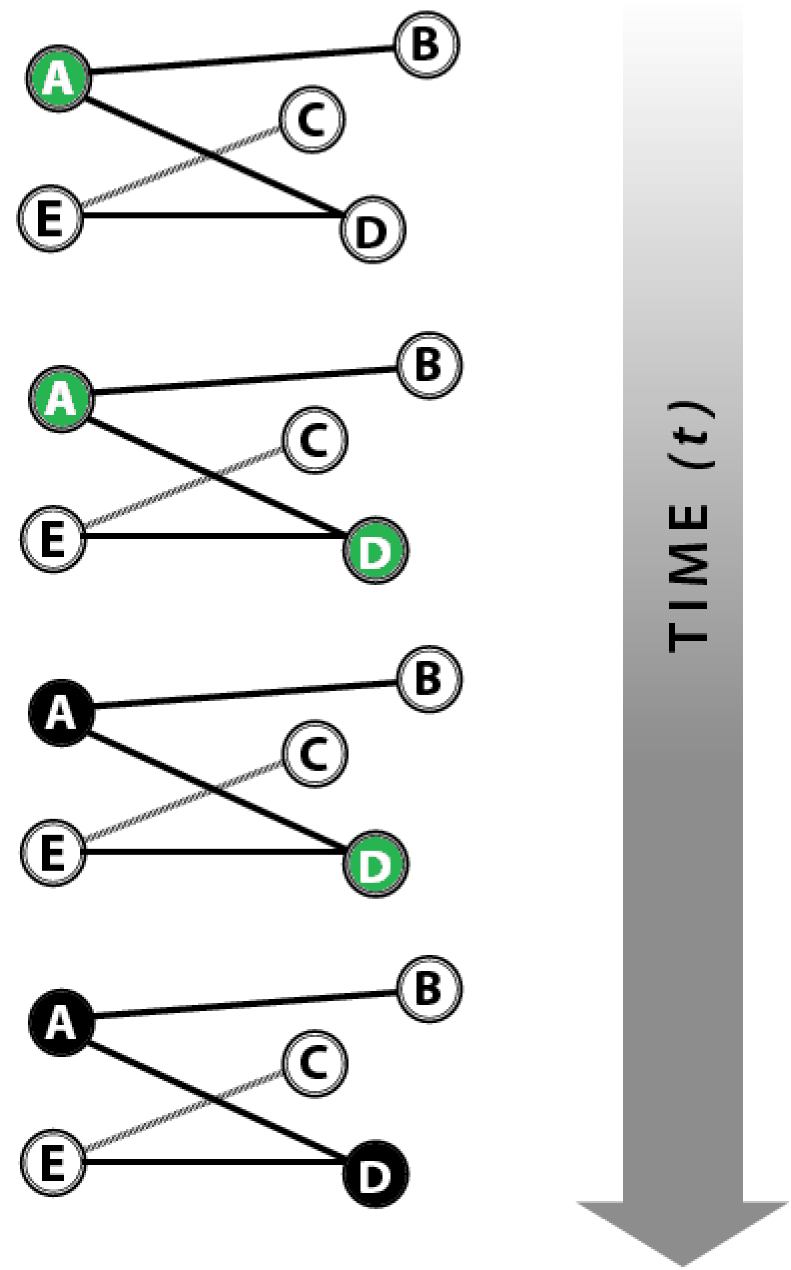
- β = virulence, probability to infect a neighbor

- t_{rec} = recovery time

  No reinfection

- Infection dies out.

  How many were infected?



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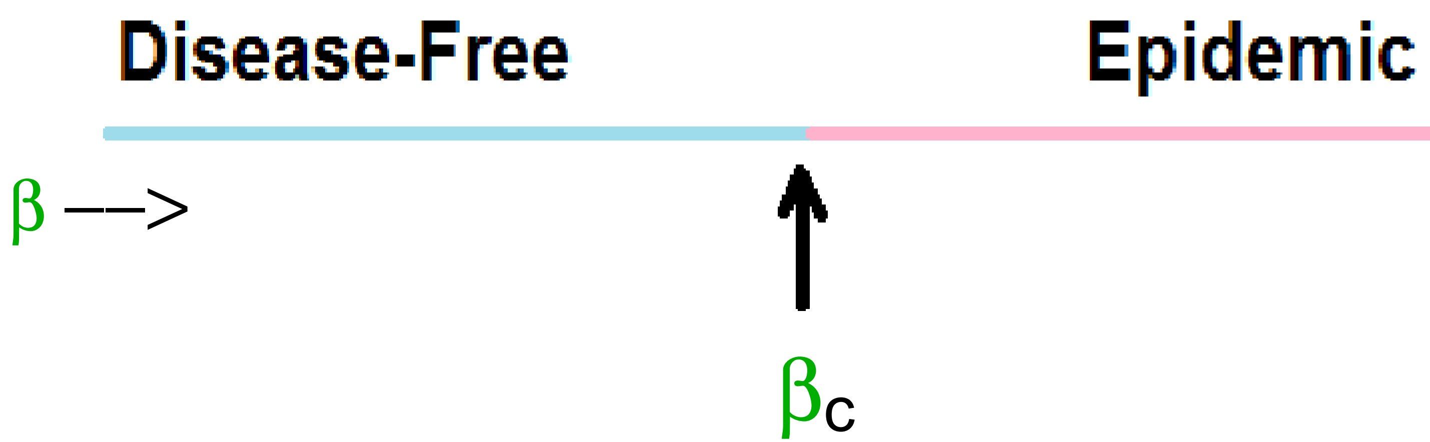
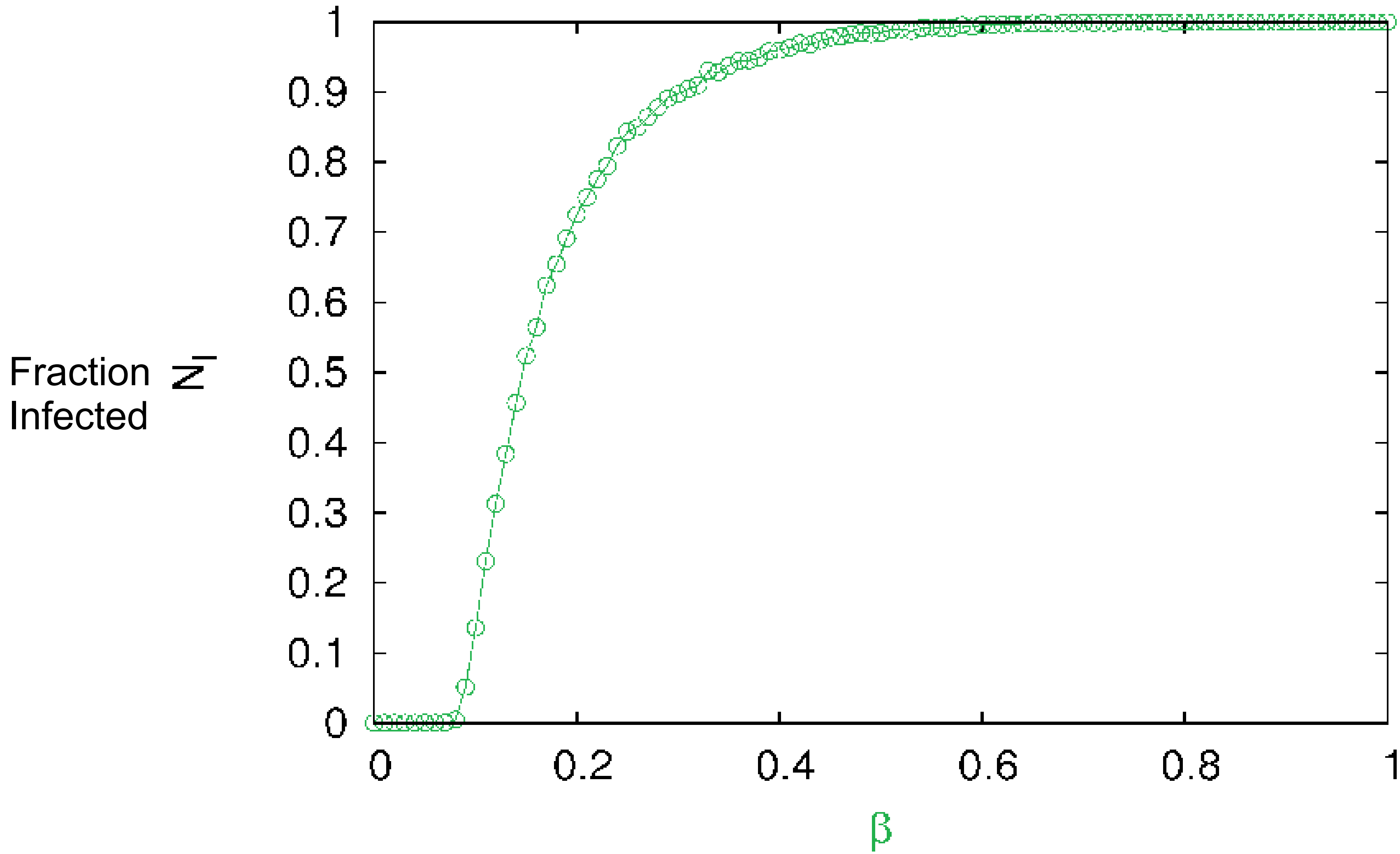
Epidemic thresholds



Critical value of virulence, β_c, epidemic abruptly occurs

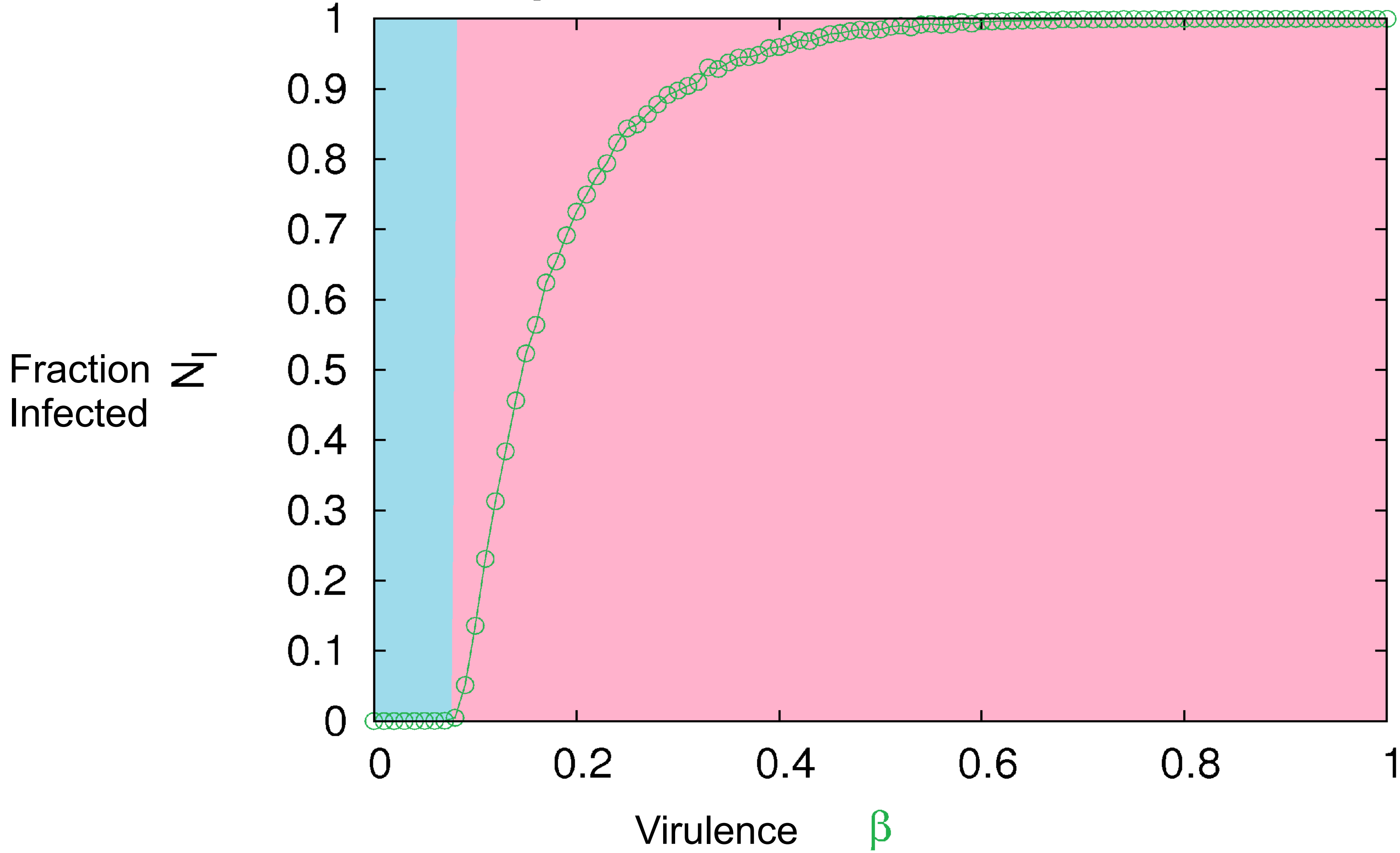
Phase transition!

Epidemic Threshold

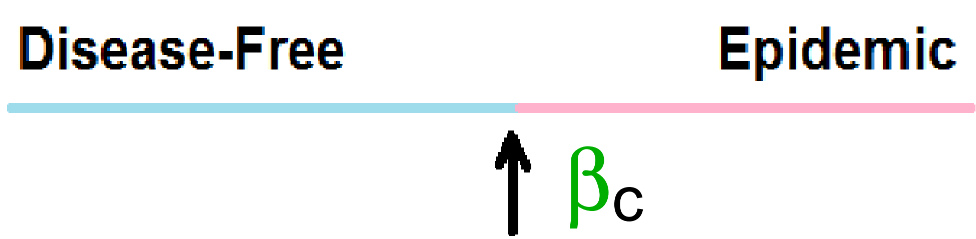


Phase transition in epidemic spreading

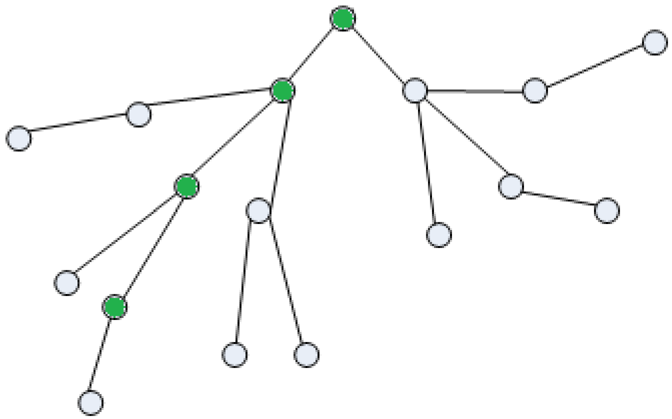
Epidemic Threshold



Phase transition in epidemic spreading



Calculating β_c



Near and below criticality, number of susceptible neighbors is

$$n_s(t) = (\kappa - 1)(1 - \beta)^t$$

Branching factor κ = $\langle k^2 \rangle / \langle k \rangle$ = number of nodes reachable following a randomly selected link.

Sum for expected number of infected neighbors

$$n_I(t_{rec}) = \beta \sum_{t=0}^{t_{rec}-1} (\kappa - 1)(1 - \beta)^t$$

$$= (\kappa - 1)[1 - (1 - \beta)^{t_{rec}}]$$

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Calculating β_c

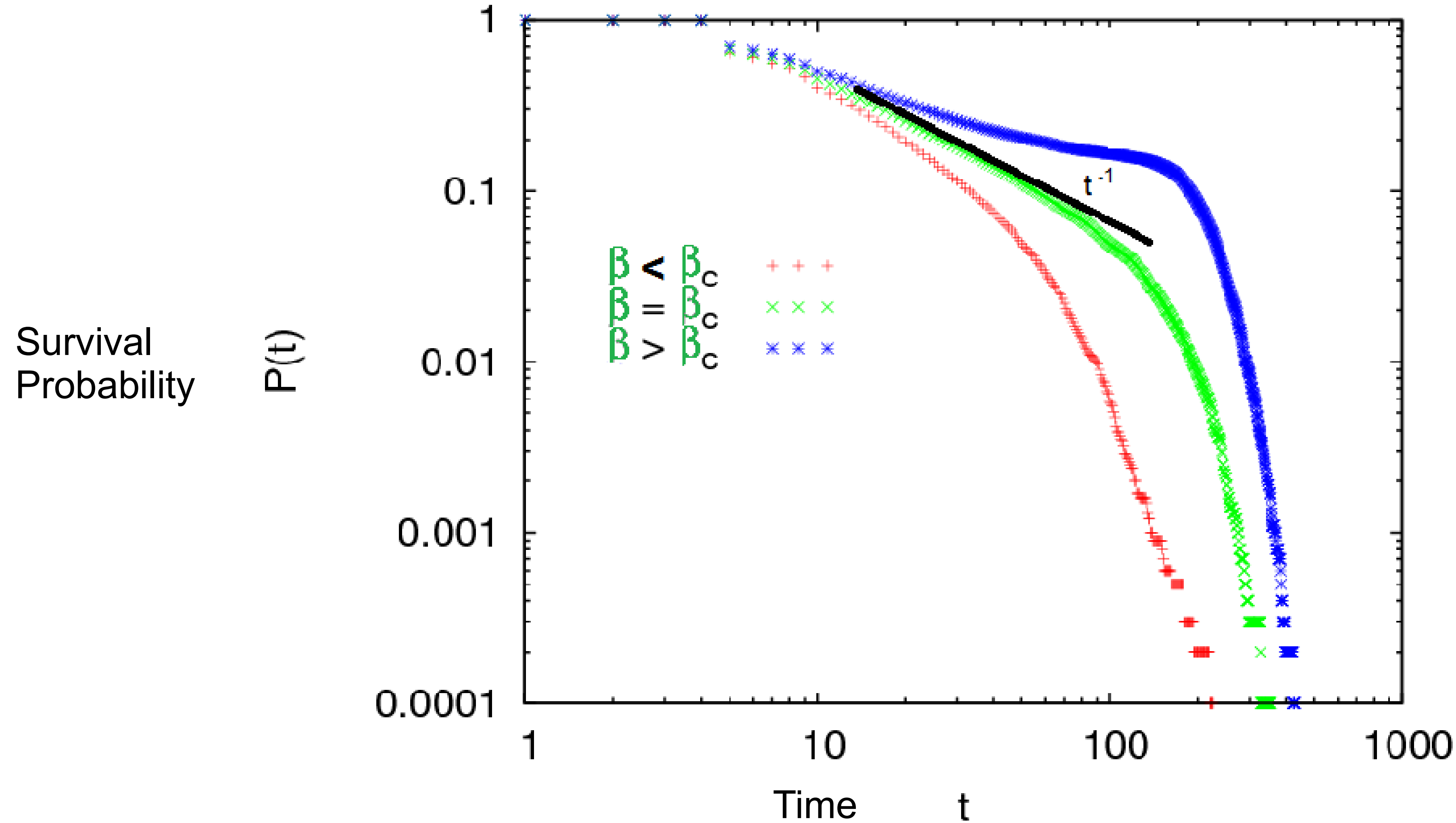
Epidemic threshold when each infected node infects one neighbor:

$$n_I = (\kappa - 1)[1 - (1 - \beta)^{t_r}] = 1$$

For given network parameters and t_r

$$\beta_c = 1 - [1 - (\kappa - 1)^{-1}]^{1/t_r}$$

Critical Point Survival Scaling



$P(t) \sim t^{-1}$ at the critical value

(with finite size cutoff)

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Quarantine Motivation

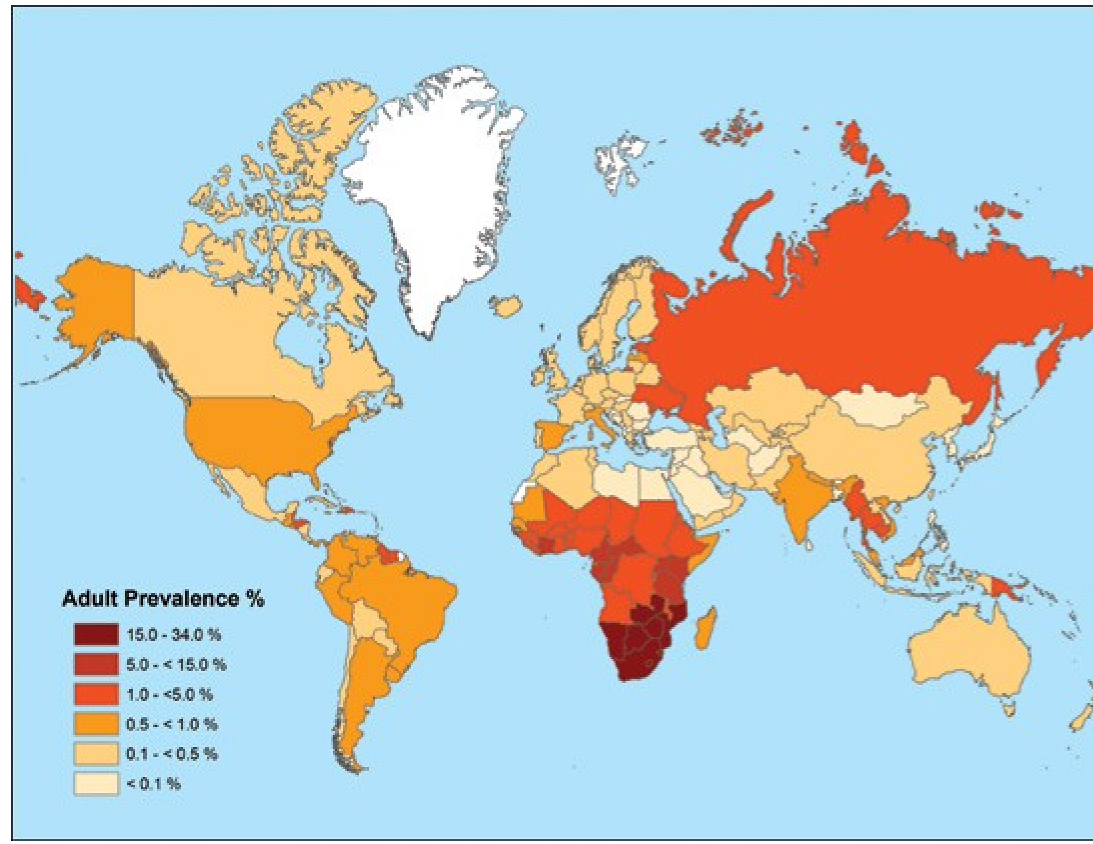
- Vaccination?

- Expensive, difficult to deploy

- Spontaneous (media driven) quarantine- substantial effect in real world cases (H1N1)

Quarantine Motivation

- Vaccination?
- Expensive, difficult to deploy
- Spontaneous (media driven) quarantine--substantial effect in real world cases (H1N1)
- Vaccines not always available



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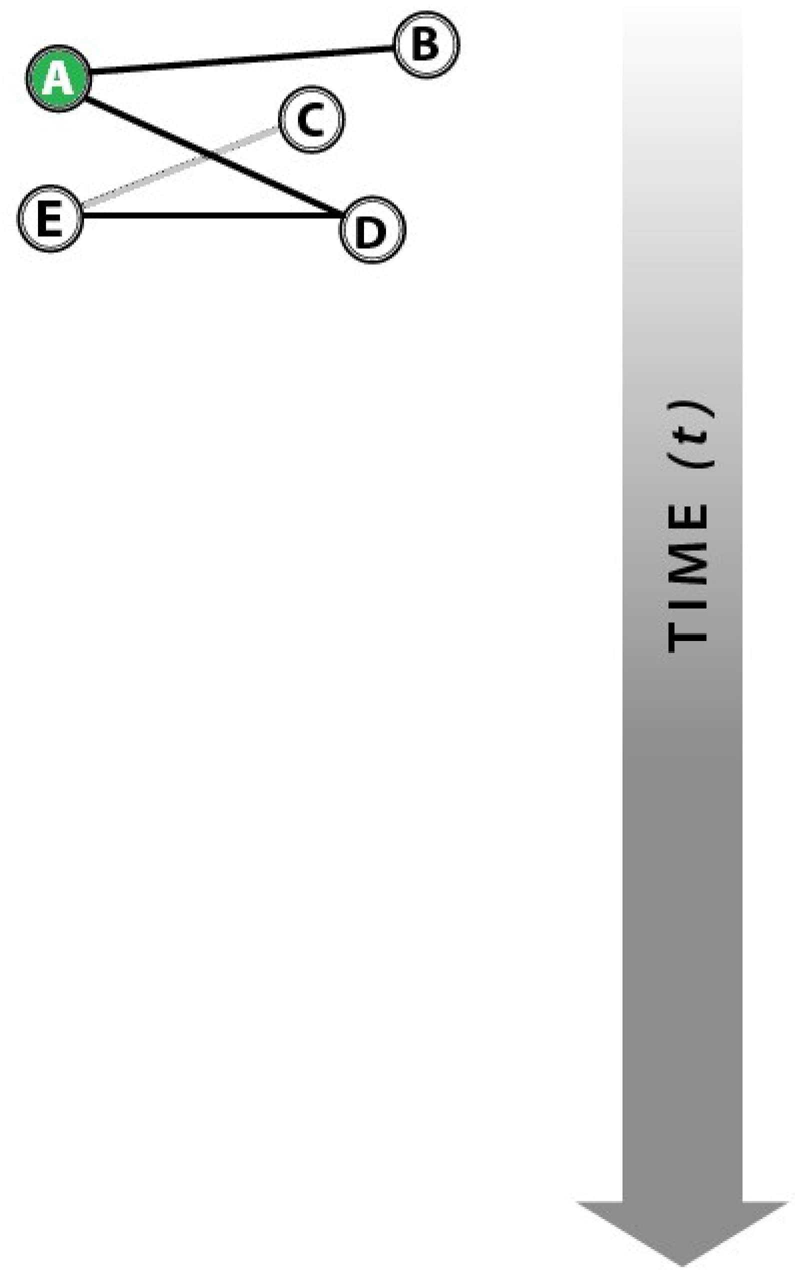
Our Quarantine Model

- Dynamic alteration of network topology



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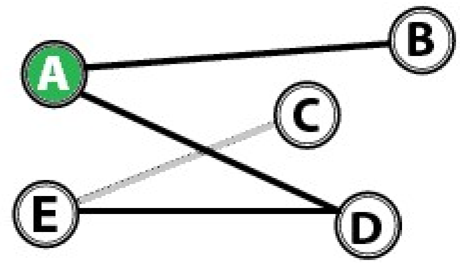
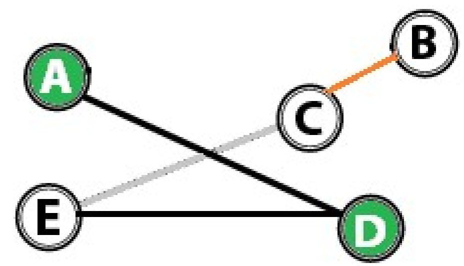
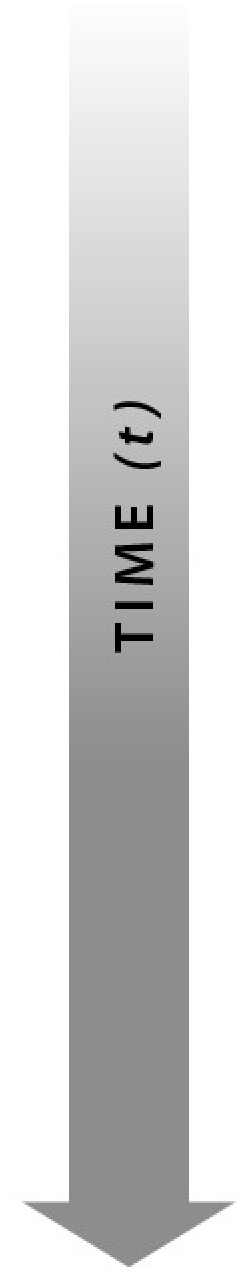


- w = quarantine probability, chance for susceptible site to change link away from infected site at each time step





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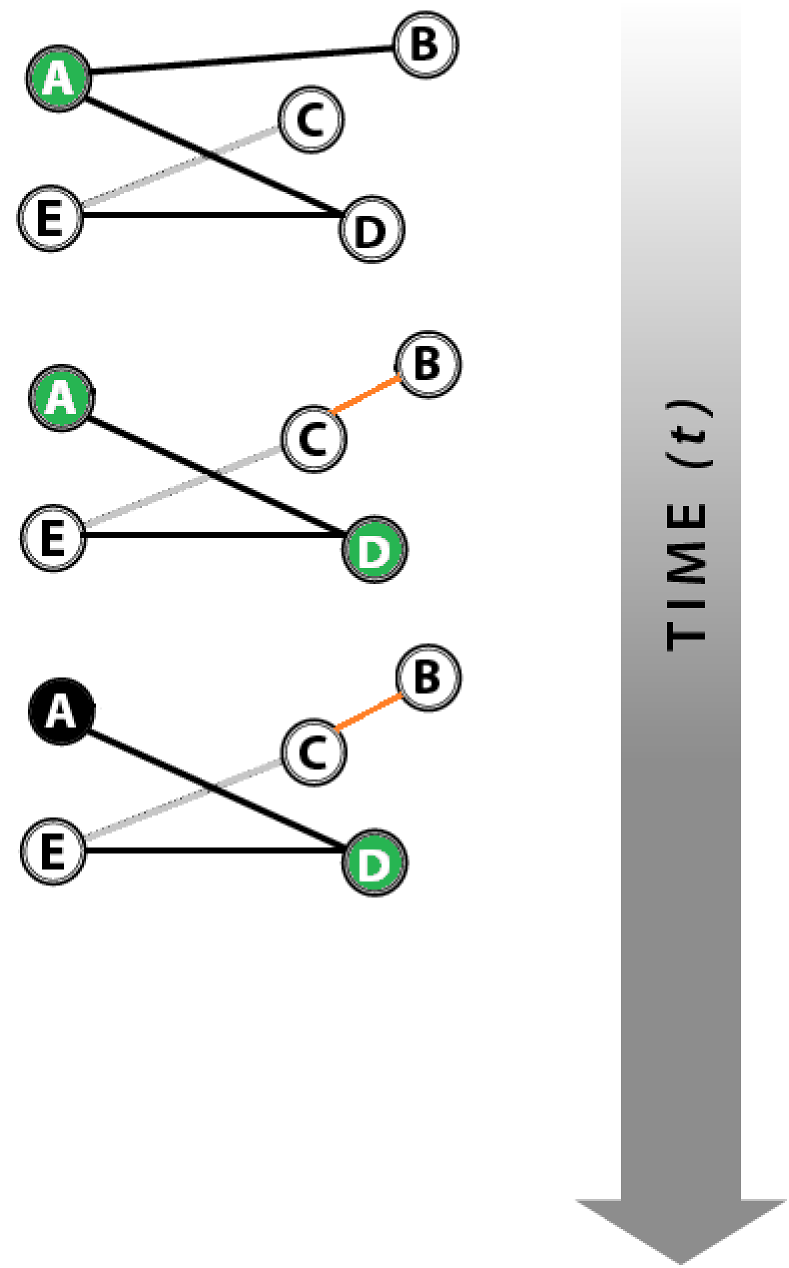

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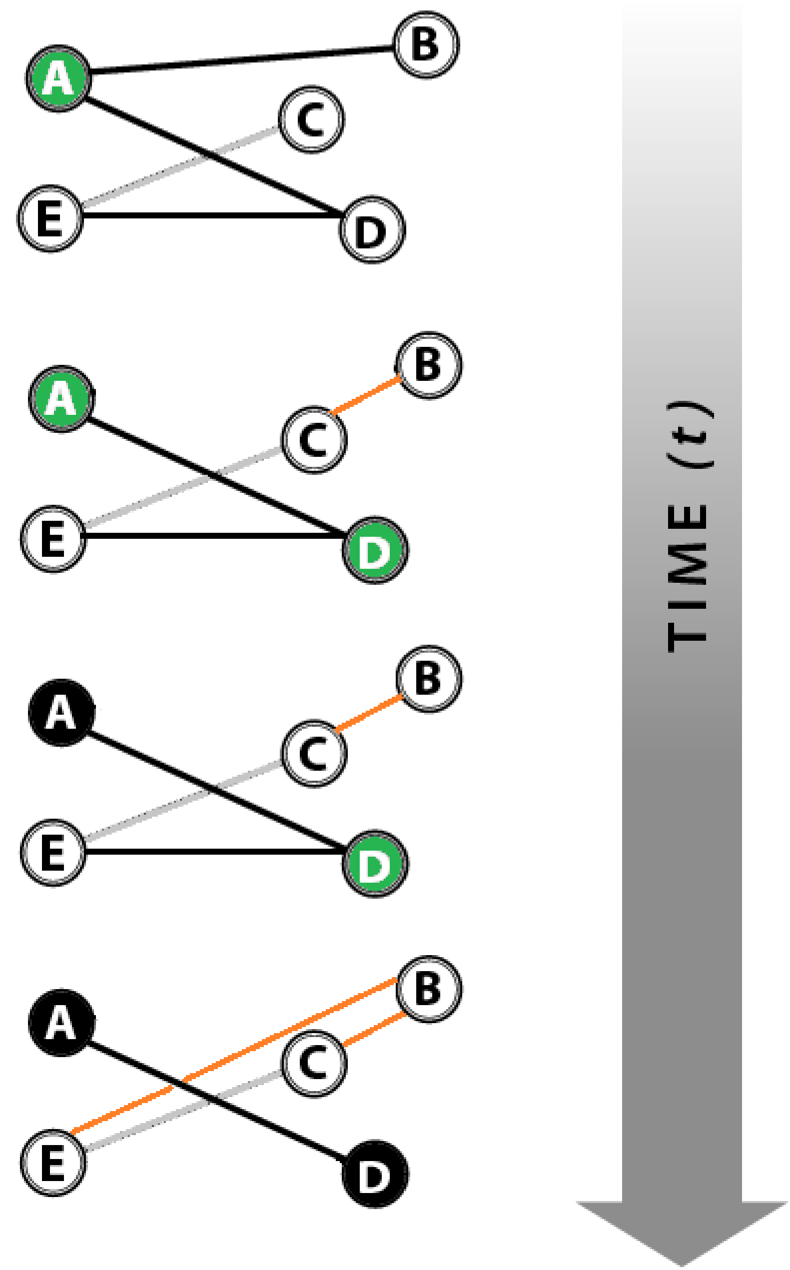
Our Quarantine Model

- Dynamic alteration of network topology
- w = quarantine probability, chance for susceptible site to change link away from infected site at each time step
- Total link number is preserved unless no healthy sites are available



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Theory: Reactionary Quarantine

Add new term reflecting quarantine parameter *w* to the original equation for number of susceptible neighbors

$$n_s(t) = (\kappa - 1)(1 - \beta)^t (1 - w)^t$$

Sum over time for number infected

$$n_I(t_{rec}) = \beta \sum_{t=0}^{t_{rec}-1} (\kappa - 1)(1 - \beta)^t (1 - w)^t$$

$$= \frac{(\kappa - 1)\beta \left\{1 - [(1 - \beta)(1 - w)]^{t_{rec}}\right\}}{1 - (1 - \beta)(1 - w)}$$

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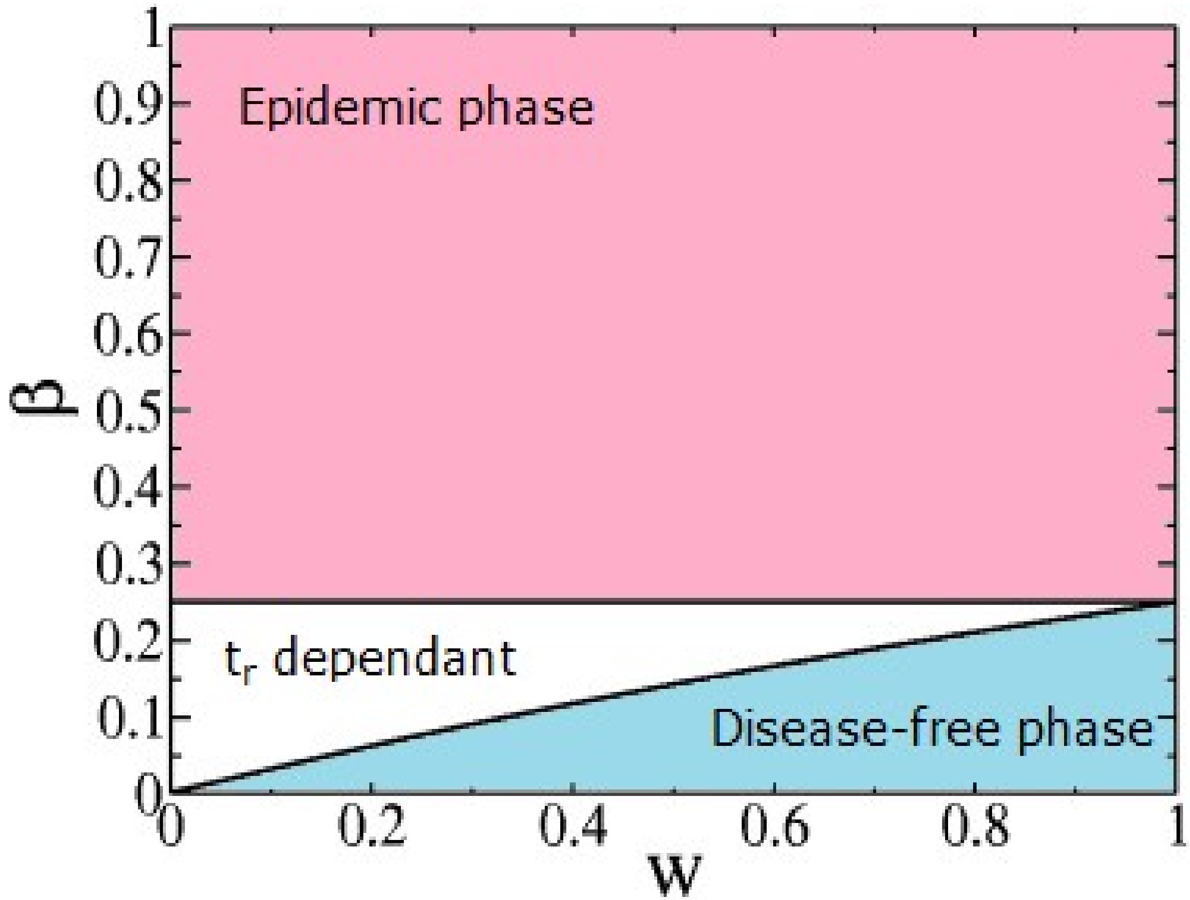
Theory: Reactionary Quarantine

Setting n_I to one gives critical condition

$$\frac{(\kappa - 1)\beta\{1 - [(1-\beta)(1-w_c)]^{t_R}\}}{1-(1-\beta)(1-w_c)} = 1$$

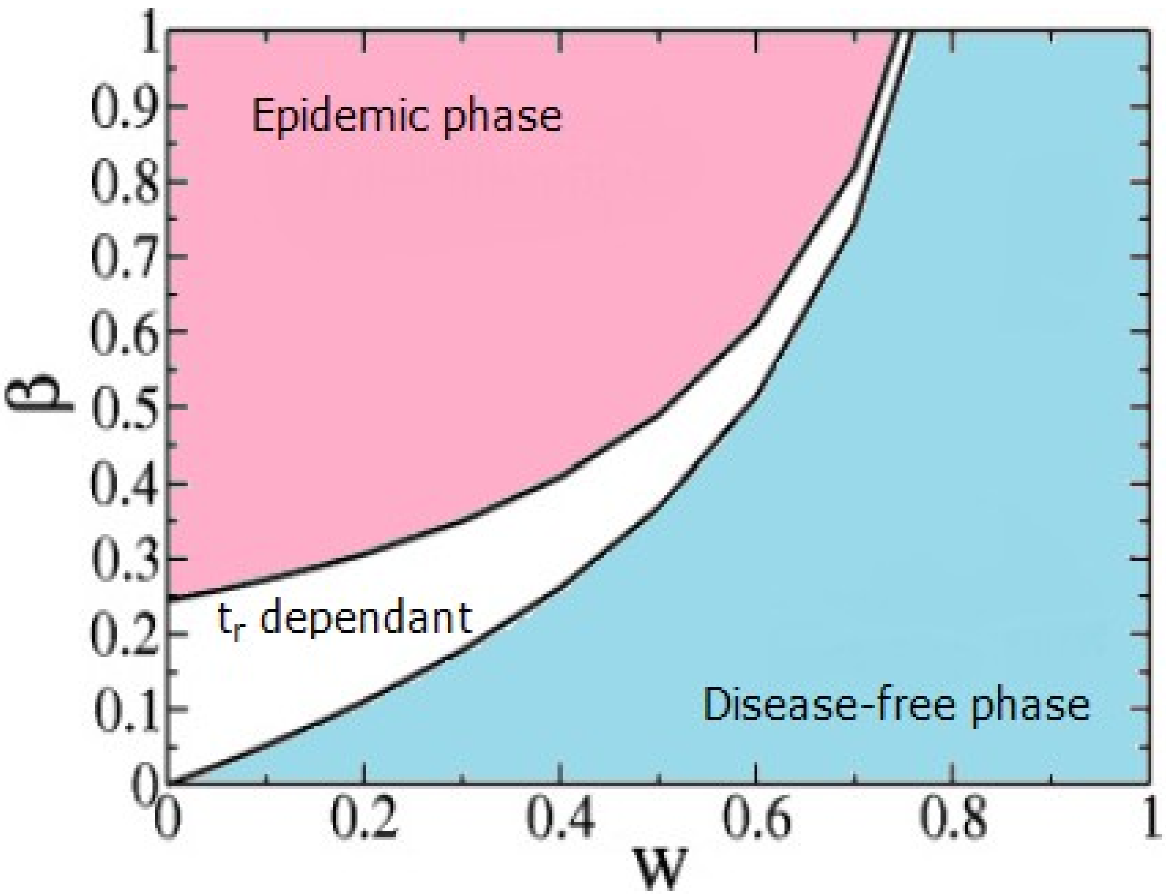
phase transition between disease-free and epidemic phase



Theory: Preemptive Quarantine

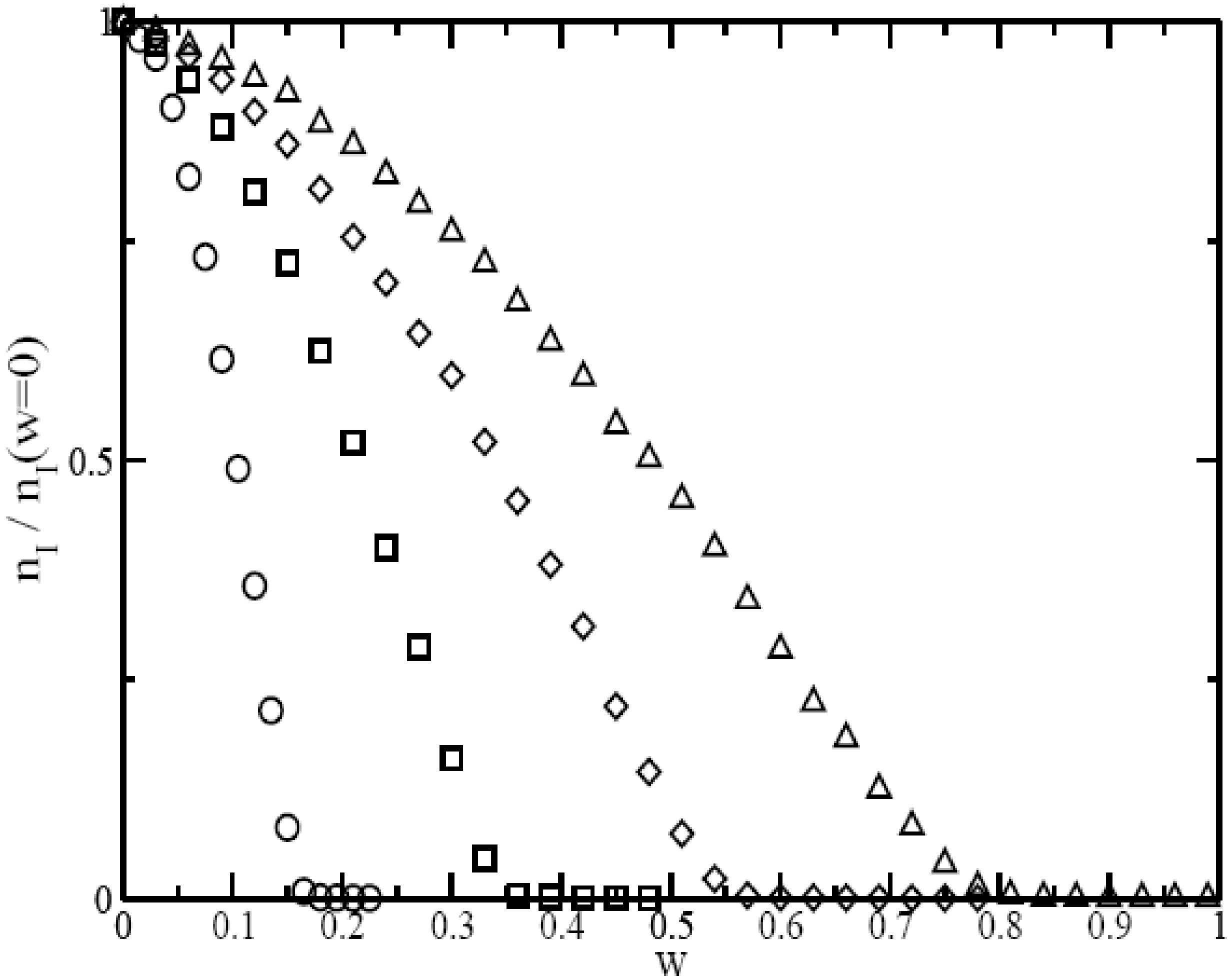
With non-local channels of information, quarantine occurs without initial contact: new critical condition

$$\frac{(\kappa-1)\beta(1-w_c)\{1-[(1-\beta)(1-w_c)]^{t_R}\}}{1-(1-\beta)(1-w_c)} = 1$$



Random Network

Fraction of nodes infected for a given quarantine scaled by the unquarantined case

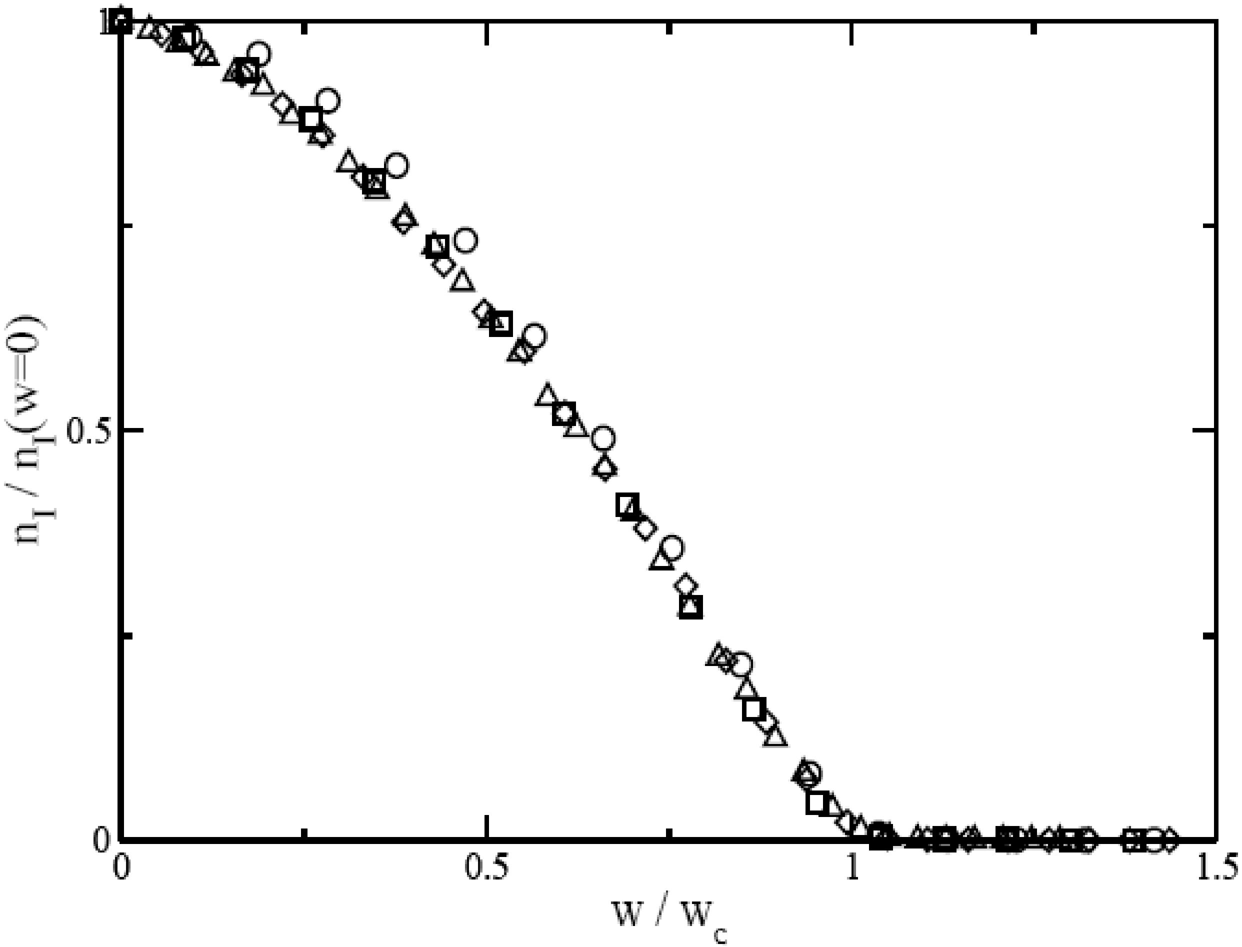


Quarantine rate

Simulations for a random network with β = (0.05,0.1,0.15,0.2)
For each β, a critical w can be seen.

Random Network

Fraction of nodes infected for a given quarantine scaled by the unquarantined case

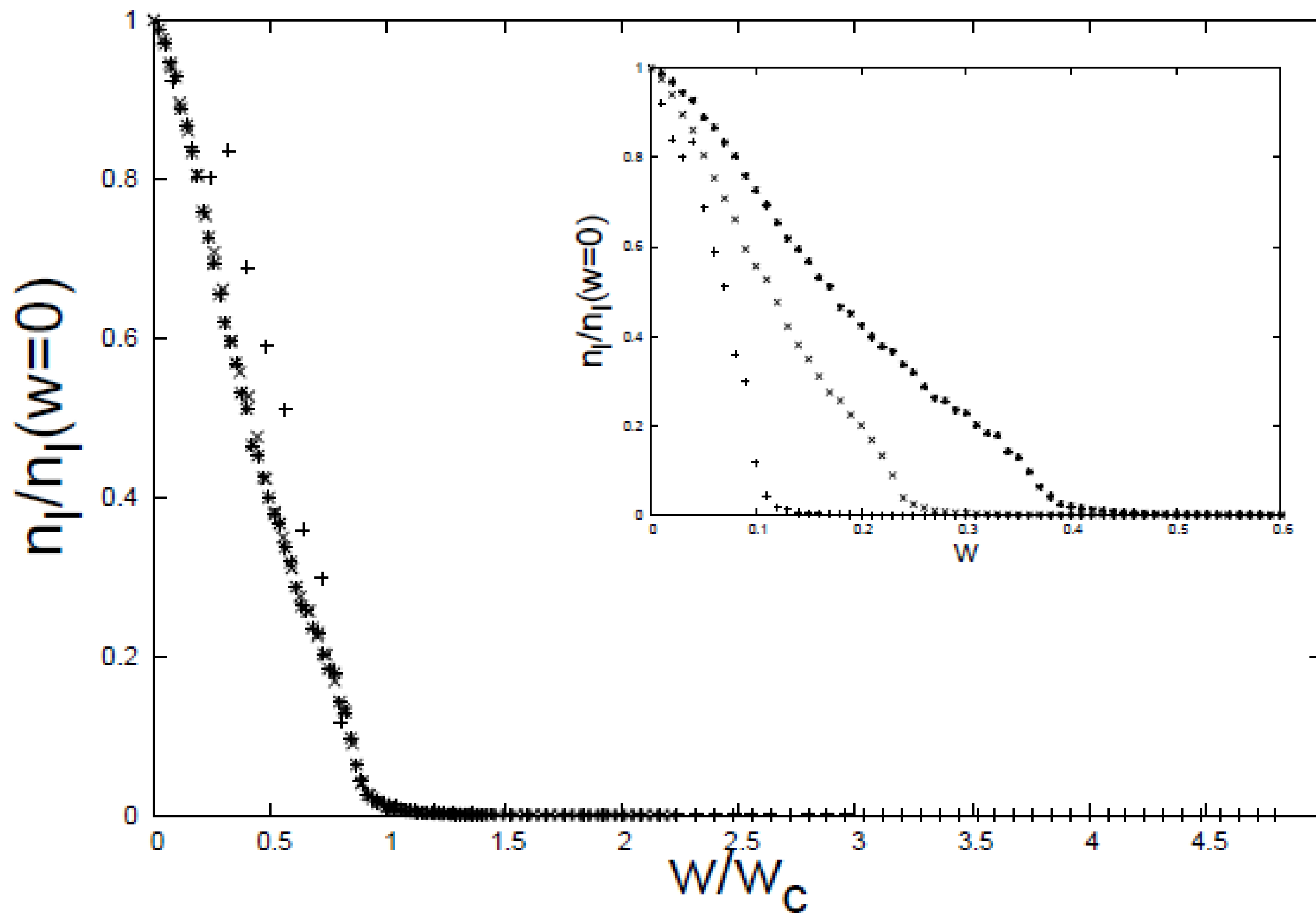


Quarantine rate scaled by theoretical critical value

Previous simulations rescaled by the predicted w_c The data collapses, showing universal behavior.

Scale-free Network

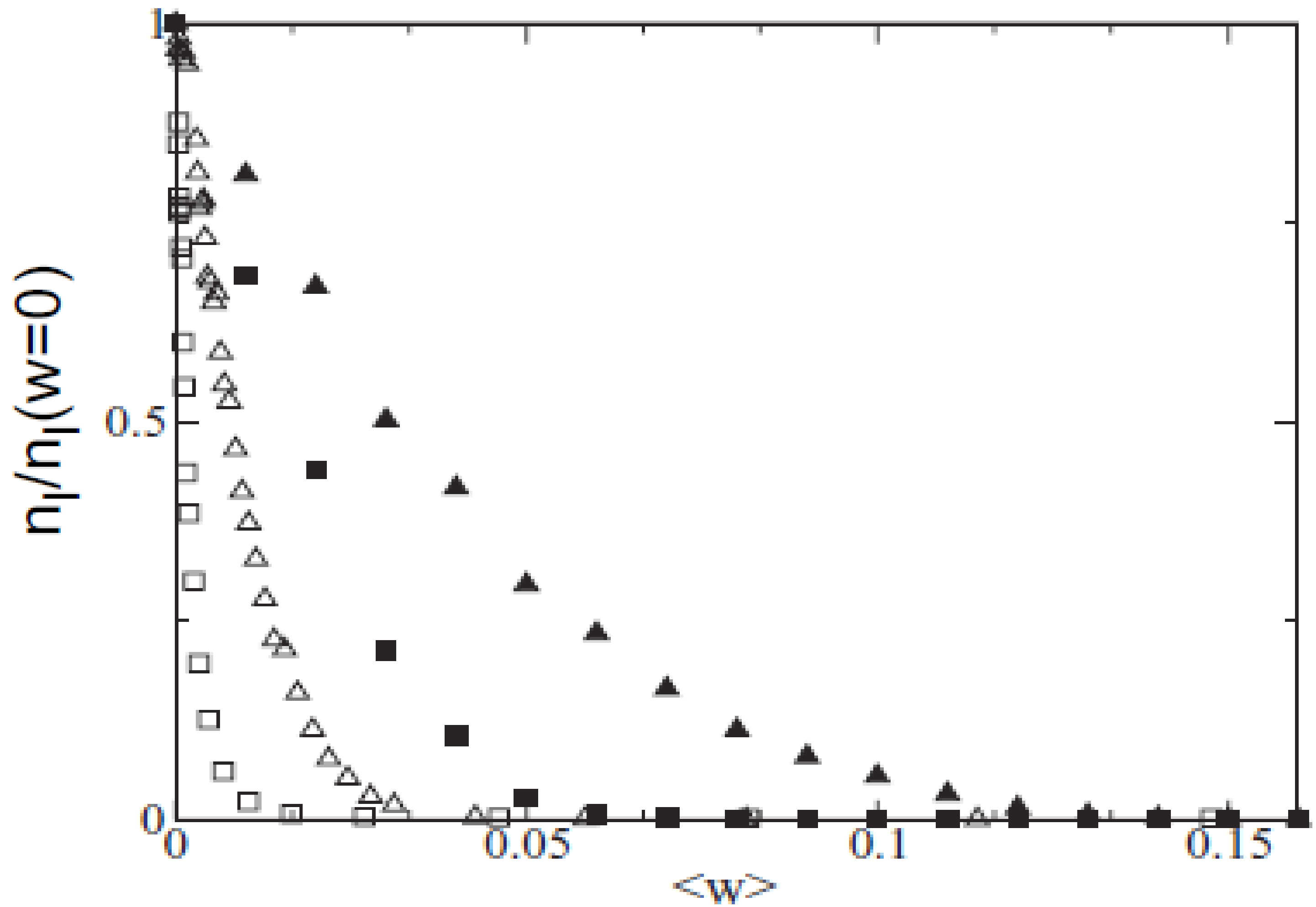
Fraction of nodes infected for a given quarantine scaled by the unquarantined case



Quarantine rate scaled by theoretical critical value

Simulations for a scale-free network with $\beta = (0.05, 0.1, 0.15)$
The data collapses when rescaled.

Targeted Quarantine



Fraction of nodes infected for a given quarantine scaled by the unquarantined case

Average quarantine rate

Open symbols have a node dependent quarantine rate of the form $w_k = \gamma k^{\alpha}$. With $\gamma = 1/k_{max}$ and α varying. Targeted quarantine can be seen to be significantly more effective.

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Summary

- Preemptive quarantine can be effective for all disease parameters
- Finite critical parameter even in scale-free networks
- Targeted quarantine significantly better in strongly heterogeneous networks

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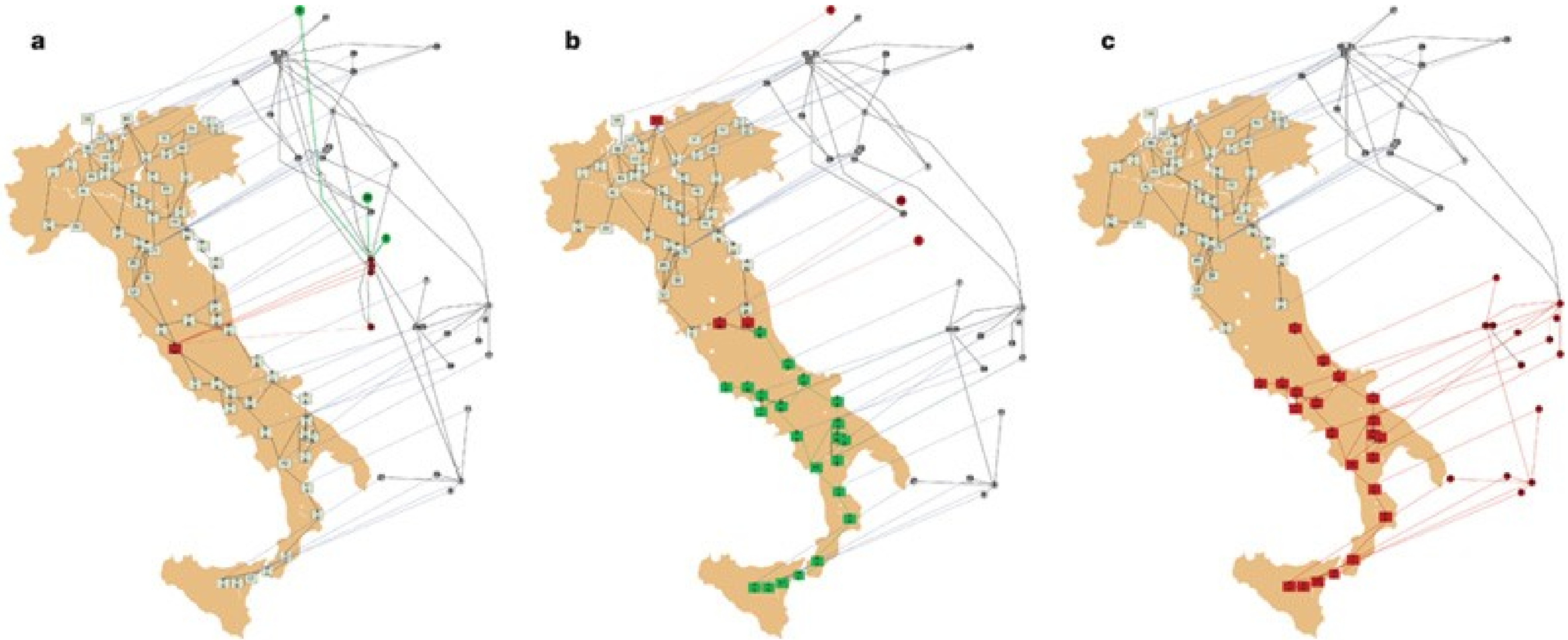
Interacting Network Systems: Background



Italian national blackout

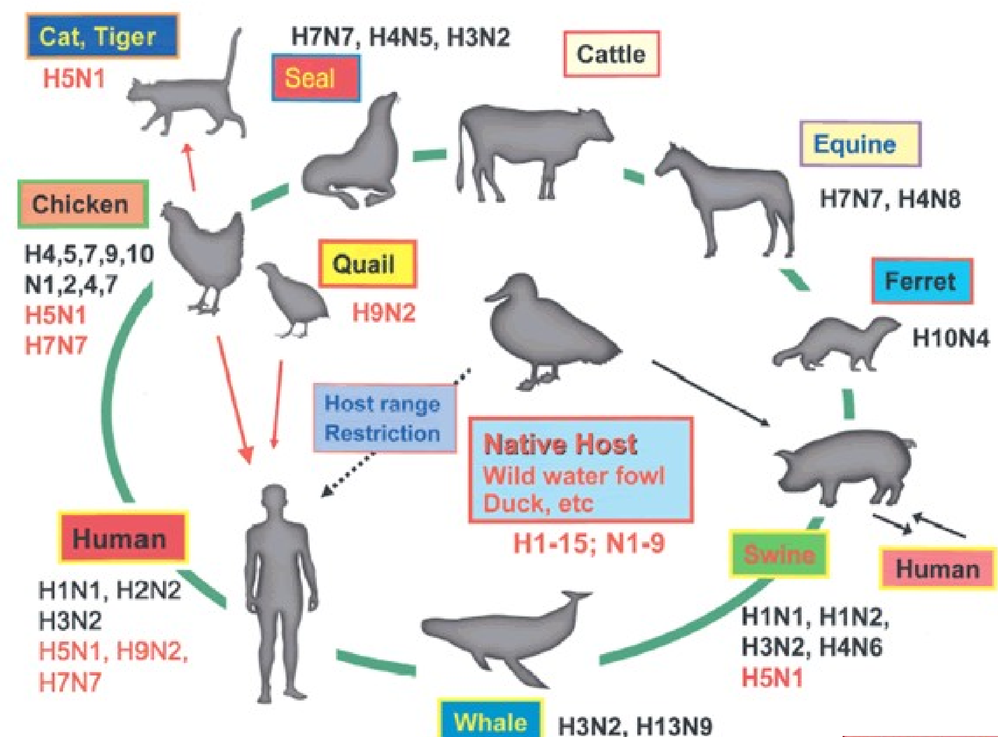
Vulnerability of networks of networks

Interacting Network Systems

Increasing danger of animal threats: malaria, flu

“The possibility of human-to-human transmission ... poses an imminent threat of a global pandemic”

-JP Liu, J. Microbiology Immunology and Infection



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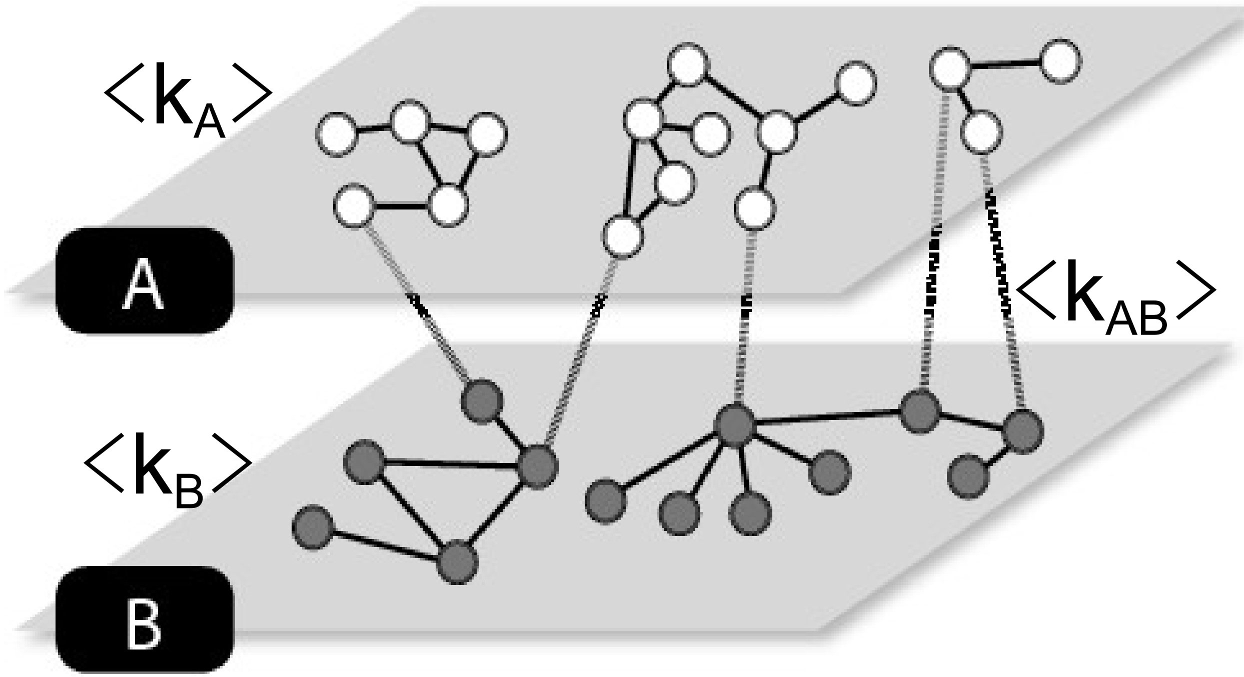
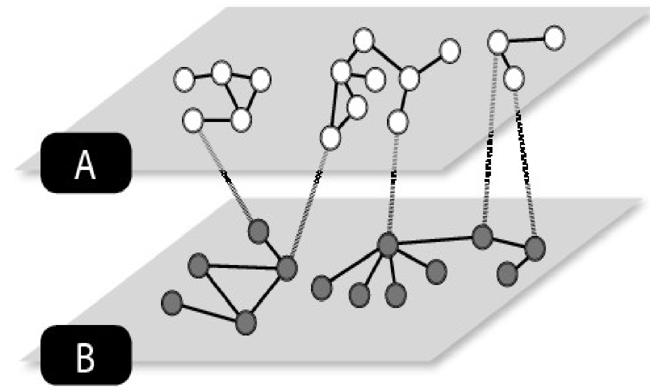
Interacting Network Systems: Defined



- Two (or more) networks
- Internal average link numbers: $\langle k_A \rangle$, $\langle k_B \rangle$
- Interacting average link number: $\langle k_{AB} \rangle$

Interacting Network Systems: Strongly- vs Weakly-Coupled

Q: Do epidemics always spread throughout the entire coupled network system, or can they remain localized?



A: Depends on the number of interacting links

Lots of links: Strongly-coupled, disease only global

Few links: Weakly-coupled, mixed phase

Critical condition on $\langle k_{AB} \rangle$:

$$\langle k_{AB} \rangle_c = \frac{\sqrt{2\langle k_A \rangle \langle k_B \rangle - \langle k_A \rangle^2} - \langle k_A \rangle}{2}$$

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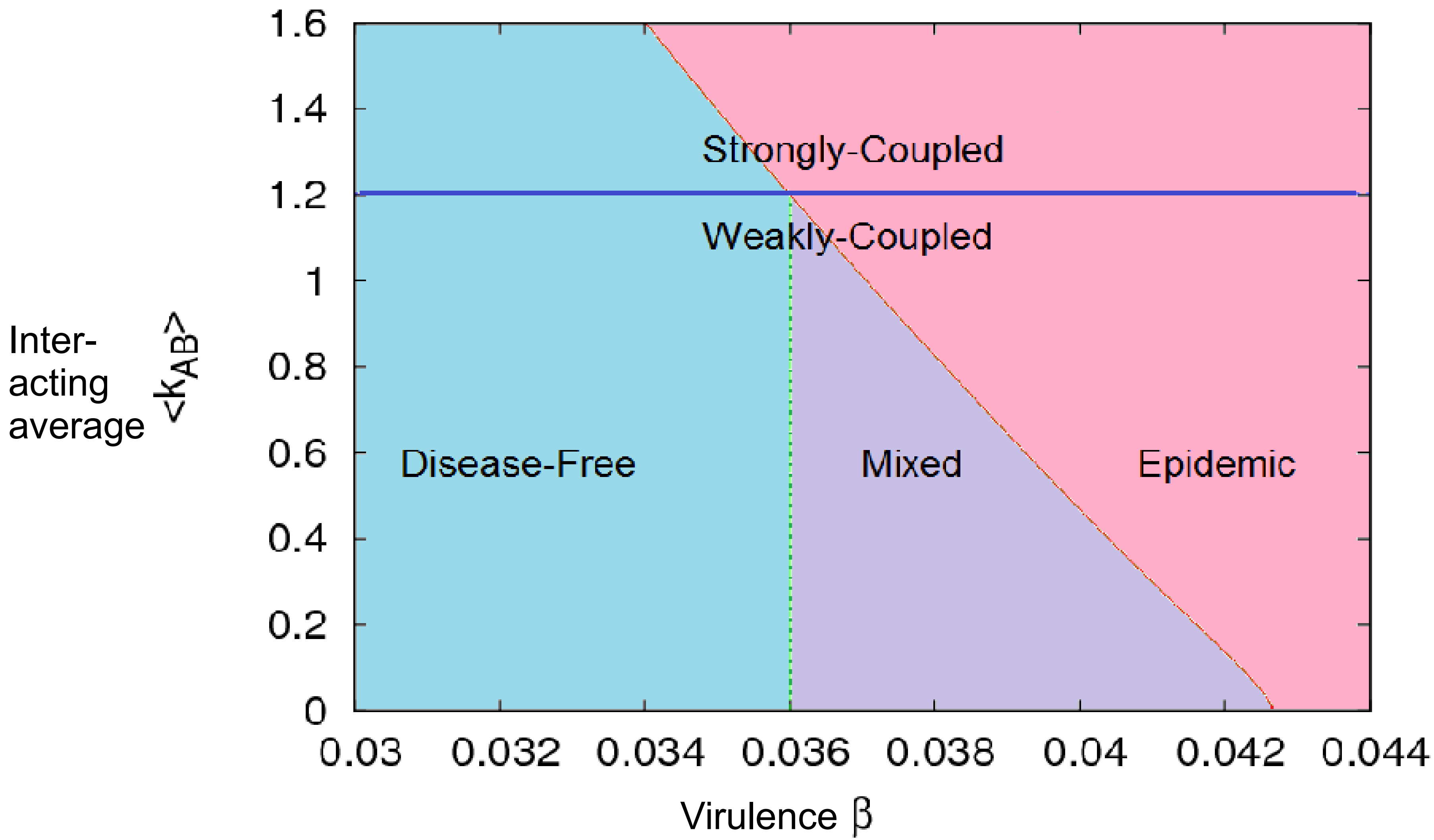
Interacting Network Phases

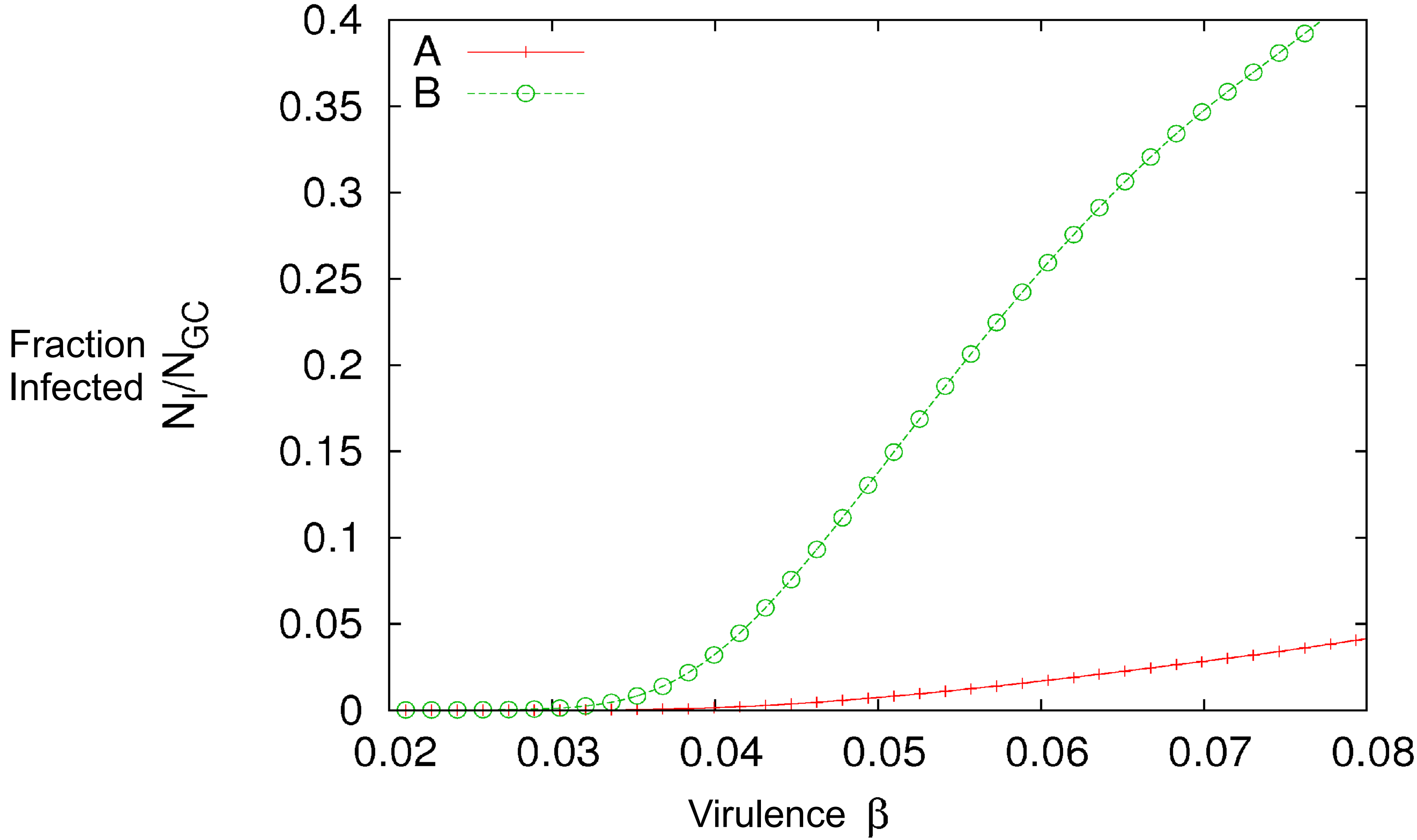


Full phase diagram for $<k_A>=1.5$ $<k_B>= 6.0$

Weakly-coupled networks have 3 phases

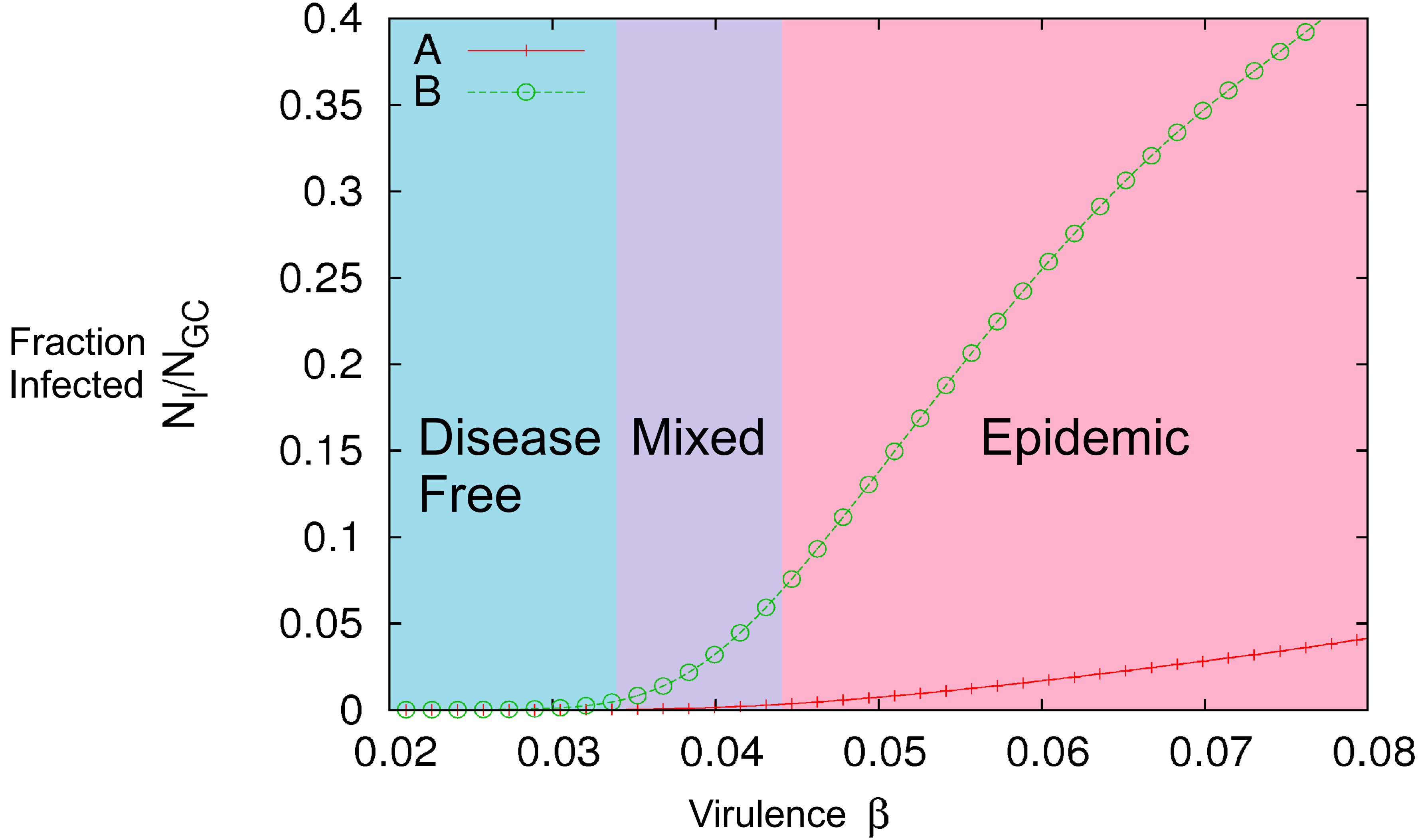
Strongly-coupled networks have 2 phases

Weakly-Coupled Simulation Data



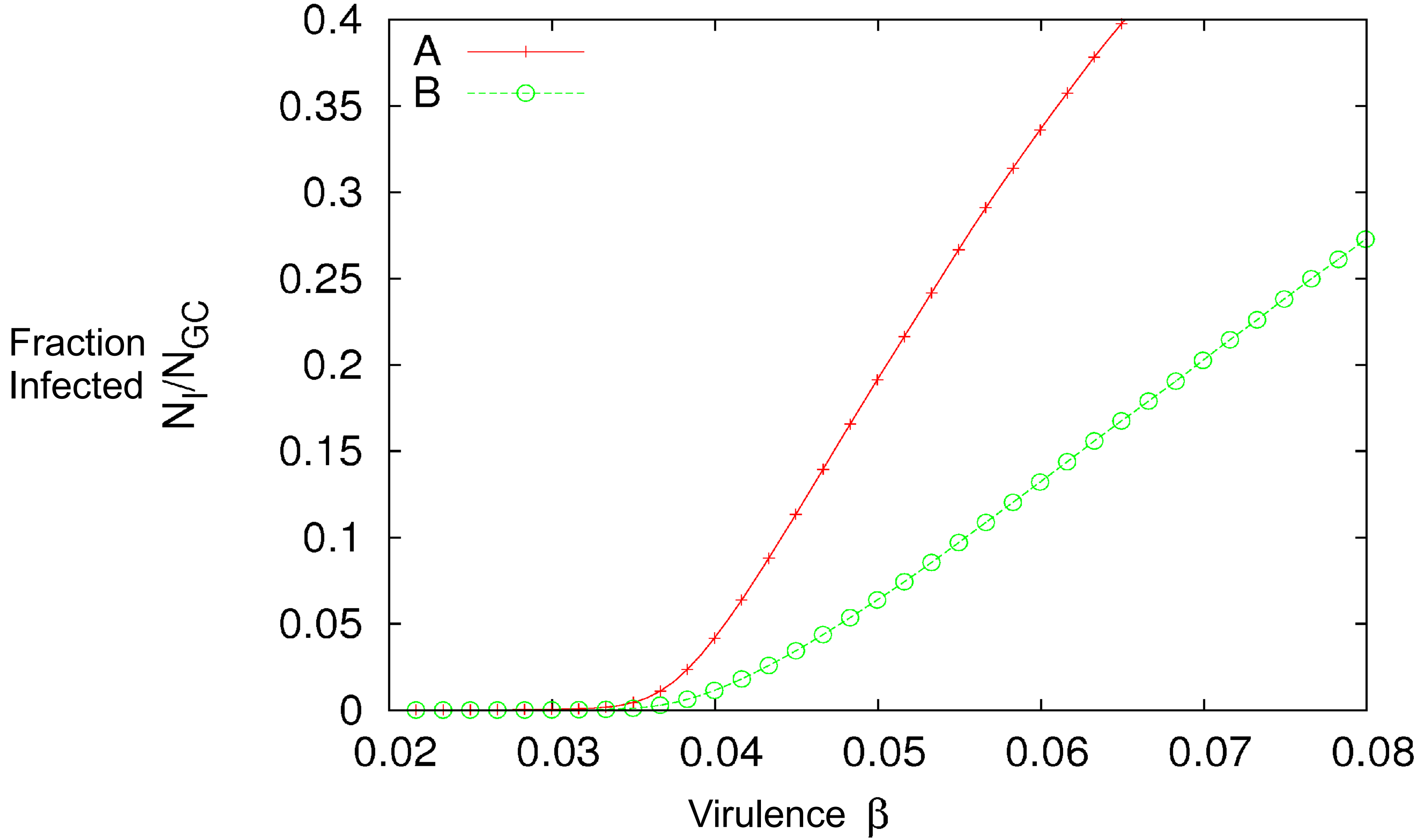
Horizontal slice across the phase diagram at $\langle k_{AB} \rangle = 0.1$

Weakly-Coupled Phases



Horizontal slice across the phase diagram at $\langle k_{AB} \rangle = 0.1$

Near the Strongly-Coupled transition



Horizontal slice across the phase diagram at $<k_{AB}>=1.0$

Difficult to see mixed phase

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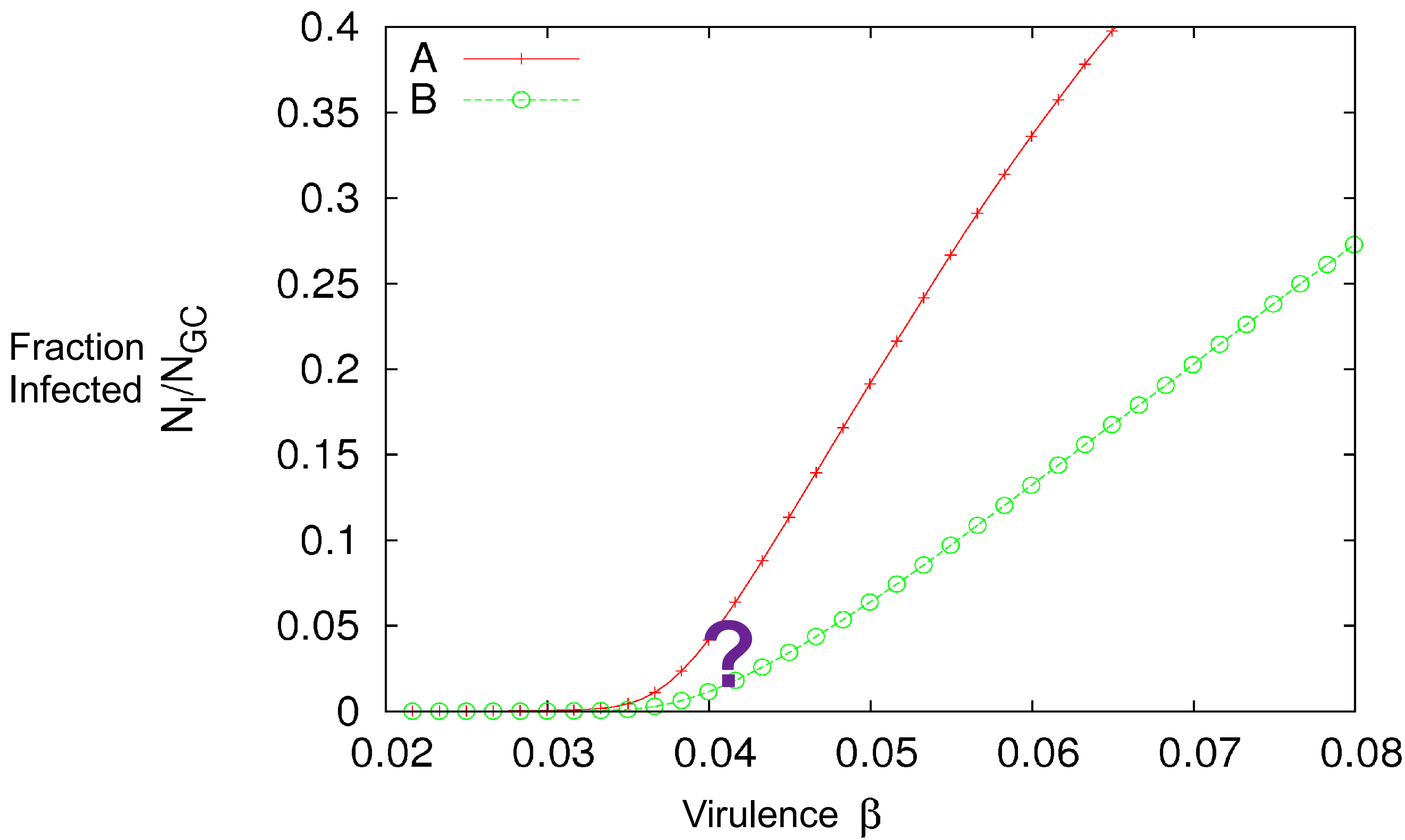
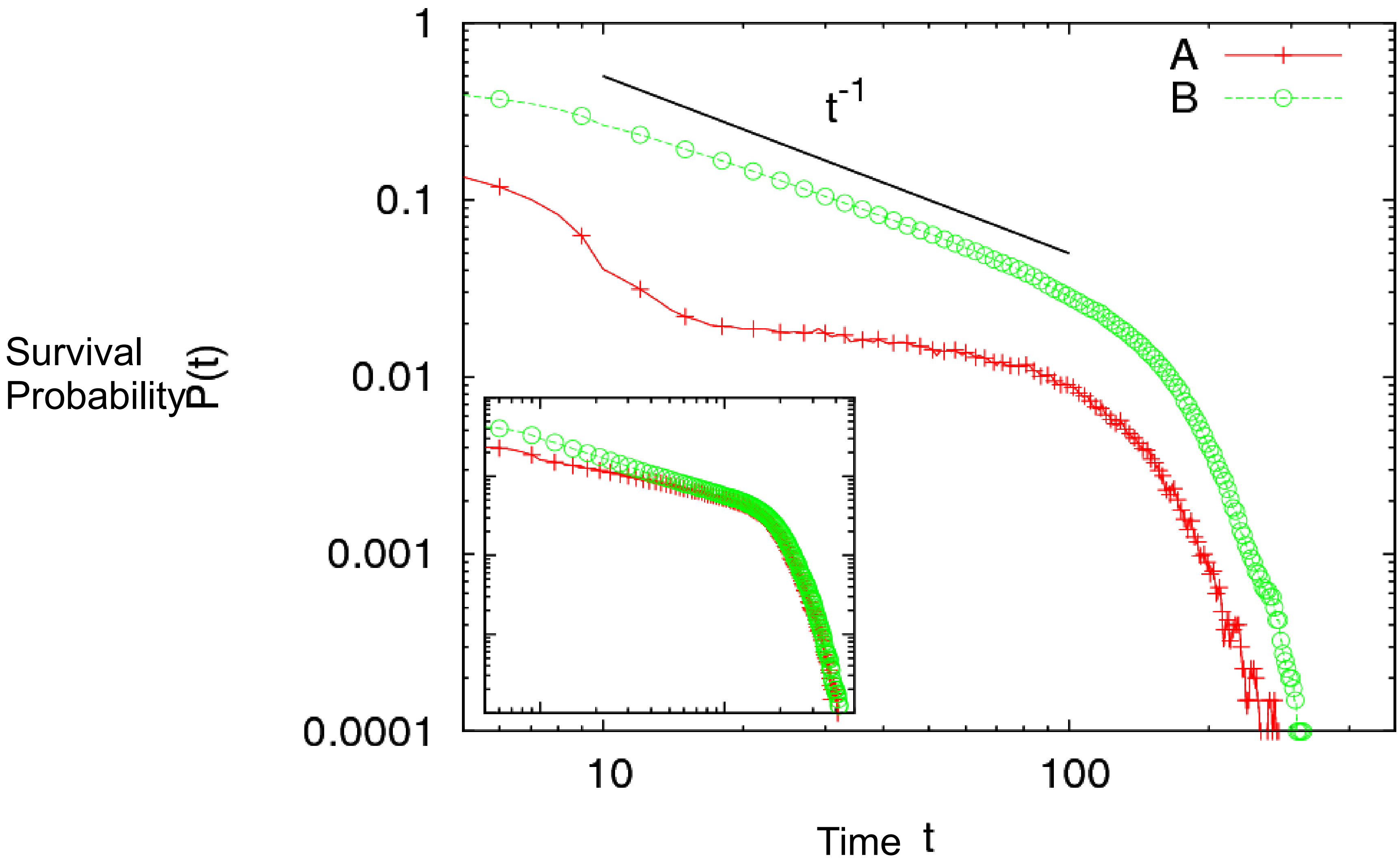
Near the Strongly-Coupled transition



Horizontal slice across the phase diagram at $\langle k_{AB} \rangle = 1.0$

Difficult to see mixed phase

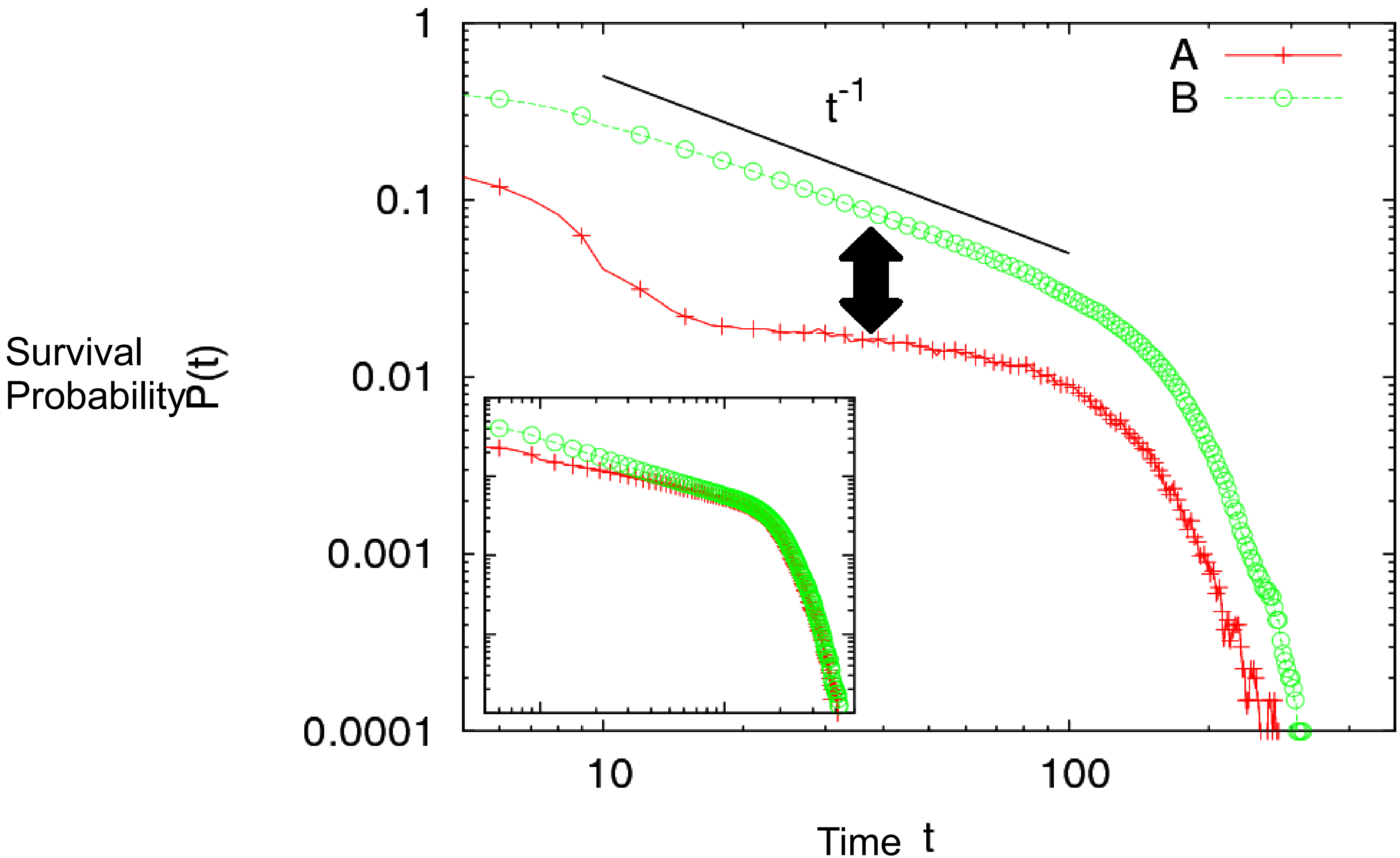
Identifying the Mixed Phase



Use critical scaling!

Mixed phase, only one network scales critically

Identifying the Mixed Phase



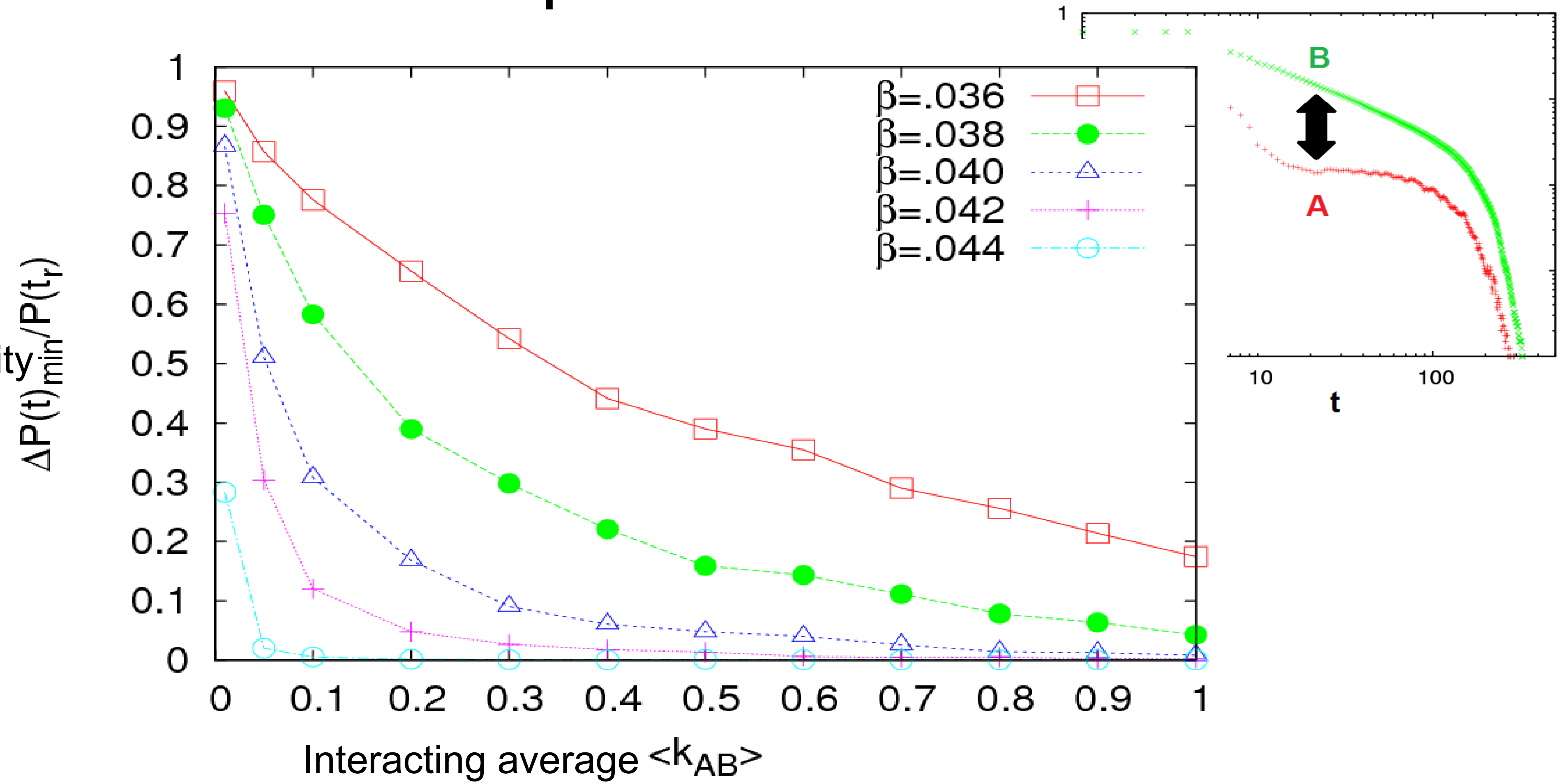
Use critical scaling!

Mixed phase, only one network scales critically

Look at smallest gap between two curves

Survival Gap as Phase Indicator

Survival probability gap

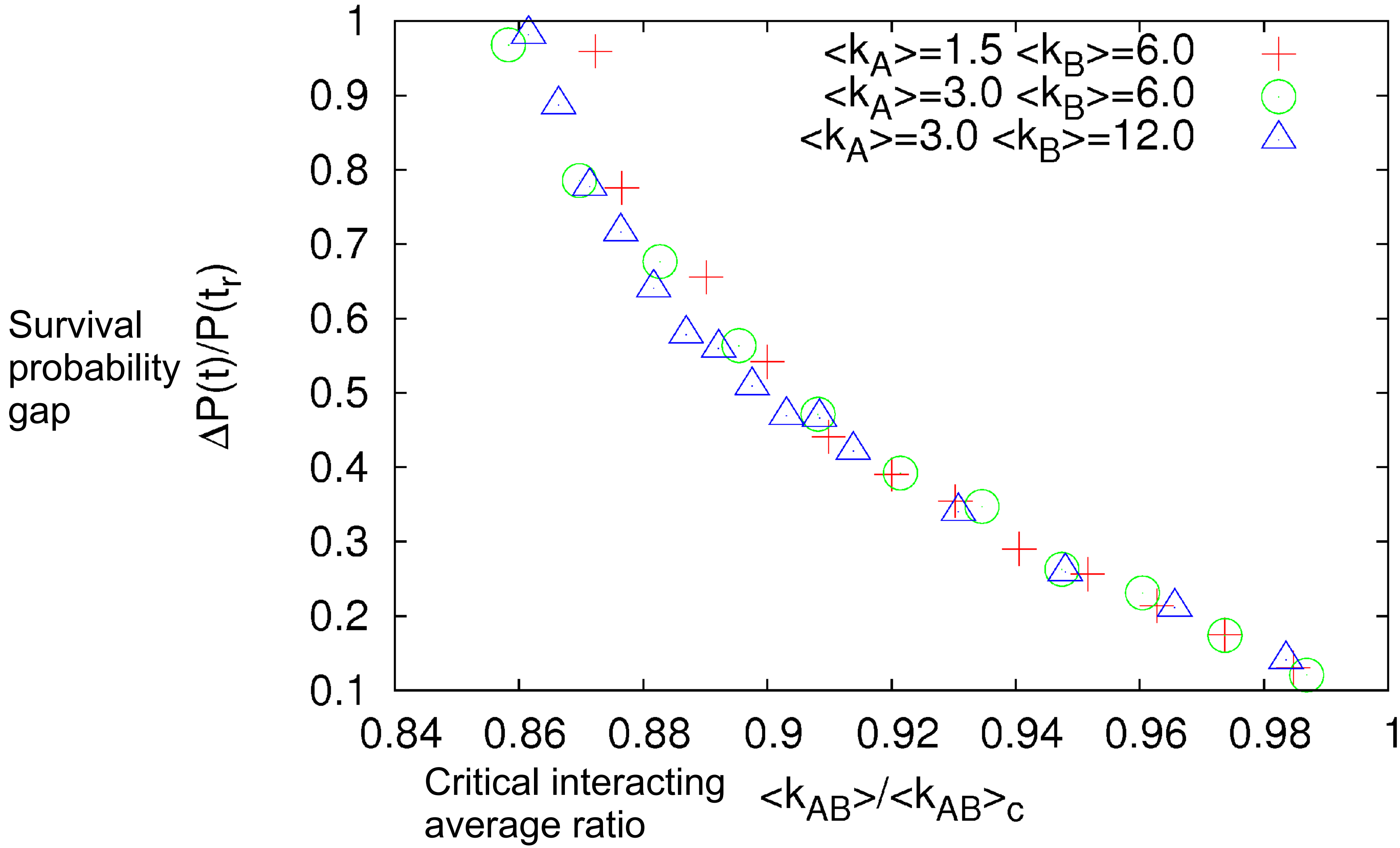


Vertical slices across the phase diagram
Gap drops to zero outside mixed phase region

Disease-Free **Mixed** **Epidemic**

$\langle k_{AB}\rangle=0$ ↑ **β=.036** ↑ **β=.044**

Universal Scaling of the Survival Gap



Approaching strong coupling along the $\beta = \beta_c$ line

Universal behavior as $\langle k_{AB} \rangle \rightarrow \langle k_{AB} \rangle_c$

Phase gap persists for $\langle k_{AB} \rangle > 1$

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Conclusions

- Two classes of network systems emerge:

  strongly-coupled and weakly-coupled

- Weakly-coupled network systems:

  new “mixed” phase

- Transition line between mixed and disease-free phase:

  universal behavior

- See arXiv:1201.6339 for more details

Outline

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Networks, our Disease Model and Epidemics

- Quarantine Generated Phase Transitions
- Epidemics in Interacting Networks
- Conclusions and Ongoing Work

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Collaborators

H.E. Stanley Shlomo Havlin Lidia Braunstein

Roni Parshani, Reuven Cohen – Bar Illan University

Cecelia Lagorio, Pablo A. Macri, and M. V. Migueles – Universidad del Mar de Plata

Federico Vazquez – Max Plank Institute

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Conclusions/Ongoing Work

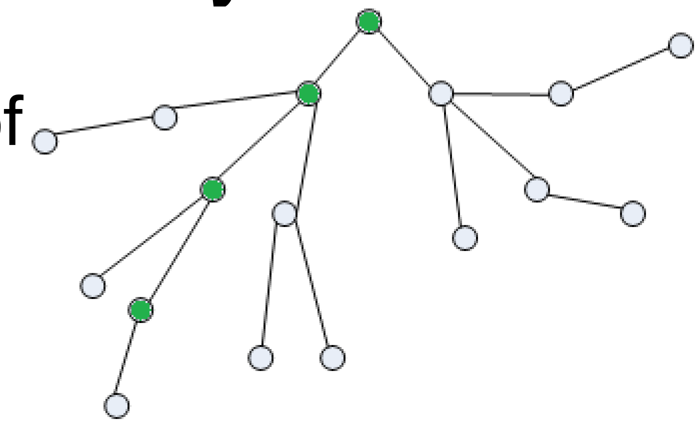
- Network Physics can offer insight into important real world problems: When and how well quarantine works, How network of network structures impacts epidemic spreading

- Still rich areas of work to explore, combine two concepts: Quarantine on Interacting Networks

- Other works not in Dissertation/Published:
  - Preferential Attachment in the Interaction between Dynamically Generated Interdependent Networks
  - Are your friends who you think they are? Implicit vs Explicit Social Networks in Online Forums

Supplemental Material Follows

Mean Field Epidemic Theory



Near and below criticality the number of susceptible neighbors an infected site has is

$$n_s(t) = (\kappa - 1)(1-\beta)^t$$

Branching factor $\kappa = \langle k^2 \rangle / \langle k \rangle$, number of nodes reachable following a randomly selected link.

Sum for expected number of infected neighbors

$$n_I(t_r) = \beta \sum_{t=0}^{t_r - 1} (\kappa - 1)(1-\beta)^t$$

$$= (\kappa - 1)[1 - (1-\beta)^{t_r}]$$

Epidemic thresholds

Epidemic occurs when each infected node infects at least one neighbor:

$$n_I = (\kappa - 1)[1 - (1 - \beta)^{t_r}] = 1$$

For given network parameters and t_r

$$\beta_c = 1 - [1 - (\kappa - 1)^{-1}]^{1/t_r}$$

Epidemic thresholds

Epidemic occurs when each infected node infects at least one neighbor:

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For given network parameters and t_r

$$\beta_c = 1 - [1 - (\kappa - 1)^{-1}]^{1/t_r}$$

Critical Point time scaling

$$\frac{ds}{dt} = -\beta_I is + \gamma i, \frac{di}{dt} = \beta_I is - \gamma i,$$

making use of the condition $i + s = 1$, we arrive at a single rate equation

$$\frac{di}{dt} = (\beta_I - \gamma)i - \beta_I i^2.$$

Approaching criticality ($\beta_I = \gamma$) from above ($0 < i \ll 1$), the stationary density of i vanishes as $i^{stat} \sim \beta_I$. The mean field density is thus linear in the order parameter, and the density exponent is $\beta' = 1$. Approaching criticality from below, we have

$$\frac{di}{dt} \approx -\delta i. \qquad (1.37)$$

Which shows the density in the inactive phase decaying with time as $i(t) \sim e^{-t} = e^{-t/\xi_\|}$,

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System Size Scaling Relation

To learn about the size-dependent properties of dynamic networks we determine the DP properties as a function of the network size N, rather than as a function of t. In DP at criticality, the infinite dimensional relationship between w, the width in the transverse axes, and t, the length in the longitudinal axes, is $w \sim t^{1/2}$. The upper critical dimension d_c is the lowest dimension for which the system has the properties of an infinite dimensional system. For DP this value is $d_c = 4 + 1$ (1 corresponds to the longitudinal axis), so the relation between the system size at the upper critical dimension and the size of a dynamic network is given by $N \sim w^4$ (the power 4 comes from the 4 transverse dimensions of d_c). Since $w \sim t^{1/2}$ we conclude that:

$$t \sim N^{1/2}$$

Therefore, for a dynamic network of size N at criticality, $P_s(t)$ decays exponentially after a time $t_\times$, with $t_\times \sim N^{1/2}$

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