DEPARTMENTAL SEMINAR

Spontaneous recovery and metastability in single and interacting networks

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Outline

- 1. Introduction: failures & recoveries
- 2.1 **Single networks** phase diagram
- 2.2 Finite size effects (single networks)
- 3.1 **Interacting networks** phase diagram
- 3.2 Finite size effects & empirical support

MOTIVATION
Let’s start with one mystery:

Phenomenon:
Some **networks**, after they fail,are able to **become spontaneously active again**.

Examples:
-TRAFIC NETWORK: traffic jams suddenly easing
-BRAIN: people waking from a coma, or having seizures
-**FINANCIAL NETWORKS:** flash crashes in finance

→The process often occurs repeatedly: ***collapse, recovery, collapse, recovery,...***

We need: metastable states and nontrivial phase diagrams

We need a network model with failures and recoveries.

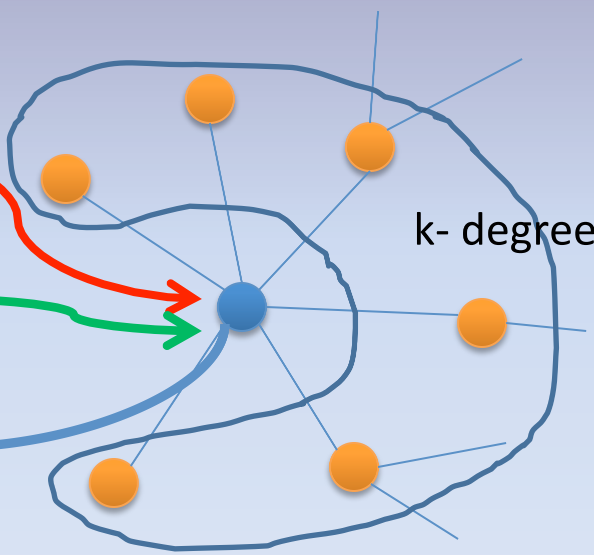
2.1. SINGLE NETWORK

• Each node in a network can be **active** or **failed**.

• We suppose there are **TWO possible reasons for the nodes' failures**: INTERNAL and EXTERNAL.

1. INTERNAL failure: intrinsic reasons inside a node

2. EXTERNAL failure: damage "imported" from neighbors

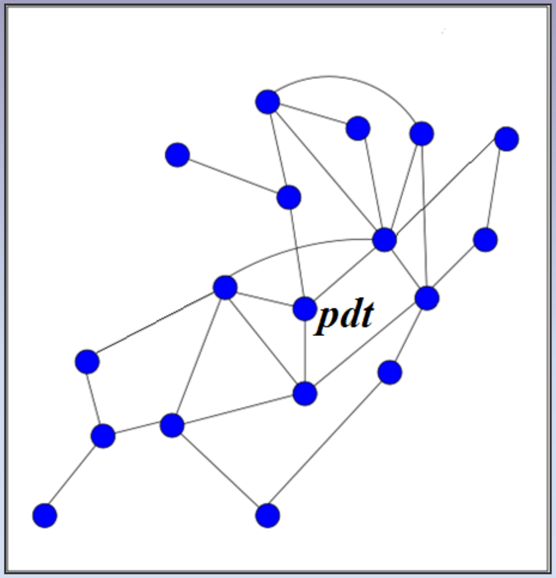


RECOVERY: A node can also **recover** from each kind of failure.

LET'S SPECIFY/MODEL THE RULES.

1. INTERNAL FAILURES

p- rate of internal failures (per unit time, for each node). During interval *dt*, there is probability **p***dt* that the node fails.



Recovery: A node *recovers from* an *internal failure after a time period* ***τ*** *.*

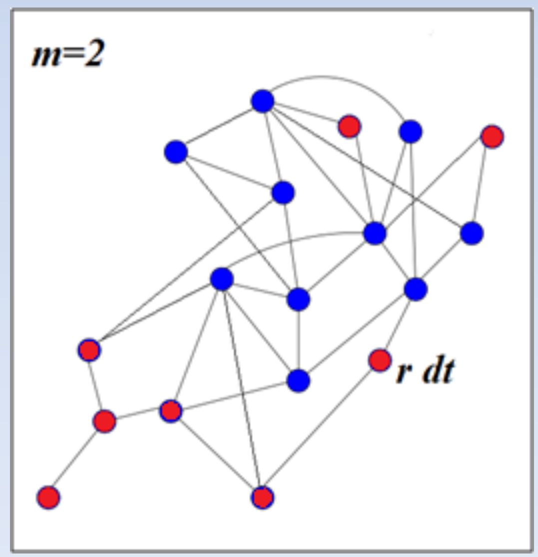
2. EXTERNAL FAILURES – if the neighborhood of a node is too damaged

IF: "CRITICALLY DAMAGED neghborhood": **less than or equal to m active neighbors, where m is a fixed treshold parameter.**

THEN: There is a probability $r\,dt$ that the node will experience externally-induced failure during *dt*.

r - external failure rate

A node recovers from an external failure after time τ'.



FAILURE TYPE	RULE	RECOVERY
Internal failure	With rate ***p*** on each node	After time ***τ***
External failure	IF(<= m active neighbors) THEN Extra rate **r** on each node	After time ***τ'***

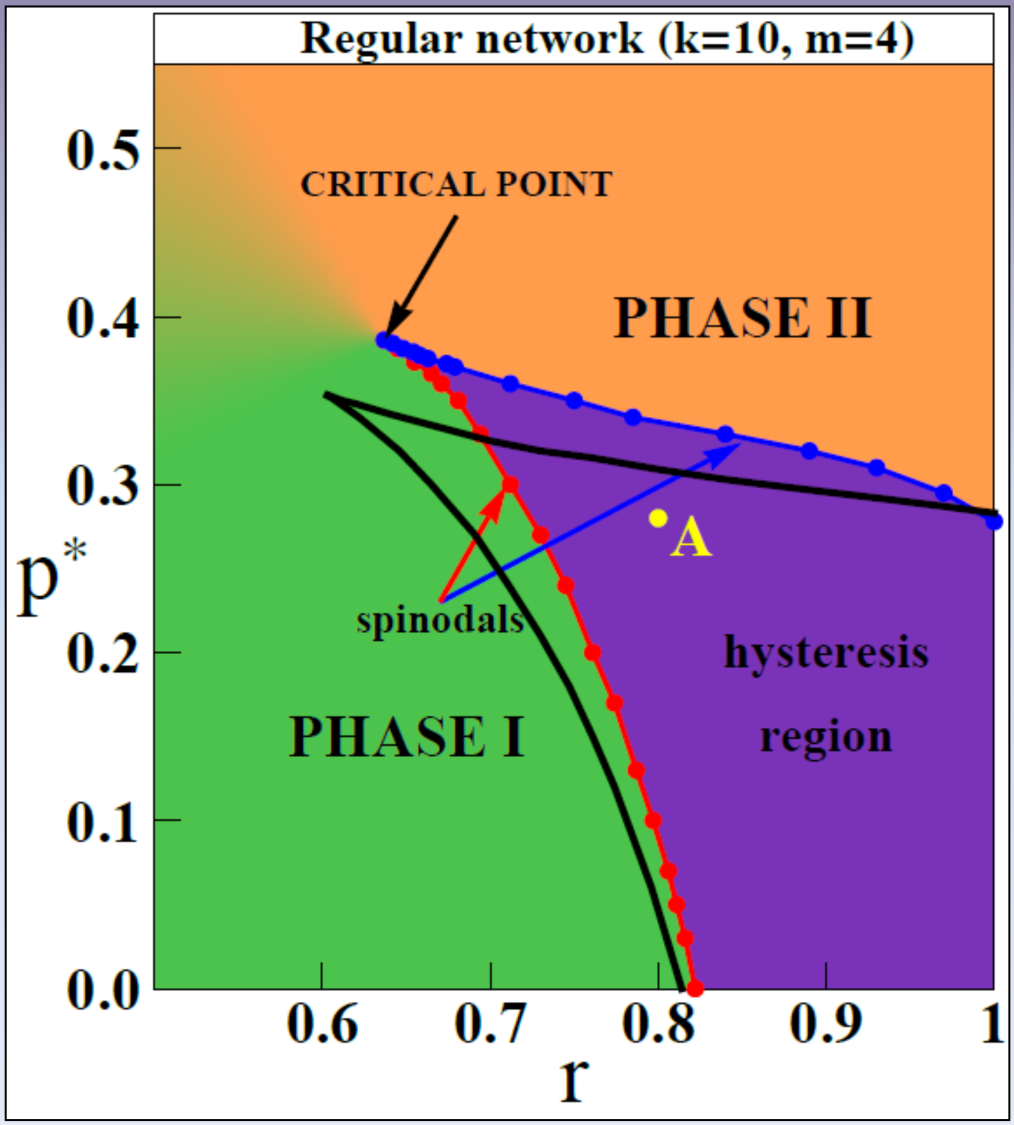
Out of these 5 parameters, we fix three of them:
m=4, *τ =100 and τ '=1*.

We let (p,r) to vary.

It turns out it is convenient to define *p*=exp(-pτ)*.
So we use (p*,r) instead of (p,r).

We measure activity Z of the network as a function of (p*,r).

Phase diagram (single network, random regular)



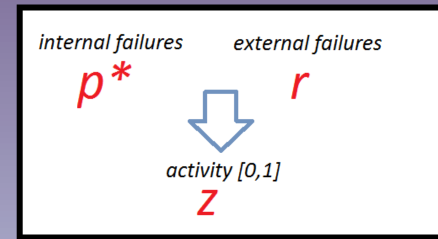
GREEN; High activity Z
ORANGE: Low activity Z

In the hysteresis region both phases exist, depending on the initial conditions or the memory/past of the system.

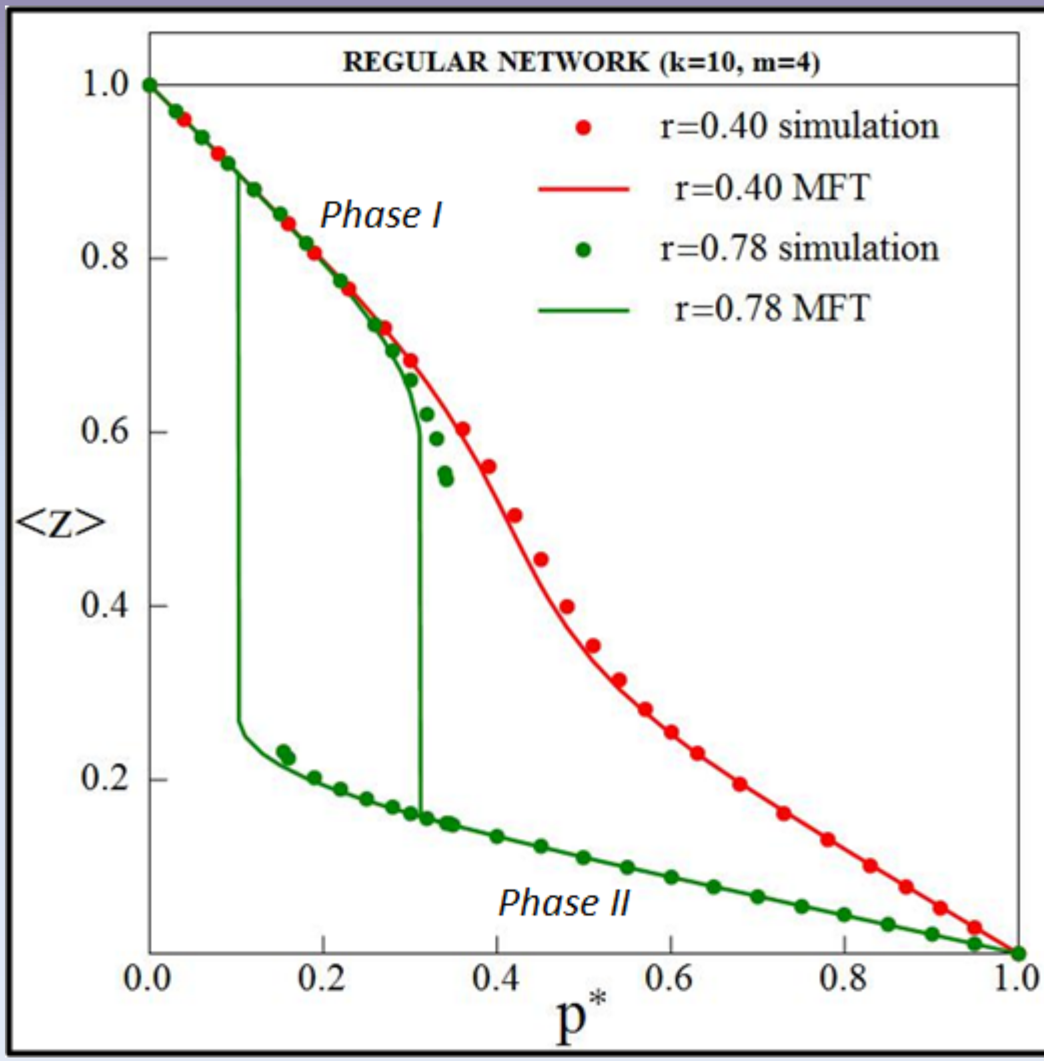
Blue line: critical line (spinodal) for the abrupt transition
I→ II

Red line: critical line (spinodal) for the abrupt transition
II→I

Model simulation [Random regular networks]



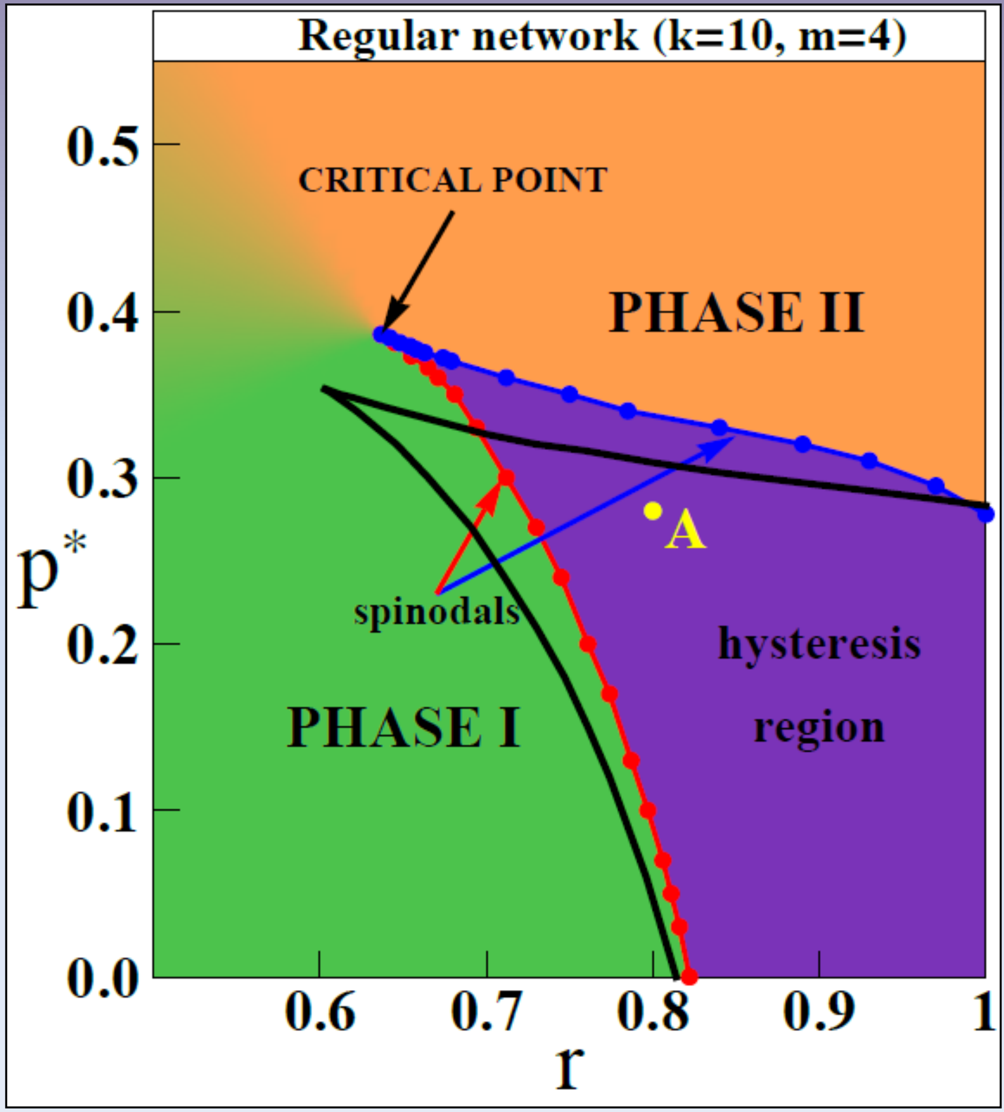
<z>- average fraction of active nodes (Z fluctuates)



We fix r, and measure <z>(p*)

For some values of *r* we have a hysteresis loop.

Let's pick point A, take a small system N=100, and run the simulation

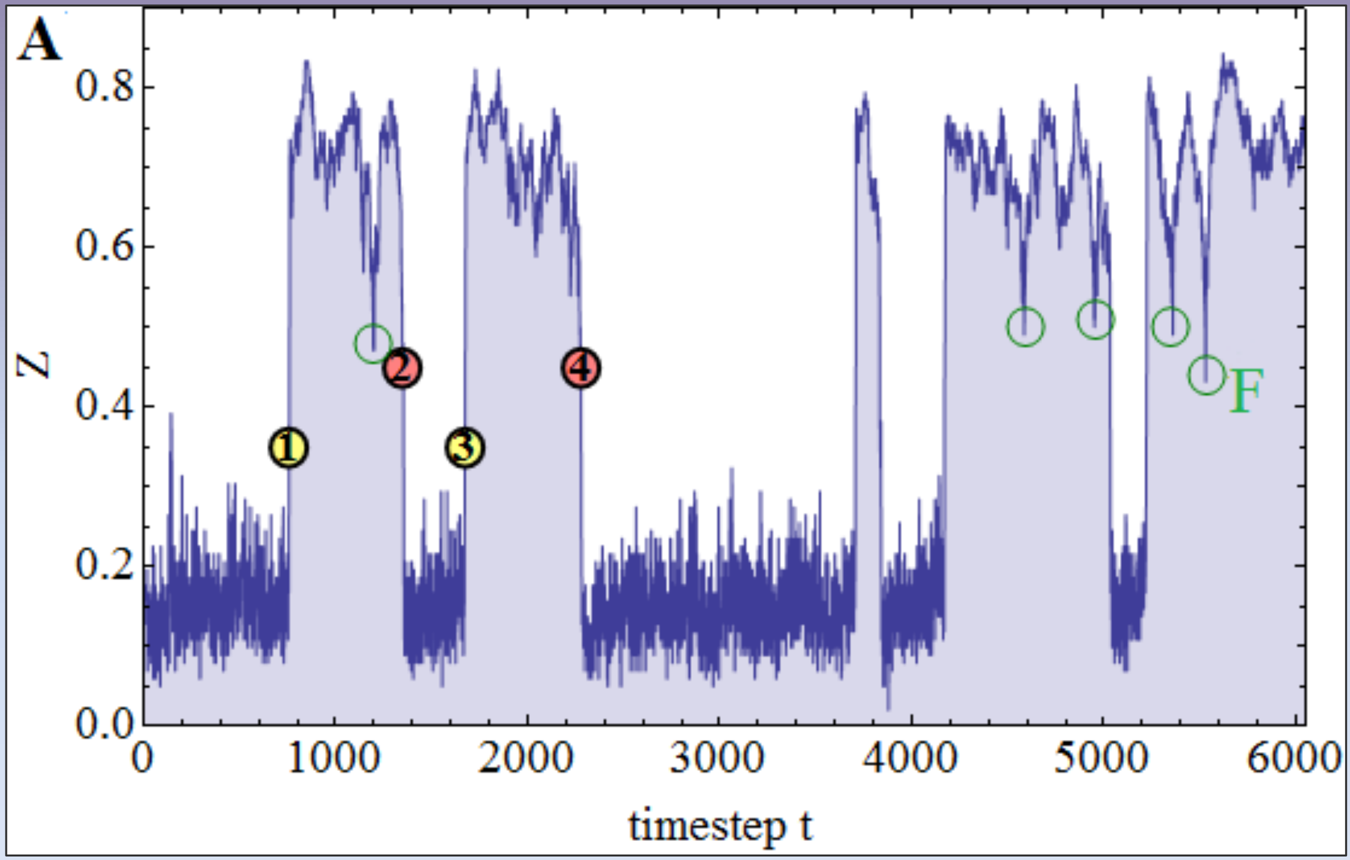


Finite size effects

Sudden transition!

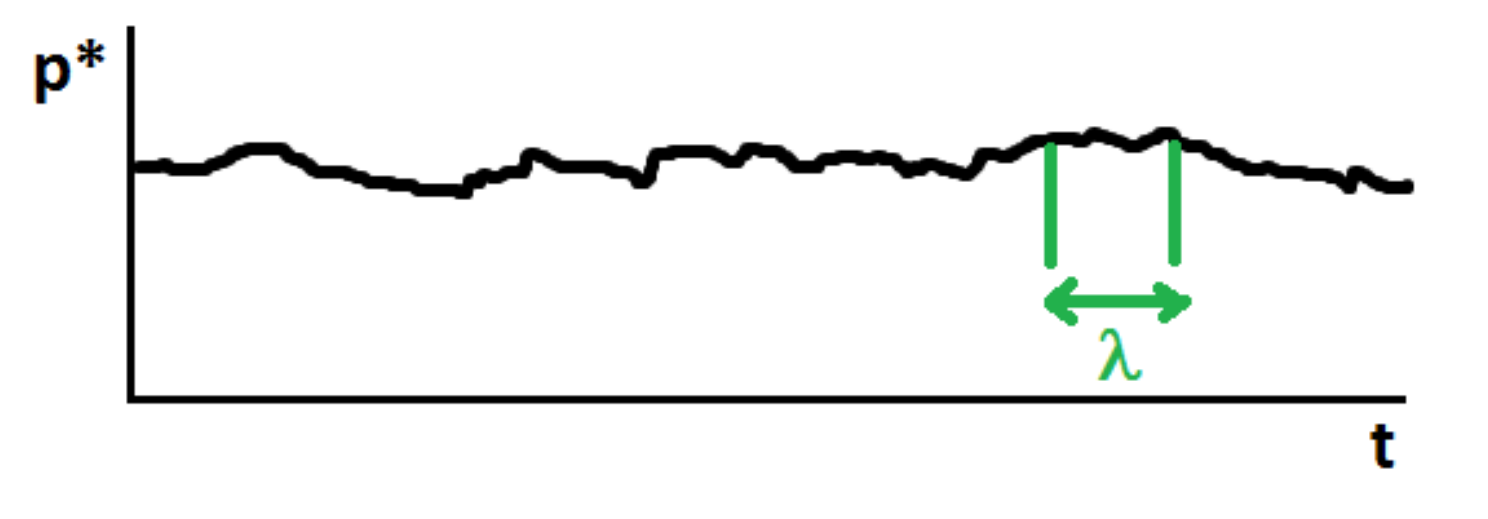
1. Why? How?

2. Is there any forewarning?



(Remember : Z = Fraction of active nodes)

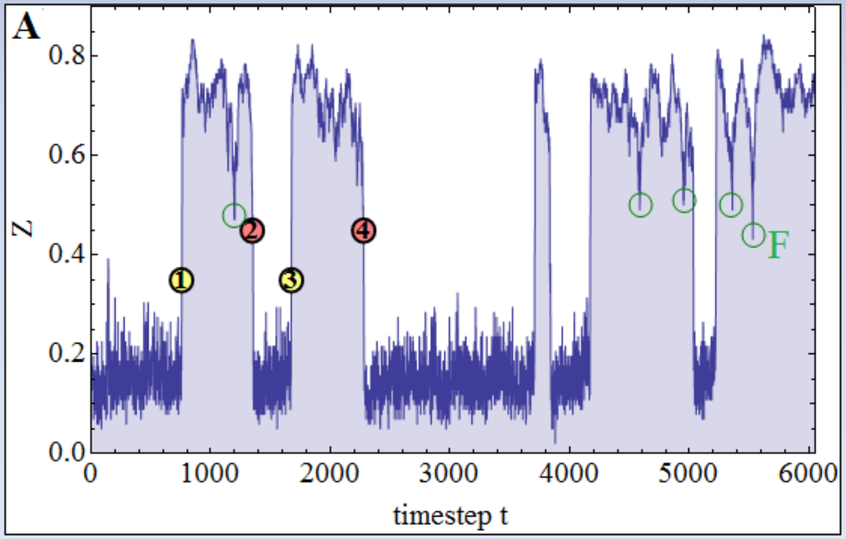

GEORGE WASHINGTON
1st PRESIDENT 1789-1797

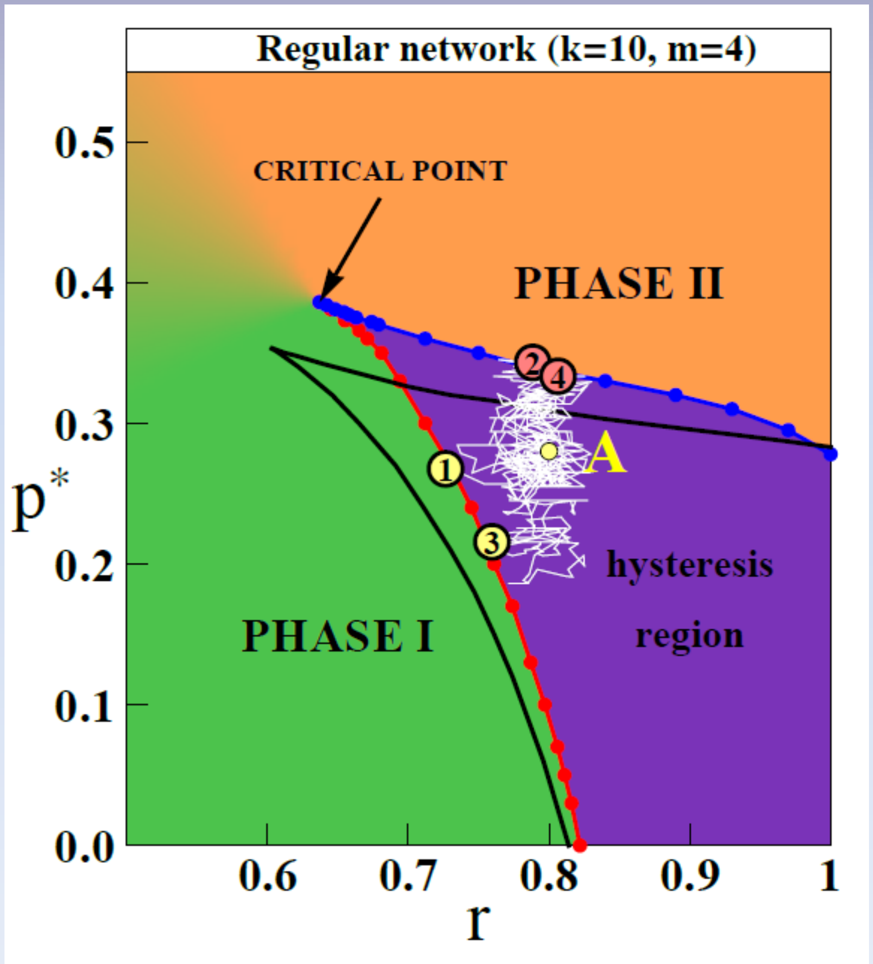

p*
λ
t

It turns out it can be predicted.

Trajectory $(r_\lambda(t),\ p_\lambda{}^*(t))$ in the phase diagram (white line, see below).

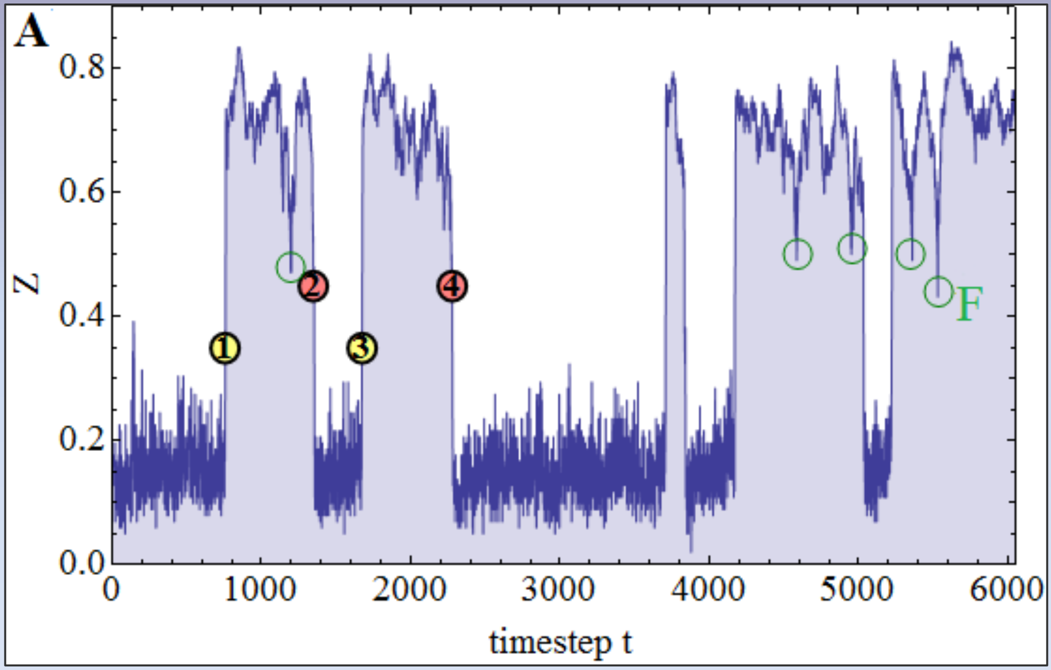
The trajectory crosses the spinodals (critical lines) interchangeably, and causes the phase flipping.





Second finite size phenomenon: Flash crashes

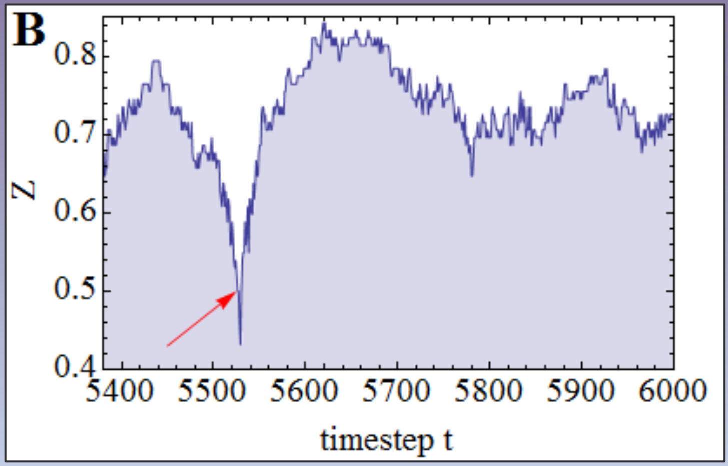
An interesting (and unexpected) by-product of the model:



Sometimes the network rapidly crashes, and then quickly recovers (green circles).

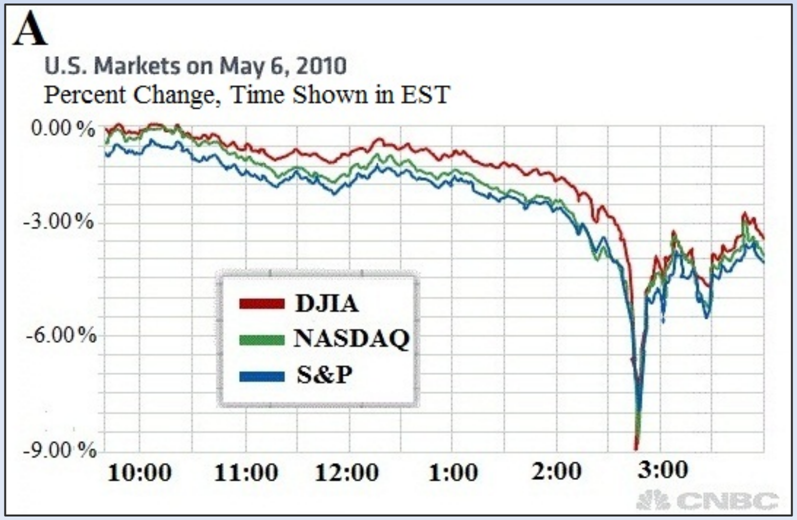
Model predicts the existance of “flash crashes”.

Explanation: Unsuccessful transitions to a lower state.



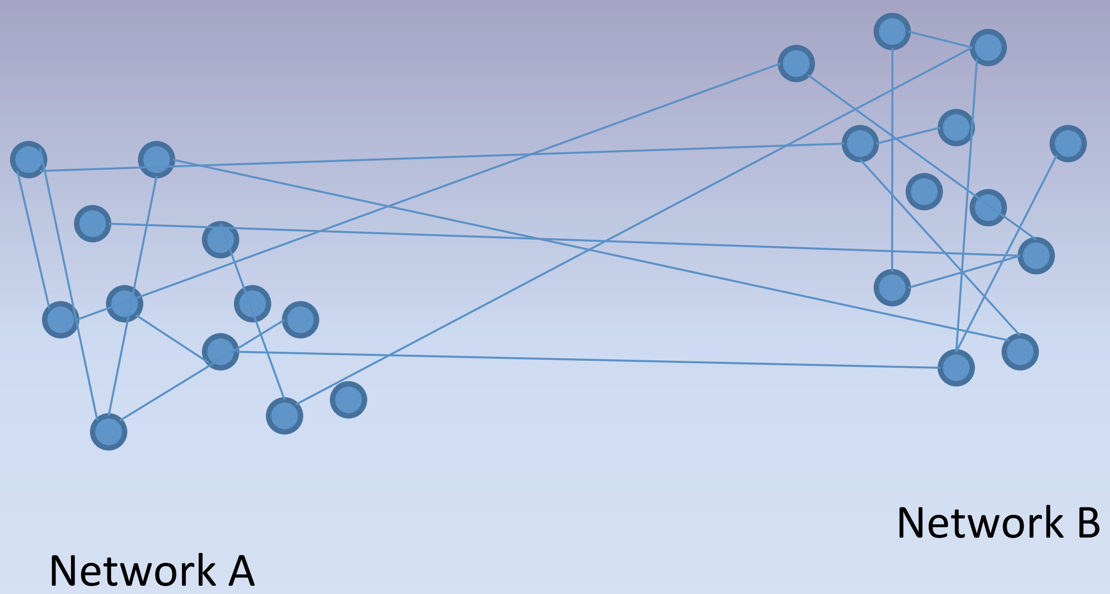
Real stock markets also show a similar phenomenon.

Q: Possible relation?

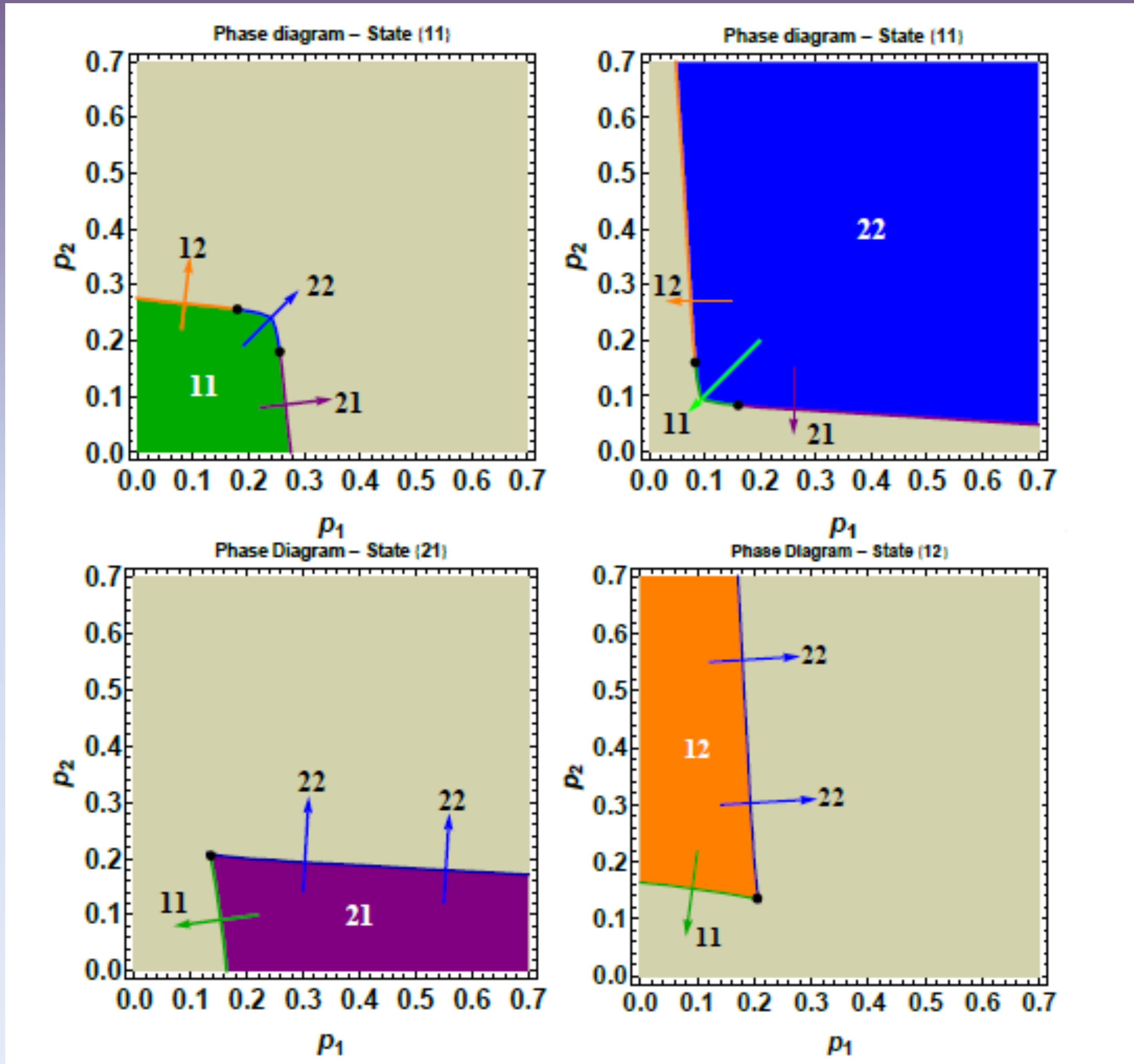


“Flash Crash 2010”

3.1. Interacting networks



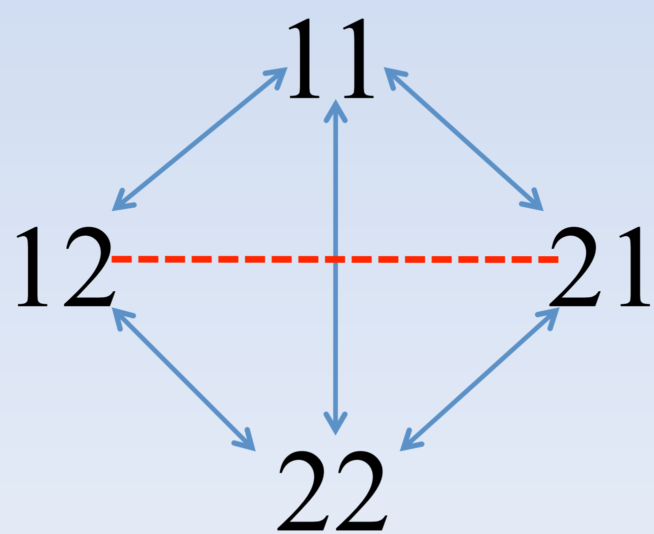
FAILURE TYPE	RULE	RECOVERY
Internal failure	With rate ***p*** on each node	After time τ
External failure	IF(<= m active neighbors) THEN Extra failure rate **r**	After time τ'
Dependency failure	IF(companion node from the opposite network failed) THEN Extra failure rate $\mathbf{r_d}$	After time τ''

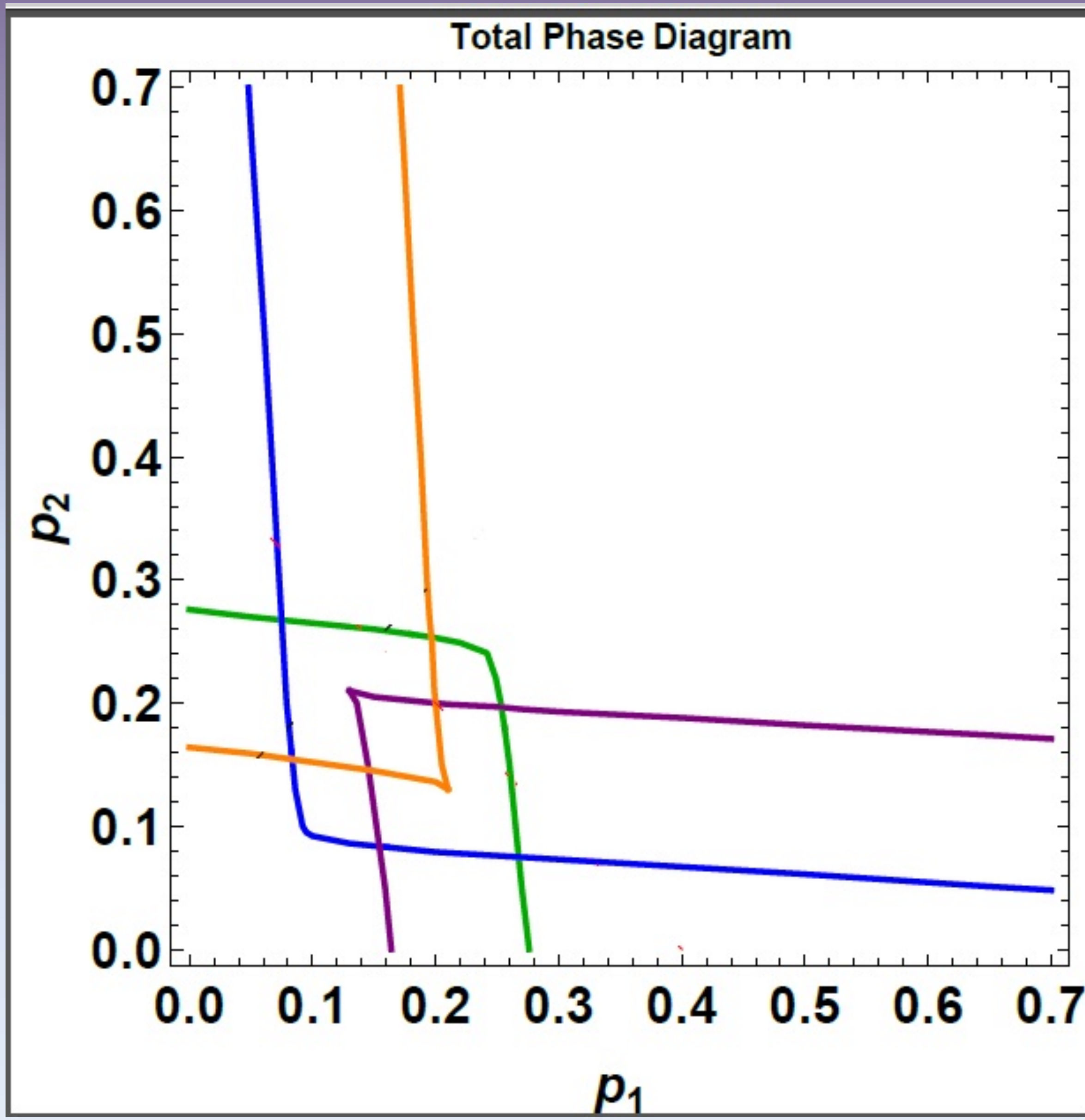


Phase diagram – State |11⟩
12
22
11
21
Phase diagram – State |11⟩
22
12
11
21
Phase Diagram – State |21⟩
22
22
11
21
Phase Diagram – State |12⟩
22
12
22
11
p_1
p_2

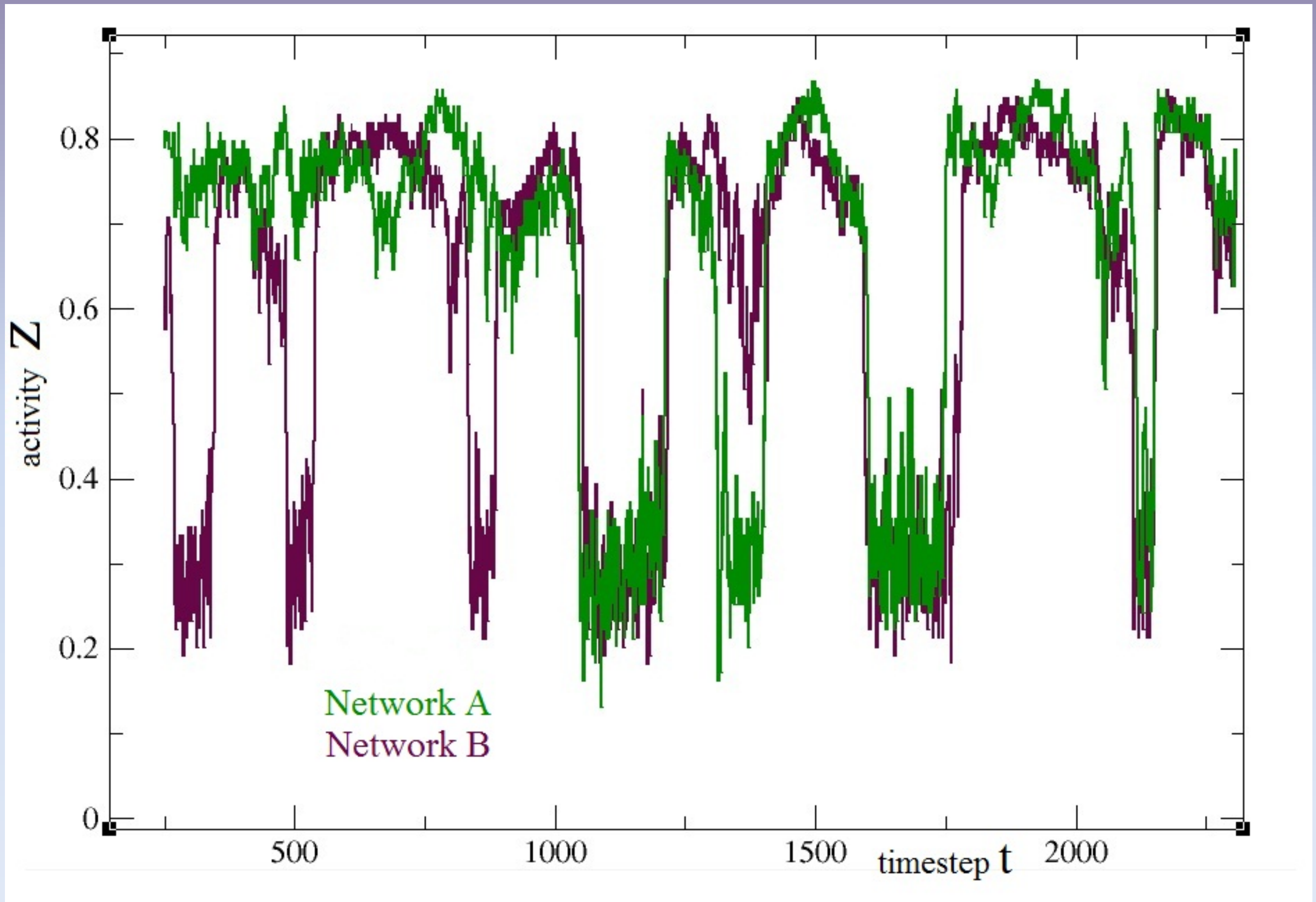
Elements of the phase diagram

- 2 critical points
- 4 triple points
- 10 allowed transitions
- 2 forbidden transitions

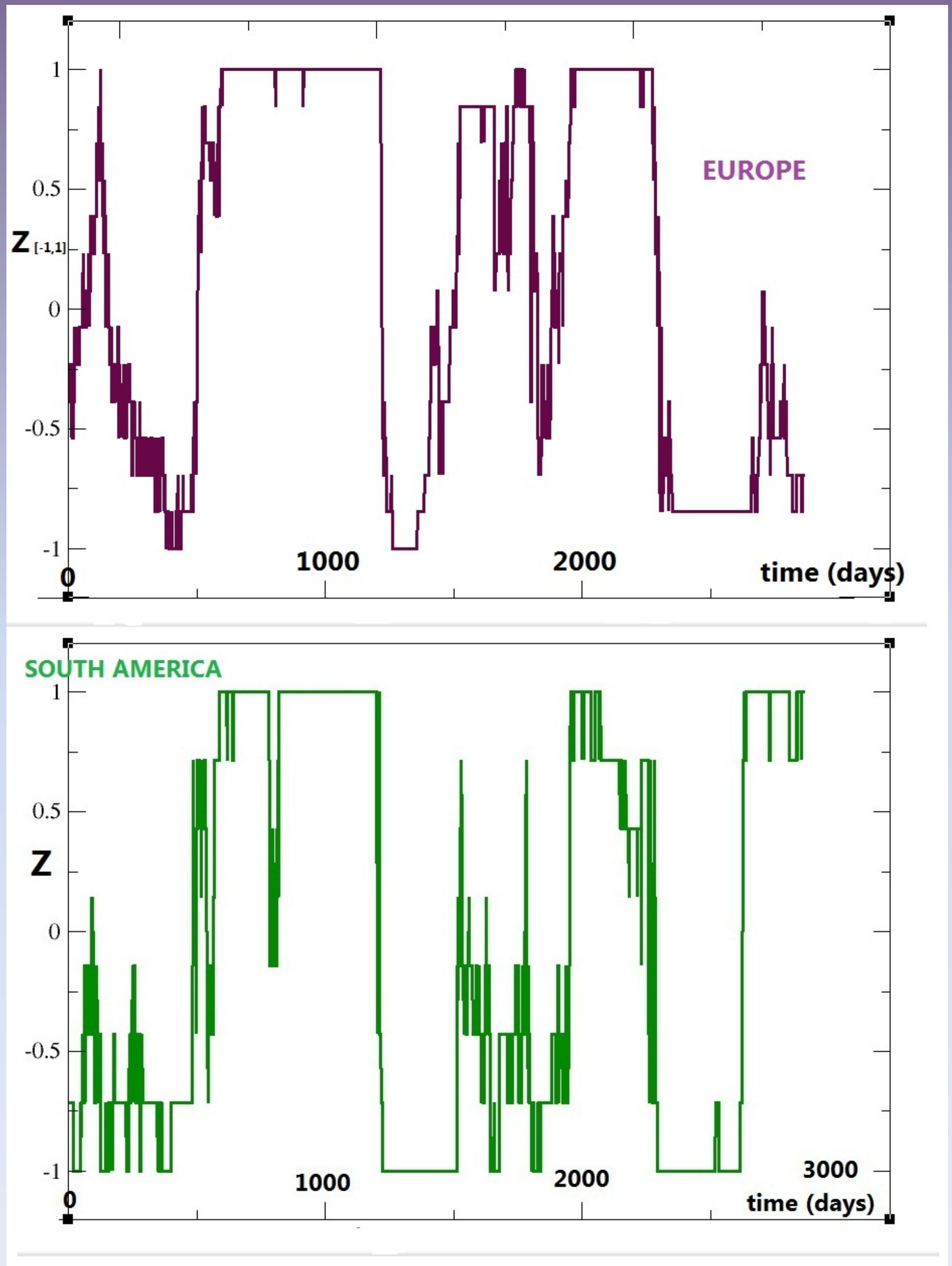



Total Phase Diagram
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
p_2
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
p_1

Two interacting networks: phase switching (MODEL)



CDS
(real data)



- Thank you for your time.

BONUS: Problem of optimal treatment