Spontaneous recovery and metastability in single and interacting networks

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1. Introduction: failures & recoveries

2.1 Single networks phase diagram

2.2 Finite size effects (single networks)

3.1 Interacting networks phase diagram

3.2 Finite size effects & empirical support
MOTIVATION
Let’s start with one mystery:

Phenomenon:
Some networks, after they fail, are able to become spontaneously active again.

Examples:
- TRAFFIC NETWORK: traffic jams suddenly easing
- BRAIN: people waking from a coma, or having seizures
- FINANCIAL NETWORKS: flash crashes in finance

→The process often occurs repeatedly: collapse, recovery, collapse, recovery,...

We need: metastable states and nontrivial phase diagrams
We need a network model with failures and recoveries.

2.1. SINGLE NETWORK

• Each node in a network can be active or failed.

• We suppose there are **TWO possible reasons for the nodes’ failures:** INTERNAL and EXTERNAL.

  1. **INTERNAL failure:** intrinsic reasons inside a node

  2. **EXTERNAL failure:** damage “imported” from neighbors

**RECOVERY:** A node can also recover from each kind of failure.

LET’S SPECIFY/MODEL THE RULES.
1. INTERNAL FAILURES

**p- rate of internal failures** (per unit time, for each node).
During interval $dt$, there is probability $pdt$ that the node fails.

**Recovery**: A node *recovers from an internal failure after a time period* $\tau$. 
2. EXTERNAL FAILURES – if the neighborhood of a node is too damaged

IF: “CRITICALLY DAMAGED neighborhood”: less than or equal to $m$ active neighbors, where $m$ is a fixed threshold parameter.

THEN: There is a probability $r \, dt$ that the node will experience externally-induced failure during $dt$.

$r$ - external failure rate

A node recovers from an external failure after time $\tau'$. 
<table>
<thead>
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<th>FAILURE TYPE</th>
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<th>RECOVERY</th>
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<td>Internal failure</td>
<td>With rate $p$ on each node</td>
<td>After time $\tau$</td>
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<td>External failure</td>
<td>IF($\leq m$ active neighbors) THEN Extra rate $r$ on each node</td>
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Out of these 5 parameters, we fix three of them: $m=4$, $\tau =100$ and $\tau' =1$.

We let $(p,r)$ to vary.

It turns out it is convenient to define $p^* = \exp(-p\tau)$.
So we use $(p^*,r)$ instead of $(p,r)$.

We measure activity $Z$ of the network as a function of $(p^*,r)$. 
Phase diagram (single network, random regular)

In the hysteresis region both phases exist, depending on the initial conditions or the memory/past of the system.

Blue line: critical line (spinodal) for the abrupt transition $I \rightarrow II$

Red line: critical line (spinodal) for the abrupt transition $II \rightarrow I$

GREEN; High activity $Z$

ORANGE: Low activity $Z$
Model simulation  [Random regular networks]

We fix $r$, and measure $<z>(p^*)$.

For some values of $r$ we have a hysteresis loop.

$<z>$ - average fraction of active nodes ($Z$ fluctuates)
Let’s pick point A, take a small system N=100, and run the simulation.
Finite size effects

Sudden transition!

1. Why? How?

2. Is there any forewarning?

( Remember : $Z = \text{Fraction of active nodes} $)
It turns out it can be predicted.

Trajectory \((r_x(t), p_x^*(t))\) in the phase diagram (white line, see below).

The trajectory crosses the spinodals (critical lines) interchangeably, and causes the phase flipping.
Second finite size phenomenon: Flash crashes

An interesting (and unexpected) by-product of the model:

Sometimes the network rapidly crashes, and then quickly recovers (green circles).
Model predicts the existence of “flash crashes”.

Explanation: Unsuccessful transitions to a lower state.

Real stock markets also show a similar phenomenon.

Q: Possible relation?

“Flash Crash 2010”
3.1. Interacting networks
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<td>Dependency failure</td>
<td>IF(comppanion node from the opposite network failed) THEN Extra failure rate $r_d$</td>
<td>After time $\tau''$</td>
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Elements of the phase diagram

- 2 critical points
- 4 triple points
- 10 allowed transitions
- 2 forbidden transitions
Two interacting networks: phase switching (MODEL)
CDS
(real data)
• Thank you for your time.
BONUS: Problem of optimal treatment