Assortativity Decreases the Robustness of Interdependent Networks

Preliminary Qualifying Oral Examination

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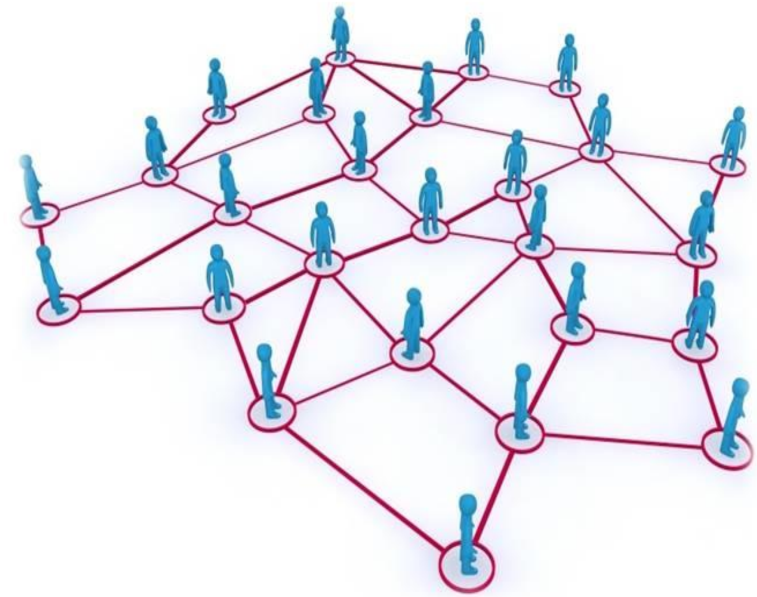
Outline

- Motivation
- Background Knowledge
- Generating Networks with Assortativity
- Phase Behavior of Interdependent Assortativity Network under random attack
- Conclusion
- Future Work

Motivation

- Why we study networks?

Computer Network (Internet, WWW)



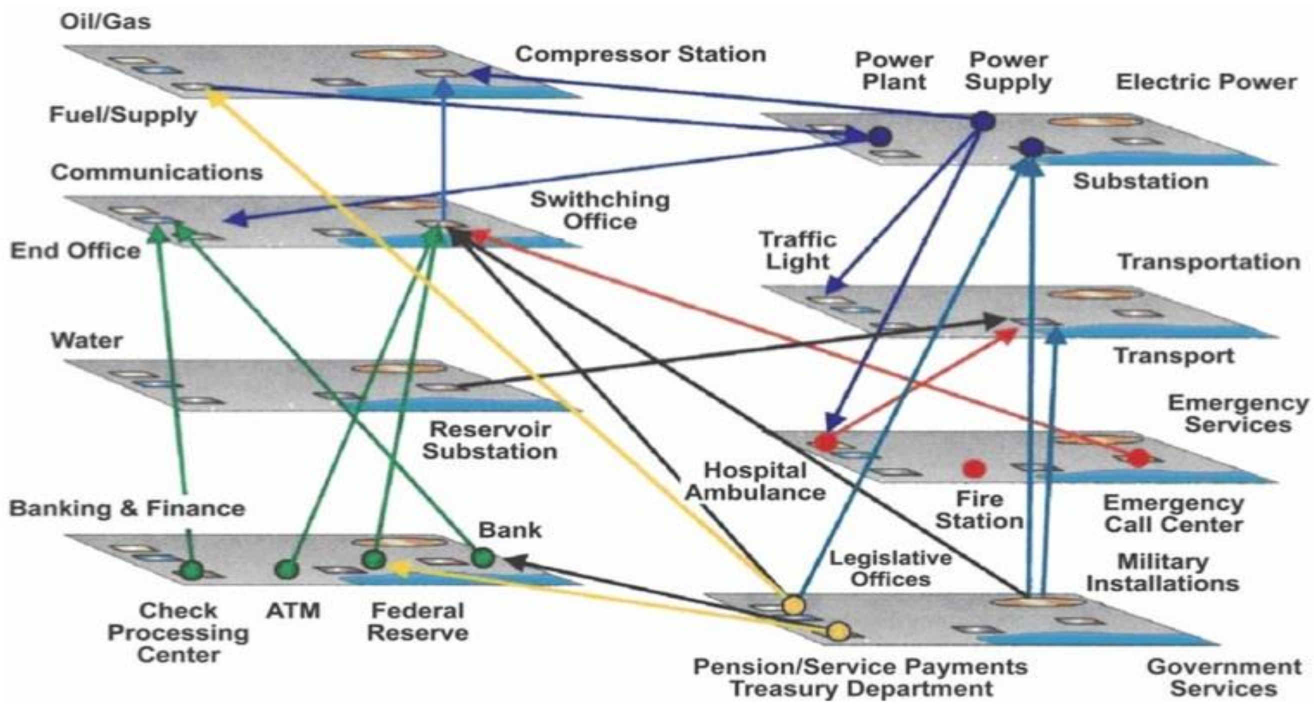
Social Network

Networks are everywhere around us!

Better understanding of networks helps to better utilize / protect them.

Motivation

- Why we study INTERDEPENDENT networks?



Infrastructures (actually, all networks) more or less depend and interact with each other.

Same as single network?

Motivation

- How we study interdependent networks?
  - Different dynamic models:

    SI, SIS, SIR … (Epidemic Model)

    NCO model, majority rules model … (Opinion Model)

    Link / site removing Model … (Percolation Model)

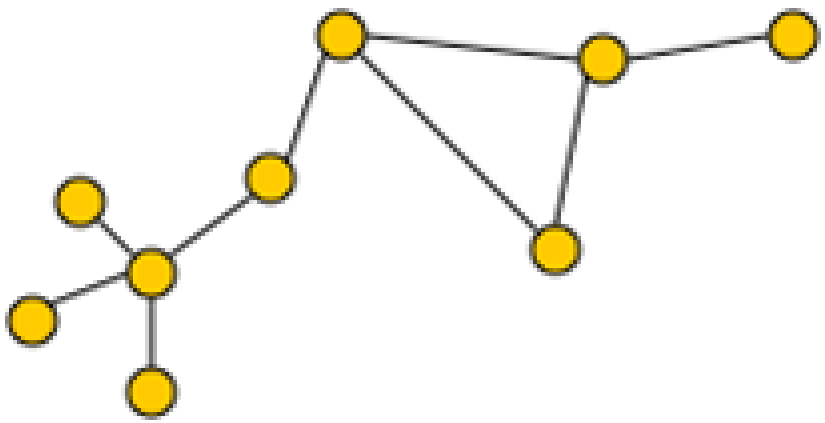
    … …
  - Here we use Site Removing Percolation Model, because it's a better model to study the structural robustness of networks under attack.

Background Knowledge

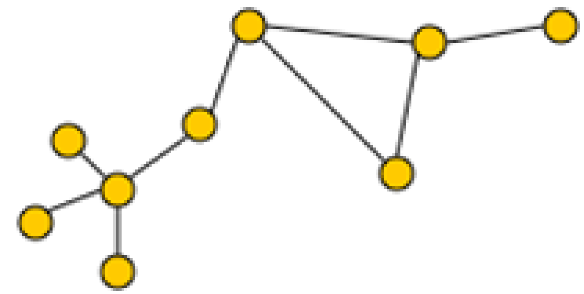
- Network :

  Nodes and Links

  Degree

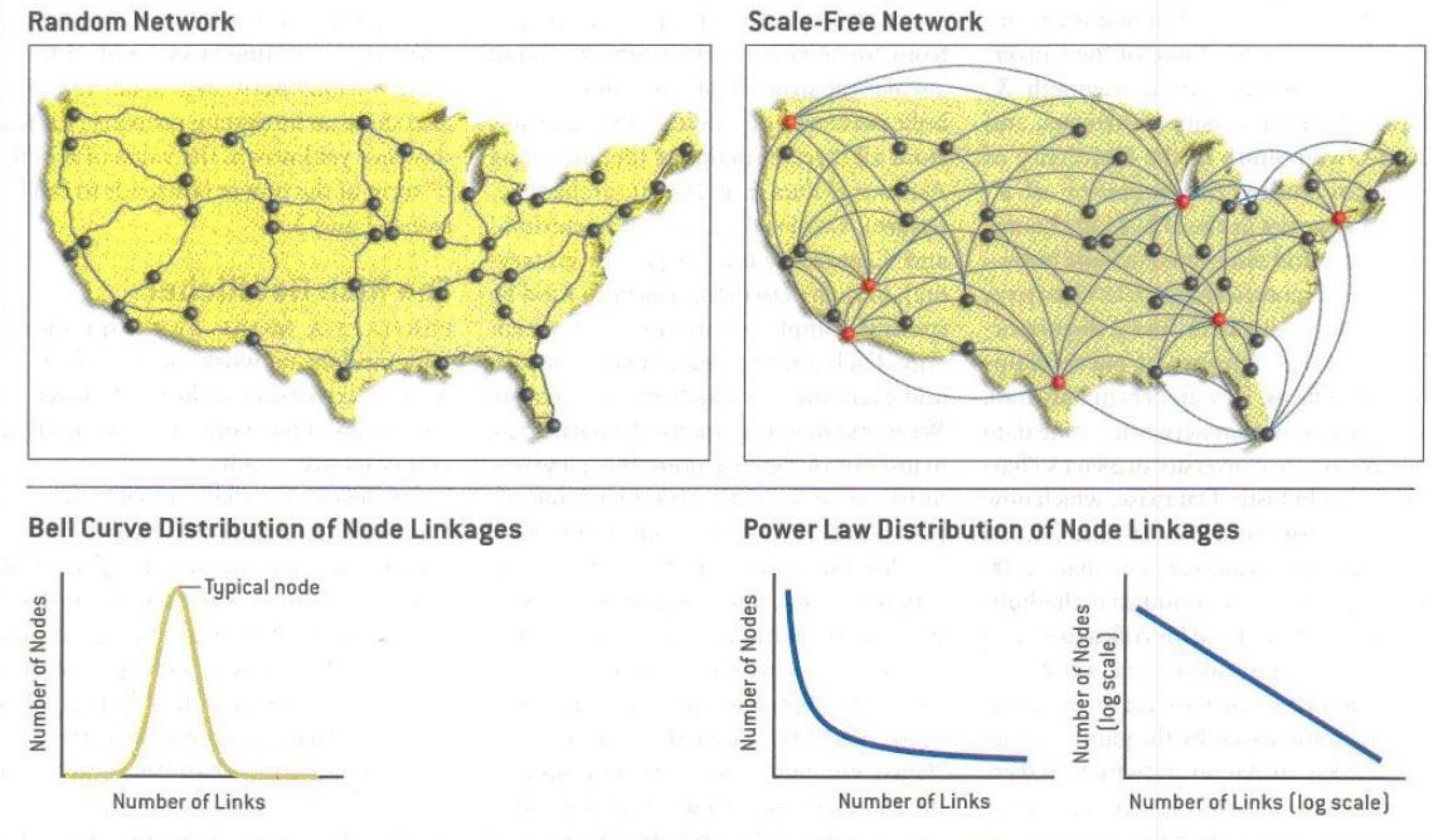


Background Knowledge



- Degree k, average degree <k>
- Degree Distribution P(k)
- Two Major Kinds of Networks:
  - Erdos-Renyi (ER) network
    - $P(k) = \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$, Poisson Distribution
    - Most nodes have about same number of links
  - Scale-Free (SF) network
    - $P(k) = ck^{-\lambda}$, Power-Law Distribution
    - Most nodes have few number of links, but few nodes (hubs) have large number of links (no-scale)

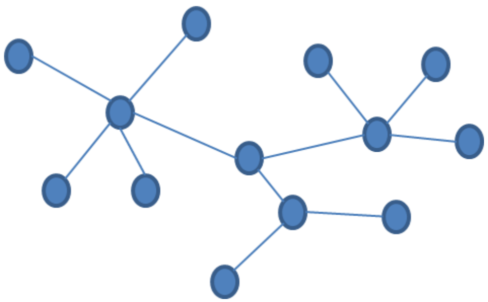
Background Knowledge



A-L Barabasi, Scientific American 2003

Background Knowledge

- Assortativity (degree-degree correlation)
- Giant component (largest cluster)

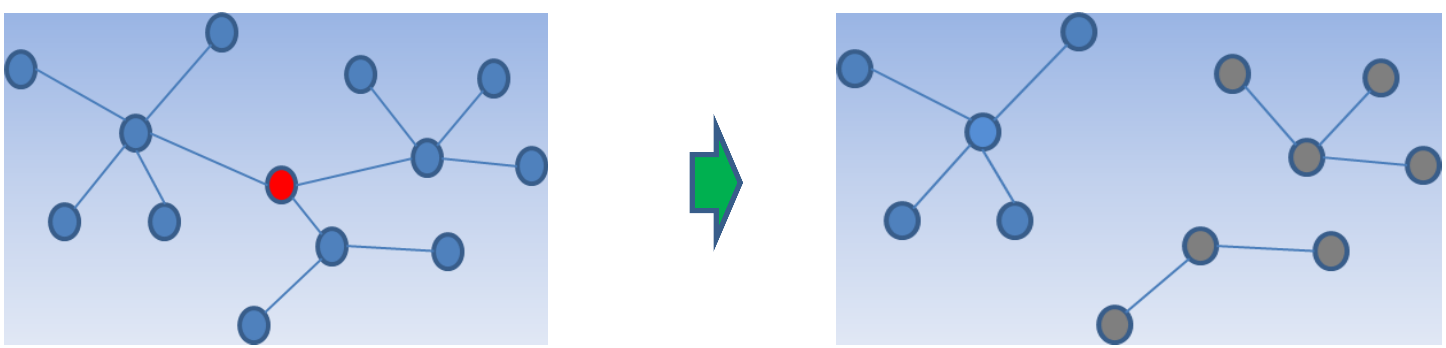


S: Number of nodes in giant cluster

N: Total number of nodes

$s = S / N$: fraction of nodes in S

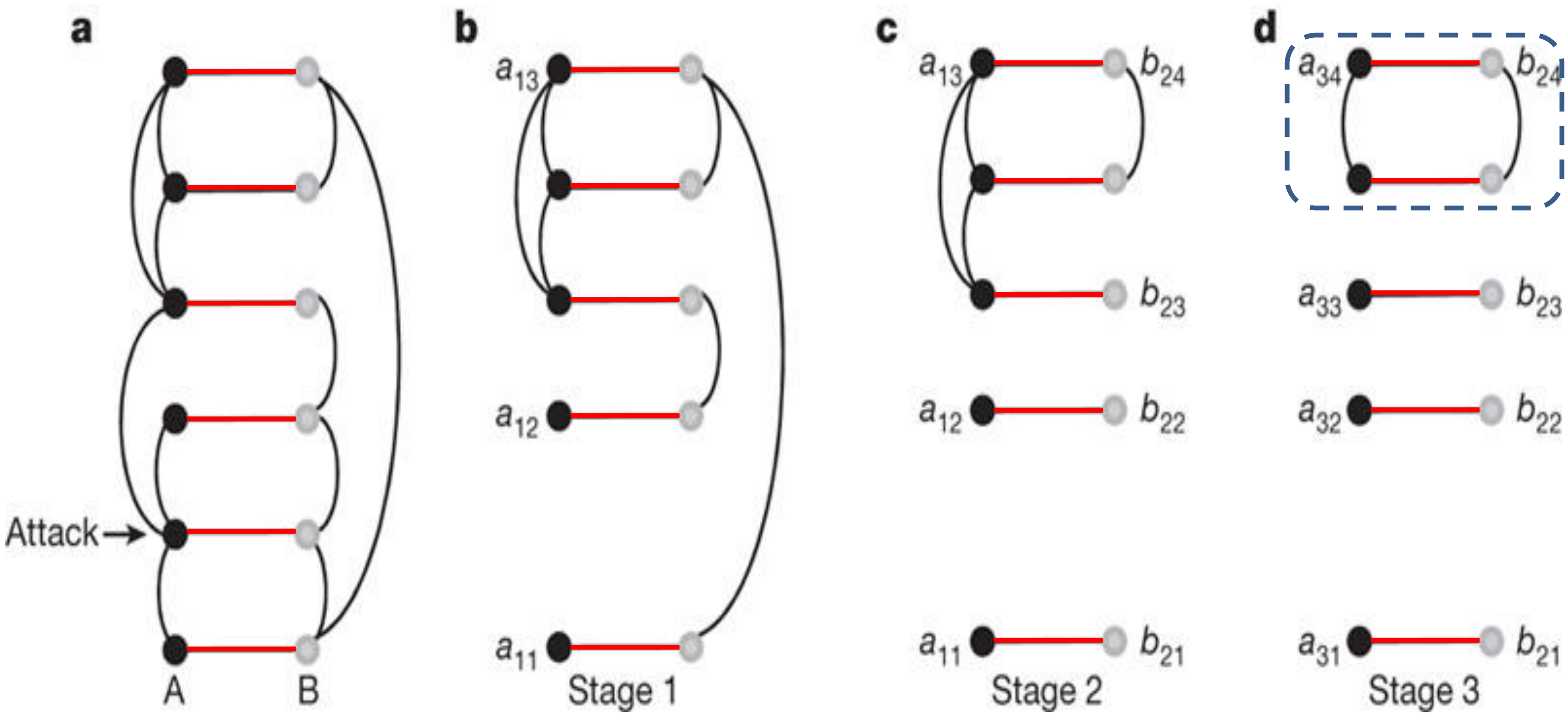
- Under attack : a network with 13 nodes



S=13/13=1 S=5/13

Background Knowledge

- Cascade failure in interdependent networks



Sergey Buldyrev, et al, Nature 2010

Generating Assortativity Network

- Assortativity Coefficient r

$$r \equiv \frac{\langle k_i k_j \rangle_e - \left[\langle (k_i + k_j)/2 \rangle_e\right]^2}{\langle ({k_i}^2 + {k_j}^2)/2 \rangle_e - \left[\langle (k_i + k_j)/2 \rangle_e\right]^2}$$

- Define Hamiltonian H

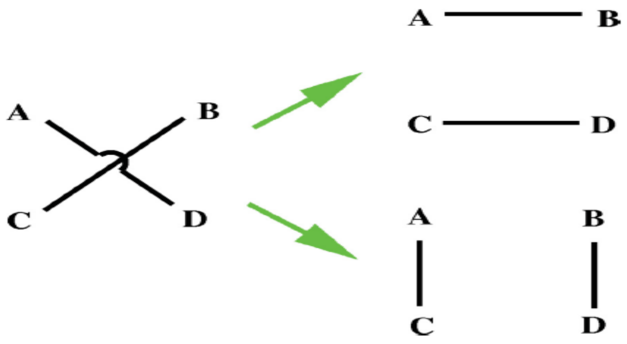
$$H(G) \equiv -J \sum_{i,j} k_i A_{ij} k_j$$

- Monte-Carlo link swapping probability

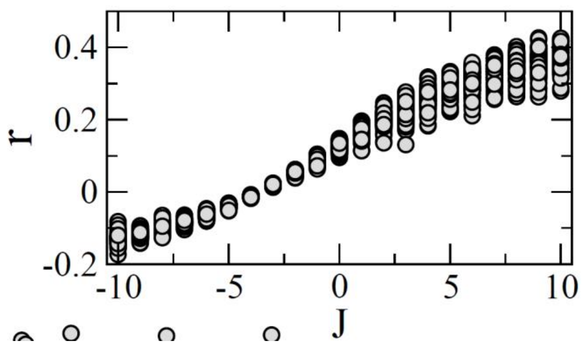
$$P_{swap}(G) = e^{-\Delta H}$$

Generating Assortativity Network

- Swap the link

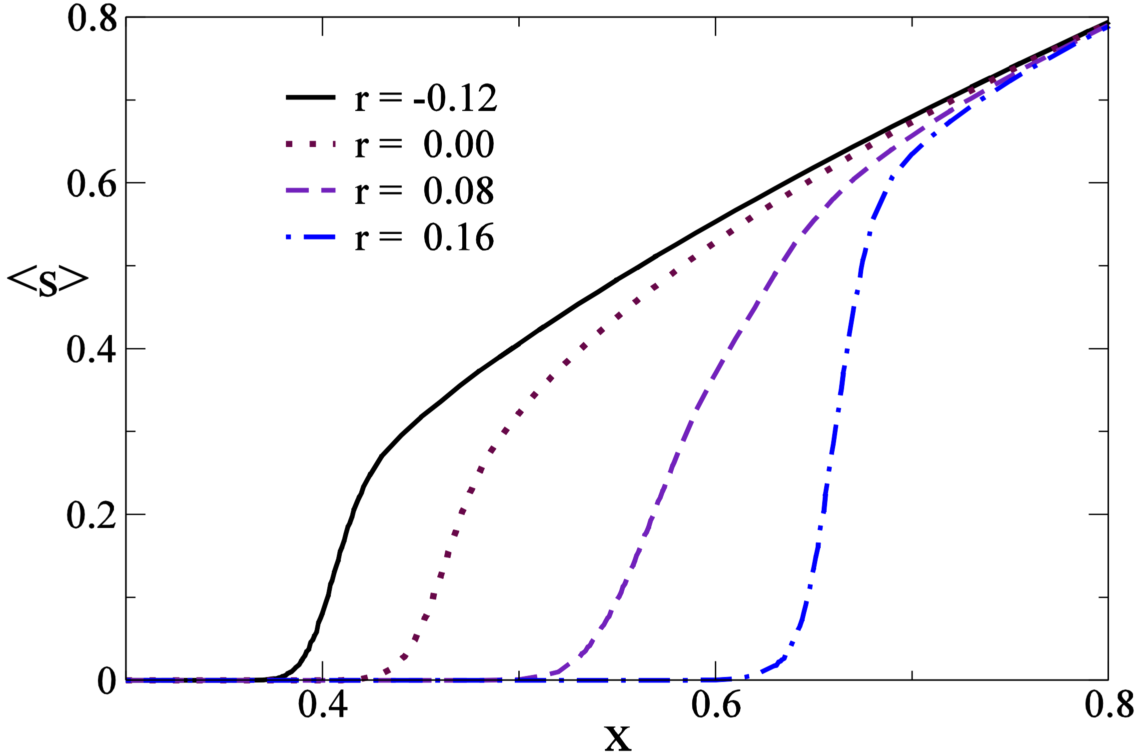


- The P(k), and degree of each node are kept constant
- r is related to H, since $H \propto \langle k_i k_j \rangle$
- If J >0, assortative ; J<0, dis-assortative



Percolation Behavior

- Randomly attack (remove) 1-x fraction of nodes
- <s> as a function of fraction of remaining nodes, x

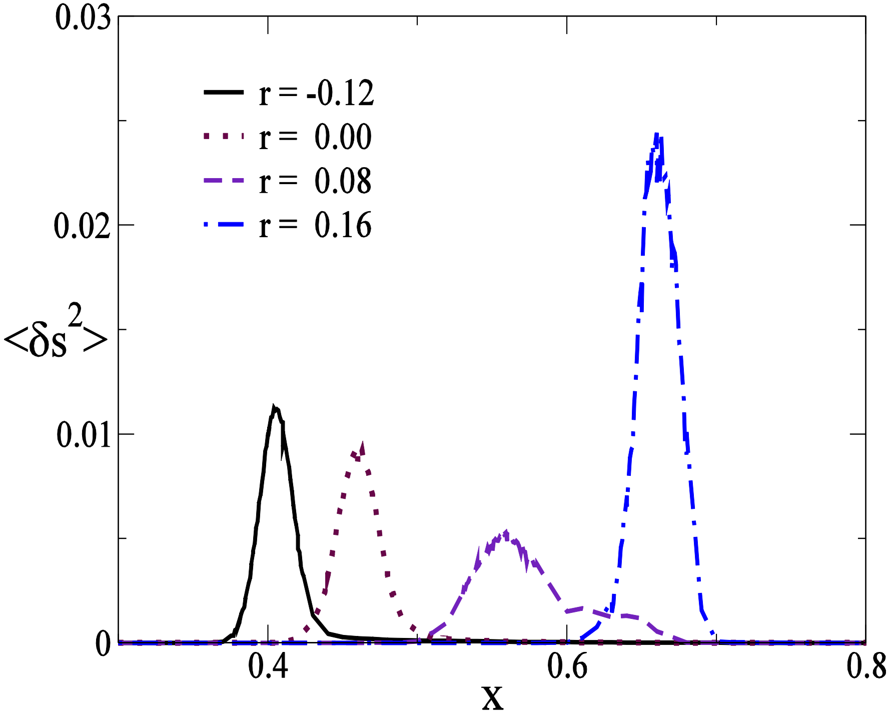


Scale-Free network, <k> =6, λ=3

N=10000; 100 networks for each r; 1000 realizations each network

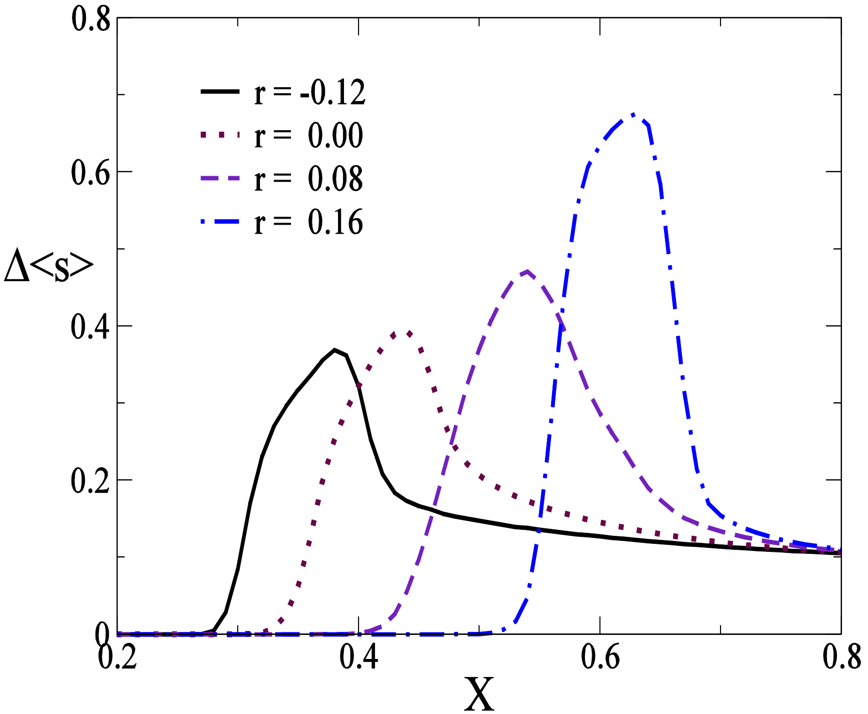
Percolation Behavior

- Determine the position of critical point x_c



Maximum fluctuation

$$\langle (\delta s)^2 \rangle \equiv \langle s^2 \rangle - \langle s \rangle^2$$

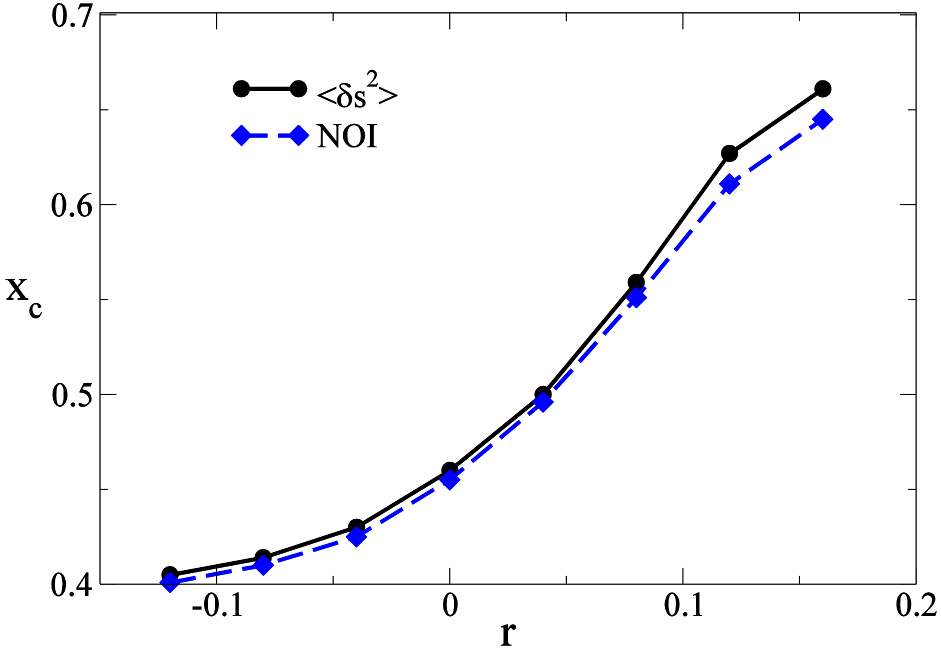


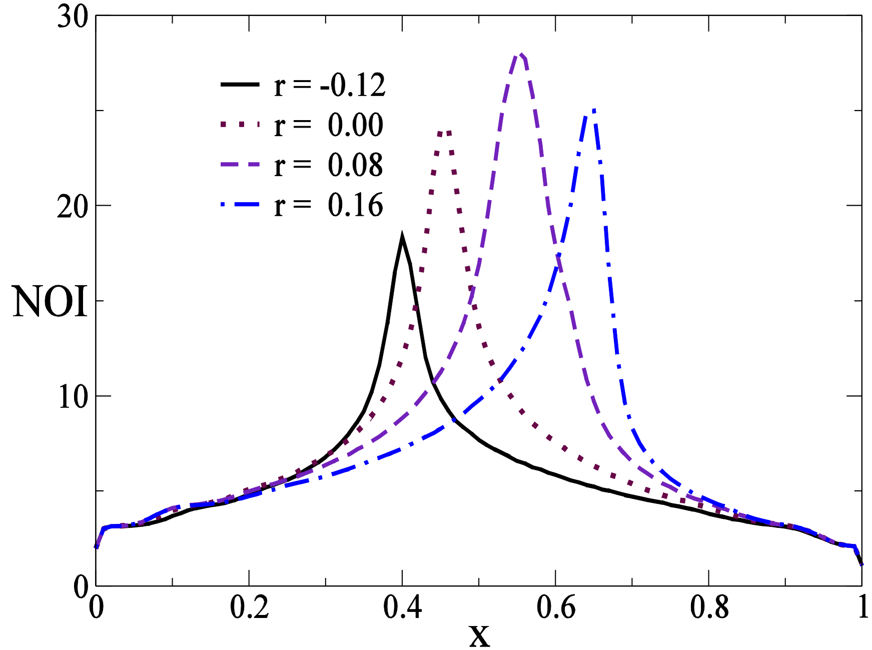
Numerical derivative

$$\Delta\langle s \rangle \equiv \frac{\langle s(x+\epsilon) \rangle - \langle s(x-\epsilon) \rangle}{2\epsilon}$$

Percolation Behavior

- Use quadratic fit to find the peak position
- Verify

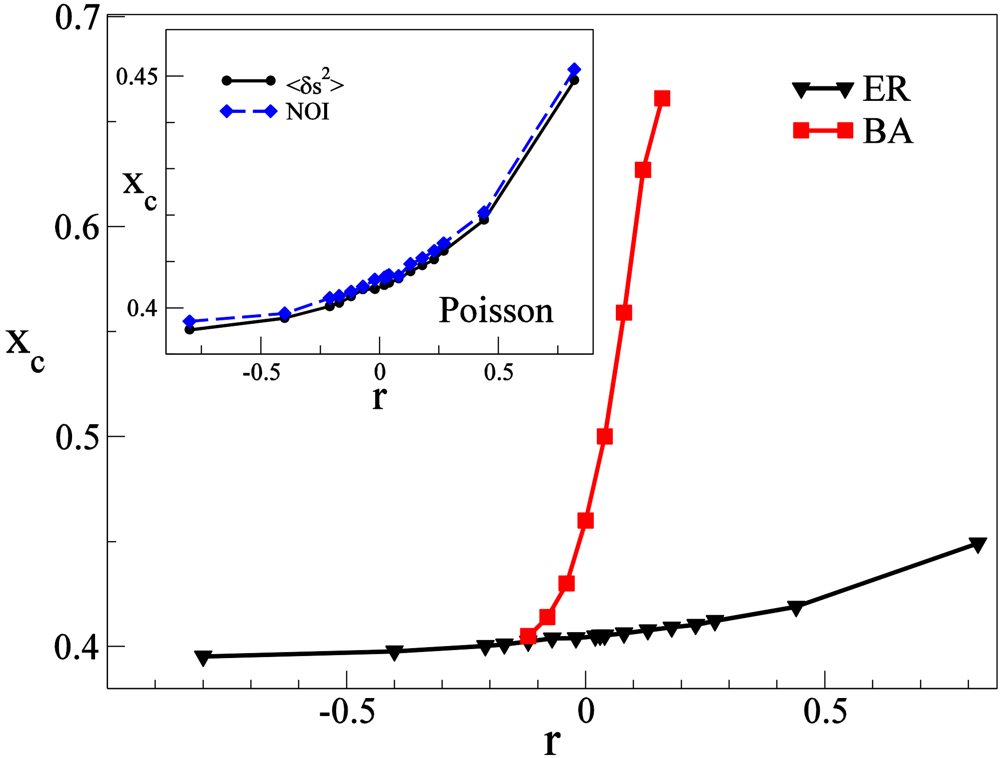




Number-Of-Iteration: number of simulation steps to reach the equilibrium of each x

Percolation Behavior

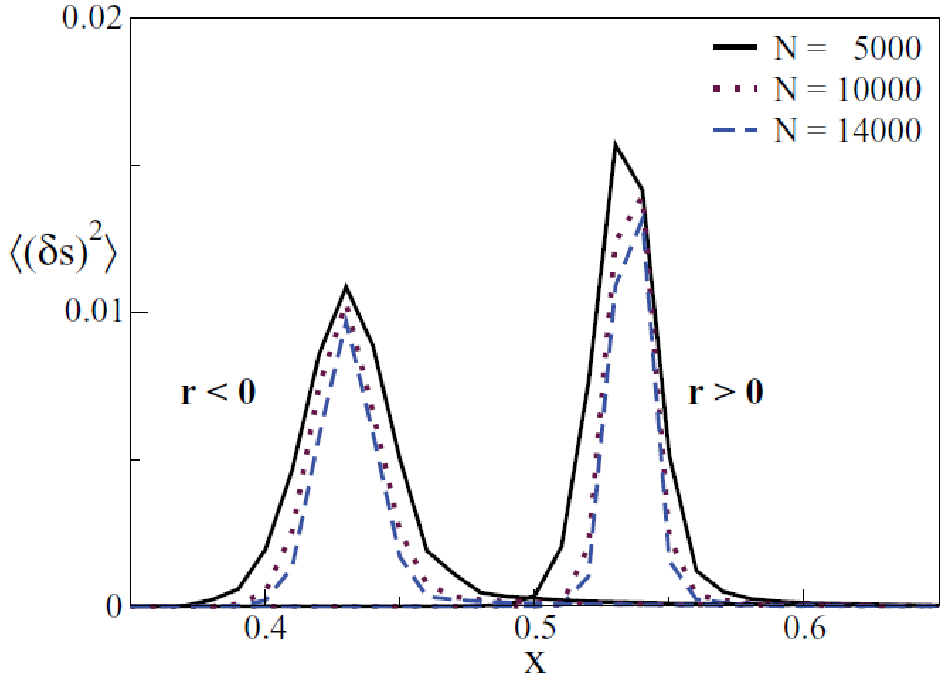
- x_c as a function of r



- SF networks are more sensitive to assortativity change compare to ER network

Percolation Behavior

- First or second order?
- Size effect check



- No second-largest-cluster-peak around x_c
- Thus it's a FIRST-order transition

Conclusion

- Random attacks to a interdependent two-layer system cause cascade failure.
- The percolation phase transition is a first-order transition when q=1.
- The percolation threshold decreases with increasing assortativity (in a single network, increasing assorativity makes it more robust).
- SF networks are less robust than ER interdependent pairs.

D. Zhou, **H. E. Stanley**, G. D'Agostino, and A. Scala, "Assortativity Decreases the Robustness of Interdependent Networks," Phys. Rev. E **86**, 066103 (2012).

Future Work

- Partial interdependence coupling q<1?
- Interdependence links have assortativity (inter-net)?
- Analytical solutions

- Interdependent Global Financial Networks

THANK YOU!

- Probability of a randomly choosing node has degree k : p_k
- the degree distribution for the vertex at the end of a randomly chosen edge is kp_k
- the distribution the number of edges leaving the vertex other than the one we arrived along is $(k+1)p_{k+1}$
- Normalized distribution q_k of the remaining degree is $q_k = \frac{(k+1)p_{k+1}}{\sum_j jp_j}$
- joint probability distribution of the remaining degrees of the two vertices at either end of a randomly chosen edge e_{jk}, we have $e_{jk} = e_{kj}, \sum_{jk} e_{jk} = 1, \sum_j e_{jk} = q_k$
- If no assortative/dis-assortative, independent, $e_{jk} = q_j q_k$
- If has, degree-degree correlation $\langle jk \rangle - \langle j \rangle \langle k \rangle = \sum_{jk} jk(e_{jk} - q_j q_k)$
- Divide by maximum value (when $e_{jk} = q_k \delta_{jk}$) :

$$\sigma_q^2 = \sum_k k^2 - \left[\sum_k k q_k\right]^2$$

- Assortativity coefficient

$$r = \frac{1}{\sigma_q^2} \sum_{jk} jk(\, e_{jk} - q_j q_k)$$

- For observed network

$$r \equiv \frac{\left\langle k_i k_j \right\rangle_e - \left[\left\langle \left(k_i + k_j\right)/2 \right\rangle_e\right]^2}{\left\langle \left({k_i}^2 + {k_j}^2\right)/2 \right\rangle_e - \left[\left\langle \left(k_i + k_j\right)/2 \right\rangle_e\right]^2}$$