Properties of Networks of Interacting Stochastic Agents

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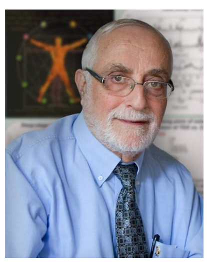
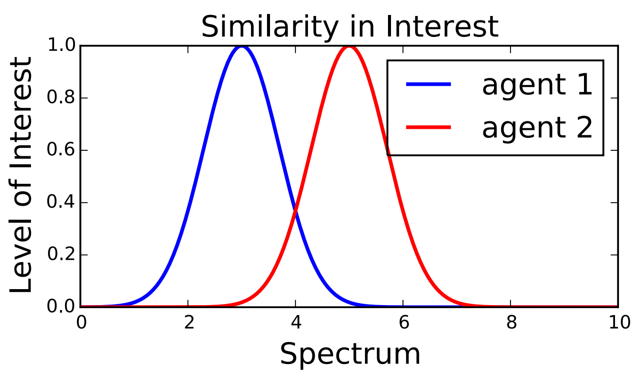
Collaborators

Navid Dianati



Asher Mullakandov Shlomo Havlin Eugene Stanley Zhi-Qiang Jiang

Motivation
Similarity in a Parameter Space



- Many networks are primarily formed from overlap or similarity in interests, location, ...

Example 1: Friendship through Face-to-Face Interaction

Co-location increases chance of friendship "close to 70% of users who call each other frequently (at least once per month on average) have shared the same space at the same time ("co-location").*"

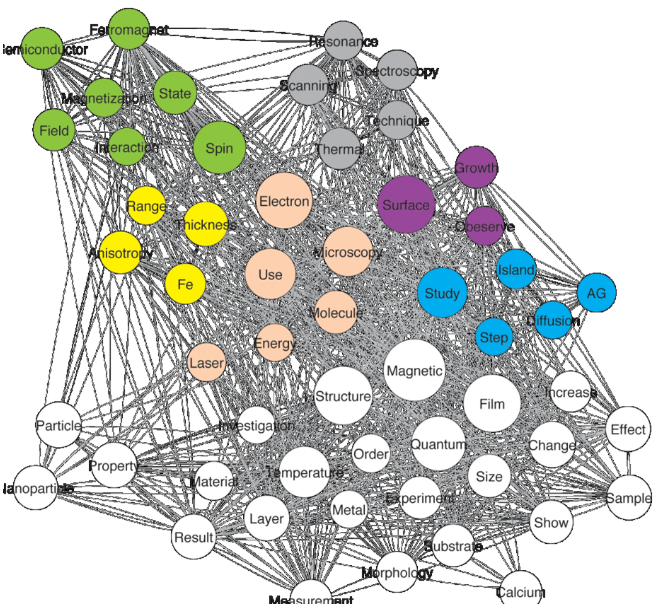


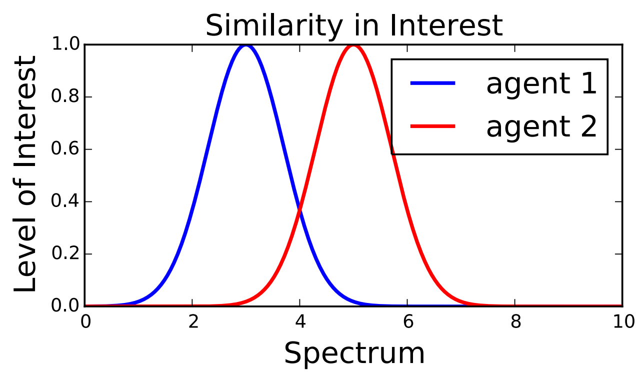
* Interplay between Telecommunications and Face-toFace Interactions: A Study Using Mobile Phone Data, *Calabrese et al.*

Example 2: Interactions in Abstract Parameter Space

Co-authorship Based on Research Interest Overlap

If we, by some means, quantify research interests (e.g. number of times the word "networks", or "spin" appear in one's papers)





* Knowledge sharing example cases from Germany

https://sisobproject.wordpress.com/2013/10/28/knowledge-sharing-example-cases-from-germany/

Network Theory Primer

- A Network has nodes, labeled $i, j, ..$ and links. We consider undirected links here.
- A_{ij}: "Adjacency matrix"
- $k_i = \sum_j A_{ij}$: Degree (# of neighbors of i)
- $P(k)$: Degree distribution.

First network moment: Degrees

$$k_i = \sum_j A_{ij},$$

analyzed through $P(k)$, the degree distribution

2nd network moment: First neighbor degrees

$$k_i^{(1)} = \frac{1}{k_i} \sum_j \left[A^2\right]_{ij},$$

analyzed by degree-degree correlation $\langle k^{(1)}(k) \rangle$

Network Theory Primer

Network Characteristics and "Moments" of A_{ij}

3rd network moment: 2nd neighbors and triangle counts

$$k^{(2)} = \frac{1}{k_i^{(1)}} \sum_j \left[A^3\right]_{ij}.$$

We'll look at local clustering $c(k)$

$$c_i \equiv \frac{2 \times \# \text{ of triangles involving } i}{k_i(k_i - 1)}$$

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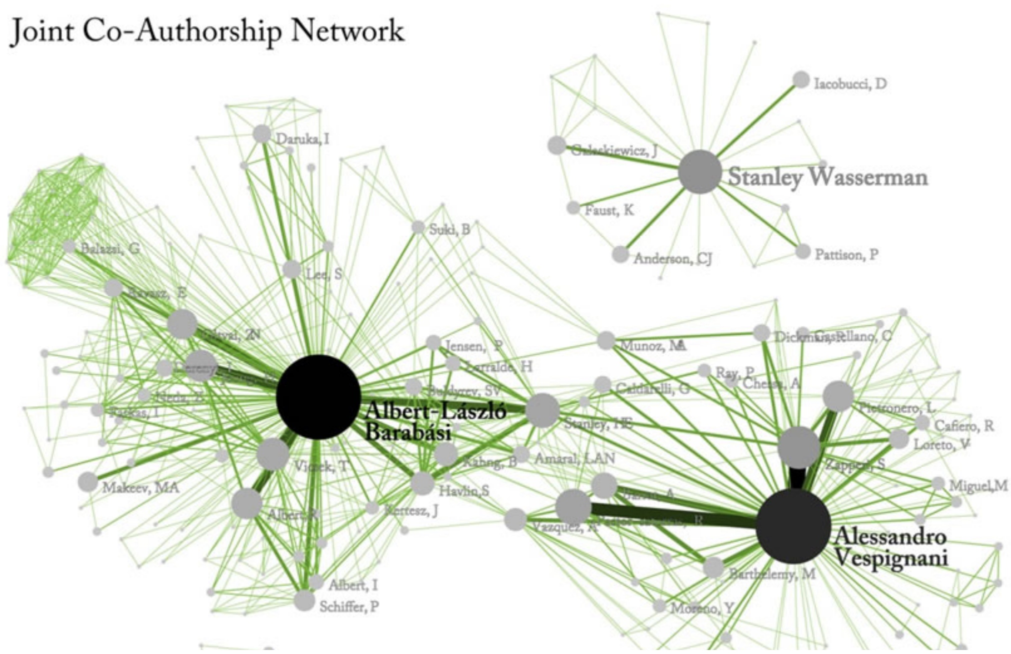
Example: Brabasi-Albert: “rich gets richer” or “Preferential Attachment”

Yields a “scale-free” network

$$P(k) \sim k^{-3}$$

Co-authorship Network

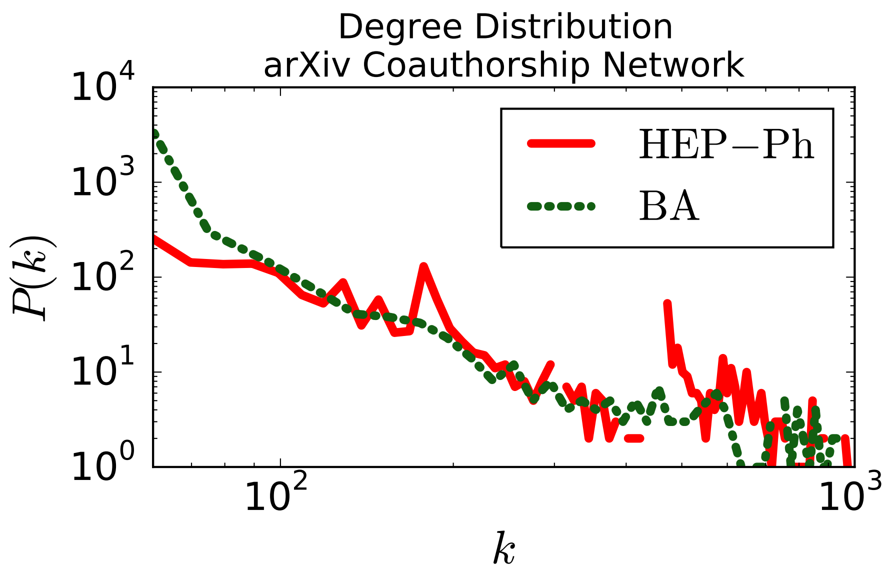
Nodes: Scientists, Links: Wrote paper together



http://wiki.cns.iu.edu/display/SCI2TUTORIAL/5.1+Individual+Level+Studies+-+Micro

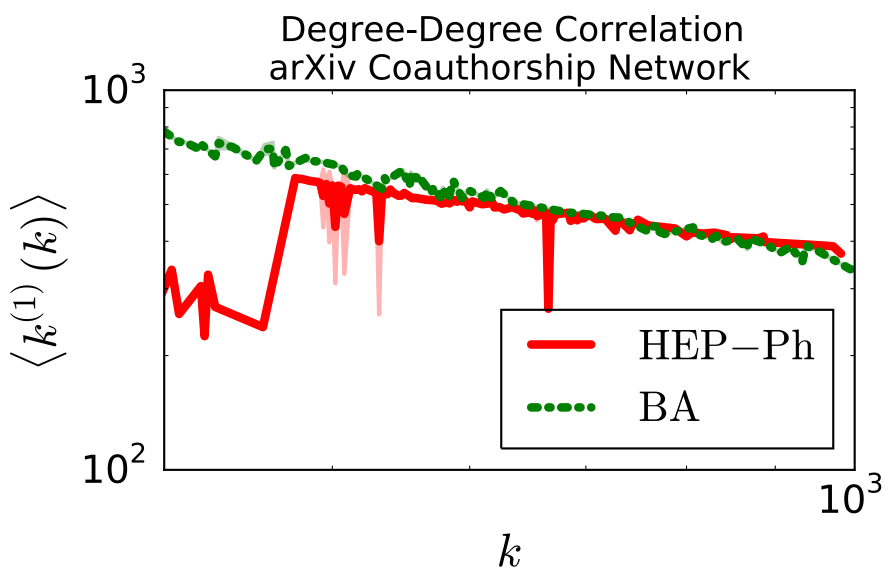
An Example: Success of Preferential Attachment
1st Moment: $P(k)$

The Barabasi-Albert Model: "rich gets richer" Comparison to a real network:



An Example: Success of Preferential Attachment
2nd Moment: $k^{(1)_i} = \frac{1}{k_i} \sum_j [A^2]_{ij}$

The Barabasi-Albert Model: "rich gets richer" Comparison to a real network:



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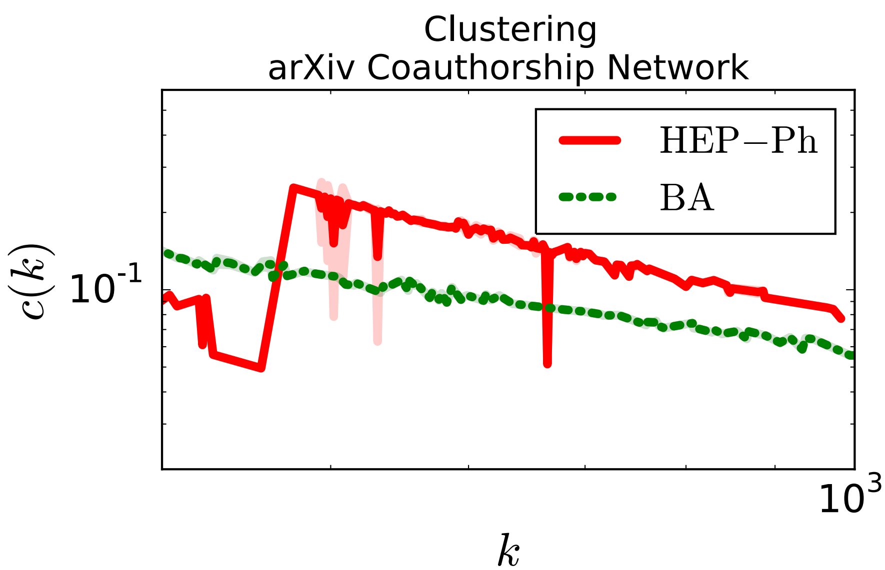

Really?...

Except: Success of Preferential Attachment?

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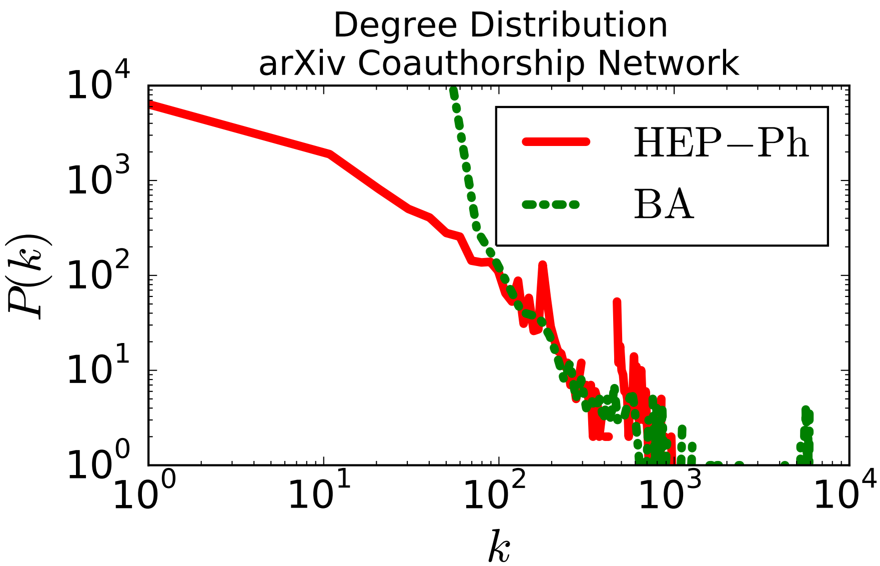

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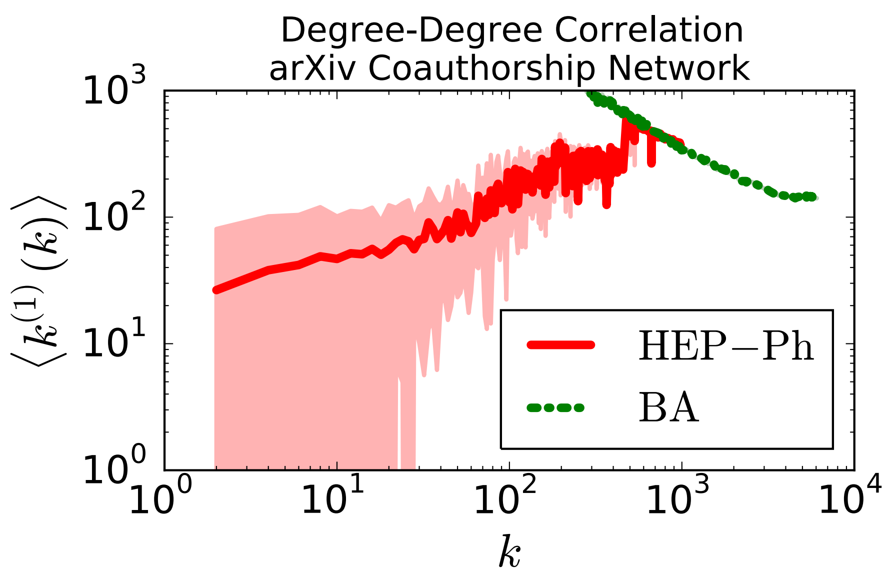

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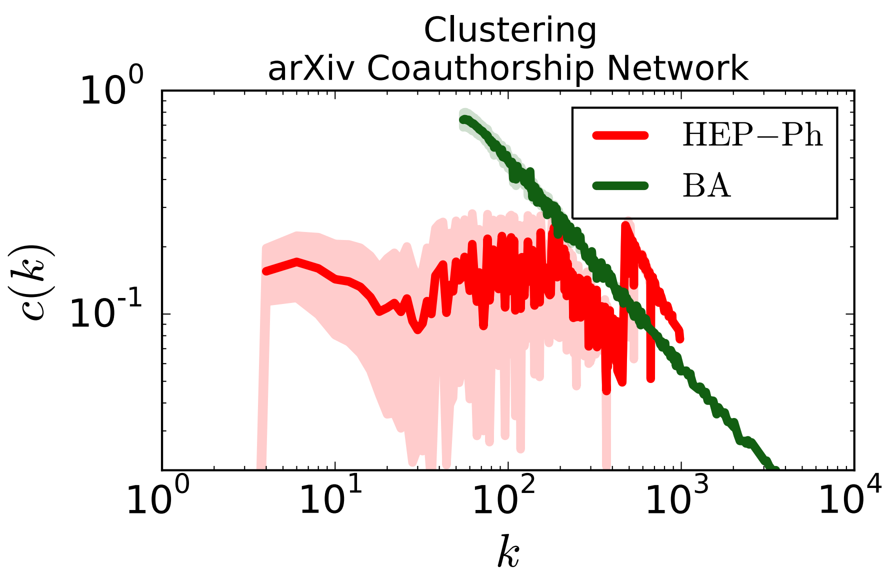
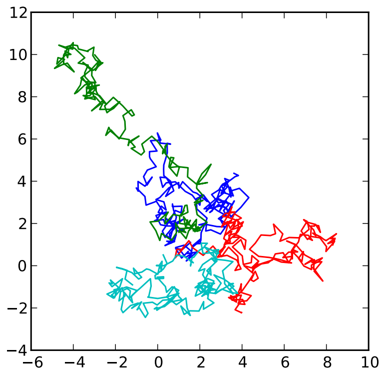


Central Question

Does locality impose anything on the network structure?

Can similarity-based networks with local interactions in a parameter space explain the features observed?

The Model
Stochastic Agents Interacting in a Parameter Space



Random Walkers

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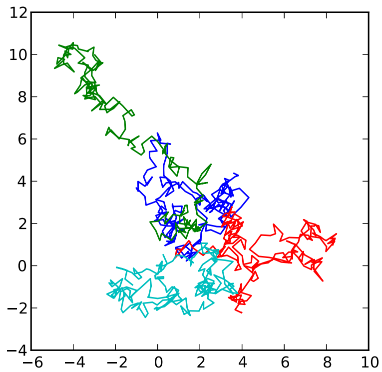

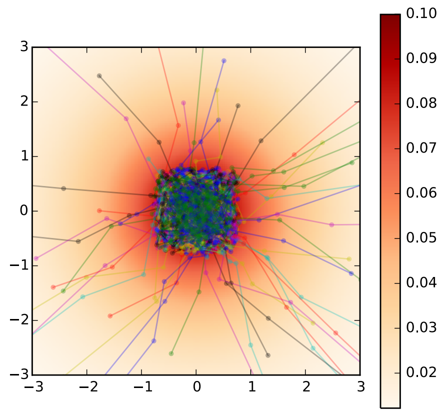

Random Walkers   Inside an Attractive Potential

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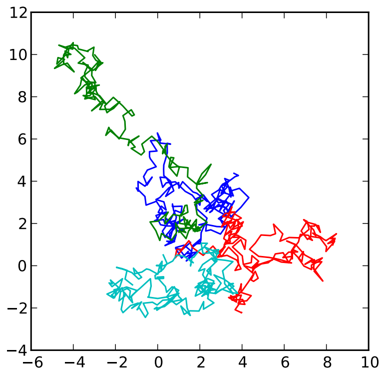

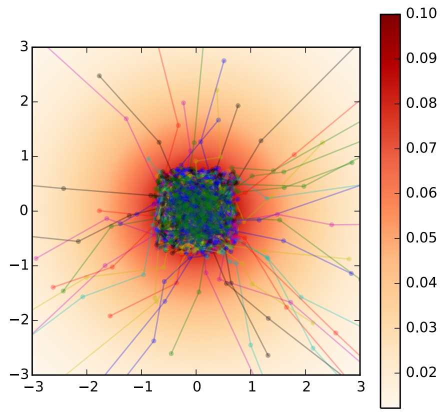

Random Walkers   Inside an Attractive Potential

Probability densities obey a nonlinear Fokker-Planck equation

$$\mathscr{L}_{x,t}\phi_i(x,t) = J_i(x,t) - \frac{\delta \mathcal{V}}{\delta \phi_i} \tag{1}$$

where $\mathcal{V}[\phi]$ denotes interaction

Network of Correlations

Correlations are a natural candidate for adjacency:

$$A_{ij} \equiv \langle \phi_i \phi_j \rangle - \langle \phi_i \rangle \langle \phi_j \rangle$$

Analytical Results

Defining the Green's function $G_{xy} \equiv G(x, t_x; y, t_y)$ and a new operator $\overline{\mathscr{L}}_x$ through

$$\mathscr{L}_y G_{xy} = \overline{\mathscr{L}}_x G_{xy} = \delta^n(x - y)$$

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We have

$$A_{ij} = G_{ix}\Gamma_{ij}(x, t_x)G_{xj} + O\left(\frac{1}{N}\right)$$

$$\Gamma_{ij} = \frac{\delta^2 \mathcal{V}}{\delta\phi_i \delta\phi_j}$$

Analytical Results

Recursive relation for mth neighbor degree

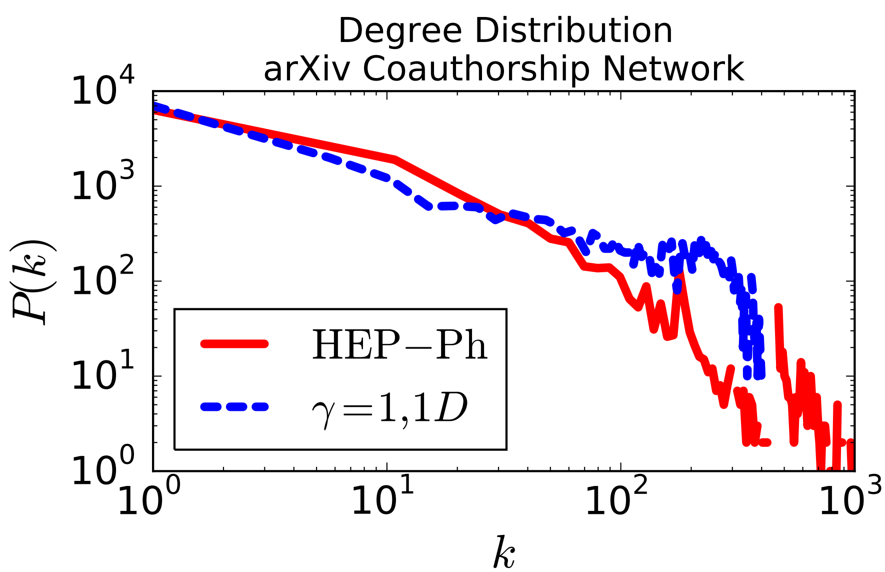
$J =$ initial node distribution

$$\overline{\mathscr{L}}_{ii}(x_i)\left(k_i^{(m)}k_i^{(m-1)}\right) = \Gamma_{ii}J(x_i)k_i^{(m-1)}$$

Simulation Results

Our Results

Interactions through existing literature $\Gamma = \langle \phi \rangle$



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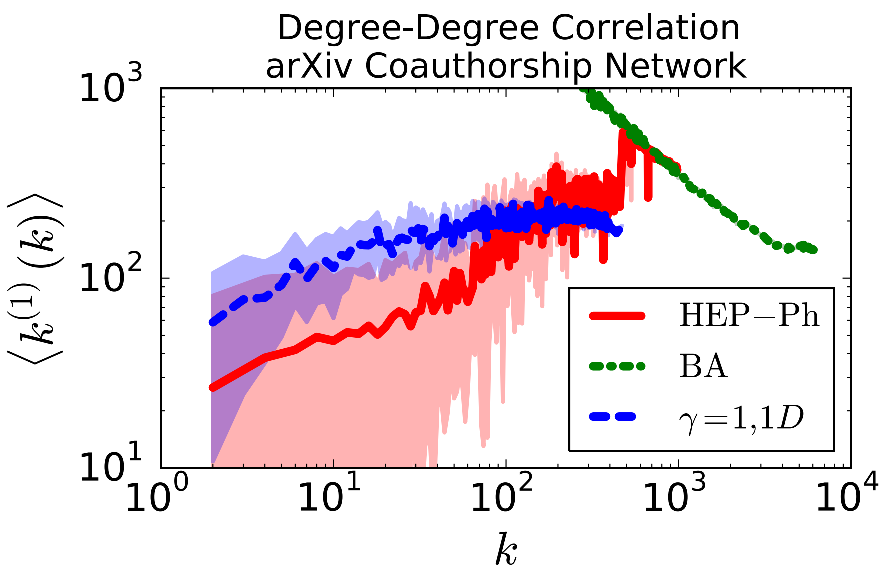

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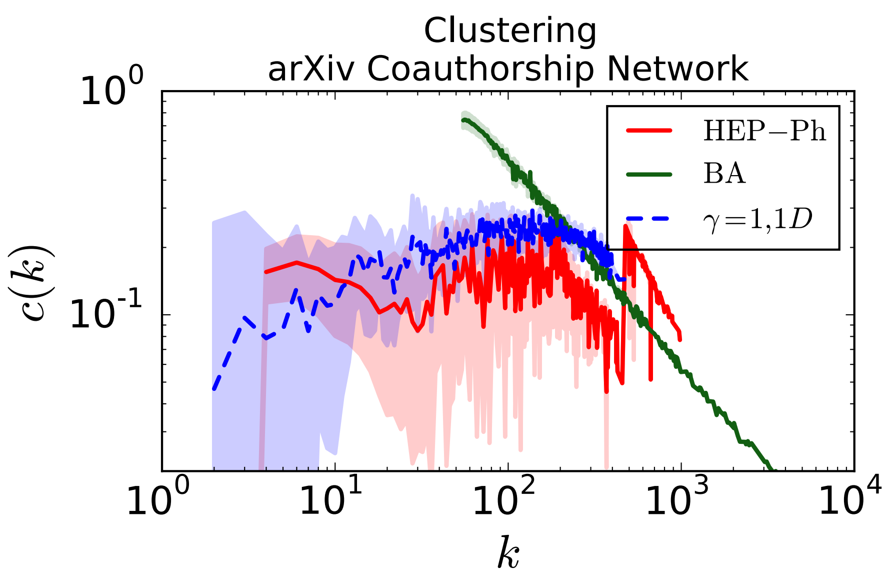
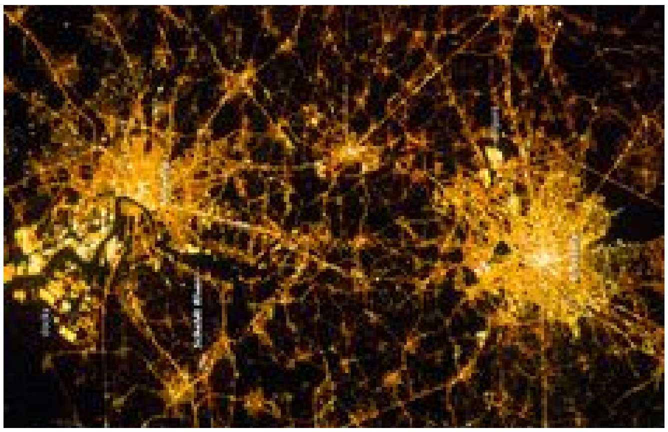
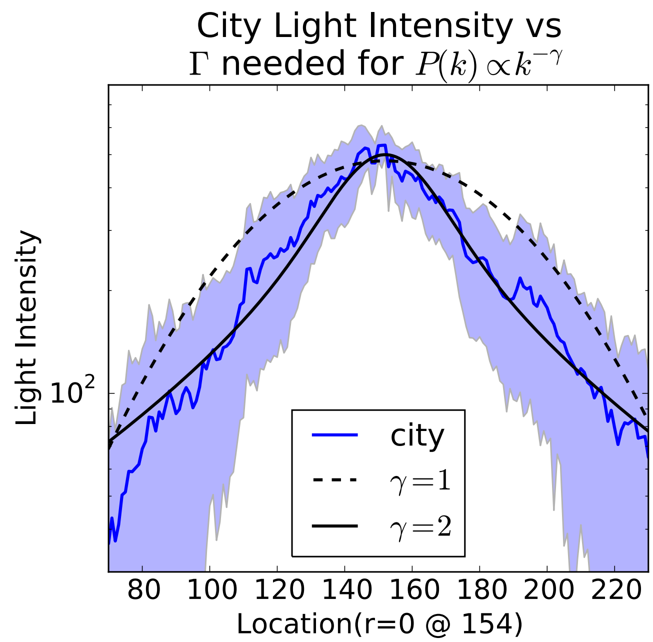


Cities as collections of “Rendezvous Points”




Density of “Rendezvous Points”



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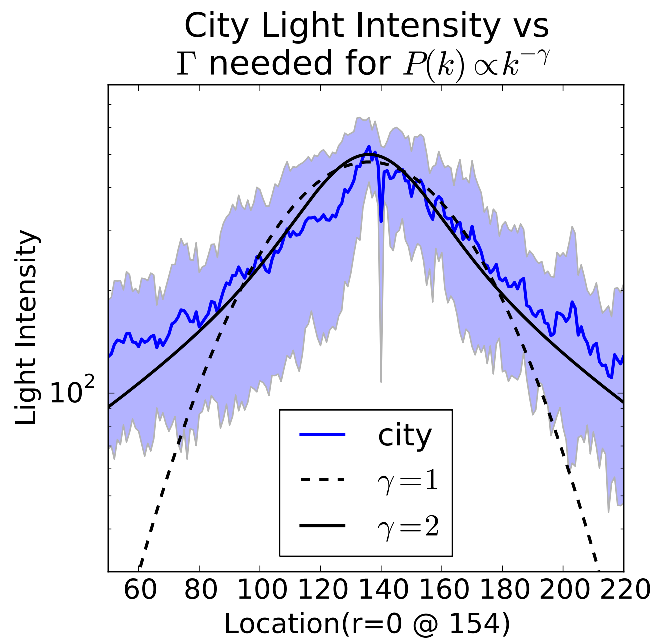
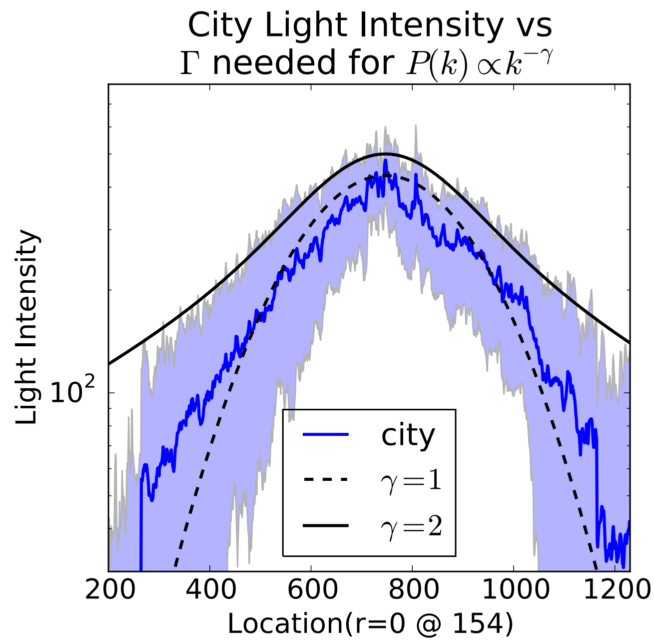


Density of "Rendezvous Points"

Paris!



Conclusion

- Local Interactions, whether in real space or in an abstract parameter space, have important implications for the network structure
- Our model based on locally interacting stochastic agents can reproduce some features of real-world networks.

Thank You!