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Inter-similarity between coupled networks

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Abstract – Recent studies have shown that a system composed of several randomly interdependent networks is extremely vulnerable to random failure. However, real interdependent networks are usually not randomly interdependent, rather pairs of dependent nodes are coupled according to some regularity which we coin inter-similarity. For example, we study a system composed of an interdependent world wide port network and a world wide airport network and show that well-connected ports tend to couple with well-connected airports. We introduce two quantities for measuring the level of inter-similarity between networks: i) the inter degree-degree correlation (IDDC); ii) the inter-clustering coefficient (ICC). We then show both by simulation models and by analyzing the port-airport system that as the networks become more inter-similar the system becomes significantly more robust to random failure.

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Recently, an American Congressional Committee highlighted the intensified risk in an attack on national infrastructures, due to the growing interdependencies between different infrastructures [1]. However, despite the high significance and relevance of the subject, only a few studies on interdependent networks exist and these usually focus on the analyses of specific real network data [2–6]. The limited progress is mainly due to the absence of theoretical tools for analyzing interdependent systems. Very recently, several studies [7,8] presented for the first time a framework for studying interdependency between networks and showed that interdependencies significantly increase the vulnerability of the networks to random attack. These studies assumed that pairs of interdependent nodes from the two networks are randomly connected, *i.e.*, a randomly selected node from network A is connected and depends on a randomly selected node from network B and vice versa. Due to the dependencies an initial failure of even a small fraction of nodes from one network can lead to an iterative process of cascading failures that can completely fragment both networks in the form of a first-order phase transition.

However, the restriction of random interdependencies is a strong assumption that usually does not occur in many interdependent systems. As a first example consider the Italian power grid and SCADA communication networks [1,2,7,8]. A power node depends on a

communication node for control while a communication node depends on a power node for electricity. It is highly unlikely that a central (high-degree) communication node will depend on a small (low-degree) power node. Rather, it is much more common that a central communication node depends on a central power station. Moreover, coupled networks usually also pose some similarity in structure, for instance, an area that is overpopulated is bound to have many power stations as well as many communication nodes. Another real example is the world wide port and airport networks that we study in this paper. We find that well-connected ports tend to couple to well-connected airports therefore supporting our assumption that real interdependent networks are usually not randomly interdependent.

In this letter we show that inter-similar coupled networks, *i.e.* coupled networks in which pairs are coupled according to some regularity rather than randomly, are significantly more robust to random failure. Moreover, increasing the inter-similarity between the networks leads to a fundamental change in the networks behavior. While randomly interdependent networks disintegrate in the form of a first-order phase transition [7,8], networks with high levels of inter-similarity disintegrate in the form of a second-order phase transition.

We start by explaining in details the studied model. Following the model for a system composed of two interdependent networks presented in [7] we assume two networks A, B where a) each node A_i from network A depends on a

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node B_i from network B and vice versa; b) each node from one network can only depend on one node from the other network. In such a system an initial removal of a fraction of nodes in network A leads to the failure of the dependent nodes in network B . The failure of the dependent nodes in network B causes additional nodes to disconnect (and fail) from network B . These nodes are coupled with new nodes in network A that also fail due to dependencies and so on [7,8]. For randomly interdependent networks such an iterative process of cascading failures leads to a first-order phase transition in, P_∞ , the size of the largest connected cluster remaining at the end of the cascade process.

Here we show that when the coupled networks become inter-similar the cascade process is significantly reduced. Figure 1(a) presents simulation results showing that for low levels of inter-similarity P_∞ abruptly drops to zero at criticality characterizing a first-order phase transition while for high levels of inter-similarity P_∞ continuously decreases characterizing a second-order transition. Figure 1(b) presents simulation results showing how the number of iterations (NOI) in the iterative process of cascading failures is reduced when the networks become more inter-similar.

We develop two measures to assess the level of inter-similarity between interdependent networks. We show that these measures can also determine the robustness of coupled networks. The first quantity, r^{AB} , measures the inter degree-degree correlation (IDDC) between a pair of dependent nodes. The two networks A and B have a degree distribution of p_k^A and p_k^B , respectively. Similar to assortative mixing in a single network [9], we define by e_{jk} the joint probability that a dependency link is connected to an A -node with degree j and to a B -node with degree k . For networks with no IDDC, $e_{jk} = p_j^A p_k^B$. For networks with IDDC, the level of correlation can be defined by $\sum_{jk} jk(e_{jk} - p_j^A p_k^B)$. Normalizing by the maximum value of IDDC we obtain a general measure, r^{AB} , in the range $-1 \leq r \leq 1$. The value 1 is achieved for a system with maximum IDDC, the value of zero for no IDDC and a value -1 for a system with maximum anti-IDDC. If the two networks have the same degree distribution ($p_k = p_k^A = p_k^B$) the maximum IDDC value is given by $\sigma_q^2 = \sum_k k^2 p_k - (\sum_k k p_k)^2$ and we obtain

$$r^{AB} = \frac{1}{\sigma_q^2} \sum_{jk} jk(e_{jk} - p_j p_k). \quad (1)$$

Positive values of r^{AB} indicate that high-degree nodes from network A tend to couple with high-degree nodes from network B and vice versa. Negative values of r^{AB} indicate that high-degree nodes from network A tend to couple with low-degree nodes from network B and vice versa. Randomly interdependent networks correspond to the case of $r^{AB} = 0$.

The second measure is the inter-clustering coefficient (ICC), c^{AB} , that evaluates for a pair of dependent nodes $\{A_j, B_j\}$ how many of the neighbors of A_j depend on

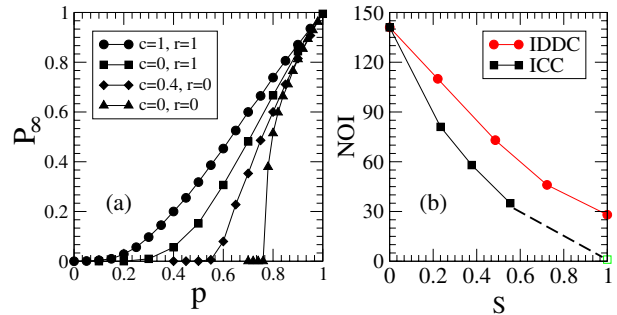


Fig. 1: (Colour on-line) (a) Simulation results for P_∞ , the fraction of nodes remaining in the largest cluster of network B after a random failure of a fraction $1 - p$ of the nodes in network A . The simulations compare four different configurations (see text) of two interdependent SF networks each of size $N = 100000$ and $\lambda = 2.7$ showing that inter-similar coupled networks are significantly more robust to random failure compared to randomly interdependent networks (the value of p for which P_∞ approaches zero is much smaller). (b) Simulation results showing the number of iterations (NOI) in the iterative process of cascading failures, at p_c . The NOI is plotted as a function of the similarity (S) between the networks which is measured either by the IDDC (circles), or by the ICC (squares). As the networks become more inter-similar the NOI is reduced indicating that less nodes fail. When measuring the effect of the IDDC on the NOI the ICC is kept zero. Similarly, when the effect of ICC is measured the IDDC is kept zero. The dashed line marks the region that cannot be properly simulated since the ICC is too high to generate networks with no IDDC.

neighbors of B_j and vice versa. Analogous to a single network [10], we define the local inter clustering coefficient, c_j^A of node A_j as

$$c_j^A = \frac{t_j}{k_j^A}, \quad (2)$$

where t_j is the number of links connecting the neighbors of A_j to the neighbors of B_j and k_j^A is the degree of A_j (see footnote ¹). Note that c_j^A is not equal to c_j^B . The global ICC can be defined as the average of all the local clustering coefficient, $c^A = \frac{1}{N} \sum_j c_j^A$. But in this case $c^A \neq c^B$. We therefore prefer to define the global clustering as

$$c^{AB} = \frac{1}{M} \sum_j t_j, \quad (3)$$

where M is the total number of dependency links between the two networks and $0 \leq c^{AB} \leq 1$. For increasing values of c^{AB} more of the neighbors of A_i depend on the neighbors of B_i and the two networks become more inter-similar. For $c^{AB} = 1$ the two networks must be identical.

¹According to the assumption made in [7,8] that a node from A depends on at most one node from B , k_j^A is the maximum number of neighbors of A_j that can depend on neighbors of B_j . However, for a more general model, where a node can depend on more than one node, k_j^A must be replaced by the new maximum.

The effect of inter-similarity between networks is dramatically influenced by the network topology. In a case where two interdependent networks have a broad degree distribution, an interdependent pair $\{A_j, B_j\}$ can greatly differ in their degree. As a result the diversity in the correlation between the networks (IDDC) is significantly increased. We therefore apply our theory to two important and very different network topologies. The first is the Erdős-Rényi (ER) network model [11–13], in which all links exist with equal probability leading to a Poisson degree distribution $P(k) = e^{-\langle k \rangle} \langle k \rangle^k / k!$. The ER network model has become a classic model in random graph theory and was intensively studied in the past few decades. The other model is that of scale-free (SF) [14,15] networks with a broad degree distribution, usually in the form of a power law, $P(k) \sim k^{-\gamma}$ with $\gamma > 2$. It was found that many real networks are scale free [14,15].

To show the effect of our measures on the robustness of inter-dependent networks, we compare between the following four systems: i) two randomly interdependent SF networks ($r^{AB} = 0$ and $c^{AB} = 0$). ii) Two SF networks where every pair of dependent nodes $\{A_j, B_j\}$ has the same degree, $k_j^A = k_j^B$ ($r^{AB} = 1$ and $c^{AB} = 0$). iii) Two interdependent SF networks with $r^{AB} = 0$ and $c^{AB} = 0.4$, which is the maximum ICC we were able to obtain without inserting IDDC. iv) Two identical interdependent SF networks ($r^{AB} = 1$ and $c^{AB} = 1$). Figure 1(a) presents P_∞ , the fraction of nodes remaining in the largest cluster of network B after a random failure of a fraction $1 - p$ of network's A -nodes. The figure shows that high IDDC even with no ICC or high ICC even with no IDDC, significantly increases the fraction of failing nodes (smaller p_c) that will fragment the system ($P_\infty = 0$), indicating that the system is more robust. Moreover, for high IDDC or ICC the jump in the size of P_∞ that characterizes a first-order phase transition changes to a gradual decrease identified with a second-order transition. Figure 1(b) provides additional support for our claim that inter-similarity increases the robustness of interdependent networks. The number of iterations (NOI) in the process of cascading failures at p_c for a network of size N , scales with $N^{1/4}$ for randomly interdependent networks [7] and is equal to 1 for identical networks. The figure shows that indeed when the inter-similarity is increased either via the IDDC measure or via the ICC measure, the NOI decreases, respectively.

Next, we study a real interdependent system that is composed of a world wide port network and a world wide airport network. The airport network is composed of 1767 airports and records the majority of the air traffic around the world. The port network is composed of 1076 ports and records the flow of commodities around the world. Previous studies have shown [16–18] that different transportation systems that are located in the same city (or area) depend on each other through their common influence on the economic prosperity of that city. In terms of our two networks, the development of an airport in a city will lead to an increase in air-traffic that in

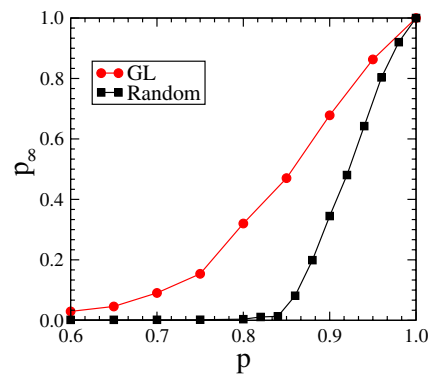


Fig. 2: (Colour on-line) The sea-air interdependent system is composed of a world wide port network and a world wide airport network. The simulations present the fraction of nodes remaining in the largest cluster of the airport network, P_∞ , after a fraction p of the ports in the port network are randomly removed (similar results are obtained for the opposite case). The results are compared between two different configurations: i) (squares) the networks are randomly coupled; ii) (circles) the networks are coupled according to geographic locations (GL), *i.e.*, an airport depends on the nearest port and vice versa. When the networks are coupled by GL the system is significantly more robust to random failure (the value of p for which P_∞ approaches zero is much smaller).

turn will result in economic prosperity. The prosperity of that city will have a positive effect on the development and increase of traffic to that city's port and vice versa. Accordingly, for our mapping we assume that a port depends on a nearby airport and vice versa. However, since the networks are not of the same size we first renormalized the networks so that they corresponds to the model presented in [7,8]. We first match between pairs of ports and airports with the minimal distance between them under the condition that a port only depends on one airport and vice versa. The remaining airports that do not depend on any port are merged with the closest airport such that the new renormalized node includes the accumulated traffic of these airports. A similar process is applied to the port network. At the end of this process we obtain two networks both of size 992 that are coupled based on geographical location (GL). We find that for the GL interdependent port-airport networks the coupling between the networks is not random, the IDDC parameter is $r^{AB} = 0.2$ (compared to $r^{AB} \rightarrow 0$ for randomly interdependent networks) indicating that high-degree ports tend to couple with high-degree airports. These findings support our theory regarding inter-similarity between real interdependent networks.

The next step is to enquire how the high level of IDDC in the port-airport networks affects the robustness of the system. Figure 2 presents P_∞ of the port network for an increasing fraction of failing nodes in the airport network (similar results are obtained when the initial nodes fail from the port network). The figure compares between two

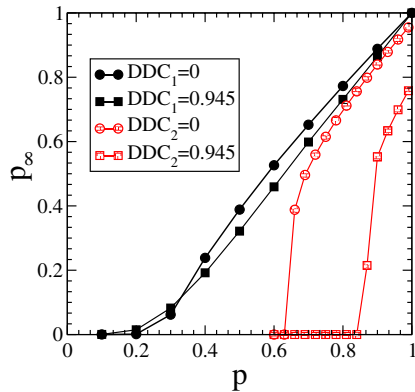


Fig. 3: (Colour on-line) Simulation results for ER networks each of size $N=100000$ and $\langle k \rangle=3.75$ showing the effect of changing the degree-degree correlation (DDC) within a single network (solid symbols) compared to the effect in two interdependent networks (open symbols). For the case of interdependent networks the DDC is measured within each network and the IDDC and ICC are kept zero. Even though the effect of the DDC on a single network is minor (solid circles) it significantly decreases the robustness of interdependent networks (open circles).

configurations of the port-airport system: i) The networks are randomly coupled. ii) The networks are coupled based on geographical location (GL). The results support our theory that a systems with high IDDC is more robust to random attack (P_∞ is larger) and that the phase transition becomes smoother as the networks become more inter-similar. However, since each of the networks has a very high average degree (within each network the nodes are well connected) and as a result the networks are very hard to fragment, we have made the reasonable assumption that when 75 percent of the traffic to a certain port (or airport) is disabled that port becomes non functional.

Until now we have shown the critical effect of inter-similarity on the robustness of a system composed of interdependent networks. But what is the effect of the local properties within each of the networks on the robustness of the interdependent system? Here we show that the degree-degree correlation (DDC) [9] within a network that has only a minor effect on the robustness of single networks greatly affects the robustness of an interdependent system. While for single networks a higher DDC usually slightly increases the robustness of the network for an interdependent system a higher DDC significantly increases the vulnerability of the system. In fig. 3 we demonstrate the effect for the case of ER networks. For ER networks that are characterized by a very narrow degree distribution, the DDC is expected to have a very limited effect on a single network. But, when two such ER networks with high DDC become randomly interdependent the effect of the DDC becomes dramatic, as shown in fig. 3.

After showing that real coupled networks are indeed inter-similar, we present a mechanism for generating

inter-similar coupled networks. The model we present can be regarded as a generalization of the Barabási-Albert (BA) preferential attachment model [14,15] to two interdependent networks, that naturally incorporates inter degree-degree correlations between the nodes of the two networks. According to the BA model, a single network with an initial set of m_0 randomly connected nodes is grown by adding on each step a new node that is connected to m different nodes from the already existing network. The probability of the new node to connect to a specific node is proportional to that node's degree. Generalizing the model to two interdependent networks A and B , we start with two initial sets of nodes m_0^A and m_0^B of the same size. The two sets are each internally randomly connected and in addition each node from m_0^A is randomly connected to one node in m_0^B . On each step t , a pair of dependent nodes $\{A_t, B_t\}$ are added to the networks, A_t to network A and B_t to network B , independently, according to the preferential attachment model. Since the two nodes are added independently, there is no correlation between the neighbors of node A_t in network A and the neighbors of B_t in network B . This process mimics a natural process of two interdependent growing network. In terms of our initial example of a power network and a communication network, at different times new developing areas are populated and connected to infrastructures. Every such area adds a pair of dependent nodes, a power node and a communication node. Even though A_t and B_t are differently connected within each network, because of the preferential attachment process, the fact that they were added at the same time significantly increases the probability that they have a similar degree. When simulating a system of two interdependent networks according to our generalized BA model we obtained two SF networks with $\lambda=3$ as obtained by the BA model for a single SF network [14,15]. We also obtain a very high level of IDDC ($r^{AB}=0.6$) without any change in the ICC value ($c^{AB}=0$). Our model therefore provides a natural mechanism for generating SF inter-similar coupled networks with high inter degree-degree correlation but without inter-clustering coefficient. Our simulations also confirm that an interdependent system generated according to the generalized BA model is significantly more robust to random failure than a randomly interdependent system.

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Additional remark: After submitting this manuscript Buldyrev *et al.* [19] presented an analytical solution for the specific case where all interdependent pairs have the same degree ($c^{AB}=1$).

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