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Design of survivable networks in the presence of aging

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Abstract – Networks are designed to satisfy given objectives under specific requirements. While the static connectivity of networks is normally analyzed and corresponding design principles for static robustness are proposed, the challenge still remains of how to design survivable networks that maintain the required level of connectivity during their whole lifespan, against component aging. We introduce network survivability as a new concept to evaluate the networks overall performance during their whole lifespan, considering both network connectivity and network duration. We develop a framework for designing a survivable network by allocating the expected lifetimes of its components, given a limited budget. Based on percolation theory and simulation, we find that the maximal network survivability can be achieved with a quantitative balance between network duration and connectivity. For different survivability requirements, we find that the optimal design can be separated into two categories: strong dependence of lifetime on node's degree leads to larger network lifetime, while weak dependence generates stronger network connectivity. Our findings could help network design, by providing a quantitative prediction of network survivability based on network topology.

Introduction. – Robustness [1–8] is a prerequisite condition for system functionality in various types of network system designs, including critical infrastructures (*e.g.*, power grids [9,10], communication networks [11] and transportation networks [12,13]) and natural systems (*e.g.*, biological systems [14,15], ecological systems [16,17] and social networks [18,19]). Robustness enables a system to perform fully or at least at an acceptable minimum function after a failure of a portion of its components, *e.g.*, due to internal faults and/or external hazards. On the other hand, the structure of a system evolves during its life and its components fail due to aging, an effect that has been rarely considered. Indeed, aging processes occur in engineering systems [20,21], biological systems [22,23], and even online social systems [24,25], where users may finally quit an online community after an active period.

Static robustness of a network can be defined as its ability to withstand losses of nodes or links under random failures or targeted attacks [5–7,26]. Based on percolation theory, network static robustness can be characterized by a percolation critical threshold, which is the critical fraction of failed network elements at which the system collapses. It has been shown [5–7] that scale-free networks are usually more robust than Erdős-Rényi (ER) networks with respect to random failures, but they are more fragile to targeted attacks. Efforts [27–29] have been made to find optimal designs of network structures that are robust to both random failures and targeted attacks. However, the optimization of robustness by connectivity design is not sufficient because the connectivity of real networks is not static and robustness changes also due to the aging process of the components [30–34]. For example, cellular networks decline and may collapse due to the aging of several cells every minute and online social networks suffer from the daily loss of users.

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Whereas previous studies mostly focus on static network robustness at a given snapshot of its life [5–7,26,34], the effect of network evolution due to the aging of the components has rarely been addressed. In this paper, network survivability is proposed as a concept to evaluate the networks overall performance during their whole lifespan, thus considering both network connectivity and network lifetime. Then, the following question becomes fundamental: given a constrained budget, how can we optimally allocate resources of components' lifetimes in order to design a survivable network for its whole lifespan?

Network performance in the whole lifespan depends on the health of the system components, and also the connection between these healthy components. We develop an objective function W (eq. (1) below) for the optimization of network survivability during its whole lifespan. In our case, we assume to have only the information on the degrees of the network components in the design stage. For example, for a Peer-to-Peer (P2P) file sharing network, it is hard to know the whole topology [35], but it would be easier for a peer to have the information about its neighbors based on the communication mechanisms. Reference [36] finds that the nodes residual lifetime correlates with the number of nodes neighbors in a P2P systems. Thus, in our model, each node in the network is allocated with a lifetime expectancy, depending on its number of connected direct links. We, then, show how to design a survivable network, using both theoretical analysis, and simulations performed on Erdős-Rényi (ER) networks with Poisson degree distribution $P(k) = e^{-\lambda}\lambda^k/k!$, where λ is the expectation of the node's degree, and on scale-free (SF) networks which is characterized by a power-law degree distribution $P(k) \sim k^{-\gamma}$. For this, we consider two exponents: α, which characterizes the power-law relation between the node expected lifetime and degree, and β, which measures the users requirement for network survivability.

Model. – In practice, the lifetimes of the components, *e.g.*, electronic components [37,38] are not accurately determined and in reliability analysis it is common to assume a lifetime exponential distribution with mean value τ [39]. Another realistic lifetime distribution is the Weibull distribution, which is realistic for many mechanical components [40]. In our model, as mentioned earlier, we assume that the mean value of the node lifetime depends only on its degree, according to a power-law relation $\tau = k^\alpha$, where α is the exponent to be optimized for network survivability.

As demonstrated in fig. 1(a), for a given node i with degree k_i, we allocate a characteristic lifetime, according to the relation $\tau_i = k_i^\alpha$. In fig. 1(b) and fig. 1(c), we demonstrate the aging process showing the evolving snapshots of the network presented by the fraction of nodes in the giant component of the network denoted by G, at successive times. In fig. 1(b), the network has in the beginning a large connected network component but a short lifetime,

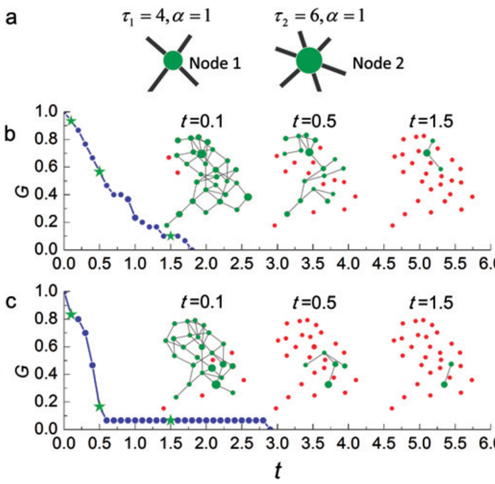

Fig. 1: (Colour online) Evolution of a network (giant component, G) with aging time t. (a) Nodes characteristic lifetime (mean value) is assumed to depend on their degree as $\tau_i = k_i^\alpha$. (b) Time evolution of a network with large G but short lifetime: the figure demonstrates the case of $\alpha = 0.5$. In the figure, the size of a node (green circle) is proportional to its remaining lifetime. The shown scale-free (SF) network contains 30 nodes, and has a power-law exponent $\gamma = 2.5$ and average degree 4. (c) The same SF network but with $\alpha = 2.5$; in this case, initially the network has a small G but relatively longer lifetime compared to (b).

while in fig. 1(c) the network is comparatively smaller than that in fig. 1(b) but has a longer lifespan.

Determining the optimal α will tell us how to distribute lifetimes between the nodes of different degrees. Depending on the application, the objective of network survivability is to have a large G as long as possible. However, the large size of G and its long lifetime are in competition due to the limited total cost. To describe these competing processes, we define the survivability function W,

$$W = \int G(t)^\beta \, \mathrm{d}t, \tag{1}$$

which evaluates the network capacity of maintaining the connectivity function through the whole lifespan. The value of W integrates the overall performance of the network in terms of survivability. The exponent β controls the importance of the size based on the design requirement for network survivability. For example, $\beta = 0$ corresponds to the case where a maximal duration of the network is required, regardless of the connectivity performance during this lifespan (*i.e.*, having a giant component). When $\beta = 1$, the size of the giant component during the network lifespan is also taken into account. As β increases, more weight is given to the network giant component size as design requirement, compared to the network lifetime. When β approaches large values ($\beta \gg 1$), the design requirement for network survivability is focused on the size of the giant component. Our aim is then to find the optimal α (for a given β) that maximizes W.

Survivability of random networks. –
Theory. Theoretical analysis for network survivability is obtained by using the generating functions method. We define $f(t|k)$ to be the conditional density probability of a node to have a lifetime t, given its degree is k. The expected lifetime of a node of degree k, that is also the expectation of $f(t|k)$, is $\tau(k)$, which assumed to be proportional to k^α. We also assume that the total budget of lifetime of all components is equal to the network size N. Thus, $\tau(k) = Nk^\alpha/\sum_{k=0}^{\infty} k^\alpha p(k)N = k^\alpha/\sum_{k=0}^{\infty} k^\alpha p(k)$. We define $q(k,t)$ as the probability that a randomly chosen node that has a degree k survives at time t. We can calculate it as follows (take the exponential distribution for example):

$$q(k,t) = p(k)\int_t^\infty f(t'|k)\mathrm{d}t' = p(k)\int_t^\infty \frac{1}{\tau(k)}e^{-t'/\tau(k)}\mathrm{d}t'. \tag{2}$$

Next, we add a time parameter, t, to the percolation generating functions [41], and the generating function of a node that survives at time t is

$$F_0(x,t) = \sum_{k=0}^{\infty} q(k,t)x^k. \tag{3}$$

The probability that a randomly chosen edge leads to a node which survives at time t, is $(k+1)q(k+1,t)/\langle k\rangle$, where $\langle k\rangle$ is the network mean degree. And the corresponding generating function is

$$F_1(x,t) = \sum_{k=0}^{\infty} \frac{kq(k,t)}{\langle k\rangle}x^{k-1}. \tag{4}$$

Hence, the generating functions for the component size that all its nodes survive at time t are

$$\begin{aligned} H_1(x,t) &= 1 - F_1(1,t) + xF_1\left[H_1(x,t),t\right], \\ H_0(x,t) &= 1 - F_0(1,t) + xF_0\left[H_1(x,t),t\right]. \end{aligned} \tag{5}$$

The size of the giant component at time t is $G(t) = 1 - H_0(1,t)$, since $H_0(1,t)$ contains only finite-size components that survive in time t. Thus, using eqs. (5) and (3) we get

$$G(t) = F_0(1,t) - F_0\left[H_1(1,t),t\right] = \sum_{k=0}^{\infty} q(k,t)\left(1-u(t)^k\right), \tag{6}$$

where $u(t) \equiv H_1(1,t)$. From eqs. (6) and (5) we calculate the survivability W (eq. (1)), by solving numerically the following equations:

$$\begin{aligned} W &= \int_{t=0}^{\infty}\left[\sum_{k=0}^{\infty} q(k,t)\left(1-u(t)^k\right)\right]^\beta \mathrm{d}t, \\ u(t) &= 1 - \sum_{k=0}^{\infty}\frac{kq(k,t)}{\langle k\rangle}\left[1-u(t)^{k-1}\right]. \end{aligned} \tag{7}$$

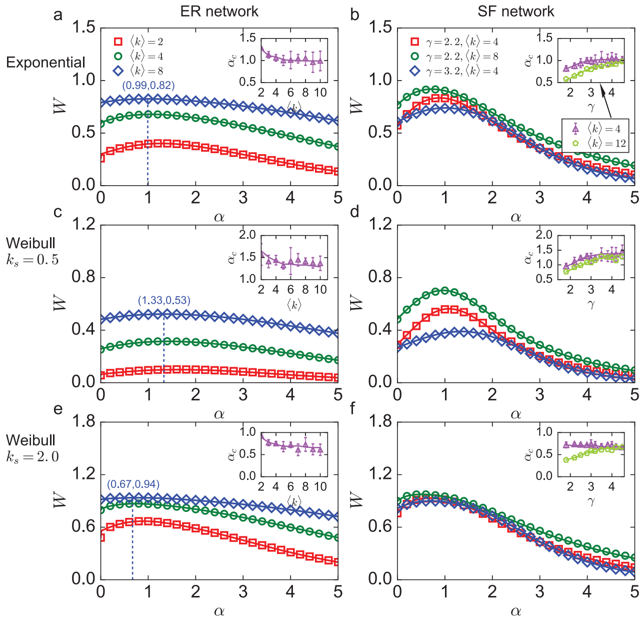

Fig. 2: (Colour online) The dependence of survivability W on α for different networks and component lifetime distributions ($\beta = 1$). (a), (b): the case of exponential lifetime distribution for component lifetime. (a) Results for W of ER networks with different average degree k. Open symbols represent the simulation results in networks with $N = 10^4$ nodes and averaged over 500 realizations, while solid lines are obtained from the theoretical predictions, eqs. (7). Both simulations and theoretical solutions were implemented by a summation of the giant component size over time in time steps of 0.01 units (the integral in eqs. (7) was replaced by a summation). Inset: the relationship between α_c and k is shown. The error bars are the standard deviations for α_c, calculated from 10 single realizations. (b) Results for SF networks with different power-law exponents but same average degree ($k = 4$ or $k = 8$). The network contains $N = 10^4$ nodes. Inset: the relationship between α_c and γ for scale-free networks. Changing the value of $\langle k\rangle$ was implemented by controlling the maximum connectivity and the probability of the minimum connectivity in the network. (c), (d): Weibull lifetime distribution case with shape parameter, $k_s = 0.5$, for ER and SF networks, which generates a broader distribution compared to the exponential distribution. (e), (f): Weibull lifetime distribution case with shape parameter $k_s = 2.0$ for ER and SF networks: the lifetime is narrower than for exponential distribution. The reason for $\alpha_c > 1$ for small k (in (a), (c) and (e)) is due to the fact that some isolated clusters exist, where some long-lifetime allocation investment is wasted. Since high-degree nodes are less probable to be in small clusters, the network functionality will gain more when allocating longer lifetimes to them.

Results. All the followings simulations results, were received by implementing stochastic simulations in a network of $N = 10^4$ nodes averaged over 500 realizations. We begin by analyzing the case where the lifetime of components follows an exponential distribution. We consider firstly the case of $\beta = 1$, and study how the exponent α affects the performance of network survivability W. In fig. 2(a), we show the survivability W for the ER network as a function of α, where (in this figure as well as in the rest of the figures in this paper), symbols represent the numerical results and solid lines represent the theoretical results. W has been calculated by a summation of the giant component size over time in time steps of 0.01 units (the integral in eq. (7) was replaced by a summation). We can see that for an ER network with a given average degree, W increases gradually as α increases and reaches the maximum at $\alpha = \alpha_c$. For example, for an ER network with $k = 4$, $\alpha_c \cong 1$. After the maximum, investing more resources on nodes with large degree will lead to the early failure of nodes with small degree, which will decrease the network survivability. Furthermore, fig. 2(a) shows that ER networks with larger average degrees have larger giant components, leading to larger network survivability. Interestingly, as seen in the inset of fig. 2(a), for $\beta = 1$, the optimal α saturates at a constant value around 1, for average degree above 4. This stable design configuration is reached due to the competition between high-degree nodes and low-degree nodes in the network design. On the one hand, high-degree nodes are more critical than low-degree nodes for the network integration; on the other hand, low-degree nodes may play a role of weak ties connecting different components to form a giant component [42].

A SF network has a more heterogeneous structure than an ER network: some nodes could have very large degree (hubs), but most have only a few connected neighbors. To optimize SF network survivability, the balance between hubs and lower-degree nodes becomes more sensitive. As shown in fig. 2(b), SF networks display a sharper maximum for W, compared to ER networks. We find that for SF networks with small power-law exponent γ, the optimal α is smaller than 1.0 found in ER networks (see inset of fig. 2(b)). This is because for similar α values, hubs in SF networks are usually much larger and will receive much longer lifetime than in ER networks, while low-degree nodes with shorter lifetime will fail in the early stages of network evolution. Moreover, α_c increases within a narrow range between 0.5 and 1.0 in SF networks with increasing power-law exponent γ, and finally approaches 1 (as for ER networks) when γ is close to 4.0. We also find that α_c increases with decreasing average degree.

For Weibull lifetime distributions, at time t the survival probability for a component follows $e^{-(t/\lambda)^{k_s}}$, where λ is the scale parameter, k_s is the shape parameter, and the lifetime expectancy is $\lambda\Gamma(1 + 1/k_s)$ (in our case it is proportional to k^{α}), where Γ is the Γ function. The broadness of the Weibull distribution is controlled by the shape parameter k_s. From fig. 2(c), for $k_s = 0.5$, where the lifetime distribution is relatively broad, we find that for $\beta = 1$, ER networks achieve optimal survivability for α larger than 1.0. For SF networks, in fig. 2(d), we also see that their survivability is also optimized at larger values of α_c. In these cases, in order to achieve the optimal resources allocation for network survivability, nodes with high degrees deserve more allocated resources. In fig. 2(d),

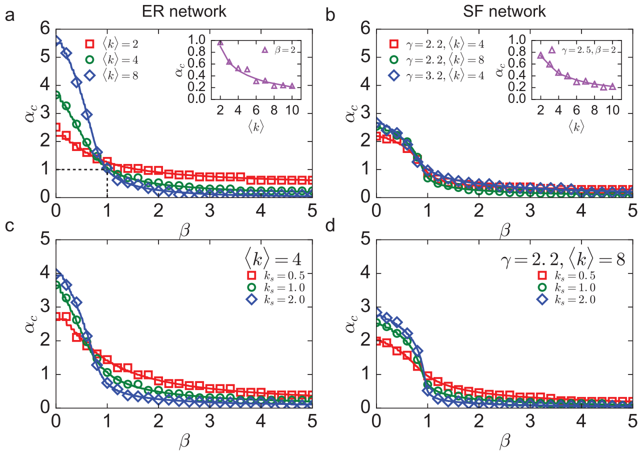

Fig. 3: (Colour online) The dependence of α_c on β. For the case of exponential lifetime distribution. (a) Simulation (symbols) and theoretical (lines) results for ER networks with different values of average degree k and the network size is 10^4; inset: for $\beta = 2$, the relationship between α_c and k is shown. (b) Simulation (symbols) and theoretical (lines) results for different SF networks. The SF network contains 10^4 nodes; inset: for $\beta = 2$, the relationship between α_c and k for SF networks with $\gamma = 2.5$ is shown. We compare the effects of different component lifetime distributions on the values of α_c: (c) ER network, $k = 4$, and (d) SF network, $\gamma = 2.2, k = 8$.

we also find that the peak of network survivability for SF networks is sharper than for ER networks (fig. 2(c)).

For $k_s = 2$, where the lifetime distribution is narrower, we find that the optimal survivability for ER and SF networks is obtained at smaller values of α_c, as shown in fig. 2(e) and fig. 2(f), suggesting that networks resources should be shared more equally to optimize survivability.

Next, we study how the users' requirements, represented by β, influence the optimal design α_c. In fig. 3 we plot graphs of α_c *vs.* β. We find in fig. 3(a) and fig. 3(b) that the optimal α_c decreases with increasing β for both ER and SF networks. When we put more weight on the network connectivity by increasing β, resources should be more uniformly invested among nodes of different degrees, which is represented by lower α_c. Indeed, when we are interested in only the network connectivity at large values of β, we obtain very small values for α_c (see fig. 3), approaching the design result for a static network, neglecting the effect of aging. Meanwhile, when the network lifetime is also considered important with decreasing β, large-degree nodes need to function in order to bridge different components in the giant component. Therefore, α_c increases continuously with decreasing β. Interestingly, we find that in both ER and SF networks the optimal α_c is close to 1 for β close to 1.0 for different combinations of network parameters.

Moreover, we find that different lifetime distributions of components have significant effects on the value of α_c for the same network (fig. 3(c) and fig. 3(d)): when β is small, narrow lifetime distribution (*e.g.*, $k_s = 2.0$) leads

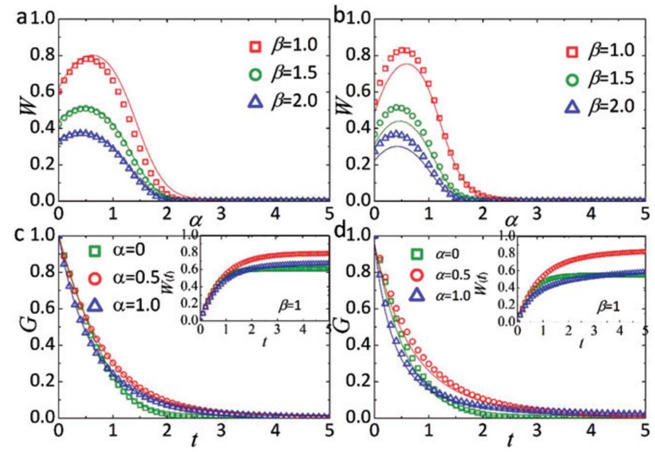

Fig. 4: (Colour online) Survivability of real-world systems with components lifetime following an exponential distribution. Open symbols show simulation results and solid lines represent theoretical results: (a) Cellular Network (Salmonella typhi) with $\gamma = 2.25$ and $k = 4.94$; (b) Internet (AS-733, January 02, 2000) Network with $\gamma = 2.13$ and $k = 3.88$. (c), (d): simulations (symbols) and theoretical (lines) results for the fraction of nodes within the giant cluster as a function of time t in the Cellular Network and Internet Network, respectively; inset: simulation results for temporal survivability of real-world systems, W, as a function of time t: $W(t) = \sum G^\beta \Delta t$, integral values of $G^\beta \Delta t$ up to t as a function of t. Here we show results for $\beta = 1$.

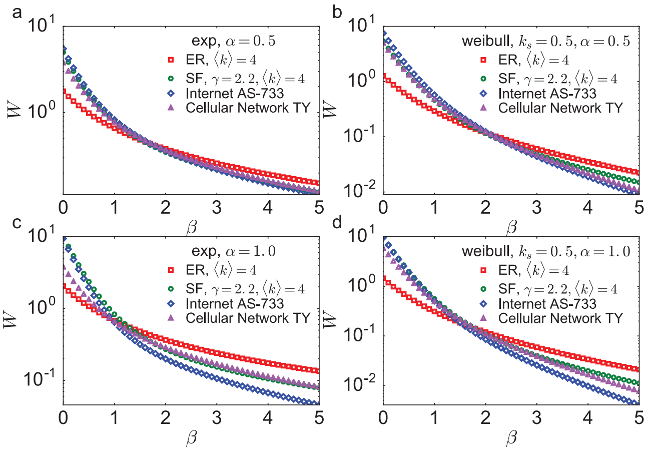

Fig. 5: (Colour online) The dependence of survivability W on β for different networks and component lifetime distributions. We calculate survivability for two network models (ER network and SF network), and two real networks (Internet AS-733 and Cellular Network (Salmonella typhi)). In (a) and (c) ($\alpha = 0.5$ and $\alpha = 1.0$, respectively), the lifetime of network components follows the exponential distribution. In (b) and (d) ($\alpha = 0.5$ and $\alpha = 1.0$, respectively), the lifetime of network components follows the Weibull distribution ($k_s = 0.5$).

to a larger value of α_c compared with broad lifetime distribution (*e.g.*, $k_s = 0.5$). However, when β increases to a certain value, the situation is reversed, the value of α_c becomes smaller for narrow lifetime distribution but higher for broad lifetime distribution.

Survivability of real complex systems. – To test the significance of our framework for real-world systems, we analyze the survivability of several real complex systems, when subject to allocation of resources according to the degrees of nodes. We calculate their survivability W as a function of α and compare them with our corresponding theoretical results. As shown in fig. 4(a) and fig. 4(b), we illustrate the simulation and theoretical results for the survivability of Cellular Network-TY [43] and Internet AS-733 [44,45], where the connectivity follows a power-law distribution. We find that their survivability for different β can be well predicted (in particular the values of α_c) by theoretical results, which indicate that survivable systems can be designed without details of network topology, but just by knowing the degrees of network components and determining their lifetime distribution. Figure 4(c) and fig. 4(d) show how the above two real systems collapse as time evolves, both in the theoretical and simulation analysis. Note that there are slight differences between theoretical and simulation results for real-systems survivability because nodes in real complex systems have properties such as clustering and degree-degree correlations, but our theory assumes that the network is completely random without correlations. As the average degree and degree heterogeneity in real systems are usually very large, for the exponential lifetime distribution case and $\beta = 1$ we find that the values of α_c for some real systems are even close to 0.5, which is similar to SF networks with small power-law exponent and large average degree.

In fig. 5, we also present how network survivability changes with users' requirements, both for network models and real networks. We find that when β increases, the network survivability decreases. Moreover, fig. 5 shows that a network with broader heterogeneous degree distribution (for example, Internet AS-733 and SF network), will have larger network survivability when users are more interested in the network lifetime. But when network connectivity is preferred, the network with homogeneous degree distribution (*e.g.*, ER network) will gain higher network survivability.

Discussion. – In this work we proposed a new realistic concept for the robustness of a random network, by considering the functionality of the network during its whole lifespan, instead of the traditional approaches valid only for networks at a given snapshot. Here the robustness is calculated in the presence of components *aging* by a survivability function W, eq. (1), which considers the way lifetime is distributed to different components (parameter α) and the functionality (parameter β) of the network during its lifetime. Our main finding is that for large β (high connectivity) α_c —the α value that maximizes W— tends to 0 and a uniform lifetime division between the nodes is required. As β decreases (which requires large survival time) α_c increases, *i.e.*, there is a preference of lifetime allocation to high degree nodes. These findings could be useful for recognizing the actual users' requirements and correspondingly improving the network survivability in the design stage.

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