Statistical Physics Approaches to Financial Fluctuations

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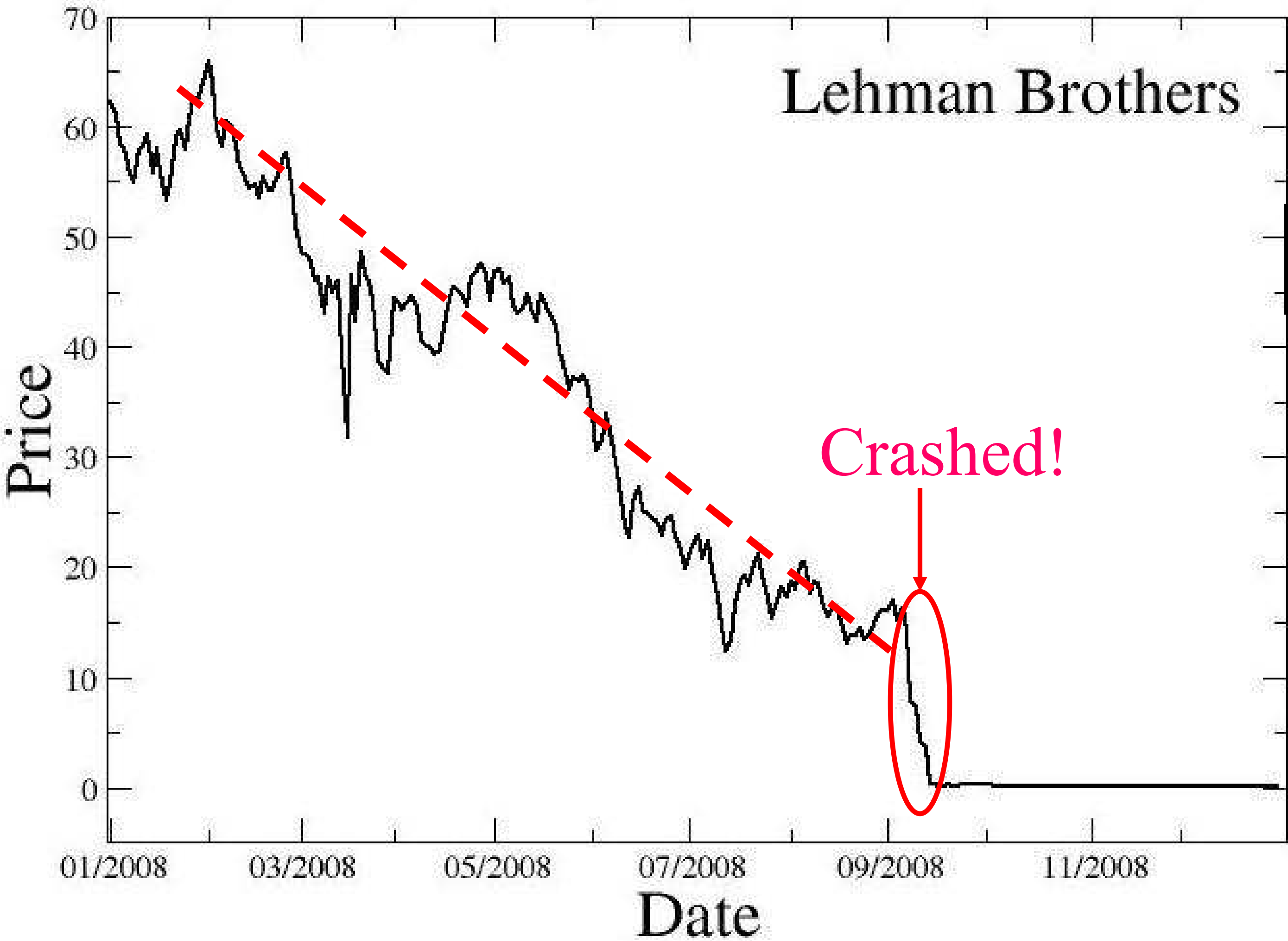
Outline

- Motivation
  - Q1: What are financial fluctuations?
  - Q2: Why study?
- Method:
  - New approach: return interval analysis
  - Ideas from statistical physics
- Results:
  - Scaling
  - Universality
  - Memory

Motivation

Q1: What are financial fluctuations?



Q2: Why study?

- Risk Control
- Complexity

Source: http://www.yahoo.com/finance

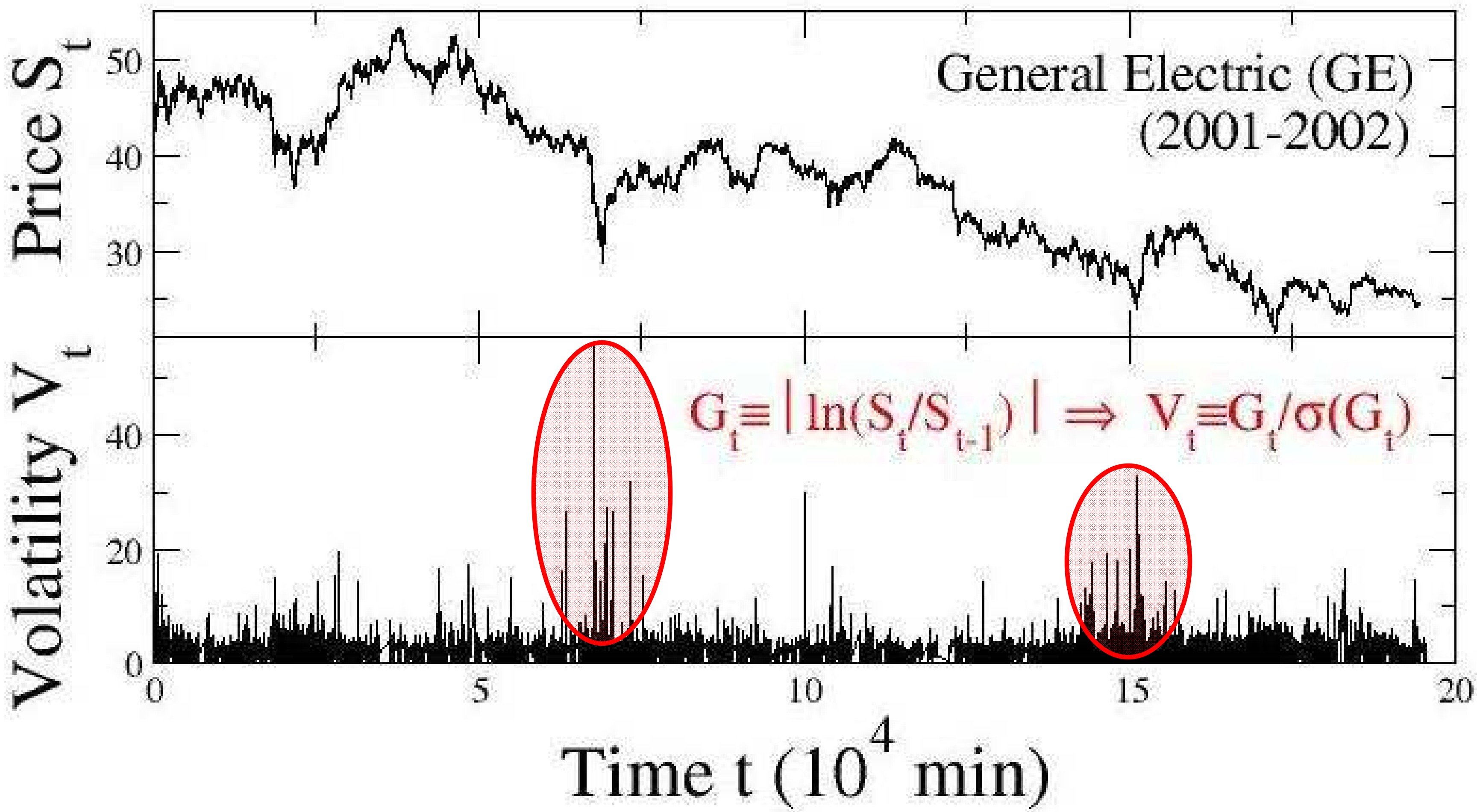
TAQ Database

- **All** transactions in U.S. market, 2001-2002
- **10^4** stocks and **10^9** records
- Dow Jones Industrial Average (DJIA) index, 30 stocks
- Sampling time:1 sec => **1 min**

Ex: General Electric

Time		Price
12:51:52		26.30000000
12:51:56		26.33000000
12:51:58	S_t	26.30000000
12:52:02		26.32000000
12:52:06		26.33000000
12:52:15		26.32000000
12:52:15		26.33000000
12:52:25		26.32100000
12:52:30		26.33000000
12:52:33		26.32000000
12:52:42		26.32000000
12:52:50		26.31000000
12:52:53		26.32000000
12:52:57	S_{t+1}	26.31000000
12:53:01		26.30000000
12:53:06		26.32000000

How to Measure Fluctuations?



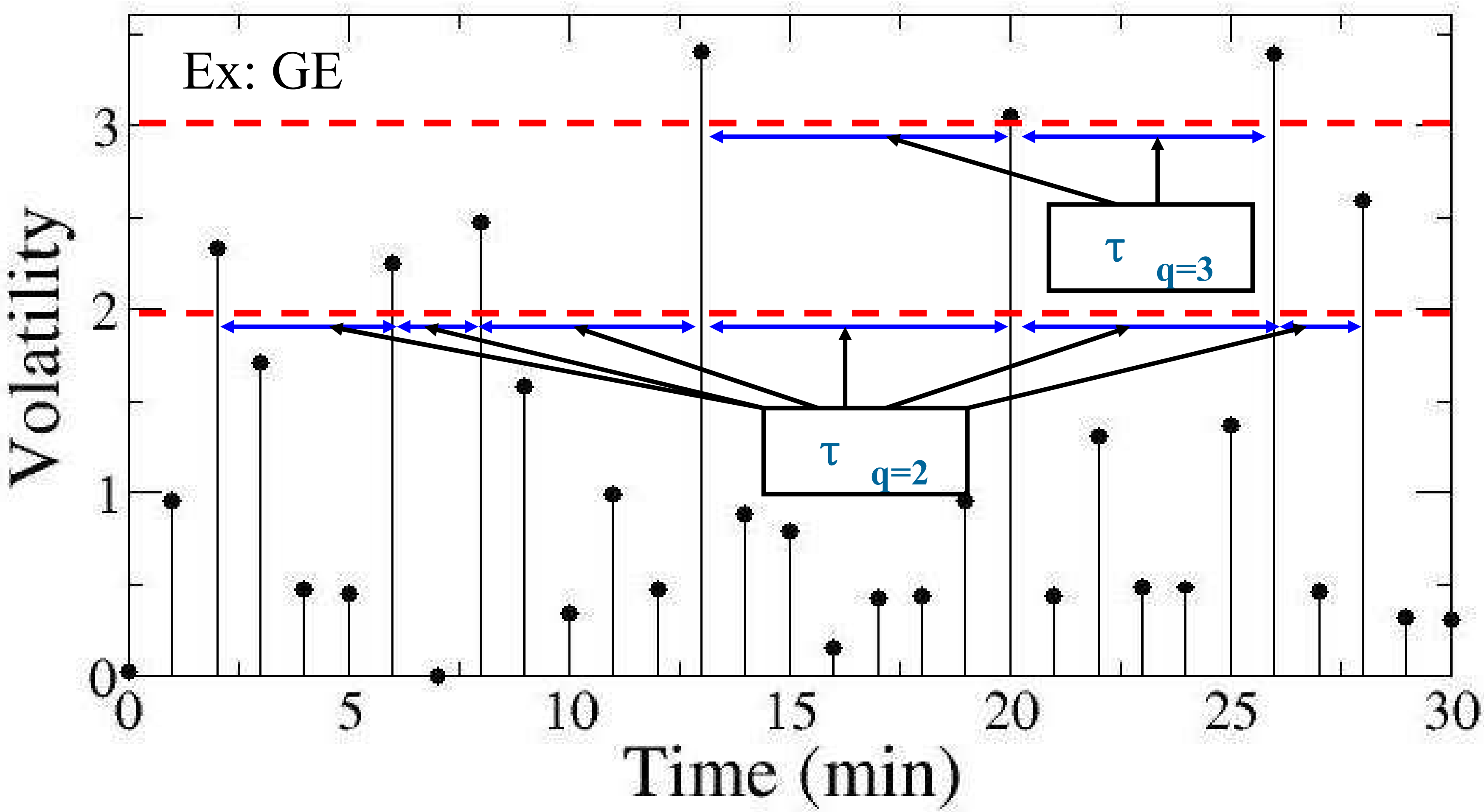
- **Fat tail:** large values (more than a normal distribution) outliers?
- **Volatility clustering:** large values tend to be together, why?

How to Test Time Organization for Large Volatilities?

New approach: **Return Interval** τ :

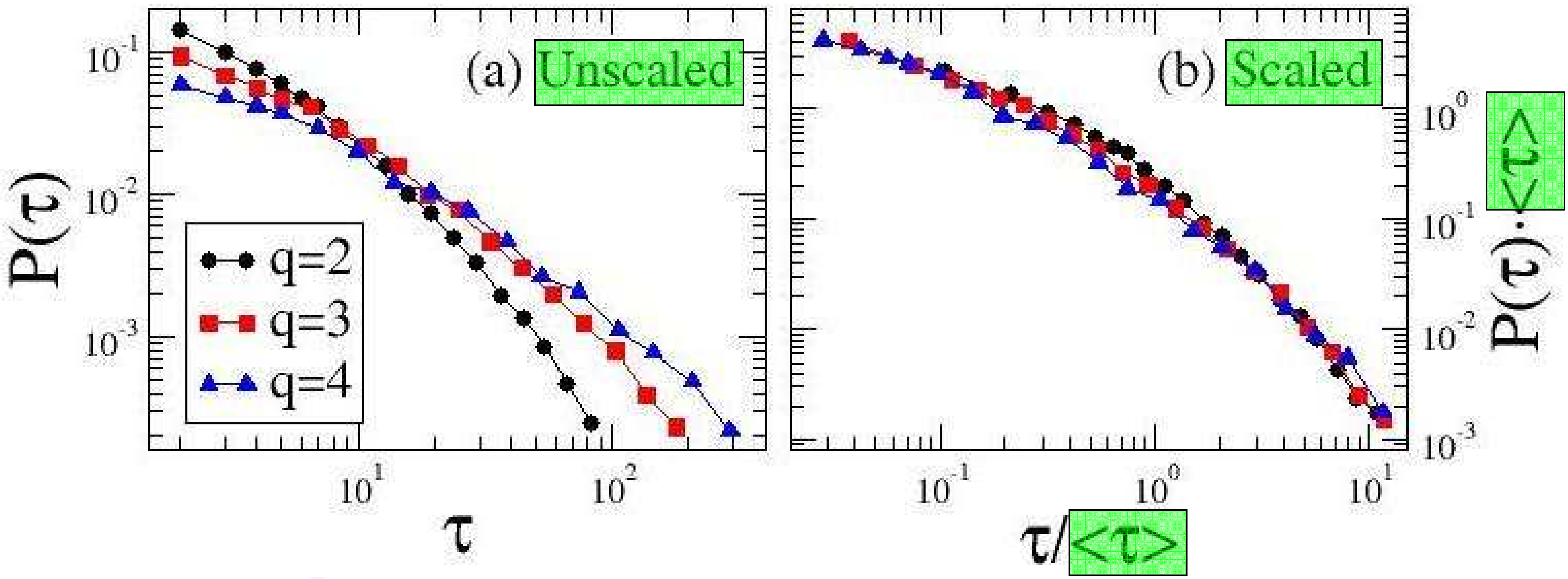
Step 1: Choose a **threshold q**

Step 2: Calculate **time intervals** between volatilities **above q**



Result: Scaling in Distributions

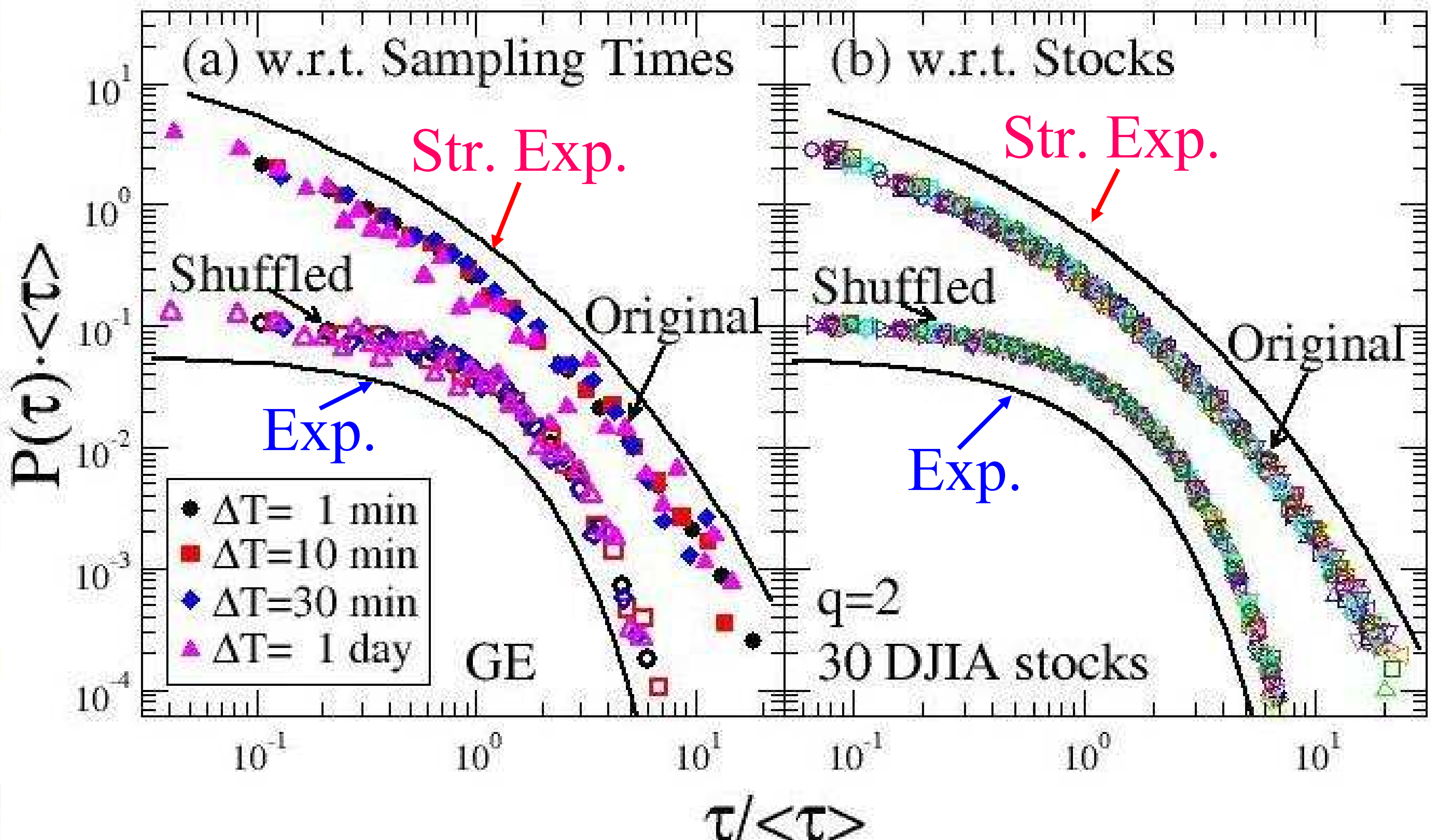
Ex: GE τ : return interval, P: distribution function



- $P(\tau)\cdot\langle\tau\rangle = f\left(\tau/\langle\tau\rangle\right) \Rightarrow P(\tau) = f\left(\tau/\langle\tau\rangle\right)/\langle\tau\rangle$
- f : scaling function

Wang et. al, Phys. Rev. E 73, 026117 (2006)

Result: Universality of Scaling



Scaling function follows a **Stretched Exponential**, $f(x) \sim \exp(-x^{\gamma})$

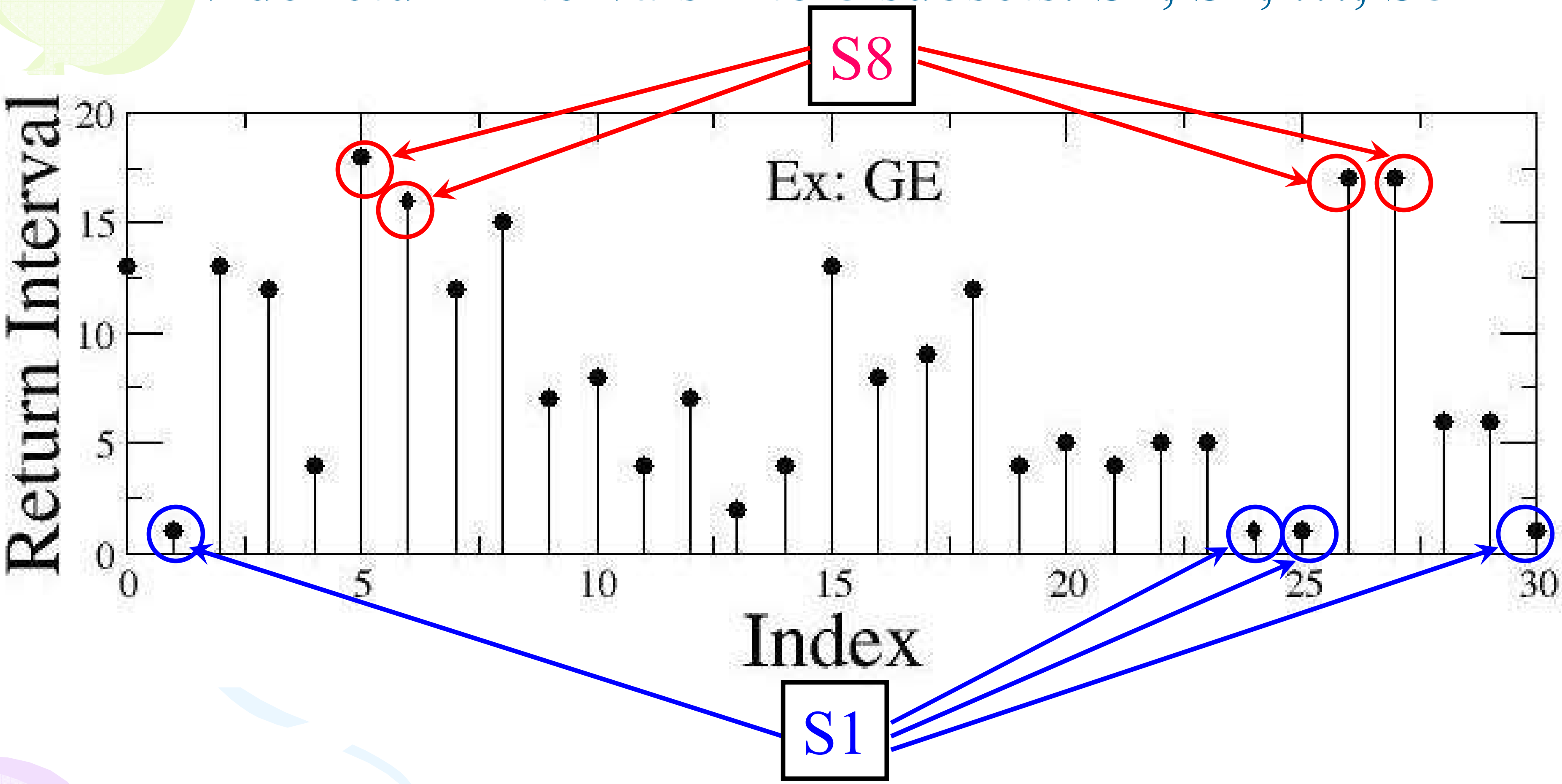
Original (w. correlations): $\gamma \sim 0.4$ (**all** stocks)

Shuffled (w/o correlation): $\gamma = 1$ (Exponential)

Wang et. al, Eur. Phys. J. B 55, 123 (2007)

How to Analyze Memory?

Divide return intervals into 8 subsets: S1, S2, …, S8

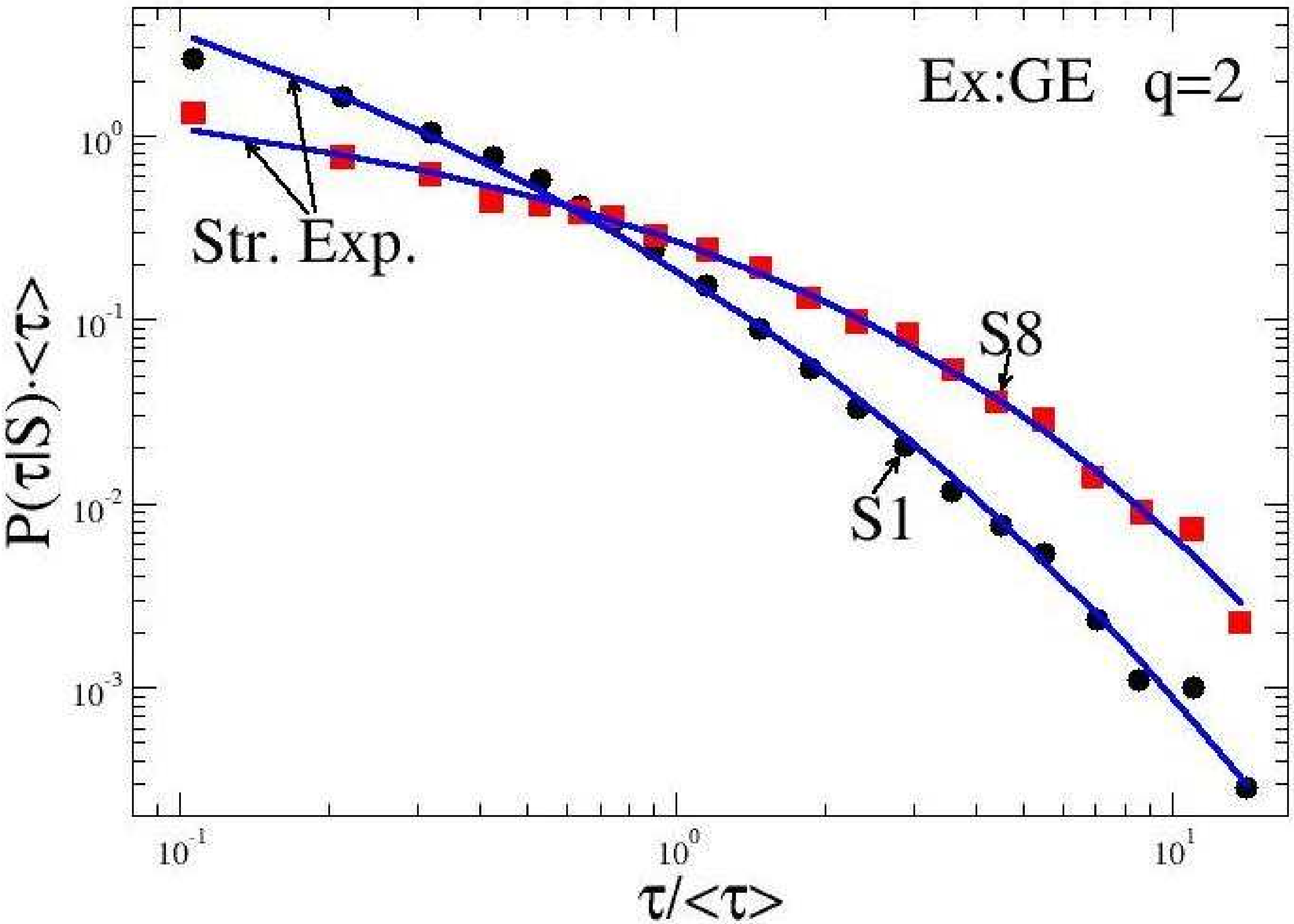


Collect all return intervals after points of one subset S

➔Conditional Distribution P(τ |S)

No memory => P(τ |S)=P(τ), or P(τ |S1)= P(τ |S8)

Result: Conditional Distribution

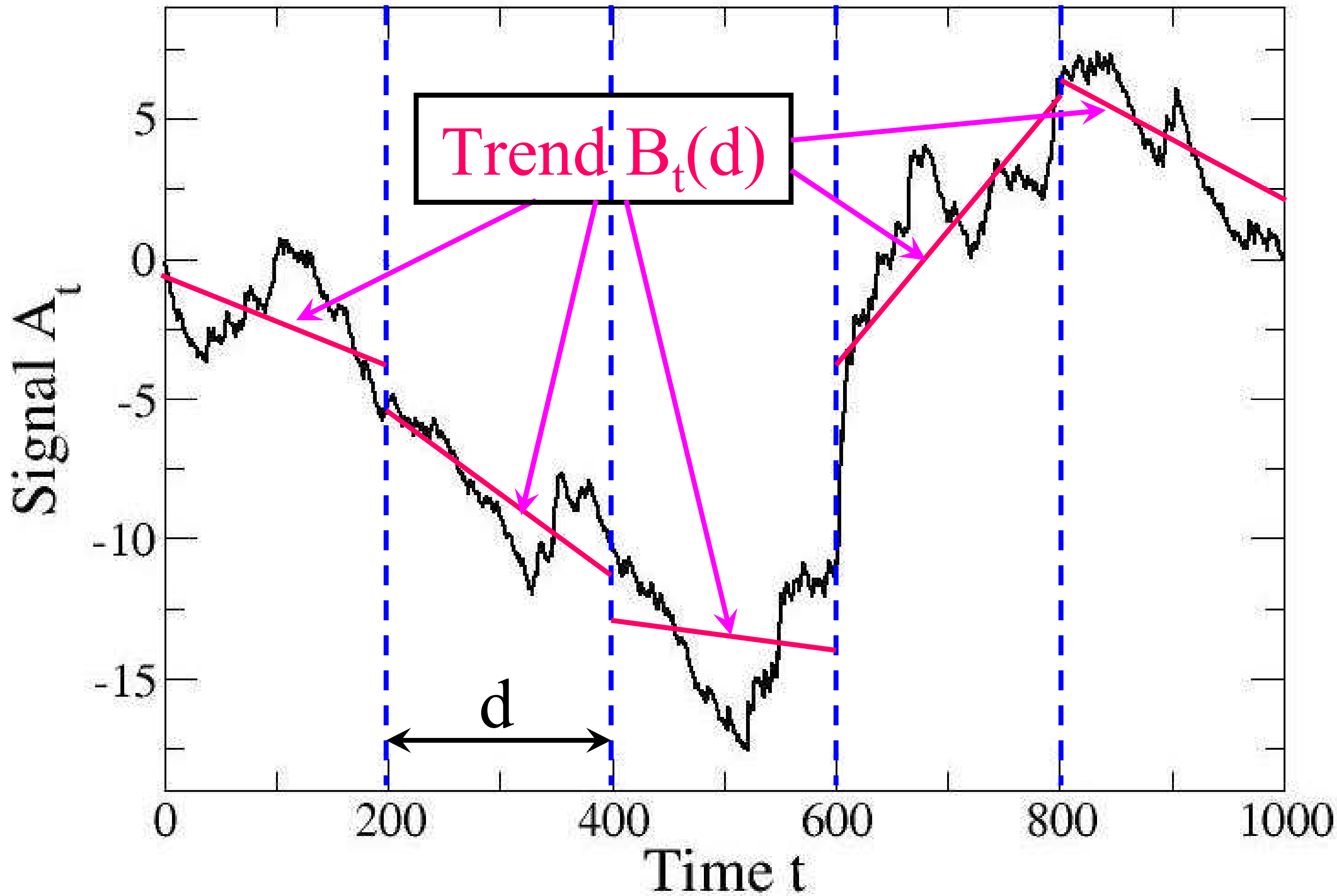


Significant discrepancy ➔ strong memory

Wang et. al, Phys. Rev. E 73, 026117 (2006)

How to Measure Long-Term Correlations?

Method: Detrended Fluctuation Analysis (DFA)

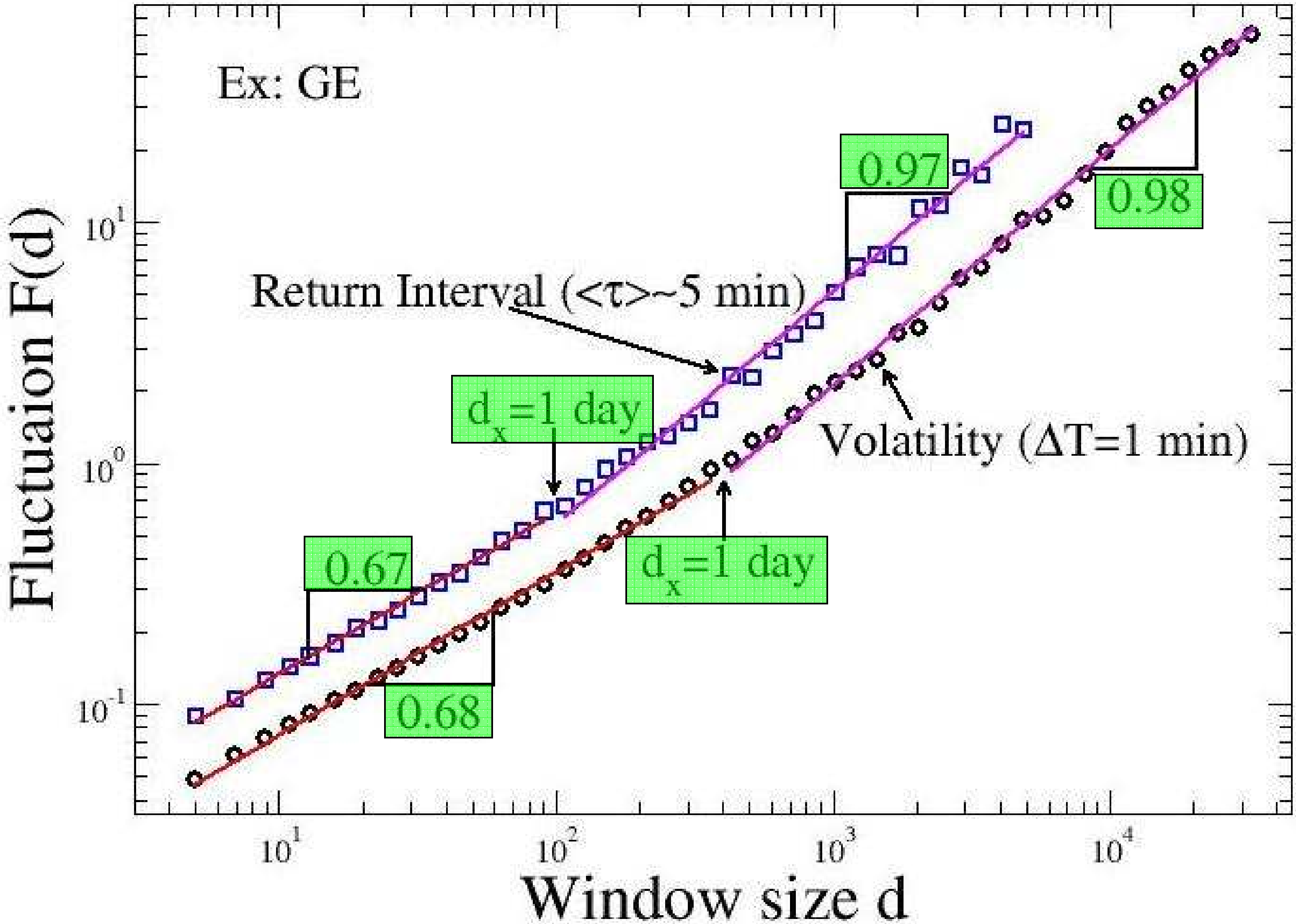


$$F(d) \equiv \sqrt{\sum_{t=1}^{N} \left(A_t - B_t(d)\right)^2 / N} \sim d^{\alpha}$$

Correlation exponent

$$\begin{cases} \alpha < 0.5 : \text{anti-correlated walk} \\ \alpha = 0.5 : \text{uncorrelated walk} \\ \alpha > 0.5 : \text{correlated walk} \end{cases}$$

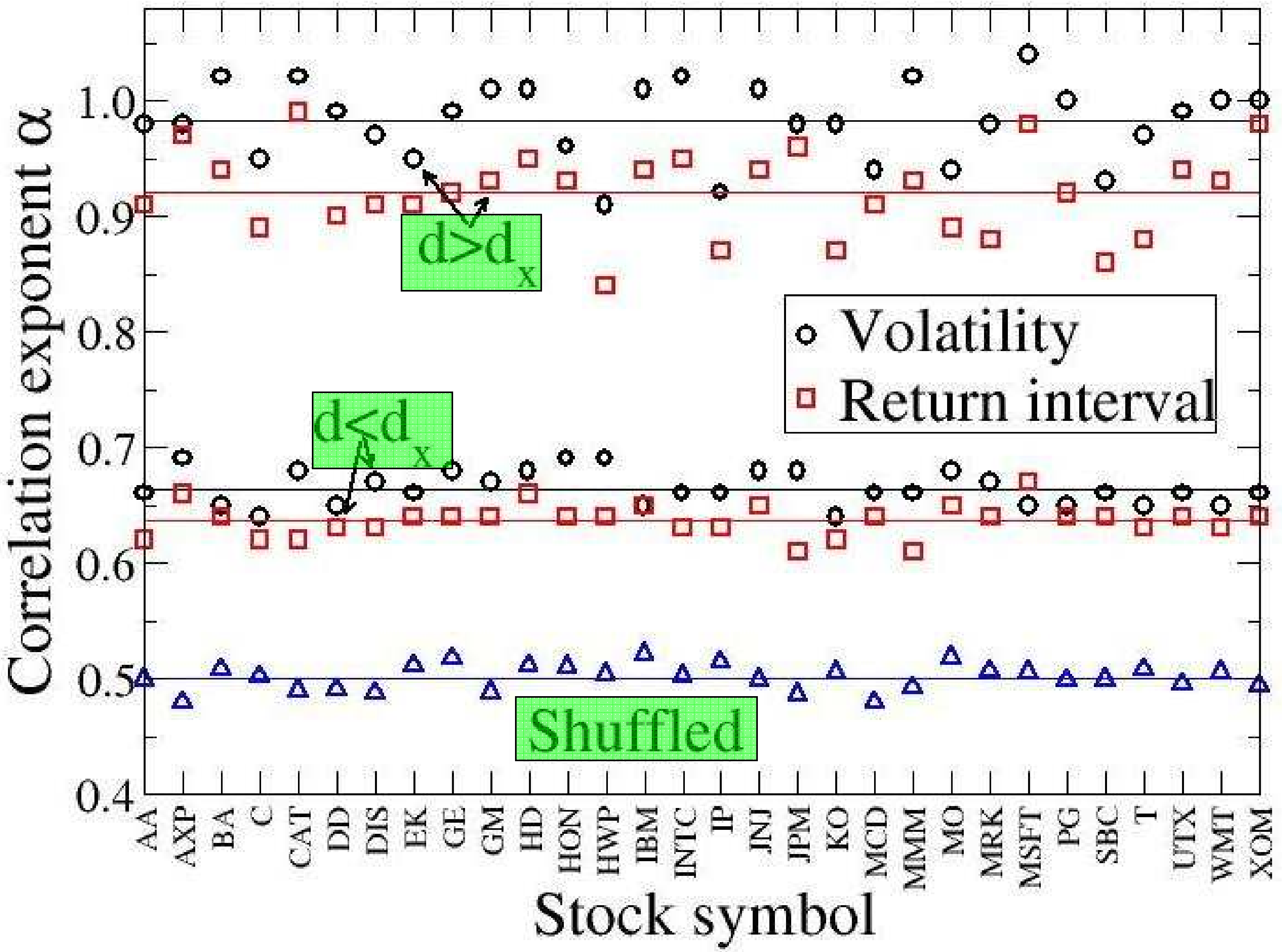
Result: Detrended Fluctuation Analysis



Surprise: Return interval correlations ⇔ Volatility correlations

Wang et. al, Phys. Rev. E 73, 026117 (2006)

Result: Universality in Correlations



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Conclusions

- Return interval distribution follows a **scaling law**.
- The scaling law is **universal** for:
  - wide range of thresholds
  - wide range of sampling times
  - various stocks
  - diverse markets
- Return interval has **long-term correlations** which are strongly related to volatility.