Statistical Physics and the "Problem of Firm Growth"

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"Statistical properties of the Internal Structure of Firms", **submitted** to EPL, March 2004.

Motivation

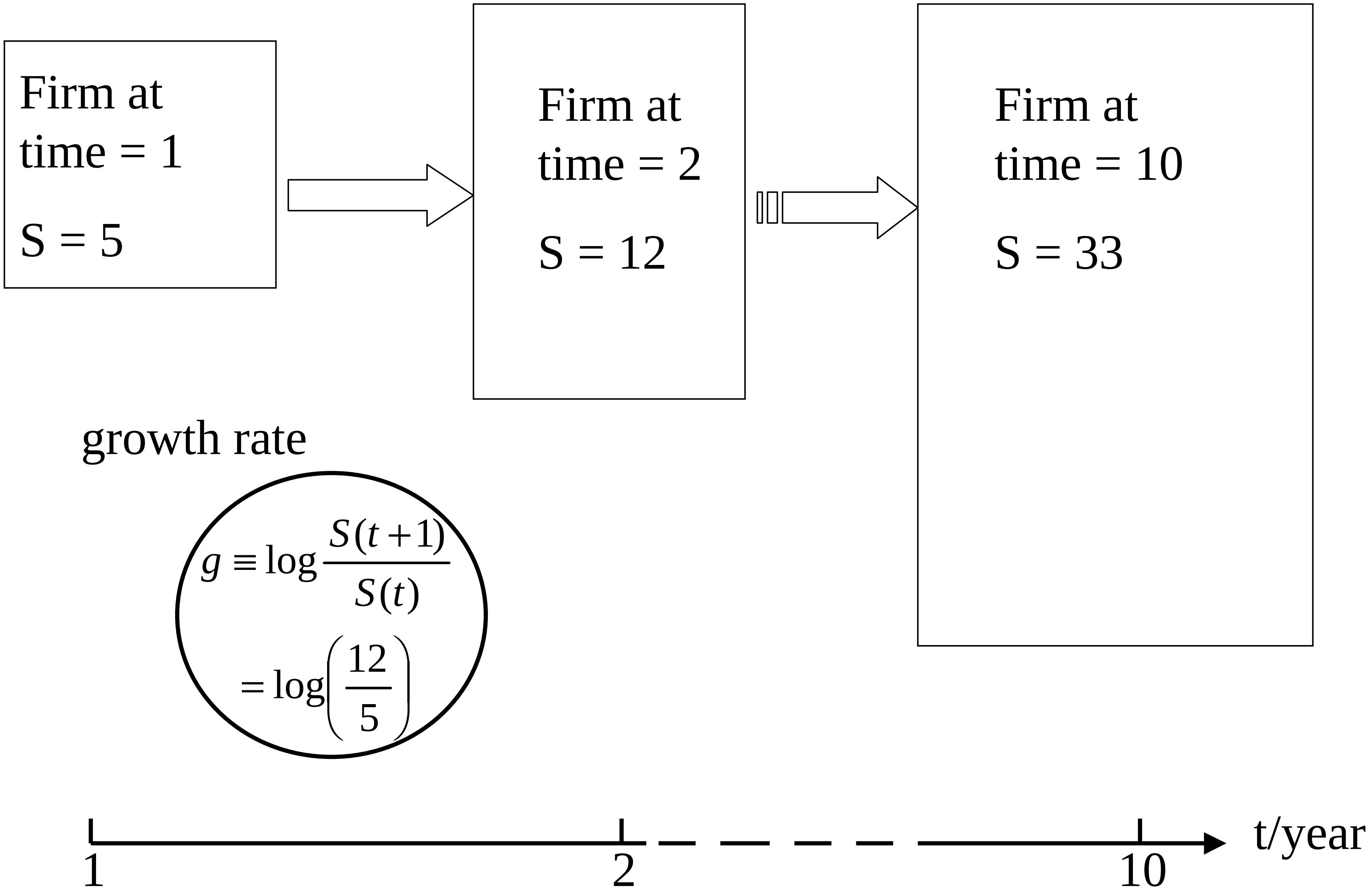
Firm growth problem $\equiv$ **statistical quantifying size changes of firms.**

1) **Firm growth problem is an unsolved problem in econophysics.**
2) **Statistical physics may help us to develop better strategies to improve economy.**
3) **People live worse if firms perform badly.**

Outline of Talk

- Apply statistical physics to "**firm growth problem"**.
- Introduce **a statistical physics model** that explains known empirical results and also predicts new results.
- Identify **3 scaling exponents** associated to firm growth process.
- Test our model by analyzing a large **NEW** Database.

Classical Problem of Firm Growth



$$g \equiv \log \frac{S(t+1)}{S(t)}$$

$$= \log\left(\frac{12}{5}\right)$$

Classic Gibrat Law & Its Implication

Question: What is pdf of growth rate?

Traditional View: Gibrat law of "Proportionate Effect" (1930's)

$$\mathbf{S(t+1) = S(t)\,(1 + \eta_t)} \quad (\eta_t \text{ is noise, } -1 < \eta_t < 1).$$

⇨ $\log S(t = T)$

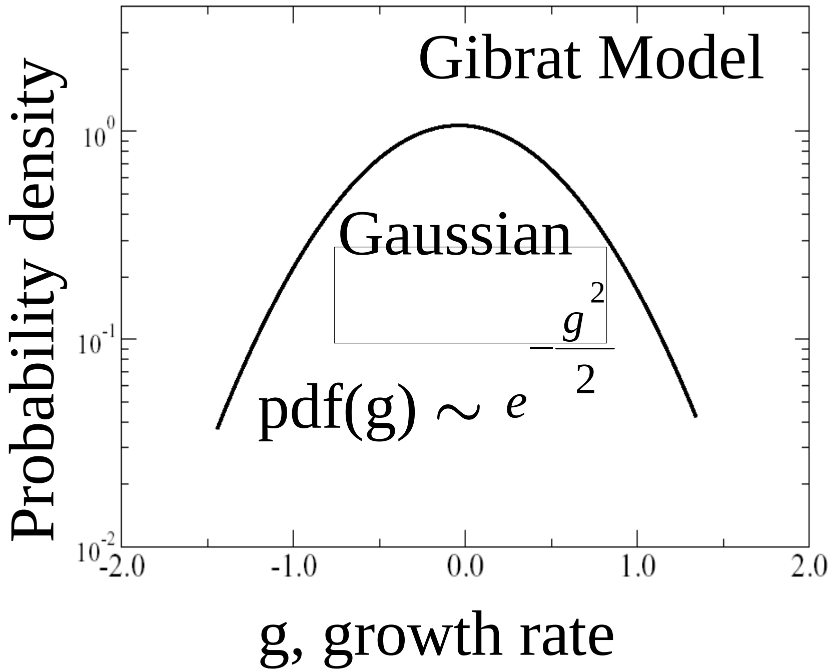
$$= \log S(t = 0) + \sum_{t'=1}^{M} \log(1 + \eta_{t'})$$

⇨ Growth rate g in T years

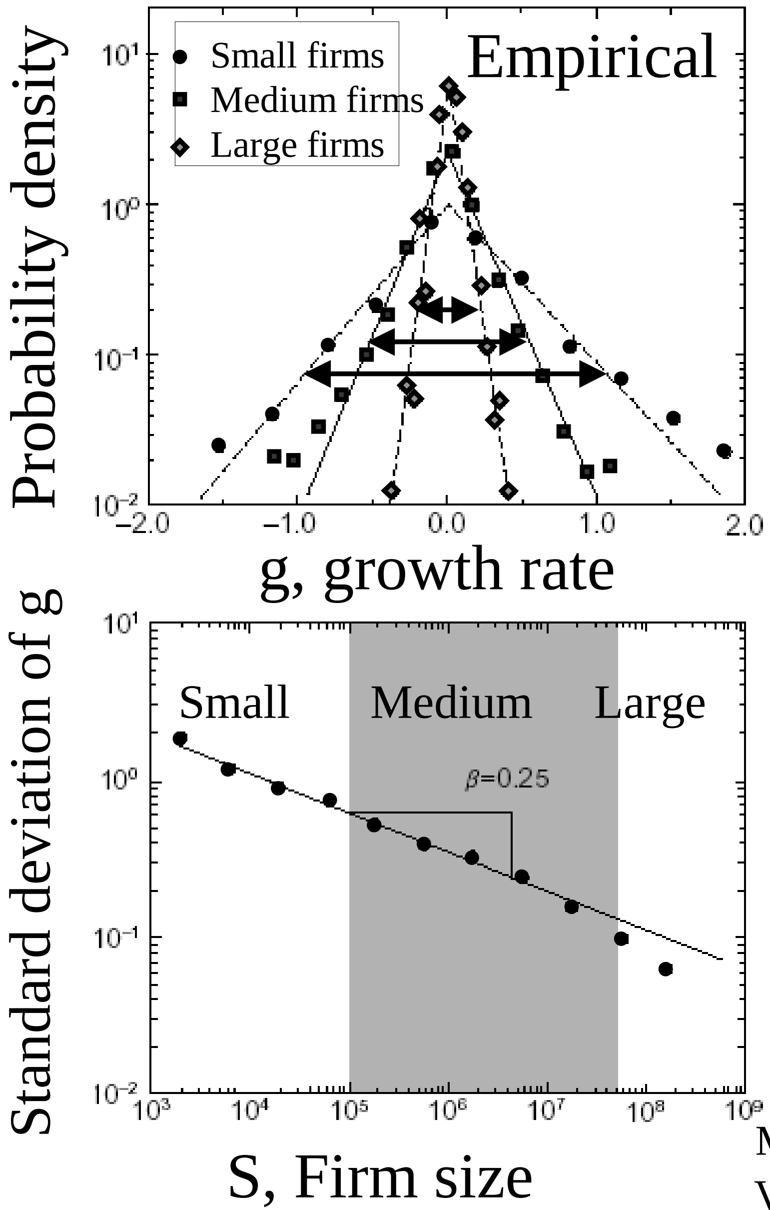
$$= \log \frac{S(T)}{S(0)}$$

$$= \sum_{t'=1}^{M} \log(1+\eta_{t'}) \approx \sum_{t'=1}^{M} \eta_{t'}$$

Gibrat: pdf of g is Gaussian.



Empirical Observations (before 1999)



Reality: it is "tent-shaped"!

$$\textbf{pdf(g|S)} \sim e^{-\frac{|g|}{\sigma(S)}}$$

$$\boldsymbol{\sigma_g(S) \sim S^{-\beta}, \quad \beta \approx 0.2}$$

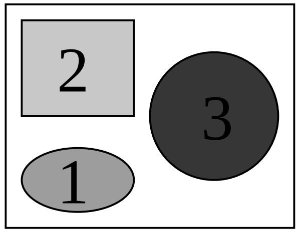
Michael H. Stanley, *et.al.* Nature **379**, 804-806 (1996).
V. Plerou, et.al. Nature **400**, 433-437 (1999).

Two Models: Gibrat Model (1934) & Amaral *et al* Model (1998)

Gibrat Model: $S(t+1) = S(t)\,(1 + \eta_t)$

Proportionate Effect ⇨ **pdf(g|S) is Gaussian & $\beta = 0$**

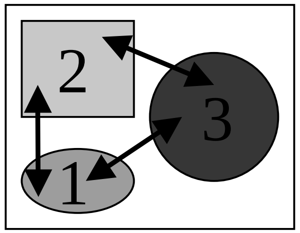
Like ideal gas, firms may be composed of independent fluctuating divisions (e.g. products). $\xi_i(t+1) = \xi_i(t)\,(1 + \eta_i(t))$



If each division independently grows,

⇨ **pdf(g|S) is Gaussian & $\beta = 0.5$**

Amaral *et al* Model:

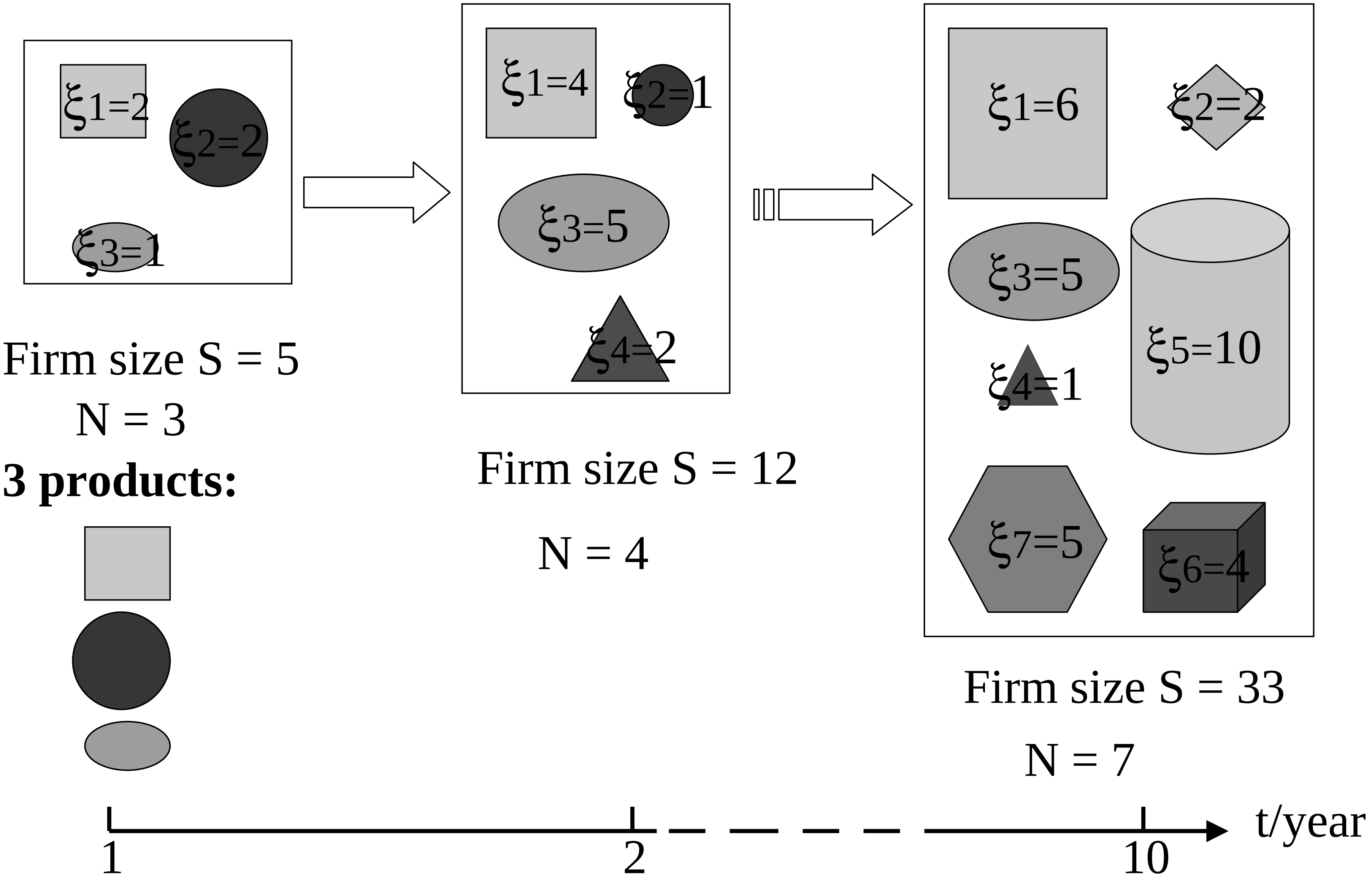


Add weak interactions between divisions,

⇨ **pdf(g|S) is like a "tent" & $\beta \approx 0.2$**

L. A. N. Amaral, et.al., "Power Law Scaling for a System of Interacting Units with Complex Internal Structure," Phys. Rev. Lett. **80**, 1385-1388 (1998).

New: Focus on Products within a Firm



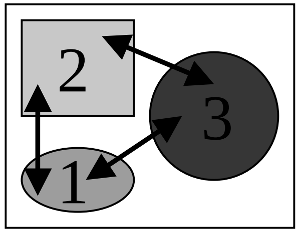
Predictions of Amaral *et al* Model

Never tested

Prediction 1

Firm size = S

Product sale = ξ



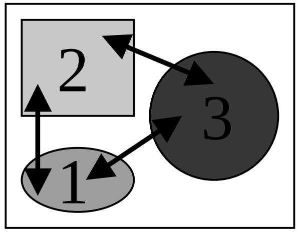
$$\boldsymbol{\rho_1(\xi|S) \sim S^{-\alpha} \cdot f_1(\xi/S^{\alpha})}$$

$$\alpha = 0.66 \pm 0.05$$

Prediction 2

Firm size = S

Number of products = N



$$\boldsymbol{\rho_2(N|S) \sim S^{-\gamma} \cdot f_2(N/S^{\gamma})}$$

$$\gamma = 0.34 \pm 0.05$$

$$\mathbf{S = N \cdot \xi}$$

$$\boldsymbol{\sigma_g(S)} \sim \frac{1}{\sqrt{N}}$$

$$\boldsymbol{\alpha + \gamma = 1}$$

$$\boldsymbol{\beta = \gamma / 2}$$

Specific Question

Are these predictions of Amaral *et al* model valid or not?

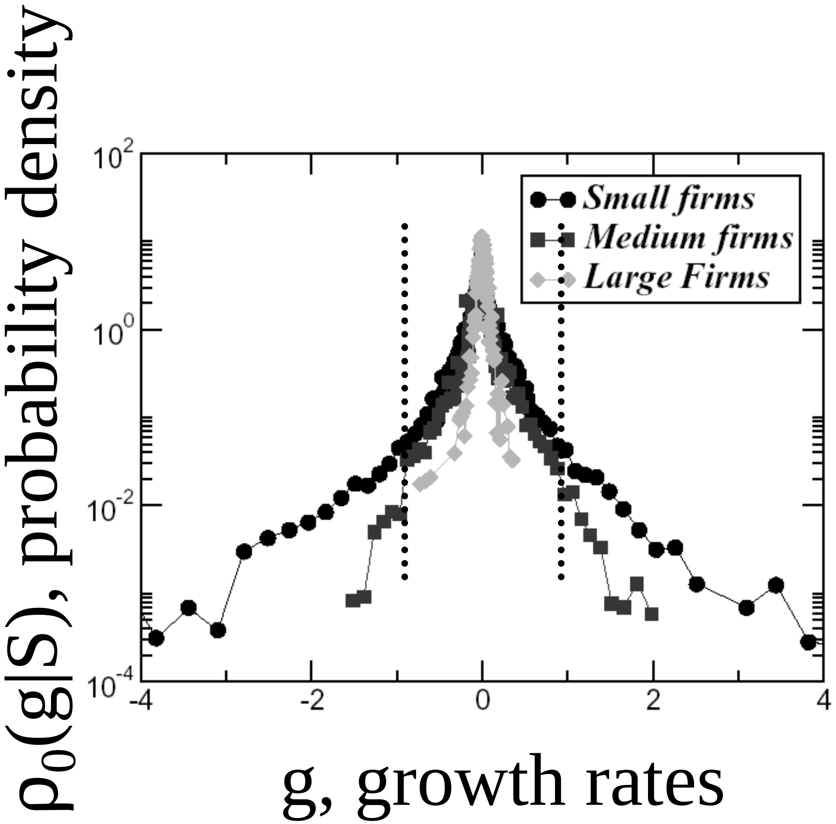
We analyze a new database of Pharmaceutical Industry (PHID).

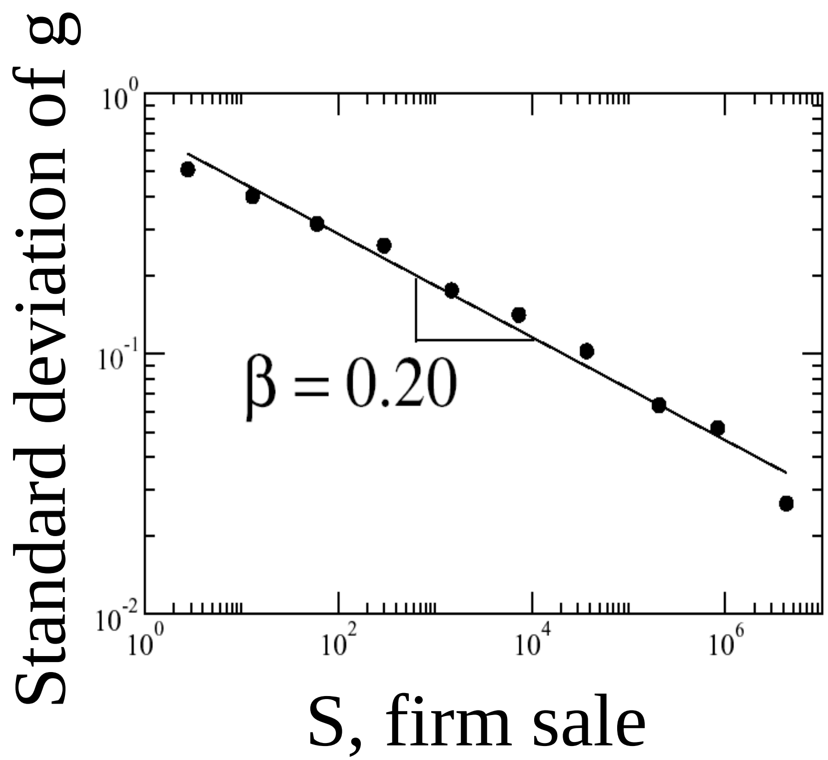
It records quarterly sales of 55624 pharmaceutical products commercialized by 3939 companies in the European Union and North America, from 1992 to 2001.

Our work is the first time to test the predictions of Amaral *et al* model so far.

New Empirical Results on Growth Rate g (1st variable)

Our Database





Empirical result :

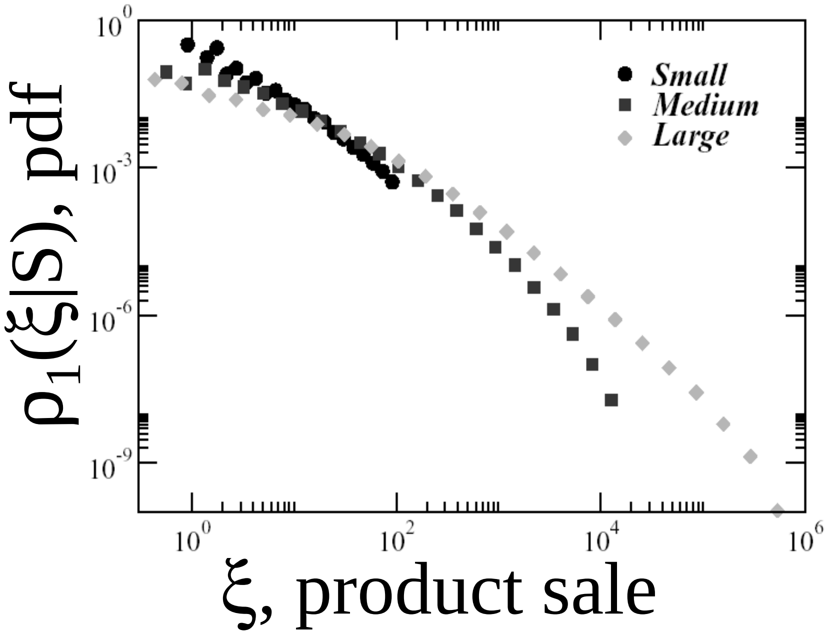
"tent shaped"

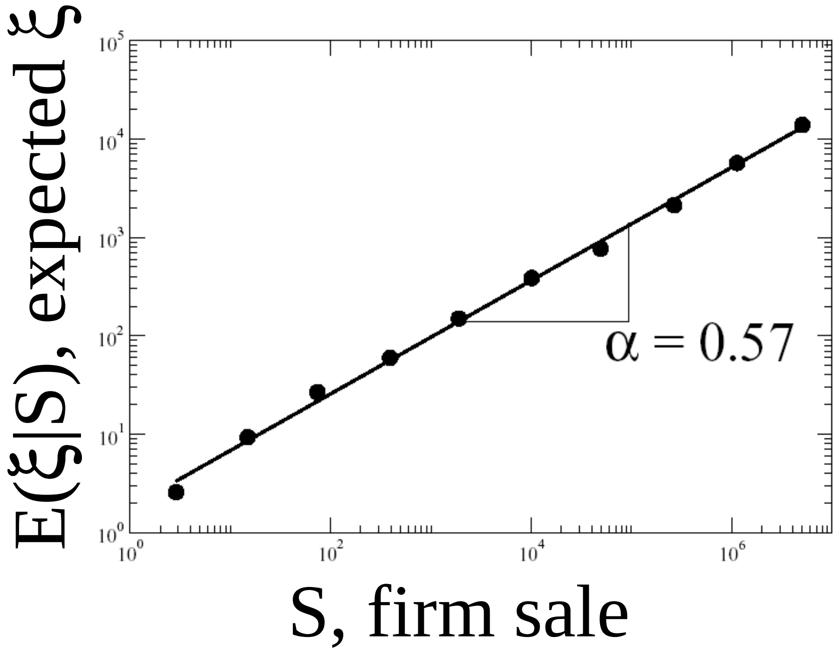
$\sigma_g(S) \sim S^{-\beta}, \quad \beta = 0.2 \pm 0.01$

New Empirical Results on Product Sale ξ (2nd variable)

$\rho_1(\xi|S) \sim S^{-\alpha} \cdot f_1(\xi/S^{\alpha})$

$E(\xi|S) \sim S^{\alpha}$





Prediction 1:

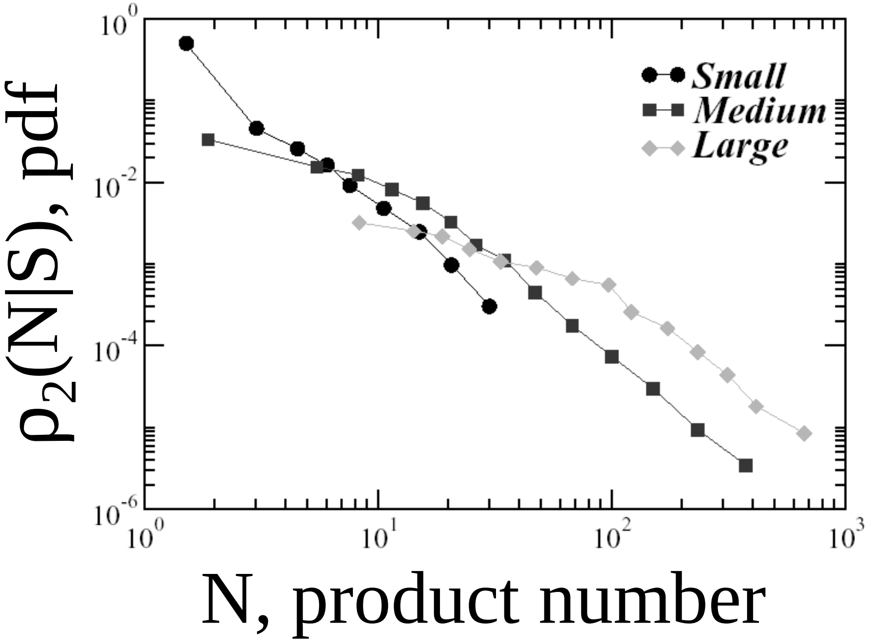
$\alpha = 0.66 \pm 0.05$

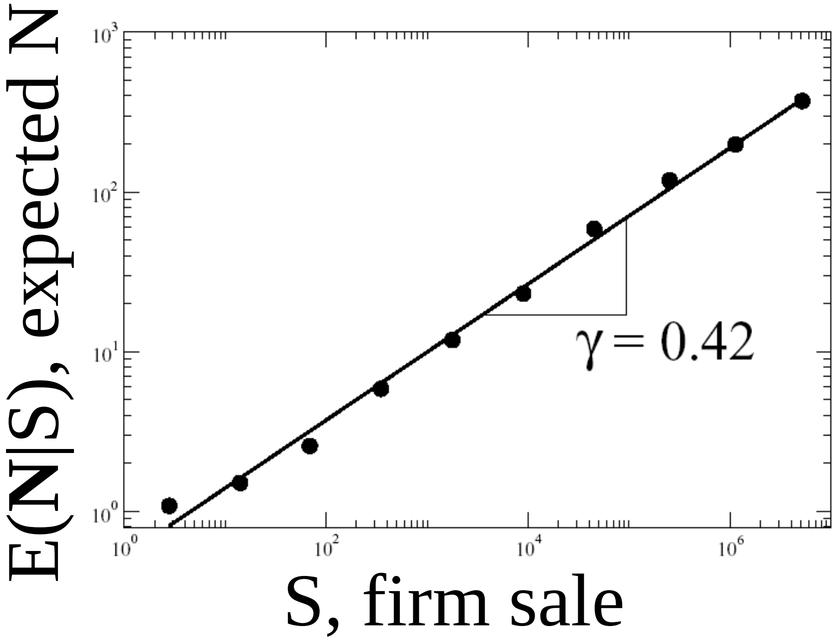
Empirical result :

$\alpha = 0.57 \pm 0.01$

New Empirical Results on Number of Products N (3rd variable)

$\rho_2(N|S) \sim S^{-\gamma} \cdot f_2(N/S^{\gamma})$ $E(N|S) \sim S^{\gamma}$





Prediction 2:

$\gamma = 0.34 \pm 0.05$

Empirical result:

$\gamma = 0.42 \pm 0.01$

Summary of New Results

Amaral *et al* model

Predictions

$\alpha = 0.66 \pm 0.05$

$\beta = 0.17 \pm 0.03$

$\gamma = 0.34 \pm 0.05$

$$\beta = \gamma / 2$$

$$\alpha + \gamma = 1$$

Our Database

Empirical results

$\alpha = 0.57 \pm 0.01$

$\beta = 0.20 \pm 0.01$

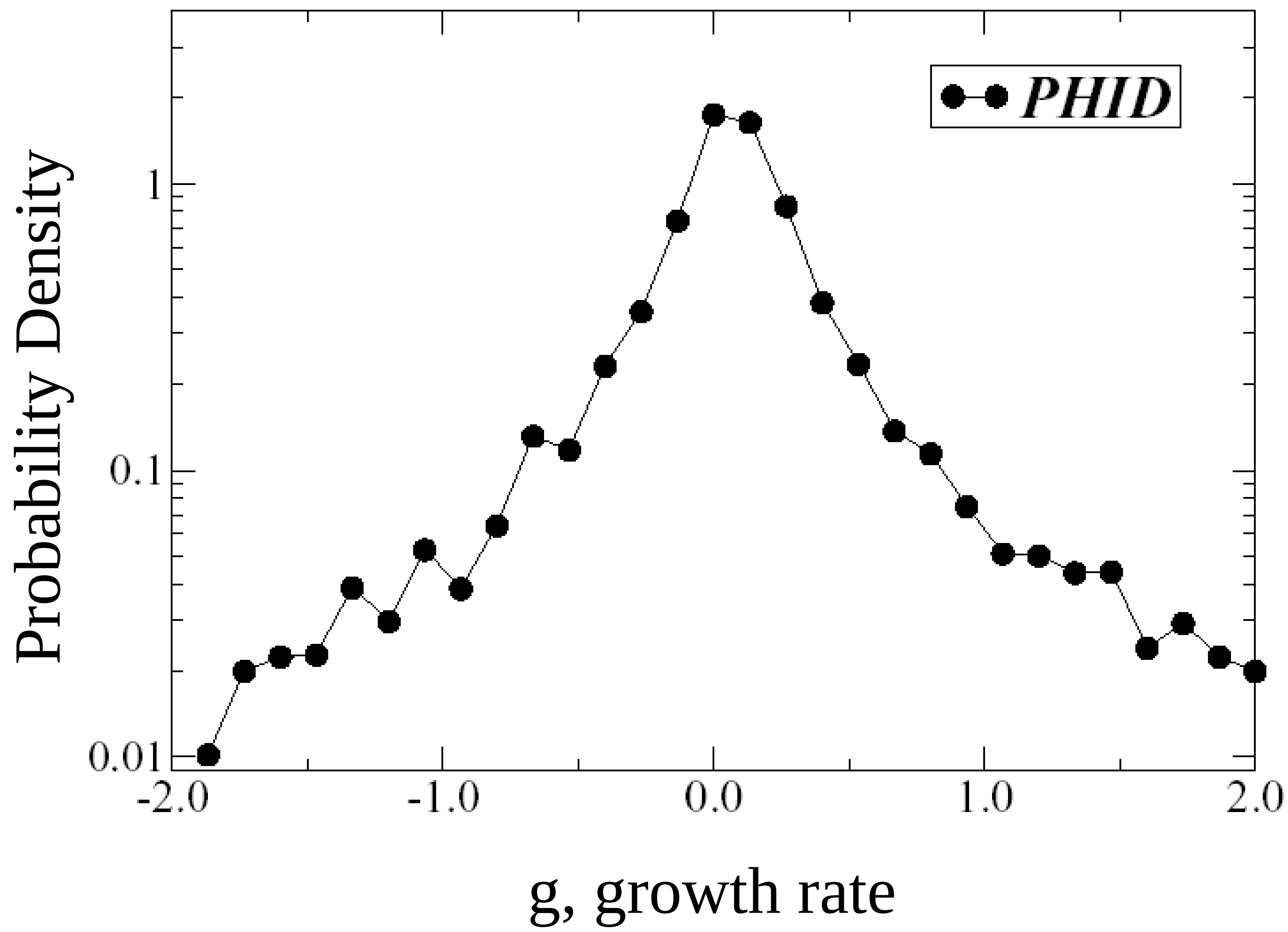
$\gamma = 0.42 \pm 0.01$

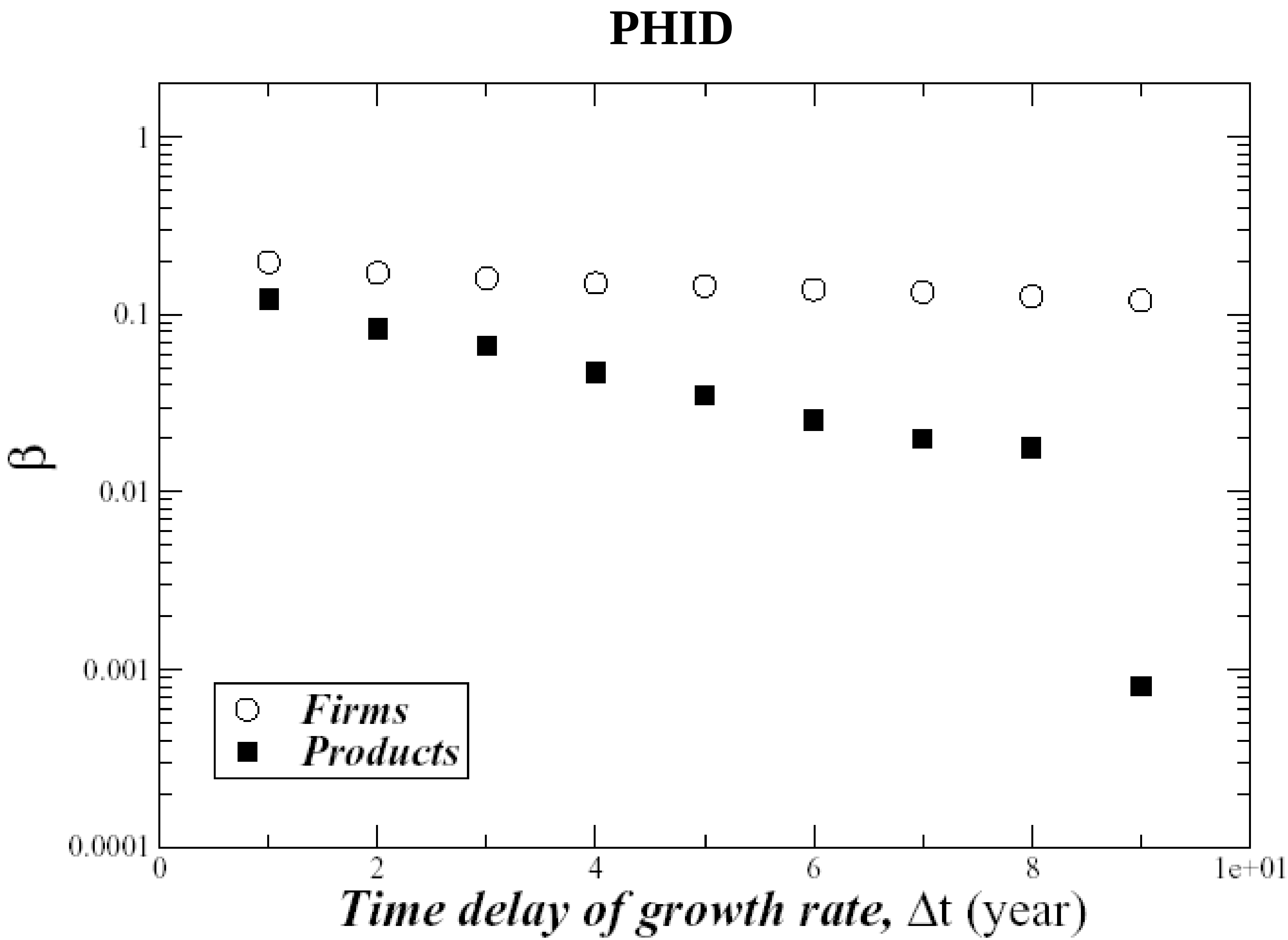
$$\beta \approx \gamma / 2$$

$$\alpha + \gamma \approx 1$$

Conclusions

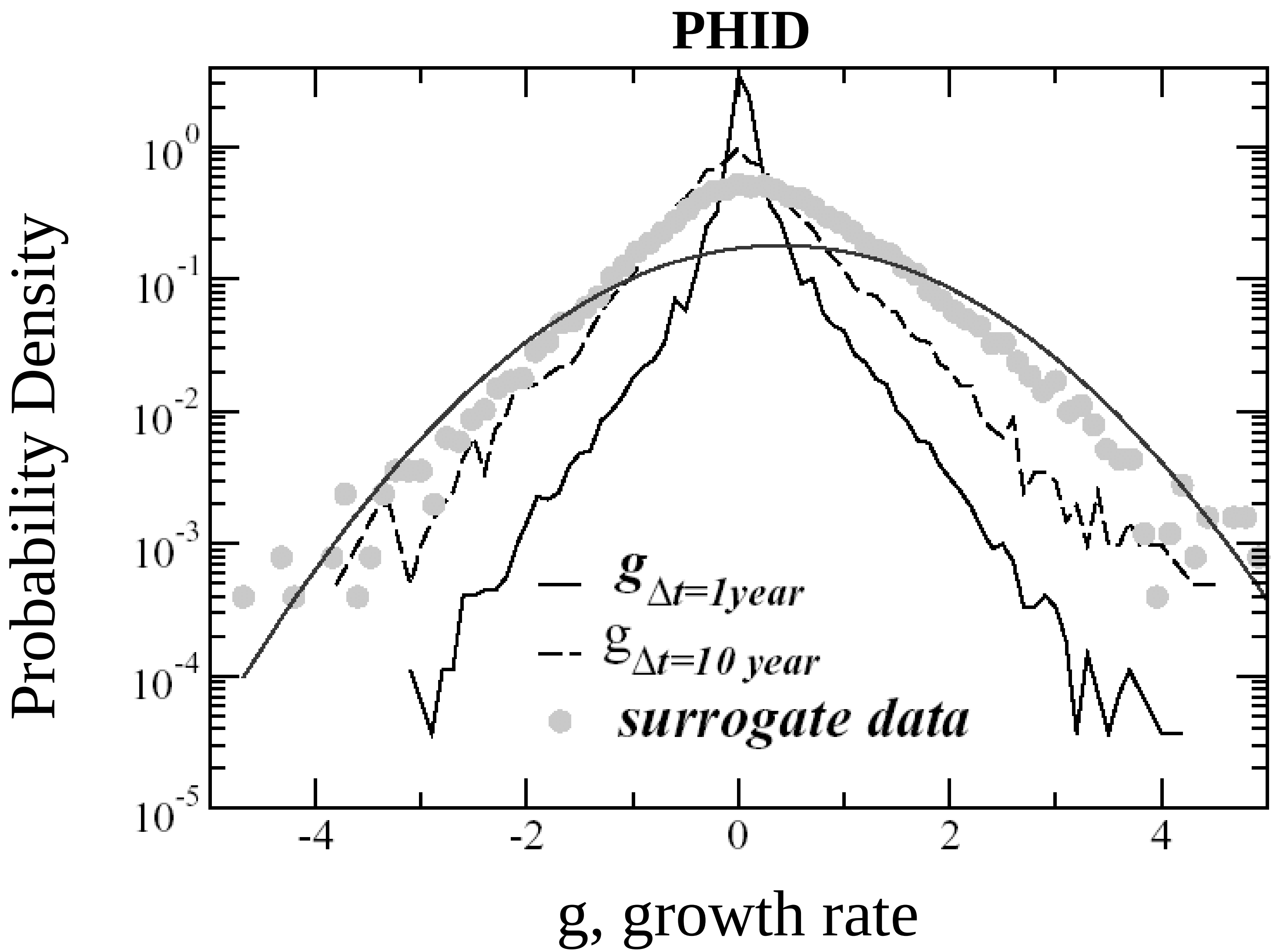
- The growth rate of pharmaceutical firms also follows **``tent-shaped''** distribution, and universal scaling exponent β is tested again.
- We test Amaral *et al* model with our database. The existence of scaling exponents α and γ are verified for the first time.
- Future work: to take into account the temporal correlations of firm growth and correlations between divisions present in the database.


PHID
Probability Density
1
0.1
0.01
-2.0
-1.0
0.0
1.0
2.0
g, growth rate



PHID
β
1
0.1
0.01
0.001
0.0001
0
2
4
6
8
1e+01
Firms
Products
Time delay of growth rate, Δt (year)

Temporal Correlation of **Product** Growth



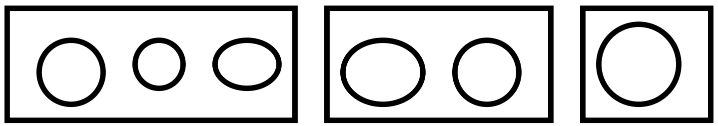
Current Status on the Models of Firm Growth

Issues \ Models	Gibrat	Simon	Sutton	Bouchaud	Amaral
p(N) is power law		√			
p(S) is log-normal	√				√
p(ξ) is log-normal	√				√
$\sigma(S) \sim S^{-\beta}$	$\beta = 0.5$		$\beta = 0.22$	depends	0.17
p(g|S) is "tent"				√	√
p(ξ|S) & scaling					√
p(N|S) & scaling					√

The Models to Explain Some Empirical Findings

Simon's Model explains the distribution of the division number is power law.

The probability of growing by a new division is proportional to the division number in the firm. Preferential attachment.



3 firms industry

⇨ The distribution of division number is power law.

Sutton's Model

Based on **partition theory**

1 1 1
1 2
3

S = 3

$$\sigma^2(\Delta S) = 1/3(1^2 + 1^2 + 1^2) + 1/3(1^2 + 2^2) + 1/3(3^2) = 17/3$$

$$\sigma^2(g) = \sigma^2(\Delta S/S) = \sigma^2(\Delta S)/S^2 = 0.63 \sim S^{-2\beta}$$

$$\Rightarrow \beta = -\ln(0.63)/2\ln(3) = 0.21$$

Bouchaud's Model:

Firm S evolves like this:

$$\frac{d\xi_i}{dt} = \gamma\left(\frac{1}{K}\sum_{j=1}^{K}\xi_j(t) - \xi_i(t)\right) + \eta_i(t)\xi_i(t)$$

assuming ξ follows power-law distribution:

$$p(\xi) \approx \frac{\mu\xi_o^{\mu}}{\xi^{1+\mu}}$$

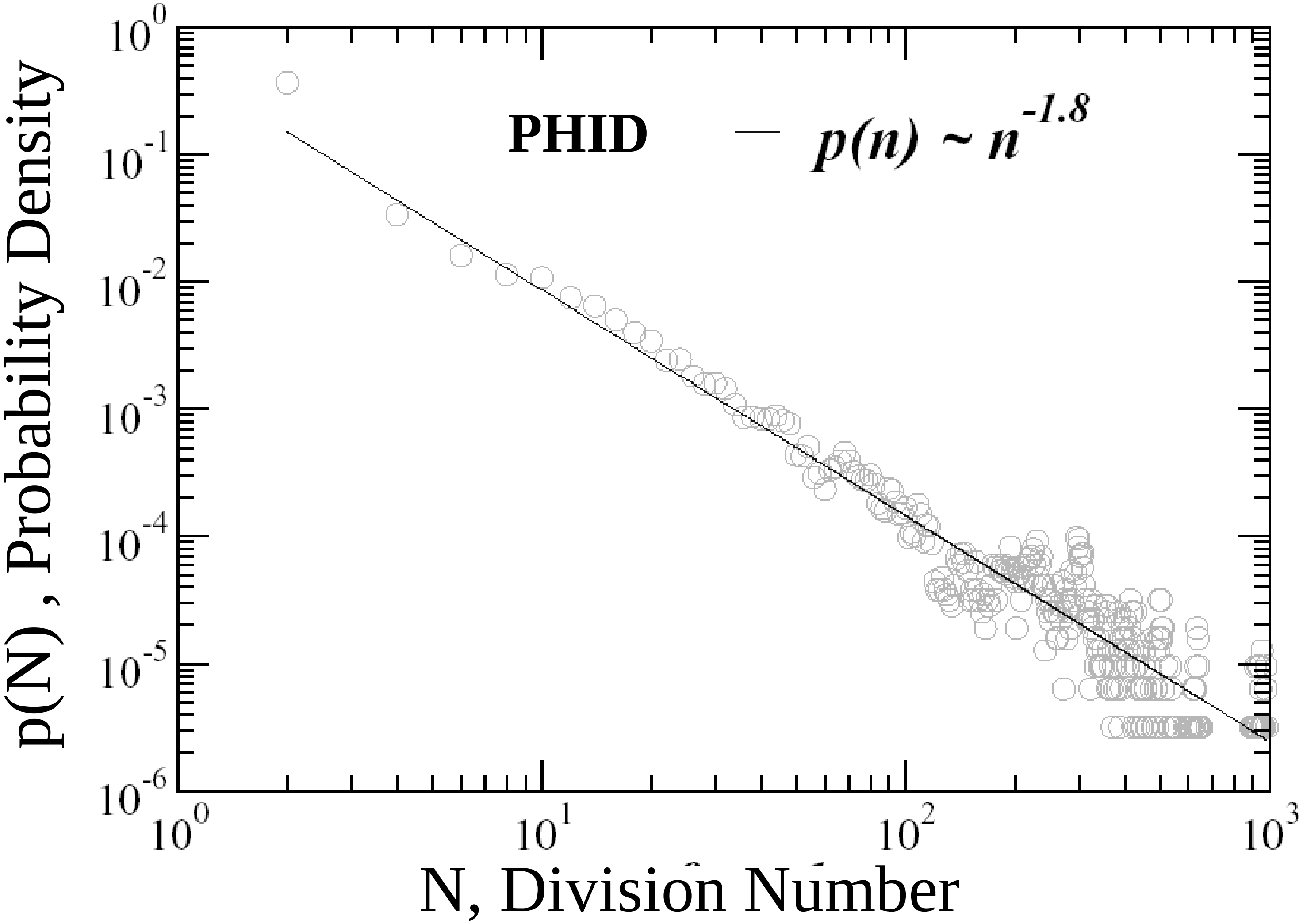
Conclusion:

1. $\beta = \frac{\mu - 1}{2}$ $\quad 1 \le \mu \le 2$

2. $\beta = 0.5$ $\quad \mu \ge 2$

3. $\beta = 0$ $\quad \mu \le 1$

The Distribution of Division Number N



Example Data (3 years time series)

A, B are firms. A1, A2 are divisions of firm A;
B1, B2, B3 are divisions of firm B.

A1	0	3	4
A2	2	1	2
B1	0	10	1
B2	11	4	7
B3	5	6	7

S	$g \equiv \log(S(t+1)/S(t))$
2	log(4/2)
4	log(6/4)
16	log(20/16)
20	log(15/20)

In the 1st year:

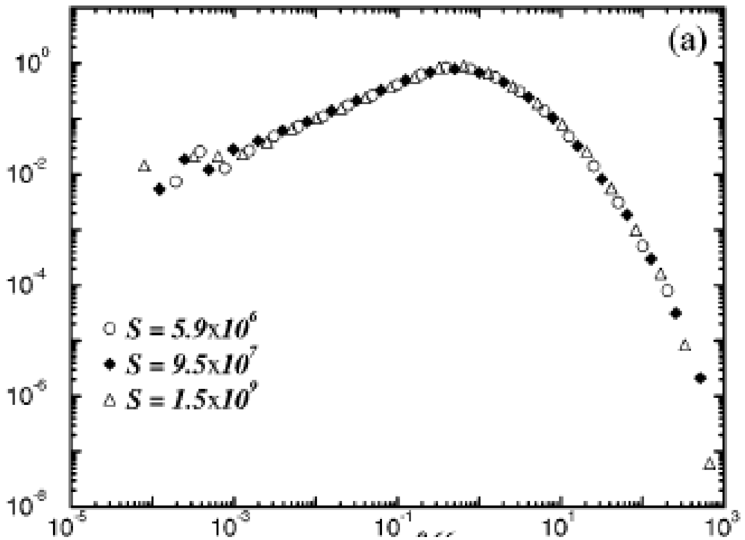
Firm	S	N	ξ
A	2	1	2
B	16	2	11, 5

S	N
2	1
16	2

S	ξ
2	2
16	11
16	5

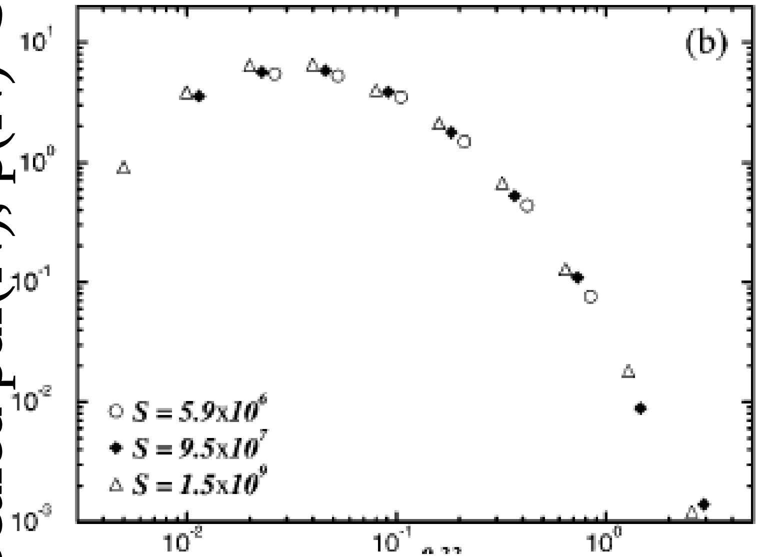
Predictions of Amaral *at al* model

Scaled pdf(ξ), p(ξ)*S^{α}



Scaled division size ξ, ξ/S^{α}

Scaled pdf(N), p(N)*S^{γ}



Scaled division number N, N/S^{γ}

$$\rho_1(\xi|S) \sim S^{-\alpha} f_1(\xi/S^{\alpha})$$

$$\rho_2(N|S) \sim S^{-\gamma} f_2(N/S^{\gamma})$$