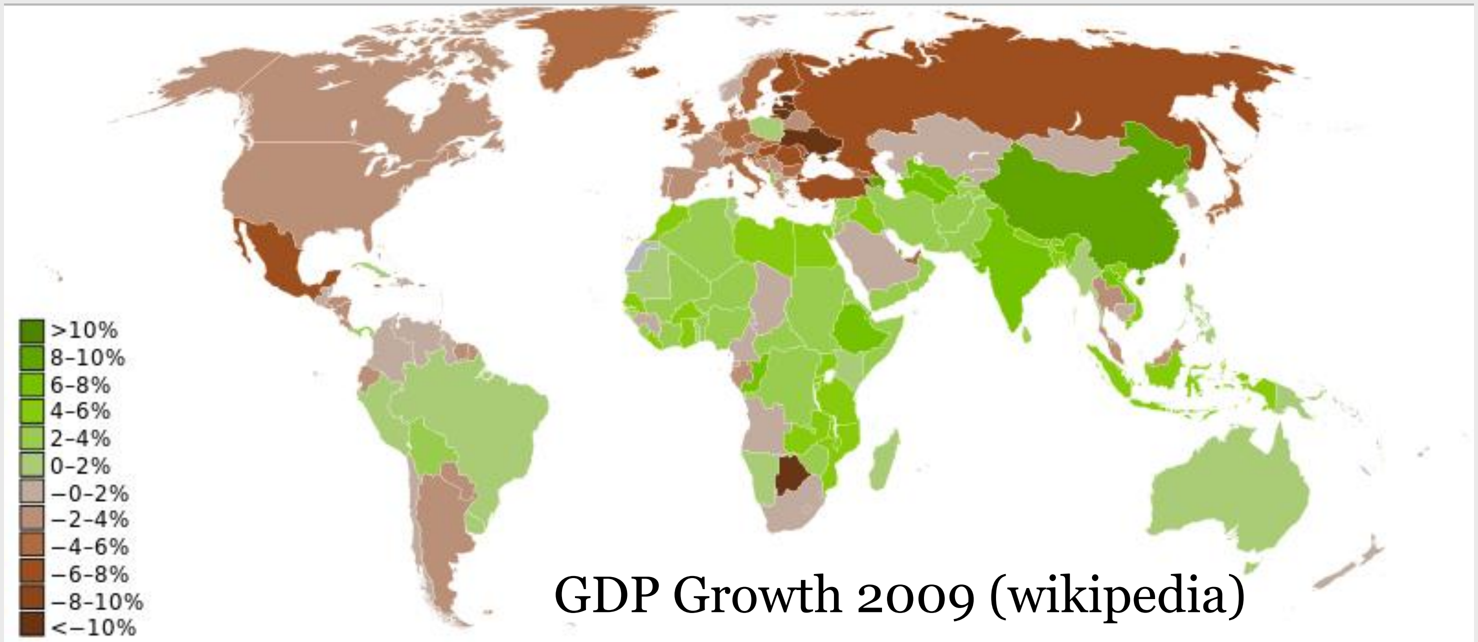
Minimal Dynamical Model of Financial Networks

Nima Dehmamy
Preliminary Oral Presentation
Work done with
Irena Vodenska, Shlomo Havlin and Gene Stanley

Financial Crisis and Recession



GDP Growth 2009 (wikipedia)

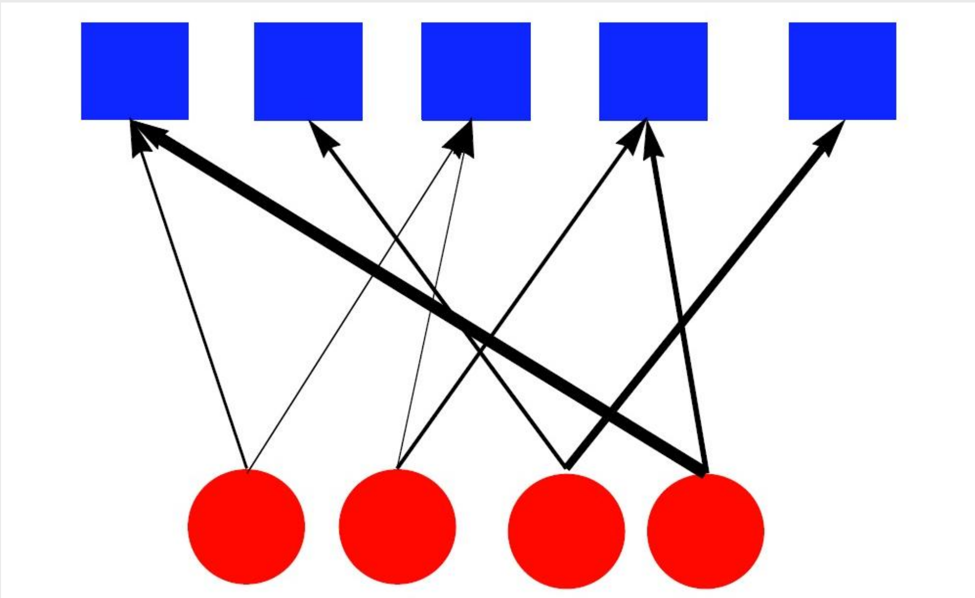
- Less economic activity: 2008–2012 global recession
- Governments can't pay back debt: European debt crisis.*

*Williams, Carol J. (May 22, 2012). "Euro crisis imperils recovering global economy, OECD warns". Los Angeles Times. Retrieved May 23, 2012.

European debt crisis

- Government debt are "assets" of banks (bonds)
- Country can't pay debt → Banks sell → Bonds (assets) lose value → Banks lose money →sell …

Banks



Assets

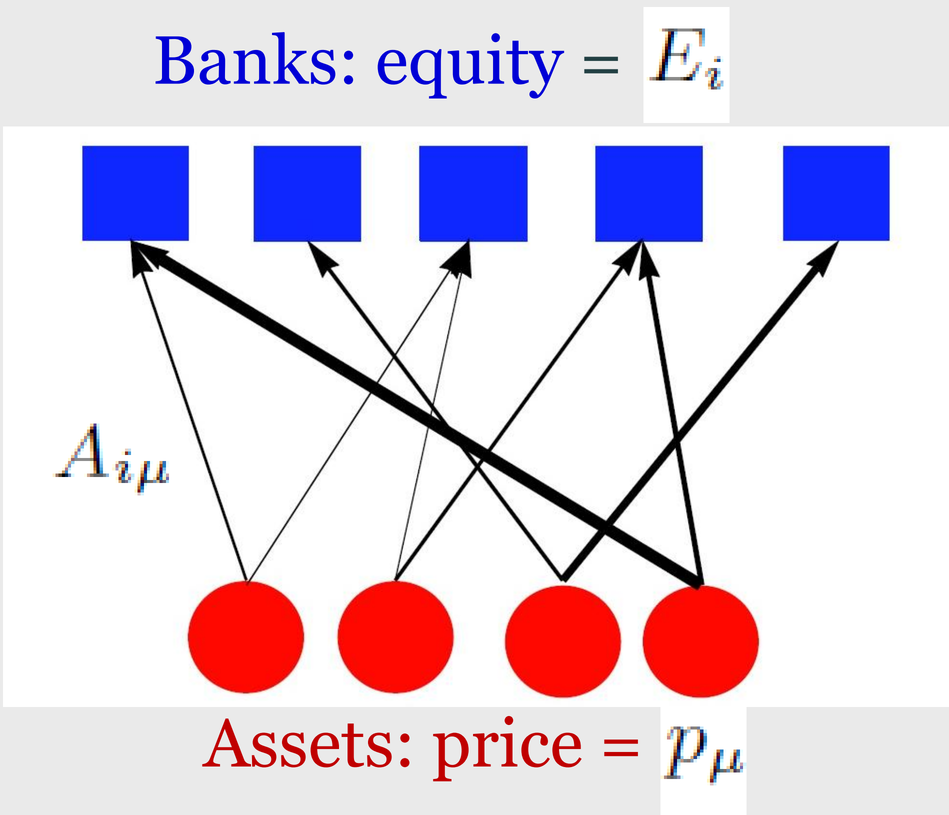
Problem

- Because of the network, failure of a bank might make the whole system collapse
- PROBLEM: Need to identify banks causing system-wide failure!

Which banks are the most important during a crisis?

Simplified Network

- Bipartite graph: No connections within banks or within assets.
- Links are weighted by number of asset owned



Challenges

Which banks are the most important during a crisis?

- Links in financial networks are changing in time
  - ➔Most standard approaches have fixed networks

1. How does the network evolve in time?
2. How do we find most important banks during crisis?

Model: network dynamics in crisis

- Equity gain/loss comes from asset price change

$$\Delta E_i(t) = \sum_{\mu} A_{i\mu}(\mathrm{t}) \Delta p_{\mu}(t)$$

- Banks trade according to equity gain/loss

$$\Delta A_{i\mu}(t+1) = \beta \frac{\Delta E_i}{E_i} A_{i\mu}(t)$$

Sensitivity of trading to profit

- Asset price changes when they are traded

$$\Delta p_{\mu}(t+1) = \alpha \frac{\sum_{\mathrm{i}} \Delta A_{i\mu}}{\sum_i A_{i\mu}} p_{\mu}(t)$$

Sensitivity of price to trading

Yet another model or…?

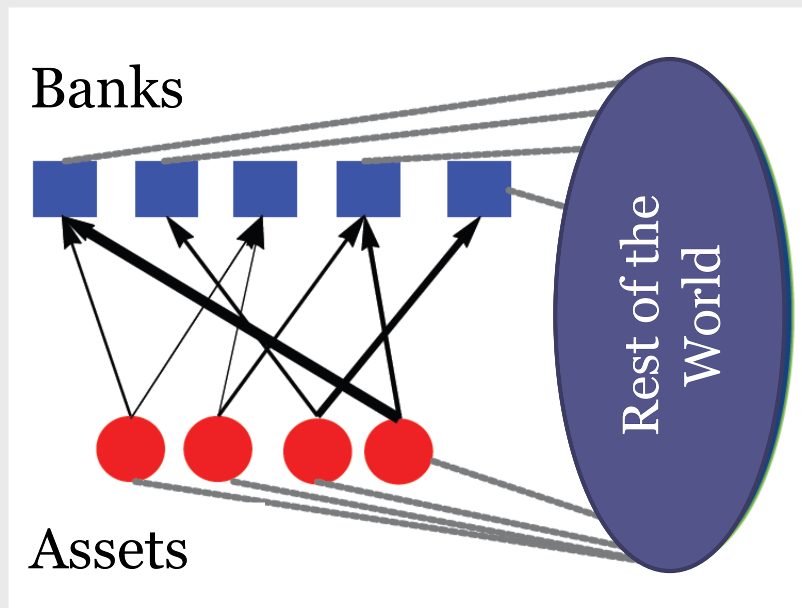
- Is there anything special about this model?

- Lagrangian: Most general Lagrangian to lowest order in E, A, p shows our model is *almost* the most general one for such a system.

Consequences of the Lagrangian

α	Sensitivity of trading to profit
β	Sensitivity of price to trading

- When system is isolated α and β are not independent (one free parameter)
- What if it's not isolated?
- Dissipation: Coupling to the rest of the world makes α and β independent



Ranking the Banks

Which banks are the most important during a crisis?

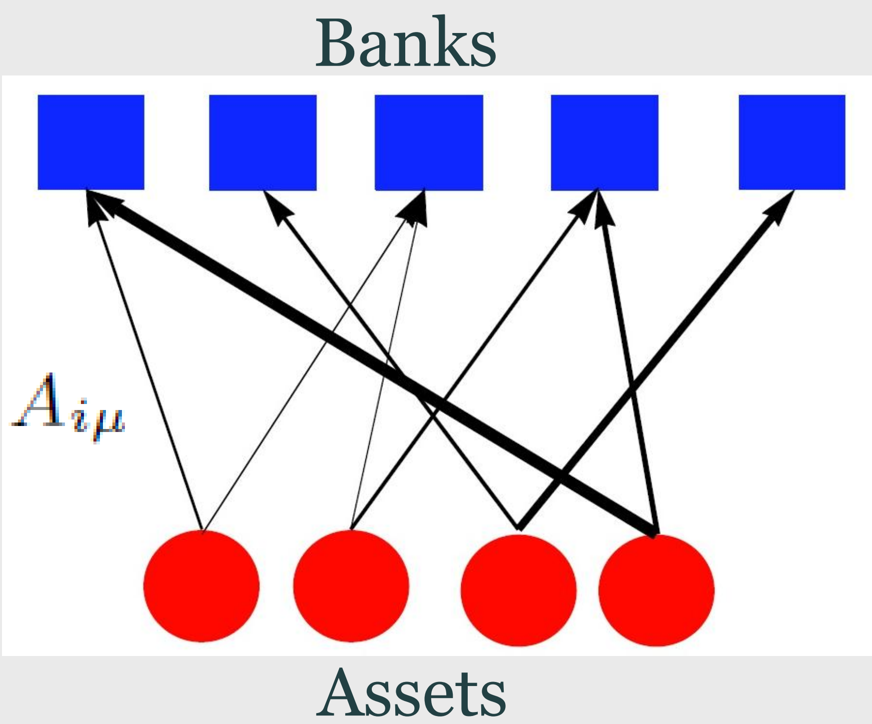
- BankRank*: A measure of money lost from failing of a bank.

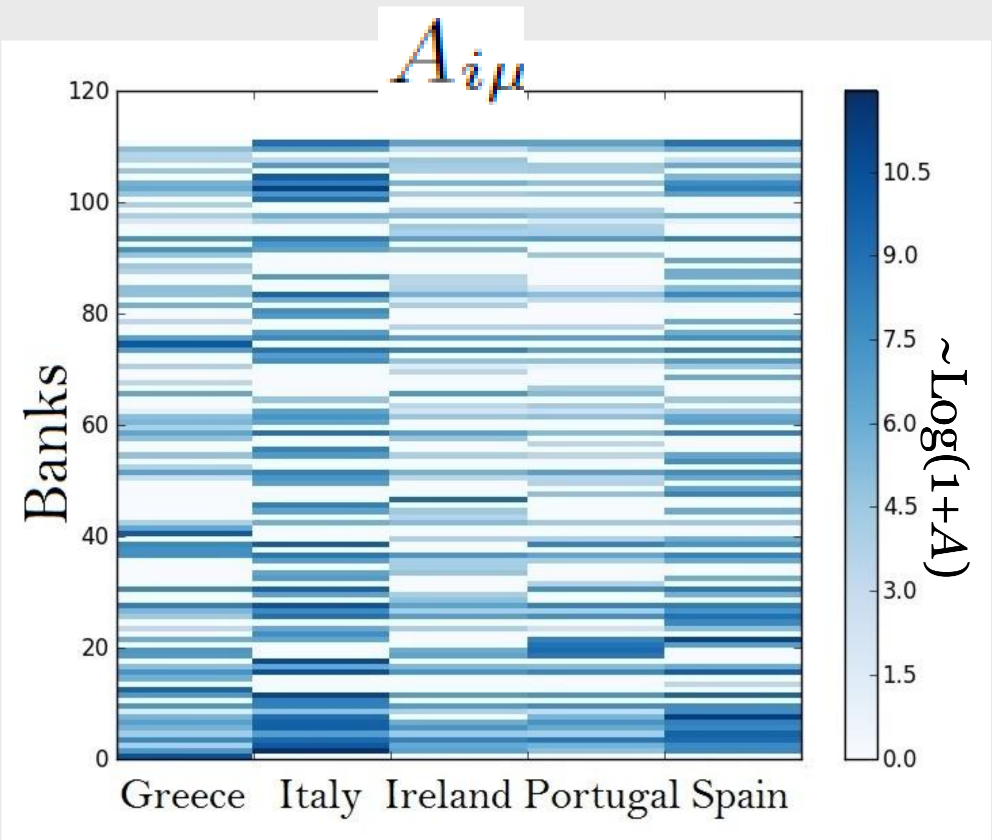
*Based on DebtRank by Battiston et al.

Battiston, S. , Puliga, M. , Kaushik, R. , Tasca, P., & Caldarelli, G. "DebtRank: Too Central to Fail? Financial Networks, the FED and Systemic Risk" , Nature, Scientific Reports, Published 02 August 2012

Application: European debt crisis

- Top Banks (big banks, Insurance co. & funds) owning government debt of Greece, Italy, Ireland, Portugal and Spain.



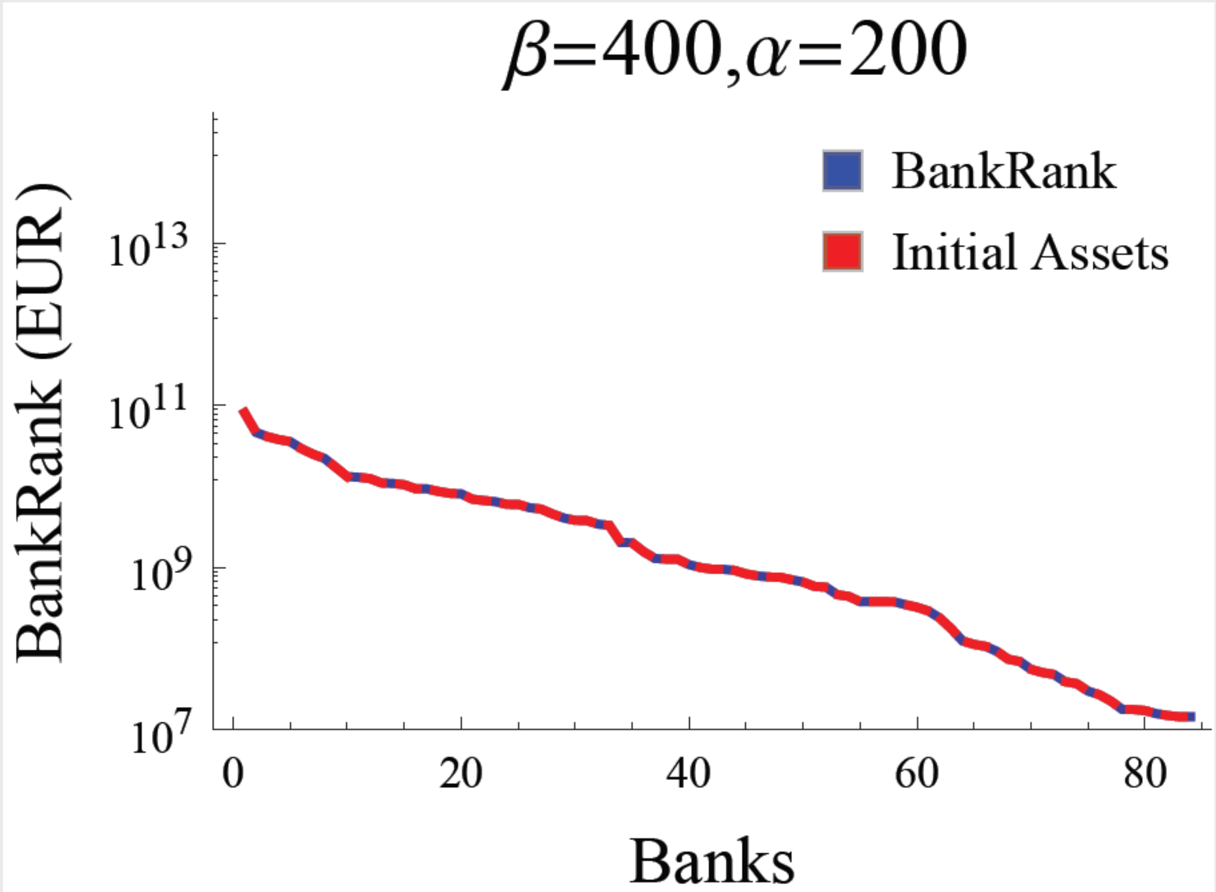


BankRank:

Money lost if bank fails

α Sensitivity of trading to profit

β Sensitivity of price to trading



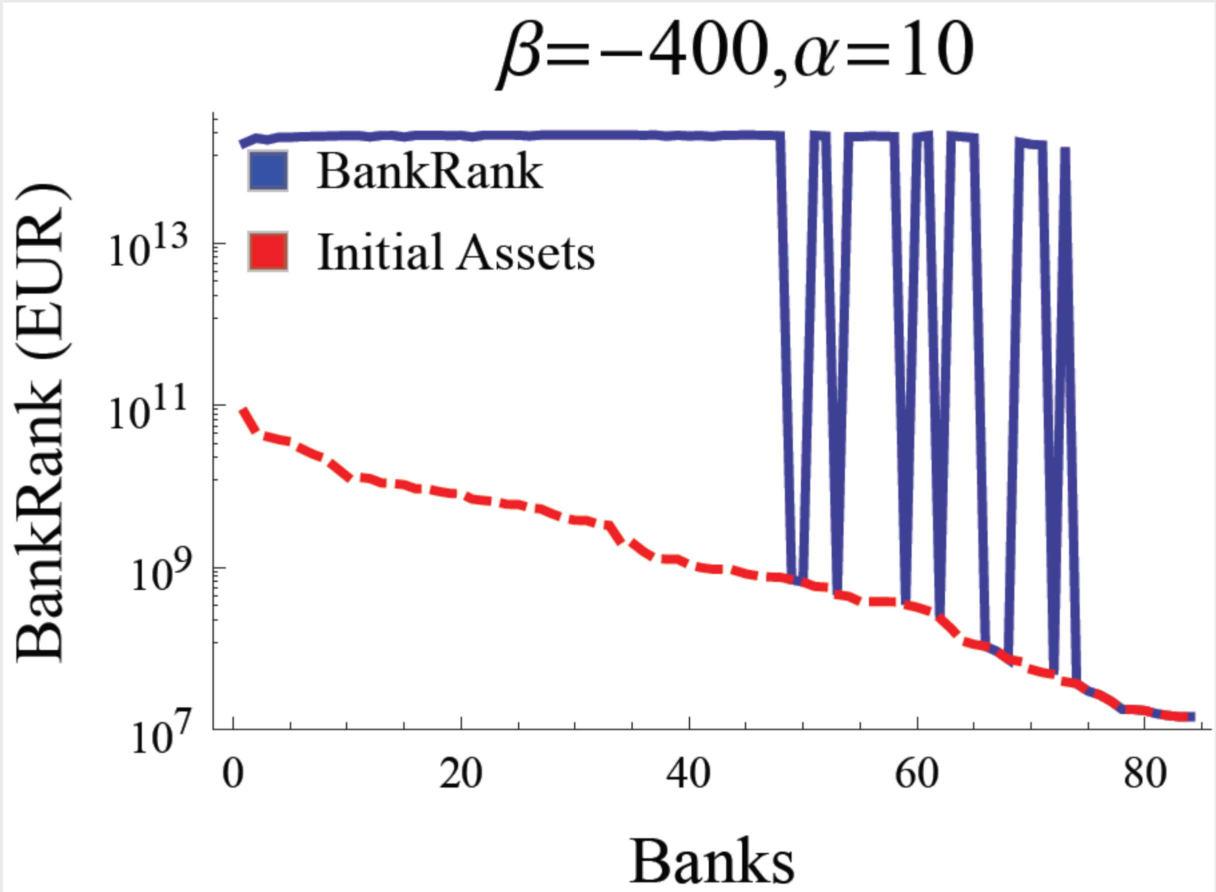
- No cascading: Losses equal to banks' assets.

BankRank:

Money lost if bank fails

α Sensitivity of trading to profit

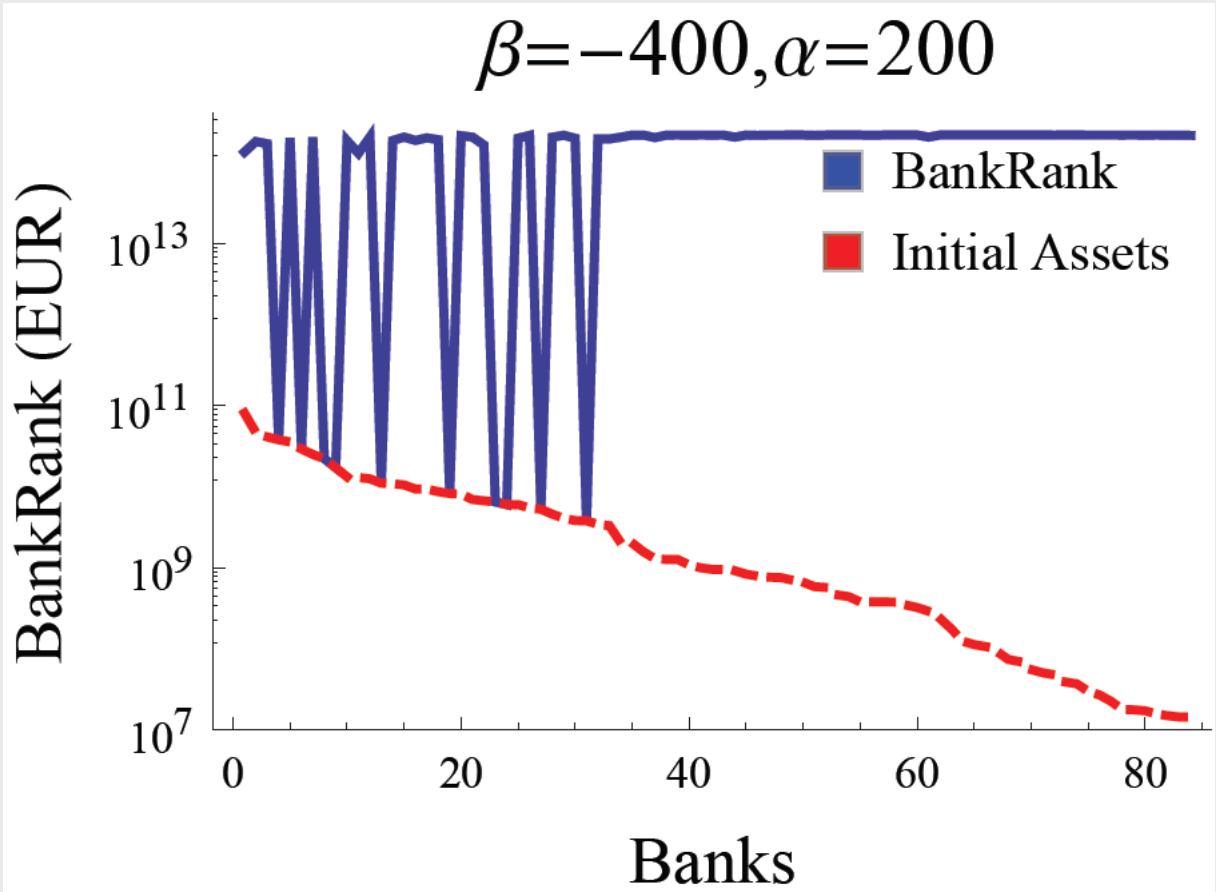
β Sensitivity of price to trading



- Banks with Large assets all caused system collapse.

BankRank:
Money lost if bank fails

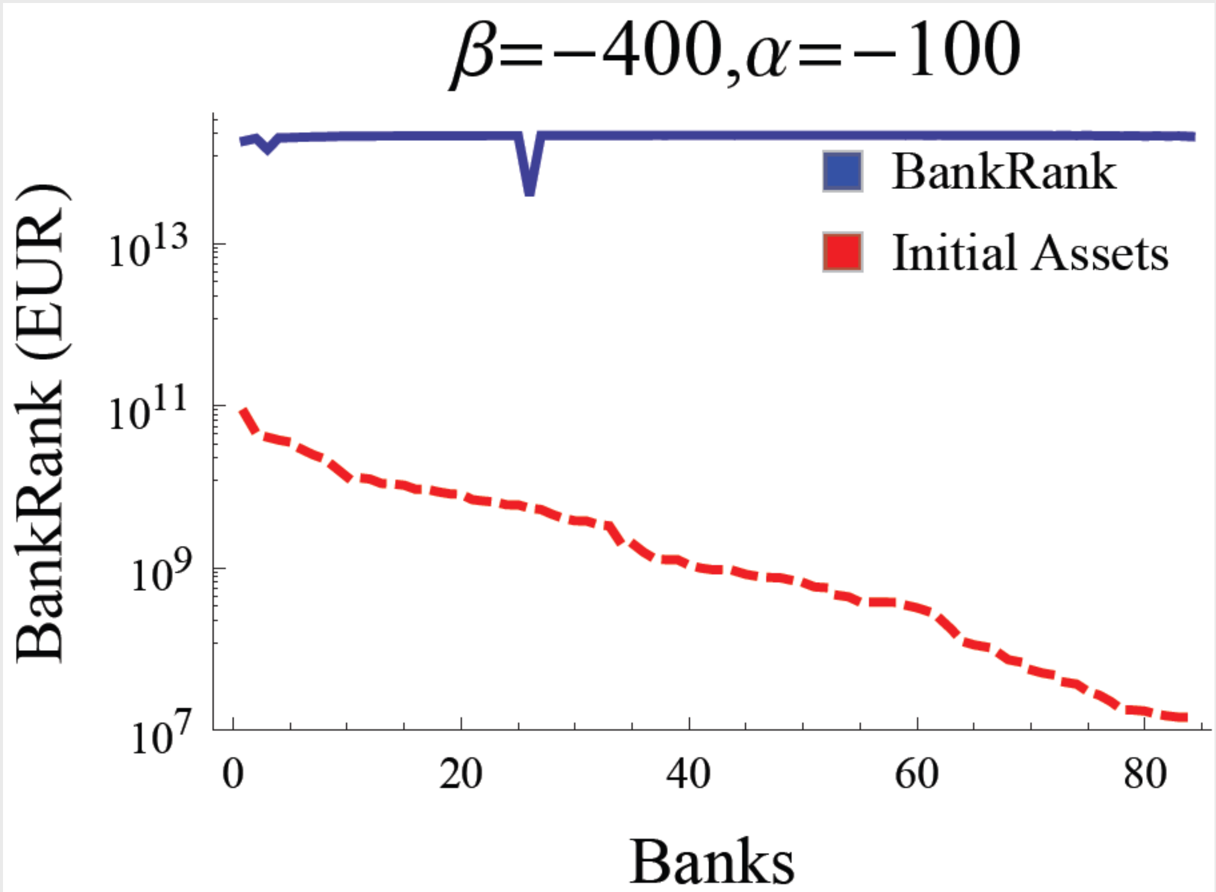
α Sensitivity of trading to profit
β Sensitivity of price to trading



- Banks with small assets are now more important!!

BankRank:
Money lost if bank fails

α	Sensitivity of trading to profit
β	Sensitivity of price to trading



- All banks cause system-wide failure.

BankRank:
Money lost if bank fails

α Sensitivity of trading to profit
β Sensitivity of price to trading

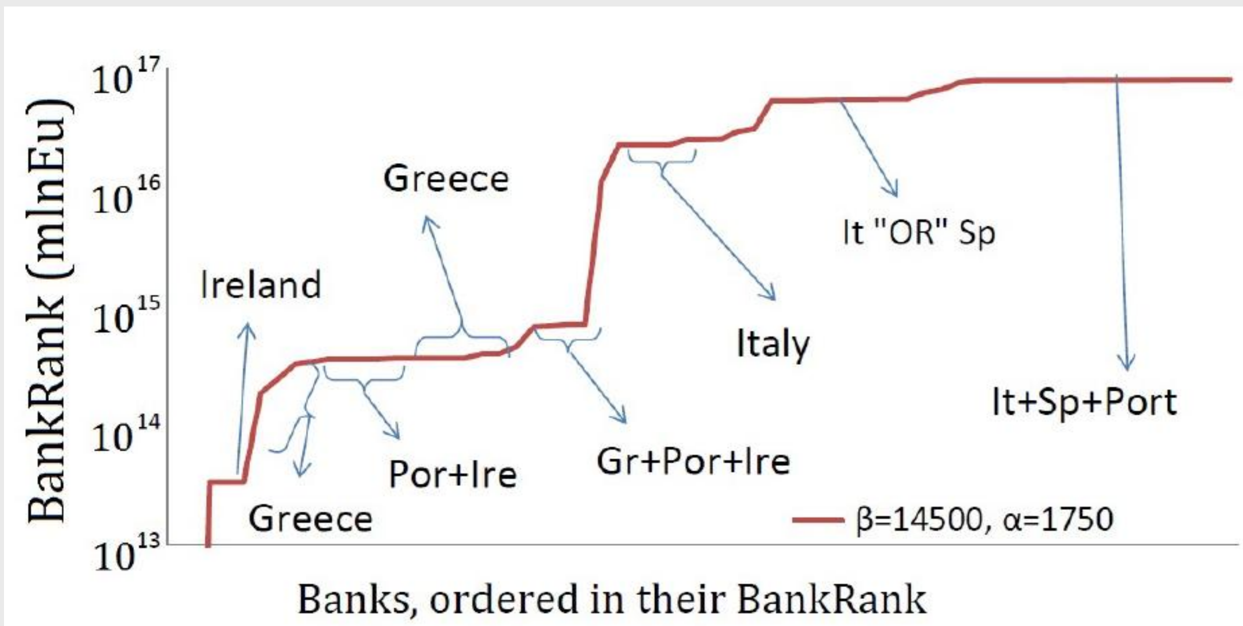
Which banks are the most important during a crisis?

- Verdict: Depends on parameters…
- BUT!!! There’s more to it!

BankRank:
Money lost if bank fails

α	Sensitivity of trading to profit
β	Sensitivity of price to trading

Which banks are the most important during a crisis?

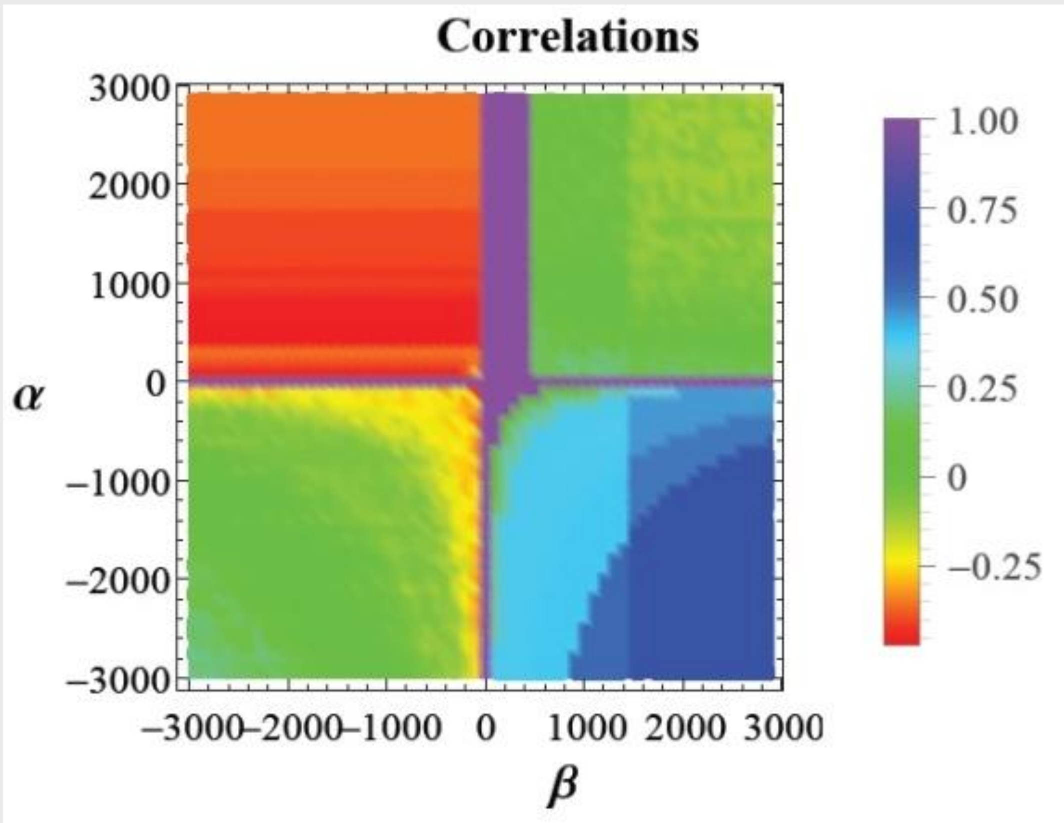


Verdict: Damage to system depends more on the group of assets a bank is connected and less on initial asset value.

α	Sensitivity of trading to profit
β	Sensitivity of price to trading

Phase diagram

- Correlation of BankRank and initial assets.



Results

- Banks with largest assets are not always the most important ones.
- Similar diversification ➔ similar damage to system.
- Diversification does not reduce damage to system.

- Values of parameters can be inferred from real world once more data is out.

Future direction

- Straightforward generalizations:
  - Networks with links within layers.
  - Multilayer
- Noise:
  - Fluctuations dissipation theorem?
  - Temperature?

Thank you.

- Especially

Adam
Alexis
Andreas
Antonio
Boris
Carlos
Chester
Erik
Gene
Irena
Joao
Nagendra
Shuai
And my parents

Conluding remarks

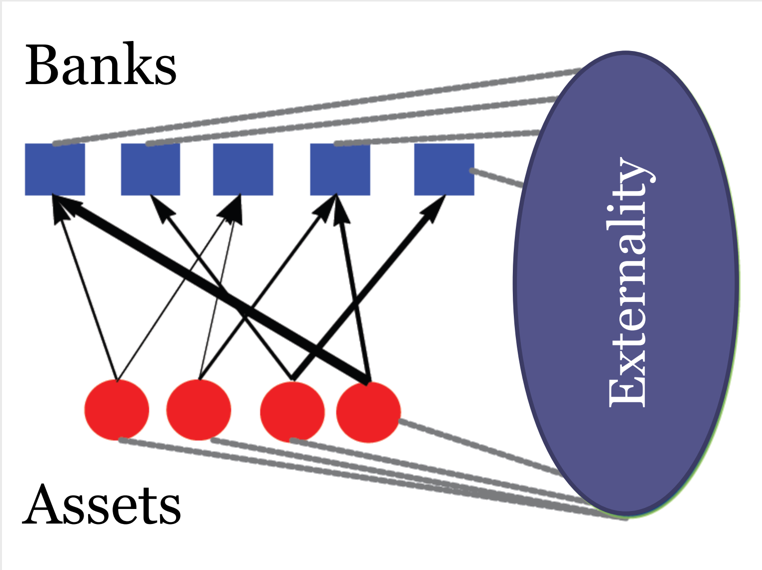
- Close to the most general dynamical model to lowest order in variables
- Isolated: one side ⇔ other
  ➔ single coupling
- Open system: Externalities = dissipation
  ➔ two couplings

Derive the model?

- Phenomenological Equations are OK, but is it possible to derive the dynamics
- Relation to known physical systems?
- Dynamics:
  - ➔ Lagrangian?
  - ➔ not Lagrangian, dissipative?

Challenges

- Links in financial networks change in time
  - ➔Standard centrality measures not appropriate for "systemic importance".
- Usually full network is unknown
  - ➔Need a way to include unknown "externalities"



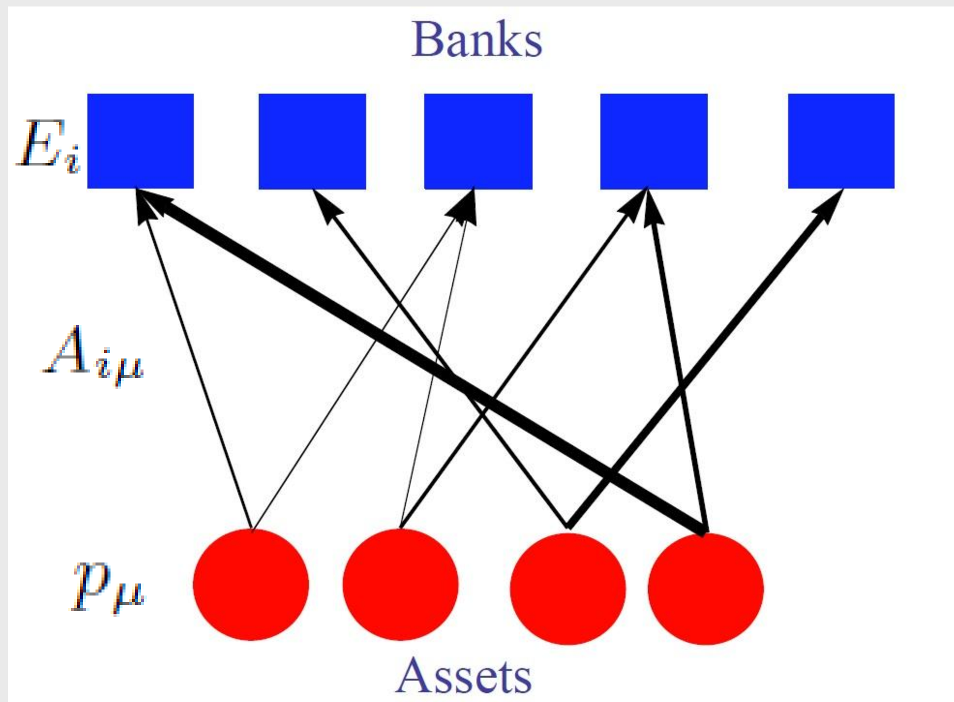
Minimal Dynamical Model

- Lagrangian: Possible terms?
  - Max 2 time-derivatives,
  - Least number of variables *E*, *A* and *p*.

- Only possible terms:

$$E^T A p$$

- And various time derivatives



Lagrangian

- Kinetic + single time-derivative + potential

$$L = L_K + L_1 + V$$

- 3 possible terms for L_1, only one coupling.

$$L_1 = \gamma \partial_t E^T A p - E^T A \partial_t p$$

Equations of motion

$$L = L_K + L_1 + V$$

- Assuming near equilibrium:
  - Accelerations are small $\delta L_K \approx 0$
  - Potential at minimum $\nabla V \approx 0$
- Equations of motion s for L_1 :
- Ratio of price change

$$A \partial_t p = \alpha (\partial_t A) p$$

$$E^T \partial_t A = \beta (\partial_t E^T) A$$

$$\gamma \equiv \alpha\beta,$$

$$\beta = \frac{-1}{\alpha + 1}.$$

Equations of motion

$$L = L_K + L_1 + V$$

- Assuming near equilibrium:
  - Accelerations are small $\delta L_K \approx 0$
  - Potential at minimum $\nabla V \approx 0$

- Equations of motion for L_1:

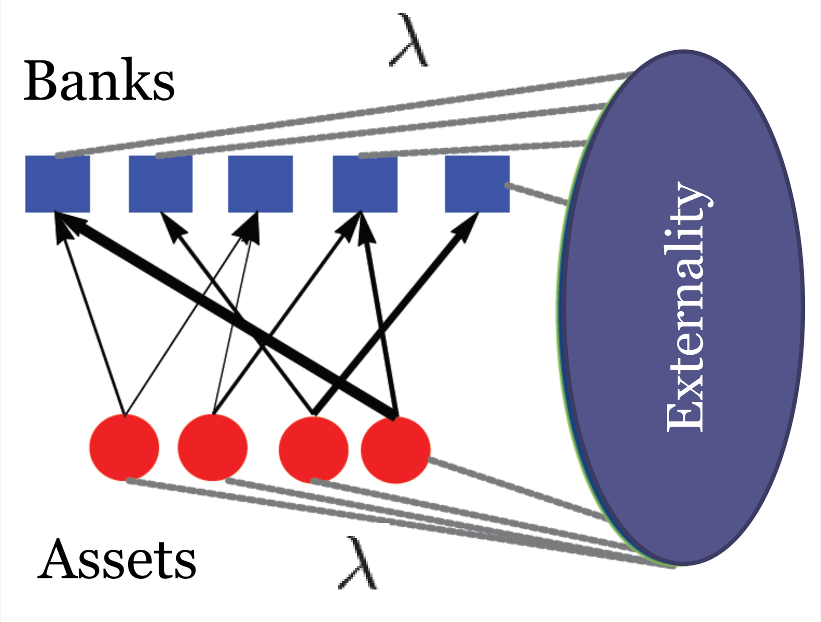
$\alpha(\gamma)$ Sensitivity of price to trading

$\beta(\gamma)$ Sensitivity of trading to loss/gain

$$\gamma \equiv \alpha\beta,$$

$$\beta = \frac{-1}{\alpha + 1}.$$

Unknowns (Externalities)



- Solutions: Add "Dissipation"!

$$\beta = \frac{-1}{\alpha + 1}(1 - \lambda).$$

Lagrangian

- Continuous time $\delta \to \partial_t$
- 1st order terms in first 2 eqs can be obtained from variation of Lagrangian:

$$L_\gamma = \gamma \partial_t E^T A p - E^T A \partial_t p.$$

Consequences

- Only one free parameter $\gamma \equiv \alpha\beta$, for 1st order terms:

→ α and β have to be related

$$\beta = \frac{-1}{\alpha + 1}.$$

Notation & Model

symbol	denotes
$A_{i\mu}(t)$	Holdings of bank i in asset μ at time t
$p_\mu(t)$	Normalized price of asset μ at time t ($p_\mu(0) = 1$)
$E_i(t)$	Equity of bank i at time t.
β	Bank's "Panic" factor.
α	"Market sensitivity" factor of price to a sale.

- Discrete time steps (business days)
- Phenomenological
- Ratio of trade ($\delta A/A$)

$$\delta A_{i\mu}(t+1) = \beta \frac{\delta E_i(t)}{E_i(t)} A_{i\mu}(t)$$

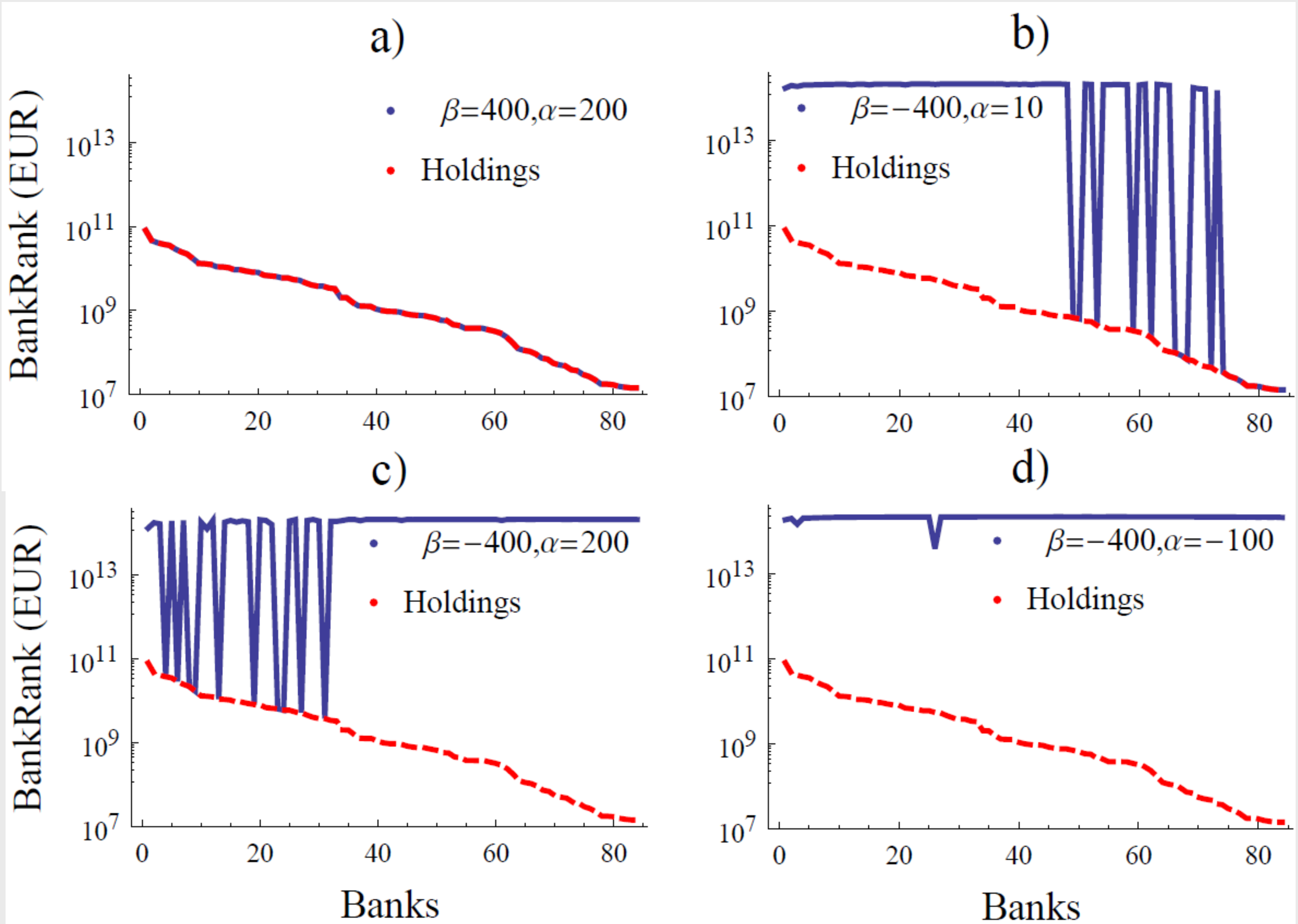
$$\delta p_\mu(t+1) = \alpha \frac{\delta A_\mu(t)}{A_\mu(t)} p_\mu(t)$$

$$\delta E_i(t) = \sum_\mu A_{i\mu}(t) \delta p_\mu(t)$$

$$\alpha = \frac{-\gamma}{\gamma + 1}, \quad \beta = -(\gamma + 1),$$

$$\alpha = \frac{\gamma}{\lambda - \gamma - 1}, \qquad \beta = \lambda - \gamma - 1,$$

BankRank


a)
b)
c)
d)
BankRank (EUR)
β=400,α=200
Holdings
β=−400,α=10
β=−400,α=200
β=−400,α=−100
Banks

Phase Diagram

$$\delta A_{i\mu} \equiv \psi_i A_{i\mu}, \quad \psi_i = \beta \frac{\delta E_i}{E_i}$$

$$\text{BankRank of } i : R^i = \sum_j \psi_j^{(i)}(t_f) V_j(0) \Big|_{i \text{ shocked}}$$

- BankrRank uses initial asset value *V=Ap*

➔Correlations with *V* may be a good order parameter

- Use Pearson correlation of *R* and *V* to characterize phases.

$$\rho(R, V) \equiv \sum_i \frac{\left(R^i - \langle R \rangle\right)\left(V_i - \langle V \rangle\right)}{\sigma_R \sigma_V}$$

Lagrangian

- Continuous time $\delta \rightarrow \partial_t$

➔Need to expand $\delta A_{i\mu}(t+1)$ and $\delta p_{\mu}(t+1)$ to get non-trivial dynamics (thanks Sergey!)

➔1st order terms in first 2 eqs can be obtained from variation of Lagrangian:

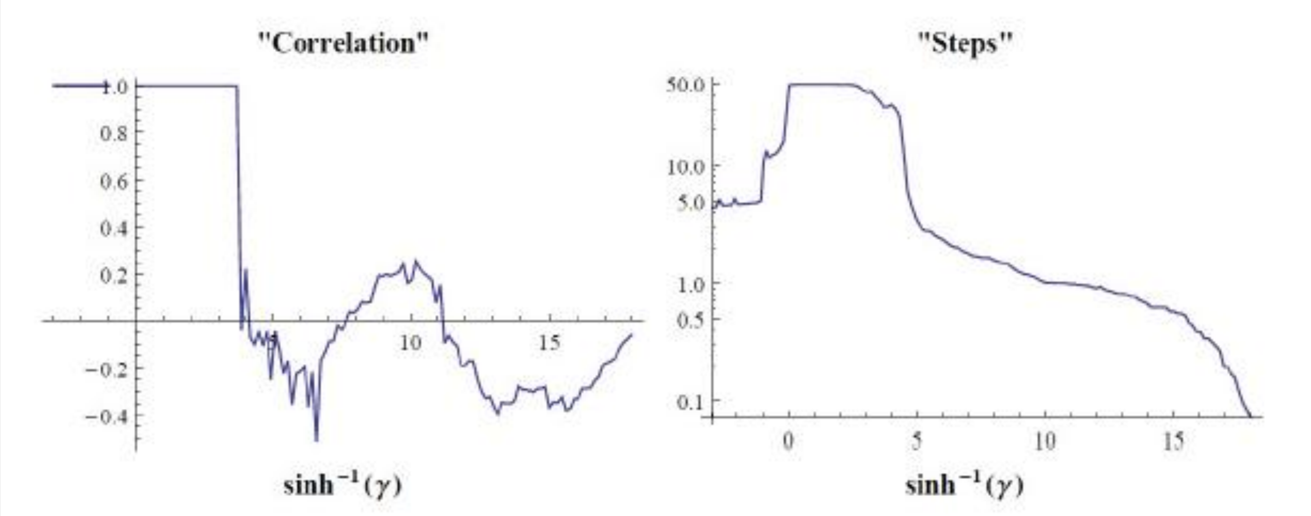
$$L_\gamma = \gamma \partial_t E^T A p - E^T A \partial_t p.$$

- And $\delta E_i(t) = \sum_\mu A_{i\mu}(t) \delta p_\mu(t)$ from a kinetic term (assuming near equilibrium accelerations ~zero)

$$S_K \equiv \frac{m}{2} \int dt |\partial_t E - A \partial_t p|^2$$

Phase diagram for γ

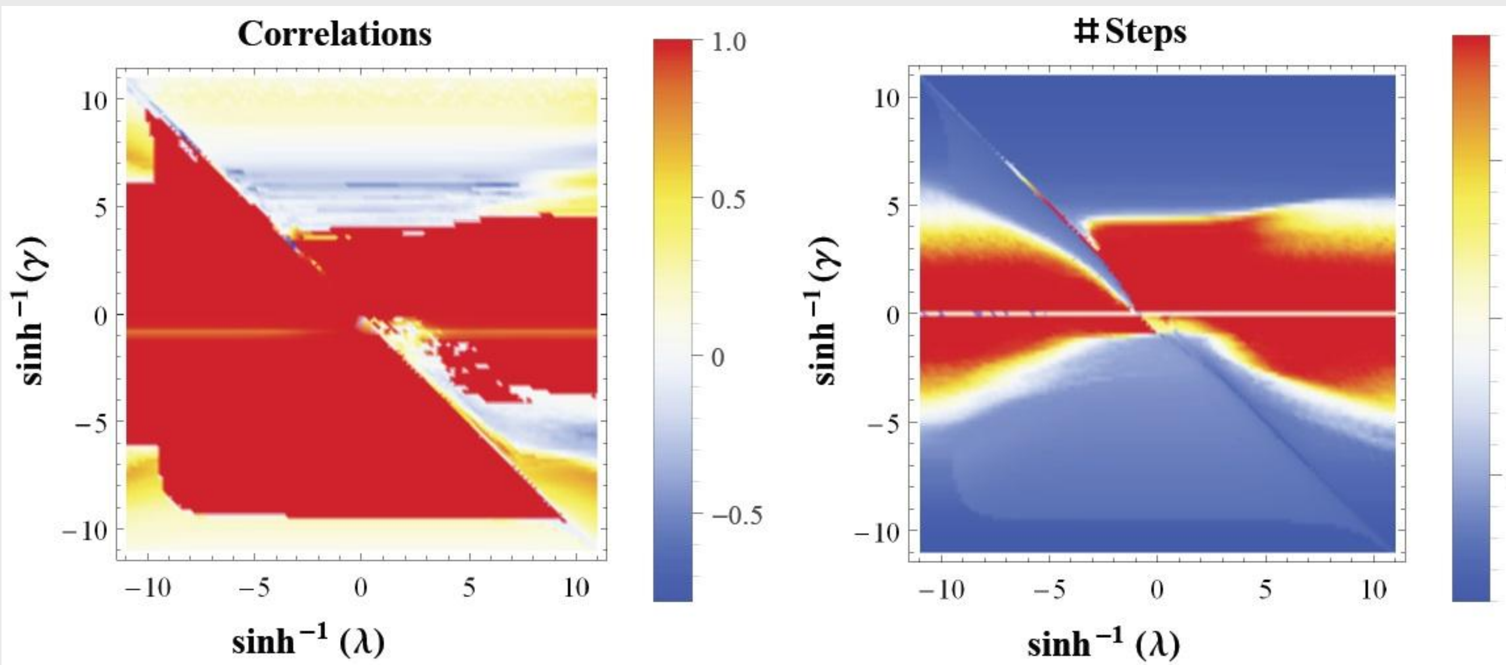
(ArcSinh used to compress a large range of the variable γ)

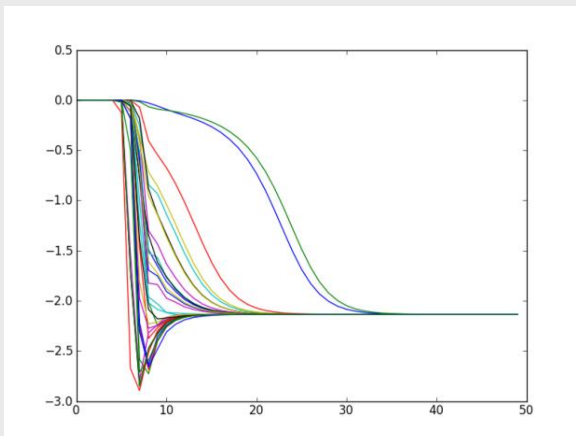


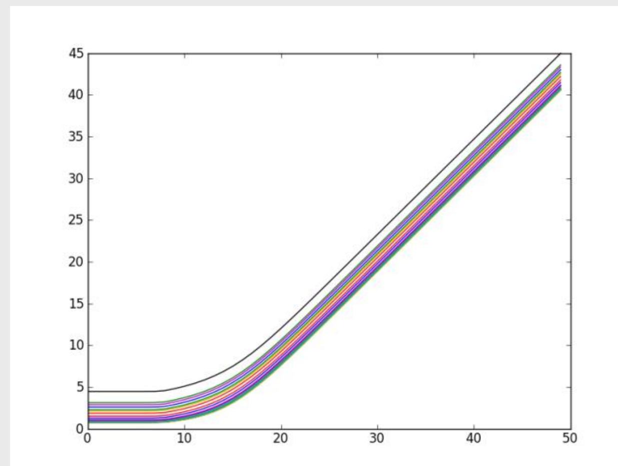
$$\beta = \frac{-1}{\alpha + 1}.$$

Phase diagram for γ and λ

$$\lambda = \beta(\alpha + 1) + 1.$$




0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
0
10
20
30
40
50


45
40
35
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25
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15
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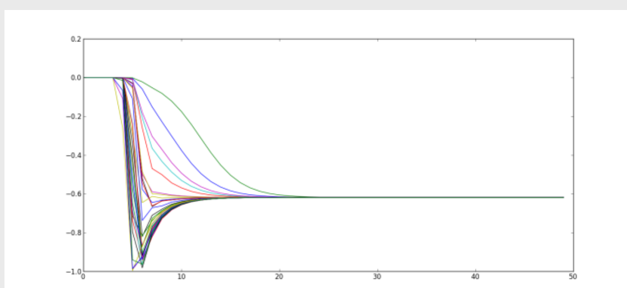
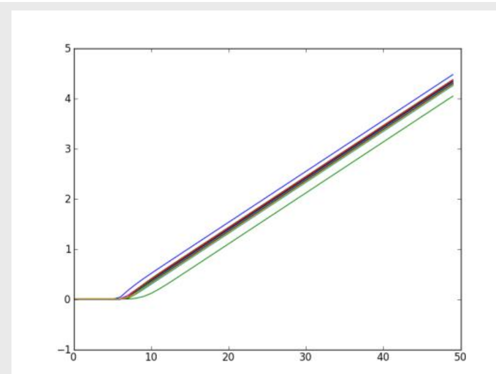
50




5
4
3
2
1
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−1
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20
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50