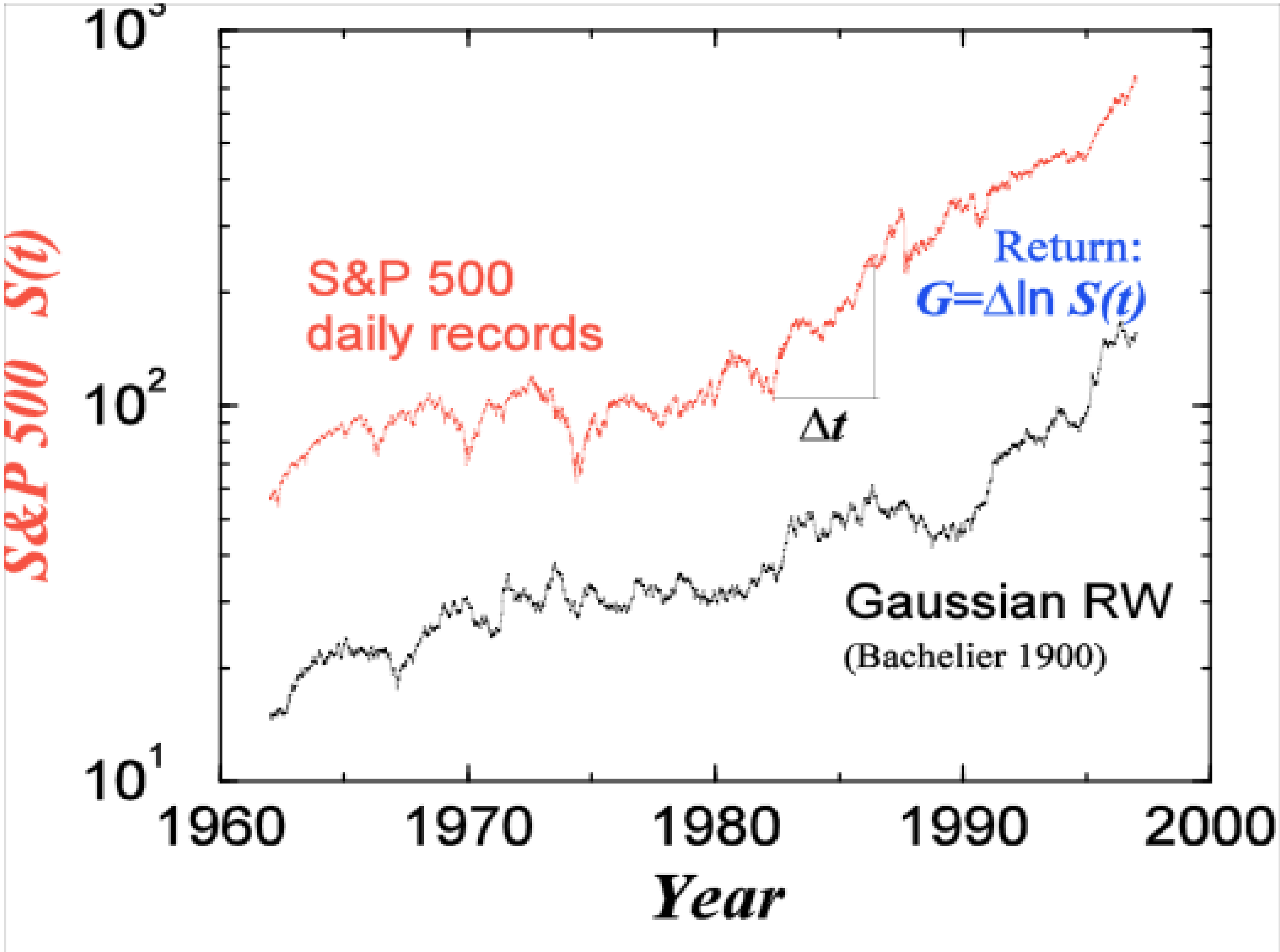
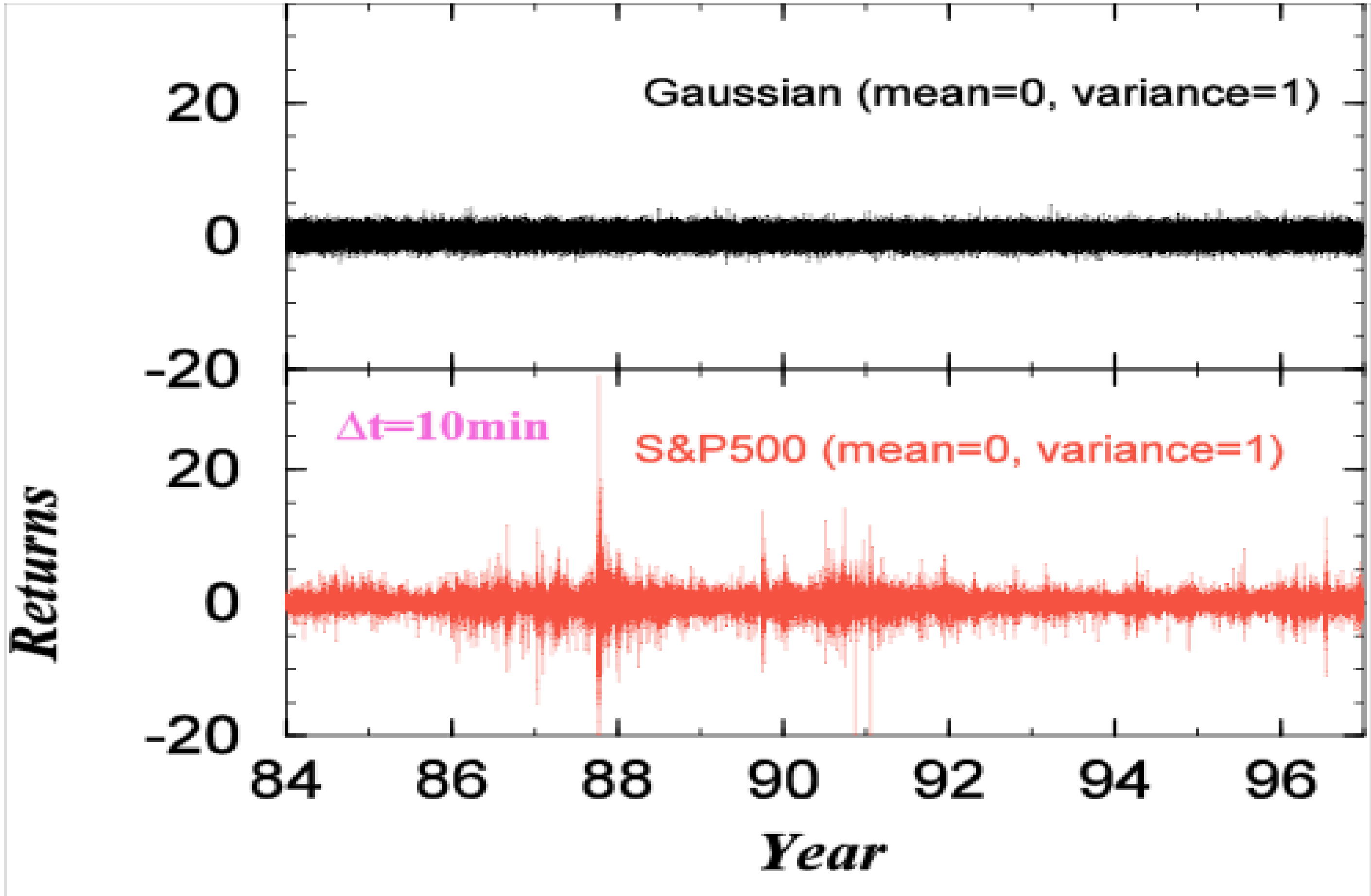
Example: S&P 500 index



<u>Two observations</u>: Returns are significantly non-Gaussian

Large events cluster

Example: S&P 500 intra-daily log-returns $R(t) = \ln S(t+\Delta t) - \ln S(t)$.



Data analyzed

Trades and Quotes (TAQ) database

- 2 years 1994-95
- 1000 stocks largest by market cap on Jan 1, ’94 (200 million records)

Center for Research in Security Prices (CRSP) database

- 35 years 1962-96
- approximately 6000 stocks

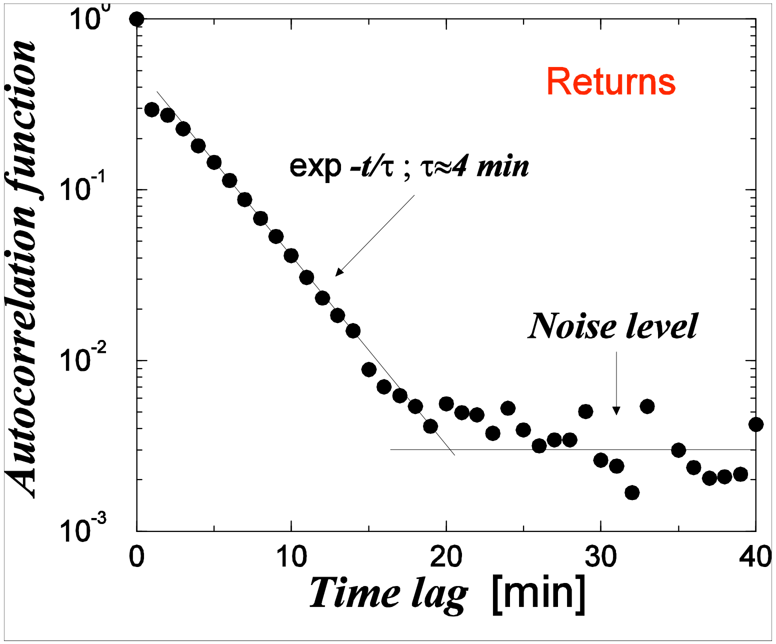
Tick data for the London Stock Exchange

- 2 yrs 2000-01
- 250 stocks.

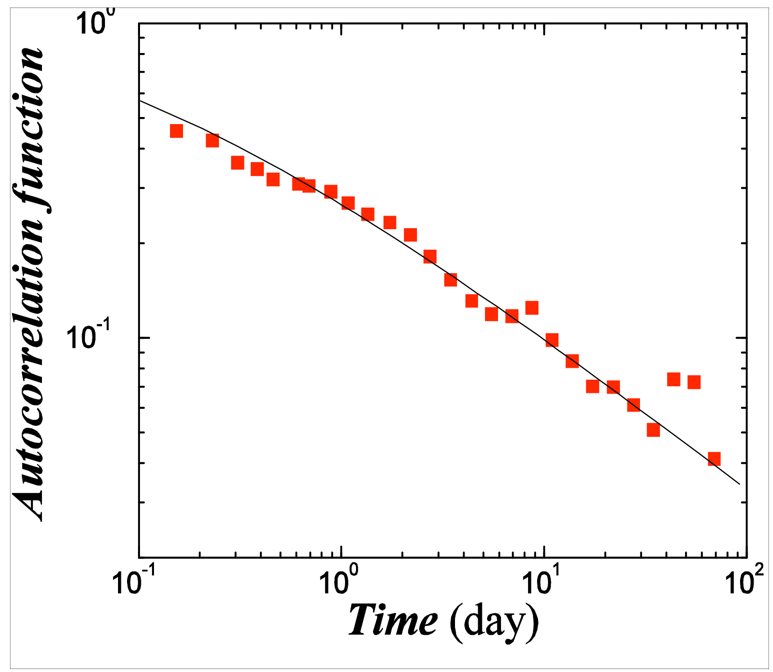
Transactions data from the Paris Bourse

- 30 stocks; 1994-95

Slow convergence indicative of long-ranged dependencies

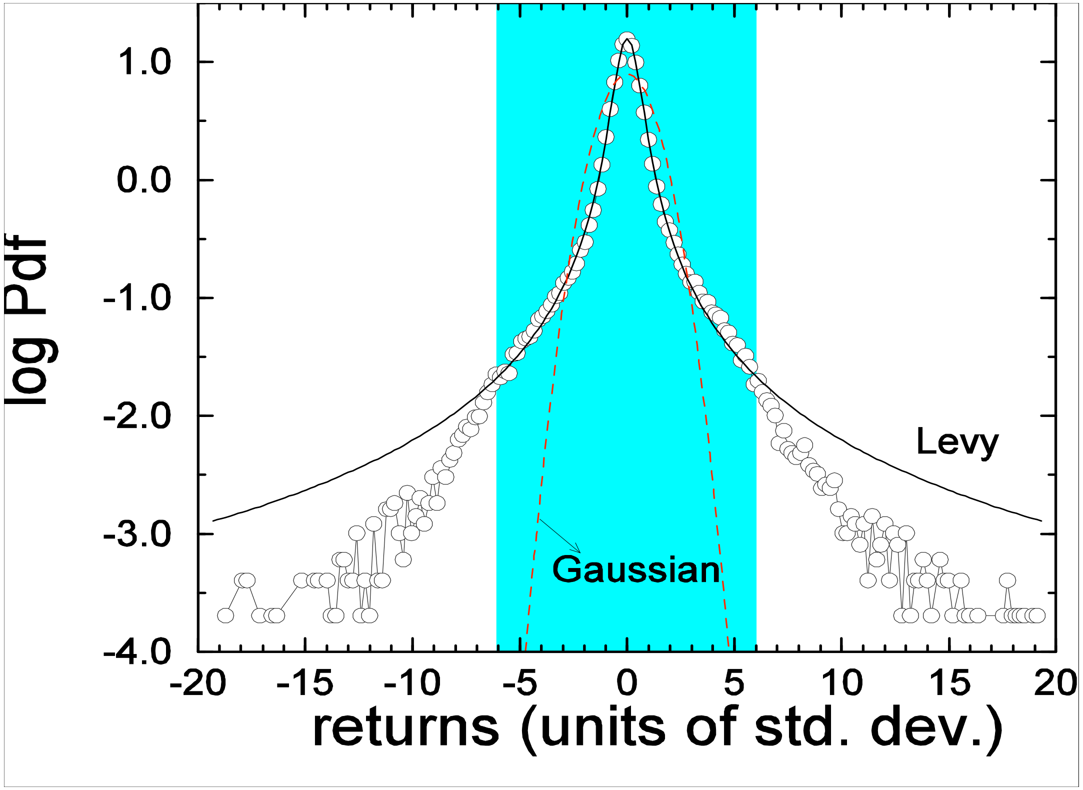


$$R_t = \mathrm{sgn}(R_t)\quad |R_t|$$

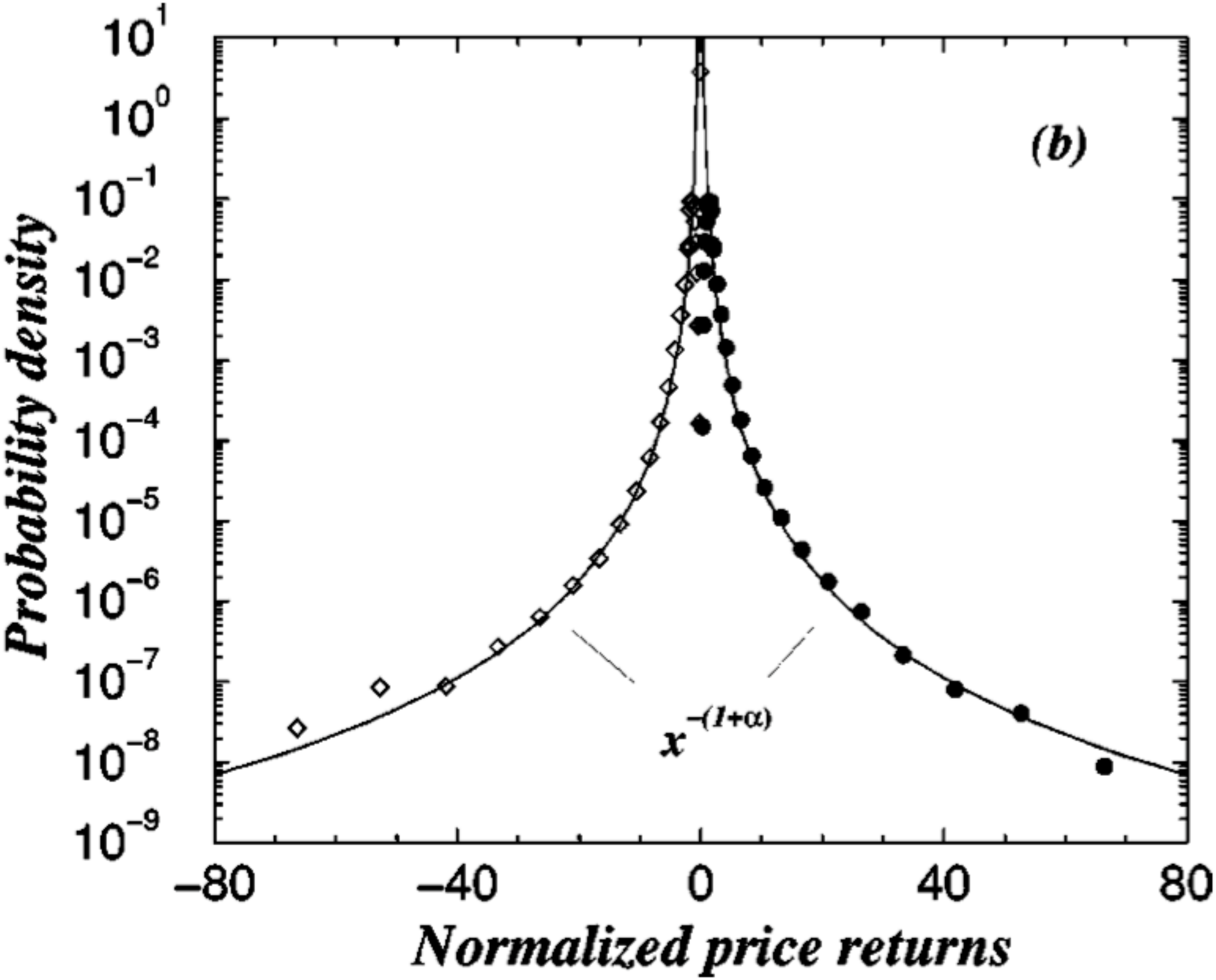


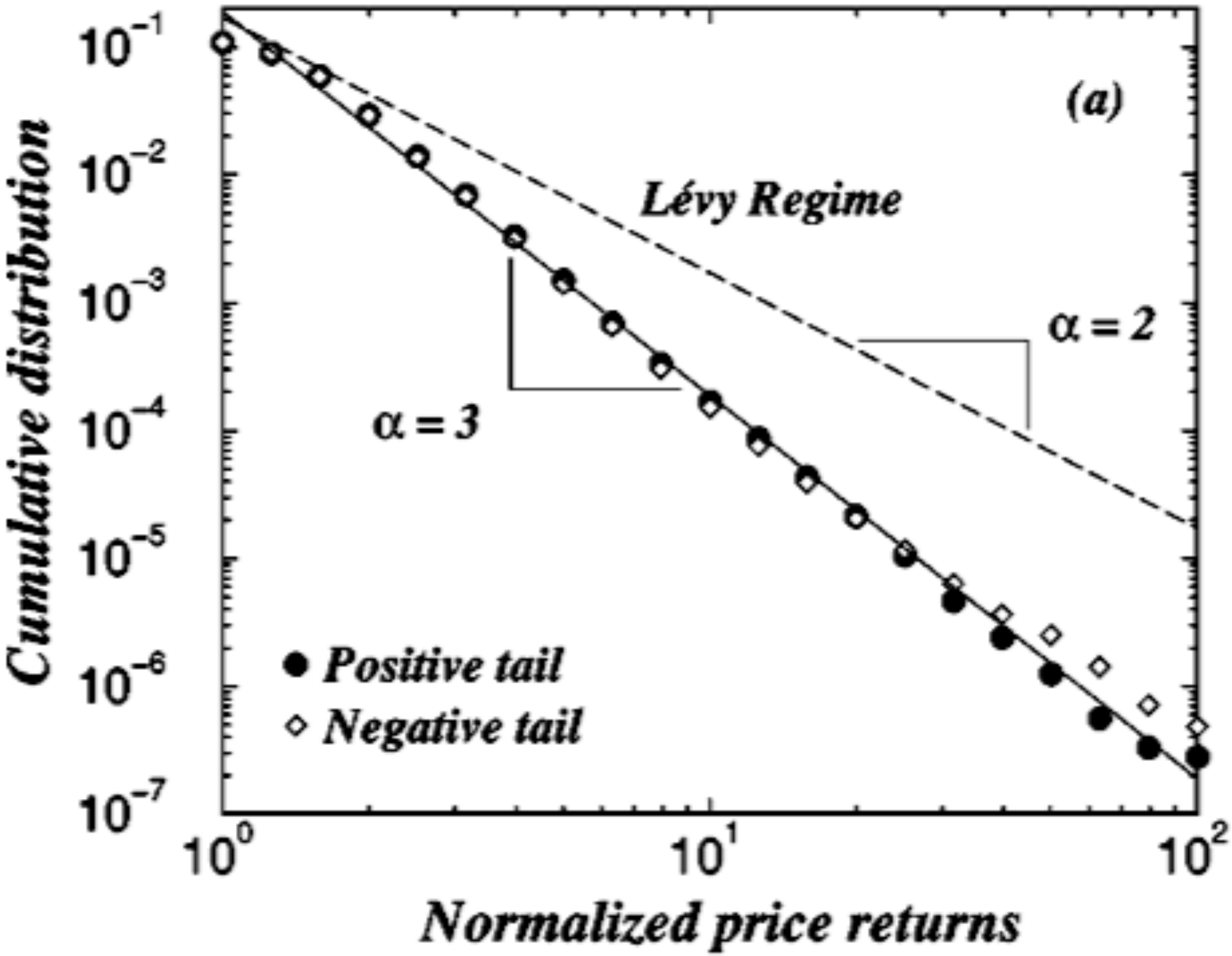
- Returns are uncorrelated.
- Absolute value of returns (vol) is long range correlated, so returns are not serially independent.

Distribution of S&P 500 returns

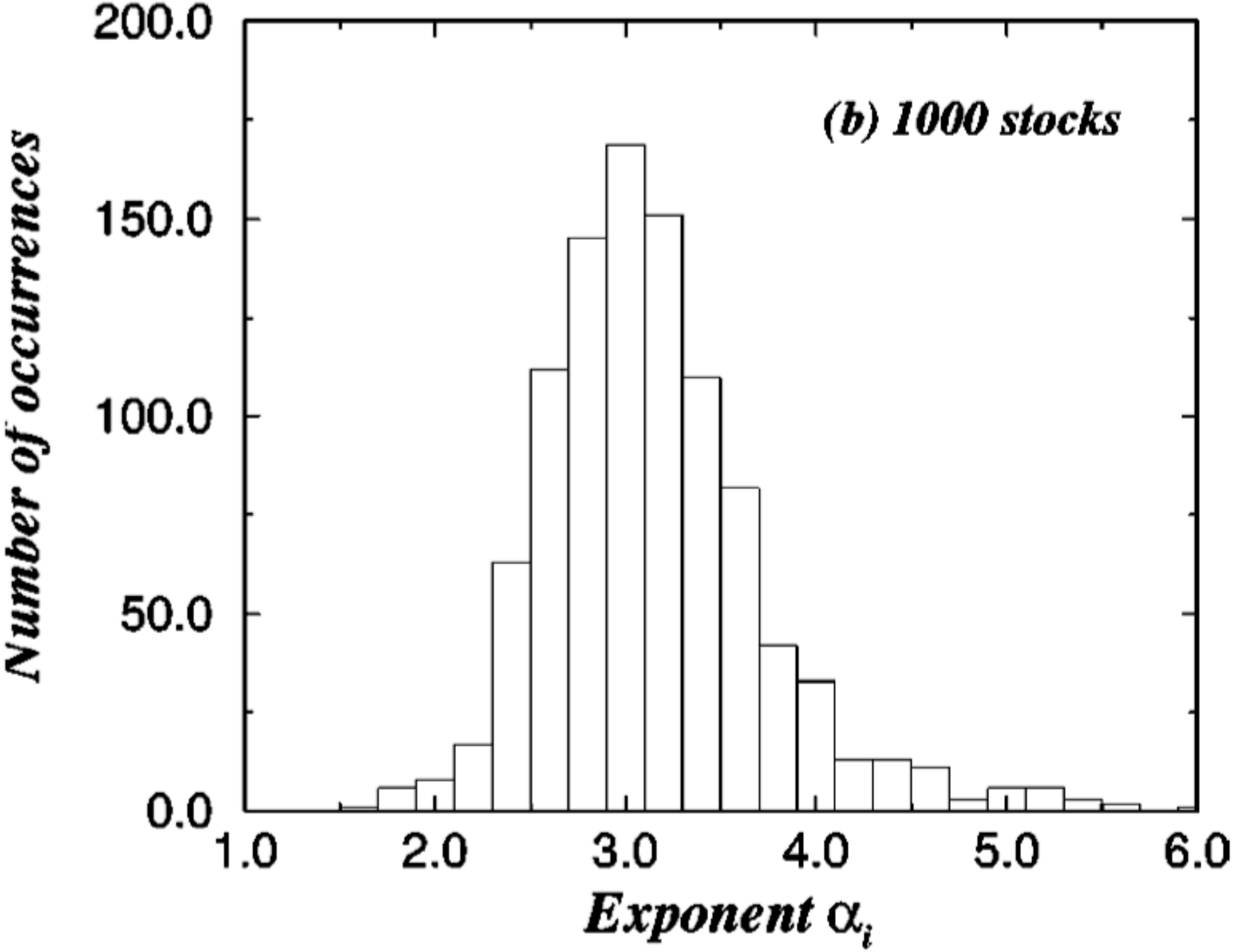


- Wings decay less rapidly than a Gaussian, but more rapidly than that of a Levy stable distribution.
- Need large amount of data to precisely quantify the tail behavior.

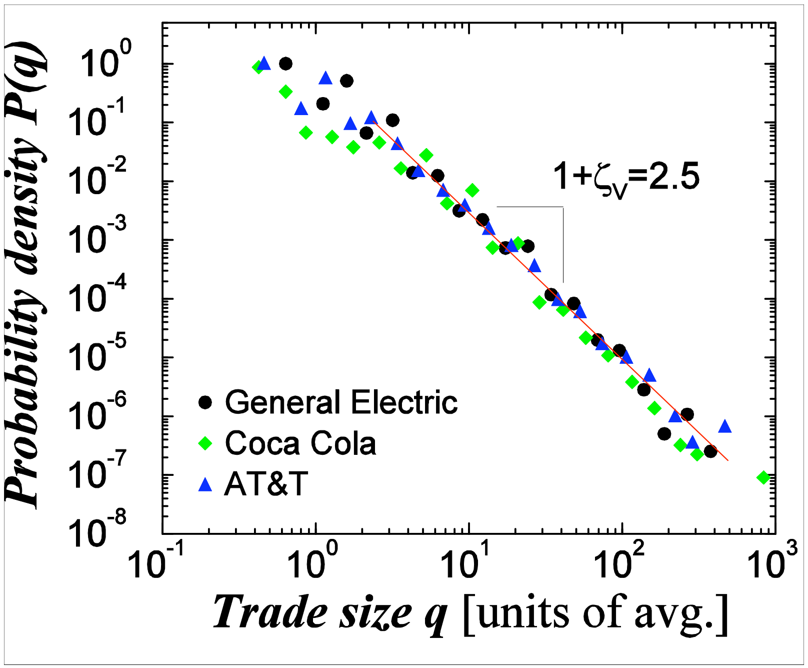

(b)
Probability density
Normalized price returns
$x^{-(1+\alpha)}$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
−80
−40
0
40
80



Cumulative distribution
Normalized price returns
(a)
Lévy Regime
α = 2
α = 3
Positive tail
Negative tail
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^0
10^1
10^2

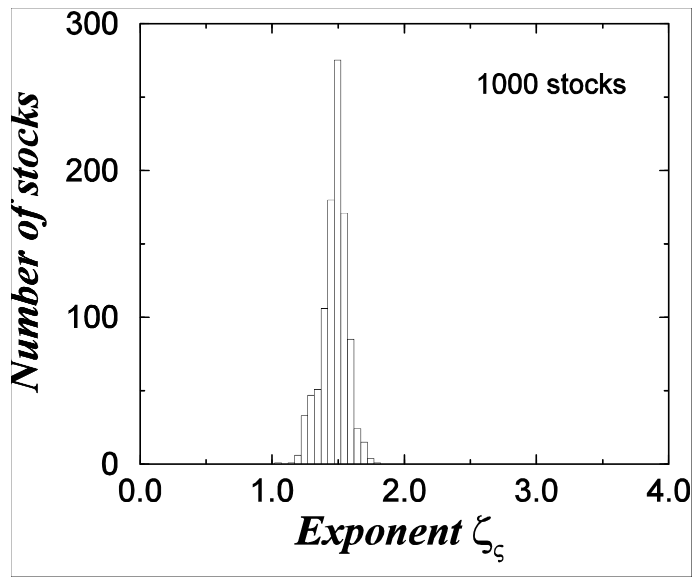

(b) 1000 stocks
Number of occurrences
0.0
50.0
100.0
150.0
200.0
1.0
2.0
3.0
4.0
5.0
6.0
Exponent α_i

Statistics of Volume Traded

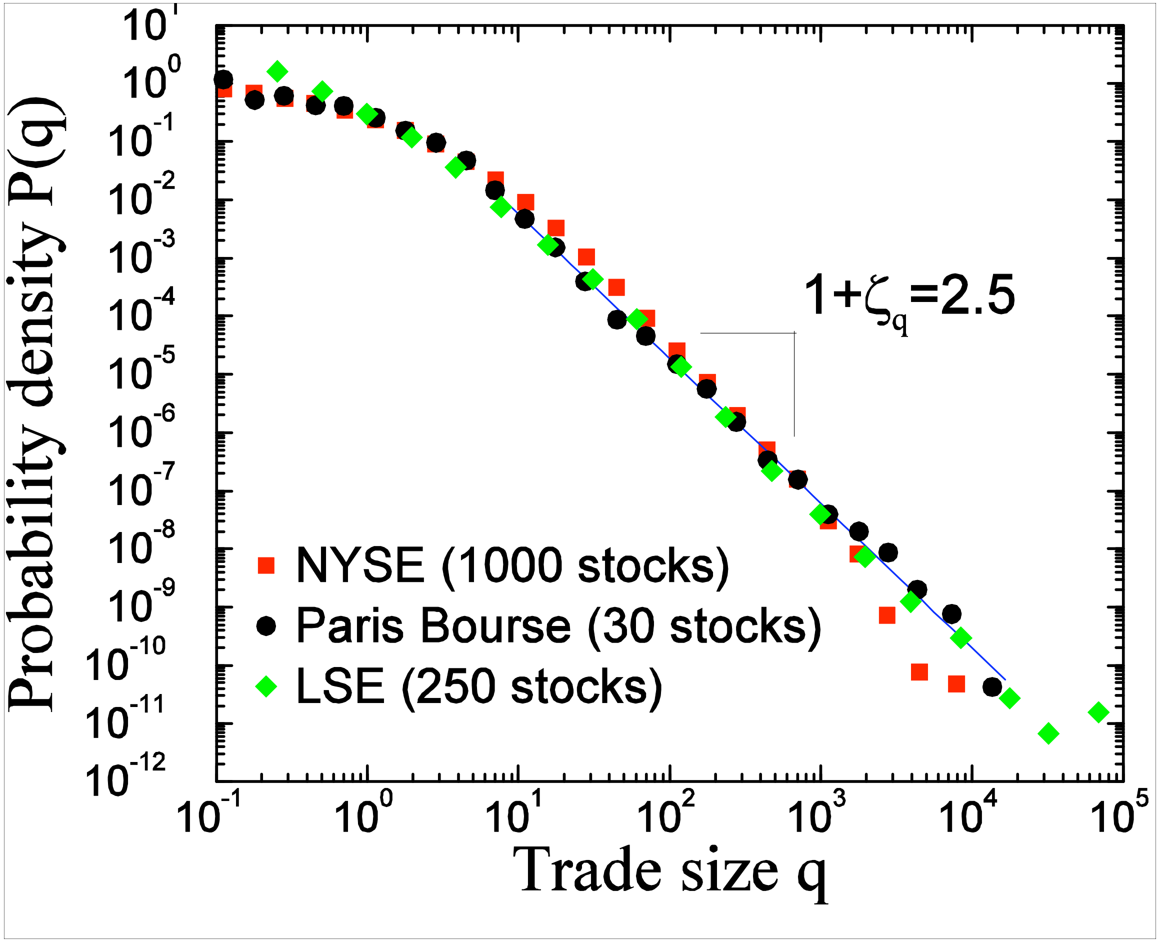


$$P(V > x) \sim x^{-\zeta_V}$$

$$\zeta_V \approx \frac{3}{2}$$

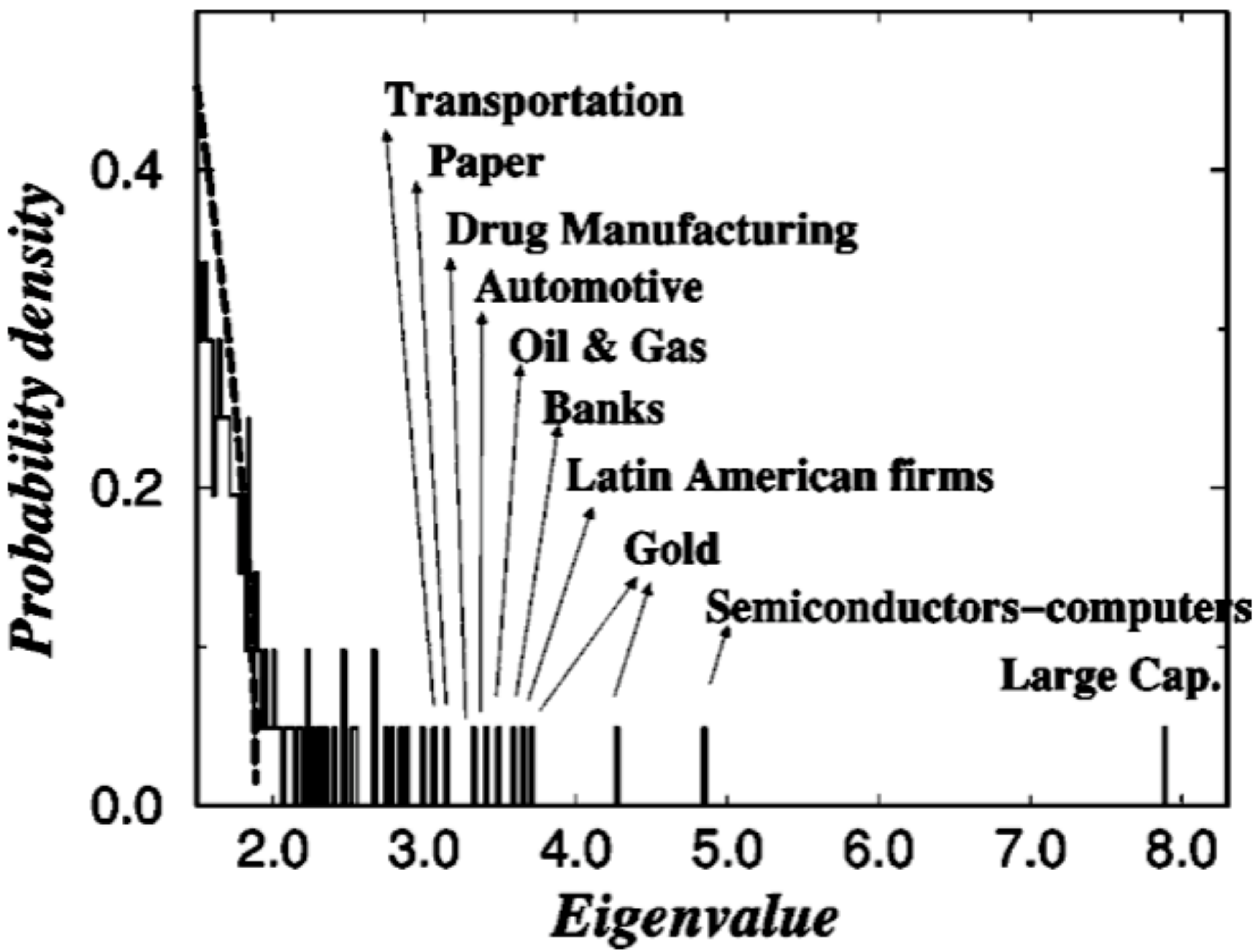


“Universal” nature of the power-law exponents



$$P(V > x) \sim x^{-\zeta_V}$$

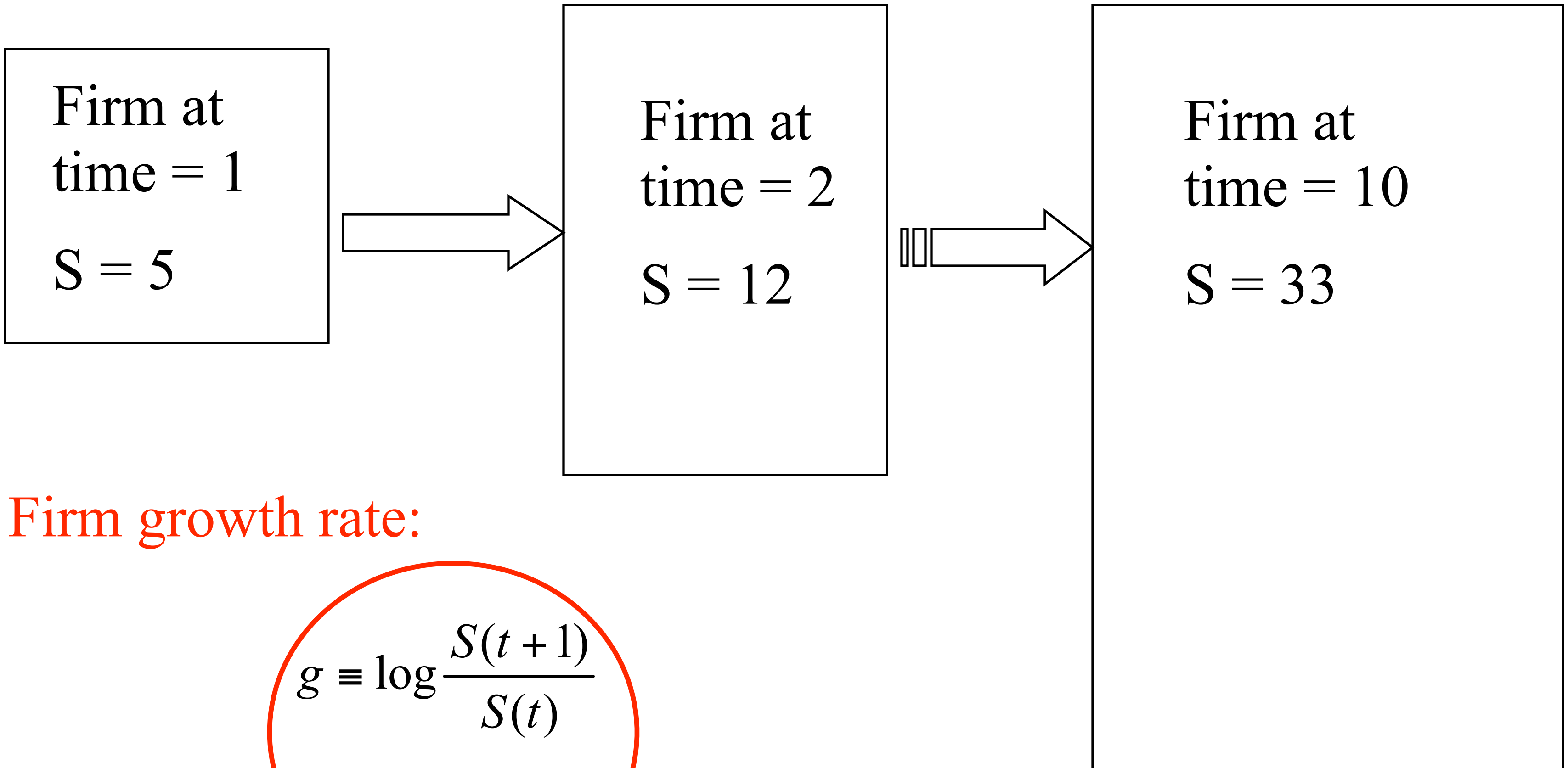
$$\zeta_V \approx \frac{3}{2}$$


Transportation
Paper
Drug Manufacturing
Automotive
Oil & Gas
Banks
Latin American firms
Gold
Semiconductors−computers
Large Cap.
0.0
0.2
0.4
2.0
3.0
4.0
5.0
6.0
7.0
8.0
Probability density
Eigenvalue

THM:Take Home Message

- “fat tails” of P(returns)obey inverse cubic law
- fat tails of other quantities also power laws
- tail exponents interrelated (a la Widom scaling laws)
- expontial decay in time correl fn (scale = 4 min)
- power law decay in volatility correlation function
- random matrix theory finds business sectors from the cross correlation function: practical applications!
- Price change = f(herd, news) = g(J,H)
- Set of all traders = complex “spin glass”

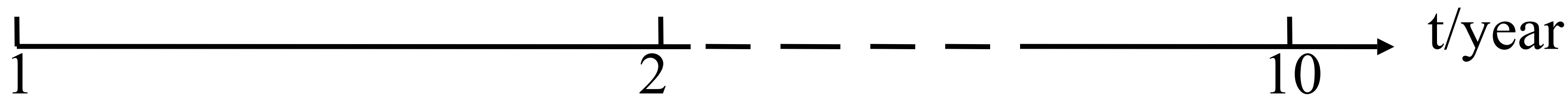
Classical Problem of Firm Growth



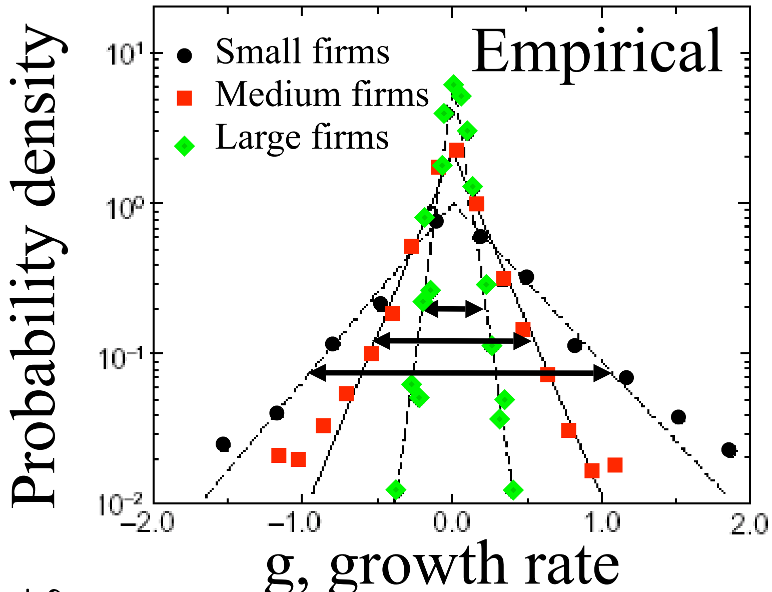
Firm growth rate:

$$g \equiv \log \frac{S(t+1)}{S(t)}$$

$$= \log\left(\frac{12}{5}\right)$$

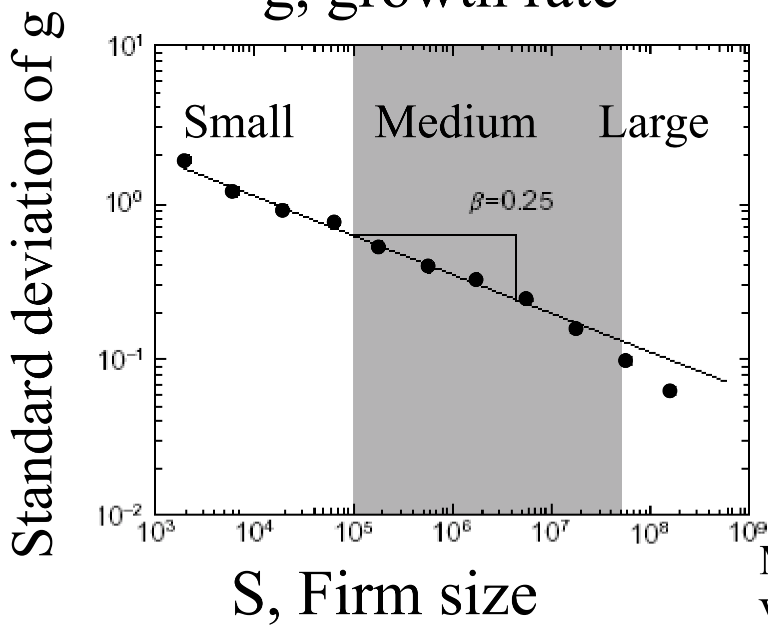


Empirical Observations (before 1999)



Reality: it is “tent-shaped”!

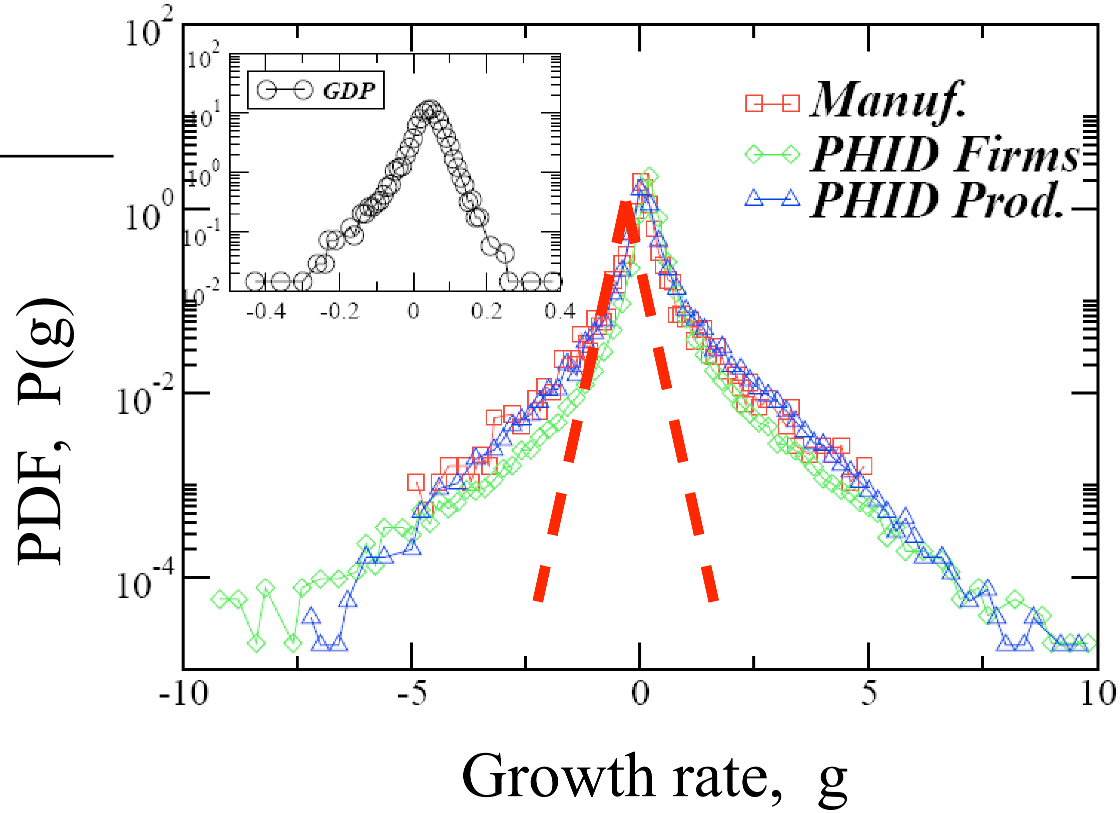
$$\mathbf{pdf(g|S)} \sim e^{-\frac{|g|}{\sigma(S)}}$$



$$\sigma_{\mathbf{g}}(\mathbf{S}) \sim \mathbf{S}^{-\beta}, \quad \beta \approx \mathbf{0.2}$$

Michael H. Stanley, *et.al.* Nature **379**, 804-806 (1996).
V. Plerou, et.al. Nature **400**, 433-437 (1999).

Empirical Findings: P(growth rate)



Laplace fitting function:

$P(g) \sim \exp(-|g-\bar{g}|/\sigma)$

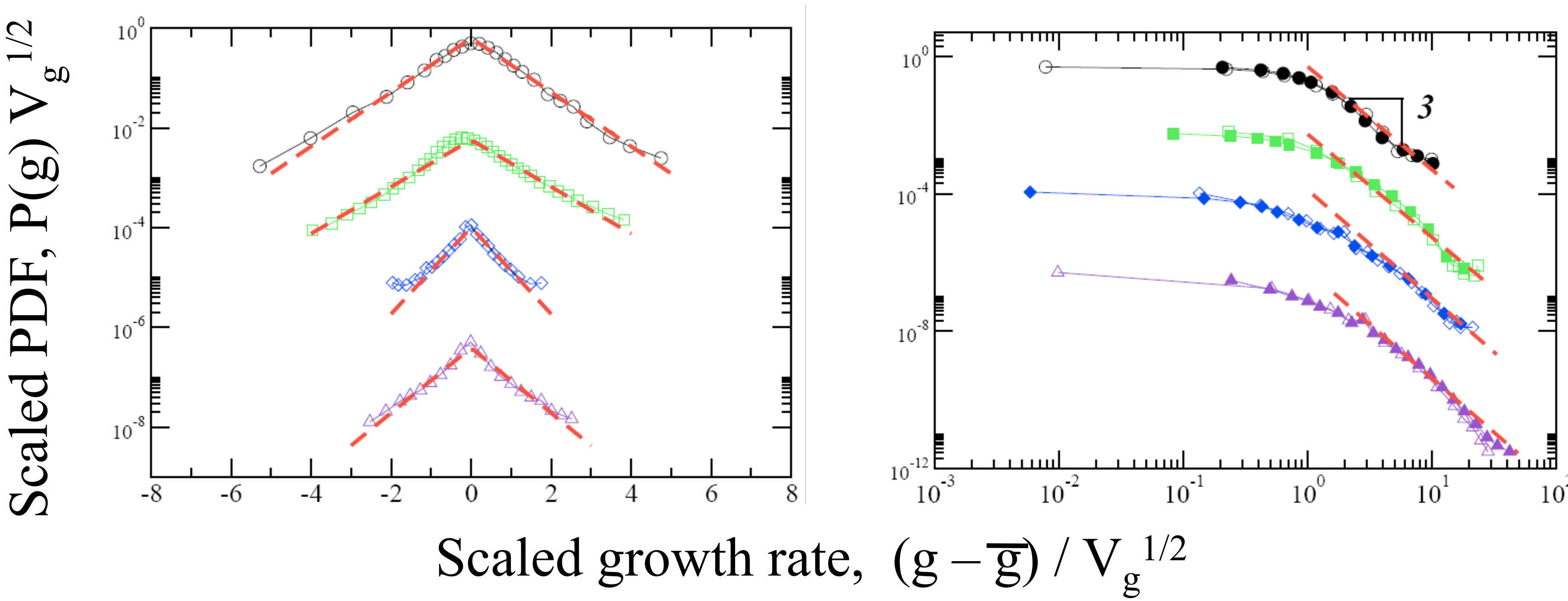
[Nature **379**, (1996)]

Our question: **What is the function P(g), the PDF of growth rate?**

Answer: Not Gaussian, [Gibrat (1930)].

Not Laplace, [M. Stanley, et al (1996)].

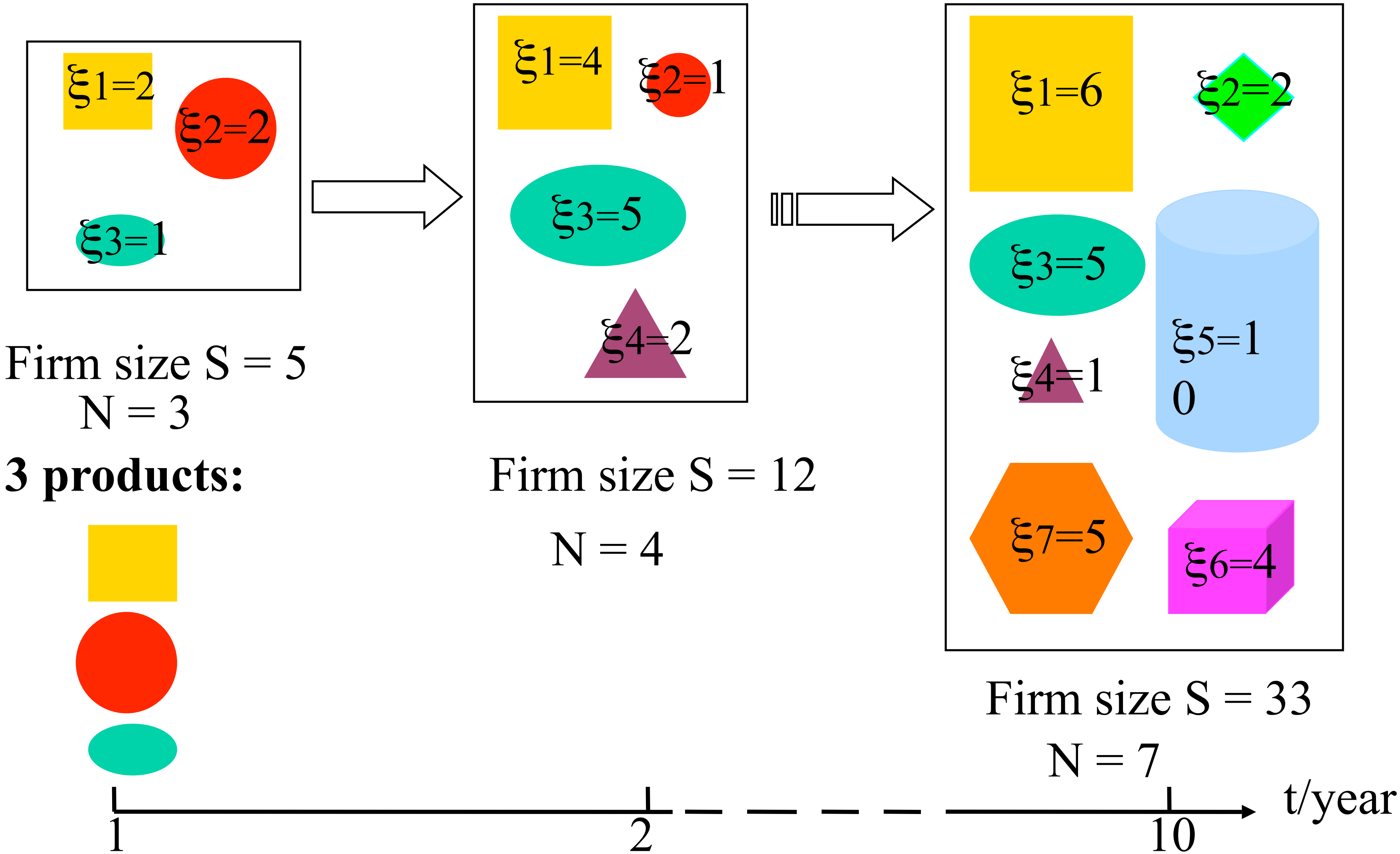
The Test of Central & Tail Parts of P(g)



Central part is Laplace.

Tail part is power-law with exponent -3.

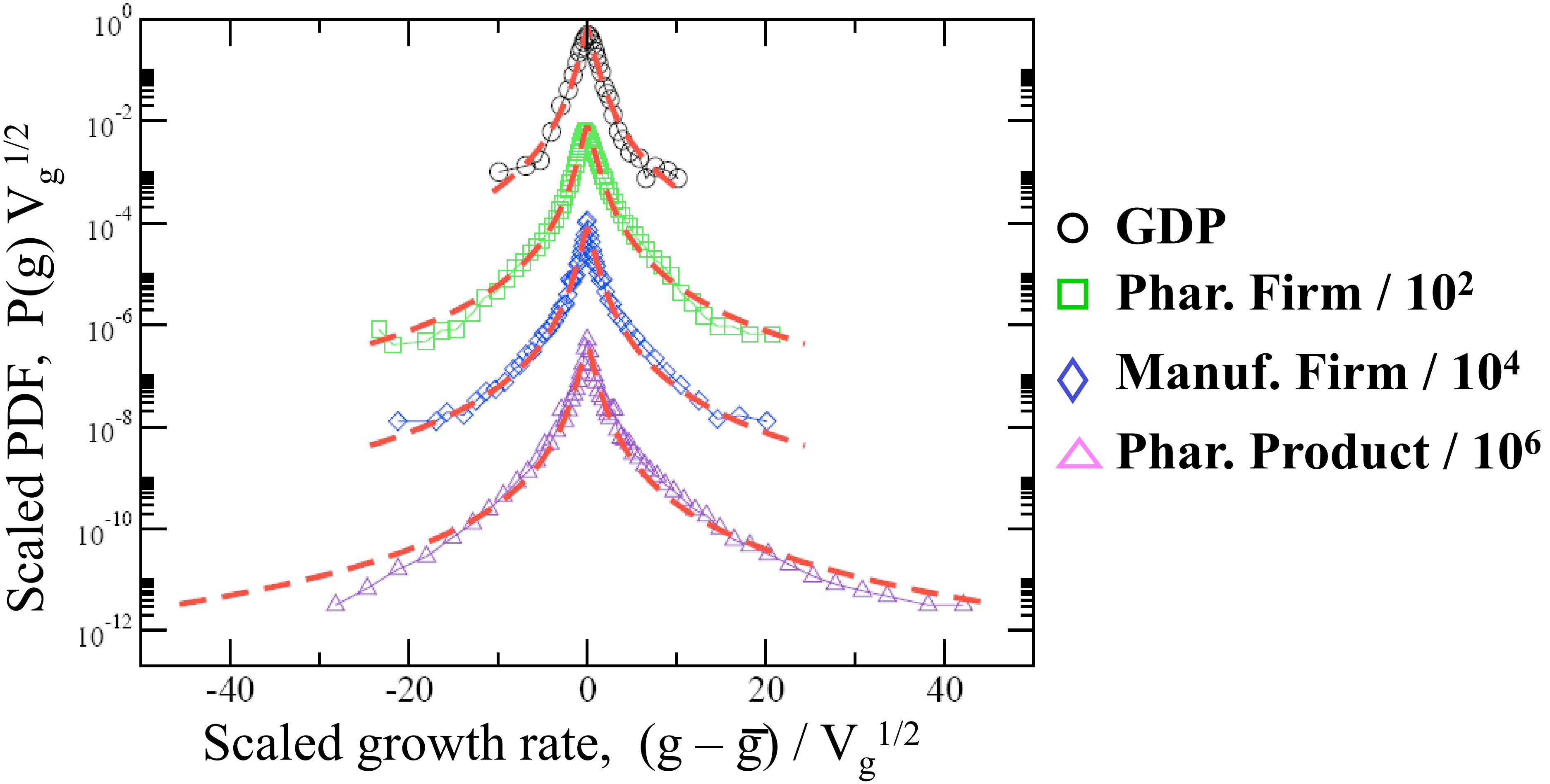
Statistical Growth of a Sample Firm



L.A.N. Amaral, et al, PRL, **80** (1998)

Test with of Empirical Data

One Parameter: V_g



Take home message

- P(growth rate) Laplace in Center: universal
- Width decreases as -1/6 power of size bin
- P(growth rate) crosses over to power law in wings
- No theory for -1/6 power law for width
- Theory (Buldyrev et al) for growth rate power law

- 20th Century: laws describing ordered systems....ex: solid state physics
- 21st Century: laws describing disordered systems....ex: liquids, gels, glasses, ...
- key question: “what matters” (ex: near critical point, correlation length matters, not Temperature, not pressure, etc)
- key hope: will uncover “unifying principles” leading to a conceptual framework linking various phenomena (ex:percolation links wide range of connectivity phenomena)

H. E. STANLEY "Mysteries of Water" Les Houches Lecture, May 1998

Collaborate?
HES@BU.EDU

THANKS TO:
Lidia B.
Gabriel C.
Sergey B.
Freddy Z.
.....
and my students who did this work so i could come to Mar del Plata today!

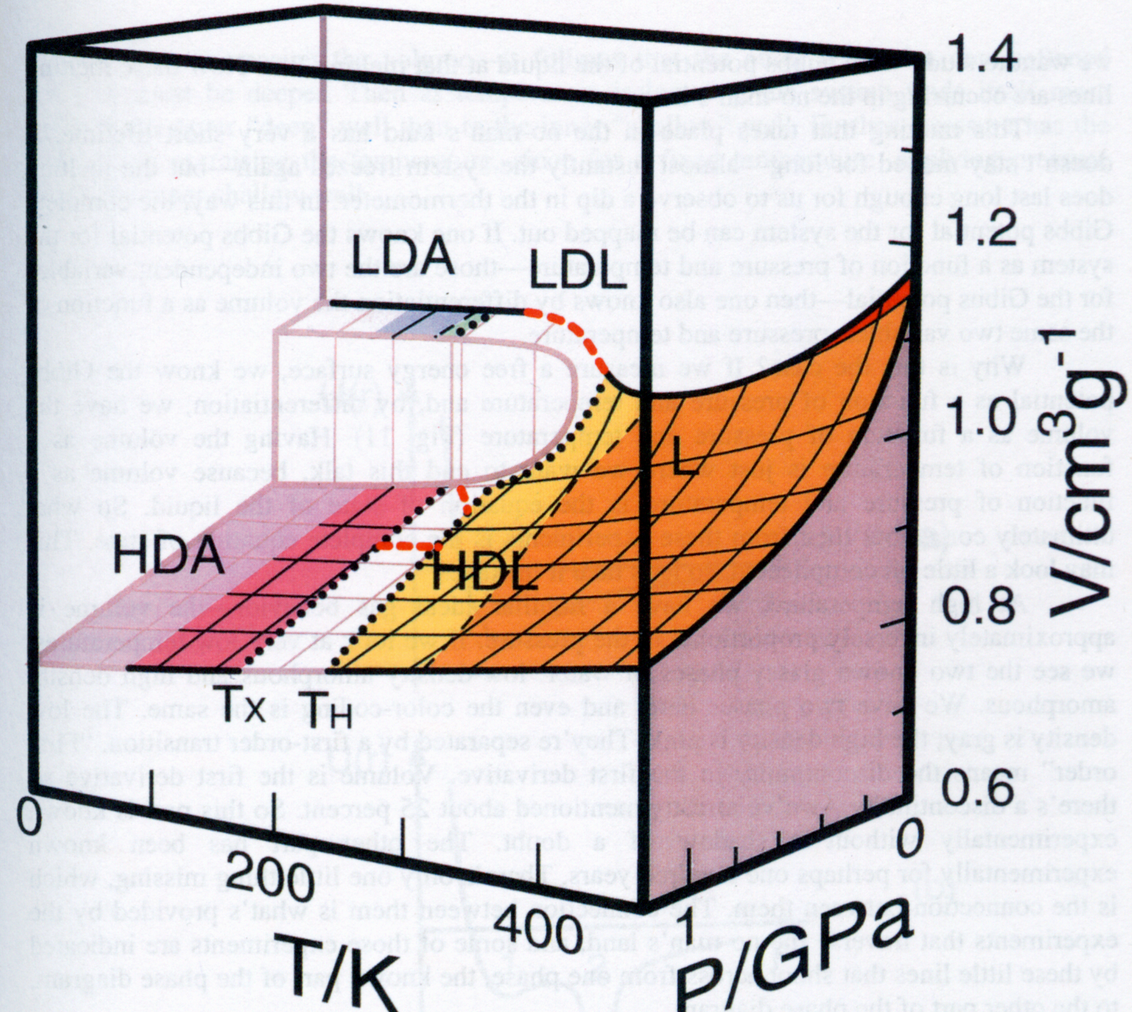


Figure 11: Experimentally-derived [56] thermodynamics equation of state V=V(P,T), using the same color coding as in Figs. 1 and 9. The specific volumes of the amorphous phases are known for the region below T_X [6]. Solid lines are the specific volume along the melting lines of ice IV and XIV. The high-temperature liquid appears to separate into two low-temperature liquid phases just below the critical point located at around 0.1 GPa and 220 K. These two liquid phases are continuous with the two amorphous phases that are known to exist below about 150 K.