APPLICATION OF STATISTICAL PHYSICS

APPROACHES TO COMPLEX ORGANIZATIONS

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Two Parts of my Thesis

- Part I: Financial Markets

Ex:

- Stock markets
- Commodity markets
- Foreign exchange markets

- Part II: Business Firms

Ex:

- Pharmaceutical Industry
- GDP of countries
- Scientific output of Countries

AIM : characterize fluctuations in the growth.

Why Fluctuations are Interesting and Important

- Interesting because:

  unsolved problem

- Important because :

  (a) Statistical physics may help us to develop better strategies to improve economy

  (b) Quantify risks in a better way

=> better living condition with better economy

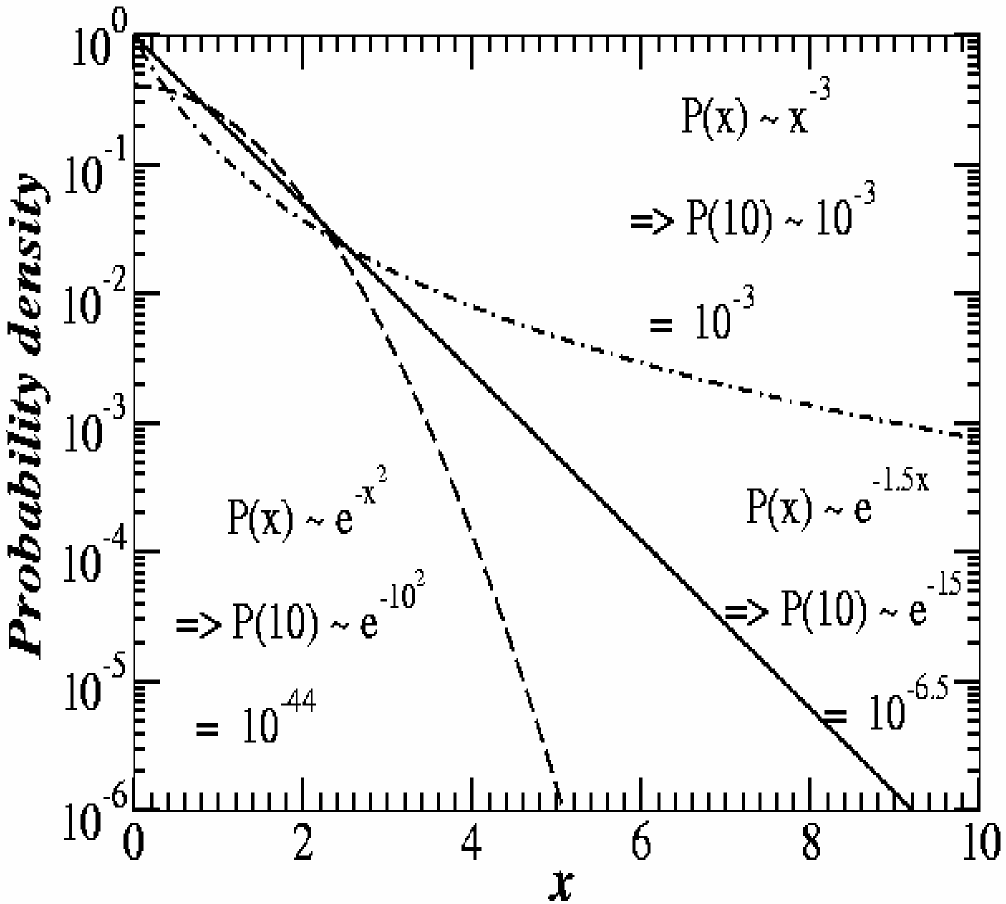
Monday October 19, 1987. Dow Jones Industrial Average fell 22.6%, largest one-day decline in recorded stock market history. This one day decline was not confined to the United States, but mirrored all over world. By the end of October, Australia had fallen 41.8%, Canada 22.5%, Hong Kong 45.8%, and the United Kingdom 26.4% : Resulted in starvation of farmers in Vietnam

How to Quantify Fluctuations ?

- Probability distribution of Fluctuations

- Correlations present in Fluctuations

Estimating Form of the Probability Distribution is Important



Note on Power Law Probability Distributions:

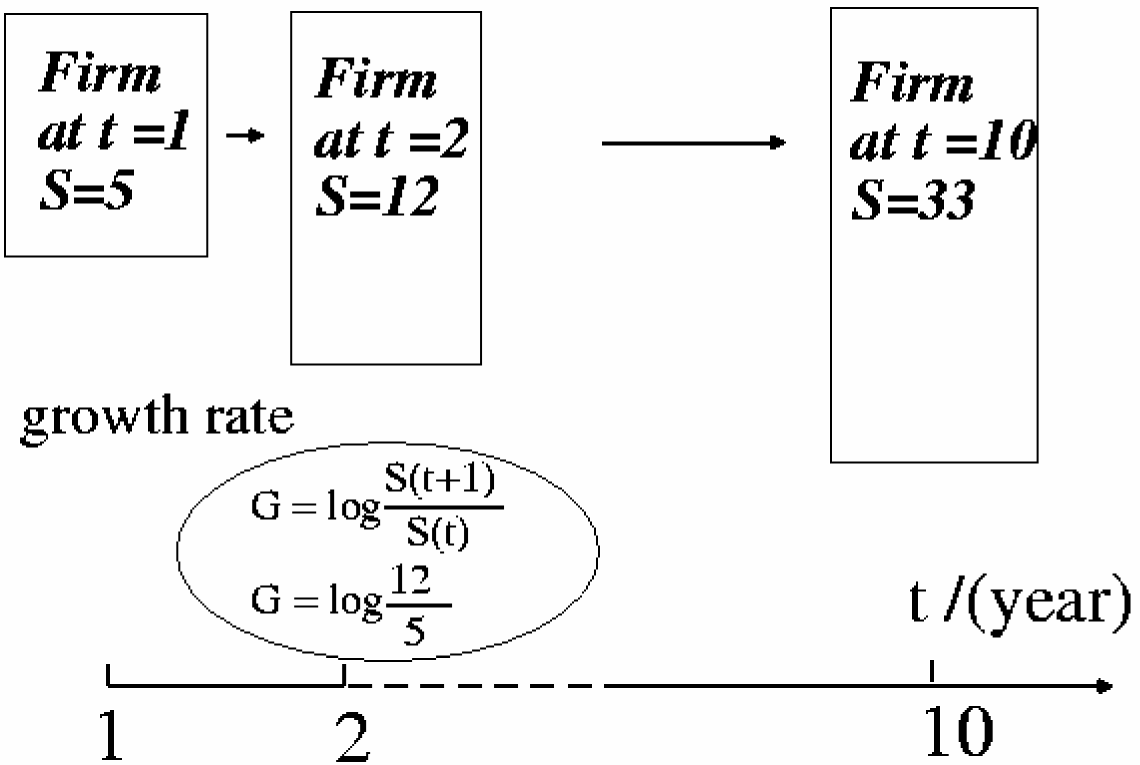
$P(x) \sim x^{-(\alpha+1)}$ with $\alpha < 2$ are called Levy stable distribution.

Properties of Levy stable distribution:

- Infinite variance
- Stable distribution **:** if you add up numbers taken from a Levy stable distribution the resulting sum will also follow Levy stable distribution

Part II : Business Firms

Classic Problem of Firm Growth



normalized growth rate $g = G/\sigma$

Classic Gibrat Law & Its Implication

Question: What is pdf of growth rate?

Traditional View: Gibrat law of "Proportionate Effect" (1930's)

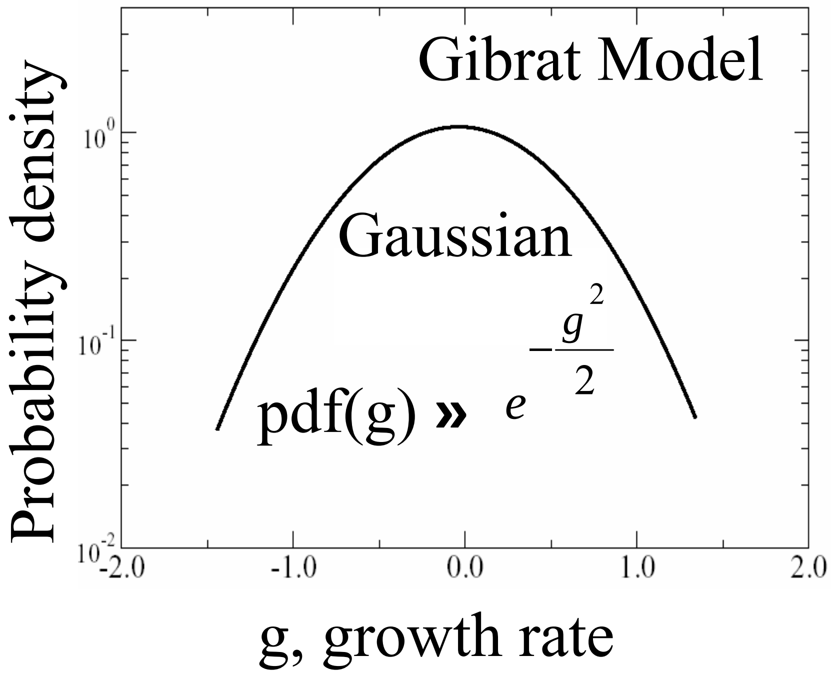
$S(t+1) = S(t)(1 + \eta_t)$ (η_t is noise, $-1 < \eta_t < 1$).

⇨ $$\log S(t = T) = \log S(t = 0) + \sum_{t'=1}^{M} \log(1 + \eta_{t'})$$

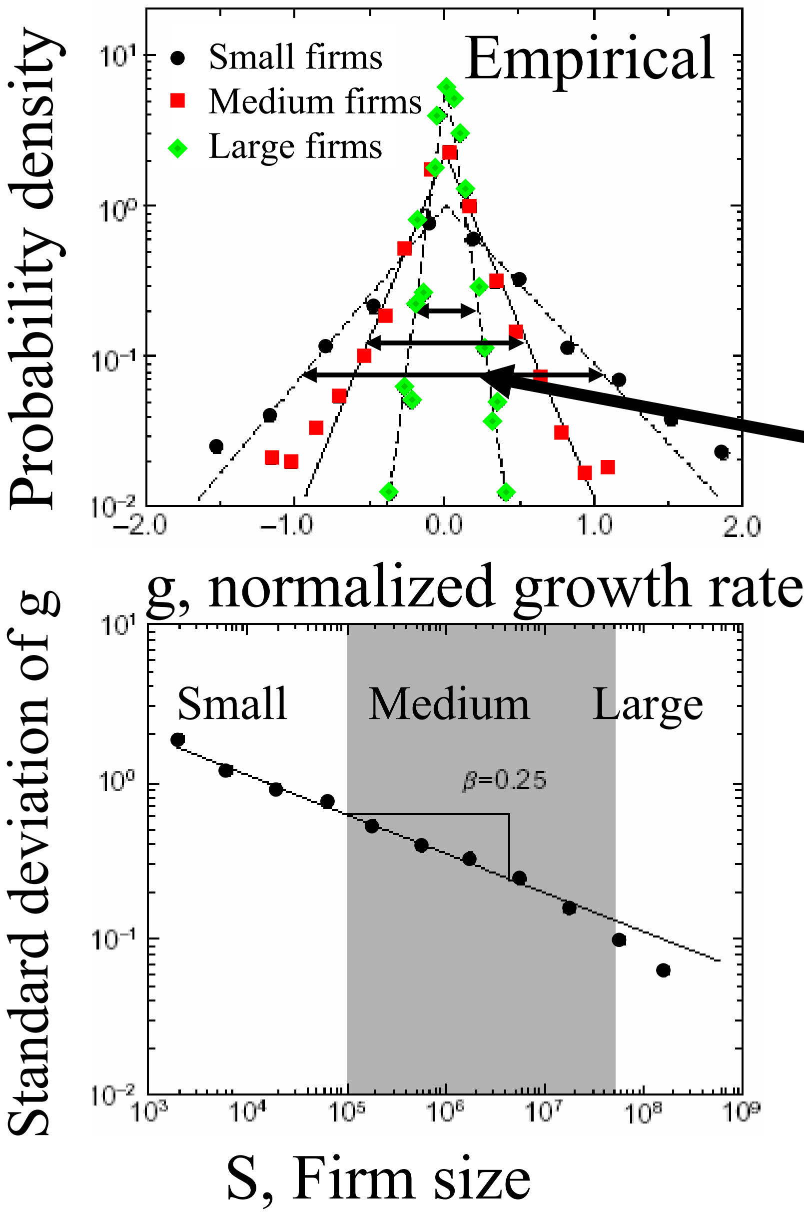
⇨ Growth rate g in T years

$$= \log\frac{S(T)}{S(0)} = \sum_{t'=1}^{M} \log(1+\eta_{t'}) \approx \sum_{t'=1}^{M} \eta_{t'}$$

Gibrat: pdf of g is Gaussian.



Empirical Observations



Reality: it is "tent-shaped"!

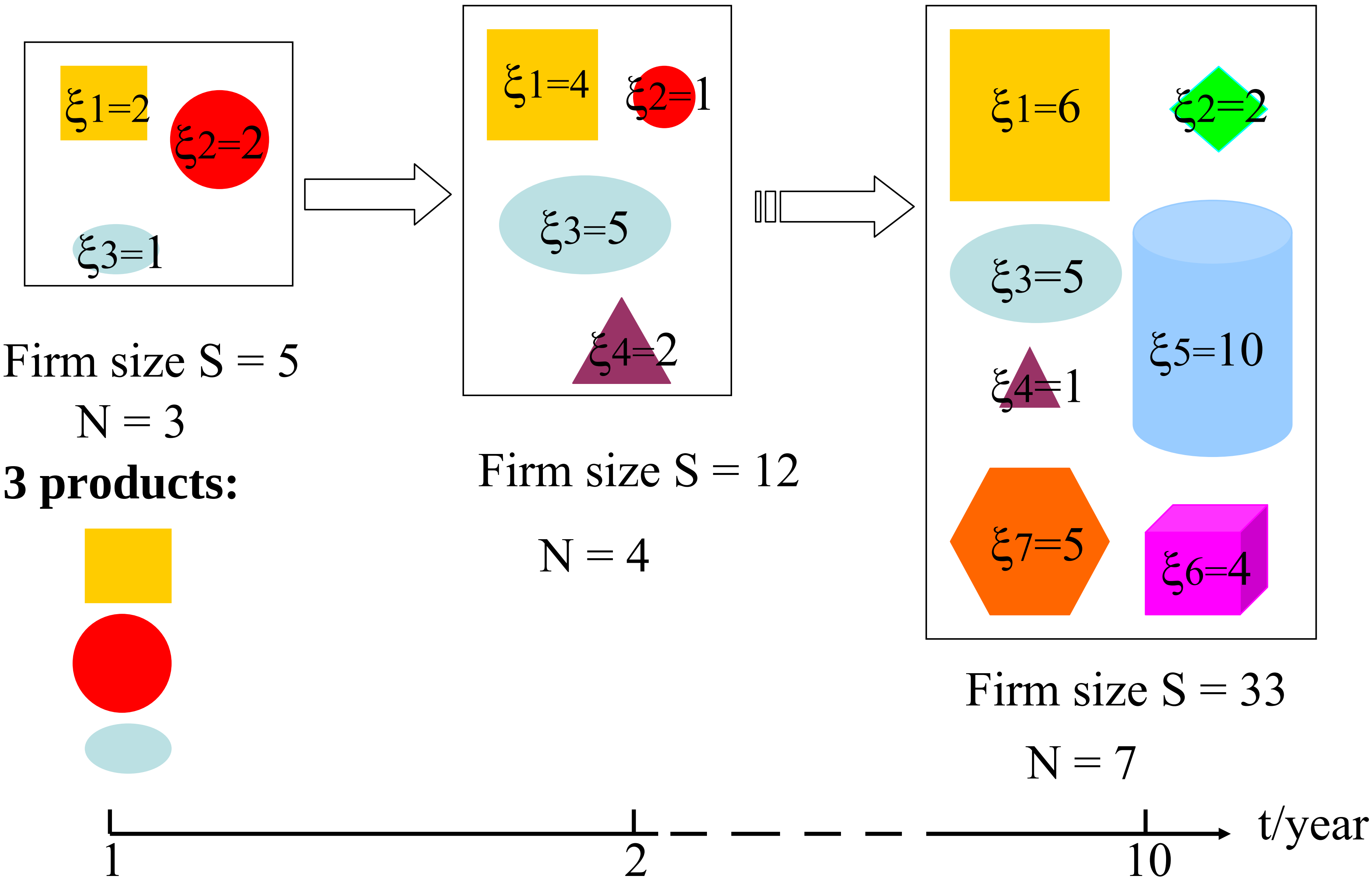
$$\mathbf{pdf(g|S)} \approx e^{-\frac{|g|}{\sigma(S)}}$$

standard deviation of g

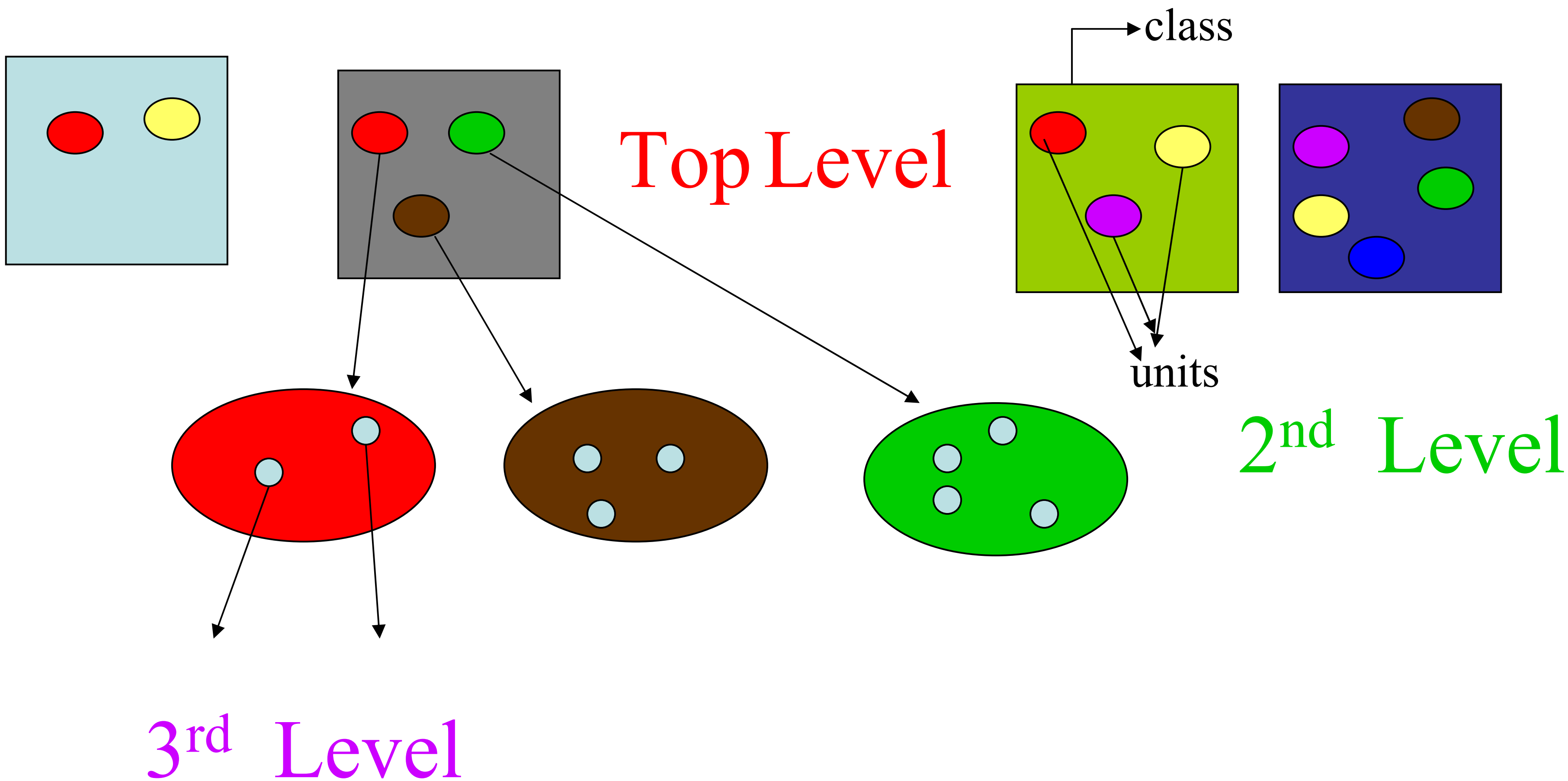
$$\sigma(g|S) \sim S^{-\beta}$$

$$\beta \approx 0.2$$

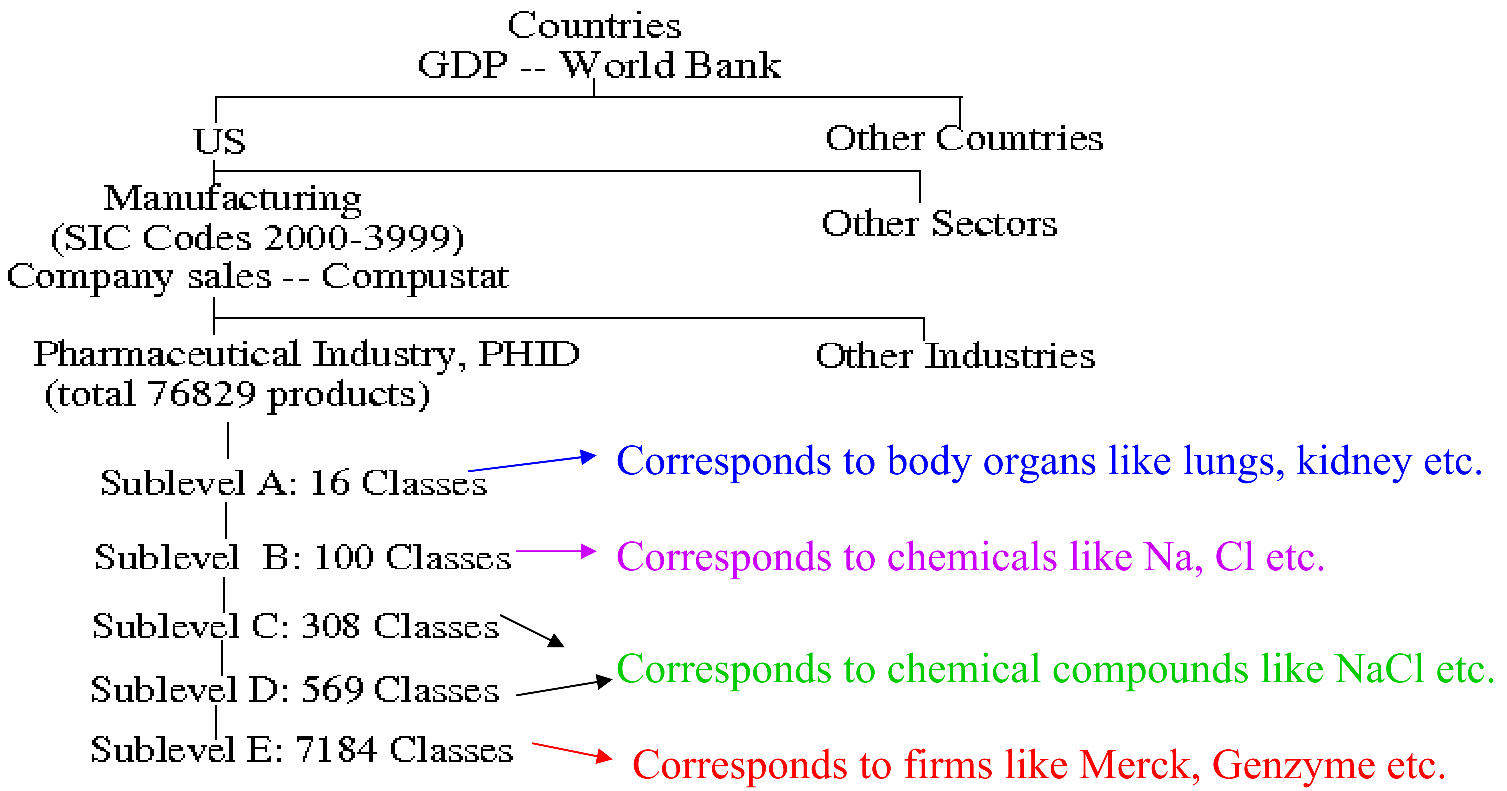
New: Focus on Products within a Firm



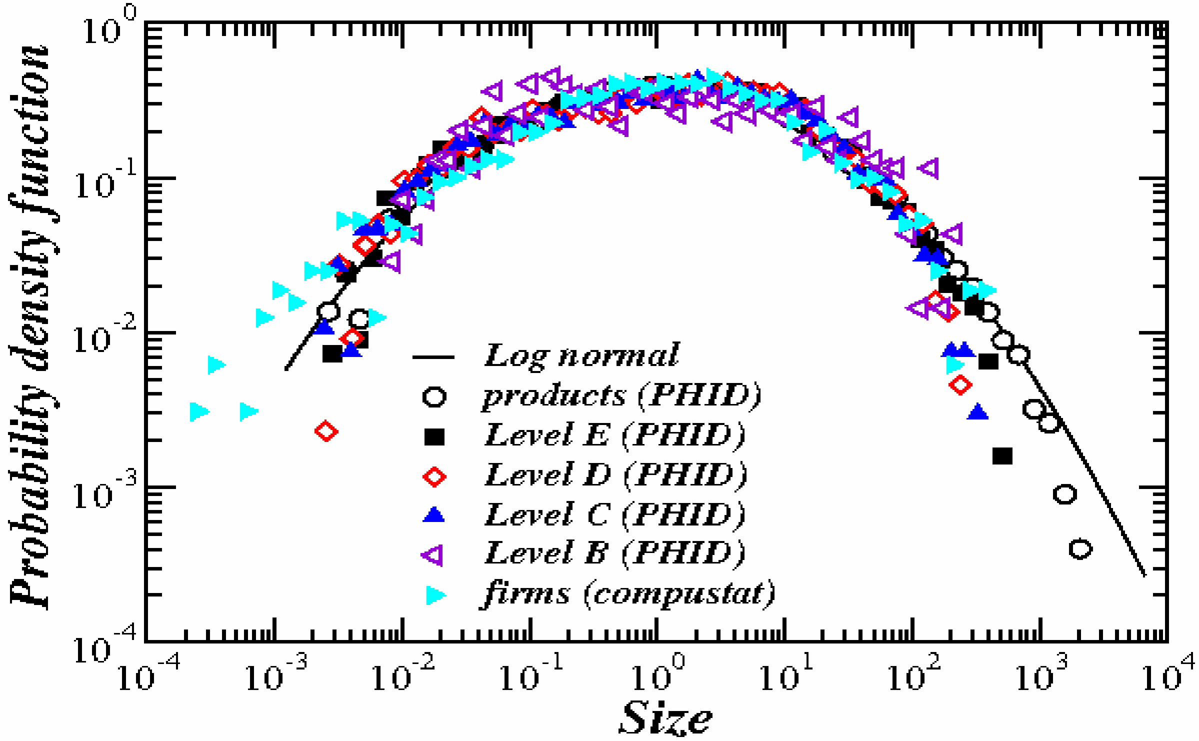
Database (Units grouped into Classes)



Database Studied (1990 – 2000)

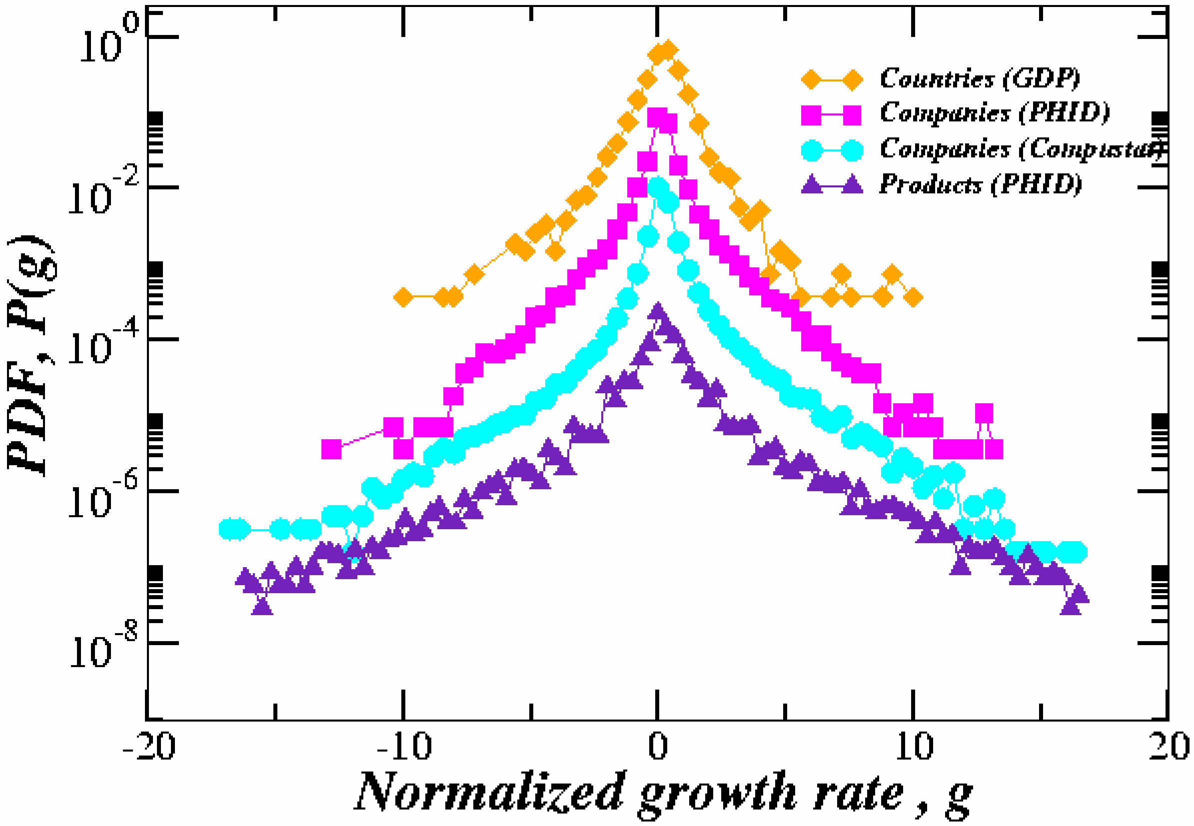


Size Distribution



Class size is log-normally distributed: ~ exp[-[log(S)]^2]

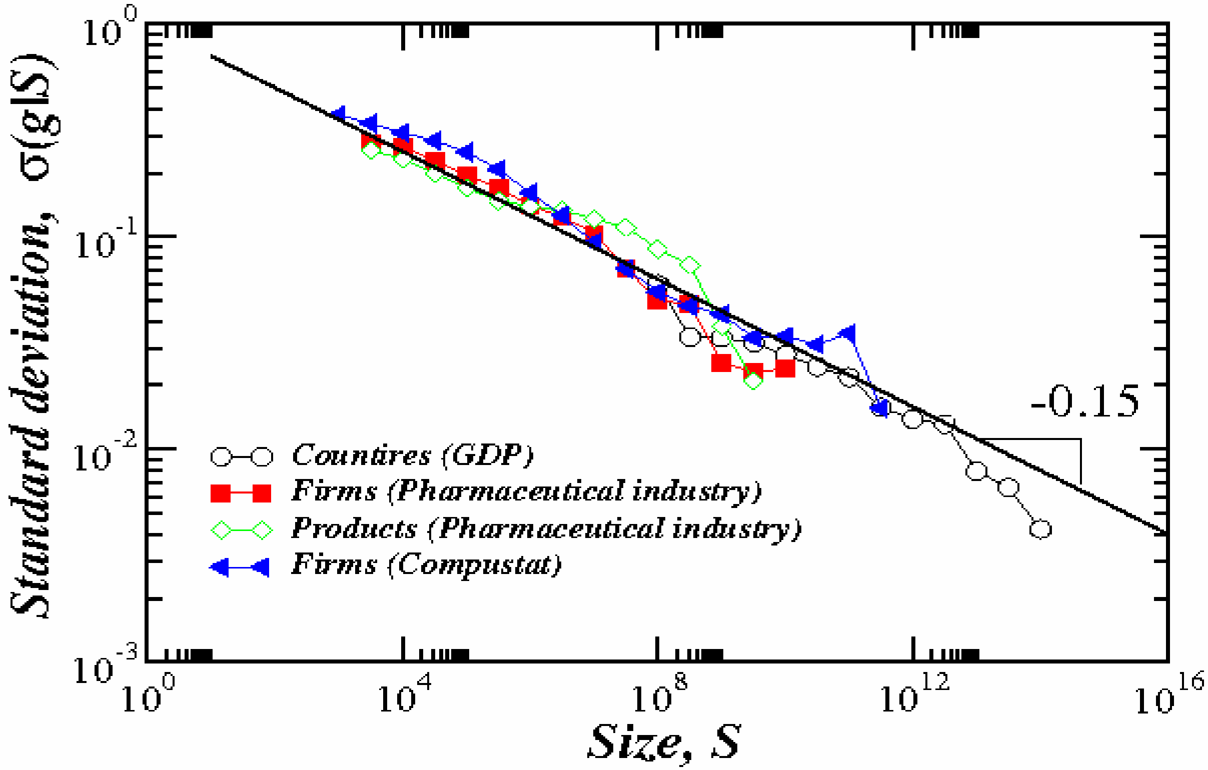
Growth Rate Distribution



Growth rate : ~ exp (-|g|) : for levels with fixed number of classes

Growth rate: ~ exp (-|g|) central part + power law wings: for levels with number of classes varying

Size Variance Relationship



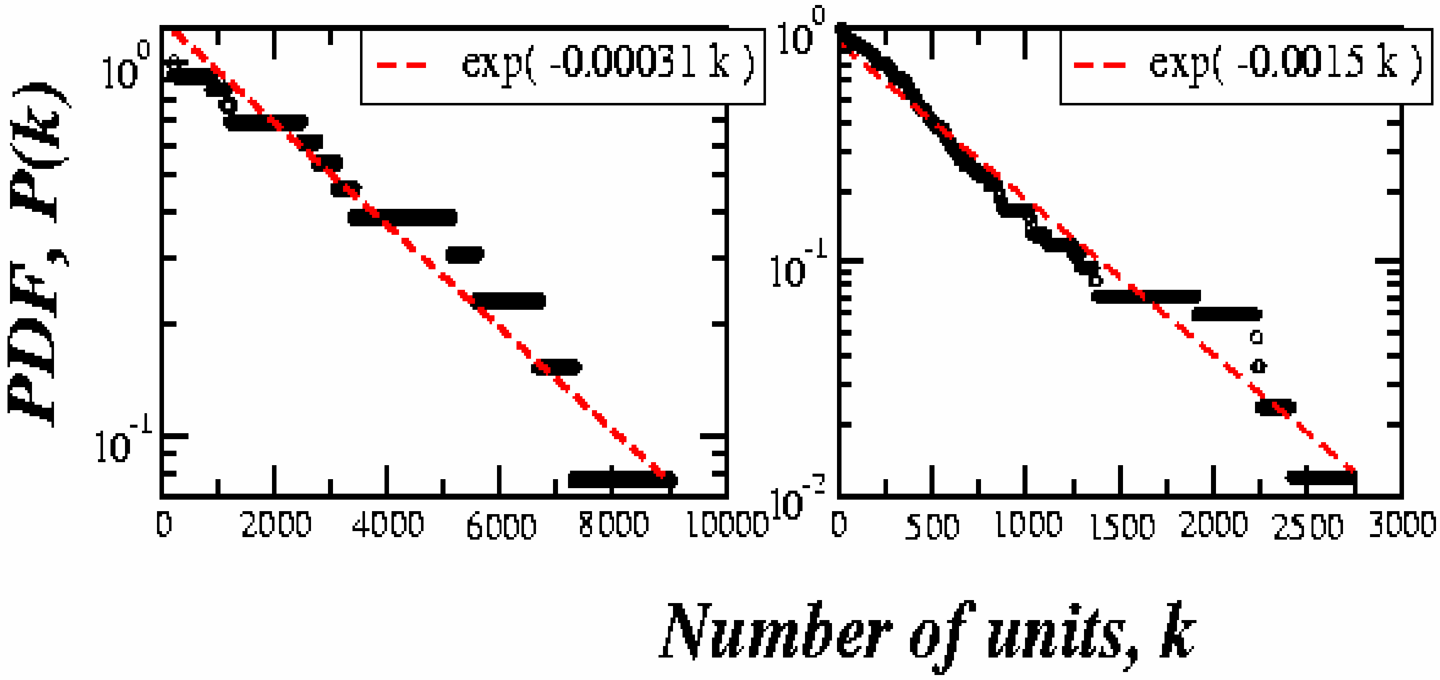
$$\sigma(g|S) \sim S^{-\beta}$$

Distribution of Number of Units in Classes

CASE I : Fixed Number of Classes over time

Sublevel A
Class corresponds to body parts

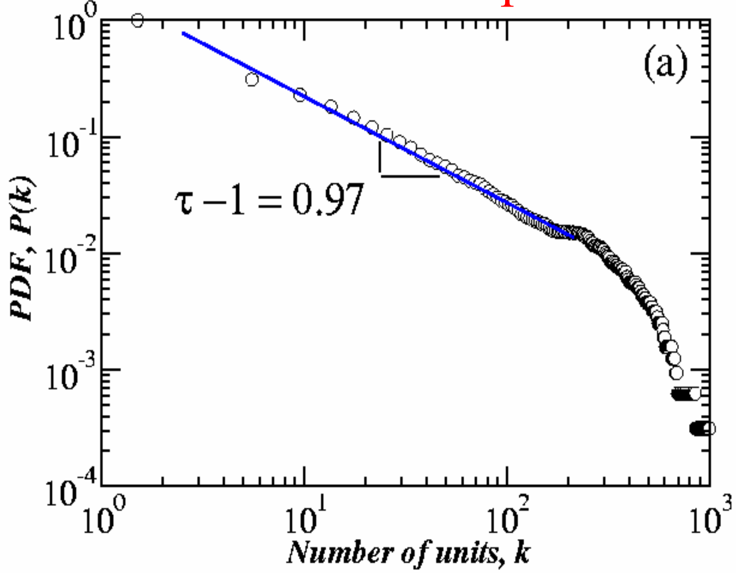
Sublevel B
Class corresponds to chemical elements

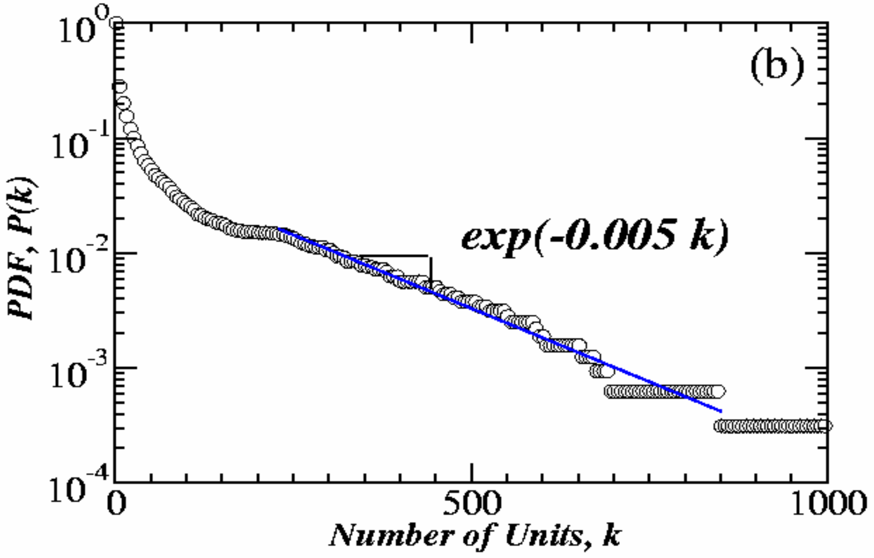


Distribution of Number of Units in Classes

CASE II : Varying Number of Classes over time

Sublevel E: Class corresponds to firms





Summary

- Number of classes remain fixed in time:

  Distribution of number of units in classes: Exponential

  Distribution of growth rate of class size: Laplace

- Number of classes change in time:

  Distribution of number of units in classes: Power law with exponential cutoff

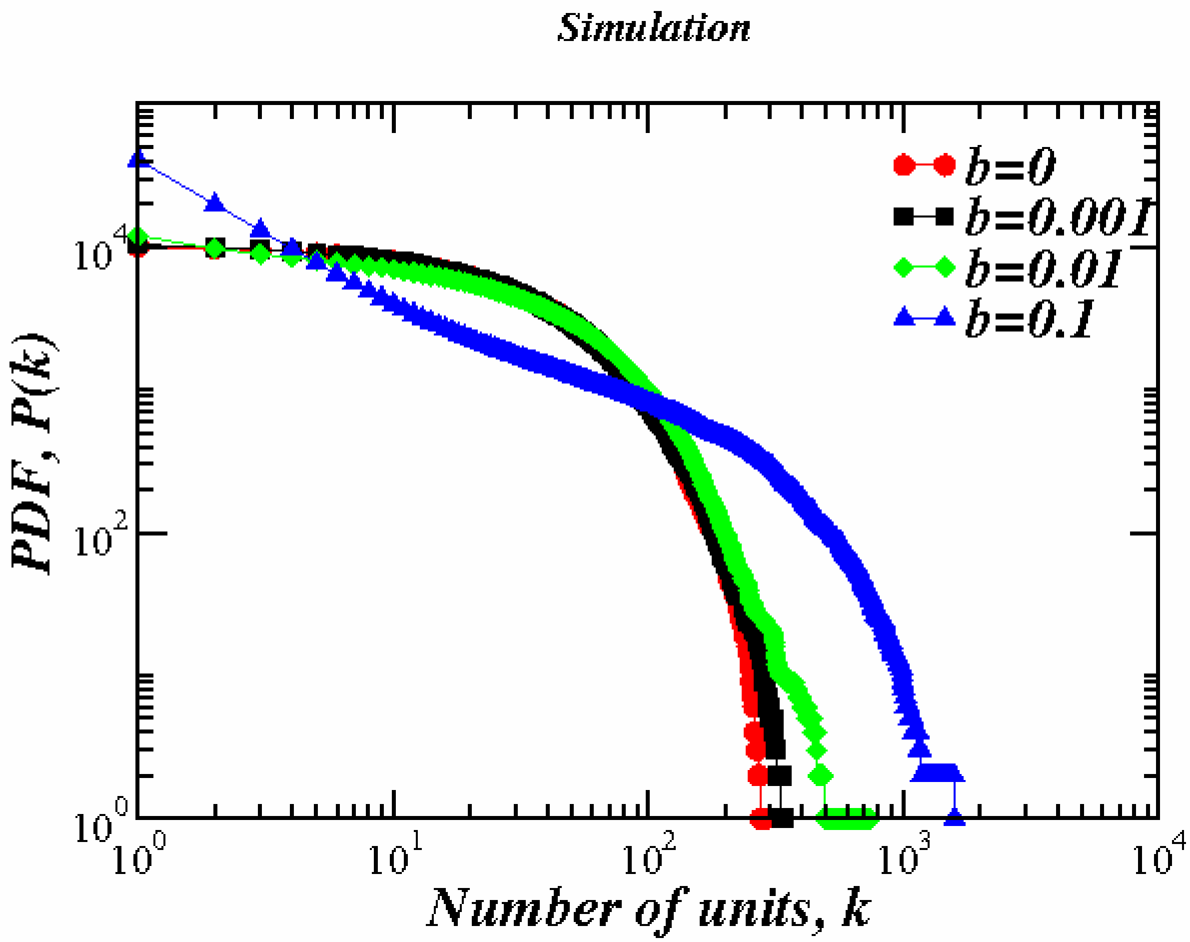
  Distribution of growth rate of class size: Laplace central part with power law wings

A Model

Rules of the Model:

- At time t=0 there exists N classes, each with a single unit.
- At each time step a unit is born and is distributed to an existing class based on “proportional effect” .
- At each time step with probability b
  (0 <b < 1) ``a class with a single unit'' is born.

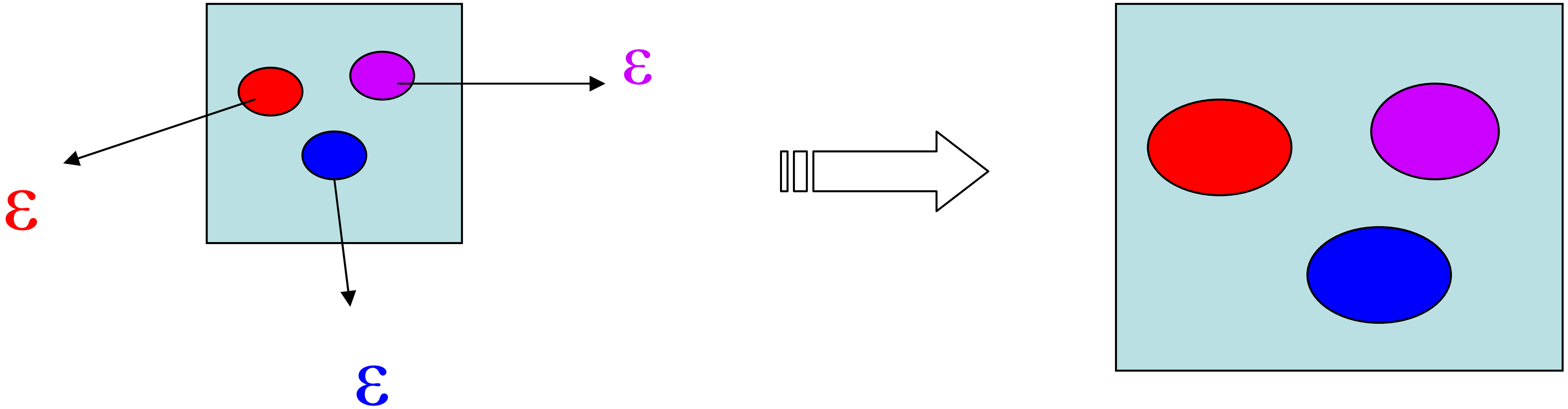
Model: Number of Units in Classes



Fixed no. of class ≡ b=0 => P(k) ~ exp(-k)

Varying no. of class ≡ b≠0 => P(k) ~ power law + exp. cutoff

Growth of Classes



$P(g|3) = \sum^{3}_{i=1} \varepsilon_i$ $\varepsilon_i \sim N(0,\sigma)$

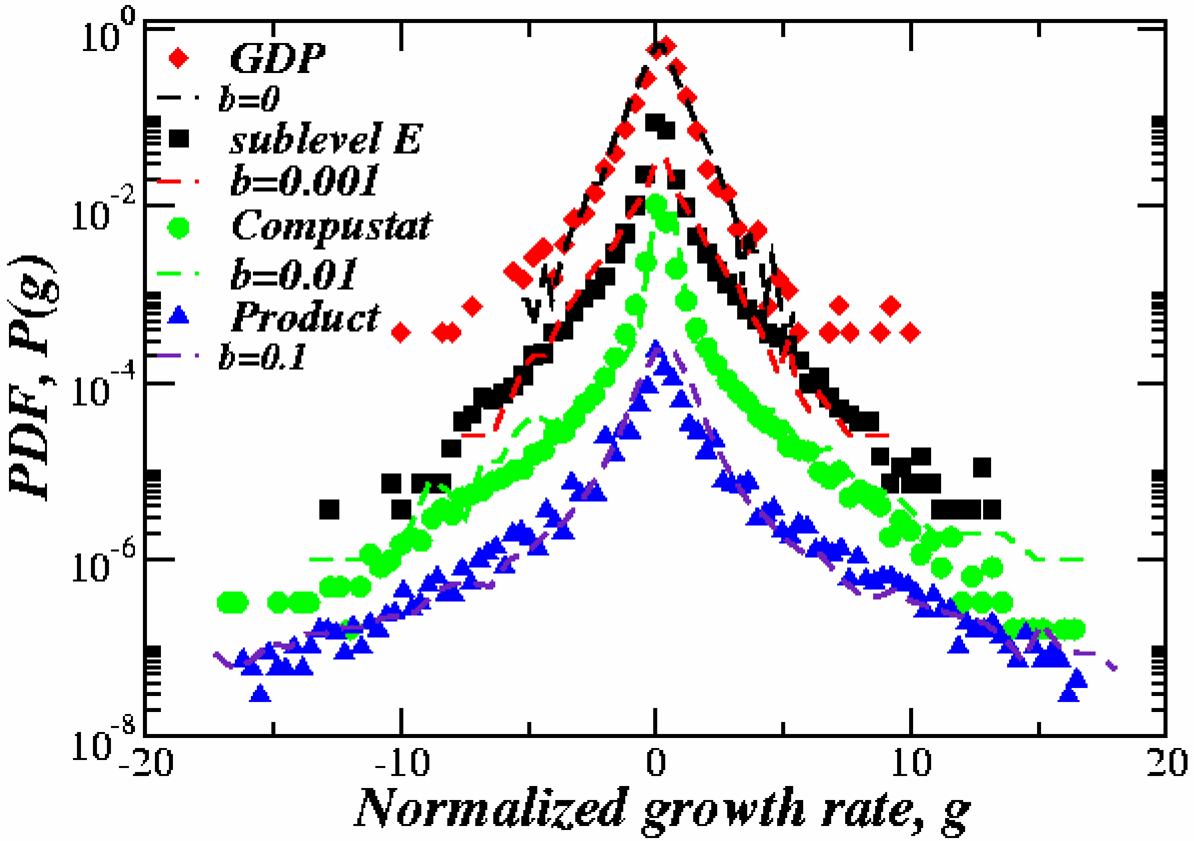
$P(g|k) = \sum^{k}_{i=1} \varepsilon_i$ $P(g) = \sum_k P(k) P(g|k)$

P(k) = exponential : classes fixed

= power law + exp. cutoff : classes varying

Growth Distribution

Simulation + Data



b=0 => fixed no. of classes => P(g) ~ Laplace distributed

b≠ 0 => varying no. of classes => P(g) ~ Laplace central part + power law wings

Conclusion

- Size distribution of Firms:

Log-normal

- Growth rate Distribution of Firms:

Laplace in the central part with power law wings

- Distribution of Number of Units in Firms:

Power law with exponential cutoff

- Proposed model can explain the observed empirical features