APPLICATION OF STATISTICAL PHYSICS APPROACHES TO COMPLEX ORGANIZATIONS

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Two Parts of my Thesis

- Part I: Financial Markets
  - Stock markets
  - Commodity markets
  - Foreign exchange markets

- Part II: Business Firms
  - Pharmaceutical Industry
  - GDP of countries
  - Scientific output of Countries

AIM: characterize fluctuations in the growth.
Why Fluctuations are Interesting and Important

• Interesting because:
  unsolved problem

• Important because:
  (a) Statistical physics may help us to develop better strategies to improve economy
  (b) Quantify risks in a better way

=> better living condition with better economy

Monday October 19, 1987. Dow Jones Industrial Average fell 22.6%, largest one-day decline in recorded stock market history. This one day decline was not confined to the United States, but mirrored all over world. By the end of October, Australia had fallen 41.8%, Canada 22.5%, Hong Kong 45.8%, and the United Kingdom 26.4% : Resulted in starvation of farmers in Vietnam
How to Quantify Fluctuations?

- Probability distribution of Fluctuations
- Correlations present in Fluctuations
Estimating Form of the Probability Distribution is Important

Note on Power Law Probability Distributions:

\[ P(x) \sim x^{-(\alpha+1)} \text{ with } \alpha < 2 \]

are called Levy stable distribution.

Properties of Levy stable distribution:

- Infinite variance
- Stable distribution: if you add up numbers taken from a Levy stable distribution the resulting sum will also follow Levy stable distribution
Part II : Business Firms

Classic Problem of Firm Growth

\[ \text{Firm at } t = 1 \quad S = 5 \rightarrow \text{Firm at } t = 2 \quad S = 12 \rightarrow \text{Firm at } t = 10 \quad S = 33 \]

growth rate

\[ G = \log \frac{S(t+1)}{S(t)} \]

\[ G = \log \frac{12}{5} \]

t / (year)

1 2 10

normalized growth rate \( g = \frac{G}{\sigma} \)
Classic Gibrat Law & Its Implication

Question: What is pdf of growth rate?

Traditional View: Gibrat law of “Proportionate Effect” (1930’s)

\[ S(t+1) = S(t) (1 + \eta_t) \quad (\eta_t \text{ is noise, } -1 < \eta_t < 1). \]

\[ \log S(t = T) = \log S(t = 0) + \sum_{t'=1}^{M} \log(1 + \eta_{t'}) \]

Gibrat: pdf of g is Gaussian.

Growth rate g in T years

\[ = \log \frac{S(T)}{S(0)} \]

\[ = \sum_{t'=1}^{M} \log(1 + \eta_{t'}) \approx \sum_{t'=1}^{M} \eta_{t'} \]
Empirical Observations

Reality: it is “tent-shaped”!

\[pdf(g|S) \approx e^{-\frac{|g|}{\sigma(S)}}\]

Standard deviation of \(g\)

\[\sigma(g|S) \sim S^{-\beta}\]

\[\beta \approx 0.2\]
New: Focus on Products within a Firm

Firm size $S = 5$
$N = 3$
3 products:

Firm size $S = 12$
$N = 4$

Firm size $S = 33$
$N = 7$

| $\xi_1=2$ | $\xi_2=2$ | $\xi_3=1$ |
| $\xi_1=4$ | $\xi_2=1$ | $\xi_3=5$ |
| $\xi_4=2$ |

$t/year$
Database (Units grouped into Classes)

Top Level

2nd Level

3rd Level
Database Studied (1990 – 2000)

- **Countries**
  - GDP -- World Bank
    - US
    - Other Countries
  - Manufacturing (SIC Codes 2000-3999)
  - Company sales -- Compustat
    - Pharmaceutical Industry, PHID (total 76829 products)
      - Sublevel A: 16 Classes
      - Sublevel B: 100 Classes
      - Sublevel C: 308 Classes
      - Sublevel D: 569 Classes
      - Sublevel E: 7184 Classes
      - Corresponds to body organs like lungs, kidney etc.
      - Corresponds to chemicals like Na, Cl etc.
      - Corresponds to chemical compounds like NaCl etc.
      - Corresponds to firms like Merck, Genzyme etc.
Class size is log-normally distributed: $\sim \exp[-[\log(S)]^2]$
Growth rate: $\sim \exp(-|g|)$ central part + power law wings: for levels with number of classes varying
Size Variance Relationship

\[
\sigma(g|S) \sim S^{-\beta}
\]
Distribution of Number of Units in Classes

CASE I: Fixed Number of Classes over time

Sublevel A
Class corresponds to body parts

Sublevel B
Class corresponds to chemical elements
Distribution of Number of Units in Classes

CASE II: Varying Number of Classes over time
Sublevel E: Class corresponds to firms

\[ \tau - 1 = 0.97 \]
Summary

• Number of classes remain fixed in time:
  Distribution of number of units in classes: Exponential
  Distribution of growth rate of class size: Laplace

• Number of classes change in time:
  Distribution of number of units in classes: Power law with exponential cutoff
  Distribution of growth rate of class size: Laplace central part with power law wings
A Model

Rules of the Model:

• At time $t=0$ there exists $N$ classes, each with a single unit.

• At each time step a unit is born and is distributed to an existing class based on “proportional effect”.

• At each time step with probability $b$ ($0 < b < 1$) “a class with a single unit” is born.
Model: Number of Units in Classes

Fixed no. of class $\equiv b=0 \Rightarrow P(k) \sim \exp(-k)$

Varying no. of classes $\equiv b\neq 0 \Rightarrow P(k) \sim$ power law + exp. cutoff
Growth of Classes

\[ P(g|3) = \sum_{i=1}^{3} \epsilon_i \]
\[ \epsilon_i \sim N(0, \sigma) \]

\[ P(g|k) = \sum_{i=1}^{k} \epsilon_i \]
\[ P(g) = \sum_k P(k) P(g|k) \]

\[ P(k) = \text{exponential} : \text{classes fixed} \]
\[ = \text{power law} + \text{exp. cutoff} : \text{classes varying} \]
b=0 ⇒ fixed no. of classes ⇒ \( P(g) \sim \) Laplace distributed

b≠0 ⇒ varying no. of classes ⇒ \( P(g) \sim \) Laplace central part + power law wings
Conclusion

• **Size distribution of Firms:**
  Log-normal

• **Growth rate Distribution of Firms:**
  Laplace in the central part with power law wings

• **Distribution of Number of Units in Firms:**
  Power law with exponential cutoff

• **Proposed model can explain the observed empirical features**