28-1 Line Spectra and the Hydrogen Atom

Figure 28.1 gives some examples of the line spectra emitted by atoms of gas. The atoms are typically excited by applying a high voltage across a glass tube that contains a particular gas. By observing the light through a diffraction grating, the light is separated into a set of wavelengths that characterizes the element.
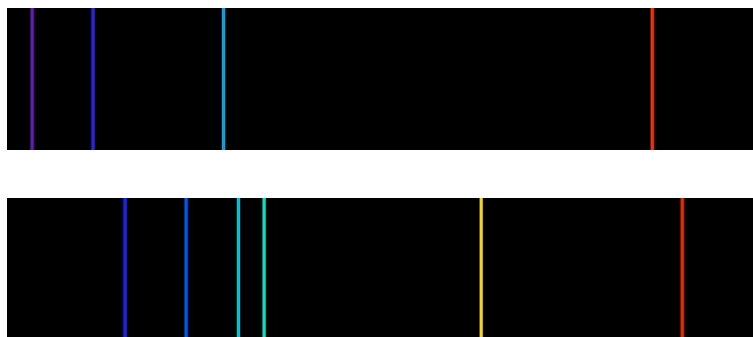

Figure 28.1: Line spectra from hydrogen (top) and helium (bottom). A line spectrum is like the fingerprint of an element. Astronomers, for instance, can determine what a star is made of by carefully examining the spectrum of light emitted by the star.

The spectrum of light emitted by excited hydrogen atoms is shown in Figure 28.1(a). The decoding of the hydrogen spectrum represents one of the great scientific mystery stories. First on the scene was the Swiss mathematician and schoolteacher, Johann Jakob Balmer (1825 – 1898), who published an equation in 1885 giving the wavelengths in the visible spectrum emitted by hydrogen. The Swedish physicist Johannes Rydberg (1854 – 1919) followed up on Balmer's work in 1888 with a more general equation that predicted all the wavelengths of light emitted by hydrogen:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right), \qquad \text{(Eq. 28.1: \textbf{The Rydberg equation for the hydrogen spectrum})}$$

where $R = 1.097 \times 10^7$ m^{-1} is the Rydberg constant, and the two n's are integers, with n_2 greater than n_1.

Neither Balmer nor Rydberg had a physical explanation to justify their equations, however, so the search was on for such a physical explanation. The breakthrough was made by the Danish physicist Niels Bohr (1885 – 1962), who showed that if the angular momentum of the electron in a hydrogen atom was quantized in a particular way (related to Planck's constant, in fact), that the energy levels for an electron within the hydrogen atom were also quantized, with the energies of the electrons being given by:

$$E_n = \frac{-13.6 \text{ eV}}{n^2}, \qquad \text{(Eq. 28.2: \textbf{Energies of the electron levels in the hydrogen atom})}$$

where n is any positive integer.

When the electron in a hydrogen atom drops down from a higher energy state to a lower energy state, a photon is given off that has an energy equal to the difference between the electron energy levels – thus, energy is conserved. Because the differences between the electron energy levels are limited, the photons that are emitted by excited hydrogen atoms are emitted at specific wavelengths, giving the few bright lines shown in Figure 28.1(a).

According to Bohr's model of the atom, the lines in Figure 28.1(a) correspond to photons emitted when electrons drop down to the second-lowest energy level in hydrogen from the levels with $n = 3, 4, 5, 6$ or 7. Bohr predicted, however, that photons should be observed at wavelengths

corresponding to electrons dropping down from an excited state to the ground ($n = 1$) state. We don't see these wavelengths with our eyes because they are in the ultraviolet region of the spectrum. When scientists using detectors sensitive in the ultraviolet region found light emitted by hydrogen at the exact wavelengths predicted by Bohr, it was a tremendous validation of the Bohr model of the hydrogen atom. Bohr received the Nobel Prize in Physics in 1922 for his work.

EXPLORATION 28.1 – Building the energy-level diagram for hydrogen
    Creating a diagram of the energy levels for hydrogen can help explain how the photon energies arise. We will start with the three lowest levels.

Step 1 – *Using Equation 28.2, determine the energies of the three lowest energy levels for hydrogen. Then create an energy-level diagram, which looks like a ladder with rungs that are unequally spaced.* The three lowest energy levels correspond to $n = 1$, 2, and 3. Substituting these values of n into Equation 28.2 gives $E_1 = -13.6$ eV, $E_2 = -3.40$ eV, and $E_3 = -1.51$ eV. The corresponding energy-level diagram is shown in Figure 28.2. Note that all the energy levels for $n > 3$ fall between $E = 0$ and the -1.51 eV of the $n = 3$ level.

Figure 28.2: An energy-level diagram, showing the three lowest energy levels for hydrogen.

Step 2 – *Let's confine ourselves to electrons that make transitions between only the three energy levels shown in Figure 28.2. Mark these transitions on the energy-level diagram with downward-pointing arrows from one level to a lower level. How many different photon energies are associated with these transitions? Determine the energies of these photons.* With three energy levels, we can get three different electron transitions, and thus three different photon energies. The transitions are shown on the energy-level diagram in Figure 28.3. In decreasing order, by photon energy, the photon energies are:

$E_{3\rightarrow 1} = E_3 - E_1 = -1.51 \text{ eV} - (-13.6 \text{ eV}) = 12.1 \text{ eV}$;

$E_{2\rightarrow 1} = E_2 - E_1 = -3.40 \text{ eV} - (-13.6 \text{ eV}) = 10.2 \text{ eV}$;

$E_{3\rightarrow 2} = E_3 - E_2 = -1.51 \text{ eV} - (-3.40 \text{ eV}) = 1.89 \text{ eV}$.

Figure 28.3: The energy-level diagram is modified to show the electron transitions that are possible between the lowest three energy levels in hydrogen. The photons emitted in the two transitions that end at the $n = 1$ level are in the ultraviolet region, while the photon associated with the $n = 3$ to $n = 2$ transition is red.

Key idea: The energy of a photon emitted by an electron that drops down from one energy level to a lower energy level is equal to the difference in energy between those two energy levels.
Related End-of-Chapter Exercises: 1, 2, 4, 13 – 18, 33 – 38.

Note that atoms can also absorb energy, in the form of photons, but they only absorb photons with an energy equal to the difference in energy between two of the atom's electron energy levels. In this case, the electrons make a transition from a lower energy level to a higher level.

Essential Question 28.1: Imagine that there is an atom with electron energy levels at the following energies: –31 eV, –21 eV, –15 eV, and –12 eV. Assuming that electron transitions occur between these levels only, (a) how many different photon energies are possible? (b) what is the (i) minimum and (ii) maximum photon energy?
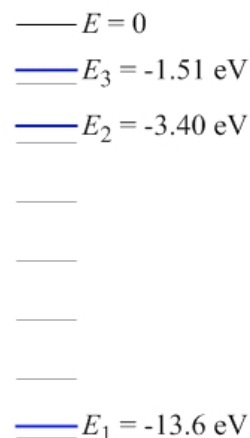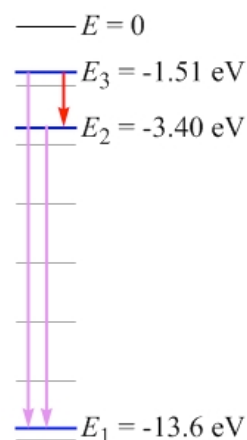
Answer to Essential Question 28.1: (a) In general, four energy levels give six photon energies. One way to count these is to start with the highest level, –12 eV. An electron starting in the –12 eV level can drop to any of the other three levels, giving three different photon energies. An electron starting at the –15 eV level can drop to either of the two lower levels, giving two more photon energies. Finally, an electron can drop from the –21 eV level to the –31 eV level, giving one more photon energy (for a total of six). (b) The minimum photon energy corresponds to the 3 eV difference between the –12 eV level and the –15 eV level. The maximum photon energy corresponds to the 19 eV difference between the –12 eV level and the –31 eV level.

28-2 Models of the Atom

It is amazing to think about how far we have come, in terms of our understanding of the physical world, in the last century or so. A good example of our progress is how much our model of the atom has evolved. Let's spend some time discussing the evolution of atomic models.

Ernest Rutherford probes the plum-pudding model
J. J. Thomson, who discovered the electron in 1897, proposed a plum-pudding model of the atom. In this model, electrons were thought to be embedded in a ball of positive charge, like raisins are embedded in a plum pudding. Ernest Rutherford (1871 - 1937) put this model to the test by designing an experiment that involved firing alpha particles (helium nuclei) at a very thin film of gold. The experiment was carried out in Rutherford's lab by Hans Geiger and Ernest Marsden. If the plum-pudding model was correct, the expectation was that the alpha particles should make it through the ball of spread-out positive charge with very little deflection. For the most part, this was the case; however, a small fraction of the alpha particles were deflected through large angles, with some even being deflected through 180°. Rutherford made a famous statement about this, which was "It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you." Through a careful analysis of the results, Rutherford determined that the positive charge of the atom was not spread out throughout the volume occupied by the atom, but was instead concentrated in a tiny volume, orders of magnitude smaller than that of the atom, which we now call the **nucleus**.

Niels Bohr provides a theoretical framework for Rydberg's equation
The next major advance in our understanding of the atom came from the Danish physicist, Niels Bohr, who incorporating ideas from Rutherford and quantum ideas. In Bohr's model, electrons traveled in circular orbits around a central nucleus, similar to the way planets travel around the Sun. It is important to understand that the Bohr model does not reflect reality, but it provides a basis for our understanding of the atom. With an analysis based on principles of physics we have discussed earlier in this book, such as the attractive force between charged particles, Bohr was able to show that the quantized energy levels in hydrogen were completely consistent with Rydberg's equation (see Equation 28.1 in section 28-1) for the wavelengths of light emitted by hydrogen. In Bohr's model, the angular momenta of the electrons are also quantized, a result we also accept today. Where the Bohr model breaks down is in the electron orbits. In the Bohr model, the electrons are confined to planar orbits of very particular radii. This is not at all the modern view of the atom, which we understand using quantum mechanics.

The modern view of the atom
Over the course of the 20th century, many people, Bohr included, contributed to furthering our understanding of the atom. We will spend some time in Chapter 29 exploring the nucleus, so for the moment let us focus our attention on the electrons in the atom. As far as the electrons are concerned, the nucleus can be thought of as a tiny ball of positive charge.

In the Bohr model of the atom, the electrons are found only in certain orbits, with the radii of the orbits being quantized, so an electron will never be found at other distances from the nucleus. Our modern understanding is rather different. Now, we talk about the probability of finding the electron at a particular distance from the nucleus. For an electron in the **ground state** (the lowest-energy state) of the hydrogen atom, for instance, the Bohr model states that the electron is a distance of 5.29×10^{-11} m from the nucleus (this is known as the **Bohr radius**). The modern view of where the electron in hydrogen's ground state is located is illustrated by Figure 28.4. Even in the modern view, the most likely place to find this ground-state electron is at a distance of one Bohr radius from the nucleus. However, as the graph shows, the electron can be found at any distance from the nucleus, aside from right at the nucleus or infinitely far away.
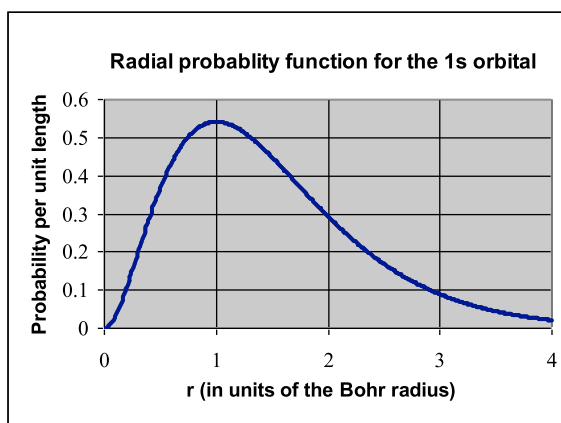

Radial probablity function for the 1s orbital

Figure 28.4: A graph of the probability, per unit length, of finding the electron in the hydrogen ground state at various distances from the nucleus. The total area under the curve, when the curve is extended to infinity, is 1 – the electron is 100% likely to be found between $r = 0$ and $r =$ infinity.

Graphs like that in Figure 28.4 come from solving the Schrödinger equation, which is essentially conservation of energy applied to the atom. Solutions to the Schrödinger equation are called wave functions. The square of a wave function gives the probability of finding a particle in a particular location. The Schrödinger equation is named for the Austrian physicist Erwin Schrödinger (1887 – 1961), who shared the 1933 Nobel Prize in Physics for his contributions to quantum mechanics.

Quantum tunneling
If you throw a tennis ball against a solid wall, the ball will never make it to the far side of the wall unless you give it enough kinetic energy to pass over the top of the wall. Such rules do not apply to quantum particles. If the wave function of a quantum particle extends through a barrier to the far side, then there is some probability of finding the particle on the far side of the barrier. Even if the particle's energy is insufficient to carry it over the barrier, the particle will eventually be found on the far side of the barrier. This process, of passing through a barrier, is known as **quantum tunneling**.

Quantum tunneling is exploited in scanning tunneling microscopes (STM's), in which a very sharp tip is scanned over the surface. By measuring the rate at which electrons tunnel across the gap between this tip and the surface, a two-dimensional picture of the surface can be created. The chapter-opening picture shows an image of a quantum corral. First, a corral of iron atoms was created on a surface by pulling the atoms into place with an STM. The STM was then used to scan the surface, to visualize electrons trapped inside the corral. Note how the electrons are wave-like in this situation, and nor particle-like.

Related End-of-Chapter Exercises: 5 – 7.

Essential Question 28.2: Return to the graph in Figure 28.4. Is the ground-state electron in hydrogen more likely to be found at a position closer than 1 Bohr radius from the nucleus, or farther than 1 Bohr radius from the nucleus? Using the graph, estimate the relative probability of finding the electron in these two ranges.

Answer to Essential Question 28.2: The area under the curve for the region beyond 1 Bohr radius from the nucleus is clearly larger than the area for the region less than 1 Bohr radius away from the nucleus – thus, the electron is more likely to be found farther than 1 Bohr radius away. A reasonable estimate for the ratio of the areas is 2:1. In other words, the electron, when in the ground state of hydrogen, is approximately twice as likely to be found at a radius farther than 1 Bohr radius from the nucleus than it is to be found at a distance less than one Bohr radius.

28-3 The Quantum Mechanical View of the Atom

In the Bohr model of the atom, only one quantum number, *n*, was used. With that single quantum number, Bohr came up with expressions for the energies of quantized energy levels, quantized angular momenta of the electrons, and quantized radii of the electron orbits. Our modern view of the atom, applying the equations of quantum mechanics, is a little more complicated, requiring four quantum numbers to completely specify the various electron energy states. These quantum numbers are:

1. *n*, the principal quantum number. To a first approximation, the energies of the quantized energy levels in an atom with an atomic number *Z* and just one electron are given by:

$$E_n = \frac{(-13.6 \text{ eV}) Z^2}{n^2} ,$$ (Eq. 28.3: **Energies of the electron levels**)

where $n = 1, 2, 3, \ldots$

2. ℓ, the orbital quantum number. This quantum number quantizes the magnitude of the electron's orbital angular momentum (this is somewhat analogous to the angular momentum associated with a planet's orbit around the Sun). The magnitude of the orbital angular momentum is given by:

$$L = \sqrt{\ell(\ell+1)}\ \frac{h}{2\pi} ,$$ (Eq. 28.4: **Orbital angular momentum**)

where $\ell = 0, 1, 2, \ldots n - 1$.

3. m_ℓ, the magnetic quantum number. This quantum number quantizes the direction of the electron's angular momentum (this is known as **space quantization**, and is illustrated in Figure 28.5). Conventionally, we say that this quantum number defines the *z*-component of the angular momentum. For a given value of the orbital quantum number, the different values of m_ℓ give electron states of the same energy unless a magnetic field is present, in which case the states have different energies (this is called the **Zeeman effect**).

$$L_z = \frac{m_\ell\ h}{2\pi} ,$$ where $m_\ell = -\ell, -\ell+1, \ldots \ell-1, \ell.$

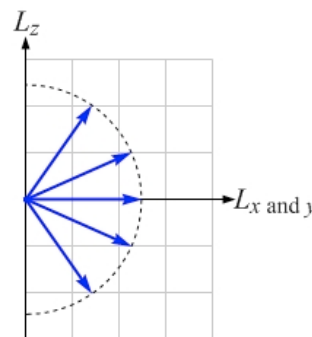(Eq. 28.5: *z*-component of the orbital angular momentum)

Figure 28.5: The allowed directions of the orbital angular momentum, when $\ell = 2$. The half-circle has a radius of $\sqrt{6}$ units. The units on both axes are angular momentum units of $h/(2\pi)$.

4. m_s, the spin quantum number. The concept of electron spin is somewhat analogous to the angular momentum associated with a planet's rotation about its own axis. The electron has two possible spin states, which we refer to as spin up and spin down. The two possible z-components of the spin angular momentum, S_z, are given by:

$$S_z = m_s \frac{h}{2\pi}, \qquad \text{(Eq. 28.6: \textbf{Spin angular momentum})}$$

where $m_s = \pm \frac{1}{2}$.

Visualizing the wave functions

Figure 28.6 shows a common way to visualize the various electron wave functions for hydrogen. Remember that there is only one electron in the hydrogen atom, but the electron has an infinite number of states that it can choose between – some of the lower-energy states are shown below. When we look at pictures like this, we again interpret them in terms of probability. The brighter the picture is at a particular point, the more likely it is to find the electron at that point. Although the pictures are two-dimensional, the wave functions are three-dimensional.

The image in the top left of Figure 28.6 corresponds to the graph in Figure 28.4. At first glance, they appear to contradict one another. For instance, the graph in Figure 28.4 shows that the probability of finding the electron at $r = 0$ is zero, while the top left image in Figure 28.6 has the brightest (highest probability) spot right in the center, where $r = 0$. The probability of finding the electron at any particular point, not just the center point, is zero, because a point has no volume. It makes more sense to discuss the probability of finding the electron within a particular volume. Both the graph in Figure 28.4 and all six diagrams below are best interpreted that way. From that perspective, the diagram at the upper left and the graph in Figure 28.4 are consistent.

Figure 28.6: A visualization of the wave functions (or, atomic orbitals) for the $n = 1, 2,$ and 3 levels for hydrogen. The brighter it is in a particular region, the more likely it is to find the electron in that region. Conversely, there is no chance of finding the electron in a region that is black. The numbers on the right side of the diagram show values of n, the principal quantum number, for each row. The letters across the top represent the values of the orbital quantum number, ℓ, for a particular column. Only one wave function is shown in the first row because, when $n = 1$, the only possible value of the orbital quantum number is $\ell = 0$. Similarly, there are only two images in the second row because there are two allowed ℓ values when $n = 2$. The wave functions are positive in the red areas and negative in the blue areas. In all cases shown, $m_\ell = 0$.
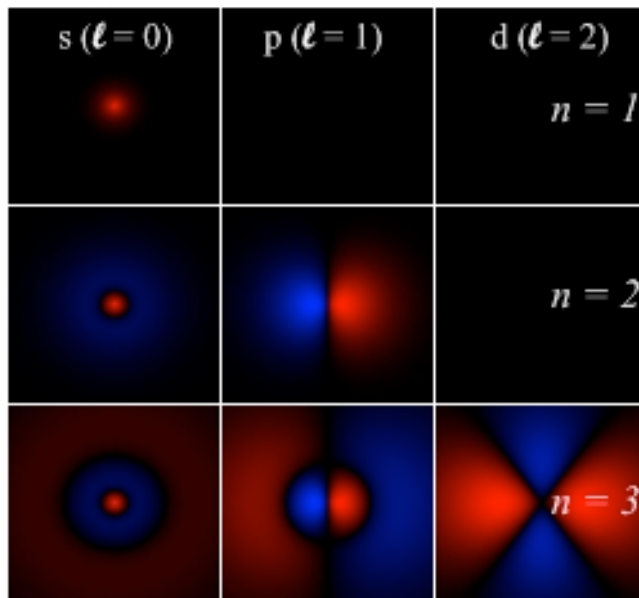
Related End-of-Chapter Exercises: 8, 9, 19 – 23.

Essential Question 28.3: How many different electron states are there that have $n = 3$?

Answer to Essential Question 28.3: If $n = 3$, then the orbital quantum number ℓ can take on 3 possible values, 0, 1, and 2. If $\ell = 0$, $m_\ell = 0$, and there are two possible spin states – thus, there are 2 states with $\ell = 0$. If $\ell = 1$, $m_\ell = -1$, 0, or +1, each with two possible spin states – thus, there are 6 states with $\ell = 1$. If $\ell = 2$, $m_\ell = -2, -1, 0, +1$, or +2, each with two possible spin states – thus, there are 10 states with $\ell = 2$. That is a total of 18 states with $n = 3$.

28-4 The Pauli Exclusion Principle

In Section 28-3, we discussed the fact that it takes four quantum numbers to completely specify the state of an electron. In this section and in Section 28-5, we examine how these four quantum numbers determine the structure of the periodic table. To understand how these four numbers can determine a structure as complex as the periodic table, we begin with the Pauli exclusion principle, which is a simple statement with far-reaching consequences.

The Pauli exclusion principle: No two electrons can simultaneously occupy the same electron state in an atom. In other words, no two electrons in an atom can simultaneously have the same set of four quantum numbers.

The exclusion principle is named for the Austrian physicist Wolfgang Pauli (1900–1958), shown in the picture in Figure 28.7. Pauli was awarded the Nobel Prize in Physics in 1945 for the exclusion principle.
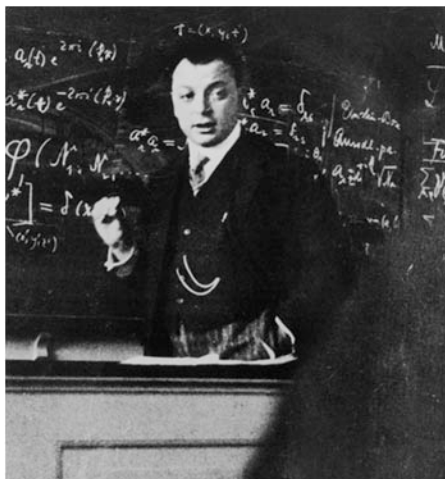
Figure 28.7: A photograph of Wolfgang Pauli, a colorful character who came up with the exclusion principle. Photo credit: Fermilab employees, via Wikimedia Commons (a public-domain image).

With the exclusion principle in mind, let's examine the ground-state (lowest energy) configurations of various atoms. In general, equation 28.3, in which the energy of an electron state is determined solely by the principal quantum number, is inadequate. The energy is also determined by the electron's orbital angular momentum. In general, for a given value of the principal quantum number, n, the higher the value of the orbital quantum number, ℓ, the higher the energy of that state. Let's begin by looking at the ground state configurations of a few elements in the periodic table. These are shown in Table 28.1. An explanation of what the number-letter-number notation means is given in Figure 28.8.

Atomic number	Name (symbol)	Ground-state configuration
2	Helium (He)	$1s^2$
6	Carbon (C)	$1s^2\,2s^2\,2p^2$
10	Neon (Ne)	$1s^2\,2s^2\,2p^6$
15	Phosphorus (P)	$1s^2\,2s^2\,2p^6\,3s^2\,3p^3$
20	Calcium (Ca)	$1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4s^2$
26	Iron (Fe)	$1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4s^2\,3d^4$

Table 28.1: Ground-state configurations for selected atoms.

the number of filled electron states for this orbital

$3p^3$

the value of n, the principal quantum number

a letter representing the value of ℓ, the orbital quantum number (see Table 28.2)

Figure 28.8: Explaining the number-letter-number notation.

Referring to Table 28.1, we see that helium, with only two electrons, can have both electrons in the $n = 1$ shell, which can only fit two electrons. For atoms that have more than 2 electrons, two electrons are in the $n = 1$ level, and then the others are in states that have larger n values. The $n = 2$ level can fit eight electrons (two with $\ell = 0$ and six with $\ell = 1$). Thus, for elements up to and including neon, which has 10 electrons in its ground-state configuration, the electrons are in the $n = 1$ and $n = 2$ levels. Beyond this, electron state with higher n values come into play.

The layout of the ground-state configurations in Table 28.1 needs further explanation. This is provided by Table 28.2, which shows what value of ℓ the various letters correspond to, and by Figure 28.9, which shows how the various orbitals compare to one another, in terms of their energy level. As you wind your way through Figure 28.9 from top to bottom, the energy of a particular orbital increases. This figure shows the order in which electrons fill states in the ground-state configuration, because the ground-state configuration minimizes the total energy of the electrons.

Value of ℓ	Letter (stands for)
0	s (sharp)
1	p (principal)
2	d (diffuse)
3	f (fundamental)
4	g (letter after f)
5	h (letter after g)
6	i (letter after h)

Table 28.2: Letters for various ℓ values.

$1s^2$
$2s^2 \quad 2p^6$
$3s^2 \quad 3p^6 \quad 3d^{10}$
$4s^2 \quad 4p^6 \quad 4d^{10} \quad 4f^{14}$
$5s^2 \quad 5p^6 \quad 5d^{10} \quad 5f^{14} \quad 5g^{18}$
$6s^2 \quad 6p^6 \quad 6d^{10} \quad 6f^{14} \quad 6g^{18} \quad 6h^{22}$
$7s^2 \quad 7p^6 \quad 7d^{10} \quad 7f^{14} \quad 7g^{18} \quad 7h^{22} \quad 7i^{26}$
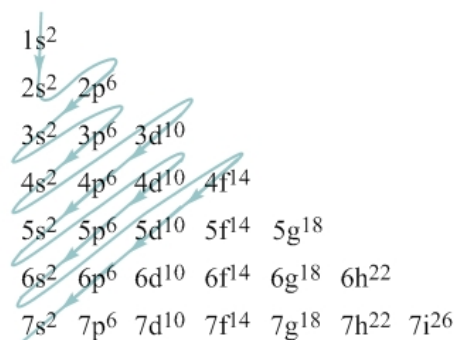
Figure 28.9: The order in which electrons fill the different subshells.

Terminology

An **atomic orbital** is both the mathematical function that describes where an electron can be, and the corresponding region in space. It takes three quantum numbers (n, ℓ, and m_ℓ) to define an orbital. Because there are two possible values of the spin quantum number, each atomic orbital can contain two electrons.

A **subshell** is the set of orbitals with the same values of n and ℓ.

A **shell** is the set of orbitals with the same value of n.

Figure 28.9 is consistent with what is known as the **aufbau principle**, derived from the German word *aufbauprinzip*, which means "building-up principle." The basic rule is that subshells are filled in order of lowest $n + \ell$ values. In cases of equal $n + \ell$ values, the subshells with the lower n value are filled first. The diagonal lines in Figure 28.9 represent subshells that have equal $n + \ell$ values, with those with the lower n values coming first.

Related End-of-Chapter Exercises: 10, 11, 57.

Essential Question 28.4: What is the minimum atomic number an atom should have before $4d^x$, where x represents a positive integer, appears in the atom's ground-state configuration? What is the range of values x can take on?

Answer to Essential Question 28.4: The atom should have an atomic number of at least 39. Following the twists and turns of Figure 28.9, we can count that there are 38 electron states up to and including the 5s subshell. Thus, the 39th electron should go into the 4d subshell. Because the 4d subshell can contain as many as 10 electrons, the x in 4d^x is an integer between 1 and 10.

28-5 Understanding the Periodic Table

In the previous two sections, we have laid the groundwork for understanding the periodic table. An idealized periodic table is laid out in Figure 28.10. Table 28.2 and Figure 28.9 are repeated here from Section 28-4 so you can more easily see the connection between the diagram that goes with the aufbau principle (Figure 28.9) and the periodic table.

Value of ℓ	Letter (stands for)
0	s (sharp)
1	p (principal)
2	d (diffuse)
3	f (fundamental)
4	g (letter after f)
5	h (letter after g)
6	i (letter after h)

Table 28.2: Letters for various ℓ values.

$1s^2$

$2s^2 \quad 2p^6$

$3s^2 \quad 3p^6 \quad 3d^{10}$

$4s^2 \quad 4p^6 \quad 4d^{10} \quad 4f^{14}$

$5s^2 \quad 5p^6 \quad 5d^{10} \quad 5f^{14} \quad 5g^{18}$

$6s^2 \quad 6p^6 \quad 6d^{10} \quad 6f^{14} \quad 6g^{18} \quad 6h^{22}$

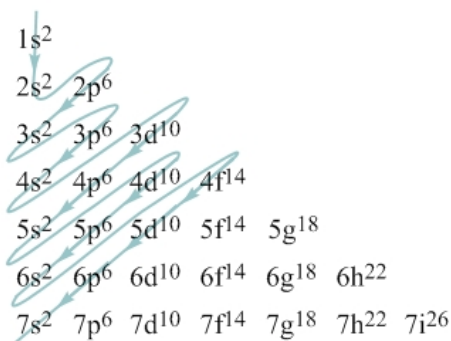$7s^2 \quad 7p^6 \quad 7d^{10} \quad 7f^{14} \quad 7g^{18} \quad 7h^{22} \quad 7i^{26}$

Figure 28.9: The order in which electrons fill the different subshells.

Figure 28.10 shows an idealized periodic table, showing the state of the last few electrons in each element's ground-state configuration. Atoms with completely filled subshells, or half-filled subshells, are particularly stable, and atoms with similar last-electron states generally have similar chemical properties. Note that the true ground-state configurations of elements with atomic numbers 105 and above are not yet known.

In reality, there are some deviations from the idealized behavior shown in Figure 28.10. These deviations can be quite instructive. Some of those deviations include:
- In column 11, the last two terms in the ground-state configurations for copper (Cu), silver (Ag), and gold (Au) are actually 4s^1 3d^{10}, 5s^1 4d^{10}, and 6s^1 5d^{10}, respectively. Each of these configurations ends with a half-full s subshell and a full d subshell, rather than a full s subshell and an almost-full d subshell, demonstrating that full and half-full subshells are particularly stable (lower energy) states.
- In column 6, the last two terms in the ground-state configurations for chromium (Cr) and molybdenum (Mo) are actually 4s^1 3d^5 and 5s^1 4d^5, respectively. Each of these configurations ends with a half-full subshell and a half-full d subshell, rather than a full s subshell and partly-filled d subshell, demonstrating that half-full subshells are particularly stable (lower energy) states.
- The last two terms in the ground-state configurations for gadolinium (Gd, element 64) and curium (Cm, element 96) are actually 4f^7 5d^1 and 5f^7 6d^1, respectively. Each of these configurations has a half-full f subshell and one electron in a d subshell, rather than partly-filled f subshell, again demonstrating that half-full subshells are particularly stable (lower energy) states.

1																	18
1 H $1s^1$	2											13	14	15	16	17	2 He $1s^2$
3 Li $2s^1$	4 Be $2s^2$											5 B $2p^1$	6 C $2p^2$	7 N $2p^3$	8 O $2p^4$	9 F $2p^5$	10 Ne $2p^6$
11 Na $3s^1$	12 Mg $3s^2$	3	4	5	6	7	8	9	10	11	12	13 Al $3p^1$	14 Si $3p^2$	15 P $3p^3$	16 S $3p^4$	17 Cl $3p^5$	18 Ar $3p^6$
19 K $4s^1$	20 Ca $4s^2$	21 Sc $3d^1$	22 Ti $3d^2$	23 V $3d^3$	24 Cr $3d^4$	25 Mn $3d^5$	26 Fe $3d^6$	27 Co $3d^7$	28 Ni $3d^8$	29 Cu $3d^9$	30 Zn $3d^{10}$	31 Ga $4p^1$	32 Ge $4p^2$	33 As $4p^3$	34 Se $4p^4$	35 Br $4p^5$	36 Kr $4p^6$
37 Rb $5s^1$	38 Sr $5s^2$	39 Y $4d^1$	40 Zr $4d^2$	41 Nb $4d^3$	42 Mb $4d^4$	43 Tc $4d^5$	44 Ru $4d^6$	45 Rh $4d^7$	46 Pd $4d^8$	47 Ag $4d^9$	48 Cd $4d^{10}$	49 In $5p^1$	50 Sn $5p^2$	51 Sb $5p^3$	52 Te $5p^4$	53 I $5p^5$	54 Xe $5p^6$
55 Cs $6s^1$	56 Ba $6s^2$	*	72 Hf $5d^2$	73 Ta $5d^3$	74 W $5d^4$	75 Re $5d^5$	76 Os $5d^6$	77 Ir $5d^7$	78 Pt $5d^8$	79 Au $5d^9$	80 Hg $5d^{10}$	81 Tl $6p^1$	82 Pb $6p^2$	83 Bi $6p^3$	84 Po $6p^4$	85 At $6p^5$	86 Rn $6p^6$
87 Fr $7s^1$	88 Ra $7s^2$	**	104 Rf $6d^2$	105 Db $6d^3$	106 Sg $6d^4$	107 Bh $6d^5$	108 Hs $6d^6$	109 Mt $6d^7$	110 Ds $6d^8$	111 Rg $6d^9$	112 Uub $6d^{10}$	113 Uut $7p^1$	114 Uuq $7p^2$	115 Uup $7p^3$	116 Uuh $7p^4$	117 Uus $7p^5$	118 Uuo $7p^6$

* Lanthanides	57 La $4f^1$	58 Ce $4f^2$	59 Pr $4f^3$	60 Nd $4f^4$	61 Pm $4f^5$	62 Sm $4f^6$	63 Eu $4f^7$	64 Gd $4f^8$	65 Tb $4f^9$	66 Dy $4f^{10}$	67 Ho $4f^{11}$	68 Er $4f^{12}$	69 Tm $4f^{13}$	70 Yb $4f^{14}$	71 Lu $5d^1$
** Actinides	89 Ac $5f^1$	90 Th $5f^2$	91 Pa $5f^3$	92 U $5f^4$	93 Np $5f^5$	94 Pu $5f^6$	95 Am $5f^7$	96 Cm $5f^8$	97 Bk $5f^9$	98 Cf $5f^{10}$	99 Es $5f^{11}$	100 Fm $5f^{12}$	101 Md $5f^{13}$	102 No $5f^{14}$	103 Lr $6d^1$

■ Alkali metals	■ Transition metals	■ Non-metals	■ Halogens
■ Alkaline earth metals	■ Poor metals	■ Metalloids	■ Noble gases

Figure 28.10: This figure lays out an idealized version of the periodic table, to show the predicted term in the ground-state configuration for each element. When viewed this way, one gets a better understanding of why the table is laid out as it is. In a given column, for instance, the elements have similar chemical properties because they have similar electron configurations. In the 17th column, for instance, these elements (known as halogens) have similar properties because in their ground-state configurations, each of the elements has 5 electrons in a p orbital, that p orbital being the highest-energy orbital that contains electrons for those elements. For most of the elements, the last term in their ground-state configurations match what is shown here, but some elements differ from the idealized version shown here, as detailed on the previous page. Also, note that helium is generally shown in the 18th column, because it is just as unreactive as the other noble gases, but you could make a strong argument for helium belonging at the top of column 2, based on its ground-state configuration.

Related End-of-Chapter Exercises: 55, 56, 59.

Essential Question 28.5: In Figure 28.10, the final term in the ground-state configuration of the element with the chemical symbol Po (element 84) is shown to be $6p^4$. What is the common name for this element? What is the complete ground-state configuration for this element, showing all the terms?

Answer to Essential Question 28.5: The common name for the element with the chemical symbol Po is polonium (not potassium, which has a chemical symbol of K). To write out the complete ground-state configuration of polonium, we can wind our way through Figure 28.9, starting from the top: $1s^2 \, 2s^2 \, 2p^6 \, 3s^2 \, 3p^6 \, 4s^2 \, 3d^{10} \, 4p^6 \, 5s^2 \, 4d^{10} \, 5p^6 \, 6s^2 \, 4f^{14} \, 5d^{10} \, 6p^4$.

28-6 Some Applications of Quantum Mechanics

Lasers

The word **laser** is an acronym, coming from the phrase light amplification by the stimulated emission of radiation. In our modern world, there are many applications of lasers. Such applications include bar-code readers in stores; surgery, particularly in eye surgery (including LASIK, Laser-Assisted in Situ Keratomileusis), where it is absolutely critical to do precision cutting; laser pointers, laser printers, and CD and DVD players; and fiber-optic communications.

The vast majority of lasers manufactured worldwide are diode lasers, in which light is produced from carefully constructed layers of semiconductors. In this section, we will focus on a different kind of laser, the helium-neon (or HeNe) laser that you may have seen in class or even used in a physics lab experiment. It is quite common to do demonstrations or experiments showing diffraction or interference of light using HeNe lasers.

As the name suggests, a helium-neon laser contains a mixture of helium and neon gas, at a relatively low pressure, with many more helium atoms than neon atoms. Coincidentally, the difference in energy between two of the electron energy levels in helium is almost the same as the difference in energy between two of the electron energy levels in neon – this is why these two elements are used. A high-voltage electrical discharge through the gas will excite helium atoms from their ground state to one of the $n = 2$ states, requiring an energy difference of 20.61 eV. These $n = 2$ states are **metastable**, which means that the electron will not immediately drop down to the ground state – it will remain in the excited state for a while.

With an extra 0.05 eV worth of kinetic energy, the excited helium atoms, when they collide with neon atoms that are in the ground state, can transfer 20.66 eV of energy to the neon atoms, just what is required to boost an electron in neon from the ground state to one of the $n = 3$ states (specifically, the 3s state). This level is also metastable, but electrons in some of these excited states will spontaneously drop down to the 2p state, emitting a photon of 632.8 nm (in air), corresponding to the wavelength of light emitted by a typical red HeNe laser. These photons interact with the excited neon atoms, which encourages them also to make the 3s to 2p transition. This part of the process is the *stimulated emission* that is part of what laser stands for. An energy-level diagram for the HeNe laser is shown in Figure 28.11.

Figure 28.11: An energy-level diagram for the HeNe laser. Because the energy difference between the two lowest energy levels in helium almost exactly equals the energy difference between the $n = 1$ and $n = 3$ levels in neon, excited helium atoms can transfer energy to ground-state neon atoms via collisions. Note that, despite what it looks like in the diagram, the E_1 levels in helium and neon are at completely different energies. What is critical, however, is that the difference between two of the helium levels almost exactly matches the difference between two of the neon levels.
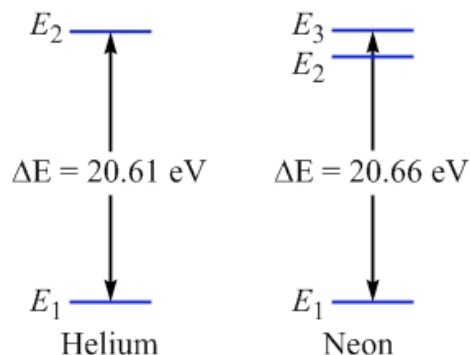
Once the light has been created, by the process of stimulated emission inside the laser, a laser beam must then be created. The photons and the low-pressure mixture of helium and neon are contained in a tube that is typically 15 – 50 cm in length. At one end of the tube is a highly reflective mirror, while at the other end of the tube (the end that the beam emerges from) is a mirror that reflects most of the light, but which allows a little light (about 1%) to pass through. Generally, the photons emerge after bouncing back and forth many times between the mirrors, resulting in a beam of light that has very little spread.

The design of the mirrors inside the laser is also interesting, because the mirrors exploit thin-film interference for a wavelength corresponding to the wavelength of light emitted by the laser. There are several different electron transitions associated with helium and neon, so a HeNe laser can actually emit several different wavelengths (a different wavelength for each transition between electron energy levels). By adjusting the thin films on the mirrors inside the laser, the laser can be optimized for the emission of the standard 632.8 nm red light, or a different wavelength. The first HeNe laser ever made, in the 1960's, for instance, emitted ultraviolet light with a wavelength of 1150 nm.

Fluorescence and phosphorescence
Two more applications of quantum mechanics, again associated with electron energy levels and the photons that are emitted when electrons make a transition from a higher-energy state to a lower-energy state, are fluorescence and phosphorescence. These two phenomena are similar, in that exposing a fluorescent or phosphorescent material to (usually) ultraviolet light will excite electrons from lower-energy states to higher-energy states. When the electrons drop back toward the lower-energy state, however, they do so by dropping down a smaller step in energy, to an intermediate level. If this smaller energy causes photons to be emitted in the visible spectrum, we can then see them. In fluorescent materials, the phenomenon is present only when the light source that excites the upward transitions is present. In phosphorescent materials, it takes a long time, on average, before the excited electrons make a transition to a lower level. Because the electron transitions occur over a long time period, visible light continues to be emitted long after the light source that excites the upward transitions is removed. A photograph of a variety of fluorescent minerals is shown in Figure 28.12.
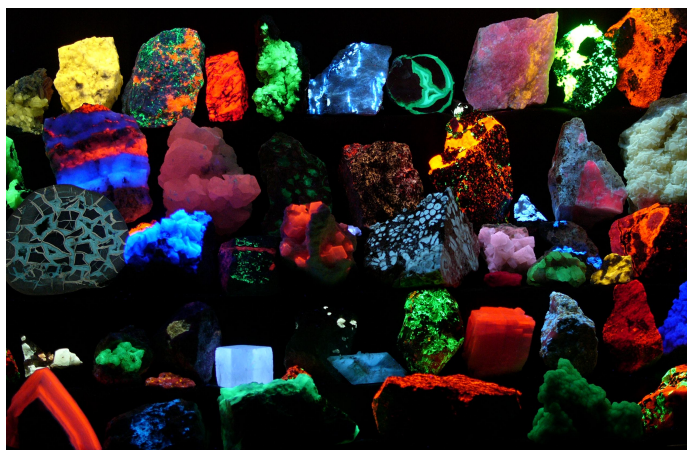

Figure 28.12: A variety of fluorescent minerals, photographed while they are being exposed to ultraviolet light. In general, these minerals are much less colorful when they are not fluorescing. Image credit: Hannes Grobe, via Wikimedia Commons.

Related End-of-Chapter Exercises: 12, 27.

Essential Question 28.6: Consider the information given in Figure 28.11. (a) If the $n = 3$ to $n = 2$ transition in neon produces a photon with a wavelength of 632.9 nm (in vacuum), what is the difference in energy between these two levels? (b) What is the difference in energy between the $n = 2$ and $n = 1$ levels in neon?

Answer to Essential Question 28.6: (a) The photon energy is the difference in energy. Converting the wavelength to energy, we get:

$E = hc/\lambda = (6.626 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})/(632.9 \times 10^{-9} \text{ nm}) = 3.141 \times 10^{-19} \text{ J}$.

Converting this energy to electron volts gives $3.141 \times 10^{-19} \text{ J} \times \dfrac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} = 1.96 \text{ eV}$.

(b) If the $n = 3$ and $n = 1$ levels are separated by 20.66 eV of energy, and the energy difference between the $n = 3$ and $n = 2$ levels is 1.96 eV, the difference between the $n = 2$ and $n = 1$ levels must be 20.66 eV – 1.96 eV = 18.70 eV.

Chapter Summary

Essential Idea: The Atom.
    In our modern view of the atom, the nucleus is modeled as a point particle with a positive charge. The chemical properties of the atom are dominated by the behavior of the atom's electrons. In contrast to the popular representation of an atom being made up of a positive nucleus orbited by electrons in well-defined circular or elliptical orbits, our modern view is that an electron behaves more like a cloud of negative charge, with the density of the charge cloud at a point corresponding to the probability of finding the electron at that point.

Atomic spectra
Exciting a gas causes light to be emitted. The resulting emission spectrum is the fingerprint of the gas – lines are seen at specific wavelengths. A photon produced by an atom comes from an electron that makes a transition from a higher energy level to a lower energy level within the atom. The energy of the photon corresponds to the difference in energy between these electron energy levels.

The four quantum numbers
    One of the early models of the atom, the Bohr model, used a single quantum number to define the quantization of the energy levels, allowed radii, and angular momenta of the electron orbits. Our modern view, the quantum mechanical view of the atom, four quantum numbers are used to completely describe the state of a particular electron. Each quantum number is associated with a physical property of the electron.

    The energy of the electron is quantized by the principal quantum number, n. Allowed values of n are $n = 1, 2, 3, \ldots$ For the hydrogen atom, the energy levels have energies given by:

$$E_n = \frac{-13.6 \text{ eV}}{n^2},$$ (Eq. 28.2: **Energies of the electron levels in the hydrogen atom**)

    The magnitude of the orbital angular momentum of the electron is quantized by the orbital quantum number, ℓ, which can take on values $\ell = 0, 1, \ldots, n-1$. The magnitude of the orbital angular momentum is given by:

$$L = \sqrt{\ell(\ell+1)}\,\frac{h}{2\pi}.$$ (Eq. 28.4: **Orbital angular momentum**)

    The direction of the orbital angular momentum is also quantized. In general, we say that the z-component can take on a limited number of directions, determined by the magnetic quantum

number, m_ℓ, which can take on the values $m_\ell = -\ell, -\ell +1,\ldots, \ell -1, \ell$. The z-component of the orbital angular momentum is given by:

$$L_z = \frac{m_\ell\, h}{2\pi},$$ (Eq. 28.5: ***z*-component of the orbital angular momentum**)

Finally, the spin angular momentum can take on one of only two values, conventionally referred to as "spin up" and "spin down." The spin angular momentum is characterized by the spin quantum number, which can take on values of $+1/2$ or $-1/2$.

Understanding the periodic table of elements
One key to understanding the periodic table is the Pauli exclusion principle – no two electrons in an atom can have the same set of four quantum numbers.

The layout of the periodic table of the elements has to do with the highest-energy filled or partly filled subshell in the ground-state electron configuration of an atom. Elements that have similar ground-state configurations (such as four electrons in a p subshell) are grouped in a column in the periodic table, and generally have similar chemical properties.

Complete ground-state configurations are written with triplets of numbers and letters, in the form $3p^4$. The first number (3, in this case) represents the value of n, the principal quantum number, for the orbital. The letter (p, in this case) stands for the value of the orbital quantum number, ℓ. The second number (4, in this case) is the number of electrons in the subshell in this configuration.

Applications of atomic physics
Two applications of atomic physics include some lasers (such as the helium-neon laser) and the properties of fluorescence and phosphorescence. These applications involve the emission of photons from electrons in atoms dropping down from one energy level to another.

End-of-Chapter Exercises

Exercises 1 – 12 are conceptual questions that are designed to see if you have understood the main concepts of the chapter.

1. In a particular line spectrum, photons are observed that have an energy of 2.5 eV. Choose the phrase below that best completes the following sentence. The element producing the light definitely has …
    (i)… an electron energy level at an energy of 2.5 eV.
    (ii)… an electron energy level at an energy of –2.5 eV.
    (iii)… two electron energy levels that are 2.5 eV apart in energy.

2. Imagine that there is an atom with electron energy levels at the following energies: –88 eV, –78 eV, and –60 eV. Confining ourselves to electron transitions between these levels only, would we expect to see photons emitted from this atom with the following energies? Explain why or why not. (a) 60 eV, (b) –60 eV, (c) 18 eV, (d) 138 eV.

3. To be visible to the human eye, photons must have an energy between about 1.8 eV and 3.1 eV. In a particular atom, one of the electron energy levels is at an energy of –60.0 eV. Can electron transitions associated with this energy level produce photons that are visible to the human eye? Explain why or why not.

number, m_ℓ, which can take on the values $m_\ell = -\ell, -\ell + 1, \ldots, \ell - 1, \ell$. The z-component of the orbital angular momentum is given by:

$$L_z = \frac{m_\ell\, h}{2\pi},$$ (Eq. 28.5: **z-component of the orbital angular momentum**)

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4. In Exploration 28.1, we determined that the energies of the three lowest energy levels in hydrogen have energies of $E_1 = -13.6$ eV, $E_2 = -3.40$ eV, and $E_3 = -1.51$ eV. Photons are produced by electrons that transition from the $n = 3$ level to the $n = 1$ level, as well as from the $n = 2$ level to the $n = 1$ level. Which of these photons have the larger (a) energy? (b) frequency? (c) wavelength?

5. Figure 28.13 shows a graph of the probability, per unit distance, of finding the electron in the *3s* orbital of hydrogen, at various distances from the nucleus. (a) At approximately what distance from the nucleus is the electron most likely to be found? Compare this to the value of 9 Bohr radii that is predicted by the Bohr model for the $n = 3$ state. (b) If we determined the area under the curve for this graph, for the region from $r = 0$ to $r =$ infinity, what value would that area turn out to be?

Radial probablity function for the 3s orbital
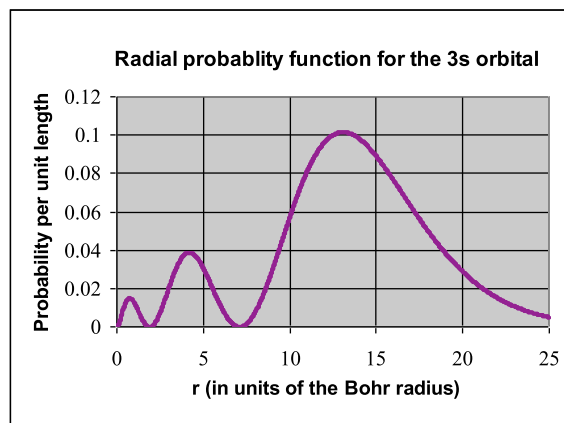
Figure 28.13: A graph of the probability, per unit length, of finding the electron in the *3s* orbital of hydrogen at various distances from the nucleus. For Exercise 5.

6. Figure 28.14 shows the relative probability per unit length as a function of position for a particle confined to a line. The particle is definitely located somewhere between $x = 0$ and $x = d$. (a) At what location(s) between $x = 0$ and $x = d$ is the particle most likely to be found? (b) At what location(s) between $x = 0$ and $x = d$ is the particle least likely to be found?
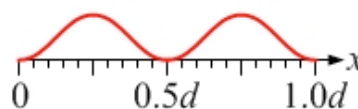

Figure 28.14: A graph of the relative probability per unit length of finding a particle confined to line at a particular position, as a function of position. For Exercise 6.

7. The Bohr model of the atom was an important stage along the path to our understanding of the atom, but it is important to recognize that the Bohr model is quite different from our modern view of the atom. Describe at least two ways in which the Bohr model differs from the modern view.

8. Each of the four quantum numbers is associated with a physical property of the electron. What physical property is associated with (a) the principal quantum number, n? (b) the orbital quantum number, ℓ ? (c) the magnetic quantum number, m_ℓ? (d) the spin quantum number, m_s?

9. If the value of the principal quantum number is $n = 2$, what are the allowed values of (a) ℓ, the orbital quantum number, (b) m_ℓ, the magnetic quantum number, and (c) m_s, the spin quantum number? (d) How many different states are there that have $n = 2$?

10. Explain why electrons tend to fill the $4s^2$ orbital before the $3d^{10}$ orbital.

11. Are the following electron configurations valid or invalid? Note that the configurations do not have to be ground-state configurations. If a configuration is valid, state which element it represents. If a configuration is invalid, explain why. (a) $1s^2 2s^3 2p^6$. (b) $1s^2 2s^1 2p^6 3s^2 3p^3$. (c) $1s^2 2s^2 2p^4 3s^2 3p^3$. (d) $1s^2 2s^2 2p^8 3s^1$.

12. Choose the phrase below that best completes the following sentence. A mixture of helium and neon was chosen to create a laser because …
   (i) … helium and neon are from the same column in the periodic table.
   (ii) … there is an energy level in helium that has almost the same energy as an energy level in neon.
   (iii) … the difference between two energy levels in helium is almost the same as the difference between two energy levels in neon.

Exercises 13 – 18 involve line spectra.

13. Two energy levels in a particular atom are at energies of –23.4 eV and –25.6 eV. When an electron makes a transition from one of these levels to the other, a photon is emitted. For the photon, find the (a) energy, (b) frequency, and (c) wavelength. (d) For the transition described here, which level does the electron transition to?

14. In Exploration 28.1, we calculated the $n = 2$ energy level for hydrogen to have an energy of –3.40 eV. (a) What are the energies of the $n = 4$ and $n = 5$ levels for hydrogen? (b) If the electron in the hydrogen atom makes a transition from the $n = 4$ level to the $n = 2$ level, what is the energy of the emitted photon? (c) Repeat part (b), with the electron transitioning from $n = 5$ to $n = 2$, instead.

15. In Exploration 28.1, we sketched an energy-level diagram with the three lowest energy levels of hydrogen. The lines in the visible region of the spectrum that are emitted by hydrogen correspond to electron transitions that come from a higher level to the $n = 2$ level. Because of this, let's focus on the part of the energy-level diagram between an energy of 0 and the $n = 2$ level. (a) Determine the energies of the $n = 4$, $n = 5$, $n = 6$, and $n = 7$ levels for hydrogen. (b) Sketch an energy-level diagram to show the energy levels for $n = 2$ through $n = 7$ for hydrogen. (c) Determine the energies of the photons emitted when electrons transition from the $n = 3$ through $n = 7$ levels down to the $n = 2$ level. (d) Calculate the wavelengths for these photons.

16. A hypothetical atom has electron energy levels at the following energies: –14 eV, –30 eV, –52 eV, and –80 eV. Assume that electron transitions occur between these levels only. (a) How many different photon energies are possible? (b) List the photon energies, from smallest to largest.

17. Figure 28.15 shows an energy-level diagram for a hypothetical atom. Sketch the corresponding emission spectrum for this atom, showing the energies of the photons associated with electron transitions between the four levels shown in the figure.

18. Figure 28.15 shows an energy-level diagram for a hypothetical atom. Sketch the corresponding emission spectrum for this atom, showing the wavelengths of the photons associated with electron transitions between the four levels shown in the figure.
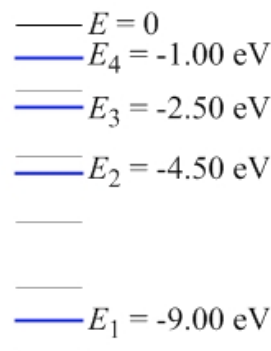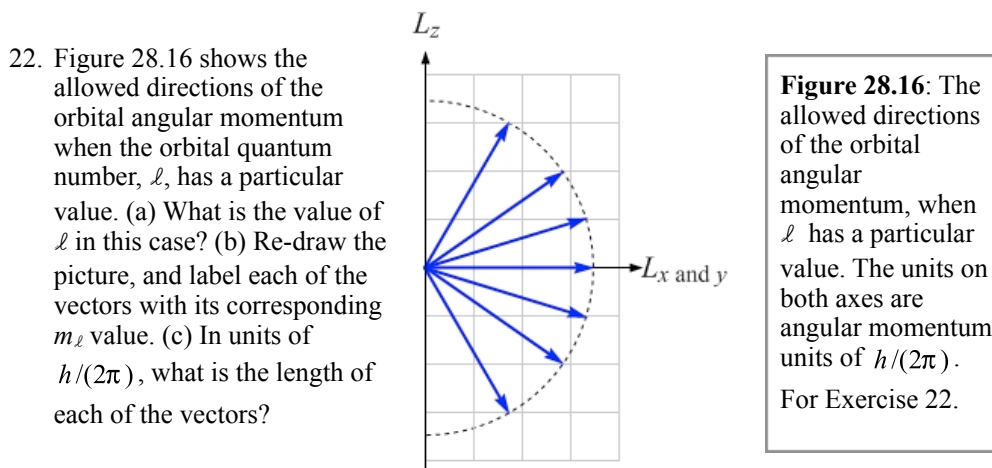
$E = 0$
$E_4 = -1.00$ eV
$E_3 = -2.50$ eV
$E_2 = -4.50$ eV
$E_1 = -9.00$ eV

Figure 28.15: An energy-level diagram for a hypothetical atom. For Exercises 17 and 18.

Exercises 19 – 24 involve the four quantum numbers that are associated with the quantum-mechanical view of the atom.

19. If the value of the orbital quantum number is $\ell = 3$, what are the allowed values of (a) m_ℓ, the magnetic quantum number, and (b) m_s, the spin quantum number? (d) How many different states are there that have $\ell = 3$?

20. If the value of the principal quantum number is $n = 4$, what are the allowed values of (a) ℓ, the orbital quantum number, (b) m_ℓ, the magnetic quantum number, and (c) m_s, the spin quantum number? (d) How many different states are there that have $n = 4$?

21. If the value of the orbital quantum number is $\ell = 4$, what are the allowed values of the principal quantum number, n?

22. Figure 28.16 shows the allowed directions of the orbital angular momentum when the orbital quantum number, ℓ, has a particular value. (a) What is the value of ℓ in this case? (b) Re-draw the picture, and label each of the vectors with its corresponding m_ℓ value. (c) In units of $h/(2\pi)$, what is the length of each of the vectors?

Figure 28.16: The allowed directions of the orbital angular momentum, when ℓ has a particular value. The units on both axes are angular momentum units of $h/(2\pi)$.

For Exercise 22.

23. The value of the magnetic quantum number for a particular electron in the 3d orbital happens to be $m_\ell = +1$. What, if anything, can we say about the values of the other three quantum numbers for this electron?

Exercises 24 – 28 involve applications of atomic physics.

24. Astronomers, particularly astronomers who use radio telescopes, have been able to examine a number of interesting features of the universe through measurements related to what is known as the "21-centimeter line" of hydrogen. The 21-centimeter line refers to photons that have a wavelength of about 21 cm, associated with a transition between two different states of hydrogen. (a) How does the energy of a photon with a wavelength of 21 cm compare to that of a photon in the visible spectrum? (b) Do some research regarding the transition in hydrogen that produces these 21-cm photons, and write a couple of paragraphs explaining the transition, and how the 21-cm line is used in astronomy.

25. One way to view emission spectra is to excite a tube of gas with high voltage, and view the emitted light through a diffraction grating that is held so that the plane of the grating is perpendicular to the direction in which the light travels from the tube to the grating. Recall that we discussed diffraction gratings in Chapter 25. The spectrum of helium gas includes lines at the following wavelengths: 447.1 nm, 501.6 nm, 587.6 nm, and 667.8 nm. When viewed through a diffraction grating in which the openings in the grating are separated by 1500 nm, at what angle (relative to the light coming from the tube) is the first-order line for the (a) 447.1 nm wavelength? (b) 667.8 nm wavelength? (c) How many complete spectra (confining ourselves to the four lines listed here) can be seen when viewing the tube through the diffraction grating?

26. One half of the 1986 Nobel Prize in Physics was awarded to Binning and Rohrer. Do some research about these two scientists, and write a couple of paragraphs regarding what they won the Nobel Prize for, and how it relates to the material covered in this chapter.

27. A neon sign, which is a glass tube filed with excited neon gas, gives off a red-orange glow, as illustrated by the photograph in Figure 28.17. If the sign is a color other than red-orange, the tube is generally not filled with neon. Explain qualitatively what the observation about the color of the neon sign tells us about the energy levels and electron transitions for neon atoms.

28. The second (the unit of time) is defined as the duration of 9 192 631 770 cycles of the radiation emitted by an electron making a transition between two ground-state energy levels of the cesium-133 atom. What is the difference between these two energy levels, in electron volts?

Figure 28.17: A neon sign, for Exercise 27. Photo credit: PhotoDisc, Inc.

General problems and conceptual questions

29. Niels Bohr, for whom the Bohr model is named, had an interesting life. Do some research about him, and write a couple of paragraphs about him. Include some descriptions about Bohr as a person as well as about his contributions to science.

30. The spectrum emitted by excited sodium gas includes two very closely spaced lines in the yellow part of the visual spectrum, at wavelengths of 588.995 nm and 589.592 nm. The photons associated with this light come from electron transitions that start from one of two closely-spaced 3p levels, and which end at the same 3s level. What is the energy difference of these two 3p levels?

31. When an electron in a particular atom transitions from one energy level to another, the atom emits a photon that has a wavelength of 623 nm. One of the energy levels associated with this transition has an energy of –30.28 eV. What is the energy of the other level?

32. Start with the electron in the ground state in the hydrogen atom, and use Equation 28.2 to calculate energy. How much energy is required to (a) ionize the atom (removing the electron completely)? (b) excite the electron from the ground state to the $n = 9$ level?

33. Which requires more energy, to excite the electron in hydrogen from the ground state to the $n = 2$ level, or to excite the electron from the $n = 2$ level to the $n = 100$ level? Explain.

34. A particular atom emits photons with wavelengths of 485 nm and 607 nm. Which wavelength is associated with the larger difference in energy between two energy levels in that atom? Briefly justify your answer.

35. The spectrum of helium gas includes lines at the following wavelengths: 447.1 nm, 501.6 nm, 587.6 nm, and 667.8 nm. Find the corresponding energy for these photons.

36. If you have the energy-level diagram for a particular atom, you can predict the emission spectrum for that atom (as we did in Exercises 17 and 18). If you have the emission spectrum for an atom, can you determine the atom's energy-level diagram? Explain.

37. When a spectral tube containing gas from a particular element is excited by applying high voltage, the spectrum shown in Figure 28.18 is observed. (a) What is the minimum number of energy levels that can be used to obtain this three-line spectrum? (b) Using the minimum number of levels, sketch an energy-level diagram that is consistent with Figure 28.18.

Figure 28.18: The lines in the visible spectrum that are seen when a particular gas is excited with high voltage, for Exercise 37.

38. When a spectral tube containing gas from a particular element is excited by applying high voltage, photons with the following energies are emitted: 3 eV, 13 eV, 16 eV, 25 eV, 38 eV, and 41 eV. No other photons in this range are observed. One of the electron energy levels in the corresponding energy-level diagram is at an energy of –20 eV. (a) What is the minimum number of energy levels that can be used to obtain the given six-line spectrum? (b) Using the minimum number of levels, sketch an energy-level diagram that is consistent with the information given here. (c) Is there only one possible answer to (b)? Explain.

39. Equation 28.3 can be used to find the energies of the energy levels for atoms that have only one electron. Let's use Equation 28.3 to examine the emission spectrum of doubly-ionized lithium, Li^{2+} (that is, lithium atoms with just one electron). (a) The energies of the electron energy levels in hydrogen are equal to the energies of some of the electron energy levels of Li^{2+}. Which n values, for Li^{2+}, give the same energies as those of the $n = 1$, $n = 2$, and $n = 3$ levels in hydrogen? (b) With matching energy levels, the energies of the photons emitted by excited hydrogen could also be produced by excited Li^{2+} atoms. Is it possible to distinguish between spectra from hydrogen and Li^{2+}? If so, how?

40. In a hypothetical atom, the energies of the electron energy levels are given by $E_n = -(144 \text{ eV})/n^2$, where n is a positive integer. (a) What are the energies of the four lowest energy levels in this atom? (b) What are the energies of the photons emitted by electrons that transition between any two of these four levels?

41. In a hypothetical atom, the energies of the electron energy levels are given by $E_n = -(60 \text{ eV})/n$, where n is a positive integer. Consider the lowest six energy levels.

For transitions between the lowest six energy levels only, determine how many different ways there are to produce photons with an energy of (a) 10 eV, and (b) 5 eV. Specify the initial and final values of n for each of these transitions.
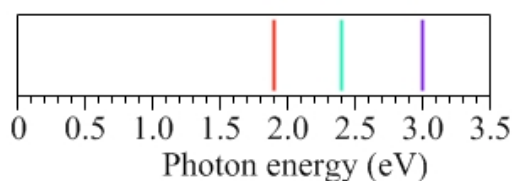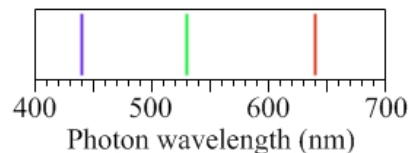
Figure 28.19: The wavelengths of photons in the visible spectrum that are seen when a particular gas is excited with high voltage, for Exercise 42.

42. The wavelengths of visible photons that are emitted by a tube of excited gas are shown in Figure 28.19. (a) Calculate the energies of these photons. (b) If these photons are emitted in electron transitions that end at an energy level of –16.5 eV, what are the energies of the energy levels that the electrons start from to produce these photons?

43. The "Balmer series" is the name given to the set of photon energies (or, equivalently, wavelengths) that correspond to electrons making the transition from higher-n levels to the $n = 2$ level in hydrogen. For the Balmer series, what is the (a) smallest photon energy? (b) largest photon energy?

44. The "Lyman series" is the name given to the set of photon energies (or, equivalently, wavelengths) that correspond to electrons making the transition from higher-n levels to a specific lower level in hydrogen. (a) What is the value of n that characterizes the lower level for the Lyman series? (b) Who was Lyman? Do some research about Lyman and write a paragraph describing this scientist.

45. Return to the situation described in Exercise 44. For the Lyman series, what is the (a) largest photon wavelength? (b) smallest photon wavelength?

46. One of the wavelengths emitted by excited hydrogen gas in a tube that is at rest on the Earth is found to be 656.28 nm. When you carefully measure the corresponding wavelength emitted from the hydrogen in a distant star, however, you find the wavelength to be 658.73 nm. Having learned about the Doppler effect for electromagnetic waves, in Chapter 22, you realize that, with this data, you can determine the velocity of the distant star with respect to the Earth (at least, you can find the component of the velocity that is along the line joining the Earth and the star). Find the value of this velocity.

47. The Hubble Space Telescope is named after the American astronomer Edwin Hubble. Do some research about Edwin Hubble and write a couple of paragraphs about what he is most famous for, and how his work relates to the principles of physics discussed in this chapter. Note that an image of part of the Eagle Nebula, as seen by the Hubble Space Telescope, is shown in Figure 28.20.

Figure 28.20: An image of a vast cloud of gas and dust that is part of the Eagle Nebula, as imaged by the Hubble Space Telescope, for Exercise 47. Image credit: Jeff Hester and Paul Scowen (Arizona State University), and NASA/ESA.

48. We discussed the photoelectric effect experiment in Chapter 27. One way the photoelectric effect experiment is often carried out is to use a mercury light source to illuminate a particular surface. The emission spectrum of mercury has strong lines at wavelengths of 365.5 nm (ultraviolet), 404.7 nm (violet), 435.8 nm (blue), and 546.074 nm (green). The light can be passed through a diffraction grating to separate the light into these different wavelengths. Assume that photons of all these wavelengths have enough energy to overcome the work function of the surface. (a) Which of these wavelengths will produce electrons with the largest kinetic energy? (b) If the maximum kinetic energy of the emitted electrons is 1.27 eV when the green light illuminates the surface, what will the maximum kinetic energy of the emitted electrons be when the violet light illuminates the surface? (c) What is the work function of this surface?

49. Figure 28.21 shows a graph of the probability, per unit distance, of finding the electron in the *2s* orbital of hydrogen, at various distances from the nucleus. (a) At approximately what distance from the nucleus is the electron most likely to be found? Compare this to the value of 4 Bohr radii that is predicted by the Bohr model for the $n = 2$ state. (b) Use the graph to estimate the probability of finding the electron at a distance of less than 2 Bohr radii from the nucleus.
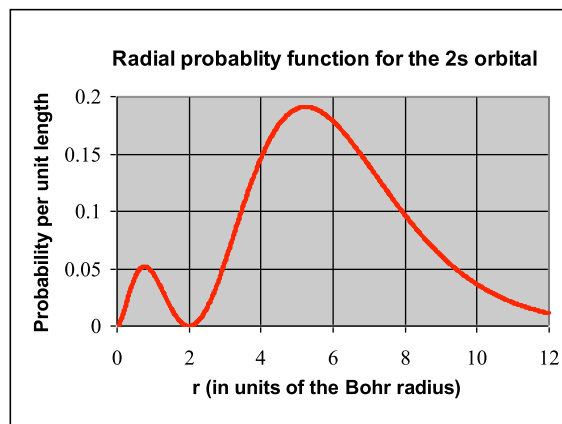

Radial probablity function for the 2s orbital

50. For the graph in Figure 28.21, the area under the curve for the region from $r = 0$ to $r = 5.8$ Bohr radii is equal to 0.5. (a) What is the area under the curve from the region from $r = 5.8$ Bohr radii to $r =$ infinity? (b) What does this mean, in terms of the probability of finding the electron at particular distances from the nucleus?

Figure 28.21: A graph of the probability, per unit length, of finding the electron in the *2s* orbital of hydrogen at various distances from the nucleus. For Exercises 49 and 50.

51. A particular electron in an atom is in a state such that its orbital quantum number is $\ell = 3$. Express your answers in terms of h (Planck's constant) and π. (a) What is the magnitude of the electron's orbital angular momentum? (b) What are the possible values of the *z*-component of the electron's orbital angular momentum?

52. A particular electron in an atom is in a state such that the *z*-component of its orbital angular momentum is $+h/2\pi$. What, if anything, does this tell us about the value of (a) the electron's orbital quantum number, ℓ ? (b) the electron's principal quantum number, *n*?

53. In Figures 28.5 and 28.16, we sketched vector diagrams to show the allowed directions of the orbital angular momentum vector for two specific values of ℓ. (a) Draw a similar diagram that applies when $\ell = 1$, labeling each vector with the corresponding value of m_ℓ. (b) What is the magnitude of each of the vectors on the diagram?

54. In the absence of an external magnetic field, states with the same values of n and ℓ have the same energy. For instance, with no external magnetic field, electrons in the 3p orbital in a particular atom all have the same energy. If such electrons drop from the 3p level to the 2s level, we would see a single line in the emission spectrum, corresponding to photons with an energy equal to the energy difference between the 3p and 2s levels. When an external magnetic field is applied, however, electrons within an orbital will have slightly different energies, experiencing a shift in energy from the zero-field value that is proportional to the strength of the magnetic field multiplied by the electron's quantum number. This splitting of the energy levels is known as the **Zeeman effect**. (a) For the example mentioned here, electrons transitioning from the 3p to 2s levels, explain why we only have to worry about the splitting of energy levels for the 3p states, not for the 2s states. (b) Sketch, qualitatively, an energy-level diagram showing the 3p and 2s energy levels in both the absence and presence of an external magnetic field. (c) What affect would applying an external magnetic field have on the line in the emission spectrum associated with the 3p to 2s transition?

55. Use the chart in Figure 28.9 to help you answer this question. Write out the complete ground-state configuration for the element with an atomic number of (a) 16. (b) 34. (c) Briefly explain why these elements are found in the same column in the periodic table.

56. A short-hand method for writing out the ground-state configuration of an element is to start from the configuration of a column-18 element, and then add whatever is missing. For instance, the ground-state configuration of zirconium (Zr) can be written as [Kr] $5s^2 4d^2$, which means that the initial terms match those of krypton (Kr), and the additional terms are $5s^2 4d^2$. (a) Write out the complete ground-state configuration for zirconium. (b) Why might a column-18 element be used as the starting point for the short-hand method, as opposed to, say, a column-16 element? Using this short-hand method, write out the ground-state configuration of (c) Nickel (Ni), and (d) Lead (Pb).

57. The ground-state configuration of a particular element is $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^5$. (a) What is the atomic number of this element? (b) What is the name of this element?

58. Reading up on the properties and applications of different elements can be quite fascinating. Do some research about the element antimony, and write up a couple of paragraphs to tell the story of antimony.

59. Elements in column 1 of the periodic table generally make strong bonds with elements of column 17. An example of this is NaCl, table salt. To form the bond, the column-1 element essentially gives up one of its electrons to the column-17 element. Explain why this generally leads to a stable structure.

60. Two students are having a conversation. Comment on each of the statements that they make below.

Julia: I'm trying to figure out how to draw the energy-level diagram from this spectrum we were given. I converted the six wavelengths to electron volts – is the energy-level diagram just that set of six energies?

Kristin: No – those are all the photon energies. You have to think about where the photons come from – how do the photons relate to the energy levels?

Julia: OK, I remember – a photon gets released when an electron changes levels, and the photon energy equals the difference between the electron levels. So, like, this 2.5 eV photon could come from an electron dropping from a 12.5 eV level to a 10 eV level.

Kristin: That sounds right. Except, how do you know those are the right energies? You could also get 2.5 eV by dropping from 10 eV to 7.5 eV.

Julia: You're right – we could get 2.5 eV an infinite number of ways. Maybe we need to look at the other photons to narrow down the possibilities. Look, I see a pattern here for these three photons, we have 2.5 eV and 4.1 eV, and they add up to 6.6 eV, and we also have a 6.6 eV photon. So, we could have energy levels at 16.6, 12.5, and 10, and that gives us all those photons.

Kristin: Don't we need 14.1 eV, too, to get the 4.1 eV photon?

Julia: No, we've got that already, because the electron can go 16.6 to 12.5 – that's 4.1. OK, so, for those three photons we had three energy levels – if we can do three more energy levels for the other photons we'll be all set. Those other photon energies are 13.4, 17.5, and 20.0. Huh, if you add 13.4 and 17.5 you don't get 20 – how do you do those ones?

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Chapter 28: Additional Resources

Pre-session Movies on YouTube

- [The de Broglie Wavelength](#)

Examples

- [Sample Questions](#)

Solutions

- [Answers to Selected End of Chapter Problems](#)
- [Sample Question Solutions](#)

Additional Links

- [*PhET* simulation: Rutherford Scattering](#)
- [*PhET* simulation: Models of the Hydrogen Atom](#)

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