27-1 Planck Solves the Ultraviolet Catastrophe

By the end of the 19th century, most physicists were confident that the world was well understood. Aside from a few nagging questions, everything seemed to be explainable in terms of basic physics such as Newton's laws of motion and Maxwell's equations regarding electricity, magnetism, and light. This confidence was soon to be shaken, however.

One of the nagging questions at the time concerned the spectrum of radiation emitted by a so-called ***black body***. A perfect black body is an object that absorbs all radiation that is incident on it. Perfect absorbers are also perfect emitters of radiation, in the sense that heating the black body to a particular temperature causes the black body to emit radiation with a spectrum that is characteristic of that temperature. Examples of black bodies include the Sun and other stars, light-bulb filaments, and the element in a toaster. The colors of these objects correspond to the temperature of the object. Examples of the spectra emitted by objects at particular temperatures are shown in Figure 27.1.

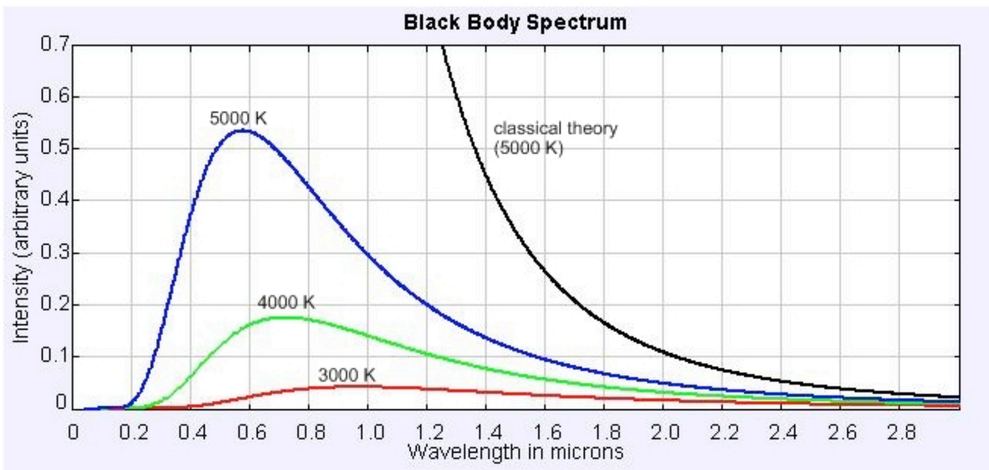

Figure 27.1: The spectra of electromagnetic radiation emitted by hot objects. Each spectrum corresponds to a particular temperature. The black curve represents the predicted spectrum of a 5000 K black body, according to the classical theory of black bodies.

At the end of the 19th century, the puzzle regarding blackbody radiation was that the theory regarding how hot objects radiate energy predicted that an infinite amount of energy is emitted at small wavelengths, which clearly makes no sense from the perspective of energy conservation. Because small wavelengths correspond to the ultraviolet end of the spectrum, this puzzle was known as the ***ultraviolet catastrophe***. Figure 27.1 shows the issue, comparing the theoretical predictions to the actual spectrum for an object at a temperature of 5000 K. There is clearly a substantial disagreement between the curves.

The German physicist Max Planck (1858 – 1947) was able to solve the ultraviolet catastrophe through what, at least at first, he saw as a mathematical trick. This trick, which marked the birth of quantum physics, also led to Planck being awarded the Nobel Prize for Physics in 1918. Planck determined that if the vibrating atoms and molecules were not allowed to take on any energy, but instead were confined to a set of equally-spaced energy levels, the predicted spectra matched the experimentally determined spectra extremely well. Planck determined that, for an atom oscillating with a frequency f, the allowed energy levels were integer multiples of the base energy unit hf, where Planck's constant h has the value 6.626×10^{-34} J s.

$E = nhf$, (Equation 27.1: **Allowed energy levels for an oscillator in a blackbody**)
where n is an integer.

Thus was born the idea of quantization, as applied to energy. If a quantity is quantized, it can take on only certain allowed values. Charge, as we discussed in chapter 18, is an example of something that is quantized, coming in integer multiples of the electronic charge e. Money is an example of an everyday item that is quantized, with quantities of money coming in integer units of a base unit, such as the penny in the United States and Canada.

Let us turn now to a second physical phenomenon that was puzzling scientists at the end of the 19^{th} century. This phenomenon is called the ***photoelectric effect***, and it describes the emission of electrons from metal surfaces when light shines on the metal. The photoelectric effect, or similar effects, have a number of practical applications, including the conversion of sunlight into electricity in solar panels, as well as the image-sensing systems in digital cameras.

Let's put the photoelectric effect experiment into context. First, recall that, beginning in 1801 with Thomas Young's double-slit experiment, physicists carried out a whole sequence of experiments that could be explained in terms of light acting as a wave. All these interference and diffraction experiments showed that light was a wave, and this view was supported theoretically by the prediction of the existence of electromagnetic waves, via Maxwell's equations. Then, in 1897, J.J. Thomson demonstrated that electrons exist and are sub-atomic particles. The stage was set for an explanation of the photoelectric effect in terms of light acting as a wave.

Predictions of the wave model of light regarding the photoelectric effect

The explanation for how light, as a wave, might interact with electrons in a metal to knock them out of the metal is fairly straightforward, based on the absorption of energy from the electromagnetic wave by the metal. Note that all metals have what is known as a ***work function***, which is the minimum energy required to liberate an electron from the metal. Essentially, the work function represents the binding energy for the most weakly bound electrons in the metal.

Remember that the intensity of an electromagnetic wave is defined as the wave's power per unit area. Predictions based on the wave model of light include:

- Light (that is, electromagnetic waves) of any intensity should cause electrons to be emitted. If the intensity is low, it will just take longer for the metal to absorb enough energy to free an electron.
- The frequency of the electromagnetic waves should not really matter. The key factor governing electron emission should be the intensity of the light.
- Increasing intensity means more energy per unit time is incident on a given area, and thus we might expect both more electrons to be emitted and that the emitted electrons would have more kinetic energy.

Amazingly, despite a century of success in explaining many experiments, the predictions of the wave model of light are completely at odds with experimental observations Again, as we will discover in Section 27-2, it took the intellect of Albert Einstein to explain what was going on.

Related End-of-Chapter Exercises: 1, 2, 36, 37.

Essential Question 27.1: In Figure 27.1, we can see that the intensity of light emitted by an object at 5000 K has a maximum at a wavelength of about 0.6 microns (600 nm). (a) What frequency does this correspond to? (b) What is difference between energy levels at this frequency?

Answer to Essential Question 27.1: (a) Assuming the wavelength is measured in vacuum, we can use the wave equation $f = c / \lambda$ to find that the frequency corresponding to a wavelength of 600 nm is $f = c / \lambda = (3.00 \times 10^8 \text{ m/s}) / (6 \times 10^{-7} \text{ m}) = 5 \times 10^{14}$ Hz. (b) Applying equation 27.1, with $n = 1$, we find that the difference between energy levels is extremely small, being $E = hf = (6.626 \times 10^{-34} \text{ J s}) \times (5 \times 10^{14} \text{ Hz}) = 3 \times 10^{-19}$ J.

27-2 Einstein Explains the Photoelectric Effect

On the previous page, we introduced the photoelectric effect (the emission of electrons from a metal caused by light shining on the metal), and discussed how the wave theory of light led to predictions about the experiment that simply did not fit the experimental observations. It was at this point, in 1905, that Albert Einstein stepped in. First, Einstein built on Planck's explanation of the spectrum of a black body. Planck had theorized that oscillators (such as atoms) in a black body could only take on certain energies, with the energy levels separated by an energy hf, where f is the oscillation frequency. Einstein went on to propose that when such an oscillator dropped from one energy level to the next lowest level, losing an energy hf, the missing energy was given off as light, but given off as a packet of energy. Such packets of energy now go by the name ***photon***. In some sense, then, a photon is like a particle of light, with an energy given by

$$E = hf \ , \qquad \text{(Equation 27.2: \textbf{Energy of a photon})}$$

where f is the frequency of the electromagnetic wave corresponding to the photon. Note how the wave and particle properties of light are brought together in this equation – the energy of a particle of light depends on the light's frequency.

In applying the photon concept to the photoelectric effect, Einstein modeled the process not as a wave interacting with a metal, but as interactions between single photons and single electrons. If the light incident on the metal has a frequency f, then the beam of light can be thought of as being made up of a stream of photons, each with an energy of hf. If each photon is absorbed by a single electron, giving up its energy to the electron, electrons are emitted from the metal as long as the energy an electron acquires from a photon exceeds the metal's work function, W_0, which represents the minimum binding energy between the electrons and the metal.

Predictions of the particle model of light regarding the photoelectric effect

For the predictions of the wave model, refer to the previous page. The particle model makes quite different predictions for the photoelectric effect than does the wave model, including:

- The frequency corresponding to $hf = W_0$ is a critical frequency (known as the *threshold frequency*) for the experiment. Below this frequency, the energy of the photons is not enough for the electrons to overcome the work function. No electrons are emitted when the frequency of the incident light is less than the threshold frequency. Electrons are emitted only when the frequency of the light exceeds the threshold frequency.
- Treating the process as a single photon – single electron interaction leads to a straightforward equation governing the process that is based on energy conservation. When the frequency of the light exceeds the threshold frequency, part of the photon energy goes into overcoming the binding energy between the electron and the metal, with the energy that remains being carried away by the electron as its kinetic energy. Thus,

$$K_{\max} = hf - W_0 . \qquad \text{(Equation 27.3: \textbf{The photoelectric effect})}$$

- Increasing the intensity of the incident light without changing its frequency means that more photons are incident per unit time on a given area. If the light frequency is below the threshold frequency, no electrons are emitted no matter what the intensity is. If the frequency exceeds the threshold frequency, increasing the intensity causes more electrons to be emitted, but the maximum kinetic energy of the emitted electrons does not change.

It took several years for these predictions to be verified; however, by 1915 experiments showed clearly that Einstein was correct, confirming that light has a particle nature. For his explanation of the photoelectric effect, Einstein was awarded the Nobel Prize in Physics in 1921.

EXPLORATION 27.2 – The photoelectric effect experiment

Let's look at one method for carrying out the photoelectric effect experiment. This will involve a review of some of the concepts from Chapter 17, such as electric potential and the workings of a capacitor.

Step 1 – ***A diagram of the experimental apparatus is shown in Figure 27.2. Light of a frequency higher than the work function shines on a metal plate (plate 1), causing electrons to be emitted. These electrons are collected by a second plate (plate 2), and the electrons travel back through a wire from plate 2 to plate 1. An ammeter measures the current in the wire, while an adjustable battery, with a voltage set initially to zero, is also part of the circuit. Explain, using conservation of energy and concepts from Chapter 17, how adjusting the battery voltage enables us to measure the maximum kinetic energy of the emitted electrons. Neglect gravity.***

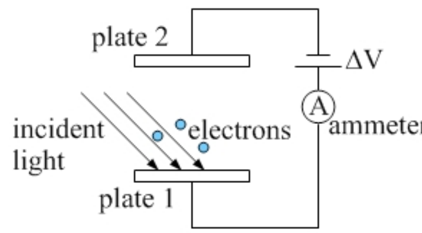

Figure 27.2: A diagram of the experimental apparatus for carrying out the photoelectric effect experiment.

If the battery is connected so plate 2 is negative and plate 1 is positive, electrons emitted from plate 1 do not reach plate 2 unless the kinetic energy they have when they leave plate 1 exceeds the change in electric potential energy, $e\,\Delta V$, associated with electrons crossing the gap from plate 1 to plate 2. The electrons that do not make it across the gap return to plate 1. Thus, as the battery voltage increases from zero, fewer electrons cross the gap and the ammeter reading drops. The smallest battery voltage ΔV_{min} needed to bring the current (and the ammeter reading) to zero is directly related to the maximum kinetic energy of the emitted electrons. By energy conservation, the potential energy of the electrons just below plate 2 is equal to the kinetic energy they have at plate 1 (defining plate 1 as the zero for potential energy). In equation form, we have $K_{max} = e\,\Delta V_{min}$.

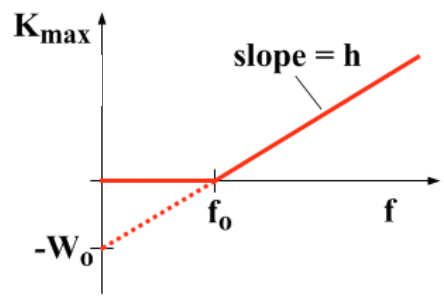

Figure 27.3: A graph of the maximum kinetic energy of electrons emitted when light shines on a metal, as a function of the frequency of the light. Below the threshold frequency, no electrons are emitted. Above the threshold frequency, the maximum kinetic energy of the emitted electrons increases linearly with frequency.

Step 2 – ***Sketch a graph showing the maximum kinetic energy of the electrons as a function of the frequency of the incident light, both above and below the threshold frequency f_0.*** This graph, which matches Equation 27.3, is shown in Figure 27.3.

Key idea: To explain the photoelectric effect experiment, we treat light as being made up of particles called photons. **Related End-of-Chapter Exercises: 4, 5, 42, 46.**

Essential Question 27.2: Plot a graph like that in Figure 27.3, but for a different metal that has a larger threshold frequency. Comment on the similarities and differences between the two graphs.

Answer to Essential Question 27.2: The new graph, corresponding to a larger threshold frequency, is shown as the rightmost graph in Figure 27.4. For frequencies above the threshold frequency, the slopes are identical because the slope of this section of the graph is Planck's constant. Also, the new line has a *y*-intercept that is more negative, consistent with the larger work function in that case.

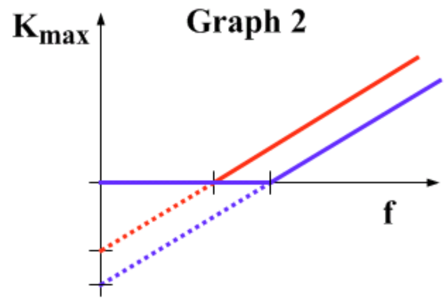

Figure 27.4: Two graphs showing the maximum kinetic energy for electrons emitted by light incident on two different metals. For the rightmost graph, the threshold frequency is larger, as is the work function of the metal, compared to that for the original graph.

27-3 A Photoelectric Effect Example

In this section, we will do an example of a photoelectric effect problem. Let us begin, though, by discussing the electron volt (eV), which is a unit of energy that is often used in settings involving the photoelectric effect.

The electron volt

How much kinetic energy does an electron gain, or lose, when it is accelerated through a potential difference of 1 volt? The electron's change in kinetic energy is equal in magnitude, and opposite in sign, to the change in electric potential energy it experiences. Thus, the electron's change in kinetic energy is $\Delta K = -(-e)\Delta V = e\Delta V$. If we used the electronic charge in coulombs ($e = 1.60 \times 10^{-19}$ C) and kept the potential difference in volts, we would obtain an energy in joules, but it would be a very small number.

As an alternative, we can define a different energy unit, the electron volt, such that an electron accelerated through a potential difference of 1 volt experiences a change in kinetic energy that has a magnitude of 1 electron volt (eV). If the potential difference is 500 volts, the electron's change in kinetic energy has a magnitude of 500 eV, etc. The conversion factor between joules and electron volts is

$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$.   (Eq. 27.4: **Conversion factor between joules and electron volts**)

We will make use of the electron volt in the following example. It is also helpful to express Planck's constant in eV s, which can be done as follows:

$$h = \frac{6.626 \times 10^{-34} \text{ J s}}{1.602 \times 10^{-19} \text{ J/eV}} = 4.136 \times 10^{-15} \text{ eV s} .$$ (Eq. 27.5: **Planck's constant in eV s**)

EXAMPLE 27.3 – Solving problems involving the photoelectric effect

Using the experimental apparatus shown in Figure 27.5, when ultraviolet light with a wavelength of 240 nm shines on a particular metal plate, electrons are emitted from plate 1, crossing the gap to plate 2 and causing a current to flow through the wire connecting the two plates. The battery voltage is gradually increased until the current in the ammeter drops to zero, at which point the battery voltage is 1.40 V.

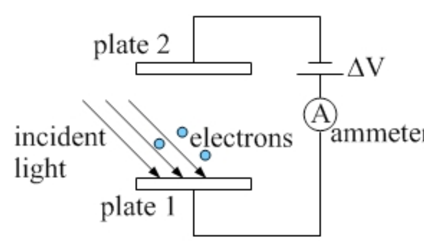

Figure 27.5: A diagram of the experimental apparatus for carrying out the photoelectric effect experiment.

(a) What is the energy of the photons in the beam of light, in eV?
(b) What is the maximum kinetic energy of the emitted electrons, in eV?
(c) What is the work function of the metal, in eV?
(d) What is the longest wavelength that would cause electrons to be emitted, for this particular metal?
(e) Is this wavelength in the visible spectrum? If not, in what part of the spectrum is this light found?

SOLUTION

(a) Assuming that the wavelength corresponds to the wavelength in vacuum, we can first convert the wavelength to the frequency using

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{2.40 \times 10^{-7} \text{ m}} = 1.25 \times 10^{15} \text{ Hz}.$$

Now, we can use Equation 26.2 to find the photon energy.

$$E = hf = (4.136 \times 10^{-15} \text{ eV s}) \times (1.25 \times 10^{15} \text{ Hz}) = 5.17 \text{ eV}.$$

(b) As we discussed in Exploration 27.2, the maximum kinetic energy of the emitted electrons is related to the minimum voltage across the two plates needed to stop the electrons from reaching the second plate (this is known as the **stopping potential**). In this case, the stopping potential is 1.40 V, so the maximum kinetic energy of the electrons is 1.40 eV.

(c) Now that we know the photon energy and the maximum kinetic energy of the electrons, we can use Equation 27.3 to find the work function of the metal.

$$W_0 = hf - K_{\text{max}} = 5.17 \text{ eV} - 1.40 \text{ eV} = 3.77 \text{ eV}.$$

(d) The maximum wavelength that would cause electrons to be emitted corresponds to the threshold frequency for this situation. Let's first determine the threshold frequency, f_0.

$$W_0 = hf_0, \quad \text{so} \quad f_0 = \frac{W_0}{h} = \frac{3.77 \text{ eV}}{4.136 \times 10^{-15} \text{ eV s}} = 9.12 \times 10^{14} \text{ Hz}.$$

Converting the threshold frequency to wavelength, assuming the light is traveling in vacuum, gives

$$\lambda_{\text{max}} = \frac{c}{f_0} = \frac{3.00 \times 10^8 \text{ m/s}}{9.12 \times 10^{14} \text{ Hz}} = 3.29 \times 10^{-7} \text{ m}.$$

(e) This wavelength is 329 nm, less than the 400 nm (violet) wavelength that marks the lower bound of the visible spectrum. This light is beyond violet, in the ultraviolet.

Related End-of-Chapter Exercises: 13 – 16, 43, 44.

Essential Question 27.3: With a particular metal plate, shining a beam of red light on the metal causes electrons to be emitted. (a) If we replace the red light by blue light, do we know that electrons will be emitted? (b) If the two beams have the same intensity and are incident on equal areas of the plate, do we get the same number of electrons emitted per second in the two cases? Assume that in both cases the probability that a photon will cause an electron to be emitted is the same in both cases (e.g., for every two photons incident on the plate, one electron is emitted).

Answer to Essential Question 27.3: (a) Blue light has a higher frequency than red light, and thus the photons in the blue light have a higher energy than the photons in the red light. We know that the photons in the red light have an energy larger than the metal's work function, because electrons are emitted, so the photons in the blue light have more than enough energy to cause electrons to be emitted, too. (b) We actually get fewer electrons emitted with the blue light. The energy per photon is larger for the blue light, so to achieve the same intensity there are fewer photons per second per unit area incident on the plate with the blue light. Fewer photons produce fewer electrons, although the electrons are emitted with, on average, more kinetic energy.

27-4 Photons Carry Momentum

In Chapter 6, we defined an object's momentum as the product of the object's mass and its velocity. Photons have no mass, so we might expect them to also have no momentum. However, we have already discussed the fact that light does carry momentum – recall the discussion of radiation pressure and solar sailboats in Chapter 22. Thus, photons do have momentum, but we need an equation that gives momentum for massless particles, keeping in mind that the units must be the familiar units of momentum that we have worked with throughout the book.

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}.$$ (Equation 27.6: **Momentum of a photon**)

One of the key pieces of evidence supporting the photon model of light is an experiment involving light interacting with matter. When light of a particular frequency is incident on matter, the light can change both direction and frequency. The shift in frequency cannot be explained in terms of the wave model of light, but the particle (photon) model provides quite a straightforward explanation. The phenomenon is known as the Compton effect after its discoverer, the American physicist A. H. Compton (1892 – 1962), for which he won the 1927 Nobel Prize in Physics.

The explanation of the Compton effect is very similar to that of the photoelectric effect. In both cases, a single photon interacts with a single electron. With the Compton effect, the analysis is similar to our analysis of a two-dimensional collision in Section 7.6, except that, in this situation, one of the objects (the electron) is initially at rest. As always in a collision situation, momentum is conserved. In a collision like this, involving a photon and an electron, there is nothing to transfer energy out of the system, so energy is also conserved. A diagram of the collision is shown in Figure 27.6.

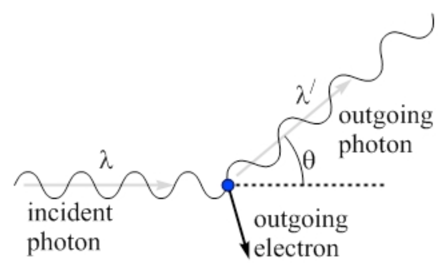

Figure 27.6: In the Compton effect, an incident photon of wavelength λ collides with an electron (the gray sphere). After the collision, the wavelength of the photon is $\lambda^{/}$.

The photon is incident with a wavelength λ, and the photon transfers some of its energy to the electron. Thus, after the collision, the photon has lower energy, which means its frequency is lower while its wavelength $\lambda^{/}$ is higher. The photon's direction changes by an angle θ. Applying energy conservation to this situation gives one equation, while applying momentum conservation in two dimensions gives us two more equations. Combining these equations is somewhat involved, but it leads to a simple relationship relating the change in wavelength experienced by the photon to the angle through which the photon has been scattered.

$$\Delta\lambda = \lambda^{/} - \lambda = \frac{h}{m_e c}(1-\cos\theta),$$ (Equation 27.7: **The Compton Effect**)

where m_e is the mass of the electron, and the quantity $h/(m_e c) = 2.43 \times 10^{-12}$ m is known as the Compton wavelength. Because $\cos\theta$ varies from +1 to –1, the quantity $(1-\cos\theta)$ varies from 0 to 2. Thus, the shift in the photon's wavelength from the Compton effect varies from 0 (for a scattering angle of 0, which is essentially no collision) to two Compton wavelengths, for a scattering angle of 180°.

EXAMPLE 27.4 – Working with the Compton effect
A photon with a wavelength of 4.80×10^{-11} m collides with an electron that is initially stationary. As shown in Figure 27.7, the photon emerges traveling in a direction that is at 120° to the direction of the incident photon.

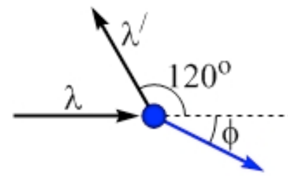

Figure 27.7: The geometry of the Compton effect situation described in Example 27.4.

(a) What is the wavelength of the photon after the collision with the electron?
(b) What is the magnitude of the momentum of the incident photon? What is the magnitude of the momentum of the outgoing photon?
(c) What is the magnitude of the electron's momentum after the collision?

SOLUTION
(a) To find the wavelength of the outgoing photon, we apply Equation 27.7. This gives:

$$\lambda^{/} = \lambda + \frac{h}{m_e c}(1-\cos\theta) = (4.80\times10^{-11}\text{ m}) + (2.43\times10^{-12}\text{ m})\times(1+0.5) = 5.16\times10^{-11}\text{ m}.$$

(b) To find a photon's momentum, we can apply Equation 27.6, $p = h/\lambda$.

For the incident photon, $p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\text{ J s}}{4.80\times10^{-11}\text{ m}} = 1.380\times10^{-23}\text{ kg m/s}$.

For the outgoing photon, $p' = \frac{h}{\lambda'} = \frac{6.626\times10^{-34}\text{ J s}}{5.16\times10^{-11}\text{ m}} = 1.284\times10^{-23}\text{ kg m/s}$

(c) Momentum is conserved in the collision, so we can apply momentum conservation to find the x and y components of the electron's momentum.

x-direction: $p = p'_x + p_{e,x}$ so

$$p_{e,x} = p - p'_x = (1.380\times10^{-23}\text{ kg m/s}) - (1.284\times10^{-23}\text{ kg m/s})\cos(120°) = 2.022\times10^{-23}\text{ kg m/s}.$$

y-direction: $0 = p'_y - p_{e,y}$ so

$$p_{e,y} = p'_y = (1.284\times10^{-23}\text{ kg m/s})\sin(120°) = 1.112\times10^{-23}\text{ kg m/s}.$$

Applying the Pythagorean theorem, we can find the magnitude of the electron's momentum.

$$p_e = \sqrt{(p_{e,x})^2 + (p_{e,y})^2} = \sqrt{(2.022\times10^{-23}\text{ kg m/s})^2 + (1.112\times10^{-23}\text{ kg m/s})^2} = 2.308\times10^{-23}\text{ kg m/s}$$

Related End-of-Chapter Exercises: 6, 7, 17 – 20, 49 – 53.

Essential Question 27.4: Return to Example 27.4. (a) What is the electron's speed after the collision? (b) If the direction of the electron's velocity after the collision makes an angle φ with the direction of the incident photon, what is φ?

Answer to Essential Question 27.4: (a) To find the electron's speed, we can divide the magnitude of the electron's momentum by the electron's mass.

$$v_e = \frac{p_e}{m} = \frac{2.308\times10^{-23} \text{ kg m/s}}{9.11\times10^{-31} \text{ kg}} = 2.53\times10^{7} \text{ m/s} .$$

This speed is less than 10% of the speed of light in vacuum, so we are safe applying the non-relativistic equations that we used in Example 27.4.

(b) We can find the direction of the electron's momentum from the components of the momentum.

$$\tan\phi = \frac{p_{e,y}}{p_{e,x}} = \frac{1.112\times10^{-23} \text{ kg m/s}}{2.022\times10^{-23} \text{ kg m/s}} \qquad \text{which gives } \phi = 28.8°.$$

27-5 Particles Act Like Waves

As we have learned in the previous two sections, light, which in many instances acts like a wave, also exhibits a particle nature. This was a rather surprising result, but the surprises kept coming. Recall that equation 27.6 relates the momentum of a photon to the wavelength of the light, $p = h/\lambda$. In 1923, Louis de Broglie (1892 – 1987) proposed turning the equation around and applying it to objects that we normally think of as particles.

The de Broglie wavelength: de Broglie proposed that Equation 27.6 could be applied to objects we think of as particles, such as electrons and neutrons, in the form:

$$\lambda = \frac{h}{p} = \frac{h}{mv} . \qquad \text{(Equation 27.8: \textbf{the de Broglie wavelength})}$$

In other words, de Broglie proposed that everything that moves has an associated wavelength. When de Broglie's idea was verified, de Broglie was awarded the Nobel Prize in Physics in 1929 for the idea.

For objects that we are used to dealing with in our daily lives, such as balls and cars and people, their de Broglie wavelength is so small that there is, effectively, no wave behavior. For instance, when you pass through a door you simply go through as a particle, rather than diffracting into the next room. For a person traveling at about 1 m/s, for instance, the de Broglie wavelength is about 1×10^{-35} m. This wavelength is so many orders of magnitude smaller than the objects and openings that we encounter everyday that our particle nature dominates.

In contrast, for tiny objects that we generally think of as particles, such as electrons and protons, their tiny mass produces a much larger wavelength. An electron with a kinetic energy of 10 eV, for instance, has a de Broglie wavelength of 3.9×10^{-10} m. That sounds small, but that wavelength is comparable in size to objects that an electron encounters. For instance, the spacing between atoms in a solid object is similar to the de Broglie wavelength of a 10 eV electron, and thus a crystal, with its regular array of atoms, can act as a diffraction grating for electrons. A sample diffraction pattern obtained by diffracting electrons from a crystal is shown in Figure 27.8. Similar patterns are obtained from diffraction with light.

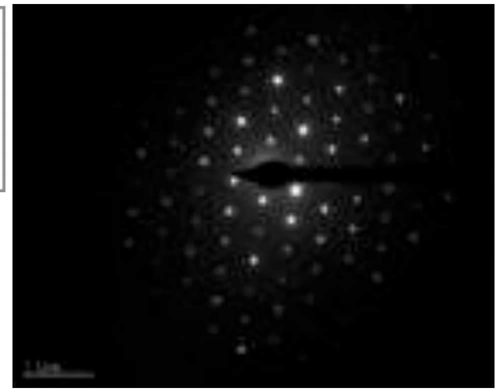

Figure 27.8: An electron diffraction image obtained from electrons incident on a sample of anthopyllite asbestos. From this pattern, the crystal structure, including the spacing between the atoms, can be deduced. Image credit: California Department of Public Health.

Experimental evidence for the de Broglie wavelength

In addition to the electron diffraction images that we just discussed, another persuasive piece of evidence supporting the idea of the de Broglie wavelength is the interference pattern obtained when electrons are incident on a double slit. Light of a particular wavelength produces a particular interference pattern when incident on a double slit. Replacing the light with electrons, which have a de Broglie wavelength equal to that of the wavelength of the light, results in the same interference pattern.

One application of the wave nature of electrons is in microscopy. One factor that limits the resolution of a light microscope is the wavelength of the light used. The same rules apply to an electron microscope, but the de Broglie wavelength of the electrons in an electron microscope can be 1000 times less than that of visible light. Such an electron microscope can resolve features 1000 times smaller than those resolved in a light microscope. An example of what an electron microscope can do is shown in Figure 27.9.

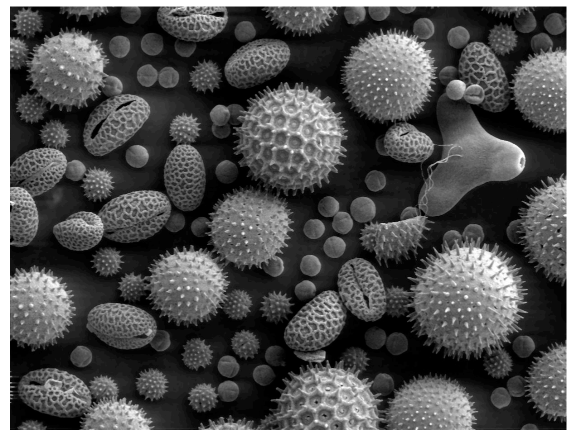

Figure 27.9: An electron microscope image showing various types of pollen. Image credit: Dartmouth Electron Microscope Facility.

Experiments with other particles, including protons, neutrons, and hydrogen and helium atoms, have also been carried out, all of which have verified that such objects exhibit both a particle nature and a wave nature, with a wavelength given by the de Broglie equation.

Wave-particle duality

The fact that everything, including ourselves, exhibits both a particle nature and a wave nature is known as wave-particle duality. Typically, to explain the result of a particular experiment, we use either the wave model or the particle model. However, recent experiments have shown interesting mixes of both. When electrons are incident on a double slit, for example, an interference pattern is produced on a screen beyond the slits – this shows the wave nature of electrons. If you add a detector that tells you which slit the electron passed through in every case, however, the pattern changes to that expected for particles. If your detector is faulty, however, and only tells you 30% of the time which slit an electron passed through, the resulting pattern is a mix of that expected from the wave nature (contributing 70% to the pattern, in this case) and that expected from the particle nature (the other 30%). This experiment also demonstrates how the act of making an observation can change the results of an experiment.

Related End-of-Chapter Exercises: 10, 21 – 25, 57.

Essential Question 27.5: Protons have a mass approximately 1800 times larger than the mass of the electron. If electrons traveling at a speed v produce a particular interference pattern when they encounter a particular double slit, with what speed would the protons have to travel to produce exactly the same interference pattern using the same double slit?

Answer to Essential Question 27.5: The interference pattern produced depends on the de Broglie wavelength of the particles incident. To produce the same pattern as the electrons, the protons must have the same wavelength. Examining Equation 27.8, we see that wavelength is determined by momentum, so the protons need to have the same momentum as the electrons. To have the same momentum with a factor of 1800 in the mass, the protons must have a speed of $v/1800$.

27-6 Heisenberg's Uncertainty Principle

The interplay between the uncertainty in an object's position, Δx, and the uncertainty in its momentum, Δp, was first quantified by the German physicist Werner Heisenberg (1901 – 1976). Heisenberg won the Nobel Prize in Physics in 1932 for his contributions to quantum theory. Heisenberg's uncertainty principle states that you cannot know something to infinite accuracy. More specifically, the uncertainty principle states that for two linked quantities, such as the position and momentum of an object, the more accurately you know one of those quantities, the less accurately you can know the other.

$$\Delta x \, \Delta p \geq \frac{h}{4\pi} = 5.273 \times 10^{-35} \text{ J s}.$$ (Equation 27.9: **Heisenberg's uncertainty principle**)

The uncertainty principle also applies to other linked quantities, such as energy and time, or to two components of angular momentum.

For any object that we are used to dealing with on a daily basis, Heisenberg's uncertainty principle is essentially irrelevant. For instance, if you could measure the position of a 1 kg water bottle, which you see as being at rest on a table, with an uncertainty of 1 nm, Heisenberg's uncertainty principle tells us that the uncertainty in the water bottle's momentum must be at least 5.273×10^{-26} kg m/s. With its mass being 1 kg, the water bottle's velocity must therefore be uncertain by at least 5.273×10^{-26} m/s, a number so small that it is essentially meaningless.

For tiny objects like electrons, however, the limitations associated with Heisenberg's uncertainty principle are quite important. For the electron bound to the nucleus of a hydrogen atom, for instance, making a reasonable assumption about the uncertainty of the electron's momentum leads to an uncertainty in position that is similar in size to the atom itself. Thus, for the hydrogen atom, we can say that the electron is in the atom, but we cannot say exactly where in the atom it is at any point in time. We will investigate this idea further in Chapter 28.

Applying the Uncertainty Principle to the single-slit experiment

As we discussed earlier in this chapter, electrons interact with single and double slits in much the same way that light does. In both cases, the resulting diffraction pattern or interference pattern can be understood in terms of the wave nature of the electrons or of the light. However, applying the uncertainty principle can also give us some insight into the experiment.

Imagine a beam of electrons traveling toward a wide slit. We have a good idea of the speed and direction of the electrons in the beam – in other words, the momentum of the electrons is well known. The slit is so wide that, while we know that the electrons pass through the slit, we do not have much information about their position. With a large uncertainty in position, according to the Uncertainty Principle, the uncertainty in momentum can be small. Thus, the momentum of each electron is essentially unchanged from what it was before encountering the slit, and the electrons pass through the slit in a straight line.

The narrower we make the slit, however, the more knowledge we have about the position of an electron when it passes through the slit. By the Uncertainty Principle, the more accurately we know an object's position, the less accurately we can know its momentum. In the experiment, this uncertainty in position manifests itself as a larger spread in the beam after the beam passes through the slit. As we discussed with light in Chapter 25, the smaller the width of the slit, the more spread there is in the beam. This concept is shown in Figure 27.10.

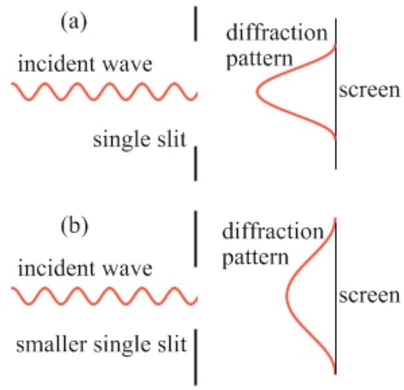

Figure 27.10: The single slit that an electron beam passes through in (a) is larger than that in (b), but the spread in the beam is smaller in (a). This is a consequence of the uncertainty principle.

EXAMPLE 27.6 – Investigating the Uncertainty Principle

Table 27.1 shows the mass, momentum, uncertainty in position, and uncertainty in momentum (assuming we are minimizing this uncertainty, according to the Uncertainty Principle) for three objects, a baseball, a virus, and an electron. In each case, the object's velocity is 10 m/s, and the uncertainty in position is 1 angstrom (Å). 1 Å = 1×10^{-10} m. (a) Some of the values in Table 27.1 are missing. Complete the table to fill in the missing data. (b) Comment on the size of the uncertainty in momentum in the three cases.

Object	Mass (kg)	Momentum (kg m/s)	Δx (m)	Δp (kg m/s)
Baseball	0.15	1.5	1×10^{-10}	5.27×10^{-25}
Virus	2.0×10^{-17}		1×10^{-10}	
Electron	9.11×10^{-31}		1×10^{-10}	

Table 27.1: The mass and the uncertainty in position for a particular situation involving three objects of very different mass. In each case, the velocity is 10 m/s. The momentum and momentum uncertainty are not shown for the virus and the electron.

SOLUTION

(a) The missing data are shown in Table 27.2. Note that, with the uncertainty in position being the same in each case, the uncertainty in momentum is also the same in each case.

Object	Mass (kg)	Momentum (kg m/s)	Δx (m)	Δp (kg m/s)
Baseball	0.15	1.5	1×10^{-10}	5.27×10^{-25}
Virus	2.0×10^{-17}	2.0×10^{-16}	1×10^{-10}	5.27×10^{-25}
Electron	9.11×10^{-31}	9.11×10^{-30}	1×10^{-10}	5.27×10^{-25}

Table 27.2: The mass, momentum, position uncertainty, and momentum uncertainty in a particular situation involving three objects of very different mass.

(b) For a baseball, the limitations imposed by the uncertainty principle are so small as to be meaningless. Even for a tiny object like a virus, the momentum uncertainty is much smaller than the momentum. However, for the electron, the momentum uncertainty is orders of magnitude larger than its momentum, giving us little confidence in the stated value of the electron's velocity.

Related End-of-Chapter Exercises: 26 – 30.

Essential Question 27.6: What if Planck's constant, h, had a value around 1 J s instead of its actual value of 6.6×10^{-34} J s? Would this change how we interact with the world?

Answer to Essential Question 27.6: The fact that Planck's constant is tiny means that the uncertainty principle is important for objects around the size of an atom or less. If Planck's constant was a great deal larger, such as 1 J s, the uncertainty principle (and the de Broglie wavelength) would be relevant for all objects we deal with on an everyday basis. The world would be a much stranger place than it already is. Cars would diffract from tunnels, batters in baseball would have to deal with the wave nature of the baseball, etc.

Chapter Summary

Essential Idea: The Quantum World.

The quantum world deals with interactions between very small particles like electrons, protons, and atoms. Because of the tiny value of Planck's constant ($h = 6.626 \times 10^{-34}$ J s), the behavior of such particles is very different from the behavior of everyday objects. Among other things, these tiny objects exhibit both a wave nature and a particle nature.

Black body radiation

Black body radiation is the radiation, in the form of electromagnetic waves, which emanates from a warm object. An example is the red-orange glow given off by a toaster element. Black body radiation is historically significant, because Max Planck explained it in terms of quantized energy levels, the first time quantized energy levels had been used.

The Photoelectric effect

When light is incident on a metal, electrons can be emitted from the metal, in a process known as the photoelectric effect. Electrons are emitted only if the frequency of the light is larger than a particular threshold frequency that depends on the metal. Increasing the intensity of the incident light does not change the maximum kinetic energy of the emitted electrons. The photoelectric effect cannot be explained in terms of light acting as a wave. Instead, it is explained in terms of light being made up of particles, which we call photons, with the electrons being emitted because of single photon – single electron interactions.

$E = hf$, (Equation 27.2: **Energy of a photon**)

where f is the frequency of the electromagnetic wave corresponding to the photon.

$K_{\max} = hf - W_0$, (Equation 27.3: **The photoelectric effect**)

where $K_{\max}$ is the maximum kinetic energy of the electrons and W_0 is the work function.

The Compton effect

Despite having no mass, photons carry momentum, given by

$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$. (Equation 27.6: **Momentum of a photon**)

The fact that photons have momentum is demonstrated by the Compton effect, a single photon – single electron collision in which both momentum and energy are conserved.

$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$, (Equation 27.7: **The Compton Effect**)

where m_e is the mass of the electron, and the quantity $h/(m_e c) = 2.43 \times 10^{-12}$ m is the Compton wavelength. θ is the angle between the directions of the incident and outgoing photons, while λ' is the wavelength of the outgoing photon and λ is the wavelength of the incident photon.

Wave-particle duality

Light is not the only thing that exhibits both a wave nature and a particle nature – everything exhibits such wave-particle duality. The wavelength of an object is inversely proportional to its momentum.

$$\lambda = \frac{h}{p} = \frac{h}{mv} .$$ (Equation 27.8: **the de Broglie wavelength**)

To explain the results of a particular experiment, usually either the wave nature or the particle nature is used.

Heisenberg's uncertainty principle

Quantum physics actually puts a limit on how accurately we can know something. More specifically, the uncertainties in two related quantities, such as the position and momentum of an object, are related in such a way that the smaller the uncertainty in one of the quantities, the larger the uncertainty has to be in the other quantity.

$$\Delta x \, \Delta p \geq \frac{h}{4\pi} = 5.273 \times 10^{-35} \text{ J s} .$$ (Equation 27.9: **Heisenberg's uncertainty principle**)

End-of-Chapter Exercises

Exercises 1 – 12 are conceptual questions that are designed to see if you have understood the main concepts of the chapter.

1. Astronomers can determine the temperature at the surface of a star by looking at the star's color. Explain how the color of a star corresponds to its temperature, and comment on whether a blue star has a higher surface temperature than does a red star, or vice versa.

2. An incandescent light bulb gives off a bright yellow-white glow when it is connected to a wall socket. If the potential difference across the bulb is reduced, however, not only does the bulb get dimmer, the emitted light takes on a distinct orange hue. Explain this.

3. With a particular metal plate, shining a beam of blue light on the metal causes electrons to be emitted via the photoelectric effect. If we reduce the intensity of the light shining on the metal, without changing its wavelength, what happens? Explain your answer.

4. The work functions of gold, aluminum, and cesium are 5.1 eV, 4.1 eV, and 2.1 eV, respectively. If light of a particular frequency causes photoelectrons to be emitted when the light is incident on an aluminum surface, explain if we know whether this means that photoelectrons are emitted from a gold surface or a cesium surface when the light is incident on those surfaces.

5. With a particular metal plate, shining a beam of green light on the metal causes electrons to be emitted. (a) If we replace the green light by blue light, do we know that electrons will be emitted? (b) If the two beams have the same intensity and are incident on equal areas of the plate, do we get the same number of electrons emitted per second in the two cases? Assume that the probability that a photon will cause an electron to be emitted is the same in both cases (e.g., for every two photons incident on the plate, one electron is emitted).

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Light is not the only thing that exhibits both a wave nature and a particle nature – everything exhibits such wave-particle duality. The wavelength of an object is inversely proportional to its momentum.

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$$\Delta x \, \Delta p \geq \frac{h}{4\pi} = 5.273 \times 10^{-35} \text{ J s} .$$ (Equation 27.9: **Heisenberg's uncertainty principle**)

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6. The diagram in Figure 27.11 represents the Compton effect collision between a photon and an electron. The diagram shows the paths followed by the incident and outgoing photons, and the wavelengths of these photons. Which is the incident photon and which is the outgoing photon?

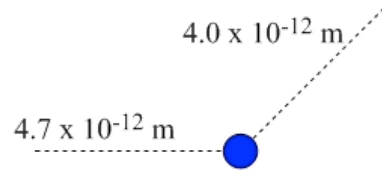


Figure 27.11: The diagram shows a representation of a particular Compton effect collision. The blue circle represents the initial position of the electron, which is at rest before the collision. The dashed lines represent the direction of the incident and outgoing photons – the lines are labeled with the photon wavelengths. For Exercise 6.

7. A photon is incident on an electron that is initially at rest. The photon experiences a Compton effect collision with the electron such that the photon, after the collision, is traveling in a direction exactly opposite to that of the incident photon. In what direction is the electron's velocity after the collision? Explain your answer.

8. One problem with solar sailboats is that the force they experience, because of photons from the Sun reflecting from the sails, cannot be directed toward the Sun. In an effort to overcome this problem, an inventor proposes a solar sailboat with a built-in light source. Instead of using sunlight to drive the sailboat, the inventor proposes attaching a high-power laser to the sailboat, and shining the light from the laser onto the sails. According to the inventor, by adjusting the angle of the sails, the sailboat can be made to turn in any direction, and, once pointed in the correct direction, the light from the laser can then propel the sailboat in the desired direction. What, if anything, is wrong with this idea?

9. Electrons are accelerated from rest and are then incident on a double slit. The pattern the electrons make on the screen is shown in Figure 27.12. If the potential difference through which the electrons are accelerated is reduced and a new pattern is observed on the screen, describe how the new pattern differs from that shown in Figure 27.12.

Figure 27.12: The pattern of dots created by electrons striking a screen a distance 2.40 m from a double slit, when an electron beam is incident on the slits. For Exercise 9.

10. The interference pattern shown in Figure 27.12 for electrons that pass through a double slit is something of an idealization. If we take a close up view of the pattern on the screen, we see that it is really more like the pattern shown in Figure 27.13. How does this figure support the idea of wave-particle duality? In particular, comment on whether any part of the pattern is associated with the wave nature of electrons, and whether any part is associated with the particle nature of electrons.

Figure 27.13: A close up view of dots created by electrons striking a screen after passing through a double slit, for Exercise 10.

11. Two experiments are carried out, using the same double slit. In the first experiment, a beam of electrons is incident on the double slit. In the second experiment, the electron beam is replaced by a beam of protons. In which experiment are the peaks in the interference patterns farther apart if the electrons and protons have equal (a) de Broglie wavelengths? (b) momenta? (c) speed? (d) kinetic energy?

12. To create a beam of fast-moving electrons, you accelerate electrons from rest by placing them between two metal plates that have a large potential difference across them. The electrons emerge from this system by means of a tiny hole you have drilled in the plate the electrons accelerate toward. You find, however, that instead of a narrow, well-defined beam, that the electrons are spread over a range of angles. (a) Come up with an explanation, based on the principles of physics covered in this chapter, to explain your observations. (b) Will the problem (the spread in the beam) get better or worse when you make the hole smaller? Explain. (c) Will the problem get better or worse when you increase the potential difference across the metal plates? Explain.

Exercises 13 – 16 involve the photoelectric effect.

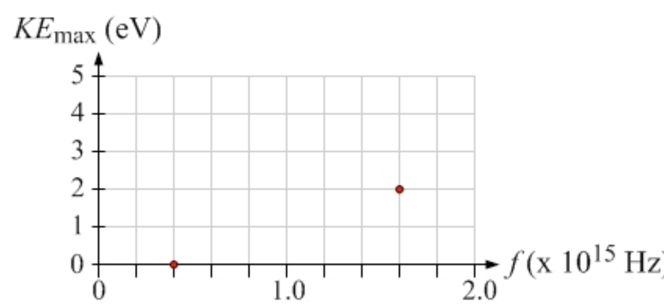
13. Gold has a work function of 5.1 eV. (a) What is the threshold (minimum) frequency of light needed to cause electrons to be emitted from a gold plate via the photoelectric effect? (b) If the frequency of the light incident on the gold plate is twice the threshold frequency, what is the maximum kinetic energy of the emitted electrons?

14. When ultraviolet light with a wavelength of 290 nm is incident on a particular metal surface, electrons are emitted via the photoelectric effect. The maximum kinetic energy of these electrons is 1.23 eV. (a) What is the work function of the metal? (b) What is the threshold frequency for this particular metal?

15. Iron has a work function of 4.5 eV. Plot a graph of the maximum kinetic energy (in eV) of photoelectrons emitted when light is incident on an iron plate as a function of the frequency of the light, which can vary from 0 to 2×10^{15} Hz.

16. An incomplete graph of the maximum kinetic energy (in eV) of photoelectrons emitted when incident light is incident on a particular metal is shown in Figure 27.14. Assume the two points shown are accurate. (a) What is the work function of the metal? (b) What is the threshold frequency in this case? (c) Complete the graph, to show the maximum kinetic energy at all frequencies up to 2×10^{15} Hz.

Figure 27.14: An incomplete graph of maximum kinetic energy of photoelectrons emitted from a particular metal plate, as a function of the frequency of the incident light. For Exercise 16.



Exercises 17 – 20 involve the Compton effect.

17. A photon with a wavelength of 5.0×10^{-12} m is incident on an electron that is initially at rest. If the photon that travels away from this collision is traveling in a direction that is at 120° to that of the incident photon, what is its wavelength?

18. A photon with a wavelength of 6.14×10^{-12} m is incident on an electron that is initially at rest. If the photon experiences a Compton effect collision with the electron, what is (a) the minimum possible, and (b) the maximum possible, wavelength of the photon after the collision? (c) In which direction does the electron travel after the collision in the situation described in (a) and (b)?

19. A photon is incident on an electron that is initially at rest. The photon experiences a Compton effect collision with the electron such that the photon, after the collision, is traveling in a direction perpendicular to that of the incident photon. (a) How does the wavelength of the photon after the collision compare to that of the photon before the collision? (b) Can the electron after the collision be traveling in a direction exactly opposite to that of the photon after the collision? Explain why or why not.

20. A photon collides with an electron that is initially at rest. After the collision, the photon has a wavelength of 4.0×10^{-12} m, and it is traveling in a direction that is at 45° to that of the incident photon. What is the wavelength of the incident photon?

Exercises 21 – 25 involve the de Broglie wavelength and wave-particle duality. Review Chapter 25 for some relevant concepts and equations.

21. When photons of a certain wavelength are incident on a particular double slit, the angle between the central maximum and one of the first-order maxima in the interference pattern is 10°. If the photons are replaced by electrons, and the electrons have a de Broglie wavelength less than that of the photon wavelength, what (if anything) will happen to the angle between the central maximum and one of the first-order maxima?

22. When light from a red laser, with a wavelength of 632 nm, is incident on a certain double slit, a particular interference pattern is observed. The laser light is replaced by a beam of electrons, all with the same energy, and exactly the same interference pattern is observed. What is (a) the wavelength, (b) frequency, and (c) energy of the electrons?

23. Electrons of a particular energy are incident on a double slit, producing an interference pattern with interference maxima and minima. When the electron energy is reduced, do the interference maxima get closer together, farther apart, or remain the same? Briefly justify your answer.

24. Figure 27.15 shows the $m = 0$ and $m = 1$ lines coming from an electron beam, in which the electrons have a de Broglie wavelength of 54 nm, that is incident on a double slit. The squares in the grid measure 20 cm × 20 cm. Determine the distance between the two slits in the double slit.

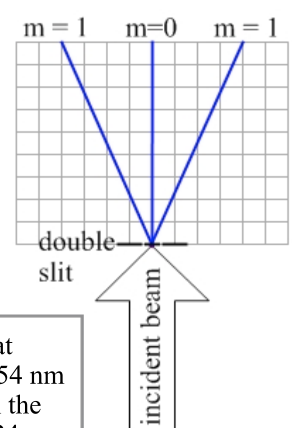


Figure 27.15: The $m = 0$ and $m = 1$ lines that result when electrons with a wavelength of 54 nm are incident on a double slit. The squares on the grid measure 20 cm × 20 cm. For Exercise 24.

25. Electrons of a particular energy are incident on a double slit. When the electrons are detected by a detector 3.0 m beyond the double slit, the distance between the central maximum and one of the first-order maxima in the interference pattern is found to be 8.0 mm. The distance between the slits in the double slit is 120 nm. What is the (a) wavelength, and (b) magnitude of the momentum of the electrons?

Exercises 26 – 30 involve the Heisenberg uncertainty principle. In all cases below, assume that the motion is one-dimensional.

26. What is the minimum uncertainty in an object's position if the object has a speed of 20 m/s and a mass of (a) 100 g? (b) 1×10^{-10} kg? (c) 1×10^{-25} kg?

27. According to the Heisenberg uncertainty principle, what is the minimum uncertainty in a proton's speed if the proton has an uncertainty in position of (a) 50 mm? (b) 500 nm? (c) 5.0×10^{-10} m?

28. Repeat Exercise 27, with an electron instead of a proton.

29. Imagine that you live in a strange world where Planck's constant h is 66.3 J s. You park your bike, which has a mass of 20 kg, in a location such that the uncertainty in the bike's location is 10 cm. What is the uncertainty in the bike's speed?

30. You place an electron on a wire that has a length of 6.0 nm, so that you know the electron is on the wire but you don't know exactly where it is. What is the uncertainty in the electron's momentum?

Exercises 31 – 34 involve applications of quantum physics.

31. In 2007, a team led by researchers at the University of Delaware announced a new world record for efficiency by a solar cell, at 42.8%. This means that the solar cell transformed 42.8% of the incident light energy into electrical energy. Estimate what area of these world-record solar cells would be required to supply the energy needs of a household of four people in the United States, based on an estimate of 2×10^{12} J of energy used annually by such a household. You can assume that the intensity of sunlight falling on the cells is 1000 W/m^2, and that, on average, the sun is shining on the solar cells for 6 hours per day.

32. Many luxury automobiles are now using xenon high-intensity discharge headlights. Compared to incandescent light bulbs, in which the filament is around 2800 K, the light given off by the high-intensity headlights has a spectrum similar to that of a black body at 4200 K. Does this help explain why the high-intensity discharge headlights have a blue tinge in comparison to the light from a standard incandescent bulb? Explain based on the principles of physics covered in this chapter.

33. One of the hot topics in research these days is the development of light-emitting diodes (LEDs) as a potential replacement for the incandescent bulbs that are commonly used for household lighting. The advantage of LEDs over incandescent bulbs is that LEDs are very efficient at transforming electrical energy into visible light, while it is commonly said that "incandescent bulbs give off 90% of their energy as heat." (a) Explain, using principles of physics, how an incandescent light bulb works. (b) Typically, the filament in an incandescent bulb is made from tungsten. Why is tungsten used instead of a cheaper, more readily available metal such as aluminum? (c) What is meant by the phrase "incandescent bulbs give off 90% of their energy as heat?"

34. Ernst Ruska was awarded a 50% share of the Nobel Prize in Physics in 1986. Write a couple of paragraphs regarding the work he did, mentioning in particular how the work is connected to the principles of physics discussed in this chapter.

35. An infrared thermometer is a thermometer that can determine an object's temperature without the thermometer needing to make contact with the object. For instance, you could aim the thermometer at the hot water flowing out of the faucet in your kitchen, and the thermometer would read the temperature of the hot water. (a) Briefly explain, using the principles of physics covered in this chapter, how such a thermometer works. (b) Explain why the thermometer is known as an infrared thermometer.

General problems and conceptual questions

36. One morning, as you wait for the toaster to finish toasting a couple of slices of bread, you measure the spectrum of light being emitted by the glowing toaster elements. The peak wavelength of this light is 600 nm. What is the temperature of the toaster elements?

37. You have probably heard the phrase "white hot" before. Approximately what temperature is a black body that is hot enough to look white?

38. A typical helium-neon laser pointer, emitting light with a wavelength of 632 nm, has a beam with an intensity of 800 W/m^2 and a diameter of 3.00 mm. How many photons are emitted by the laser pointer every second?

39. The work functions of gold, aluminum, and cesium are 5.1 eV, 4.1 eV, and 2.1 eV, respectively. When light of a particular frequency shines on a cesium surface, the maximum kinetic energy of the emitted photoelectrons is 2.5 eV. What is the maximum kinetic energy of the photoelectrons emitted when the same light shines on (a) an aluminum surface? (b) a gold surface?

40. We consider the visible spectrum to run from 400 nm to 700 nm. (a) What is the equivalent energy range, in eV, of photons for light in the visible spectrum? If white light, composed of wavelengths covering the full range of the visible spectrum but no more, is incident on a surface, will photoelectrons be emitted if the surface is (b) carbon (work function = 4.8 eV)? (c) potassium (work function = 2.3 eV)? Explain your answers to parts (b) and (c).

41. Zinc has a work function of 4.3 eV. A standard lecture demonstration involves attaching a zinc plate to an electroscope, and then charging the plate by rubbing it with a negatively charged rod. At that point, the electroscope indicates that it is charged, and we know that the charge is negative. (a) If light in the visible spectrum, from a bright incandescent light bulb, shines on the plate, do we expect photoelectrons to be emitted from the plate, causing the electroscope to discharge? Why or why not? (b) What is the maximum wavelength of light that would cause the electroscope to discharge? What part of the spectrum is this wavelength in?

42. Table 27.3 gives a set of data for a photo-electric effect experiment, showing the maximum kinetic energy of the emitted electrons at a number of different frequencies of the light incident on a particular metal plate. Plot a graph of the maximum kinetic energy as a function of frequency and determine, from your graph, (a) the threshold frequency, (b) the work function of the metal, and (c) the slope of that part of the graph above the threshold frequency.

Frequency ($\times 10^{15}$ Hz)	K_{max} (eV)
0.5	0
1.0	0
1.5	1.9
2.2	4.8
2.9	7.7
3.7	11.0

Table 27.3: A set of data from a photo-electric effect experiment, showing the maximum kinetic energy of the emitted electrons at a number of different photon frequencies, for Exercise 35.

43. In a particular photoelectric effect experiment, photons with an energy of 5.0 eV are incident on a surface, producing photoelectrons with a maximum kinetic energy of 2.2 eV. If the energy of the photons doubles, does the maximum kinetic energy of the photoelectrons also double? Explain your answer.

44. In a particular photoelectric effect experiment, photons with an energy of 5.6 eV are incident on a surface, producing photoelectrons with a maximum kinetic energy of 3.3 eV. (a) What is the work function of the metal? (b) What is the minimum photon energy necessary to produce a photoelectron in this situation? (c) What is the corresponding threshold frequency?

45. With a particular metal plate, shining a beam of green light on the metal causes electrons to be emitted. (a) If we replace the green light by red light, do we know that electrons will be emitted? (b) If the two beams have the same intensity and are incident on equal areas of the plate, do we get the same number of electrons emitted per second in the two cases, assuming that photons are emitted in both cases? Assume that the probability that a photon will cause an electron to be emitted is the same in both cases (e.g., for every two photons incident on the plate, one electron is emitted).

46. Figure 27.16 shows a graph of the maximum kinetic energy of emitted photoelectrons as a function of the energy of the photons that are incident on a particular surface. From this graph, determine (a) the work function of the surface, and (b) the threshold frequency.

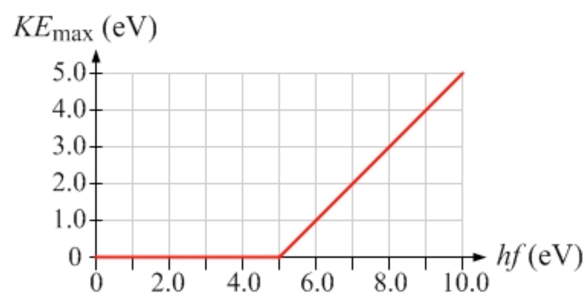


Figure 27.16: This graph shows the maximum kinetic energy of emitted electrons as a function of the energy of the incident photons, for Exercise 46.

47. A green helium-neon laser pointer emits light with a wavelength of 532 nm. The beam has an intensity of 900 W/m^2 and a diameter of 4.00 mm. (a) How many photons are emitted by the laser pointer every second? (b) What is the magnitude of the momentum of each of these photons? How much momentum is imparted to an object, in a 1.00-second interval, that (c) completely absorbs all these photons? (d) reflects all these photons straight back toward the laser pointer?

48. A laser beam that is completely absorbed by a black surface exerts a force of 2.5×10^{-8} N on the surface. (a) What is the net momentum transferred to the surface by the beam every second? (b) If the wavelength of light emitted by the laser is 632 nm, what is the magnitude of the momentum of each photon in the beam? (c) How many photons strike the surface every second?

49. A photon with a wavelength of 6.00×10^{-12} m is incident on an electron that is initially at rest. If the photon that travels away from this collision is traveling in a direction that is at 45° to that of the incident photon, what is its wavelength?

50. Return to the situation described in Exercise 49. Relative to the direction of the incident photon, in what direction does the electron travel after the collision?

51. Photons with a particular wavelength experience Compton-effect collisions with electrons that are at rest. Figure 27.17 shows a graph of the wavelength of the photons that travel away from the various collisions as a function of the cosine of the scattering angle (this is the angle between the direction of the incident photon and the direction of the photon after the collision). For the incident photons, determine the (a) wavelength, (b) frequency, and (c) energy, in eV.

52. Photons with a particular wavelength experience Compton-effect collisions with electrons that are at rest. Figure 27.17 shows a graph of the wavelength of the photons that travel away from the collision as a function of 1 minus the cosine of the scattering angle. We repeat the experiment with incident photons that have wavelengths twice as large as that of the original incident photons. If we plot a new graph, like that in Figure 27.17, how will the graphs compare in terms of their (a) slopes, and (b) *y*-intercepts?

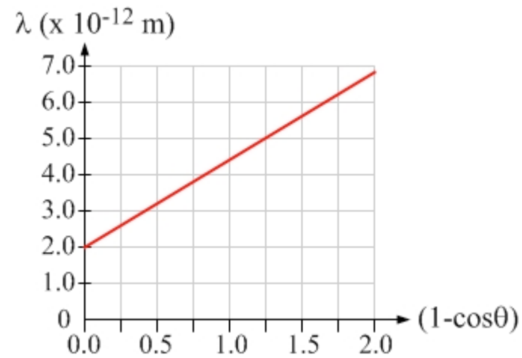


Figure 27.17: A graph of the wavelength of the photon after colliding with a stationary electron in a Compton effect collision, as a function of 1 minus the cosine of the scattering angle. The scattering angle is the angle between the direction of the incident photon and the direction of the photon after the collision. For Exercises 51 and 52.

53. A photon is incident on an electron that is initially at rest. The photon and electron experience a Compton-effect collision such that the electron, after the collision, is traveling in the same direction that the photon was before the collision. The electron's momentum after the collision is 5.0×10^{-22} kg m/s. (a) Write down an expression to show how the wavelength of the photon after the collision is related to that of the photon after the collision. (b) Write down an expression to show how the momentum of the photon before the collision is related to the momentum of the photon after the collision and the electron's momentum after the collision. Now solve your two equations to find the wavelength of the photon (c) before the collision, and (d) after the collision.

54. Electrons with an energy of 8.00 eV are incident on a double slit in which the two slits are separated by 400 nm. What is the angle between the two second-order maxima in the resulting interference pattern?

55. A beam of electrons, shining on a double slit, creates the pattern shown in Figure 27.18 at the center of a screen placed 2.40 m on the opposite side of the double slit from the electron source. If the de Broglie wavelength of the electrons is 135 nm, what is the distance between the two slits in the double slit?

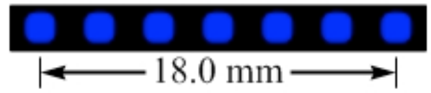


Figure 27.18: The pattern of dots created by electrons striking a screen a distance 2.40 m from a double slit, when an electron beam is incident on the slits. For Exercises 55 – 56.

56. A beam of electrons, shining on a double slit, creates the pattern shown in Figure 27.18 at the center of a screen placed 1.50 m on the opposite side of the double slit from the electron source. (a) If the distance between the two slits is 500 nm, what is the de Broglie wavelength of the electrons? (b) If the electrons achieved this wavelength by being accelerated from rest through a potential difference, what is the magnitude of the potential difference?

57. A baseball has a mass of 150 g. With what speed would a baseball have to be moving so that its de Broglie wavelength is 632 nm, the same as the wavelength of light from a red helium-neon laser?

58. Consider the opening image of this chapter, showing the head of an ant as imaged by an electron microscope. To be able to resolve tiny details, with a size much less than the wavelength of visible light, the de Broglie wavelength of the electrons also needs to be tiny. (a) What is the speed of an electron that has a de Broglie wavelength of 10 nm? (b) If the electron achieved this speed by being accelerated from rest through a potential difference, what was the magnitude of the potential difference?

59. To create a beam of fast-moving electrons, you accelerate electrons from rest by placing them between two metal plates that have a large potential difference across them. The electrons emerge from this system by means of a tiny hole you have drilled in the plate the electrons accelerate toward. You find, however, that instead of a narrow, well-defined beam, that the electrons are spread over a range of angles. To help solve this problem, make use of Equation 25.6, which states that the angle of the first minimum in the diffraction pattern from a circular opening of diameter D is $\theta_{min} = 1.22\lambda / D$. (a) If the electrons are accelerated from rest through a potential difference of 500 V, what is their de Broglie wavelength? (b) What diameter hole is required for the beam to diffract through the hole so that $\theta_{min} = 10°$? (c) If the hole diameter is reduced by a factor of 2, what is θ_{min}? (d) If, instead, the potential difference is doubled, what is θ_{min}?

60. Two students are having a conversation. Comment on the part of their conversation that is reported below. This one may require a little background reading.

Maggie: I really don't buy this wave-particle duality stuff. I mean, let's say we do this. We send an electron beam through a single slit. When the slit is wide, the electrons go straight through. The narrower we make the slit, the greater the probability that the electrons hit one of the edges of the slit, changing the direction. The narrower the slit, the greater the spread in the beam after passing through the slit – who needs waves, it's all particles.

Dipesh: That sounds sensible. Except, when you look at the pattern carefully, there are some angles where you don't get any electrons going – can you explain that just with particles?

Maggie: Oh, good point. OK, let's think about the double slit a little. What if, instead of firing a whole beam of electrons at the double slit, you just did one electron at a time. Wouldn't the electron have to choose one slit or the other? Plus, we shouldn't get interference any more - one electron wouldn't be able to interfere with itself, right?

Dipesh: What a cool idea. Let's google it and see if anyone has done an experiment like that. I'll type in "single electron interference double slit" and see what comes up.

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Solutions

- Answers to Selected End of Chapter Problems
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Additional Links

- *PhET* simulation: Blackbody Spectrum
- *PhET* simulation: Photoelectric Effect