26-1 Observers

Although Special Relativity is often thought of as applying to fast-moving objects, we can see some effects of Special Relativity at low speeds, too. Let's explore this, and get comfortable with the idea of how the same situation looks to different observers.

EXPLORATION 26.1 – Who needs magnetism?

Consider one observer, Jack, who is watching two pairs of charges, as shown in Figure 26.1. The charges are identical, and we will assume that the charges within each pair repel one another, but that the pairs of charges are far enough apart that one pair of charges does not influence the other pair. One pair of charges is initially at rest, with respect to Jack, while each charge in the other pair of charges has an initial velocity v directed to the right. There is also a

second observer, Jill, who has a constant velocity v directed to the right with respect to Jack.

Step 1 – Jack observes that the charges that were initially at rest move apart more quickly than the charges that were initially moving with respect to him. Figure 26.2 illustrates what Jack sees after a time T has passed. Using principles of electricity and magnetism, explain Jack's observations.

First, consider the electrostatic interaction between the

charges. The charges are identical, so by Coulomb's law, Jack expects the charges that were initially at rest to repel and accelerate away from one another. Jack expects the charges that were initially moving to repel each other, too, but he also expects those charges to interact magnetically, because they are like two parallel currents. When Jack reviews Chapter 19, he recalls that two parallel currents attract one another. When Jack does the calculations, he finds that the repulsive electrostatic force is larger than the attractive magnetic force, so he expects the charges on the right to move apart, but not as fast as the charges on the left. This is exactly what he observes.

Step 2 – Now draw two more pictures, one showing the initial situation from Jill's frame of reference, and the other showing the situation at a time T after the charges are released, again from Jill's frame of reference. Once again, use principles of physics to explain what Jill observes.

What Jill observes is a mirror image of what Jack sees. Initially, at t' = 0, where t' denotes times measured by Jill, Jill sees the charges on the right at rest and the charges on the left, and Jack, moving to the left with velocity v.

At a time t' = T, according to Jill, Jill's view of the situation is again a mirror image of what Jack sees. As shown in Figure 26.4, Jill sees the charges on the right, which were initially at rest with respect to her, move apart more quickly than the charges on the left, because of the attractive magnetic force associated with the charges on the left.



Figure 26.1: The situation at t = 0 involving two pair of charges, as seen from Jack's frame of reference. According to Jack, the charges in the left pair are released from rest, while each charge in the right pair have an initial velocity directed right. There is also a second observer, Jill, who is moving right at constant velocity with respect to Jack.



Figure 26.2: The situation at t = T, where the charges on the left have separated more than the charges on the right, according to Jack.







make sense to you? Possibly, it all makes sense. However, you may be bothered by the fact that the two observers disagree on which charges move apart more quickly. Jack observes that the charges on the left move apart more quickly, while Jill observes that the charges on the right move apart more quickly. For most people, that does not make sense – shouldn't everyone agree on what is happening? This is a key part of relativity – observers in different reference frames observe different things.

Step 3 – What, if anything, about the previous steps does not

Figure 26.4: The situation at t' = T, where the charges on the right have separated more than the charges on the left, according to Jill.

Step 4 – Come up with an alternate explanation for what Jack and Jill observe. Instead of using magnetism to explain why the initially moving charges move apart more slowly, come up with an alternate explanation involving how time works in different reference frames. This is an unfair question, because we're asking you to be Einstein here. However, the relativistic argument is that there is no need to resort to magnetism. The charges simply repel one another, and move apart. However, observers observe time passing differently (moving more slowly) in reference frames that are moving with respect to that observer.

In this situation, the charges on the left are in Jack's reference frame, while the charges on left are in Jill's reference frame, which is moving at a constant velocity v to the right with respect to Jack's reference frame. Thus, Jack observes Jill's wristwatch running slowly, and everything in her reference frame (such as the charges on the right) to be moving in slow motion. Jill, on the other hand, observes her wristwatch to be running just fine, and the charges on the right to move apart as she expects, but she observes Jack's wristwatch to be running slowly, and the left-hand charges, which are in Jack's reference frame, to be moving in slow motion.

Key ideas: We investigated a variety of ideas in this Exploration. First, magnetism is a relativistic effect, which means that you can change the strength of a magnetic interaction simply by changing the relative velocity between you and a set of interacting charges. Second, observers in different reference frames can disagree on what happens in certain cases. However, any constant-velocity reference frame is just as good as any other reference frame in terms of making observations. In the situation above, for instance, Jill's reference frame is no better or worse than Jack's. Third, time works in a manner that is quite different from what we're used to based on our past experience, in that it runs at different rates in different reference frames (we will explore this in more detail in the sections that follow). **Related End-of-Chapter Exercises: 1, 2.**

Postulates of special relativity: Relativity is based on two simple ideas.

- 1. The speed of light in vacuum is the same for all observers.
- 2. There is no preferred reference frame. The laws of physics apply equally in all reference frames.

One implication of these postulates is that nothing can travel faster than the speed of light in vacuum, which is $c = 3.00 \times 10^8$ m/s.

Essential Question 26.1: (a) How fast are you moving right now? (b) What is your speed associated with being on the Earth while the Earth is spinning around its axis? (c) What is your speed associated with being on Earth while the Earth is orbiting the Sun?

Answer to Essential Question 26.1: (a) The obvious answer is that you are at rest. However, the question really only makes sense when we ask what the speed is measured with respect to. Typically, we measure our speed with respect to the Earth's surface. If you answer this question while traveling on a plane, for instance, you might say that your speed is 500 km/h. Even then, however, you would be justified in saying that your speed is zero, because you are probably at rest with respect to the plane. (b) Your speed with respect to a point on the Earth's axis depends on your latitude. At the latitude of New York City (40.8° north), for instance, you travel in a circular path of radius equal to the radius of the Earth (6380 km) multiplied by the cosine of the latitude, which is 4830 km. You travel once around this circle in 24 hours, for a speed of 350 m/s (at a latitude of 40.8° north, at least). (c) The radius of the Earth's orbit is 150 million km. The Earth travels once around this orbit in a year, corresponding to an orbital speed of 3×10^4 m/s. This sounds like a high speed, but it is too small to see an appreciable effect from relativity.

26-2 Spacetime and the Spacetime Interval

We usually think of time and space as being quite different from one another. In relativity, however, we link time and space by giving them the same units, drawing what are called spacetime diagrams, and plotting trajectories of objects through spacetime. A spacetime diagram is essentially a position versus time graph, with the position axes and time axes reversed.

EXPLORATION 26.2 – A spacetime diagram

We can convert time units to distance units by multiplying time by a constant that has units of velocity. The constant we use is c, the speed of light in vacuum.

Step 1 – Plot a graph with position on the x-axis and time, converted to distance units, on the y-axis. This graph is a spacetime diagram. We are at rest in this coordinate system, which means that the spacetime diagram is for our frame of reference. At t = 0 and x = 0, we send a pulse of light in the positive x-direction. On the graph, show the trajectory of this light pulse, which travels at the speed of light in vacuum. How far does the pulse travel in 5 meters of time? The spacetime diagram is shown in Figure 26.5. Because the pulse of light travels at the speed of light, the pulse's trajectory is a straight line with a slope of 1. The pulse travels 5 m of distance in 5 m of time, which makes things easy to plot.

Step 2 – On the spacetime diagram, plot the trajectory (this is called a worldline) of a superfast mosquito that passes through x = 0 at t = 0. With respect to us, the mosquito moves in the positive x-direction at half the speed of light. What is the connection between the slope of the worldline and the velocity of the mosquito? This spacetime diagram is shown in Figure 26.6. Because the mosquito's velocity is constant, the slope of its worldline is constant. The mosquito is always half the distance from the origin that the light pulse is (at least according to us!).

The slope of an object's worldline is the inverse of the velocity, if the velocity is expressed as a fraction of c. In this case, the mosquito has a velocity of +0.5, so the slope of its worldline is +2.







Figure 26.6: The spacetime diagram, showing the worldline of a mosquito traveling in the +x-direction at half the speed of light.

Step 3 – Add two events to the spacetime diagram. According to us, Event 1 occurs at x = 0 at t = 0, and Event 2 occurs at x = +2 m at t = +4 m of time. According to us, what is the time interval between the two events, and what is their spatial separation?

Note that we define an event as something that takes place at a particular point in space and at a particular instant in time. The amended spacetime diagram with the events marked on it is shown in Figure 26.7. The time interval, converted to distance units, between the two events is $c\Delta t = 4$ m. The spatial separation is $\Delta x = 2$ m.



Figure 26.7: The spacetime diagram, now showing the space and time coordinates, as measured from our reference frame, of two events.

The spacetime interval: It turns out that observers in different constant-velocity reference frames always agree on the value of the spacetime interval between two events, as defined by

 $(c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2 = (\text{spacetime interval})^2$, (Eq. 26.1: The spacetime interval)

where $c \Delta t$ and $c \Delta t'$ are the time intervals, converted to distance units, between the two events as measured from two different frames of reference, and Δx and $\Delta x'$ are the spatial separations between the two events, as measured in the same two reference frames. Note that, if the left-hand side of the equation gives a negative number, it is appropriate to reverse the order of the terms.

Step 4 – What is the spatial separation $\Delta x'$ between Event 1 and Event 2 as measured by the mosquito? Knowing this, use Equation 26.1 to find the time interval $c\Delta t'$ between the events, as measured by the mosquito. Both of the events lie on the worldline of the mosquito, so the mosquito thinks that they both take place at x' = 0. Thus, the spatial separation between them, as measured by the mosquito, is $\Delta x' = 0 - 0 = 0$. Let's now work out the value of the spacetime interval between these events, as measured by us.

(spacetime interval)² = $(c\Delta t)^2 - (\Delta x)^2 = (4 \text{ m})^2 - (2 \text{ m})^2 = 12 \text{ m}^2$.

In the mosquito's reference frame, then, the time difference is given by $(c\Delta t')^2 = (\text{spacetime interval})^2 - (\Delta x')^2 = 12 \text{ m}^2 - 0 = 12 \text{ m}^2$.

Thus, we observe that 4 m of time pass between the events, while the mosquito observes only $\sqrt{12}$ m = 3.5 m of time passing between them. The main point here is that observers in different reference frames measure different amounts of time passing between the same two events.

Key ideas: In relativity, the emphasis is often on what is different in two different reference frames. However, as we have learned earlier (such as with energy conservation) the parameters that everyone agrees on are generally most important. In relativity, all observers agree on the value of the spacetime interval. In addition, in relativity we treat space and time as different components of a spacetime coordinate system, rather than treating them as completely different things, as we are more used to doing. **Related End-of-Chapter Exercises: 9 – 18.**

Essential Question 26.2: Return to Exploration 26.2. Add your worldline to the spacetime diagram shown in Figure 26.7.



26-3 Time Dilation – Moving Clocks Run Slowly

In Sections 26-1 and 26-2, we looked at situations in which time passes at different rates in different reference frames. This may be at odds with your own experience of time. However, we are not used to dealing with speeds that are significant fractions of the speed of light, so perhaps we should not be too surprised that time has such an interesting behavior when we compare reference frames that are moving at high speed with respect to one another.

The time-honored statement summarizing how time behaves is that "moving clocks run slowly." In this section, we will investigate how a clock that uses light pulses behaves when it is viewed from a moving reference frame. We will also discuss some of the experimental evidence supporting this behavior of time.

EXPLORATION 26.3 – A light clock

Figure 26.9 shows a light clock, which has a source (or emitter) of light pulses (at the bottom), a mirror at the top to reflect the light, and a detector at the bottom to record the pulses. The clock runs at a rate equal to the rate at which the pulses are received by the detector.



Step 1 – Let's say the clock above belongs to Jack. Relative to Jack, Jill moves to the right at a constant speed of 60% of the speed of light in vacuum. Jill's light clock is identical to Jack's. Sketch a diagram, similar to that in Figure 26.9, showing how the light pulses in Jill's clock travel from emitter to mirror to detector, according to Jack. According to Jack, Jill's clock is moving, so a light pulse in Jill's clock travels farther to reach the mirror after leaving the emitter than does a light pulse in Jack's clock. When Jack's clock reads 1 unit, Jill's clock is reading 0.8 units, according to Jack. This pattern continues, with Jill's clock continuing to show time passing by at 80% of the rate at which Jack's clock measures time passing, according to Jack.



Figure 26.10: What Jill's light clock looks like, according to Jack. Compare this figure to Figure 26.9 to see what Jack sees on his clock and Jill's clock at the same instants.

Step 2 – Sketch a diagram showing how the light pulses in Jack's clock travel from emitter to mirror to detector, according to Jill. According to Jill, her clock is working fine (she sees her clock as in Figure 26.9), but Jack's clock is running slowly. All observers see light traveling at the speed of light, so Jill sees Jack's clock running slowly because the light pulses in Jack's clock, shown in Figure 26.11, travel farther to reach the mirror than do the pulses in Jill's clock.



Figure 26.11: What Jack's light clock looks like, according to Jill. Compare this figure to Figure 26.9 to see what Jill sees on her clock and Jack's clock at the same instants.

Proper time and time dilation: An observer measures the proper time interval Δt_{proper} between two events when that observer is present at the location of both events. Observers for whom the events take place in different locations measure a longer time interval Δt between the events.

 $\Delta t = \frac{\Delta t_{proper}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \ \Delta t_{proper} , \qquad (Equ$

(Equation 26.2: Time dilation)

where v is the relative speed between the two reference frames.

Key ideas: The faster a clock moves with respect to an observer, the more slowly it ticks off time, according to that observer. All time-keeping devices, including beating hearts, act this way, not just the light clocks we explored here. **Related End-of-Chapter Exercises:** 19 - 22, 34.

Experimental evidence for time dilation

In 1941, Bruno Rossi and David Hall compared the rate at which muons (essentially heavy electrons) entering the Earth's atmosphere passed through their detectors when they were at the top of Mt. Washington in New Hampshire, at an altitude of 6300 feet. They also measured the rate at which muons passed through the detectors at sea level. For an observer at rest on the Earth, these muons take 6.4 μ s to cover 6300 feet, considerably longer than the 2.2 μ s average lifetime of muons created in the laboratory. Thus, one would expect almost all the muons to decay before reaching sea level. Rossi and Hall's measurements showed that significantly more muons reached sea level than would be expected. This can be explained by time dilation. According to an Earth-based observer, time passes more slowly for these muons, which travel at over 99% of the speed of light. When 6.4 μ s has elapsed for us, we observe clocks in the muons' frame of reference to be running more slowly, explaining why they last longer than we expect.

Another experiment was carried out in 1971 by J. C. Hafele and R. E. Keating. They flew four atomic clocks eastward around the world, and then flew the clocks westward around the world. After each trip, the clocks were compared to an identical clock that remained at the United States Naval Observatory. Both the effects of general and special relativity were important in this experiment, but the observed results (a loss of 59 ± 10 ns for the eastward trip, and a gain of 273 \pm 7 ns for the westward trip) were in agreement with the predictions of relativity.

Essential Question 26.3: Return to Exploration 26.3. Who is aging more slowly, Jack or Jill?

Answer to Essential Question 26.3: Who ages more slowly depends on who you ask. According to Jack, time passes more slowly for Jill, who is moving with respect to Jack, and thus Jack says that Jill is aging more slowly. According to Jill, time passes more slowly for Jack, who is moving with respect to Jill, and thus Jill says that Jack is aging more slowly. This brings us to the famous **twin paradox**, in which one twin goes off at high speed to explore a distant galaxy, and returns some years later to find that the twin who remained behind on Earth is considerably older than the traveling twin. The apparent paradox is why there is no symmetry in this situation. The twin who stayed behind should see the traveling twin to be aging more slowly. Dut the traveling twin should see the traveling twin changed reference frames halfway through, from a frame of reference in which she travels away from the Earth to a new frame of reference in which she travels down the Earth. In both reference frames, interestingly, she does observe the twin who remained on Earth to be aging more slowly than she is, but she also observes the age of the twin who remained behind to jump when the traveling twin switches reference frames.

26-4 Length Contraction

Time is not the only thing that behaves in an unusual way, space does too. Observers who view objects moving with respect to them, for instance, measure the length of the object to be contracted (shorter) along the direction of motion.

Let's now work through an example that ties together all of the ideas we have discussed thus far in the chapter

Length contraction: Assume that two points separated by some distance are in the same reference frame. An observer measures the proper length L_{proper} between these two points when the points are at rest with respect to that observer (that is, the observer is in the same frame of reference as the points). For an observer in a different reference frame, when the displacement between the points has a component parallel to the velocity of the points, the observer measures a contracted length between the points. This length contraction is most pronounced when the displacement from one point to the other is parallel to the velocity, in which case the length L measured by the observer is

$$L = \sqrt{1 - \frac{v^2}{c^2}} L_{proper} = \frac{L_{proper}}{\gamma},$$

(Equation 26.3: Length contraction)

where *v* is the relative speed between the two reference frames.

EXAMPLE 26.4 – Isabelle's travels

Let's say that Planet Zorg is at rest with respect to the Earth, and that in the reference frame of the Earth, the distance between Earth and Zorg is 40 light-years. Isabelle is passing by the Earth in her rocket ship traveling toward Zorg at a constant velocity of 0.8 c. At the instant she passes by, you, who are on Earth, send a light pulse toward Zorg.

- (a) According to you, how long does the light pulse take to reach Zorg?
- (b) According to you, how long does Isabelle take to reach Zorg?
- (c) According to Isabelle, what is the spatial distance between the event of Isabelle passing Earth and the event of Isabelle arriving at Zorg?
- (d) Use Equation 26.1 to find the time it takes Isabelle to reach Zorg, according to Isabelle.
- (e) What is the distance between Earth and Zorg, according to Isabelle?
- (f) How long does the light pulse take to reach Zorg, according to Isabelle?

SOLUTION

(a) According to you, the planets are separated by a distance of 40 light-years. Because light covers 1 light-year every year, it takes the light pulse 40 years to reach Zorg, according to you.

(b) According to you, Isabelle travels at 4/5 the speed that light does, so Isabelle should take 5/4 the time that light does.

 $t = \frac{40 \text{ light-years}}{0.8 c} = 50 \frac{\text{light-years}}{c} = 50 \text{ years}.$

Note that light-years divided by the speed of light in vacuum, c, gives years. Also, note that we don't have to use a relativistic equation to get the answer. We are, instead, using the basic idea that for motion with constant speed, time is simply the distance divided by the speed.

(c) For Isabelle, these two events happen at the same location, right outside Isabelle's rocket. Thus, their spatial separation, according to Isabelle, is $\Delta x' = 0$.

(d) Equation 26.1 is the equation for the spacetime interval. According to you, the time between the events is 50 years which, when multiplied by *c*, gives $c\Delta t = 50$ light-years, and the events are separated spatially by $\Delta x = 40$ light-years. The spacetime interval is given by: (spacetime interval)² = $(c\Delta t)^2 - (\Delta x)^2 = (50 \text{ light-years})^2 - (40 \text{ light-years})^2 = (30 \text{ light-years})^2$.

Solving for the time interval from Isabelle's perspective, we get: $(c\Delta t')^2 = (\text{spacetime interval})^2 - (\Delta x')^2 = (30 \text{ light-years})^2 - 0 = (30 \text{ light-years})^2$.

In Isabelle's reference frame, we have $c\Delta t' = 30$ light-years. Dividing by *c* gives a time interval of 30 years, so it only takes Isabelle 30 years to go from Earth to Zorg, according to Isabelle.

Note that 30 years is also the proper time between the two events, because Isabelle is present at both events. The proper length between Earth and Zorg, on the other hand, is the 40 light-years measured by you, because the planets are at rest in your reference frame. At this point, you may be concerned that it looks like Isabelle travels faster than light, since she travels to Zorg in 30 years while light travels there in 40 years, but we are about to resolve that.

(e) We could use the length contraction equation to determine the distance between the planets, according to Isabelle. A simpler method is that, according to Isabelle, Zorg travels toward her at a constant velocity of 0.8 c, and Zorg passes her 30 years after Earth passes her. Thus, Zorg must have covered a distance of 0.8 $c \times 30$ years, which is 24 light-years, in that time, which represents the distance between the planets, according to Isabelle.

(f) According to Isabelle, the planets are separated by a distance of 24 light-years. Isabelle sees the light pulse traveling away from her at c, and Zorg coming toward her at 0.8c, for a relative speed between them of 1.8c, according to Isabelle. Thus, according to Isabelle, the pulse and Zorg would meet at a time of 24 light-years divided by 1.8c, or 13.3 years. Both you and Isabelle agree that nothing travels faster than light in this situation, by the way.

Related End-of-Chapter Exercises: 6, 23 – 25.

Essential Question 26.4: If your clock and Isabelle's clock both read zero when Isabelle passes Earth, what will the clocks read, according to both you and Isabelle, when Isabelle reaches Zorg?

Answer to Essential Question 26.4: According to you, 50 years elapses on your clock during Isabelle's trip to Zorg. Using the time dilation equation with a relative speed of 0.8*c*, we can show that Isabelle's clock is running at 60% of the rate of yours, according to you. Thus, when 50 years passes by on your clock, only 30 years passes by on Isabelle's clock, according to you. This is consistent with the result of part (d) in Example 26-4.

As we showed in part (d), according to Isabelle, the time interval between Earth passing her and Zorg passing her to is 30 years. However, if you see her clock running at 60% of the rate of yours, Isabelle sees your clock running at 60% of the rate of hers. 60% of 30 years is 18 years, so, according to Isabelle, your clock only reads 18 years when Isabelle and Zorg meet.

26-5 The Breakdown of Simultaneity

Let's explore the behavior of time in more detail, beginning with extending our understanding of spacetime diagrams. From our work earlier in the book, we are used to being able to transform from one coordinate system to another by simply rotating the x and y axes through the same angle, as demonstrated in Figure 26.12. The two coordinate systems share the same origin, and a point such as x = +3 m, y = +4 m (a distance of 5 m from the origin) transforms into x' = +5 m, y' = 0 (not coincidentally also 5 m from the origin). For any point a distance r from the origin, we have the equivalent of the spacetime interval equation,

$$x^{2} + y^{2} = (x')^{2} + (y')^{2} = r^{2}$$
.

We can something similar with the spacetime diagram, except that the space and time axes rotate in opposite directions. The angle of rotation depends on the relative velocity between one reference frame and the other. With the super-fast mosquito we looked at in Exploration 26-2, the mosquito's time axis, as viewed from your frame of reference, was rotated as it was because the mosquito was moving at half the speed of light with respect to you. Similarly, the mosquito's *x*-axis (as seen from your reference frame) is rotated through the same angle, but in the opposite direction, so that the slope of the mosquito's *x*-axis is equal to the mosquito's speed, with respect to you, expressed as a fraction of the speed of light. The mosquito's coordinate system, according to you, is shown in Figure 26.13.

Examining events 1 and 3 in Figure 26.13, we can see that events which are simultaneous in one reference frame may not be simultaneous in another. Events 1 and 3 are on the mosquito's x-axis, which means they take place at the same instant (at t' = 0, in this case) and are thus simultaneous in the mosquito's frame of reference. In your reference frame, however, Event 1 occurs before Event 3. Simultaneity is relative! In a situation like this, events are simultaneous for observers in different reference frames only when they occur at the same time and place.



Figure 26.12: Rotating the *x* and *y* axes through the same angle transforms coordinate systems in the *x*-*y* plane.



Figure 26.13: Transforming coordinate systems on a spacetime diagram is accomplished by rotating the time and space axes in opposite directions, by an angle determined by the relative velocity between the two coordinate systems.

Let's construct a spacetime diagram, from your frame of reference, for Isabelle's travels, from Example 26.4. Isabelle is traveling at 0.8*c* with respect to you so, in your reference frame, Isabelle's coordinate system is rather distorted. This is shown in Figure 26.14. If you and an observer called Yan, who is in your frame of reference but located on Zorg, synchronize your clocks, according to you and Yan, all four of the following events are simultaneous:

- Isabelle passes Earth
- Your clock reads zero
- Isabelle's clock reads zero
- Yan's clock reads zero

Isabelle agrees that the first three events are simultaneous, but she disagrees that Yan's clock reads zero at that instant. From the spacetime diagram, we can divide spacetime into locations at which Isabelle records a positive time on her clock, and locations where Isabelle records a negative time. According to Isabelle, Yan's clock reads zero long before your clock reads zero. In fact,



Figure 26.14: A spacetime diagram, from your frame of reference, for the situation of Isabelle's travels between Earth and Zorg. The boxes on the grid measure $4 \text{ ly} \times 4 \text{ ly}$.

according to Isabelle, Yan's clock is set 32 years ahead of your clock! This helps explain a mystery regarding Essential Question 26.4. You think Isabelle takes 50 years to reach Zorg, and Yan sends a message to you that, when Isabelle passed Zorg, Yan's clock read 50 years and Isabelle's read 30 years. This makes complete sense to you. Isabelle's message to you, however, states that, when Zorg passed her, Yan's clock read 50 years, her clock read 30 years, but that your clock read only 18 years. This is consistent with Isabelle observing both your clock and Yan's clock to be running slow, but with Yan's clock 32 years ahead of your clock. During her trip, according to Isabelle, 18 years ticked by on both your clock and Yan's clock.

Differences in clock readings are NOT associated with the travel time of light

You should be clear that the different times and lengths recorded by observers in different reference frames are not caused by the observers being separated by some distance and light taking some time to travel from one observer to another. When we say things like "When Isabelle reaches Zorg, from her reference frame your clock reads 18 years," that is not the reading that Isabelle would see on your clock at that instant, looking back many light-years through a powerful telescope to Earth. Instead, we imagine multiple observers at different locations in Isabelle's reference frame, their clocks synchronized with Isabelle's. Each of these observers, when they pass Earth, can send a message to Isabelle to tell her what their clock reading was, and what your clock was reading, when they passed Earth. Many years after passing Zorg, Isabelle will finally get the message confirming what your clock was reading when she passed Zorg, according to an observer in her reference frame. Thus, the statement "When Isabelle reaches Zorg, from her reference frame your clock reads 18 years" is interpreted as "When Isabelle reaches Zorg, the observer in Isabelle's frame of reference who is next to your clock records that your clock reads 18 years." The observer is right there, so light travel time is not an issue.

Related End-of-Chapter Exercises: 44 – 46.

Essential Question 26.5: According to Isabelle, how many years after passing Zorg would she get the message stating what your clock read when she passed Earth, from the observer in Isabelle's frame of reference who sees Earth pass by when Isabelle sees Zorg pass by?

Answer to Essential Question 26.5: According to Isabelle, she and the relevant observer in her reference frame are separated by 24 light-years, the distance between Earth and Zorg in her reference frame. The fastest the message could be sent would be at light speed, in which case the message would take 24 years to reach Isabelle.

Chapter Summary

Essential Idea: Special Relativity.

The closer relative speeds get to the speed of light, the more time and space exhibit unusual behaviors. This includes (but is not limited to) clocks in different reference frames running at different rates, lengths measured to be different by different observers, and events that are simultaneous for one observer taking place at different times for other observers.

The Postulates of Special Relativity

Relativity is based on two simple ideas, or postulates. One implication of these postulates is that nothing can travel faster than the speed of light in vacuum, which is $c = 3.00 \times 10^8$ m/s.

- 1. The speed of light in vacuum is the same for all observers.
- 2. There is no preferred reference frame. The laws of physics apply equally in all reference frames.

The Spacetime Interval

Observers in different constant-velocity reference frames always agree on the value of the spacetime interval between two events, as defined by

$$(c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2 = ($$
spacetime interval $)^2$, (Eq. 26.1: the spacetime interval)

where $c \Delta t$ and $c \Delta t'$ are the time intervals, converted to distance units, between the two events as measured from two different frames of reference, and Δx and $\Delta x'$ are the spatial separations between the two events in the same two reference frames. If the left-hand side of the equation gives a negative number, it is appropriate to reverse the order of the terms.

Proper time and time dilation

An observer measures the proper time interval Δt_{proper} between two events when that observer is present at the location of both events. Observers for whom the events take place in different locations measure a longer time interval Δt between the events.

$$\Delta t = \frac{\Delta t_{proper}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \ \Delta t_{proper} \,,$$

(Equation 26.2: **Time dilation**)

where *v* is the relative speed between the observers.

Length contraction

An observer measures the proper length L_{proper} between two points when the points are at rest with respect to that observer (that is, the observer is in the same frame of reference as the points). For an observer in a different reference frame, the observer measures a contracted length L between the points along the direction of motion. v is the relative speed between the observers.

$$L = \sqrt{1 - \frac{v^2}{c^2}} L_{proper} = \frac{L_{proper}}{\gamma}.$$

(Equation 26.3: Length contraction)

End-of-Chapter Exercises

Exercises 1 – 8 are primarily conceptual questions designed to see whether you understand the main concepts of the chapter.

- 1. Return to the Jack and Jill situation in Exploration 26.1. Let's now add a third observer, Martin, who, according to Jack, is traveling in the same direction as Jill but at a different speed. From Martin's point of view, the charges in Jack's reference frame move apart at the same rate that the charges in Jill's reference frame move apart. Explain qualitatively how this is possible.
- 2. If magnetism is a relativistic phenomenon, then we should be able to explain effects that we previously attributed to magnetic interactions without resorting to magnetism. An example is the situation shown in Figure 26.15, in which an object with a negative charge has an initial velocity v directed parallel to a long straight wire. The current in the long straight wire is directed to the left. (a) First, use the principles of physics we discussed in Chapter 19 to explain how to determine the direction of the force exerted by the wire on the moving charge, and state the direction of that force. (b) Now, we'll work through the process without magnetism. Imagine that the current in the wire is associated with electrons flowing to the right with the same speed, v, that the charged object has. The wire, when no current flows, has an equal amount of positive and negative charge. We can imagine the positive charges to be at rest in the original frame of reference, shown in Figure 26.15(b). If we look at the situation from the point of view of the charged object, however, which charges in the wire are moving, the positive charges or the negative charges? (c) From the frame of reference of the charged object, will the distance between the electrons in the wire be length-contracted? What about the distance between the positive charges in the wire? (d) Again from the frame of reference of the charged object, will the wire appear to be electrically neutral, or will it have a net positive or net negative charge? (e) Looking at the electric interactions only, will the wire exert a net force on the charged object? If so, in what direction is it, and is this direction consistent with the answer to part (a)?



Figure 26.15: (a) A negatively charged object near a long straight current-carrying wire, and (b) a close-up of the wire showing the moving electrons and stationary positive charges making up the wire, for Exercise 2.

- 3. According to Michael, two events take place at the same location, but are separated in time by 5 minutes. Jenna is moving at a reasonable fraction of the speed of light with respect to Michael. According to Jenna, (a) do the events take place at the same location, and (b) is the time interval between them longer than, equal to, or shorter than 5 minutes?
- 4. When you and your rocket are at rest on the Earth, you measure the length of your rocket to be 60 m. When you and your rocket are traveling at 80% of the speed of light with respect to the Earth, how long do you measure your rocket to be?

- 5. Consider events A and B on the spacetime diagram shown in Figure 26.16. Do the events occur at the same time, or at the same location, for any of the four observers? Explain.
- 6. You are on a train that is traveling at 70% of the speed of light relative to the ground. Your friend is at rest on the ground, a safe distance from the track, watching the train go past. Which of you measures the proper length for the following? (a) The length of the train. (b) The distance between the two rails the train is riding on. (c) The distance between the two cities the train is traveling between.



Figure 26.16: This spacetime diagram shows worldlines for four observers, whose motion is confined to the *x*-axis, as well as the spacetime coordinates of two events, in Sam's frame of reference. For Exercise 5.

- 7. Return to the Earth, Zorg, you and Isabelle situation from Exploration 26-4, and the spacetime diagram for this situation that was drawn for it, from your frame of reference, in Figure 26.14. Now sketch the spacetime diagram for this situation from Isabelle's frame of reference.
- 8. One way that observers in the same reference frame can synchronize their clocks is to use a light pulse. For instance, you and your friend, who is located some distance from you, might agree to both set your clocks to read zero when you each observe a light pulse that is emitted from a source exactly halfway between you and which spreads out uniformly in all directions. Comment on this possible alternate method of clock synchronization. Your friend can bring his clock to where you are, you can synchronize them, and then your friend can carry his clock back to his original location. What, if anything, is wrong with that method?

Exercises 9 – 13 involve the spacetime interval.

- 9. According to you, two events take place at the same location, with a time interval of 60 meters of time between them. According to a second observer, the time interval between the events is 75 meters of time. (a) How many nanoseconds does 60 meters of time represent? (b) What is the distance between the locations of the two events, according to the second observer?
- 10. Four observers watch the same two events. The time interval and spatial separation between the events, according to each of the observers, is shown in Table 26.1. Fill in the blanks in the table.

Table 26.1: The time interval and
spatial separation between the same
two events, according to four
different observers, for Exercise 10.

Observer	Time interval	Spatial separation
Anna		7.0 m
Bob		14.0 m
Caroline	35.7 meters of time	21.0 m
Dewayne	50.0 meters of time	

11. According to you, two events take place simultaneously at locations that are 100 m apart. According to a second observer, the spatial separation between the locations of the events is 150 m. What is the time interval between the events, according to the second observer?

- 12. According to you, two events take place 100 ns apart with a spatial separation between them of 18 m. (a) What is the spacetime interval for the two events? (b) For a second observer, the two events occur at the same location. What is the time interval between them, in nanoseconds, according to the second observer?
- The spacetime diagram in Figure 26.17 shows three events. Determine the spacetime interval between (a) Event 1 and Event 2, (b) Event 1 and Event 3, and (c) Event 2 and Event 3.



Figure 26.17: A spacetime diagram showing three events, for Exercise 13.

Exercises 14 – 18 involve drawing and interpreting spacetime diagrams.

- 14. Consider the spacetime diagram shown in Figure 26.18, for four people whose motion is confined to the *x*-axis. The squares on the grid measure $1 \text{ m} \times 1 \text{ m}$. (a) Which worldline is impossible? Why? (b) What is Keith's velocity with respect to Erica? (c) What is Sai's velocity with respect to Erica?
- 15. Consider the spacetime diagram shown in Figure 26.18. (a) What are the coordinates of events A and B, according to Erica? (b) What is the spacetime interval for these two events? (c) According to Sai, what is the spatial separation between events A and B? (d) According to Sai, what is the time difference between the events?
- 16. Consider the spacetime diagram in Figure 26.19, which shows worldlines for you and for two other astronauts. All the motion is confined to the *x*-axis. (a) What is Sasha's velocity with respect to you? (b) What is Michel's velocity with respect to you? (c) What is your velocity with respect to Michel?
- 17. Consider the spacetime diagram shown in Figure 26.19. (a) Add two events to the spacetime diagram that, according to you, happen at a distance of 5 light-years from you, but which are separated by a time interval of 4 years. (b) Add two events to the spacetime diagram that happen at different times, but at the same location, according to Sasha.







Figure 26.19: A spacetime diagram showing worldlines for three astronauts, for Exercises 16 and 17.

18. Draw a spacetime diagram, with units of meters, showing (a) your worldline, and (b) the worldline of a rocket that, according to you, passes through x = +100 m at t = 0 and travels in the positive *x*-direction at 60% of the speed of light. (c) According to you, what is the rocket's location at t = +600 meters of time?

Exercises 19 – 22 involve time dilation.

- 19. According to you, an astronaut's trip from Earth to a distant planet takes 50 years. The distant planet is in the same frame of reference as the Earth, and the astronaut travels at constant velocity between the two planets. (a) How long does the trip take, according to the astronaut, if the astronaut's speed with respect to you is (i) 5% of the speed of light, (ii) 50% of the speed of light, and (iii) 95% of the speed of light? (b) What is the distance between the planets, according to you, for the three different cases in part (a)?
- 20. Light takes 500 s to travel from the Sun to Earth. In the frame of reference of the light, how much time does the trip take?



21. Figure 26.20 shows the path followed by a light pulse in Randi's light clock. Randi and her clock are moving at constant velocity to the right with respect to you. What is Randi's speed with respect to you?

Figure 26.20: The path followed by a light pulse in Randi's clock, as observed by you. For Exercise 21. The grid on the diagram is square, according to you.

22. According to you, the distance between the Earth and a distant star is 1000 light-years. You and the star are in the same reference frame as the Earth. (a) According to you, what is the shortest time a message could be sent from the Earth to the star? (b) In the frame of reference of Megan, who is traveling at constant velocity between the Earth and the star, could the trip take only 10 years? If not, explain why not. If so, determine Megan's speed with respect to you. (c) Which observer, you or Megan, measures the proper time between the two events of Megan passing Earth and Megan passing the distant star? Explain.

Exercises 23 – 25 involve length contraction.

- 23. According to you, the distance between the Earth and a distant star is 1000 light-years. You and the star are in the same reference frame as the Earth. (a) In the frame of reference of Rajon, who is traveling at constant velocity between the Earth and the star, could the Earth and the star be separated by a distance of only 5 light-years? If not, explain why not. If so, determine Rajon's speed with respect to you. (b) Which observer, you or Rajon, measures the proper length between the Earth and the star? Explain.
- 24. A rocket is passing by the Earth traveling at 80% of the speed of light in a direction parallel to the length of the rocket. As the rocket passes, you measure the rocket's length to be precisely 100 m, using a tape measure in which lines are marked every millimeter. According to an observer moving with the rocket, (a) what is the length of the rocket, and (b) how far apart are the marks on your tape measure?
- 25. If you examine Figures 26.9 and 26.10 carefully, you will notice that the images of the mirror, and the images of the emitter/detector, are shorter in Figure 26.10 than they are in Figure 26.9. (a) What is the explanation for this? (b) For the particular situation described in Exploration 26.3, how much shorter are the mirrors in Figure 26.10 compared to those in Figure 26.9?

Exercises 26 – 31 involve ideas from General Relativity. In case you are intrigued by Einstein's ideas and you would like to know more, these exercises will give you some starting points for further reading.

- 26. These days, GPS (Global Positioning System) units can be carried by hikers and sailors, and are built into cars and airplanes, to provide accurate information about someone or something's position on Earth. Do some research regarding how and why the clocks on the GPS satellites are corrected for effects associated with General Relativity, and write a couple of paragraphs about this.
- 27. In 1960, Robert Pound and Glen Rebka did an interesting experiment at Harvard University to prove that the frequency of light is affected by gravity. Do some reading about their experiment, describing how gravity affects the frequency of light, and how the Pound and Rebka experiment verified the effect.
- 28. Joseph Taylor and Russell Hulse won the Nobel Prize for Physics in 1993. Do some reading about their research, and write a couple of paragraphs about it. Be sure to mention how their work is connected to General Relativity.
- 29. LIGO and LISA are acronyms for two large physics experiments that relate to General Relativity. Do some research about them, first to determine what these acronyms stand for, and then so you can write up a couple of paragraphs about how the experiments are designed to work and what they are trying to find.
- 30. A famous experiment in 1973 to test the predictions of relativity theory involved atomic clocks, which are incredibly accurate time-keepers. One atomic clock was placed on an airplane that circled the Earth going eastward. A second clock was placed on a plane that went westward around the Earth. A third clock was left at the US Naval Observatory. When the clocks were all brought back together, they showed that slightly different time intervals had gone by. Could the principles of physics be used to explain the observations? Was it Special Relativity or General Relativity that was more important in the experiment? Do some reading about the experiment, and write a couple of paragraphs describing the implications of the results.
- 31. In 1919, Arthur Eddington led an expedition to the island of Príncipe in an effort to test one of Einstein's predictions regarding how light is influenced by gravity. What did the expedition determine, and why did they have to go to Príncipe to do this?

General problems and conceptual questions

- 32. The spacetime interval between two events is 40 m. The events have a spatial separation of (1) 18 m, according to Gary, (2) 8 m, according to Megan, and (3) 0 m, according to Shawn. What is the time interval, in meters of time, between the events, according to (a) Gary, (b) Megan, and (c) Shawn.
- 33. Review the discussion of the Rossi and Hall experiment in Section 26.3, and assume that the muons are traveling at 99% of the speed of light. (a) From the point of view of an observer at rest on the Earth, how much time elapses on the clock of an observer who is in the reference frame of the muons? (b) From the point of view of an observer in the reference frame of the muons, what is the vertical distance between the top of Mt. Washington and sea level, and how long does it take a muon to traverse that distance?

- 34. Let's investigate a light clock that is moving with respect to you, but moving in a direction perpendicular to the mirror instead of parallel to it as in Exploration 26.3. The clock is shown in Figure 26.21. You observe the emitter and the mirror to be separated by 90 cm, and the clock to be moving at 50% of the speed of light. Note that this situation parallels the Brandi and Mia situation from Chapter 4, in which Mia ran on a moving sidewalk. (a) How long does a light pulse take to travel from the emitter to the mirror, according to you? (b) How long does the pulse take to travel back from the mirror to the detector, according to you? (c) What is the total round trip time for the light pulse in the clock, according to you? (d) If you have an identical clock, what is the round trip time for a light pulse in your clock? (e) Is the time dilation equation (Equation 26.2) valid for this situation? Explain why or why not.
- 35. A spacetime diagram, showing worldlines for four observers whose motion is confined to the *x*-axis, is shown in Figure 26.22. (a) Which two observers have the same velocity? What is the spatial separation between the two observers with the same velocity, according to (b) Sam, (c) Jen, and (d) Marco?
- 36. (a) What is the spacetime interval between events A and B on the spacetime diagram in Figure 26.22? According to Jen, what is (b) the spatial separation between the two events, and (c) the time interval between the two events? (d) Do any of the other three observers agree with Jen on the answers to (b) and (c)? If so, who agrees with Jen?
- 37. The spacetime diagram in Figure 26.23 shows the spacetime coordinates of three events, as measured by a particular observer. Determine the spacetime interval between (a) events A and B, (b) events A and C, and (c) events B and C.
- 38. The spacetime diagram in Figure 26.23 shows the spacetime coordinates of three events, as measured by a particular observer whose worldline follows the *ct* axis. (a) According to a second observer, who is traveling at constant velocity along the *x*-axis with respect to the first observer, events A and B occur at the same location. What is the speed of the second observer with respect to the first observer, who is traveling at constant velocity along the *x*-axis with respect to the first observer, events A and B occur at the same location. What is the speed of the second observer with respect to the first observer, who is traveling at constant velocity along the *x*-axis with respect to the first observer, events C and B occur simultaneously. What is the speed of the third observer?



Figure 26.21: A light clock that is moving in a direction perpendicular to the plane of its mirror. For Exercise 34.



Figure 26.22: This spacetime diagram shows worldlines for four observers, whose motion is confined to the *x*-axis, as well as the spacetime coordinates of two events, in Sam's frame of reference. For Exercises 35 - 36.





- 39. You are standing on a train platform that has a length of 150 m, according to you. According to a passenger on the train, the train has a length of 250 m. As the train passes through the station at very high speed, however, you observe the length of the train to be exactly the same as the length of the platform, 150 m. If we define event A to be the front of the train passing one end of the platform, and event B as the rear of the train passing the other end of the platform, you observe that the two events are simultaneous. (a) What is the speed of the train, according to you? (b) What is the spacetime interval between the events A and B? (c) What is the time interval between the two events, according to the passenger on the train?
- 40. You are on a train that is traveling at 70% of the speed of light relative to the ground. Your friend is at rest on the ground, a safe distance from the track, watching the train go past. According to you, the train has a length of 300 m, and the distance between the two cities the train is traveling between is 1200 km. What does your friend measure for (a) the length of the train, and (b) the distance between the two cities the train is traveling between.
- 41. Return to Exercise 40. Assuming the velocity of the train is constant, what is the time taken by the train to travel between the two cities according to (a) you, and (b) your friend?
- 42. *An introduction to the pole-and-barn paradox.* A particular pole is 10 m long, according to Paul, who is at rest with respect to the pole. According to you, the distance between the front and back doors of a barn is only 5 m. However, Paul runs fast enough, while holding the pole parallel to the ground, that the pole appears to be just under 5 m long, according to you. Thus, with the barn's front door open and back door closed, Paul can run through the front door of the barn, and you can close the front door and your assistant can simultaneously open the back door so that Paul passes through the barn without any trouble. The apparent paradox comes when the situation is looked at from Paul's perspective. According to Paul, (a) what is the length of the pole, and (b) what is the distance between the two barn doors? (c) Is it possible for Paul's pole, according to Paul, to be entirely inside the barn, as you saw it to be? (d) According to you, Paul can pass through the barn without the pole hitting either door. According to Paul, is this possible? Either explain this apparent paradox yourself, or do some background reading about the barn and pole paradox to resolve it.
- 43. Table 26.2 gives the readings on the clocks of three observers, as recorded by one of the three observers. The relative velocities between the observers remain constant at all times. (a) Complete the table, filling in the missing readings. (b) Which of the three observers is recording the clock readings? Explain. (c) What is Sarah's speed with respect to Josh?

	Josh	Rachel	Sarah
Event A	2 hours	0	2 hours
Event B	5 hours		6 hours
Event C	11 hours	3 hours	

 Table 26.2: A table of clock readings for various events, as recorded by one of the three observers.

- 44. Let's return to the Isabelle, you, and Yan situation that was introduced in Section 26-4, and elaborated on in Section 26-5. You and Yan, who are in the same reference frame but separated by 40 light-years, according to you, agree that you will set your clocks to zero when a light pulse, emitted from the point midway between you and Yan, reaches you. The light travels at the speed of light in vacuum. Isabelle, who is traveling at 80% of the speed of light while traveling from Earth, where you are, to Zorg, where Yan is, happens to pass you at the same time the light pulse reaches you, so she sets her clock to read zero at the same time you do. (a) According to you, how long does it take the light pulse to reach you after it is emitted? (b) According to Isabelle, how far is it from the point where the light pulse was emitted to you? Isabelle agrees that the point where the light pulse is emitted is halfway between you and Yan. (c) According to Isabelle, how long does the light pulse take, after being emitted, to reach Yan? Note that, according to Isabelle, the light pulse and Yan are both moving. (d) According to Isabelle, how long does the light pulse take, after being emitted, to reach you? (e) On Isabelle's clock, how much time elapses between the light pulse reaching Yan and the light pulse reaching you? (f) According to Isabelle, how much time elapses on Yan's clock between the light pulse reaching Yan and the light pulse reaching you? Note that this answer should agree with the information in Section 26-5.
- 45. Return to Exercise 44. Sketch a spacetime diagram, from your frame of reference, for the situation described in Exercise 44. Draw worldlines for you, Yan, Isabelle, the light pulse that travels from the midpoint between you and Yan to you, and the light pulse that travels from the midpoint between you and Yan to Yan.
- 46. Repeat Exercise 45, but now sketch the spacetime diagram for Isabelle's frame of reference.
- 47. You are traveling along the intergalactic freeway in your personal rocket. A large transporter in the next lane, which has the same velocity as you, is exactly 4 times the length of your rocket. An identical transporter in the oncoming lane, however, appears to be the same length as your rocket. Relative to you, at what speed is transporter in the oncoming lane traveling?
- 48. Two students are considering a particular question. Comment on the part of their conversation that is reported below.

Erin: The question says "As the rocket goes by at 75% of the speed of light, observers in the Earth's reference frame mark the locations of the tip and tail of the rocket at the same time. They then measure the distance between the marks to be 60 meters. What is the length of the rocket?" Uh, 60 meters, right?

Katie: Doesn't the speed have something to do with it? Don't we have to use the length contraction equation to find the length of the rocket in the rocket's frame of reference?

Erin: Well, the question doesn't really say, right? It asks for the length of the rocket, but according to who? Who's the observer?

Katie: What if we did two answers, one 60 meters, and the other the length that's seen by somebody in the rocket?

Erin: OK, I'll go with that. Now, length gets contracted, so the other length should be less than 60 meters, right?

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Chapter 26: Additional Resources

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Additional Links

• <u>Time Dilation Simulation, by Walter Fendt and Taha Mzoughi</u>

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