25-1 Interference from Two Sources

In this chapter, our focus will be on the wave behavior of light, and on how two or more light waves interfere. However, the same concepts apply to sound waves, and other mechanical waves. We will begin by considering two sources, separated by some distance, which are broadcasting identical single-frequency waves in phase with one another. The sources could be two speakers of sound, or could be two sources of light waves. We briefly discussed this situation at the end of section 21-7, and we will now investigate this quantitatively and in detail.

To begin with, we can represent the waves emitted by each source by a set of concentric circles, with a dark region corresponding to a trough in the wave, and a white region corresponding to a peak in the wave. If we then overlap the circles, as shown in Figure 25.1, we get an interesting pattern that is the result of interference between the two sets of the waves. The dark lines radiating out from the center of the pattern correspond to destructive interference, while the bright areas correspond to constructive interference. If you set up two speakers broadcasting identical single-frequency sounds, you can create an interference pattern like this, and you can walk through it to hear areas of constructive and destructive interference.

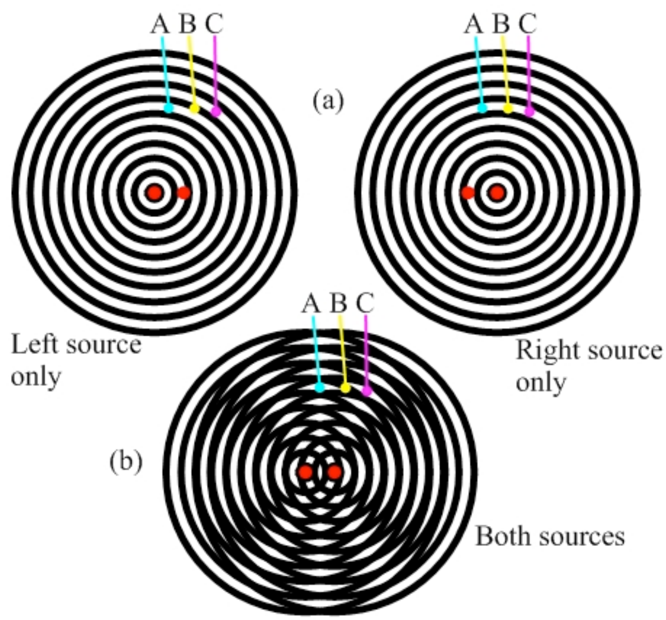


Figure 25.1: (a) The waves emitted by two sources can be represented by a pattern of concentric circles (in three dimensions they are a set of spherical shells) that expand out from the source at the wave speed. (b) The pattern of constructive and destructive interference that results when both sources emit waves simultaneously can be seen when the two sets of concentric circles are overlapped.

The interference pattern in Figure 25.1 (b) looks complicated, but we can understand it using interference ideas. First, let's define the path-length difference as the difference between the distance a point is from one source and the distance the point is from the second source. For point *A* in Figure 25.1 (b), which is on the perpendicular bisector of the line connecting the sources, the path-length difference is zero because point *A* is equidistant from both sources. Because the sources are in phase with one another, at the instant a peak in the wave is emitted by the left source, a peak is also emitted by the right source. These peaks travel the same distance to point *A* at equal speeds, and thus they arrive at *A* simultaneously. Two peaks arriving at the same time produce constructive interference. This argument holds for any point on the perpendicular bisector to the line connecting the speakers, because all those points have a path-length difference of zero.

Point B in Figure 25.1(b) is closer to the right source than it is to the left source, and thus the path-length difference is not zero. Point B happens to be exactly half a wavelength farther from the left source than it is from the right source. When a peak emitted by the right source reaches point B, the peak that was emitted at the same time from the left source is still half a wavelength from point B. Half a wavelength from a peak is a trough, so a trough arrives at point B from the left source at the same time a peak arrives from the right source (and vice versa), leading to destructive interference. All such points that are half a wavelength farther from one source experience destructive interference.

Point C in Figure 25.1(b) is one wavelength closer to source 2 than it is to source 1, so when a peak emitted by source 2 reaches point C, the peak that was emitted at the same time from source 1 is still a wavelength from point C. A full wavelength from a peak is another peak, so peaks arrive at C simultaneously from the two sources, leading to constructive interference. All such points that are a full wavelength farther from one source than the other experience constructive interference.

The trend continues. The bottom line is that all locations that are an integer number of wavelengths farther from one source than the other experience constructive interference, and all locations that are an integer number of wavelengths plus half a wavelength farther from one source than the other experience destructive interference. These general conditions for interference are summarized in the box below.

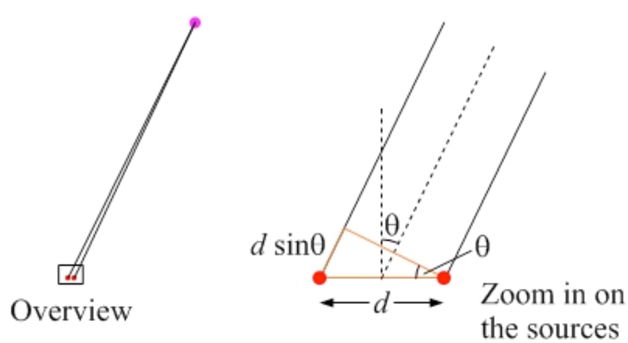
> For two sources, which are in phase with one another, that broadcast identical waves in all directions, the interference can be understood in terms of the path-length difference.
>
> $L_1 - L_2 = \Delta L = m\lambda$, (Equation 25.1: **condition for constructive interference**)
> where m is an integer.
>
> $L_1 - L_2 = \Delta L = (m + 1/2)\lambda$, (Equation 25.2: **condition for destructive interference**)
> where m is an integer.

For locations that are far from the sources, in comparison to d, the distance between the sources, the waves from the two sources are essentially parallel to one another. As illustrated in Figure 25.2, the path-length difference in this case is given by $\Delta L = d \sin\theta$, where the angle θ is shown in Figure 25.2. Thus, in this situation the angles at which constructive or destructive interference occur are:

$d \sin\theta = m\lambda$, (Equation 25.3: **constructive interference, for two sources in phase**)
where m is an integer, and

$d \sin\theta = (m + 1/2)\lambda$, (Equation 25.4: **destructive interference, for two sources in phase**)
where m is an integer.

Figure 25.2: When a point is a long way from both sources, the geometry of the situation allows us to approximate the path-length difference in terms of d, the distance between the sources, and θ, the angle between the perpendicular bisector of the line joining the sources and the straight line going from the midpoint between the sources and the point.



Related End-of-Chapter Exercises: 4, 13 – 15.

Essential Question 25.1 (a) A particular point experiences constructive interference no matter what the wavelength is of the waves sent out by the sources. Where is the point?
(b) What happens to the angles at which destructive interference occurs when (i) the wavelength of the waves is decreased, and (ii) d, the distance between the sources, is decreased?

Answer to Essential Question 25.1 (a) The only points for which changing the wavelength has no impact on the interference of the waves are points for which the path-length difference is zero. Thus, the point in question must lie on the perpendicular bisector of the line joining the sources. (b) If we re-arrange Equation 25.3 to solve for the sine of the angle, we get

$$\sin\theta = \frac{m\lambda}{d}.$$

Thus, decreasing the wavelength decreases sin θ, so the pattern gets tighter. Decreasing d, the distance between the sources, has the opposite effect, with the pattern spreading out. We can understand the wavelength effect conceptually in that, when the wavelength decreases, we don't have to go as far from the perpendicular bisector to locate points that are half a wavelength (or a full wavelength) farther from one source than the other. Figure 25.3 shows the effect of decreasing the wavelength, or of decreasing the distance between the sources.

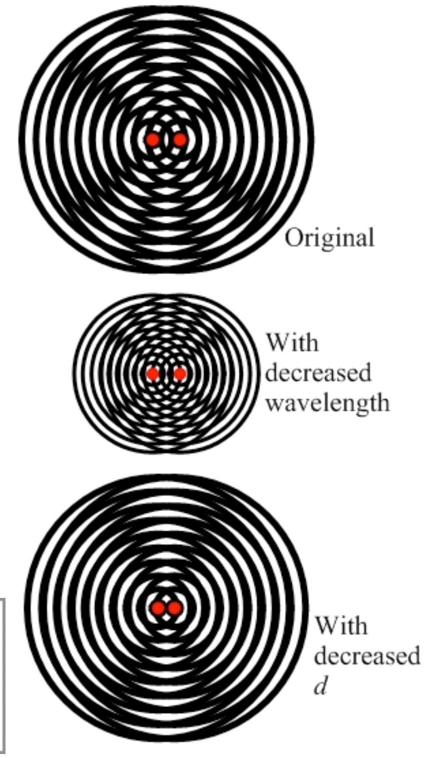


Figure 25.3: The top diagram shows the interference pattern produced by two sources. The middle diagram shows the effect of decreasing the wavelength of the waves produced by the sources, while the bottom diagram shows the effect of decreasing the distance between the sources.

25-2 The Diffraction Grating

Now that we understand what happens when we have two sources emitting waves that interfere, let's see if we can understand what happens when we add additional sources. The distance d between neighboring sources is the same as the distance between the original two sources, and the sources are arranged in a line. All the sources emit identical waves that are in phase.

EXPLORATION 25.2A – Adding sources

Step 1 – Consider a point a long way from two sources. The sources are a distance d apart. The point is one wavelength farther from one source than the other, so constructive interference occurs at the point. When we add a third source, so that we have three sources equally spaced in a line, separated by d, do we still get constructive interference taking place at the point?

Yes. As the diagram in Figure 25.4 shows, the path-length difference for the third source and the source it was placed closest to will also be one wavelength. Now we get constructive interference for three waves at once, not just two, so the amplitude of the resultant wave is larger than it was with only two sources.

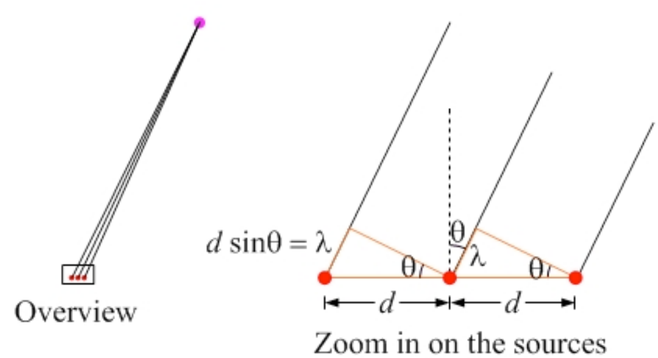


Figure 25.4: For a point that is one wavelength farther from one source than another, adding a third source results in even larger amplitude because of constructive interference.

Step 2 – If we consider a different point that is half a wavelength farther from one of two sources than the other, destructive interference occurs at the point. When we add a third source, so that we have three sources equally spaced in a line, separated by d, do we still get destructive interference taking place at the point?

No. The destructive interference at the point was caused by the cancellation between the waves from the first two sources. Adding a third source does not change the fact that the first two waves cancel one another, so there is nothing to cancel the third wave.

Step 3 – For three sources, what path-length difference (between zero and one wavelength) between neighboring sources results in completely destructive interference? With three sources, it turns out that there are two path-length differences between 0 and one wavelength that result in completely destructive interference, these being one-third and two-thirds of a wavelength.

Step 4 – What if we have N sources, where N is any integer greater than 1. Is there a general rule for predicting the angles at which constructive interference occurs? What about destructive interference? Constructive interference occurs at the same points for N sources that it does for 2 sources, so the equation $d \sin\theta = n\lambda$ still applies for situations with $N > 1$ sources.

There are $N - 1$ places where destructive interference happens in between each interference maximum, so we generally dispense with an equation for destructive interference when $N > 2$.

Key idea: The equation $d \sin\theta = m\lambda$ applies to any number of sources > 1, as long as the sources are equally spaced. With multiple sources, it is much easier to produce destructive interference than it is to produce completely constructive interference, so there is no simple equation for destructive interference. **Related End-of-Chapter Exercises: 7, 16 – 18, 38, 39, 48.**

The Diffraction Grating

A diffraction grating is essentially a large number of equally spaced sources, and thus the $d \sin\theta = m\lambda$ equation applies.

One application of diffraction gratings is in spectroscopy, which involves separating light into its different wavelengths, a process that astronomers, or chemists, can use to determine the chemical makeup of the source producing the light. In actuality, a diffraction grating is typically a glass or plastic slide with a large number of slits (long thin openings between long thin lines). A diffraction grating (which should probably have been named an interference grating) offers two main advantages over a double slit. First, the more openings the light passes through, the brighter the interference maxima are. Second, the more openings there are, the narrower the bright lines are in the interference pattern, which is important when trying to resolve two similar wavelengths. Figure 25.5 shows the increased sharpness that results from adding slits.

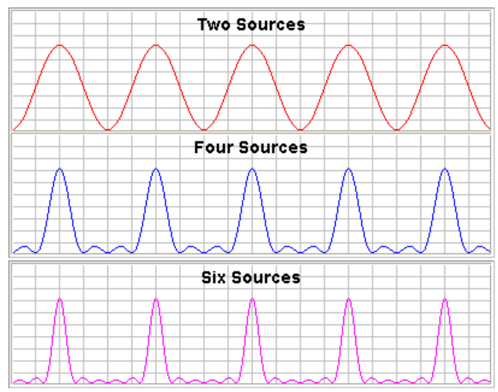


Figure 25.5: Adding sources (or slits that light goes through) results in sharper interference maxima. Each case shows the relative intensity at various points. The amplitude of the peaks also grows as sources are added.

EXPLORATION 25.2B – Double-slit geometry

When light of a single wavelength (say, from a laser) is incident on a double slit (or a diffraction grating, which gives a sharper pattern), we get a pattern of bright and dark fringes on a screen beyond the double slit. Such a pattern is shown in Figure 25.6. The bright fringes come from constructive interference, and the dark fringes come from destructive interference.

Step 1 – If we wanted to increase the distance between the bright spots on the screen, what would we change about d or λ?

This should be something of a review. If we re-arrange the equation for constructive interference, we get:

$$\sin\theta = \frac{m\lambda}{d} .$$

Increasing the value of $\sin\theta$ will increase the value of θ, which means that the lines of constructive interference will be further apart, in terms of the angles between them. This will spread out the pattern on the screen. To increase $\sin\theta$, we can either replace the first laser with a laser that emits light of a longer wavelength (switch from blue to red, for instance), or we can switch to a double slit that has a smaller value of d. These two options are illustrated in Figure 25.7.

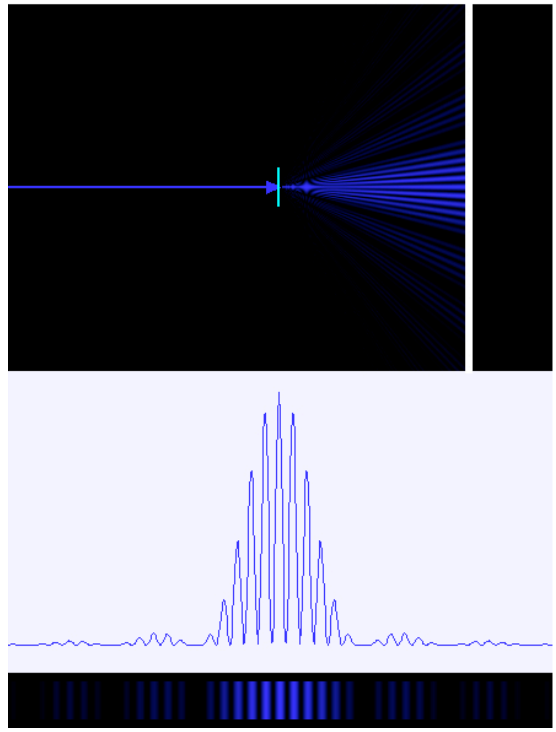


Figure 25.6: The geometry of the double-slit pattern, for blue light. The top shows an overview of the interference pattern. The bottom has the pattern of bright and dark fringes on the screen. In between is a graph of the intensity of the fringes in the pattern, as a function of position.

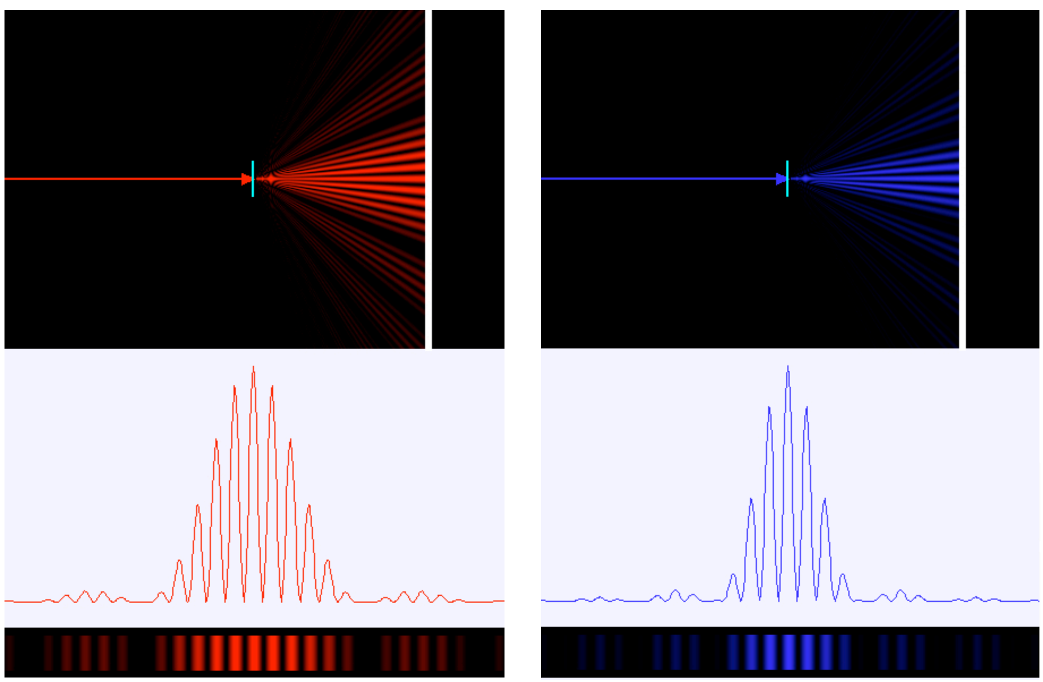


Figure 25.7: The pattern on the left shows how the situation of Figure 25.6 changes if we change to a larger wavelength, while the one on the left shows the change of switching to a smaller *d*.

Step 2 – What if we only have one laser and one double slit, so we can't change the wavelength or d. Is there a different way to increase the distance between the bright spots on the screen?
Yes. If we move the screen farther from the double slit, the screen will intercept the light from the grating after the bright lines in the pattern have been able to spread out farther, increasing the distance between the bright spots on the screen. This is illustrated in Figure 25.8.

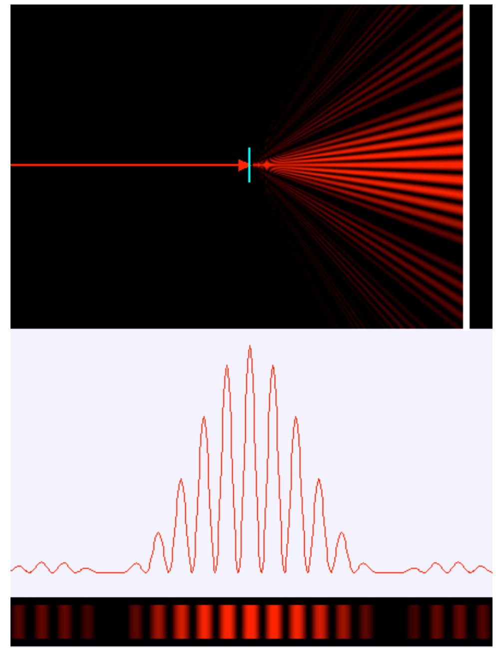


Figure 25.8: Spreading the dots out by moving the screen farther away, starting from the left-hand picture in Figure 25.7.

Step 3 – As shown in Figure 25.9, let's use L to denote the distance from the double slit to the screen, and y_m to denote the distance from the central bright spot on the screen to the m^{th} bright spot. For instance, y_4 is the distance from the center of the pattern to one of the m = 4 bright spots on the screen. If the angle is small (say, 10˚ or less), we can use the approximation $\sin\theta \approx \tan\theta$. Using that assumption, derive an expression for y_m in terms of d, m, λ, and L .
We have two equations to work with here, one for $\sin\theta$, from above, and then one for $\tan\theta$, from the geometry of right-angled triangles.

$$\sin\theta = \frac{m\lambda}{d} \quad \text{and} \quad \tan\theta = \frac{y}{L} .$$

The small-angle approximation enables us to set these two equations equal to one another. Doing that and solving for y gives:

$$y_m = \frac{m\lambda L}{d} .$$

(Eq. 25.5**: The distance from the center of the pattern to the m'th bright spot)**

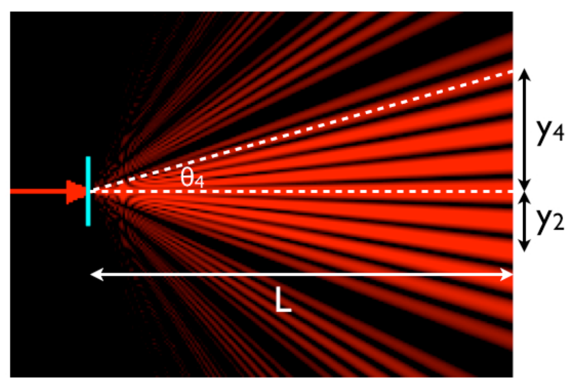


Figure 25.9: The geometry of the pattern, based on a right-angled triangle.

Step 4 – What would you do if you wanted to predict the position of a particular bright spot on the screen, but you could not use the small-angle approximation?
If we could not use the small-angle approximation, we could first use the $\sin\theta$ equation to find θ, and then take the tangent of that angle when we were using the $\tan\theta$ equation to find y.

Essential Question 25.2: A beam of light made up of three wavelengths, 660 nm (red light), 530 nm (green light), and 400 nm (violet light) is incident on a diffraction grating that has a spacing of d = 1300 nm. The first order spectrum, consisting of a violet line, a green line, and a red line, produced by the grating is shown in Figure 25.10. What are the colors of the other three beams (1 – 3) that come from the grating?

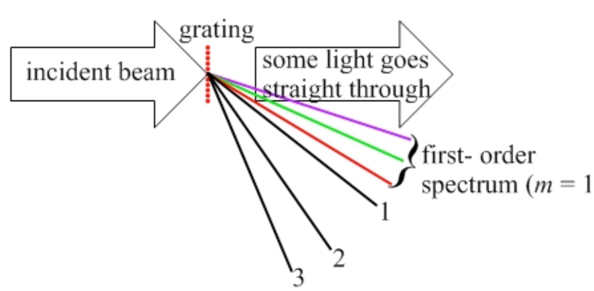
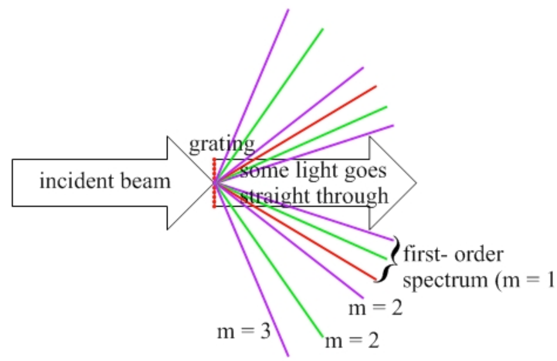


Figure 25.10: The first-order (m = 1) spectrum for the situation of Essential Question 25.2.

Answer to Essential Question 25.2: You might think that beams 1, 2, and 3 would also be violet, green, and red, respectively. However, in the equation $\sin\theta = m\lambda / d$, the right-hand side cannot exceed 1, because that is the limit on sin θ. If we use the three wavelengths with m = 2 or m = 3, we get the values shown in Table 25.1. It is possible to see the second-order violet and green lines, and the third-order violet lines, but none of the others because they correspond to values of sin θ that are greater than 1, and are thus not possible. The beams are violet, green, and violet.



Order	Violet (400 nm)	Green (530 nm)	Red (660 nm)
m = 2	$\sin\theta = 800/1300$	$\sin\theta = 1060/1300$	$\sin\theta = 1320/1300$
m = 3	$\sin\theta = 1200/1300$	$\sin\theta = 1590/1300$	$\sin\theta = 1980/1300$

Table 25.1: Values of sinθ for m > 1 for the situation of Essential Question 25.2.

Figure 25.11: The entire spectrum for the situation of Essential Question 25.2, showing how the light splits because of passing through the diffraction grating.

25-3 Diffraction from a Single Slit

In Section 25-1, we considered what happens when two sources broadcasting identical waves interfere. With light, we typically shine a laser beam through two closely-spaced slits (a double slit, in other words). Each slit acts as a source of waves, but it turns out that each slit does not send out light uniformly in all directions. Instead, a wave passing through a slit (or striking an object) experiences **diffraction**. Each point on the opening, or on the object, acts as a source of waves, and the resulting diffraction pattern is the result of the interference between all these waves. As Figure 25.12 shows, the waves interfere constructively in the forward direction, with more destructive interference in most other directions.

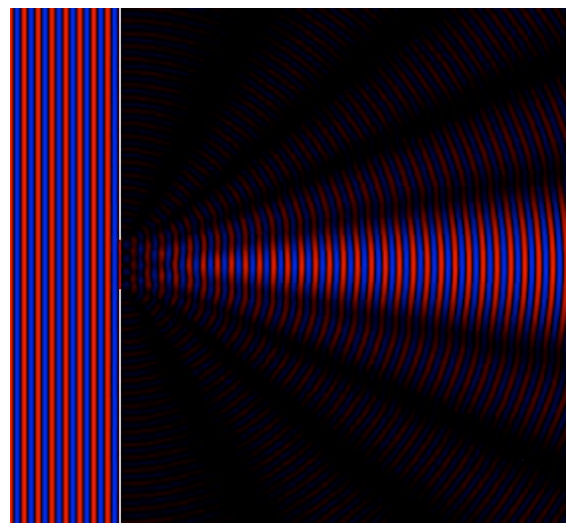


Figure 25.12: The diffraction pattern that results from a plane wave striking an opening that is four wavelengths wide. In color, the picture has alternating red (representing peaks), and blue (represents troughs) regions, separated by black regions denotes zero, or small, amplitude.

The graph in Figure 25.13, and the corresponding picture underneath the graph that shows the diffraction pattern from a laser shining through a single slit, show how much of the wave's energy is concentrated in the forward direction. The secondary peaks have much less intensity than the central maximum. The central maximum is also twice as wide as are the secondary peaks, at least at small angles.

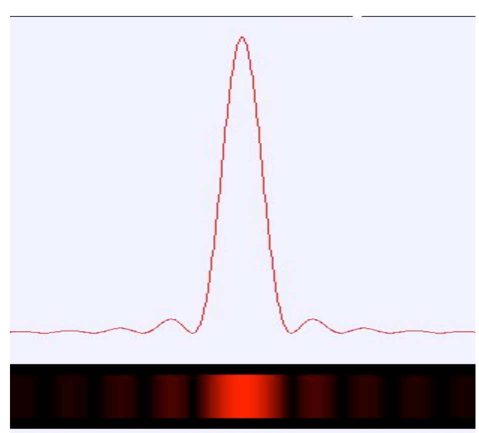


Figure 25.13: A graph of the intensity as a function of angle that corresponds to the diffraction pattern below the graph. The diffraction pattern comes from a laser shining on a single slit.

EXPLORATION 25.3 – An equation for the single slit

Step 1 – ***Return to Figure 25.2, which illustrates how waves from two sources can interfere constructively at a particular point if the path length difference is, for instance, one wavelength. Now turn this two-source situation into a single-slit situation by filling in the space between the original two sources with more sources.*** Figure 25.14 models a single-slit as being made up of a number of sources of waves laid out across the width of the opening. Note that while we use *d* to represent the distance between the two sources, we generally use *a* to represent the width of the single opening.

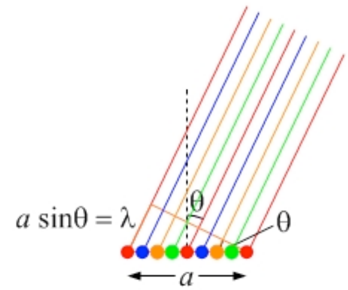


Figure 25.14: A modification of Figure 25.2, turning the double-source situation into a single-slit situation by filling the space between the original sources with additional sources.

Step 2 – ***The two sources in red that are at either end of the line of sources constructively interfere, in the situation shown, because their path-length difference is a full wavelength. What is the path-length difference for the source colored red that is in the middle of the set of sources, and the source at the left end of the line? What kind of interference is associated with these two sources?*** The path length for the left source is half a wavelength longer than the path length for the middle source, which corresponds to destructive interference.

Step 3 – ***What kind of interference results for the two blue sources, or the two orange sources, or the two green sources?*** The path-length difference for all these pairs of sources is half a wavelength, corresponding to destructive interference. For the point in question, the waves from the left half of the opening cancel the waves arriving from the right half of the opening.

Step 4 – ***If the equation d sinθ = mλ gives the angles at which constructive interference occurs for two sources, what does the equation a sinθ = mλ correspond to for the single slit?*** The equation $a \sin\theta = m\lambda$ gives the angles at which destructive interference occurs for the single slit.

$a\sin\theta = m\lambda$, (Equation 25.6: **destructive interference for a single slit**)

where *m* is an integer, and *a* is the width of the slit.

Key idea: The diffraction of waves passing through a single opening can be understood in terms of interference between waves leaving all points on the opening. The narrower the opening, the more the waves spread out. **Related End-of-Chapter Exercises: 19 – 21.**

Diffraction for sound waves

Diffraction takes place for other waves, such as water waves and sound waves, just as it does for light waves, with Equation 25.6 even applying for these other kinds of waves. Horn speakers, such as those shown in Figure 25.15, are often shaped to exploit diffraction, causing the waves to spread out in a particular dimension when they emerge from the end of the speaker.

Figure 25.15: These speakers are shaped to take advantage of diffraction for sound waves. The narrower the opening, the wider the diffraction pattern. The speakers are designed to diffract sound waves horizontally, where they can be heard by people on a train platform. Photo credit: A. Duffy.

Essential Question 25.3: As you are walking toward the open door to a room, you can hear the conversation between two people inside, even though you can't see the people. Explain why the sound waves are diffracted by the doorway, while the light is not.

Answer to Essential Question 25.3: A big difference between the sound waves and the light waves is the wavelength. The sound waves have wavelengths that are on the order of a meter, while the wavelengths of the light waves are about six orders of magnitude smaller. The width of the doorway is comparable to the wavelength of the sound waves, and so the sound waves experience significant diffraction. The doorway is so large compared to the wavelength of light, however, that the light goes in a straight line out the door, with negligible diffraction.

25-4 Diffraction: Double Slits and Circular Openings

The bottom graph in Figure 25.16 shows the relative intensity, as a function of position, of the light striking a screen after passing through a double slit. If each slit acted as a source of light, emitting waves uniformly in all directions, we would expect the peaks on the screen to be equally bright, as shown in the "Double Source" picture. Instead, each opening emits a diffraction pattern, as shown in the "Single Slit" picture. The interference between the two diffraction patterns results in the "Double Slit" pattern at the bottom, with the amplitude of the peaks predicted by the double-source equation being reduced by a factor given by the single-slit equation.

The "Double Slit" pattern exhibits a phenomenon known as **missing orders**. Peaks that are predicted in the pattern by the double-source equation, $d \sin\theta = m_d \lambda$, coincide with zeros from the single-slit equation, $a \sin\theta = m_s \lambda$, and are thus missing from the pattern.

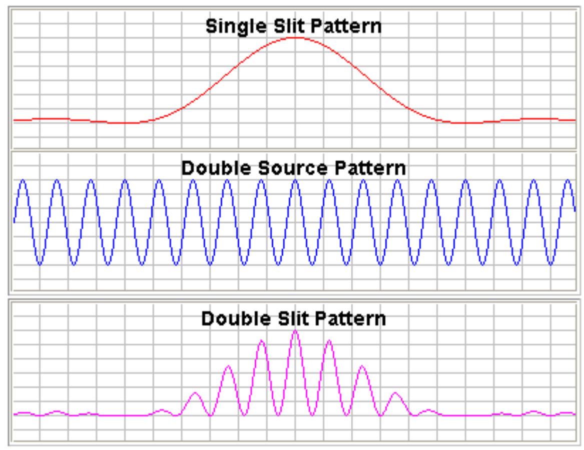


Figure 25.16: When light passes through a double slit, the interference maxima are not equally bright, but drop off quite dramatically in brightness as you move away from the center of the pattern, as shown at the bottom. The double-slit pattern is a combination of the single-slit pattern (at the top) and the double-source pattern (in the middle).

A bit of history

Prior to 1800, there was a big debate in physics about the nature of light. The Dutch scientist Christiaan Huygens (1629 – 1695) came up with a way to explain many optical phenomena (such as refraction) in terms of light acting as a wave. The main proponent of the particle theory, however, was Sir Isaac Newton (1643 – 1727), who called it the corpuscular theory. With the weight of Newton behind it, the particle model of light won out until Thomas Young's double-slit experiment in 1801, followed by the work of the Frenchman, Augustin Fresnel, who studied diffraction in the early 1800's.

In 1818, Siméon Poisson realized that if light acted as a wave, the shadow of a round object should have a bright spot at its center. The light would leave all points on the edge of the object, and constructively interfere to produce a bright spot at the center of the shadow, because that point has a path-length difference of zero. Poisson actually put forward the idea of the bright spot as a way to disprove the wave theory, so he was somewhat taken aback when Dominique Arago did an experiment to show that there really is such a bright spot. These days, it is easy to create the bright spot at the center of a shadow by diverging a laser beam with a lens and then shining the beam onto a smooth metal ball. The shadow produced by such an arrangement is shown in Figure 25.17.

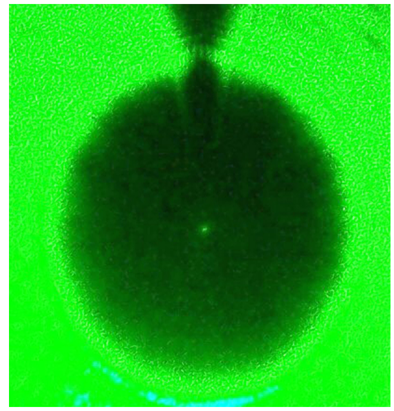


Figure 25.17: The bright spot at the center of the shadow of a ball bearing, demonstrating that light acts as a wave. Photo credit: A. Duffy.

Diffraction by a circular opening
A related and common phenomenon is diffraction by a circular opening (commonly called a circular aperture), such as the one we all look through, the pupil in each of our own eyes. For a circular opening, the angle at which the first zero occurs in a diffraction pattern is given by:

$$\theta_{min} = \frac{1.22\lambda}{D}$$ (Eq. 25.7: **The first zero in a diffraction pattern from a circular aperture**)

where D is the diameter of the opening. Note that the larger the diameter of the opening, the narrower the width of the central peak in the diffraction pattern. This dependence on the diameter of the opening has implications for how close two objects can be before you cannot resolve them. For instance, when you look up at the sky at night, two stars that are very close together may appear to you to be a single star. If you look at the same patch of sky through binoculars, or through a telescope, however, you can easily tell that you're looking at two separate stars. The light enters binoculars or telescopes through an aperture that is much larger than your pupil, and thus experiences much less diffraction.

It turns out that you can just resolve two objects when the first zero in the diffraction pattern associated with the first object coincides with the maximum in the diffraction pattern associated with the second object. Hence, Equation 25.7 gives the minimum angular separation between two objects such that you can just resolve them. Figure 25.18 illustrates the issue, where two objects are too close to be resolved by a human eye in bright sunlight, when the pupil is small, but can be resolved by the same eye when it is dark out, and the pupil has become larger to let in more light.

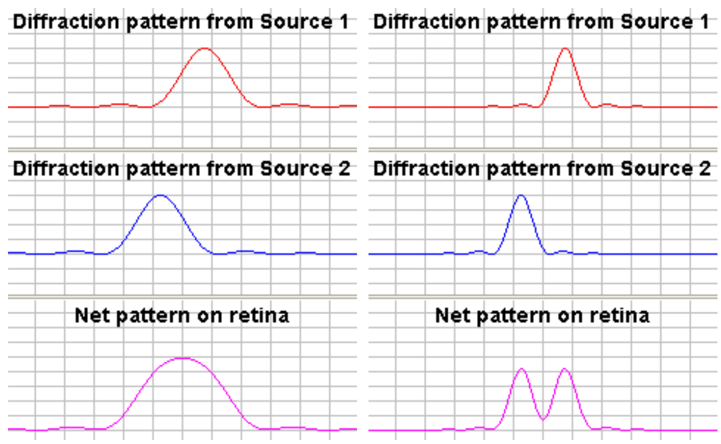


Figure 25.18: On the left, we see that the angular separation between two objects is too small for them to be resolved, with the patterns overlapping too much on the retina. The images on the left correspond to a human eye in bright sunlight, when the pupil is small. The images on the right correspond to the same situation, but viewed in the dark. In the dark, the pupil expands to let in more light, reducing the spreading associated with diffraction.

Related End-of-Chapter Exercises:
9, 22, 23, and 46.

Essential Question 25.4: Consider the double-slit pattern in Figure 25.16. Noting the location of the missing orders in the pattern, what is the ratio of d to a for this double slit? That is the ratio of the center-to-center distance between the two openings (d) to the width of each opening (a).

Answer to Essential Question 25.4: One way to answer this question is to set up a ratio of the double-source equation to the single-slit equation:

$$\frac{d\sin\theta}{a\sin\theta} = \frac{m_d\lambda}{m_s\lambda} \quad \Rightarrow \quad \frac{d}{a} = \frac{m_d}{m_s}\text{, for the same angle, }\theta.$$

The position corresponding to $m_s = 1$ is where we find the first zero on one side of the central maximum in the single-slit pattern. Looking at the double-slit pattern in Figure 25.19, and counting the peak in the center of that pattern as $m_d = 0$, we see that the peak at $m_d = 5$ lines up with the first zero in the single-slit pattern, and is thus a missing order. With $m_d / m_s = 5/1$, we have $d/a = 5$ here.

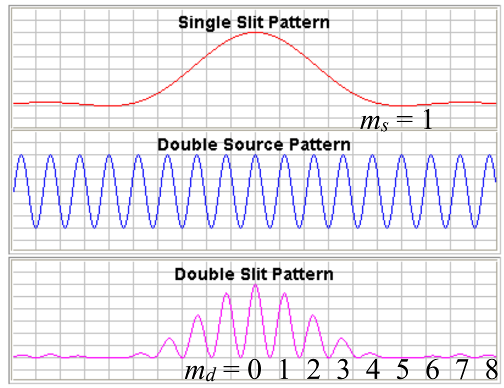


Figure 25.19: In this case, the first zero in the single-slit pattern corresponds to the same position, and therefore the same angle, as the $m_d = 5$ peak in the double-source pattern, leading to a missing order in the double-slit pattern.

25-5 Reflection

As we have discussed in Chapter 21 for waves on a string, when a wave reflects from the fixed end of a string, the reflected wave is inverted. When a wave reflects from the free end of a string, the reflected wave is upright.

What happens when the end of the string is neither perfectly free nor perfectly fixed, such as when a light string is tied to a heavy string? As shown in Figure 25.20 (a) and (b), when a wave is traveling along the light string, the point where the strings meet acts more like a fixed end than a free end. Part of the wave is transmitted onto the heavy string, and the part that reflects back along the light string is inverted. Conversely, as in Figure 25.20 (c) and (d), when a wave is traveling along the heavy string, the point where the strings meet acts more like a free end. Part of the wave is again transmitted into the second medium, while the part that reflects is upright.

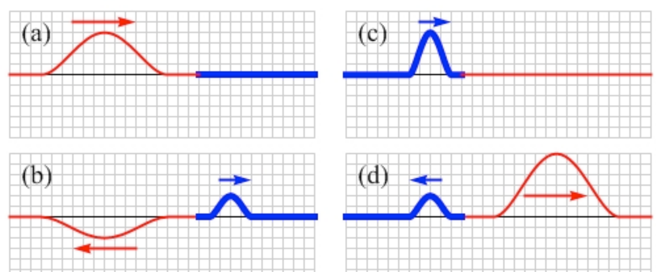


Figure 25.20: (a) When a wave traveling on a light string encounters the boundary between the light string and a heavy string, part of the wave is transmitted onto the heavy string, and part reflects back onto the light string, as in (b). The boundary acts like a fixed end, so the reflected wave is inverted. (c) If the wave is traveling along the heavy string before striking the boundary, the part of the wave that reflects is reflected upright, as in (d). In this situation, the speed of the wave on the light string is three times the speed of the wave on the heavy string.

An analogous process happens for light, or for any other electromagnetic wave. When a light wave traveling in one medium (medium 1) encounters an interface between that medium and a second medium (medium 2) with a different index of refraction, part of the light wave is transmitted into the second medium, and part is reflected back into the first medium. Whether the reflected wave is inverted or not depends on how the indices of refraction compare, as summarized in the box below, and as shown pictorially in Figure 25.21.

A light wave reflecting from a medium with a higher index of refraction than the medium the wave is traveling in ($n_2 > n_1$) is inverted upon reflection. If the second medium has a smaller index of refraction than the first ($n_2 < n_1$), the wave is reflected upright. For a sine wave, inverting the wave has the same effect as shifting the wave by half a wavelength, so we will treat an inversion upon reflection as a half wavelength shift.

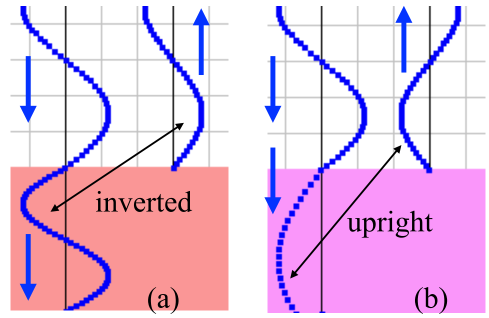


Figure 25.21: (a) When light traveling in one medium reflects from a medium with a larger index of refraction, the part of the wave that is reflected is inverted upon reflection. (b) If the second medium has a smaller index of refraction than the first, the reflected part of the wave reflects upright. In both cases, the reflected wave has been shifted to the right to distinguish it from the incident wave.

EXPLORATION 25.5 – Double-source interference with a single source

Figure 25.22 shows a situation in which a single source of sound waves is located above the floor. At any point, such as at point *A* in the figure, waves are received directly from the source, but waves are also received after being reflected from the floor.

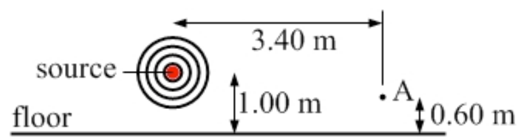


Figure 25.22: A source of sound waves, in air, located above a flat floor.

Step 1 – ***We can treat this situation as if there are two sources of waves. Where, effectively, is the second source located?*** The second source is where the image of the first source is located. Treating the floor like a plane mirror, reflecting the first source in the mirror gives the second source at the position shown in Figure 25.23.

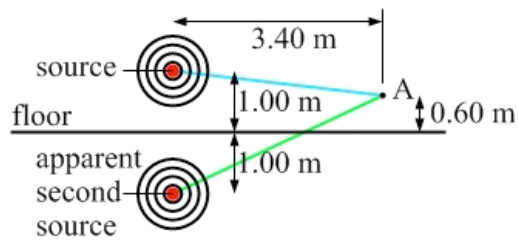


Figure 25.23: The floor acts like a plane mirror for sound waves. Effectively, there is a second source of waves where the image of the first source is created by the mirror.

Step 2 – ***To analyze the interference between the waves from the two sources, we consider the mirror-image source to be 180° out of phase with the first source. Explain why we do this.*** The 180° phase shift comes from the fact that the wave does not actually originate at the second source. Instead, it originates at the first source, and reflects from the floor, producing an inversion of the wave upon reflection. Inverting the wave is equivalent to shifting the wave half a wavelength, which is equivalent to a 180° phase shift.

Key idea: Even reflecting sound waves experience an inversion upon reflection.
Related End-of-Chapter Exercises: 8, 50 – 52.

Essential Question 25.5: Return to the situation described in Exploration 25.5, and assume the speed of sound is 340 m/s. Using the geometry of right-angled triangles, we can determine that point A is a distance of $\sqrt{(3.40\text{ m})^2 + (0.40\text{ m})^2} = 3.42\text{ m}$ from the source, and a distance of $\sqrt{(3.40\text{ m})^2 + (1.60\text{ m})^2} = 3.76\text{ m}$ from the apparent second source. What is the lowest frequency sound wave from the source that will produce completely constructive interference at point A?

Answer to Essential Question 25.5: In this situation, the condition for constructive interference is that the path-length difference is half a wavelength. Using an integer number of wavelengths plus a half-wavelength would also produce constructive interference, but it would also decrease the wavelength. To get the lowest frequency, we need the longest wavelength. The wave that reflects from the floor is inverted upon reflection, and an inversion is equivalent to traveling an additional half wavelength, so the net shift is a full wavelength. This gives a path-length difference of 3.76 m – 3.42 m = 0.34 m. The path-length difference is half a wavelength, so a full wavelength is 0.68 m, corresponding to a frequency of v/λ = 340 m/s / 0.68 m = 500 Hz.

25-6 Thin-Film Interference: The Five-Step Method

The photograph in Figure 25.24 shows some colorful soap bubbles. The beautiful colors of the bubbles are caused by thin-film interference, interference between light reflecting from the outer surface of a soap bubble and light reflecting from the inner surface of the bubble. The colors we see are directly related to the thickness of the bubble wall. The basic process of thin-film interference is illustrated in Figure 25.25.

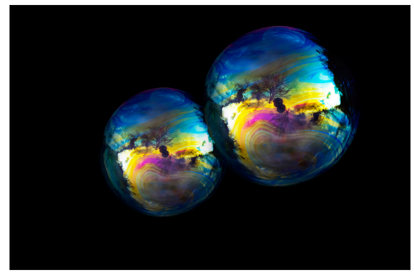


Figure 25.24: The colors in these soap bubbles are produced by thin-film interference - interference between light reflecting from the outer and inner surfaces of a bubble. Photo credit: George Horan, from publicdomainpictures.net

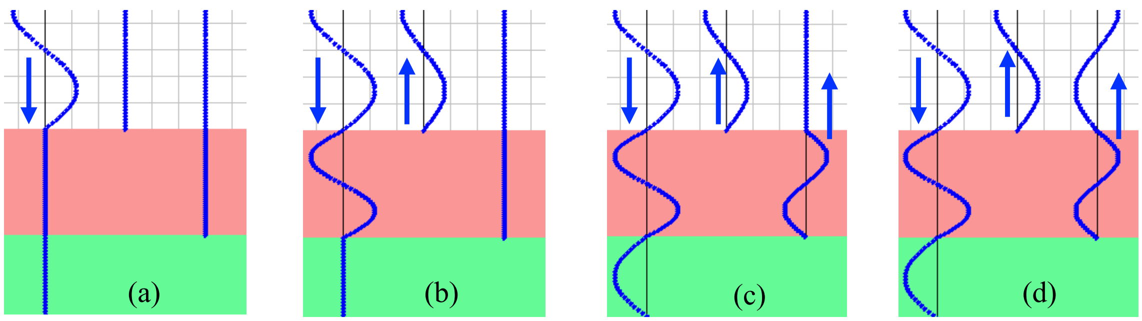


Figure 25.25: These successive images are separated in time by one period of the wave. The thin film, shown in pink, is characterized by an index of refraction n_2. (a) A wave traveling in medium 1 is incident on the interface separating media 1 and 2 along the normal to the interface. (b) Part of the wave is transmitted into medium 2, while part reflects back into medium 1. The reflected wave is shifted to the right for clarity. The thin film happens to be exactly one wavelength thick. In general, the wave reflecting from the top surface of the film can be inverted upon reflection, or reflect without being inverted. In this case, the reflected wave is inverted because $n_2 > n_1$. (c) At the interface separating media 2 and 3, the wave is also partly reflected and partly transmitted. This reflected wave is shown on the far right of the diagram. In this case, the reflected wave is not inverted upon reflection because $n_3 < n_2$. (d) The two reflected waves interfere with one another in medium 1. By adjusting the thickness of the thin film, this interference can be completely constructive, completely destructive, or something in between.

Note that, in Figure 25.25, the wave that reflects off the bottom surface of the film travels a total extra distance of 2 wavelengths, compared to the wave that reflects off the film's top surface. What kind of interference occurs between the two reflected waves? As we can see from Figure 25.25(d), the waves interfere destructively. The extra path length is an integer number of wavelengths, but the inversion upon reflection at the top surface introduces a half wavelength shift that causes peaks in one reflected wave to align with troughs in the other, and vice versa.

Let's take a systematic approach to analyzing a thin-film situation. The basic idea is to determine the effective path-length between the wave reflecting from the top surface of the film and the wave reflecting from the bottom surface. For a film of thickness t, and with a wave incident along the normal, the effective path-length difference accounts for the extra distance of $2t$ traveled by the wave that reflects from the bottom surface of the film, as well as for any inversions that occur when a wave reflects from a higher-n medium.

For a wave that does get inverted by reflecting from a higher-n medium, we will treat the inversion as an extra half-wavelength contribution to the wave's path-length. We do this because for a sine wave, inverting the wave is equivalent to shifting the wave by half a wavelength. However, a general thin-film situation involves three different media, and hence three different wavelengths! Which wavelength is it that matters? The wave that reflects from the bottom surface of the film is the one that travels the extra distance. Because the extra distance traveled is in the thin film, the wavelength that matters is the wavelength in the thin film. It is helpful to remember the relationship between the wavelength in the film and the wavelength in vacuum:

$$\lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n_{\text{film}}} .$$ (Eq. 25.8: **An expression for the wavelength of light in the thin film)**

The Five-Step Method for analyzing thin films

Our approach will assume that the wave starts in medium 1, and is normally incident on a thin film (medium 2) of thickness t that is on a third medium (medium 3), as shown in Figure 25.25.

Step 1 – Determine Δ_t, the shift for the wave reflecting from the top surface of the film. This contribution to the path length is non-zero only if the wave is inverted upon reflection. If $n_2 > n_1$, $\Delta_t = \lambda_{\text{film}} / 2$. If $n_2 < n_1$, $\Delta_t = 0$.

Step 2 – Determine Δ_b, the shift for the wave reflecting from the bottom surface of the film. This contribution to the path length is at least $2t$, because that wave travels an extra distance t down through the film, and t back up through the film. There is an extra half-wavelength contribution if the wave is inverted upon reflection. If $n_3 > n_2$, $\Delta_b = 2t + \lambda_{\text{film}} / 2$. If $n_3 < n_2$, $\Delta_b = 2t$.

Step 3 – Determine Δ, the effective path-length difference. Simply subtract the results from the previous steps to find the relative shift between the two waves. $\Delta = \Delta_b - \Delta_t$.

Step 4 – Bring in the interference condition appropriate to the situation. If the interference is constructive, such as when we see a particular color reflecting from the thin film, we set the effective path-length difference equal to an integer number of wavelengths ($\Delta = m\lambda_{\text{film}}$, where m is an integer). If the interference is destructive, $\Delta = (m+0.5)\, \lambda_{\text{film}}$.

Step 5 – Solve the resulting equation. In general, the equation relates the thickness of the thin film to the wavelength of the light.

Related End-of-Chapter Exercises: 24 – 28.

Essential Question 25.6: Fill in Table 25.2, which summarizes the various possibilities for what Δ_t and Δ_b can be.

	$n_2 > n_1$	$n_2 < n_1$
$n_3 > n_2$	$\Delta_t =$ $\Delta_b =$	$\Delta_t =$ $\Delta_b =$
$n_3 < n_2$	$\Delta_t =$ $\Delta_b =$	$\Delta_t =$ $\Delta_b =$

Table 25.2: A table for summarizing the various results for Δ_t and Δ_b.

Answer to Essential Question 25.6: The shift for the wave reflecting from the top surface, Δ_t, depends on how n_2 compares to n_1. The shift for the wave reflecting from the bottom surface, Δ_b, depends on how n_3 compares to n_2.

	$n_2 > n_1$	$n_2 < n_1$
$n_3 > n_2$	$\Delta_t = \lambda_{film}/2$ $\Delta_b = 2t + (\lambda_{film}/2)$	$\Delta_t = 0$ $\Delta_b = 2t + (\lambda_{film}/2)$
$n_3 < n_2$	$\Delta_t = \lambda_{film}/2$ $\Delta_b = 2t$	$\Delta_t = 0$ $\Delta_b = 2t$

Table 25.3: Summarizing the various results for Δ_t and Δ_b.

25-7 Applying the Five-Step Method

EXPLORATION 25.7 – Designing a non-reflecting coating
High-quality lenses, such as those for binoculars or cameras, are often coated with a thin non-reflecting coating to maximize the amount of light getting through the lens. We can apply thin-film ideas to understand how such a lens works. Explaining why such lenses generally look purple will also be part of our analysis. In this example, we will assume light is traveling through air before it encounters the non-reflective coating (n = 1.32) that is on top of the glass (n = 1.52). Figure 25.26 shows the arrangement. The coating is completely non-reflective for just one wavelength, so we will design it to be non-reflective for light with a wavelength in vacuum of 528 nm, which is close to the middle of the visible spectrum.

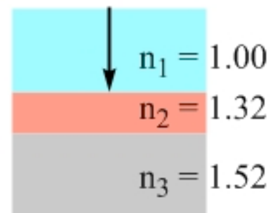


Figure 25.26: The arrangement of air (top), coating (middle), and glass (bottom) for a typical situation of a non-reflective coating on a glass lens.

Step 1 – ***Determine Δ_t, the shift for the wave reflecting from the air-coating interface.*** Because the coating has a higher index of refraction than the air, this wave is inverted upon reflection, giving $\Delta_t = \lambda_{film}/2$.

Step 2 – ***Determine Δ_b, the shift for the wave reflecting from the coating-glass interface.*** The glass has a higher index of refraction than the coating, so this wave is also inverted upon reflection. For a coating of thickness t, $\Delta_b = 2t + (\lambda_{film}/2)$.

Step 3 – ***Determine Δ, the effective path-length difference.*** $\Delta = \Delta_b - \Delta_t = 2t$.

Step 4 – ***Bring in the appropriate interference condition.*** In this situation, we do not want light to reflect from the coating. We can accomplish this by having the reflected waves interfere destructively. Setting the effective path-length difference equal to (m + 1/2) wavelengths gives:

$$2t = (m + 1/2)\lambda_{film} .$$

Step 5 – ***Solve for the minimum possible coating thickness.*** To solve for the smallest possible coating thickness, we choose the smallest value of m that makes sense, remembering that m is an integer. In this case, m = 0 gives the smallest coating thickness.

$$2t_{min} = (0 + 1/2)\lambda_{film} \qquad \Rightarrow \qquad t_{min} = \frac{\lambda_{film}}{4} = \frac{\lambda_{vacuum}}{4n_{film}} = \frac{528 \text{ nm}}{4 \times 1.32} = 100 \text{ nm} .$$

Step 6 – ***If a 100-nm-thick film produces completely destructive interference for 528 nm green light, what kind of interference will it produce for the violet end of the spectrum (400 nm) and the red end of the spectrum (700 nm)? Why does this make the lens look purple in reflected light?*** In the coating, 400 nm violet light has a wavelength of 400 nm / 1.32 = 303 nm. Thus, an effective path-length difference of $2t$ = 200 nm shifts one reflected violet wave relative to another by 200 nm / 303 nm, a shift of about 2/3 of a wavelength. The interference is partly destructive, so some violet light reflects from the coating. For red light of 700 nm, with a wavelength in the film of 700 nm / 1.32 = 530 nm, the relative shift is 200 nm / 530 nm = 0.38 wavelengths. Again, this produces partly destructive interference, so some red light reflects. When white light shines on the film, therefore, almost no green light is reflected, small amounts of yellow and blue are reflected, a little more orange and indigo are reflected, and even more red and violet are reflected. Thus, the reflected light is dominated by red and violet, which makes the film look purple.

Key ideas: The five-step method can be applied in all thin-film situations, to help us relate the film thickness to the wavelength of light. **Related End-of-Chapter Exercises: 31, 32, 54.**

EXAMPLE 25.7 – A soap film

A ring is dipped into a soap solution, creating a round soap film. (a) When the ring is held vertically, explain why horizontal bands of color are observed, as seen in Figure 25.27(a). (b) As time goes by, the film gets progressively thinner. Where the film is very thin, no light reflects from the film, so it looks like the film is not there anymore, as in the top right of Figure 25.27(b). Apply the first three steps of the five-step method to explain why, in the limit that the film thickness approaches zero, the two reflected waves interfere destructively.

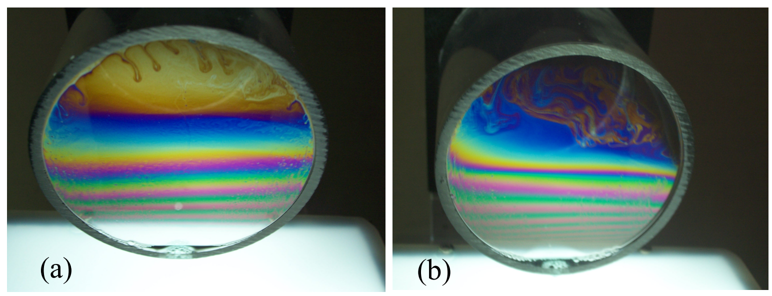


Figure 25.27: (a) A vertical soap film generally has horizontal bands. (b) When the film gets very thin, it does not reflect any light whatsoever, as is happening at the top right of the film in this case. Photo credit: A. Duffy.

SOLUTION

(a) The film thickness is approximately constant at a given height, with that thickness corresponding to constructive interference for a particular wavelength (color). Gravity pulls the fluid down toward the bottom of the film, so the film thickness decreases as the vertical position increases, changing the wavelength (color) associated with a particular height.

(b) The index of refraction of the soap film is essentially that of water (n = 1.33), with the film being surrounded by air (n = 1.00). The wave reflecting from the front surface of the film is in air, reflecting from the higher-n film, so it experiences a half-wavelength shift: $\Delta_t = \lambda_{film}/2$. The wave reflecting from the back surface of the film reflects from a lower-n medium, so the effective path-length is simply $\Delta_b = 2t$, where t is the film thickness. The effective path-length difference is therefore $\Delta = \Delta_b - \Delta_t = 2t - \lambda_{film}/2$. In the limit that the film thickness t approaches zero, the effective path-length difference has a magnitude of half a wavelength. Shifting one wave with respect to the other by half a wavelength produces destructive interference, and the interference is destructive for all wavelengths, so no light is reflected when the film is very thin.

Related End-of-Chapter Exercises: 10, 12.

Essential Question 25.7: For the situation shown in Exploration 25.7, the non-reflective coating on glass, what kind of interference results as the thickness of the coating approaches zero?

Answer to Essential Question 25.7: Returning to the result of Step 3 in Exploration 25.7, the effective path-length difference between the two waves is $\Delta = 2t$. Thus, in the limit that the thickness, t, approaches zero, the effective path-length difference approaches zero and the interference is constructive.

Chapter Summary

Essential Idea: Interference and Diffraction.
In many situations, light acts as a wave. In general, waves diffract through narrow openings, and waves interfere with one another. Examples of this behavior with light occur when a laser beam is incident on one or more narrow openings, when light passes through the pupil of your eye, and when light interacts with thin films such as those in soap bubbles.

Constructive Interference – from Double Slits to Diffraction Gratings
For a wave of wavelength λ that is incident on a number of equally spaced narrow openings, where the number of openings is at least two, the angles at which constructive interference occurs are given by

$d \sin\theta = m\lambda$, (Equation 25.3: **constructive interference, for $N > 1$ sources)**
where m is an integer, and d is the distance between neighboring openings.

Destructive Interference – Single and Double Slits
For a wave of wavelength λ that is incident on a single slit of width a, the angles at which destructive interference occurs are given by

$a \sin\theta = m\lambda$, (Equation 25.5: **diffraction minima for a single slit)**
where m is an integer, and a is the distance between neighboring openings.

For a double slit, the interference minima occur at angles given by

$d \sin\theta = (m + 1/2)\lambda$, (Equation 25.4: **destructive interference, for two sources in phase**)
where m is an integer.

Limits imposed by diffraction
For a circular opening, the angle at which the first zero occurs in a diffraction pattern is given by

$\theta_{min} = \dfrac{1.22\lambda}{D}$, (Eq. 25.6: **The minimum angle between two sources to be resolvable**)

where D is the diameter of the opening. This equation can be applied to our own eyes.

Thin-film interference
The colorful patterns exhibited by thin films, such as soap bubbles, can be understood by following the five-step method outlined in Section 25.6. Such patterns result from the wave reflecting from one surface of the film interfering with the wave reflecting from the other surface of the film. A key part of the analysis is accounting for the fact that when waves in one medium reflect from a second medium that has a lower index of refraction, the reflected wave is upright, while if the second medium has a higher index of refraction, the reflected wave is inverted. This inversion upon reflection is like an extra half-wavelength distance traveled by the wave.

End-of-Chapter Exercises

Exercises 1 – 12 are conceptual questions designed to see whether you understand the main concepts in the chapter.

1. Red laser light shines on a double slit, creating a pattern of bright and dark spots on a screen some distance away. State whether the following changes, carried out separately, would increase, decrease, or produce no change in the distance between the bright spots on the screen, and justify each answer. (a) Replace the red laser with a green laser. (b) Decrease the spacing between the slits. (c) Decrease the distance between the slits and the screen. (d) Immerse the entire apparatus in water.

2. Light of a single wavelength shines onto a double slit. A particular point on the opposite side of the double slit from the light source happens to be 1800 nm farther from one slit than the other. Assume that the point receives some light from each slit, and that the beams arriving at the point from each slit are of equal intensity. For the following wavelengths, determine whether the interference at the point is constructive, destructive, or something in between. (a) 400 nm violet light, (b) 500 nm green light, (c) 600 nm orange light, (d) 700 nm red light. Explain each of your answers.

3. The graph in Figure 25.24 shows sinθ as a function of wavelength for different orders of light. The red line corresponds to the first-order (m = 1) spectrum. (a) What does the blue line correspond to? (b) Copy the graph and draw the line corresponding to the third-order spectrum. (c) What is the largest wavelength for which there is a third-order spectrum for this grating?

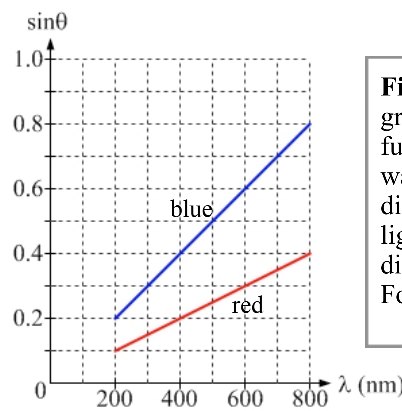


Figure 25.24: The graph shows sinθ as a function of wavelength for different orders of light shining on a diffraction grating. For Exercise 3.

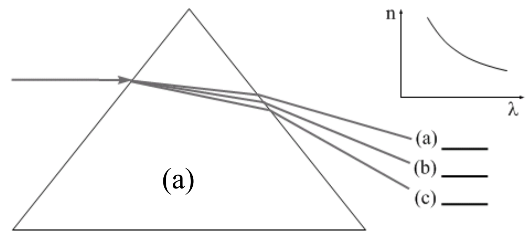
4. The pattern in Figure 25.25 represents the interference pattern set up in a room by two speakers (the red circles) broadcasting identical single-frequency sound waves in phase with one another. (a) If you walk slowly along the line shown in yellow, from one end to the other, what will you hear? Explain your answer. (b) If the wavelength of the sound waves is 1.5 m, how far apart are the speakers?

Figure 25.25: An interference pattern set up in a room by two speakers broadcasting identical single-frequency waves. For Exercise 4.

5. A red laser shining on something creates the pattern shown in Figure 25.26. Is the laser shining on a single slit, double slit, or a diffraction grating? Explain your answer.

Figure 25.26: The pattern at the center of a screen, produced by a red laser beam shining on a single slit, a double slit, or a diffraction grating. The distance between neighboring tick marks, shown on the screen below the pattern, is 8.00 mm. For Exercise 5.

6. A beam of white light strikes a glass prism, as shown in Figure 25.27(a). The white light is made up of only three colors. These are, in alphabetical order, green, red, and violet. The graph of the index of refraction vs. wavelength for the glass is shown below to the right of the prism. For each of the three rays labeled (a) – (c) on the diagram, label the ray with its color. Use W for white, G for green, R for red, and V for violet. (b) The prism is now replaced by a diffraction grating with a grating spacing of d = 1300 nm. The three colors in the beam of white light have, in order of increasing wavelength, wavelengths of 400 nm, 500 nm, and 700 nm. For the seven rays labeled (d) – (j) in Figure 25.27(b), label the ray with its color. Use W for white, G for green, R for red, and V for violet.



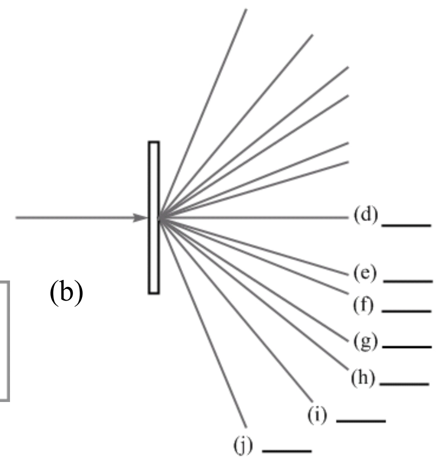


Figure 25.27: (a) A beam of violet, green, and red light is incident on a prism. (b) The same beam is incident on a diffraction grating. For Exercise 6.

7. Figure 25.28 shows the $m = 0$ through $m = 2$ lines that result when green light is incident on a diffraction grating. The squares on the grid in the figure measure 10 cm × 10 cm. The horizontal blue line at the top of the figure represents a screen, which is 1.0 m long and 90 cm from the grating. Approximately how far from the grating should the screen be located so that the two second-order green lines are just visible at the left and right edges of the screen?

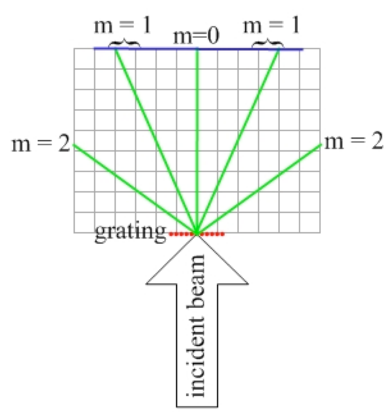


Figure 25.28: The $m = 0$ through $m = 2$ lines that result when green light with a wavelength of 540 nm is incident on a diffraction grating. The horizontal line at the top represents a screen, which is 1.0 m long. The squares on the grid measure 10 cm × 10 cm. For Exercise 7.

8. Two speakers send out identical single-frequency sound waves, in phase, that have a wavelength of 0.80 m. As shown in Figure 25.29, the speakers are separated by 3.6 m. Three lines, labeled A through C, are also shown in the figure. Line A is part of the perpendicular bisector of the line connecting the two sources. If you were to walk along these lines, would you observe completely constructive interference, completely destructive interference, or something else? Answer this question for (a) line A, (b) line B, and (c) line C. Briefly justify each of your answers.

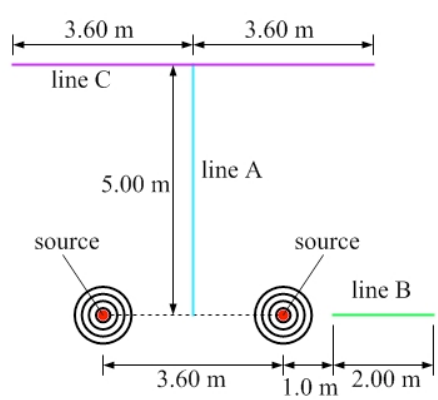


Figure 25.29: Three lines located near two speakers (each labeled "source") that are broadcasting identical single-frequency sounds, for Exercise 8.

9. Figure 25.30 shows, at the top, the pattern resulting from light with a wavelength of 480 nm passing through only one slit of a double slit. In the middle of the figure is the pattern that would result if both slits were illuminated and the slits sent out light uniformly in all directions. At the bottom of the figure is the actual pattern observed when the light illuminates both slits. What is the ratio of the distance between the slits to the width of one of the slits?

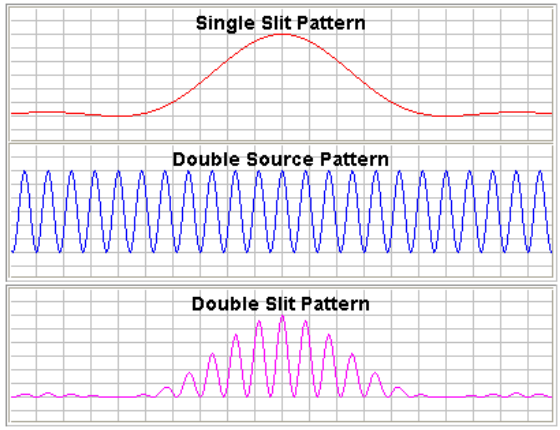


Figure 25.30: The pattern on a screen that results from 480 nm light illuminating a double slit is shown at the bottom. This pattern is a combination of the single slit pattern (top) and double source pattern (middle). For Exercise 9.

10. Figure 25.31 shows four situations in which light is incident perpendicularly on a thin film (the middle layer in each case). The indices of refraction are $n_1 = 1.50$ and $n_2 = 2.00$. In the limit that the thickness of the thin film approaches zero, determine whether the light that reflects from the top and bottom surfaces of the film interferes constructively or destructively in (a) case A, (b) case B, (c) case C, and (d) case D.

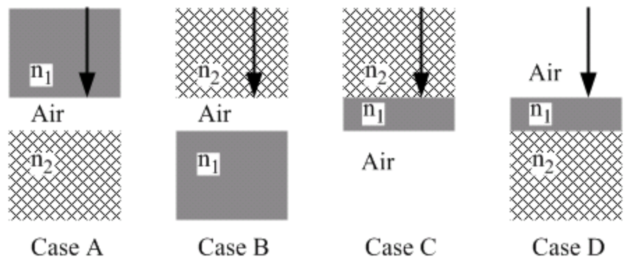
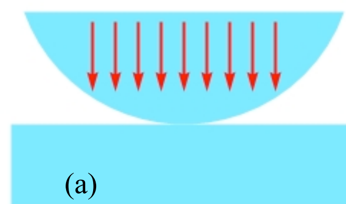


Figure 25.31: Four thin-film situations involving different arrangements of the same three media, for Exercise 10.

11. A soap film, surrounded by air, is held vertically so that, from top to bottom, its thickness varies from a few nm to a few hundred nm. The film is illuminated by white light. Which is closer to the top of the film, the location of the first band of red light, produced by completely constructive thin-film interference, or the first band of blue light? Explain.



12. Figure 25.32 illustrates a phenomenon known as Newton's rings, in which a bull's–eye pattern is created by thin-film interference. The film in this case is a thin film of air that is between a piece of glass with a spherical surface (such as a watch glass) that is placed on top of a flat piece of glass. Two possible patterns, one with a dark center and one with a bright center, are shown in the figure. (a) Which pattern would you see when you look down on the rings from above, and which would you see when you look up at them from below? Explain.

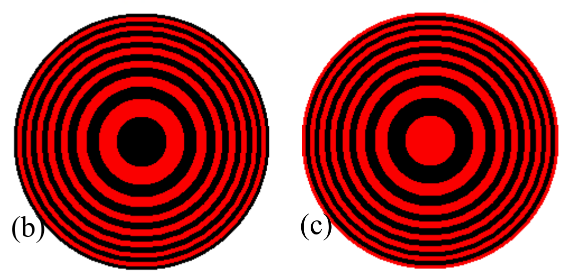


Figure 25.32: The phenomenon of Newton's rings comes from light shining through an object with a spherical surface that rests on a flat surface (a). Interference between light reflecting from the top surface of the film (between the spherical surface and the flat surface) and light reflecting from the bottom surface produces a bull's-eye pattern. Two possible patterns are shown in (b) and (c). For Exercise 12.

Exercises 13 – 15 deal with the interference from two sources.

13. Two speakers broadcasting identical single-frequency sound waves, in phase with one another, are placed 4.8 m apart. The speed of sound is 340 m/s. You are located at a point that is 10.0 m from one speaker, and 8.4 m from the other speaker. What is the lowest frequency for which you observe (a) completely constructive interference? (b) completely destructive interference?

14. Two speakers broadcasting identical single-frequency sound waves, in phase with one another, are placed 6.5 m apart. The wavelength of the sound waves is 3.0 m. You stand directly in front of the speaker on the left (along the dashed line in Figure 25.33), at a distance of 4.5 m from it. Your friend than changes the wavelength of the identical waves being emitted by the speakers. What are the two largest wavelengths that, at your location, result in (a) completely constructive interference, and (b) completely destructive interference?

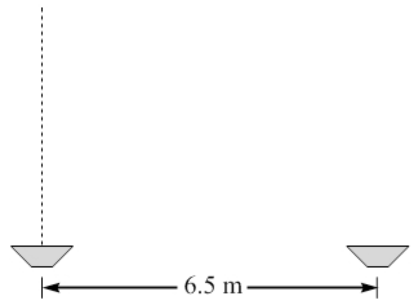


Figure 25.33: Two speakers broadcasting identical single-frequency waves, for Exercises 14 and 15.

15. Two speakers broadcasting identical single-frequency sound waves, in phase with one another, are placed 6.5 m apart. The wavelength of the sound waves is 3.0 m. You stand directly in front of the speaker on the left (along the dashed line in Figure 25.33), but some distance from it. How far are you from that speaker if the interference at your location is (a) completely constructive? (b) completely destructive? Find all the possible answers in each case.

Exercises 16 – 18 involve double slits and diffraction gratings.

16. When the beam of a red laser is incident on a particular diffraction grating, the $m = 1$ bright fringe is observed at an angle of 28.0°. At what angle is the (b) $m = 2$ bright fringe, and (c) the $m = 3$ bright fringe?

17. Light with a wavelength of 540 nm shines on two narrow slits that are 4.40 μm apart. At what angle does the fifth dark spot occur on a screen on the far side of the slits from the light source?

18. Laser light shines onto a diffraction grating, creating the pattern of bright lines shown in Figure 25.34. The lines strike a screen (in blue in the figure) that is a distance L away from the grating, creating some bright spots on the screen. The distance between the central spot and the mth bright spot to either side is denoted y_m. (a) What is the relationship between θ_m (the angle between the mth bright line and the $m = 0$ line) and y_m? (b) Show that, in the limit that θ_m is small, y_m is given by $y_m = \frac{m\lambda L}{d}$.

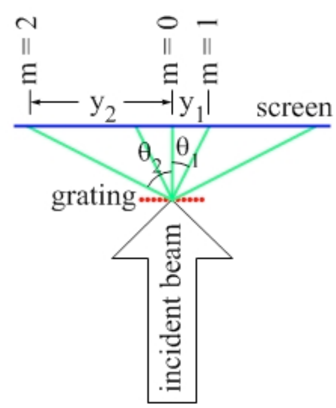


Figure 25.34: Single-frequency light shining on a double slit, for Exercise 18.

Exercises 19 – 23 involve single slits and diffraction by circular openings.

19. Light of a particular wavelength is incident on a single slit that has a width of 2.00 μm. If the second zero in the diffraction pattern occurs at an angle of 30°, what is the wavelength of the light?

20. The label on a green laser pointer states that the wavelength of the laser is 532 nm. You shine the laser into an aquarium filled with water, with an index of refraction of 1.33, and onto a single slit (in the water) that has a width of 1.60 μm. At what angle is the first zero in the diffraction pattern?

21. Light with a wavelength of 600 nm shines onto a single slit, and the diffraction pattern is observed on a screen 2.5 m away from the slit. The distance, on the screen, between the dark spots to either side of the central maximum in the pattern is 25 mm. (a) What is the distance between the same dark spots when the screen is moved so it is only 1.5 m from the slit? (b) What is the width of the slit?

22. On a dark night, you watch a car drive away from you on a long straight road. If the car's red tail lights are LED's emitting a wavelength of 640 nm, the distance between the lights is 1.50 m, and your pupils are 6 mm in diameter, what is the maximum distance the car can get away from you before the two individual lights look like one light to you?

23. A spy satellite takes in light through a circular opening 2.0 m in diameter. (a) If the wavelength of the light is 540 nm, and the satellite is 250 km above the ground, how close together can two small objects be on the ground for the satellite to be able to resolve them? (b) If the pupils in your eyes are 4.0 mm in diameter, how far above the ground would you be to achieve the same resolution as the satellite?

Exercises 24 – 28 are designed to give you practice with applying the five-step method for thin-film interference. For each of these problems, carry out the following steps. (a) Determine Δ_t, the shift for the wave reflecting from the top surface of the film. (b) Determine Δ_b, the shift for the wave reflecting from the bottom surface of the film. (c) Determine Δ, the effective path-length difference. (d) Bring in the interference condition appropriate to the situation. (e) Solve the resulting equation to solve the problem.

24. When you shine red light, with a wavelength of 640 nm, straight down through air onto a thin film of oil that coats a water surface, the film looks dark because of destructive interference. The index of refraction of the oil is 1.60, while that of water is 1.33. The goal of the problem is to determine the smallest non-zero film thickness. Carry out the five-step method as outlined above.

25. A ring is dipped into a soap solution, resulting in a circular soap film in the ring. When the plane of the ring is horizontal, the film looks green to you when you look straight down onto the film from above. The soap film is surrounded on both sides by air, and the index of refraction of the film is that of water, 1.33. If the film thickness is such that it produces completely constructive interference for green light with a wavelength, in vacuum, of 532 nm, what is the minimum non-zero thickness of the film? Carry out the five-step method, as outlined above, to solve the problem.

26. A thin film of glass, with an index of refraction of 1.5, is used to coat diamond, which has an index of refraction of 2.4. The thickness of the thin film is 200 nm. Light, traveling through air, shines down along the normal to the film, as shown in Figure 25.35. If we define the visible spectrum as extending from 400 nm to 700 nm (measured in air), for which wavelength in the visible spectrum (measured in air) does the film produce completely constructive interference? Carry out the five-step method, as outlined above, to solve the problem.

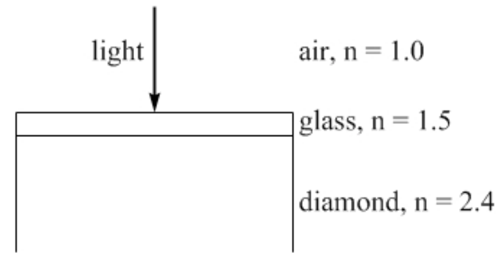


Figure 25.35: A 200-nm-thick film of glass is placed on top of diamond. For Exercises 26 and 27.

27. Return to the situation shown in Figure 25.35 and described in Exercise 26. Now determine for which wavelength in the visible spectrum (measured in air) the film produces completely destructive interference. Carry out the five-step method, as outlined above, to solve the problem.

28. A thin film with an index of refraction of 1.70 is used as a non-reflective coating on a glass lens that has an index of refraction of 1.50. What are the three smallest non-zero thicknesses of the film that will produce completely destructive interference for light that has a wavelength of 510 nm in vacuum? Assume the light is traveling in air before encountering the film, and that it strikes the film at normal incidence. Carry out the five-step method, as outlined above, to solve the problem.

Exercises 29 – 33 involve practical applications of the interference of light.

29. Understanding a particular spectrum is important in many areas of science, including physics and chemistry, where it can be used to identify a gas, for instance. To create a spectrum, light is generally sent through a diffraction grating, splitting the light into the various wavelengths that make it up. (a) The first step in the process is to calibrate the grating, so we know the grating spacing. Sodium has two yellow lines that are very close together in wavelength at 590 nm. When light from a sodium source is passed through a particular diffraction grating, the two yellow lines overlap, looking like one line at an angle of 33.7° in the first-order spectrum. What is the grating spacing? (b) The hydrogen atom is the simplest atom there is, consisting of one electron and one proton, and it has thus been well studied. When hydrogen gas is excited by means of a high voltage, three of the prominent lines in the spectrum are found at wavelengths of 658 nm, 487 nm, and 435 nm. When the light is passed through the diffraction grating we calibrated with sodium, at what angles will these three lines appear in the first-order spectrum? Scientists observing these lines can be confident that the source of the light contains hydrogen.

30. Return to the situation described in Exercise 29. Another application of spectra produced by a diffraction grating is in astrophysics, where the Doppler shift of a particular galaxy can be measured to determine the velocity of the galaxy with respect to us. (a) If the red hydrogen line in the galaxy's spectrum is observed at 690 nm instead of 658 nm, is the galaxy moving toward us or away from us? (b) Recalling that the Doppler equation for electromagnetic waves states that the magnitude of the shift in frequency associated with relative motion between a source and observer is $|f' - f| = vf / c$, determine v, the relative speed of the galaxy with respect to us.

31. Thin coatings are often applied to materials to protect them. In a particular manufacturing process, a company wants to deposit a 200–nm–thick coating onto glass mirrors to protect the mirrors during shipping. The coating material has an index of refraction of 1.30, while that of the glass is 1.53. White light in air is incident on the film along the normal to the surface, and the film looks the color of the wavelength that is experiencing completely constructive interference. If the technician observing the coating as it is being deposited, and gradually increasing in thickness, views the light reflecting from the coating, at which of the following points should the technician stop the deposition process? When the reflected light is violet (400 nm), green (520 nm), orange-red (612 nm), or none of these? Explain.

32. As shown in Figure 25.36(a), two flat pieces of glass are touching at their left edges, and are separated at their right edges by a cylindrical wire. This apparatus can be used to determine the diameter of the wire. When the apparatus is illuminated from above with yellow light with a wavelength of 590 nm, you see the thin-film interference pattern shown in Figure 25.36(b) when you look down on the apparatus from above. Note that the third dark fringe from the left is exactly halfway between the left and right edges of the pieces of glass. At the point where the third dark film from the left appears, (a) how many wavelengths thick is the film, and (b) how thick is the film? (c) How is the diameter of the wire related to the answer to part (b)? (d) What is the diameter of the wire?

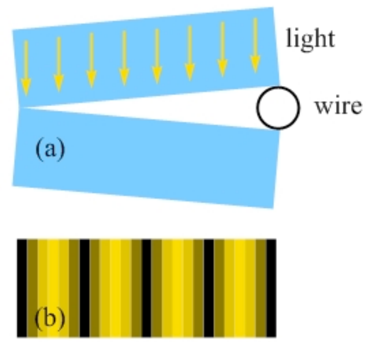


Figure 25.36: (a) A thin film of air is trapped between two flat pieces of glass. The pieces of glass are in contact at their left edges, and are separated at their right edges by a thin wire. (b) The interference pattern you observe when you look down on the film from above, when the film is illuminated from above with 590-nm light, for Exercise 32.

33. In a compact disk (CD) player, to read the information on a CD an infrared laser, with a wavelength of 780 nm in air, reflects from flat-topped bumps and the flat surroundings (known as the land) on the CD. When the laser beam reflects solely from the top of a bump, or solely from the land, a significant signal is reflected back. However, when the beam is moving from a bump to the land, or vice versa, destructive interference between the two parts of the beam, one part which travels a shorter distance than the other, results in a low signal. Thus, music can be encoded as a binary (two-state) signal. What is the height of the bumps on a CD, if the transparent polycarbonate coating on the CD has an index of refraction of 1.55? The bump height is designed to be the smallest needed to produce completely destructive interference between waves reflecting from the bumps and waves reflecting from the land.

General problems and conceptual questions

34. Christiaan Huygens made a number of important contributions to our understanding of the wave nature of light. Do some research about him and his contributions, and write a couple of paragraphs about what you find.

35. The graph in Figure 25.37 shows sinθ as a function of wavelength for different orders of light. Let's say that the red line corresponds to the first-order (m = 1) spectrum. At what angle is (a) the third-order spot for 500 nm light? (b) the fourth-order spot for 400 nm light?

36. The graph in Figure 25.37 shows sinθ as a function of wavelength for different orders of light. What is the grating spacing if the red line corresponds to (a) the first-order (m = 1) spectrum? (b) the fifth-order (m = 5) spectrum?

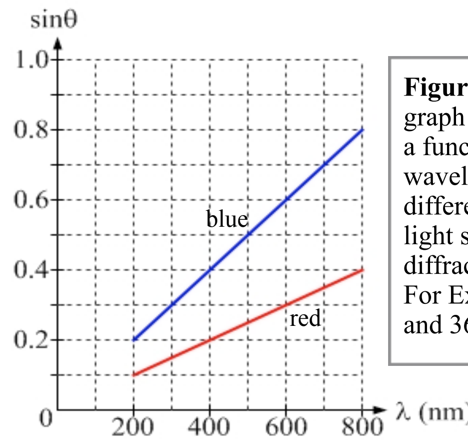


Figure 25.37: The graph shows sinθ as a function of wavelength for different orders of light shining on a diffraction grating. For Exercises 35 and 36.

37. A laser with a wavelength of 600 nm is incident on a pair of narrow slits that are separated by a distance of $d = 3.00 \times 10^{-5}$ m. The resulting interference pattern is projected onto a screen 2.00 m from the slits. (a) How far is one of the first-order bright spots from the central bright spot on the screen (measuring from the center of each spot)? Note that for small angles sinθ ≈ tanθ. (b) Does the answer change if the entire apparatus is immersed in water, which has an index of refraction of 4/3? If so, how does it change?

38. Figure 25.38 shows the m = 0 through m = 2 lines that result when green light with a wavelength of 540 nm is incident on a diffraction grating. Also shown, as dashed lines, are the two m = 1 lines for a second wavelength. The squares on the grid in the figure measure 10 cm × 10 cm. (a) What is the second wavelength? (b) What is the grating spacing? (c) Will there be m = 3 lines for either the green light or the second wavelength in this situation? Explain.

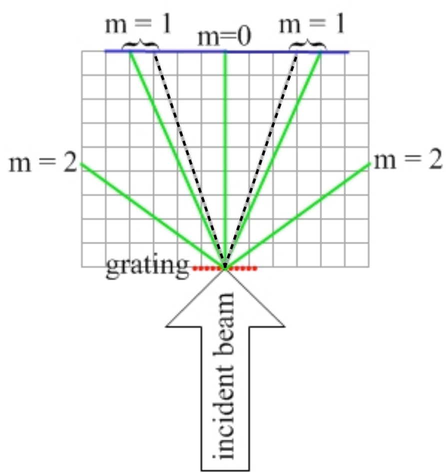


Figure 25.38: The m = 0 through m = 2 lines that result when green light with a wavelength of 540 nm is incident on a diffraction grating. The two m = 1 lines (dashed lines) are for a second wavelength. The horizontal line at the top represents a screen, which is 1.0 m long. The squares on the grid measure 10 cm × 10 cm. For Exercise 38.

39. Light with a wavelength of 400 nm shines onto a double slit. A particular point on the far side of the double slit from the light source happens to be exactly 6 wavelengths farther from one slit than the other. (a) At this particular point, do we expect to see constructive interference or destructive interference? (b) For which wavelengths in the visible spectrum (400 – 700 nm) will the interference be completely constructive at the point? (c) For which wavelengths in the visible spectrum will the interference be completely destructive at the point?

40. Figure 25.39 shows the m = 0 and m = 1 lines coming from a red laser beam, with a wavelength of 632 nm, that shines on a diffraction grating. The squares in the grid measure 10 cm × 10 cm. Duplicate the figure, and show all the lines resulting from 450 nm blue light shining on the same grating.

41. Return to the situation described in Exercise 40, and shown in Figure 25.39. (a) Determine the grating spacing. (b) For what range of grating spacings would there be three, and only three, orders to either side of the central maximum with 632 nm red light?

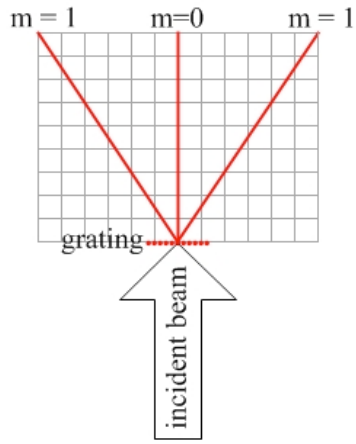


Figure 25.39: The m = 0 and m = 1 lines that result when red light with a wavelength of 632 nm is incident on a diffraction grating. The squares on the grid measure 10 cm × 10 cm. For Exercises 40 and 41.

42. A red laser, shining on a double slit, creates the pattern shown in Figure 25.40 at the center of a screen placed 2.00 m on the opposite side of the double slit from the laser. If the laser wavelength is 632 nm, what is the distance between the two slits in the double slit?

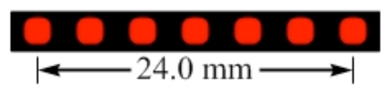


Figure 25.40: The pattern of dots created on a screen a distance 2.00 m from a double slit, when a red laser shines on the slits. For Exercises 42 and 43.

43. Return to the situation described in Exercise 42, and shown in Figure 25.40. When the red laser is replaced by a second laser, exactly 5 dots are observed within a distance of 24.0 mm at the center of the screen, instead of having exactly 7 dots in that distance, as in Figure 25.33. What is the wavelength of the second laser?

44. Laser light with a wavelength of 632 nm is incident on a pair of identical slits that are 5.60 μm apart. (a) If the slits are very narrow, how many bright fringes would you expect to see on one side of the central maximum? (b) In the pattern on a screen, you notice that, instead of a bright fringe where you expect the fourth bright fringe to be, there is a dark spot. The first three bright fringes are where you expect them to be, however. What is the width of each slit? (c) Are there any other fringes missing, in addition to the fourth one on each side?

45. Repeat Exercise 44, but, this time, use a wavelength of 440 nm.

46. Red light, with a wavelength of 650 nm, is incident on a double slit. The resulting pattern on the screen 1.2 m behind the double slit is shown in Figure 25.41. If the slits are 2.40 μm apart, what is the width of each of the slits?

Figure 25.41: The pattern on a screen resulting from red light illuminating a double slit. For Exercise 46.

47. Light with a wavelength of 440 nm illuminates a double slit. When you shine a second beam of light on the double slit, you notice that the 4th-order bright spot for that light occurs at the same angle as the 5th-order bright spot for the 440 nm light. (a) What is the wavelength of the second beam? (b) If the angle of these beams is 40.0°, what is the distance between the two slits in the double slit?

48. When light from a red laser, with a wavelength of 632 nm, is incident on a diffraction grating, the second-order maximum occurs at an angle of 15.4°. (a) What is d, the grating spacing for the diffraction grating? (b) At what angle is the second-order maximum if the red laser is replaced by a green laser with a wavelength of 532 nm?

49. Figure 25.42 shows the pattern at the center of a screen, produced by a red laser beam shining on either a single slit, a double slit, or a diffraction grating. If the laser has a wavelength of 632 nm, and the screen is 1.4 m away, determine the width of the single slit (if the laser shines on a single slit) or the distance between the slits in the double slit (if the laser shines on a double slit) or the grating spacing (if the laser shines on a diffraction grating).

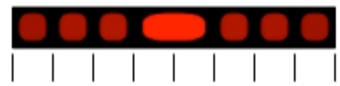


Figure 25.42: The pattern at the center of a screen, produced by a red laser beam shining on a single slit, a double slit, or a diffraction grating. The distance between neighboring tick marks, shown on the screen below the pattern, is 8.00 mm. For Exercise 49.

50. Figure 25.43 shows a source of sound that is located 0.800 m from a wall. The source is emitting waves of a single frequency. Point A is located some distance from the source, as shown. If the speed of sound in air is 340 m/s, find the three lowest frequencies that produce, at A, (a) completely constructive interference, and (b) completely destructive interference.

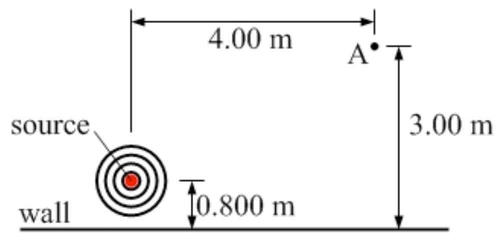


Figure 25.43: A source of single-frequency waves is located 0.800 m from a wall, for Exercises 50 and 51.

51. Return to the situation described in Exercise 50, and shown in Figure 25.43. If the source is emitting the lowest frequency sound wave to produce completely destructive interference at point A, what is the minimum distance the source can be moved, directly toward the wall, so the interference becomes completely constructive at A? It is acceptable to answer this by approximating that A is a long way from the source.

52. Two speakers send out identical single-frequency sound waves, in phase, that have a wavelength of 0.80 m. As shown in Figure 25.44, the speakers are separated by 3.6 m. Three lines, labeled A through C, are also shown in the figure. Line A is part of the perpendicular bisector of the line connecting the two sources. (a) How many points are there along line C at which the interference between the waves from the two speakers is completely destructive? (b) Relative to the midpoint of line C, approximately where are the points of destructive interference?

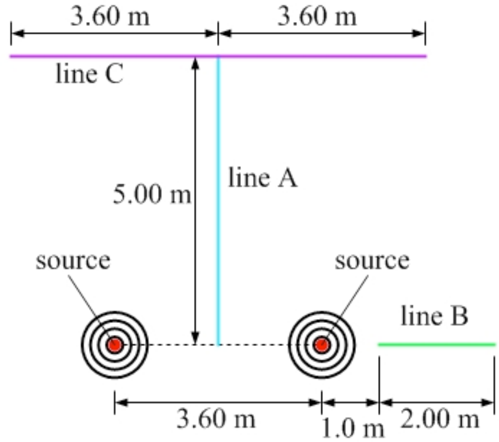


Figure 25.44: Three lines located near two speakers (each labeled "source") that are broadcasting identical single-frequency sounds, for Exercise 52.

53. Figure 25.45 shows four situations in which light is incident perpendicularly on a thin film (the middle layer in each case). The indices of refraction are $n_1 = 1.50$ and $n_2 = 2.00$. (a) Determine the minimum non-zero thickness of the film that results in constructive interference for 450 nm light (measured in vacuum) that reflects from the top and bottom surfaces of the film in case C. (b) Does this film thickness also produce constructive interference for 450 nm light in any of the other cases? Explain why or why not.

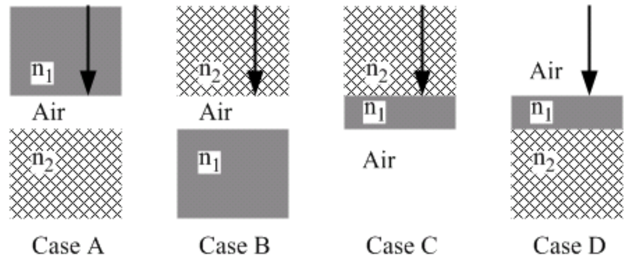


Figure 25.45: Four thin-film situations involving different arrangements of the same three media, for Exercise 53.

54. A thin piece of glass with an index of refraction of $n_2 = 1.50$ is placed on top of a medium that has an index of refraction $n_3 = 2.00$, as shown in Figure 25.40. A beam of light traveling in air ($n_1 = 1.00$) shines perpendicularly down on the glass. The beam contains light of only two colors, blue light with a wavelength in air of 450 nm and orange light with a wavelength in air of 600 nm. What is the minimum non-zero thickness of the glass that gives completely constructive interference for (a) the blue light reflecting from the film? (b) BOTH the blue and orange light simultaneously?

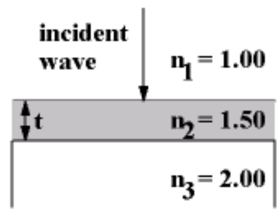


Figure 25.46: A thin film of glass on top of a medium that has an index of refraction of $n_3 = 2.00$. For Exercise 54.

55. Light traveling in air is incident along the normal to a thin film of unknown material that sits on a thick piece of glass ($n = 1.50$). The index of refraction of a typical medium is in the range 1.0 – 2.4. Confining ourselves to this range, what is the index of refraction of the unknown material if the film is 120 nm thick, and it produces completely destructive interference for light, in air, with a wavelength of 540 nm? Find all the possible answers.

56. Sound waves traveling in air encounter a mesh screen. Some of the waves reflect from the screen, while the rest pass through and reflect from a wall that is 30.0 cm behind the mesh screen. You observe completely destructive interference between the two reflected waves when the frequency of the sound waves is 275 Hz. (a) Do the sound waves experience an inversion when they reflect from the mesh screen and from the wall? Explain. (b) What is the speed of sound in this situation?

57. Return to the situation described in Exercise 56. What are the next two frequencies, above 275 Hz, that will also produce completely destructive interference?

58. It is somewhat ironic that the phenomenon of Newton's rings (see Exercises 12 and 59), which provide evidence for the wave behavior of light, are named after Newton, because Sir Isaac Newton was a firm believer in the particle model of light. Do some research on Newton's contributions to optics, and write a couple of paragraphs about it.

59. Figure 25.47 illustrates a phenomenon known as Newton's rings, in which a bull's–eye pattern is created by thin-film interference. The film in this case is a thin film of air that is between a piece of glass with a spherical surface (such as a watch glass) that is placed on top of a flat piece of glass. The spherical surface of the top piece of glass has a radius of curvature of 500 cm. (a) How many wavelengths of 500 nm light (measured in air) fit in the film of air at a point 1.00 cm from the point where the top piece of glass makes

contact with the bottom piece? (b) Would you expect constructive or destructive interference to occur at this point? (c) Would your answers change if the air was replaced with a fluid with an index of refraction of 1.25? If so, how? Assume that the two pieces of glass have indices of refraction of about 1.5.

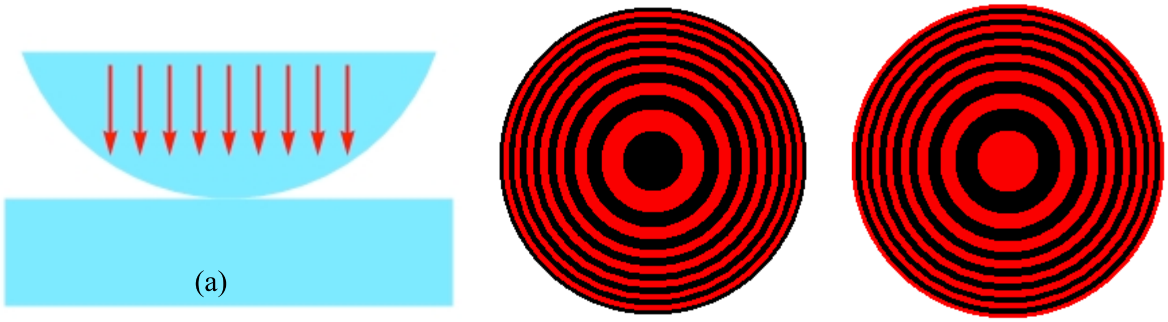


Figure 25.47: The phenomenon of Newton's rings comes from light shining through an object with a spherical surface that rests on a flat surface. Interference between light reflecting from the top surface of the film (between the spherical surface and the flat surface) and light reflecting from the bottom surface produces a bull's-eye shaped pattern. For Exercise 59.

60. A particular metal ruler has thin lines on it every half millimeter. As shown in Figure 25.48, laser light is incident on the ruler at an angle of $\alpha = 4.00°$ with respect to the ruler. (a) For the situation shown in the figure, find the relationship between the wavelength of the incident light and the angles (β values) at which constructive interference occurs. Use d to represent the spacing between the lines on the ruler. (b) If the first-order maximum occurs at an angle of $\beta = 4.93°$, what is the wavelength of the laser light?

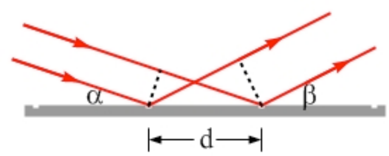


Figure 25.48: Laser light is incident on a metal ruler. The light is incident at an angle of 4.00°, measured from the ruler. After interacting with the ruler, the rays of light leaving the ruler interfere constructively with one another when the rays make an angle β with the ruler surface. The dashed lines on the left and right are perpendicular to the incoming and outgoing rays, respectively. For Exercise 60.

61. Three students are working together on a problem involving thin-film interference. Comment on the part of their conversation that is reported below.

Evan: Do you know the equation for constructive interference in a thin-film situation?

Alison: There isn't one equation that works all the time – it depends on how the different indices of refraction compare.

Christian: Here's one, though, 2t equals m plus a half wavelengths. That's what we worked out in class when we did the soap film.

Evan: Isn't m plus a half for destructive interference?

Christian: Usually, it is, but with thin films you always get one of the waves flipping upside down when it reflects, which is like shifting it half a wavelength.

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