24-1 Refraction

To understand what happens when light passes from one medium to another, we again use a model that involves rays and wave fronts, as we did with reflection. Let's begin by creating a short pulse of light – say we have a laser pointer and we hold the switch down for just an instant. We shine this light onto the interface between two transparent media so that the beam of light is incident along the normal to the interface. For instance, Figure 24.1 shows a beam of light in air incident on a block of glass. Part of the beam is reflected straight back, and the rest is transmitted along the normal into the glass.

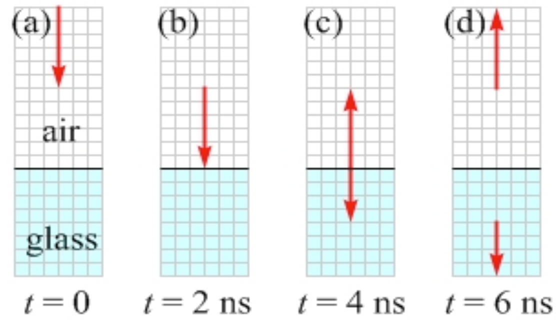


Figure 24.1: When a beam of light is incident along the normal to the interface between two transparent media, part of the beam is reflected straight back, while the rest is transmitted along the normal into the second medium. The squares on the grid measure 10 cm × 10 cm, and the images are shown at 2 ns intervals.

Figure 24.1 shows a sequence of images, showing the pulse of light at regular time intervals. Note that the light in the glass travels with an average speed that is about 2/3 of the average speed of the light in air. The average speed at which light travels through a medium depends on the medium. We can quantify this dependence of average speed on the medium by defining a unitless parameter known as the index of refraction, n.

$$n = \frac{c}{v}, \qquad \text{(Equation 24.1: \textbf{Index of refraction})}$$

where c = 3.00×10^8 m/s is the speed of light in vacuum. Table 24.1 gives indices of refraction for several media, and the corresponding speed of light in the media.

Medium	Index of refraction	Speed of light ($\times 10^8$ m/s)
Vacuum	**1.000**	**3.00**
Water	**1.33**	**2.25**
Glass	**1.5**	**2.0**
Diamond	**2.4**	**1.25**

Table 24.1: Typical indices of refraction for various media, and their corresponding speeds of light.

We can make sense of what happens in Figure 24.1 by looking at the wave fronts associated with the beam of light (see Figure 24.2). Because the wave slows down when it is transmitted into the glass, the wave fronts get closer together, shortening the distance from the front of the pulse to the end of the pulse. Going further, we can apply the wave equation (Equation 21.1, $v = f\lambda$) to re-write equation 24.1. The frequency of the wave in the two media is the same, so:

$$n = \frac{c}{v} = \frac{f\lambda}{f\lambda'} = \frac{\lambda}{\lambda'},$$

(Eq. 24.2: **Index of refraction, in terms of wavelength**)

where λ is the wavelength in vacuum, and λ' is the wavelength in the medium.

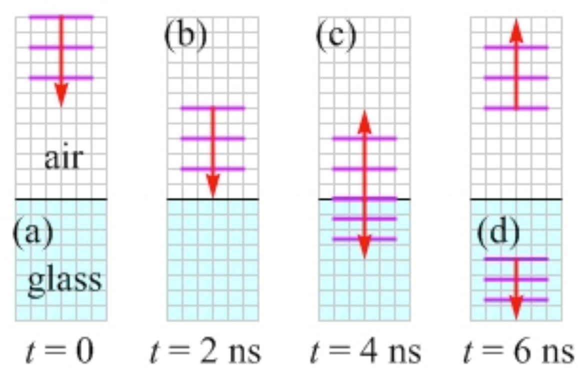


Figure 24.2: Looking at the wave fronts associated with the light as it passes from one medium to another, we can see that when the wave slows down, the wavelength decreases.

What happens when the beam of light is not incident along the normal? In this case, as shown in Figure 24.3, we observe that part of the wave is reflected from the interface, obeying the law of reflection, while the rest is transmitted into the second medium. However, the transmitted light travels in a different direction from the direction the incident light was traveling in the first medium. This change in direction experienced by light transmitted from one medium to another is known as **refraction**.

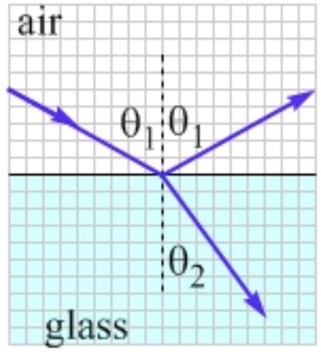


Figure 24.3: When light is not incident along the normal, the light changes direction when it is transmitted into the second medium, as long as the two media have different indices of refraction.

We can understand refraction by looking at the wave fronts as the beam of light passes from one medium to another, as shown in Figure 24.4. The part of the wave front that enters the glass first slows down, while the part still traveling in air maintains its speed. As can be seen, this causes the wave to change direction.

What happens if the light is traveling in the glass instead, and is then incident on the glass-air interface? The reflected ray is quite different from what we had above, but if the angle between the normal and the ray of light in the glass is the same in Figure 24.2 and Figure 24.4, the angle between the normal and the ray in air is also the same in the two cases. Snell's Law applies, no matter which direction the light is going.

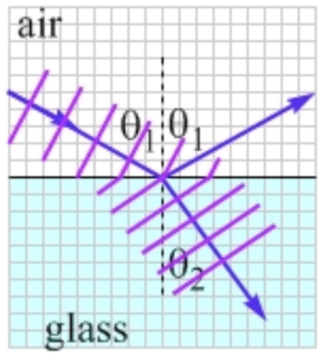


Figure 24.4: The part of the wave front that enters the slower medium (the glass) first slows down before the rest of the wave front, causing the ray to change direction. Wave fronts associated with the reflected wave have been omitted for clarity.

When light is transmitted from one medium to another, the angle of incidence, θ_1, in the first medium is related to the angle of refraction, θ_2, in the second medium by:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$ (Equation 24.3: **Snell's Law**)

These angles are measured from the normal (perpendicular) to the surface. The n's represent the indices of refraction of the two media.

In general, when light is transmitted from one medium to a medium with a higher index of refraction, the light bends toward the normal (that is, the angle for the refracted ray is less than that for the incident ray). Conversely, the light bends away from the normal when light is transmitted from a higher-n medium to a lower-n medium.

Figure 24.5: The photograph shows a picture of a pencil in a glass of water. Refraction of the light leaving the water makes the pencil appear to bend. Photo credit: A. Duffy.

Related End-of-Chapter Exercises: 1, 2, 3 – 5.

Essential Question 24.1: Figure 24.6 shows a beam of light as it is transmitted from medium 1 to medium 3. Rank these media based on their index of refraction.

Figure 24.6: A beam of light passing through three media.

Answer to Essential Question 24.1: Because all of these interfaces are parallel to one another, we can apply a rule-of-thumb that the smaller the angle between the normal and the beam, within a medium, the larger is that medium's index of refraction. Based on this rule-of-thumb, the ranking by index of refraction is 1 > 3 > 2.

24-2 Total Internal Reflection

Let's now explore an implication of Snell's Law that has practical applications. Under the right conditions, light incident on an interface between two media is entirely reflected back into the first medium. This phenomenon, that none of the light is transmitted into the second medium, is known as **total internal reflection**. Let's begin by examining light being transmitted from air to glass, a situation in which total internal reflection does not occur.

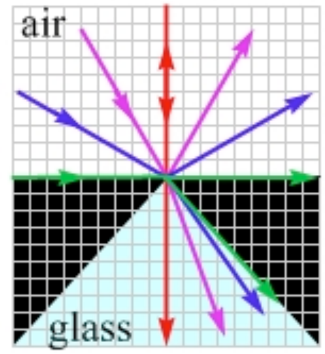


Figure 24.7: Four rays of light, incident on the same point on an air-glass interface. When the light strikes the interface, part of the beam is reflected back into the air, and part is refracted into the glass. The glass is rectangular, but the part of the glass that can not be reached by the light incident on this point has been shaded black.

Figure 24.7 shows various beams of light traveling in air and incident on the same point on an air-glass interface. The beams are shown in different colors to make it clear what path the reflected and transmitted beams follow. Four beams are shown, corresponding to angles of incidence of 0°, 30°, 60°, and almost 90°. As the angle of incidence increases, so does the refracted angle (the angle between the beam refracted into the glass and the normal). By choosing an appropriate angle of incidence, any point in the region shaded in blue in the glass can be illuminated by the light refracted into the glass from the air. Note, however, that no point in the region shaded in black in the glass can be illuminated by the light coming from the air, no matter what angle of incidence is used, if the beam of light is always incident on the same point on the interface.

Table 24.2 shows the angles of the refracted rays in the glass. These angles are determined by applying Snell's Law, using indices of refraction of 1.00 for air and 1.50 for glass.

Incident angle	Refracted angle
0°	0°
30.0°	19.5°
60.0°	35.3°
90.0°	41.8°

Table 24.2: The angles of the incident rays and the refracted rays in Figure 24.7.

EXPLORATION 24.2 – Total internal reflection

Let us now draw a similar picture to that in Figure 24.7, but the light will now travel through the glass before it is incident on the air-glass interface.

Step 1 – ***First, draw the rays in glass incident at angles of 0°, 19.5°, 35.3°, and 41.8°, the same values as the refracted angles in Table 24.2 . Show both the reflected rays and the rays that refract into the air.***

This figure is very similar to Figure 24.7, with the incident and refracted rays reversing directions. The values in Table 24.2 apply again, with the refracted angles in Table 24.2 now the incident angles, and vice versa.

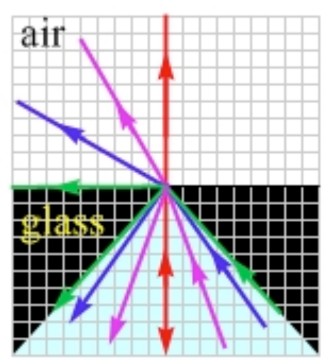


Figure 24.8: Four rays of light, incident on the same point on a glass-air interface. When the light strikes the interface, part of the beam is reflected back into the glass, and part is refracted into the air.

Step 2 – ***Now draw two additional rays, with angles of incidence of 60° and 75°, incident on the same point as the incident rays in Figure 24.8, and originally traveling in glass. What happens when you apply Snell's Law for these rays? What do these rays do when they encounter the interface?*** Let's apply Snell's Law to the situation of the 60° angle of incidence:

$$n_1 \sin(60°) = n_2 \sin\theta_2 .$$

$$\sin\theta_2 = \frac{n_{glass}\sin(60°)}{n_{air}} = \frac{1.5\times 0.866}{1.0} = 1.3 .$$

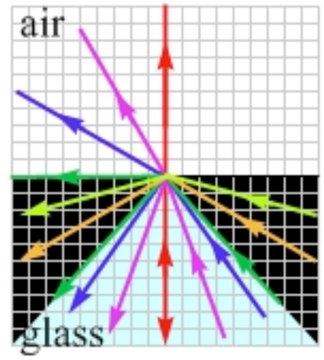


Figure 24.9: When the incident angle for light rays incident on the glass-air interface from the glass side exceed a particular critical angle (in the region colored black), 100% of the light reflects back into the glass. This is known as total internal reflection.

This equation cannot be solved, because the largest value that sin *θ* can be is 1. So, according to Snell's Law, there is no angle of refraction in the air for an angle of incidence of 60° in glass. A similar argument holds for all incident angles greater than 41.8° for rays traveling in glass and striking a glass-air interface. This is consistent with the forbidden region shaded in black in Figure 24.7. No light can refract into the glass from air at angles of refraction that exceed 41.8°, and so no refraction can occur in the opposite direction if the angle of incidence exceeds this critical angle of 41.8°. If the light does not refract into the air, it is entirely reflected back into the glass, obeying the law of reflection. This is known as **total internal reflection**.

Step 3 – ***In general, the critical angle of incidence beyond which total internal reflection occurs is the angle of incidence, in the higher-n medium, that results in a 90° angle of refraction in the lower-n medium. Apply Snell's Law to this situation to derive an equation for the critical angle, θ_c.*** Applying Snell's Law, and recognizing that sin(90°) = 1, gives:

$$n_1 \sin\theta_c = n_2 \sin(90°) = n_2 .$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right).$$ (Eq. 24.4: **The critical angle beyond which total internal reflection occurs**)

Key ideas: Total internal reflection can only occur when light in one medium strikes an interface separating that medium from a medium with a lower index of refraction. If the angle of incidence exceeds a particular critical angle, no light is transmitted into the second medium. Instead, all the light is reflected back into the first medium. **Related End-of-Chapter Exercises: 16 – 20.**

An important application of total internal reflection

Fiber optic cables, which are very important for communications, exploit total internal reflection. Data, as well as voice signals from telephone conversations, are sent through optical fibers by means of laser signals encoded to carry the information. The fibers can even bend around corners and, as long as the bend is not too abrupt, total internal reflection keeps the light inside the fiber.

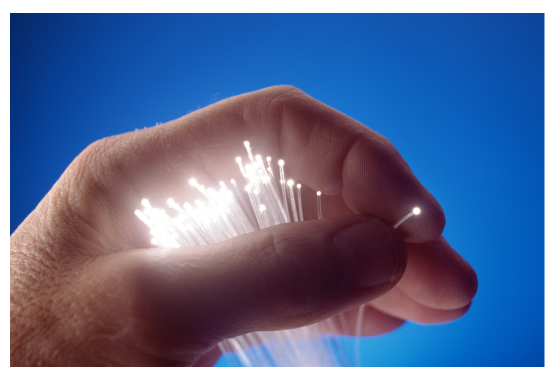


Figure 24.10: A photograph of a collection of optical fibers. Photo credit: PhotoDisc/Getty Images.

Essential Question 24.2: Light traveling in medium 1 reaches the surface of the medium, which is surrounded by air (*n* = 1.00). The angle of incidence is 39°, and the light experiences total internal reflection. What can you say about the index of refraction of medium 1?

Answer to Essential Question 24.2: We know that 39° exceeds the critical angle for total internal reflection. If the critical angle is 39°, equation 24.4 gives $n_1 = 1.59$. (n_2 in the equation is 1.00, because medium 2 is air.) For the critical angle to be less than 39°, the index of refraction must be larger than 1.59. Thus, we know that the index of refraction of the medium is at least 1.59.

24-3 Dispersion

When a beam of white light shines onto a prism, a rainbow of colors emerges from the prism. Such a situation is shown in Figure 24.11. What does this tell us about white light? What does this tell us about the glass that the prism is made from?

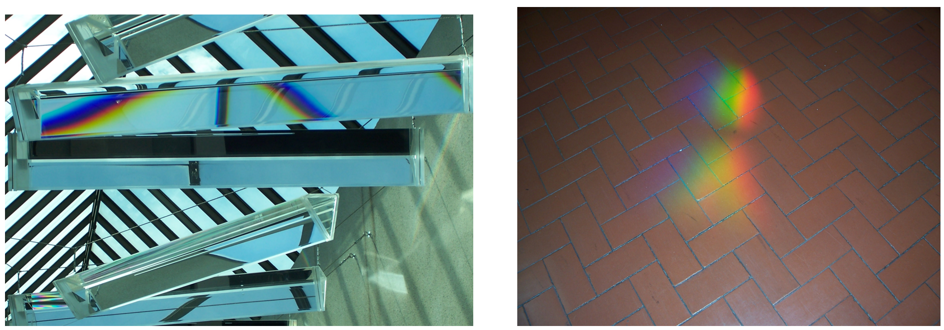


Figure 24.11: The triangular prisms in the picture on the left are suspended beneath the skylights at the Albuquerque Convention Center. Sunlight passing through these prisms produces beautiful rainbows on the floor, as in the photo on the right. Photo credits: A. Duffy.

The fact that a prism splits white light up into various colors of the rainbow tells us that white light consists of all the colors. For the prism to be able to separate out these individual colors, however, the index of refraction of the glass must depend on the wavelength of the light passing through the prism. This phenomenon is known as **dispersion**. Because white light contains all wavelengths of light in the visible spectrum, the different wavelengths (corresponding to different colors) refract at different angles, spreading out the colors.

In general, what we observe for a prism is that violet light, with a wavelength of 400 nm, experiences the largest change in direction, while red light, with a wavelength of 700 nm, experiences the smallest change in direction. As the wavelength increases from 400 to 700 nm, the change in direction decreases, and thus the index of refraction of the glass must also decrease. A graph of the index of refraction as a function of wavelength is shown in Figure 24.12.

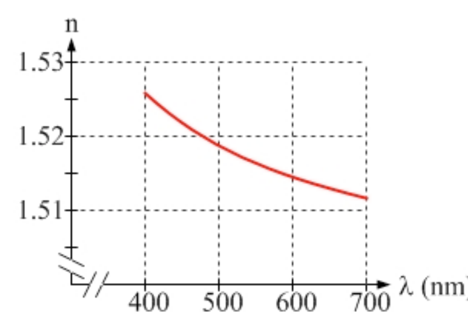


Figure 24.12: A graph of the index of refraction, as a function of wavelength, for a typical sample of glass. The graph is confined to the visible spectrum, from 400 nm (violet) to 700 nm (red).

Refraction, dispersion, and total internal reflection are all important in the formation of a rainbow, such as that in the photograph on the opening page of this chapter. To understand how a rainbow is formed, remember that you can generally only see a rainbow after it has rained, when the Sun is fairly low in the sky, and when you are looking away from the Sun. The rain is important because it produces a lot of water droplets in the sky. Let's begin by looking at how the red light, which is part of the white light coming from the Sun, interacts with a spherical water droplet.

Figure 24.13 shows how red light refracts into a spherical water droplet. Some of the red light reflects from the back of the droplet, and refracts again as it exits the droplet.

Figure 24.13: The path followed by red light inside a water droplet.

Let us now consider the path followed by violet light through the droplet. Water exhibits some dispersion, so when the violet light refracts into the water droplet the angle of refraction for the violet light is a little different from that for red light. Like the red light, some of the violet light reflects from the back of the water droplet, and experiences more refraction as it exits the droplet. As shown in Figure 24.14, the end result is that the beam of violet light coming from that droplet is higher in the sky than the beam of red light from the same droplet. All of the other colors of the visible spectrum lie between those of red and violet, because red and violet are at the two extremes of the visible spectrum.

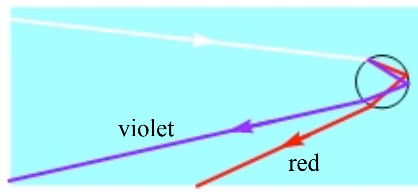


Figure 24.14: The path followed by violet light inside a water droplet, compared to that followed by the red light.

Compare Figure 24.14 to the photograph of the rainbow shown on the first page of this chapter. In Figure 24.14, the beam of violet light that leaves the droplet is higher in the sky than the red light that leaves the droplet. When you look at a rainbow, however, you see the red on the upper arc of the rainbow, and the violet on the lower arc. Is Figure 24.14 at odds with the photo?

Let's try adding the eye of an observer, as well as a second water droplet below the first, to the diagram. These additions are shown in Figure 24.15. When the observer looks at the upper water droplet, what color does it appear to be? It appears to be red, because red light from that droplet enters the observer's eye, while the violet light from that same droplet passes well over the observer's head. The lower droplet appears violet, however, because only violet light from that droplet enters the observer's eye.

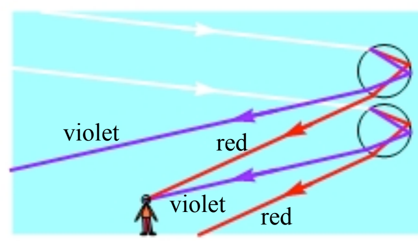


Figure 24.15: Adding an observer, and a second water droplet, to the diagram in Figure 24.14 shows why the top arc of the rainbow is red, while the bottom arc is violet. The other colors come from droplets in between these two. The size of the water droplets is exaggerated so that the path taken by the beams is visible.

Other issues related to rainbows

Under the right conditions, you can see a double rainbow (or even a triple rainbow). As can be seen in the chapter-opening photograph, the second rainbow is dimmer than the primary rainbow, higher in the sky, and the order of colors is reversed. The secondary rainbow comes from light that reflects twice inside each water droplet, rather than once for the primary rainbow. Another issue is that rainbows are generally half-circles. Is it possible to see a rainbow that is a complete circle, or at least more than a half circle? Yes, it is. If you view a rainbow from a high vantage point, such as out the window of a plane, you may be able to see the entire circle.

Related End-of-Chapter Exercises: 7, 40.

Essential Question 24.3: After a light rain shower, you look out your window and see a beautiful rainbow in the sky. Your neighbor happens to look out her window at the same time, and also observes a rainbow. Does your neighbor see exactly the same rainbow that you do?

Answer to Essential Question 24.3: Everyone has their own rainbow! The light you see enters your eye only. In addition, the light making up the colors of the rainbow that you see comes from droplets that are at just the right position in the sky to refract and reflect sunlight back toward you. Your neighbor would see a very similar rainbow, but at least some of the droplets responsible for her rainbow are different from the water droplets that create your rainbow.

24-4 Image Formation by Thin Lenses

Lenses, which are important for correcting vision, for microscopes, and for many telescopes, rely on the refraction of light to form images. As with mirrors, we draw ray diagrams to help us to understand how such images are formed. Let's first begin by looking at what a lens does to a set of parallel rays of light, such as the five rays in Figure 24.16.

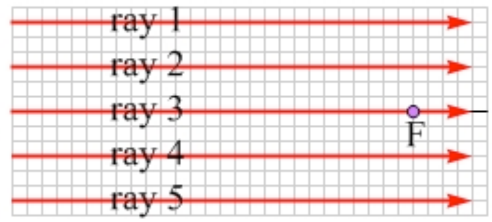


Figure 24.16: Five parallel rays of light.

How can we change the direction of the five rays of light in Figure 24.16 so that they all pass through the point labeled F? Ray 3 already passes through point F, so we don't need to change its direction at all. We can deflect ray 2 with a triangular piece of glass, as shown in Figure 24.17. Passing from air into the glass prism, the ray deflects toward the normal at that surface, while when it emerges back into the air the deflection is away from the normal at the second surface. We can follow a similar process for ray 1, except that we need to produce a larger change in direction for ray 1 compared to ray 2. The glass prism we use for ray 1 thus has its sides at a greater angle from the vertical. For ray 4, we use an identical prism to that used for ray 2, except that we invert it, and for ray 5 the prism is identical to the prism for ray 1, but inverted.

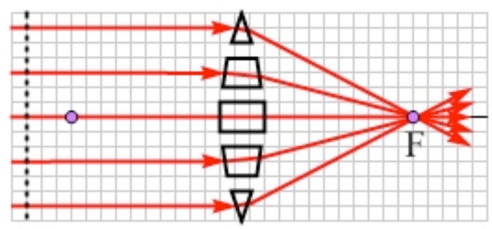


Figure 24.17: We can deflect four of the five rays, so that all five rays meet at point F, by using prisms of the appropriate shapes to produce the deflection required for each beam.

Now, not only do we want all five rays to converge on point F, but we want them to take equal times to travel from the vertical dashed line in Figure 24.17 to point F. Remember that the light travels more slowly in glass than in air. Because rays 1 and 5 travel the greatest distance, they need to pass through the least amount of glass. Ray 3 travels the shortest distance, so we need to delay ray 3 by having it pass through the thickest piece of glass. We can do this without deflecting ray 3 by using a piece of glass with vertical sides, so that ray 3 is incident along the normal. Rather than using various individual glass rectangles and prisms to do the job, we can use a single piece of glass that is thickest in the middle. This piece of glass gets thinner, and its surfaces curve farther away from the vertical, as you move away from ray 3 (that is, as you move away from the principal axis). This is shown in Figure 24.18 – we use a lens. Point F is a focal point of the lens. Lenses allow light to pass through from left-to-right or from right-to-left, so a lens has two focal points, one on each side of the lens.

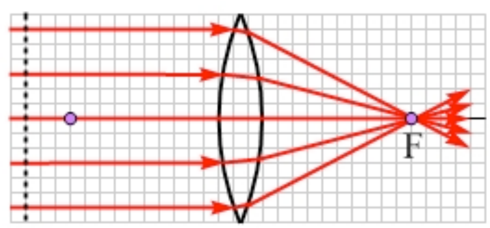


Figure 24.18: A convex lens with surfaces that are spherical arcs brings all parallel rays to one of the focal points, and ensures that the parallel rays take the same time to travel from the vertical line to this focal point.

How does a concave lens, which is thinner in the center than at the edges, affect parallel rays? As shown in Figure 24.19, such lenses generally diverge parallel rays away from the focal point that is on the same side of the lens that the light comes from.

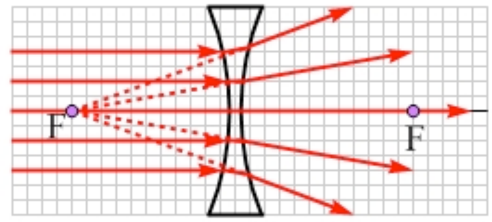


Figure 24.19: The influence of a diverging lens on a set of parallel rays.

EXPLORATION 24.4 – Using a ray diagram to find the location of an image

When drawing ray diagrams for lenses, we follow a process similar to that for mirrors.

Step 1 – ***Figure 24.20 shows an object in front of a converging lens. Sketch two rays of light, which travel in different directions, that leave the tip (the top) of the object and pass through the lens. Show the direction of these rays after they are refracted by the lens.*** Figure 24.21 shows three rays that leave the tip of the object and which are incident on the lens. Because the ray in red is parallel to the principal axis, it is refracted by the lens to pass through the focal point that is to the right of the lens. The ray in green travels along the line connecting the tip of the object to the focal point on the left of the lens. This ray is refracted so that it is parallel to the principal axis. We know this because of the reversibility of light rays – if we reversed the direction of the ray, it would come from the right and be refracted by the lens to pass through the focal point on the right. Finally, the ray in blue passes through the center of the lens without changing direction. This is an approximation, which is valid as long as the lens is thin so the ray enters and exits the lens very close to the principal axis.

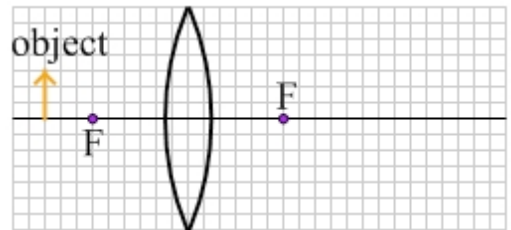


Figure 24.20: An object located some distance in front of a converging lens.

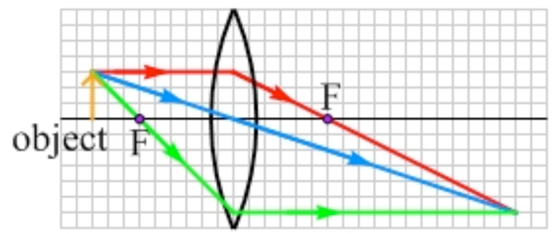
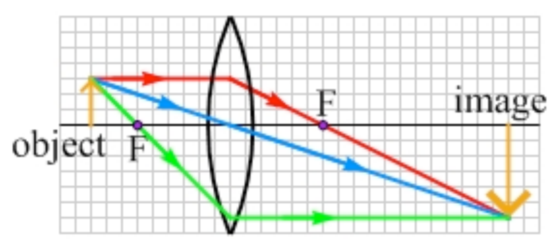


Figure 24.21: Three of the rays that are refracted by the lens. In general, we draw a ray changing direction once, inside the lens. Each ray really changes direction twice, once at each air-glass interface. We are drawing the ray's path incorrectly within the lens, but it is correct outside of the lens, which is where it really matters.

Step 2 – ***The point where the refracted rays meet is where the tip of the image is located. Use this information to sketch the image of the object.*** In this situation, the refracted rays meet at a point to the right of the lens, below the principal axis. Thus, we draw an inverted image between the point where the image of the tip is (where the rays meet) and the principal axis. This is a real image, because the rays pass through the image. All the rays take the same time to travel from the tip of the object to the tip of the image.

Figure 24.22: Because the rays originate at the tip of the object, the point where the refracted rays meet is the location of the tip of the image of the object. The base of the object is on the principal axis, so the base of the image is on the principal axis, too.



Key idea for ray diagrams: The location of the image of any point on an object, when the image is created by a lens, can be found by drawing rays of light that leave that point on the object and are refracted by the lens. The point where the refracted rays meet (or where they appear to diverge from) is where the image of that point is. **Related End-of-Chapter Exercises: 5, 49**

Essential Question 24.4: Are the three rays in Figure 24.22 the only rays that pass through the tip of the image? Explain.

Answer to Essential Question 24.4: The three rays in Figure 24.21 are easy to draw, because we know what they do after passing through the lens. However, as shown in Figure 24.23, all rays that leave the tip of the object and which are refracted by the lens will converge at the tip of the image.

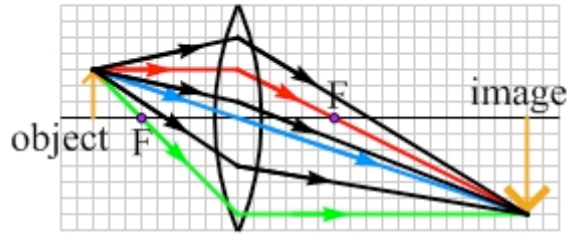


Figure 24.23: All rays that leave the tip of the object and pass through the lens will converge at the tip of the image.

24-5 Lens Concepts

In general, a lens has a larger index of refraction than the medium that surrounds it. In that case, a lens that is thicker in the center than the ends is a converging lens (it converges parallel rays toward a focal point), while a lens that is thinner in the middle than at the ends is a diverging lens (it diverges parallel rays away from a focal point). Like a concave mirror, converging lenses can produce a real image or a virtual image, and the image can be larger, smaller, or the same size as the object. Like a convex mirror, diverging lenses can only produce a virtual image that is smaller than the object.

As with mirrors, the focal point of a lens is defined by what the lens does to a set of rays of light that are parallel to one another and to the principal axis of the mirror. As we discussed in Section 24-4, a converging lens generally refracts the rays so they converge to pass through a focal point, *F*. A diverging lens, in contrast, refracts parallel rays so that they diverge away from a focal point.

Because of dispersion (the fact that the index of refraction of the lens material depends on wavelength), a lens generally has slightly different focal points for different colors of light. This range of focal points is a defect called **chromatic aberration**.

Focal length of a lens with spherical surfaces: The focal length of a lens depends on the curvature of the two surfaces of the lens, the index of refraction of the lens material, and on the index of refraction of the surrounding medium. The focal length is given by:

$$\frac{1}{f} = \left(\frac{n_{lens}}{n_{medium}} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right),$$ (Equation 24.5: **The lensmaker's equation**)

where the two *R*'s represent the radii of curvature of the two lens surfaces. A radius is positive if the surface is convex, and negative if the surface is concave.

Companies that make eyeglasses exploit the lensmaker's equation in creating lenses of the desired focal length. By choosing a lens material that has a high index of refraction, a smaller radius of curvature (and thus a thinner and lighter lens) can be used, compared to a lens made from glass with a smaller index of refraction.

The factor of $[(n_{lens}/n_{medium}) - 1]$ in Equation 24.5 has an interesting implication. First, consider the diagram in Figure 24.24, which shows a familiar situation of a lens, made from material with an index of refraction larger than that of air, surrounded by air. This lens causes the parallel rays to change direction so that they pass through the focal point on the right, and $[(n_{lens}/n_{medium}) - 1]$ is positive, so the focal length of the lens is positive.

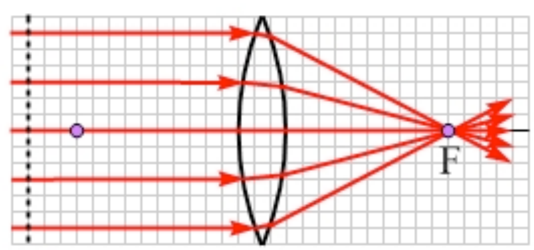


Figure 24.24: A ray diagram for a set of parallel rays encountering a convex lens, made of plastic, surrounded by air.

What happens to the light if the medium around the lens has the same index of refraction as the lens material (perhaps we immerse the lens in some kind of oil)? In this case, the factor of $[(n_{lens}/n_{medium}) - 1]$ is zero - this means that the lens does no focusing at all, which we would expect if the lens and the medium have the same index of refraction. Going further, if the surrounding medium has a larger index of refraction than the lens material, $[(n_{lens}/n_{medium}) - 1]$ is negative and so is the focal length: parallel rays would be *diverged* in this situation.

Thus, depending on the situation, a lens with a convex shape can be a converging lens, a diverging lens, or neither. To minimize any ambiguity, for the rest of this chapter we will refer to lenses by their function (converging or diverging) rather than their shape (convex or concave).

EXPLORATION 24.5 – Ray diagram for a diverging lens

We will follow a process similar to that of mirrors to draw a ray diagram for a diverging lens, starting with the situation in Figure 24.25. The ray diagram will show us where the image of an object is.

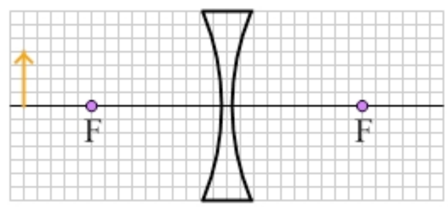


Figure 24.25: An object in front of a diverging lens.

Step 1 – ***Draw a ray of light that leaves the tip of the object (the top of the arrow) and goes parallel to the principal axis (this is known as the parallel ray). Show how this ray is refracted by the lens.*** For a diverging lens, all parallel rays appear to diverge from the focal point on the side of the lens that the light comes from, so we draw the refracted ray (see Figure 24.26) refracting along a line that takes it directly away from that focal point.

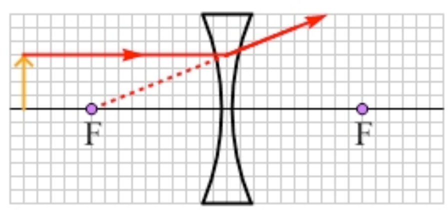


Figure 24.26: The parallel ray refracts to travel directly away from the focal point on the side of the lens the light came from.

Step 2 – ***Sketch a second ray that leaves the tip of the object and is refracted by the lens. Using the refracted rays, draw the image.*** One useful ray, shown in Figure 24.27, passes through the center of the lens without changing direction. This is something of an approximation, but the thinner the lens, the more accurate this is. Another useful ray goes straight toward the focal point on the right of the lens. This ray refracts so as to emerge from the lens going parallel to the principal axis. The refracted rays diverge to the right of the lens, but we can extend them back to meet on the left side of the lens, showing us where the tip of the image is.

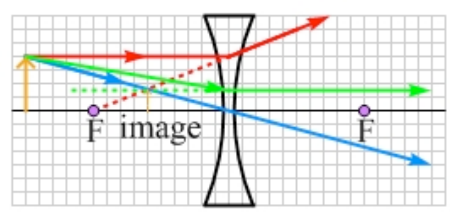


Figure 24.27: In addition to the parallel ray, two other rays are easy to draw the refracted rays for. The ray that passes through the center of the lens is undeflected, approximately. The ray that travels directly toward the focal point on the far side of the lens is refracted so it emerges from the lens traveling parallel to the principal axis. If you look at the object through the lens, your brain interprets the light as coming from the image.

Key idea: When a number of rays leave the same point on an object and are refracted by a lens, the corresponding point on the image is located at the intersection of the refracted rays.
Related End-of-Chapter Exercises: 3, 4, 52.

Essential Question 24.5: Starting with Figure 24.27, show a few more rays of light leaving the tip of the object and being refracted by the lens. How do you know how to draw the refracted rays?

Answer to Essential Question 24.5: To draw the refracted rays properly, we know that when we extend the refracted rays back, they will pass through the tip of the image, which we located in Figure 24.27. Three additional rays are shown in Figure 24.28.

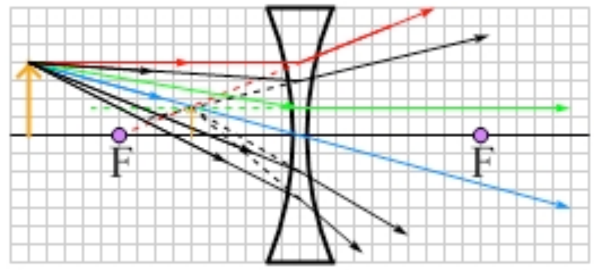


Figure 24.28: For all rays of light that leave the tip of the object and reflect from the mirror, the refracted rays can be extended back to pass through the tip of the image.

24-6 A Quantitative Approach: The Thin-Lens Equation

Even though mirrors and lenses form images using completely different principles (the law of reflection versus Snell's law), we use the same equation to relate focal length, object distance, and image distance, for both mirrors and lenses. This surprising result comes from the fact that the formation of images with both mirrors and lenses can be understood using the geometry of similar triangles. Let's look at how that works for lenses.

Let's look at the ray diagram we drew in Figure 24.22 of section 24-4, shown again here in Figure 24.29.

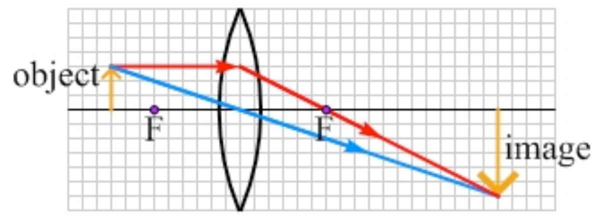


Figure 24.29: The ray diagram we constructed in section 24-4, for an object in front of a converging lens.

Remove the red rays, and examine the two triangles in Figure 24.30, one shaded green and one shaded yellow, bounded by the blue rays, the principal axis, and the object and image. The two triangles are similar, because the three angles in one triangle are the same as the three angles in the other triangle. We can now define the following variables: d_o is the object distance, the distance of the object from the center of the lens; d_i is the image distance, the distance of the image from the center of the lens; h_o is the height of the object; h_i is the height of the image.

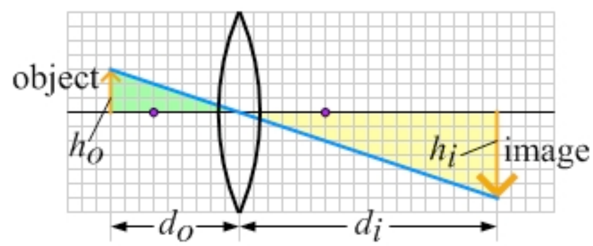


Figure 24.30: Similar triangles, bounded by the principal axis, the object and image, and the ray that passes through the center of the lens.

Using the fact that the ratios of the lengths of corresponding sides in similar triangles are equal, we find that:

$$-\frac{h_i}{h_o} = \frac{d_i}{d_o} . \qquad \text{(Equation 24.6)}$$

The image height is negative because the image is inverted, which is why we need the minus sign in the equation. Let's now return to Figure 24.29, and remove the ray that passes through the center. This gives us the shaded similar triangles shown in Figure 24.31.

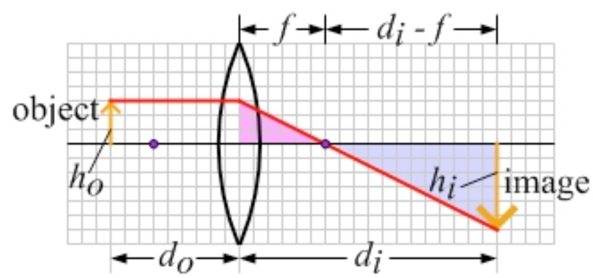


Figure 24.31: Similar triangles, with two sides bounded by the principal axis and the parallel ray, and a third side that is equal to the object height (smaller triangle) or the image height (larger triangle).

Again, using the fact that the ratios of the lengths of corresponding sides in similar triangles are equal, we find that: $\frac{d_i - f}{f} = -\frac{h_i}{h_o}$.

Simplifying the left side, and bringing in equation 24.6, we get: $\frac{d_i}{f} - 1 = \frac{d_i}{d_o}$.

Dividing both sides by d_i gives: $\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o}$, which is generally written as:

$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$. (Equation 24.7: **The thin-lens equation**)

The mnemonic "If I do I di" can help you to remember the thin-lens equation.

Often, we know the focal length f and the object distance d_o, so equation 24.7 can be solved for d_i, the image distance:

$d_i = \frac{d_o \times f}{d_o - f}$ (Equation 24.8: **The thin-lens equation, solved for the image distance**)

Sign conventions

We derived the lens equation above by using a specific case involving a convex lens. The equation can be applied to all situations involving a convex lens or a concave lens if we use the following sign conventions.

The focal length is positive for a converging lens, and negative for a diverging lens.

The image distance is positive, and the image is real, if the image is on the side of the lens the light passes through to, and negative, and the image is virtual, if the image is on the side the light comes from.

The image height is positive when the image is above the principal axis, and negative when the image is below the principal axis. A similar rule applies to the object height.

Magnification

The magnification, m, is defined as the ratio of the height of the image (h_i) to the height of the object (h_o). Making use of Equation 24.6, we can write the magnification as:

$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$. (Equation 24.9: **Magnification**)

The relative sizes of the image and object are as follows:

- The image is larger than the object if $|m| > 1$.
- The image and object have the same size if $|m| = 1$.
- The image is smaller than the object if $|m| < 1$.

The sign of the magnification tells us whether the image is upright (+) or inverted (–) compared to the object.

Related End-of-Chapter Exercises: 21 – 24.

Essential Question 24.6: As you are analyzing a thin-lens situation, you write an equation that states: $\frac{1}{f} = \frac{1}{+12 \text{ cm}} + \frac{1}{+24 \text{ cm}}$. What is the value of $1/f$ in this situation? What is f?

Answer to Essential Question 24.6: To add fractions, you need to find a common denominator.

$$\frac{1}{f}=\frac{1}{+12\text{ cm}}+\frac{1}{+24\text{ cm}}=\frac{2}{+24\text{ cm}}+\frac{1}{+24\text{ cm}}=\frac{3}{+24\text{ cm}}.$$ This gives $f=\frac{+24\text{ cm}}{3}=8.0\text{ cm}$.

24-7 Analyzing a Converging Lens

In section 24-4, we drew one ray diagram for a converging lens. Let's investigate the range of ray diagrams we can draw for such a lens. Note the similarities between a converging lens and a concave mirror. This section is very much a parallel of section 23-6, in which we analyzed the range of images formed by a concave mirror.

EXPLORATION 24.7 – Ray diagrams for a converging lens

Step 1 – ***Draw a ray diagram for an object located 40 cm from a converging lens that has a focal length of +10 cm. Verify the image location on your diagram with the thin-lens equation.*** Two rays are shown in Figure 24.32. One is the parallel ray, which leaves the tip of the object, travels parallel to the principal axis, and is refracted by the lens to pass through the focal point on the far side of the lens. The second ray passes straight through the center of the lens, undeflected.

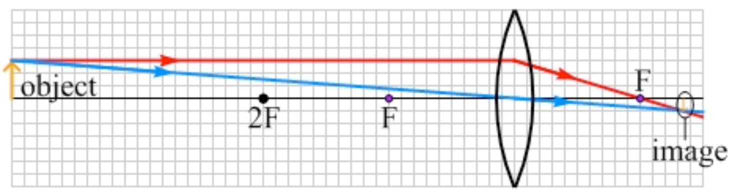


Figure 24.32: A ray diagram for the situation in which the object is far from the lens. The squares in the grid measure 1.0 cm × 1.0 cm.

Applying the thin-lens equation, in the form of equation 24.8, to find the image distance:

$$d_i=\frac{d_o\times f}{d_o-f}=\frac{(40\text{ cm})\times(+10\text{ cm})}{(40\text{ cm})-(+10\text{ cm})}=\frac{+400\text{ cm}^2}{30\text{ cm}}=+13.3\text{ cm}.$$

This image distance is consistent with the ray diagram in Figure 24.32.

Step 2 – ***Repeat step 1, with the object now twice the focal length from the lens.*** We draw the same two rays again, with the parallel ray (in red) being refracted so that it passes through the focal point on the far side of the lens, and the second ray (in blue) passing undeflected (approximately) through the center of the lens. As shown in Figure 24.33, this situation is a special case. When the object is located at twice the focal length from the lens, the image is inverted, also at twice the focal length from the lens (on the other side of the lens), and the same size as the object because the object and image are the same distance from the lens.

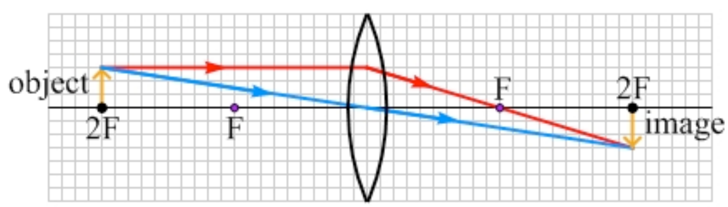


Figure 24.33: When the object is at twice the focal length from the lens, so is the image.

Applying the thin-lens equation to find the image distance, we get:

$$d_i=\frac{d_o\times f}{d_o-f}=\frac{(20\text{ cm})\times(+10\text{ cm})}{(20\text{ cm})-(+10\text{ cm})}=\frac{+200\text{ cm}^2}{10\text{ cm}}=+20\text{ cm},$$ matching the ray diagram.

Step 3 – *Repeat step 1, with the object 15 cm from the lens.* No matter what the object distance is, the parallel ray always does the same thing, being refracted by the lens to pass through the focal point on the far side. The path of the second ray, in blue, depends on the object's position. The ray diagram (Figure 24.34) shows that the image is real, inverted, larger than the object, and about twice as far from the lens as the object is.

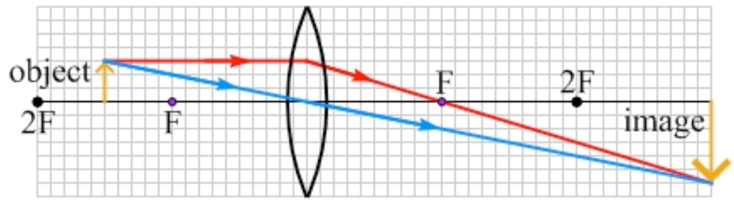


Figure 24.34: A ray diagram for a situation in which the object is between twice the focal length from the lens and the focal point.

Applying the thin-lens equation gives:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(15\text{ cm})\times(+10\text{ cm})}{(15\text{ cm})-(+10\text{ cm})} = \frac{+150\text{ cm}^2}{5.0\text{ cm}} = +30\text{ cm}$$, matching the ray diagram.

Step 4 – *Repeat step 1, with the object at a focal point.* As shown in Figure 24.35, the two refracted rays are parallel to one another, and never meet. In such a case the image is formed at infinity.

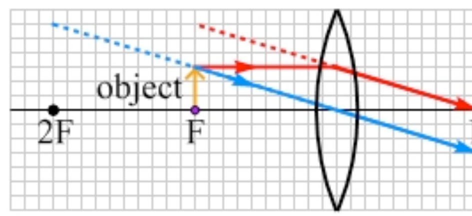


Figure 24.35: A ray diagram for a situation in which the object is at the focal point.

Applying the thin-lens equation gives:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(10\text{ cm})\times(+10\text{ cm})}{(10\text{ cm})-(+10\text{ cm})} = \frac{+100\text{ cm}^2}{0\text{ cm}} = +\infty ,$$

which agrees with the ray diagram.

Step 5 – *Repeat step 1, with the object 5.0 cm from the lens.* When the object is closer to the lens than the focal point, the refracted rays diverge to the right of the lens, and they must be extended back to meet on the left of the lens. The result is a virtual, upright image that is larger than the object, as shown in Figure 24.36. If you look at the object through the lens, your brain interprets the light as coming from the image.

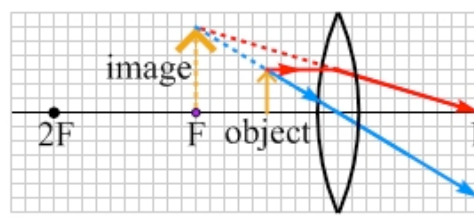


Figure 24.36: A ray diagram for a situation in which the object is between the lens and its focal point.

Applying the thin-lens equation gives:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(5.0\text{ cm})\times(+10\text{ cm})}{(5.0\text{ cm})-(+10\text{ cm})} = \frac{+50\text{ cm}^2}{-5.0\text{ cm}} = -10\text{ cm} .$$

Recalling the sign convention that a negative image distance is consistent with a virtual image, the result from the thin-lens equation is consistent with the ray diagram.

Key idea for converging lenses: Depending on where the object is relative to the focal point of a converging lens, the lens can form an image of the object that is real or virtual. If the image is real, it can be larger than, smaller than, or the same size as the object. If the image is virtual, the image is larger than the object. **Related End-of-Chapter Exercises: 44, 50, 53.**

Essential Question 24.7: When an object is placed 20 cm from a lens, the image formed by the lens is real. What kind of lens is it? What, if anything, can you say about the lens' focal length?

Answer to Essential Question 24.7: The lens must be converging, because a diverging lens cannot produce an image that is larger than the object. A converging lens produces a real image only when the object distance is larger than the focal length, so the focal length in this case must be positive but less than 20 cm.

24-8 An Example Problem Involving a Lens

Let's begin by discussing a general approach we can use to solve problems involving a lens. We will then apply the method to a particular situation.

A general method for solving problems involving a lens

1. Sketch a ray diagram, showing rays leaving the tip of the object and being refracted by the lens. Where the refracted rays meet is where the tip of the image is located. The ray diagram gives us qualitative information about the location and size of the image and about the characteristics of the image.

2. Apply the thin-lens equation and/or the magnification equation. Make sure that the signs you use match those listed in the sign convention in section 24-6. The equations provide quantitative information about the location and size of the image and about the image characteristics.

3. Check the results of applying the equations with your ray diagram, to see if the equations and the ray diagram give consistent results.

Rays that are easy to draw

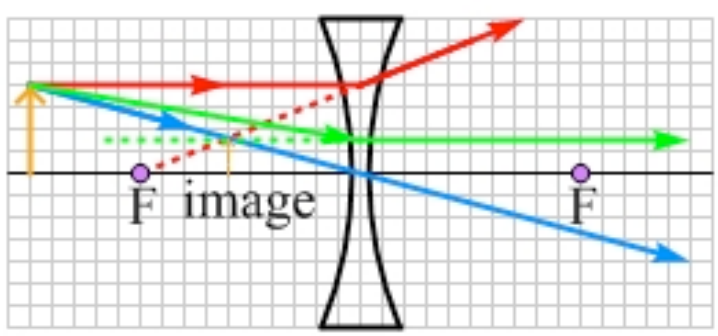
To locate an image on a ray diagram, you need a minimum of two rays. If you draw more than two rays, however, you can check the image location you find with the first two rays. You can draw any number of rays being refracted by the lens, but some are easier to draw than others because we know exactly where the refracted rays go for these rays. Such rays are shown on Figure 24.37, and include:

1. The ray that goes parallel to the principal axis, and refracts to pass through the focal point on the far side of the lens (converging lens), or away from the focal point on the near side of the lens (diverging lens).

2. The ray that passes straight through the center of the lens without changing direction.

3. The ray that travels along the straight line connecting the tip of the object and the focal point not associated with the first ray. This ray is refracted by the lens to go parallel to the principal axis.

Figure 24.37: An example of the three rays that are easy to draw the refracted rays for. When you look at the object through the lens, your brain interprets the light as traveling in straight lines, so you see the image, and not the object.



EXAMPLE 24.8 – Applying the general method

When you look at a cat through a lens that has its focal points at distances of 24 cm on either side of the lens, you see an image of the cat that is 1.5 times as large as the cat. How far is the cat from the lens? Sketch a ray diagram to check your calculations.

SOLUTION

In this case, let's first apply the equations and then draw the ray diagram. The lens is clearly a converging lens, because only converging lenses produce images that are larger than the object. One possibility is that the lens produces a virtual, upright image, so the sign of the magnification is positive. Applying the magnification equation, we get:

$m = +1.5 = -\frac{d_i}{d_o}$, which tells us that $\frac{1}{d_i} = -\frac{1}{1.5 d_o}$.

Applying the thin-lens equation:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} - \frac{1}{1.5 d_o} = \frac{3}{3d_o} - \frac{2}{3d_o} = \frac{1}{3d_o}.$$

Thus, we find that $3d_o = f = +24 \text{ cm}$, so $d_o = +8.0 \text{ cm}$ and we can show that $d_i = -12 \text{ cm}$.

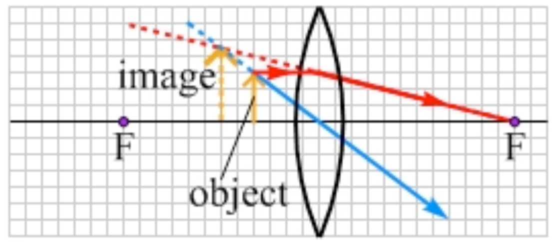


Figure 24.38: A ray diagram for the solution involving a virtual image. Each box on the grid measures 2 cm × 2 cm.

The ray diagram for this situation is shown in Figure 24.38, confirming the calculations.

The solution above is only one of the possible answers. The image could also be real and inverted, so the sign of the magnification is negative. Applying the magnification equation, we get:

$m = -1.5 = -\frac{d_i}{d_o}$, which tells us that $\frac{1}{d_i} = +\frac{1}{1.5 d_o}$.

Applying the thin-lens equation:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{1.5 d_o} = \frac{3}{3d_o} + \frac{2}{3d_o} = \frac{5}{3d_o}.$$

Thus, we find that $d_o = \frac{5}{3} f = +40 \text{ cm}$, and we can show that $d_i = +60 \text{ cm}$.

The ray diagram for this situation is shown in Figure 24.39, again confirming the calculations above. These two rays, and all rays that travel from the tip of the object to the tip of the image, take the same time to get there.

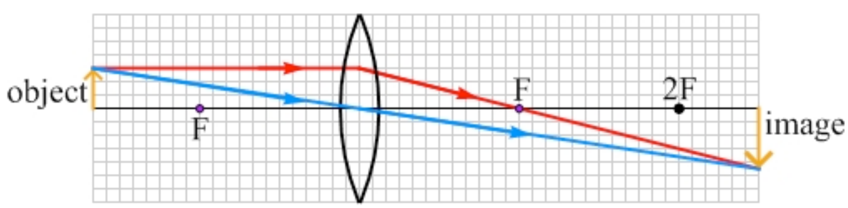


Figure 24.39: A ray diagram for the situation involving a real image. Each box on the grid measures 2 cm × 2 cm.

Related End-of-Chapter Exercises: 25 – 28.

Essential Question 24.8: Return to the situation described in Example 24.8. Would there still be two solutions if the image was smaller than the object? Explain.

Answer to Essential Question 24.8: Yes, there would still be two solutions. One solution involves a real image produced by a converging lens, while the other involves a virtual image produced by a diverging lens.

24-9 The Human Eye and the Camera

There are some interesting parallels between the human eye and a camera. Before continuing, stop and make a list of some of the similarities between the two systems, as well as some of the differences.

An important component of both the eye and a camera is the lens that is used to create a real image. Interestingly, in both systems the image is inverted.

In each case, there is a diaphragm that controls the amount of light that gets through to the lens. In the eye, the pupil plays this role, while the camera shutter does this job in the camera.

Another important component of both systems is the system for recording the focused image. In the eye, this system is the set of rods and cones that cover the retina at the back of the eye. In older cameras, the film does the job of recording the image, while in a digital camera the system is quite similar to that of the eye, with a large number (four million being typical) of tiny light sensors sensing the image.

A key difference between the human eye and the camera is in how the focusing is done. In your own eyes, when you look from one object to another that is a different distance away from you, your eyes adjust to bring the second object into focus. To understand why this adjustment is necessary, considering the thin-lens equation:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} .$$ (Equation 24.7: **The thin-lens equation**)

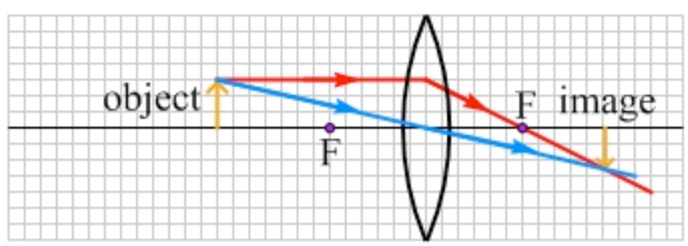
In the human eye, the distance between the lens and retina, which is the image distance d_i, is fixed. Changing the object distance thus requires a change in the focal length of the eye to produce a focused image on the retina. This adjustment is done via the ciliary muscles in the eye, which actually change the shape of the lens to change the focal length. This process, known as accommodation, occurs so quickly that we don't even notice it.

In the camera, the lens is generally a solid piece of glass with a fixed focal length. Adjusting the object distance, therefore, requires an adjustment in the image distance (the distance from the lens to the film or light sensors). This is accomplished by moving the lens. The farther the object is from the camera, the closer the lens must be to the film or light sensors.

EXPLORATION 24.9 – Correcting human vision

Consider the ray diagram shown in Figure 24.40, in which the object is quite close to the eye.

Figure 24.40: A ray diagram for a situation in which you are focusing on an object quite close to you.



Step 1 – ***Where is the retina located in Figure 24.40?*** The retina must be where the image is for you to see the focused image. Note that the image is upside down, but when you look at objects you do not see them upside down. The flipping of the image back to upright is accomplished by the image-processing system in your brain.

Step 2 – ***If the object in Figure 24.40 is moved farther away, and the eye's focal length is unchanged, where is the focused image? Sketch a new diagram to show this.*** When the object distance increases, and the focal length remains the same, the image distance decreases, as shown in Figure 24.41.

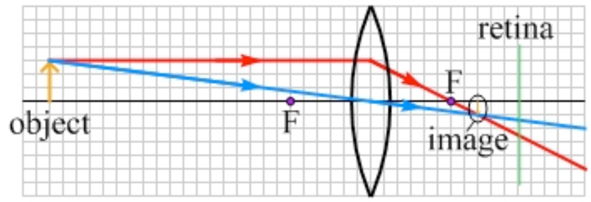


Figure 24.41: If the eye's focal length is fixed, moving the object farther from the eye produces a focused image at a point within the eye, and a blurred image on the retina.

Step 3 – ***In a normal eye, the shape of the lens is changed so that the image is formed not before the retina, as in Figure 24.41, but on the retina. To accomplish this for the situation shown in Figure 24.41, should the lens be flatter or rounder than the lens that produces the correctly focused image in Figure 24.40, when the object is closer to the eye?*** The lens in Figure 24.41 is doing too much focusing, because it is too round. The ciliary muscles flatten the lens to shift the image position farther from the lens, onto the retina, as shown in Figure 24.42.

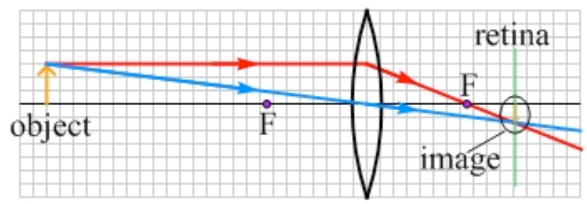


Figure 24.42: In a normal eye, when the object is farther from the eye, the curvature of the lens is reduced to focus the image onto the retina.

Step 4 – ***In the eye of a near-sighted person (someone who can only focus properly on objects close to them), the lens can not be made flat enough to focus the image of a far-off object on the retina, so a corrective lens is placed in front of the eye. Does a near-sighted person need a diverging lens or a converging lens? Support your answer with a diagram.*** A near-sighted person requires a diverging lens. Before the light from the object reaches the eye, the lens diverges it enough that the eye then deflects the rays to create a focused image on the retina, as shown in Figure 24.43. Another way to understand this is that the diverging lens creates a virtual image of the object close enough to the eye that the near-sighted person can focus on it.

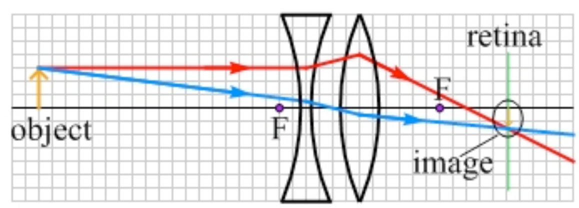


Figure 24.43: Placing a diverging lens, of the appropriate shape, in front of the eye of a nearsighted person allows the person to see far-away objects correctly.

Key ideas for corrective lenses: In a normal eye, the ciliary muscles adjust the shape of each eye's lens to form images on the retina. In a near-sighted person, the lens cannot be made flat enough to view far-away objects correctly, so a diverging lens is used to correct this deficiency. In a far-sighted person, the lens cannot be made round enough to properly focus on close objects, so the corrective lens is a converging lens. **Related End-of-Chapter Exercises: 29, 63.**

Essential Question 24.9: What if Figure 24.40 represents a camera in which the image is focused on the film? When the object is moved to the position shown in Figure 24.41, what changes in the camera to produce a focused image again? Draw a ray diagram reflecting this change.

Answer to Essential Question 24.9: If Figure 24.40 shows a focused image for the camera, the image location is where the film is in the camera. When the object is moved farther away, as in Figure 24.41, the lens must be moved to the right, closer to the film, so that the image is again focused on the film. This is shown in Figure 24.44.

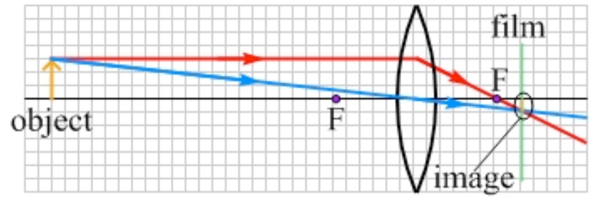


Figure 24.44: In a camera, the lens has a fixed shape, so the lens is moved to the right, closer to the film, when the object is moved farther from the camera. This produces a focused image on the film.

24-10 Multi-lens Systems

How do we handle systems in which there is more than one lens (or more than one mirror, or combinations of mirrors and lenses)? The standard approach is to do the analysis one lens (or mirror) at a time. Starting from the object, follow the light until it reaches the first lens or mirror. Apply the methods we learned earlier to find the image created by the first lens or mirror. That image is then the object for the next lens or mirror in the sequence. Continue the process, one lens or mirror at a time, until we have followed the light through every lens or mirror.

EXAMPLE 24.10 – Analyzing a two-lens system

As shown in Figure 24.45, a toy train with a height of 4.0 cm is placed 24 cm from a converging lens that has a focal length of 8.0 cm. A second converging lens, identical to the first, is placed 18 cm from the first lens, and on the opposite side of the lens from the train.

(a) Calculate the position of the image created by the first lens, and sketch a ray diagram to support your calculations.

(b) Repeat part (a), but for the second lens, to find the final image.

(c) Determine the overall magnification of this two-lens system.

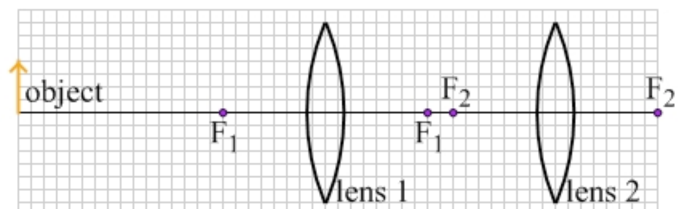


Figure 24.45: An object 24 cm in front of one lens, with a second lens 18 cm behind the first lens. Each box on the grid measures 1 cm × 1 cm.

SOLUTION

(a) To find the first image, let's apply the thin-lens equation. In this case, we get:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(+24 \text{ cm}) \times (+8.0 \text{ cm})}{24 \text{ cm} - 8.0 \text{ cm}} = \frac{192 \text{ cm}^2}{16 \text{ cm}} = 12 \text{ cm} .$$

A ray diagram for this situation is shown in Figure 24.46, confirming the calculation above and showing that the image is real, inverted, and smaller than the object.

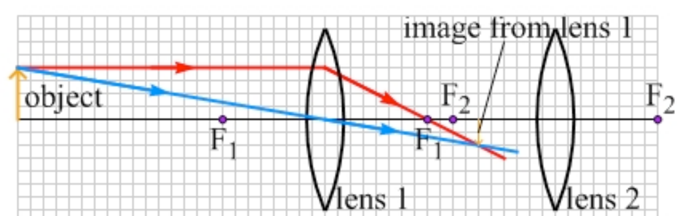


Figure 24.46: A ray diagram showing the real inverted image produced by the first lens. Each box on the grid measures 1 cm × 1 cm.

(b) We use the image produced by the first lens as the object for the second lens. The first thing we need to determine is the object distance for the second lens. If the image is 12 cm from the first lens, and the second lens is 18 cm from the first lens, then the image is only 6 cm (18 cm minus 12 cm) from the second lens (and is now the object for that lens). With an object distance of 6 cm, applying the thin-lens equation gives:

$$d_i = \frac{d_o \times f}{d_o - f} = \frac{(+6 \text{ cm}) \times (+8.0 \text{ cm})}{6 \text{ cm } - 8.0 \text{ cm}} = \frac{48 \text{ cm}^2}{-2 \text{ cm}} = -24 \text{ cm}.$$

The ray diagram for this situation is shown in Figure 24.47, confirming the calculations.

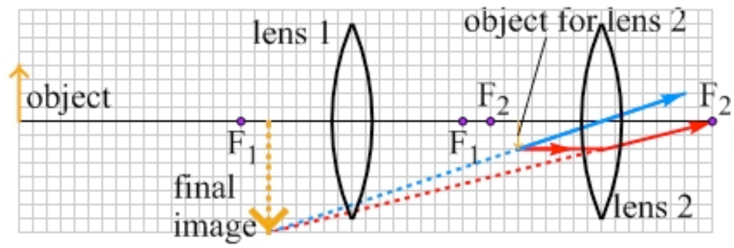


Figure 24.47: A ray diagram showing the image produced by the second lens. Each box on the grid measures 1 cm × 1 cm.

(c) We can determine the magnification in two different ways (which should give the same answer). First, the overall magnification is the ratio of the height of the final image to the height of the original object. We can find the heights of the two images by applying the magnification equation twice. For the first lens,

$m_1 = \frac{h_{i1}}{h_{o1}} = -\frac{d_{i1}}{d_{o1}} = -\frac{12 \text{ cm}}{24 \text{ cm}} = -0.5$, which, with an object height of 4.0 cm, gives an image height of –2.0 cm.

For the second lens,

$m_2 = \frac{h_{i2}}{h_{o2}} = -\frac{d_{i2}}{d_{o2}} = -\frac{(-24 \text{ cm})}{6 \text{ cm}} = +4.0$, which, with an object height of –2.0 cm, gives an image height of –8.0 cm.

Thus, the overall magnification is: $M = \frac{h_{i2}}{h_{o1}} = \frac{-8.0 \text{ cm}}{4.0 \text{ cm}} = -2.0$.

The final image is twice as large as the original object, with the minus sign telling us that the image is inverted compared to the original object.

The second way to find the overall magnification is to combine the magnifications of the individual lenses. The first lens gives an image that is inverted and half as large as the original object, while the second lens increases the size by a factor of four while maintaining the orientation. The overall magnification is the product of the individual magnifications:

$$M = m_1 \times m_2 = -0.5 \times (+4.0) = -2.0.$$

The method we applied here can be applied to any number of lenses and/or mirrors. Simply follow the light through the system, using the image created by one lens or mirror as the object for the next lens or mirror in the sequence.

Related End-of-Chapter Exercises: 64, 65.

Essential Question 24.10: In an astronomical telescope, which uses two converging lenses, the distance between the lenses is the sum of the two focal lengths. Explain why this is the case.

Answer to Essential Question 24.10: In an astronomical telescope, the object (such as a distant galaxy) is very far away, so the first lens (the objective) creates an image at the focal point of that lens. To produce as large a final image as possible, the second lens (the eyepiece) should create an image at infinity, so the first image needs to be located at the eyepiece's focal point. Because the focal points coincide, the distance between the lenses is the sum of the focal lengths.

Chapter Summary

Essential Idea: Refraction and Lenses.

When light passes from one medium to a second medium in which the speed of light is different, the change in speed is generally associated with refraction (a change in direction of the light). The phenomenon of refraction is exploited in optical fibers and corrective lenses.

Index of Refraction

The index of refraction of a medium is a unitless parameter that is equal to the ratio of the speed of light in vacuum to the speed of light in the medium. In general, the speed of light in vacuum represents the maximum speed of light, so we expect $n \geq 1$.

$$n = \frac{c}{v}.$$ (Equation 24.1: **Index of refraction**)

The index of refraction is also equal to the ratio of the wavelength of light in vacuum to the wavelength of light in the medium.

$$n = \frac{\lambda_{vacuum}}{\lambda_{medium}}.$$ (Equation 24.2: **Index of refraction, in terms of wavelength**)

Snell's Law

When light is transmitted from one medium to another, the angle of incidence, θ_1, in the first medium is related to the angle of refraction, θ_2, in the second medium by:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$ (Equation 24.3: **Snell's Law**)

These angles are measured from the normal (perpendicular) to the surface. The n's represent the indices of refraction of the two media.

Total Internal Reflection

Optical fibers are an important application of total internal reflection. When light traveling in a medium encounters an interface separating that medium from a medium with a lower index of refraction, 100% of the light will be reflected back into the first medium if the angle of incidence exceeds a particular critical angle given by:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right).$$ (Eq. 24.4: **The critical angle beyond which total internal reflection occurs**)

Ray Diagrams

To draw a ray diagram, we show rays leaving the tip of an object and being refracted by a lens. The tip of the image is where the refracted rays meet. If the rays leaving a single point on the object are refracted so they pass through a single point, a real image is formed. If, instead, such refracted rays appear to diverge from a single point behind the lens, a virtual image is formed. A summary of three rays that are particularly easy to draw is given in section 24-7.

Thin Lenses

Type of lens	Focal length	Image characteristics
Diverging (usually concave)	Negative	The image is virtual, upright, smaller than the object, and between the lens and the focal point on the side of the lens the object is on.
Converging (usually convex)	Positive	The image can be real or virtual, and larger than, smaller than, or the same size as the object. See the table below for details.

Table 24.2: A summary of the lenses we investigated in this chapter.

Images formed by a Converging Lens

Object position	Image position	Image characteristics
∞	At the focal point.	Real image with height of zero.
Moving from ∞ toward twice the focal length.	Moving from the focal point toward twice the focal length.	The image is real, inverted, and smaller than the object. The image moves closer to twice the focal length, and increases in height, as the object is moved closer to twice the focal length from the lens.
At twice the focal length.	At twice the focal length.	The image is real, inverted, and the same size as the object.
Moving from twice the focal length toward the focal point.	Moving from twice the focal length toward infinity.	The image is real, inverted, and larger than the object. The image moves farther from the lens, and increases in height, as the object is moved closer to the focal point.
At the focal point.	At infinity.	The image is at infinity, and is infinitely tall.
Closer to the lens than the focal point.	On the same side of the lens as the object.	The image is virtual, upright, and larger than the object. The image moves closer to the lens, and decreases in height, as the object is moved closer to the lens.

Table 24.3: A summary of the image positions and characteristics for a converging lens.

The thin-lens equation (same as the mirror equation from Chapter 23)

The thin-lens equation relates the object distance, d_o, the image distance, d_i, and the focal length, f. The mnemonic "If I do I di" can help you to remember the equation.

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}.$$ (Equation 24.7: **The thin-lens equation**)

$$d_i=\frac{d_o \times f}{d_o - f}$$ (Equation 24.8: **The thin-lens equation, solved for the image distance**)

Sign conventions

The image distance is positive if the image is on the reflective side of the mirror (a real image), and negative if the image is behind the mirror (a virtual image).

The image height is positive when the image is upright, and negative when the image is inverted. A similar rule applies to the object height.

Magnification

The magnification, m, is the ratio of the image height (h_i) to the object height (h_o).

$$m=\frac{h_i}{h_o}=-\frac{d_i}{d_o}.$$ (Equation 24.9: **Magnification**)

The image is larger than the object if $|m|>1$, smaller if $|m|<1$, and of the same size if $|m|=1$. The magnification is positive if the image is upright, and negative if the image is inverted.

End-of-Chapter Exercises

Exercises 1 – 12 are conceptual questions designed to assess whether you understand the main concepts of the chapter.

1. Figure 24.48 shows a beam of light in air (medium 1) incident on an interface along the normal to the interface, with some of the light refracting into medium 2 and some reflecting straight back. What, if anything, is wrong? If nothing is wrong, determine the index of refraction of medium 2. If something is wrong, explain.

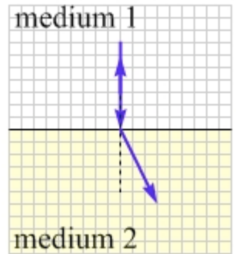


Figure 24.48: A beam of light incident along the normal to an interface, refracting into medium 2. For Exercise 1.

2. As shown in Figure 24.49, a beam of light in medium 1 is incident on an interface separating medium 1 from medium 2. Part of the beam reflects from the interface, and part refracts into medium 2. Describe, qualitatively, how the indices of refraction of the two media compare if the refracted beam follows (a) path a, (b) path b, or (c) path c.

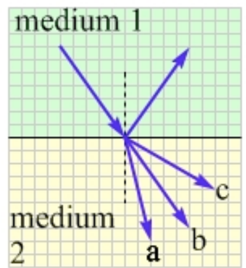


Figure 24.49: Three possible paths for the refracted beam to follow when light in medium 1 encounters the interface between two media, for Exercise 2.

3. Figure 24.50 shows an object near a diverging lens. A single ray is drawn on the ray diagram. Duplicate the diagram, and add a second ray to show the position of the image.

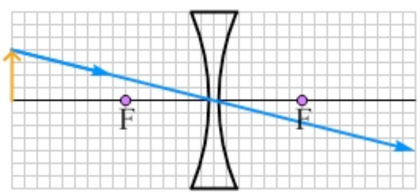


Figure 24.50: A single ray, passing undeflected through the center of a thin lens, is shown on the figure. For Exercise 3.

4. Two rays of light are shown on the ray diagram in Figure 24.51, along with two arrows representing the object and the image. (a) Could this diagram be incorrect? If so, how could it be corrected? (b) Could this diagram be correct? If so, explain how this could be possible.

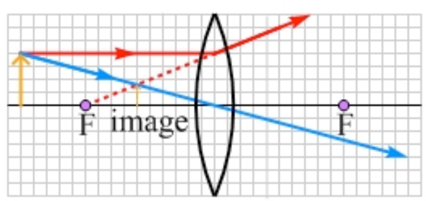


Figure 24.51: A ray diagram, for Exercise 4. Could it be correct, or is it incorrect?

5. Figure 24.52 shows an object in front of a converging lens. First, re-draw the diagram, preferably on a piece of graph paper. For parts (a) – (c), start the ray from the tip of the object and show how the ray is refracted by the lens. (a) On your diagram, draw a ray that travels parallel to the principal axis toward the lens. (b) Draw a ray that travels directly toward the center of the lens. (c) Draw a ray that travels through the focal point between the object and the lens. (d) Use the rays to locate the image on the diagram.

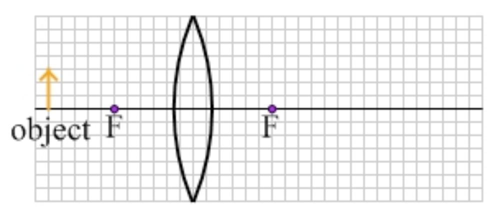


Figure 24.52: An object in front of a converging lens, for Exercise 5.

6. Figure 24.53 shows the virtual image formed by a thin-lens, as well as the locations of the two focal points of the lens. The small squares on the grid measure 1.0 cm × 1.0 cm. The goal of this exercise is to determine the type of lens creating the image, as well as the position and size of the object. (a) How many different solutions are there to this exercise? (b) For each of the solutions, describe the kind of lens that produces the image, the object distance, and the object height. (c) For each solution, sketch a ray diagram.

Figure 24.53: The virtual image created by a thin lens, and the locations of the two focal points of the lens. The small squares on the grid measure 1.0 cm × 1.0 cm. For Exercise 6.

7. Assume that the index of refraction of a particular piece of glass varies as shown in the graph in Figure 24.54. If light traveling in this piece of glass encounters a glass-air interface, is the critical angle for total internal reflection larger for violet light, red light, or is it equal for both? Explain.

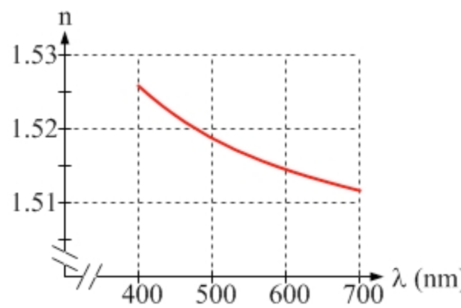


Figure 24.54: A graph of the index of refraction, as a function of wavelength, for a typical sample of glass. The graph is confined to the visible spectrum, from 400 nm (violet) to 700 nm (red). For Exercise 7.

8. In a common demonstration, a small glass beaker virtually disappears when it is placed in a larger beaker full of a particular type of oil. Using the principles of physics addressed in this chapter, explain how this could work.

9. If you wear your eyeglasses under water, can you still see clearly? Explain.

10. Identify whether the lens in this situation is a converging lens or a diverging lens. For each part below, determine which possibility you can rule out, if either. (a) First, when an object is placed 15 cm from a lens, a virtual image is observed. (b) Then, when the object is moved a little closer to the lens, the image is observed to move closer to the lens. (c) As the object is moved closer to the lens, the image is observed to decrease in size.

11. Identify whether the lens in this situation is a converging lens or a diverging lens. For each part below, determine which possibility you can rule out, if either. (a) First, when an object is placed 15 cm from a lens, a virtual image is observed. (b) Then, when the object is moved a little closer to the lens, the image is observed to increase in size. (c) What, if anything, can you conclude about the focal length of the lens?

12. You have an unknown optical device that you are trying to identify. The device could be any one of five things, a plane mirror, convex mirror, concave mirror, converging lens, or diverging lens. To identify the device you make the following observations, in order. For each part, state what, if anything, the observation tells you about what kind of mirror or lens the device is. (a) You observe that when you place an object in front of the device that the device creates an image of the object that is larger than the object. What could the device be? (b) You then observe that the image is inverted compared to the object. Based on this and the previous observation, what could the device be? (c) You then observe that the larger inverted image is on the opposite side of the device as the object. Based on this and the previous observations, what is the device? (d) If the device has a focal length of 10 cm and the object distance is 15 cm, what is the image distance?

Exercises 13 – 15 involve refraction.

13. A beam of light is incident on an interface separating two media. When the angle of incidence, measured from the normal, is 10.0°, the angle of refraction is 18.0°. What is the angle of refraction when the angle of incidence is 30.0°?

14. Return to the situation described in Exercise 13. If the speed of light in one of the media is 2.80×10^8 m/s, what is the speed of light in the other medium? Is there more than one possible answer? Explain.

15. A beam of light is incident on an interface separating two media, as shown in Figure 24.55. The squares in the grid on the diagram measure 10 cm × 10 cm. (a) Which medium has a larger index of refraction? Explain your answer. (b) If the index of refraction of one of the media is 1.10, what is the index of refraction of the other medium? Is there more than one possible answer? Explain.

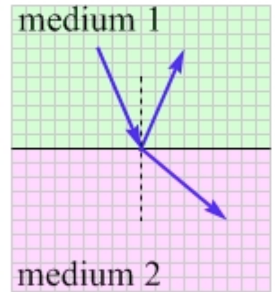


Figure 24.55: When a beam of light is incident on the interface separating two media, part of the beam is reflected back into medium 1 and part refracts into medium 2, for Exercise 15.

Exercises 16 – 20 involve total internal reflection.

16. As shown in Figure 24.56, a beam of light traveling in medium 2 experiences total internal reflection when it encounters the interface separating medium 2 from medium 1. The angle of incidence is 45°, and medium 1 is air, with an index of refraction of 1.00. What, if anything, can you say about the index of refraction of medium 2?

17. As shown in Figure 24.56, a beam of light traveling in medium 2 experiences total internal reflection when it encounters the interface separating medium 2 from medium 1. The angle of incidence is 45°, and the speed of the light in medium 2 is 1.50×10^8 m/s. What, if anything, can you say about the index of refraction of medium 1?

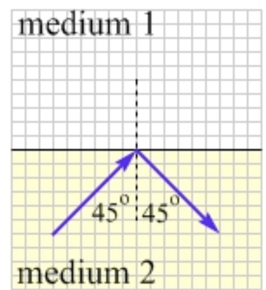


Figure 24.56: A beam of light in medium 2 experiences total internal reflection when it encounters the interface separating medium 2 from medium 1. For Exercises 16 and 17.

18. As shown in Figure 24.57, a beam of light in medium 1 is incident on an interface separating medium 1 from medium 2. Part of the beam reflects from the interface, and part refracts into medium 2, traveling along the interface between the media. Using the diagram, calculate the critical angle for total internal reflection for this situation.

19. A beam of light is incident on an interface separating two media. When the angle of incidence, measured from the normal, is 18.0°, the angle of refraction is 10.0°. What is the critical angle for this particular interface?

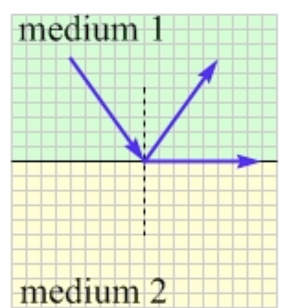


Figure 24.57: When a beam of light in one medium encounters an interface, part of it reflects and part refracts into medium 2, traveling along the interface. For Exercise 18.

20. A small red light-emitting diode (LED) is placed 12.0 cm below the surface of the water in a bathtub. A circle of red light is observed at the water surface. What is the diameter of this circle? Assume that the air above the water has an index of refraction of 1.00, and that the water has an index of refraction of 1.33.

Exercises 21 – 24 are designed to give you practice applying the thin-lens equation.

21. As you are analyzing a thin-lens situation, you write an equation that states: $\frac{1}{f} = \frac{1}{+40 \text{ cm}} + \frac{1}{+30 \text{ cm}}$. (a) What is the value of $1/f$ in this situation? (b) What is the focal length of the lens? (c) What kind of lens is this?

22. Return to Exercise 21. What is the object distance in this situation, and what is the image distance?

23. As you are analyzing a thin-lens situation, you write an equation that states: $\frac{1}{f} = \frac{1}{+15 \text{ cm}} + \frac{1}{-30 \text{ cm}}$. (a) What is the value of $1/f$ in this situation? (b) What is the focal length of the lens? (c) What kind of lens is this?

24. As you are analyzing a thin-lens situation, you write an equation that states: $\frac{1}{+24 \text{ cm}} = \frac{1}{+24 \text{ cm}} + \frac{1}{d_i}$. (a) What is the value of $1/d_i$ in this situation? (b) What is the image distance?

Exercises 25 – 28 are designed to give you practice applying the general method for analyzing a problem involving lenses.

25. An object is placed 30 cm away from a lens that has a focal length of +10 cm. (a) Sketch a ray diagram, to show the position of the image and the image characteristics. (b) Determine the image distance. (c) Determine the magnification.

26. An object is placed 30 cm away from a lens that has a focal length of –10 cm. (a) Sketch a ray diagram, to show the position of the image and the image characteristics. (b) Determine the image distance. (c) Determine the magnification.

27. You are examining an ant through a magnifying glass, which is simply a converging lens. When the ant is 10 cm from the lens and you look through the lens, you see an upright image of the ant that is 3.0 times larger than the ant itself. (a) Determine the image distance in this situation. (b) Determine the focal length of the magnifying glass. (c) Sketch a ray diagram for this situation.

28. An object is placed 48 cm from a lens. When you look through the lens, you see an image of the object that is 3.0 times larger than the object. (a) What kind of lens is it? (b) Sketch a ray diagram to check your calculations. Make sure you find all possible solutions.

Exercises 29 – 33 involve applications of refraction and lenses.

29. The lens in your digital camera has a focal length of 5.0 cm. You are using the camera to take a close-up picture of a flower that is 12.0 cm from the lens. (a) Determine how far the lens should be from the image-sensing system inside the camera. (b) Determine the magnification in this situation. (c) If you then use your camera to take a photograph of your friend, who is 3.0 m from the lens of the camera, how far should the lens be from the image-sensing system now? (d) Determine the magnification in this new situation.

30. Binoculars generally use pairs of prisms in which the light experiences total internal reflection. Each prism (in blue on the diagram) is right-angled, with the other two angles being 45°. A diagram of the path followed by light as it travels through the prisms to your eyes is shown in Figure 24.58. If the prisms are surrounded by air, determine the minimum index of refraction of the prism material.

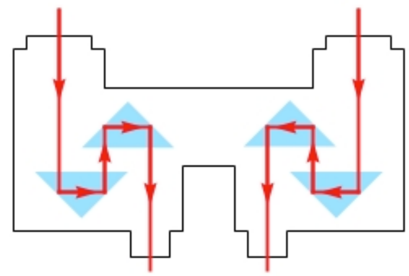


Figure 24.58: A pair of binoculars, with two right-angled prisms on each side to shift the light from the path it is following when it enters the binoculars to a path that takes it right into your eye. For Exercise 30.

31. Diamonds are particularly colorful and sparkly. One reason for this is the relatively large index of refraction of diamond of around 2.4. Another reason is that diamond does exhibit dispersion. Use these facts to explain why light that enters a diamond often experiences a number of reflections within the diamond before emerging, and why this would help spread white light out into its constituent colors.

32. When you are driving along a highway, or walking in the desert, on a hot day, you often see a mirage in the distance, where it looks like there is water on the road, or sand, ahead of you. Do a little research about this phenomenon and write a couple of paragraphs describing it, and the physics relevant to this chapter that are responsible for producing a mirage.

33. As a participant on the reality show *Survivor,* you are stranded on a sunny island with several other individuals. You get the bright idea of trying to start a fire by using your eyeglasses to focus the Sun's rays onto a piece of dry wood. (a) How far from your glasses should the wood be? (b) Do you need to be nearsighted or farsighted for this method to have a chance of working?

General problems and conceptual questions

34. A pulse of light takes 3.00 ns to travel through air from an emitter to a detector. When a piece of transparent material with a length of 40 cm is introduced into the light's path, the pulse takes 3.40 ns. What is the index of refraction of the transparent material?

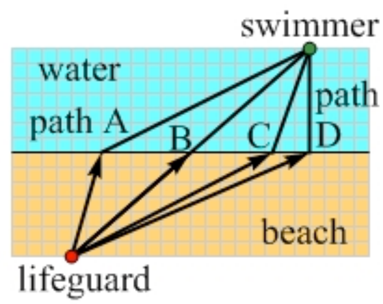


Figure 24.59: A situation involving a lifeguard trying to reach a swimmer in the shortest time, which is analogous to light as it travels from one medium to another. For Exercise 35.

35. Answer this problem by analogy with optics and refraction. A lifeguard can run at 5.0 m/s along the sandy beach, and can swim at 2.0 m/s through the water. The initial positions of the lifeguard and a swimmer who needs the lifeguard's help are shown in Figure 24.59. Four possible paths for the lifeguard to take are shown on the diagram. Which path should the lifeguard take to minimize the time it takes to reach the swimmer? Explain.

36. A particular converging lens has a focal length of +20 cm. A second lens of exactly the same shape as the first lens has a focal length of +25 cm. Is this possible? Explain.

37. As shown in Figure 24.60, light traveling in medium 2 experiences total internal reflection at the boundary with medium 1, then experiences reflection and refraction at the boundary with medium 3. Rank the media based on their indices of refraction, from largest to smallest.

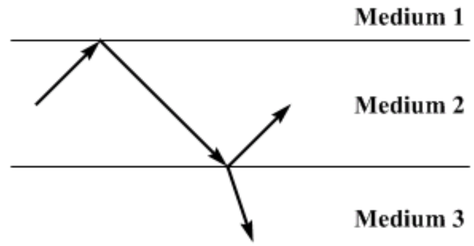


Figure 24.60: The path followed by light that is initially in medium 2. For Exercise 37.

38. A horizontal beam of monochromatic (single wavelength) laser light is incident on a block of glass, as shown in Figure 24.61. The faces of the block of glass are all either horizontal or vertical except for the face at the top right, which is inclined at 45°. The glass has an index of refraction of 1.50, while the air surrounding the glass has an index of refraction of 1.00. Copy the diagram, and sketch the path the light takes through the block, accounting for both refraction and reflection at each air-glass interface the light encounters. Label every point where light emerges from the glass back into the air and, at each of these points, determine the angle at which the light emerges from the glass.

39. Repeat Exercise 38, but now the medium surrounding the glass is water, with an index of refraction of 1.33, instead of air.

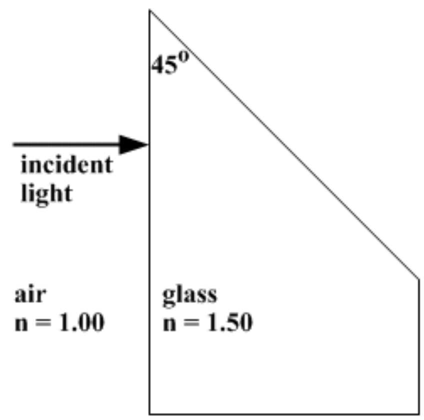


Figure 24.61: Light is incident on a piece of glass that is surrounded by air. For Exercises 38 and 39.

40. As shown in Figure 24.62, a beam of red and violet light is incident along the normal to one surface of a right-angled triangular glass prism. The glass has an index of refraction of 1.52 for red light, and 1.54 for violet light. (a) Draw a sketch showing how the red and violet beams travel from the point at which they enter the prism to the side *ab* of the prism. (b) If the angle at vertex *a* of the prism is $\theta = 30.0°$, determine the angles of refraction for the red and violet beams that emerge from the prism from the side *ab*. Show these refracted beams on your sketch. (c) On your sketch, show how the red and violet beams reflect from side *ab* of the prism.

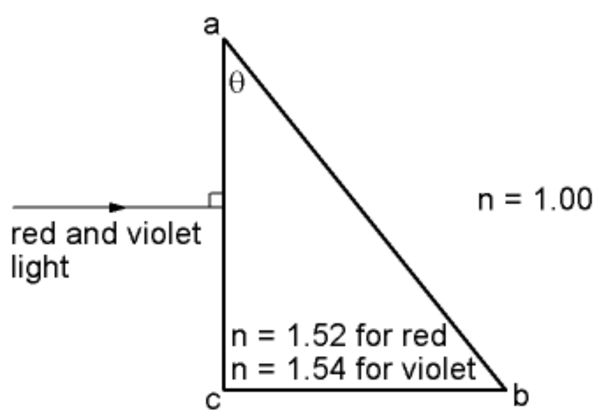


Figure 24.62: A beam of red and violet light entering a right-angled glass prism. For Exercise 40.

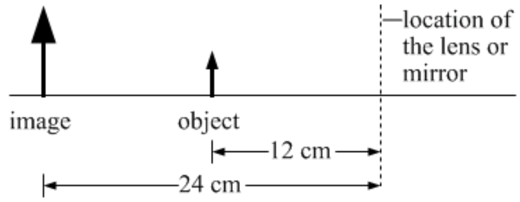


Figure 24.63: When an object is placed 12 cm in front of an optical device, which is either a lens or a mirror, a virtual image is formed 24 cm from the device. For Exercise 41.

41. As shown in Figure 24.63, when an object is placed 12 cm in front of a particular optical device (either a single mirror or a single lens) a **virtual image** is formed 24 cm from the device on the same side of the device as the object. (a) What kind of mirror or lens could this optical device be? (b) Find the focal length of the device.

42. Galileo Galilei used a telescope to carry out detailed observations of the moons of a particular planet. Do some research about Galileo's telescope, and about the observations he made with it, and write a couple of paragraphs describing the telescope and the observations.

43. In a particular situation, involving an object in front of a lens, the object distance is 20 cm and the magnification is +4.0. Find (a) the image distance, (b) the focal length of the lens.

44. In a particular situation, involving an object in front of a lens with a focal length of +20 cm, the magnification is +4.0. Find (a) the object distance, (b) the image distance.

45. An object is placed a certain distance from a lens. The image created by the lens is exactly half as large as the object. If the two focal points of the lens are 20 cm from the lens, where is the object? Where is the image? (a) Find one solution to this problem. (b) Find a second solution. (c) Sketch ray diagrams for your two solutions.

46. Figure 24.64 shows an object and a real image created by a lens. Assume the boxes on the grid measure 10 cm × 10 cm. Find the position of the lens, and its focal length.

Figure 24.64: This figure shows an object and a real image created by a lens. For Exercises 46 and 48.

47. Figure 24.65 shows an object (the larger arrow) and the virtual image of that object, created by a lens. Assume the boxes on the grid measure 10 cm × 10 cm. Find the position of the lens, and its focal length.

Figure 24.65: The larger arrow represents an object, while the smaller arrow represents the virtual image of that object, created by a lens. For Exercise 47.

48. Sketch a ray diagram for the situation shown in (a) Figure 24.64, (b) Figure 24.65.

49. In the situation shown in Figure 24.66, a small red LED (light-emitting diode) is placed on the principal axis at one of the focal points of a particular converging lens. The LED can be considered to be a point source. Draw a ray diagram to show what happens to rays of light that are emitted by the LED and are refracted by the lens.

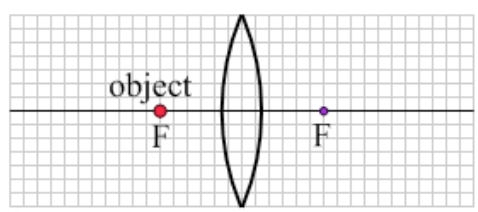


Figure 24.66: A small red LED (light-emitting diode) is placed at one focal point of a converging lens, for Exercise 49.

50. In the situation shown in Figure 24.67, a small red LED (light-emitting diode) is placed on the principal axis, 7.0 cm from a converging lens that has a focal length of 14 cm. The LED can be considered to be a point source. Find the location of the image of the LED.

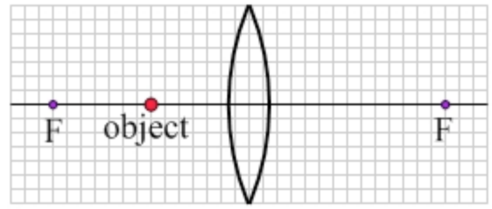


Figure 24.67: A small red LED (light-emitting diode) is placed in front of a converging lens, for Exercises 50 and 52.

51. In the situation shown in Figure 24.68, a small red LED (light-emitting diode) is placed 4.0 cm above the principal axis, and 8.0 cm away from a diverging lens that has a focal length of –12 cm. The LED can be considered to be a point source. Find the location of the image of the LED.

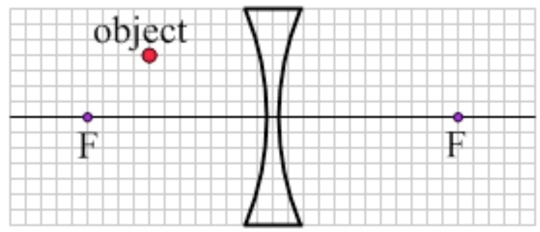


Figure 24.68: A small red LED (light-emitting diode) is placed in front of a diverging lens, for Exercises 51 and 52. The LED is 4.0 cm above the principal axis.

52. Draw several rays showing how the image of the LED is formed in (a) Figure 24.67, and (b) Figure 24.68.

53. A model of a dinosaur is placed 36 cm away from a converging lens that has a focal length of +20 cm. The model is 8.0 cm tall. Determine (a) the location of the image, (b) the height of the image, (c) whether the image is real or virtual, and (d) whether the image is upright or inverted.

54. Repeat Exercise 54, with the model of the dinosaur 15 cm from the lens instead.

55. A model of a horse is placed 36 cm away from a lens that has a focal length of –20 cm. The model is 8.0 cm tall. Determine (a) the image location, (b) the height of the image, (c) whether the image is real or virtual, and (d) whether the image is upright or inverted.

56. Return to the situation described in Exercise 55. Describe what happens to the position and size of the image if the model is moved a little bit farther from the lens.

57. Return to the situation described in Exercise 53. Describe what happens to the position and size of the image if the model is moved a little bit farther from the lens.

58. A particular lens has a focal length of +40 cm. (a) For this lens, plot a graph of $1/d_i$ as a function of $1/d_o$ for object distances between +20 cm and +80 cm. (b) How can you read the focal length directly from the graph?

59. Repeat Exercise 58, but now plot a graph of d_i as a function of d_o.

60. Figure 24.69 shows a graph of $1/d_i$ as a function of $1/d_o$ for a particular lens. What kind of lens is it, and what is the focal length of the lens?

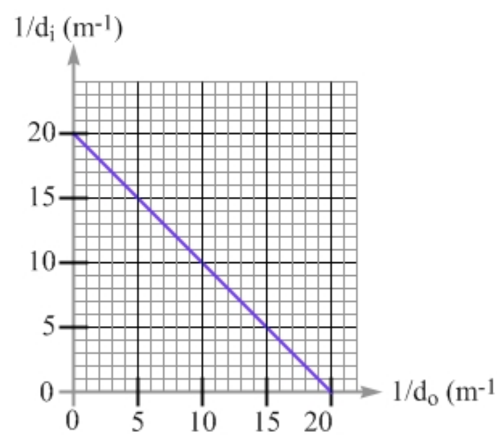


Figure 24.69: A graph of $1/d_i$ as a function of $1/d_o$ for a particular lens. For Exercise 60.

61. A particular lens has a focal length of +25 cm. (a) For this lens, plot a graph of the magnification as a function of the object distance, for object distances between +10 cm and +40 cm. (b) How can you read the focal length directly from the graph?

62. Refer to Figure 24.23, just above the start of Section 24-5. (a) What, if anything, happens to the image if you cover up the bottom half of the lens, preventing any light from reaching that part of the lens? (b) Does your answer change if you cover up the top half of the lens instead? Explain, and refer to Figure 24.23 in your explanations.

63. Refer to Figure 24.42, in Section 24-9, which shows how light from a distant object is focused on to the retina in your eye. (a) Sketch a ray diagram showing where the image is located when the object is only half the distance from the lens. Assume that neither the focal length of the lens, nor the distance from the lens to the retina, changes. (b) Sketch a second ray diagram showing a corrective lens placed in front of the eye, to correctly focus the image of the object onto the retina.

64. Figure 24.70 shows five parallel rays that are incident on a pair of lenses. The first lens is a converging lens with a focal length of +10 cm, while the second lens is a diverging lens with a focal length of -5 cm. One of the focal points of the converging lens is at the same location as one of the focal points of the diverging lens. (a) Sketch a ray diagram to show what this combination of lenses does to the parallel rays. (b) Describe the function of arranging two lenses in this way.

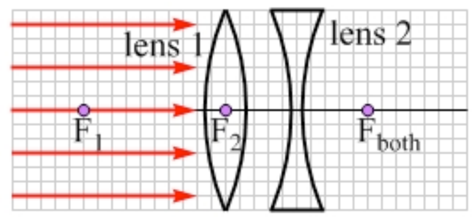


Figure 24.70: A set of parallel rays incident on a pair of lenses, for Exercise 64.

65. As shown in Figure 24.71, an object that is 4.0 cm tall is placed 10 cm to the left of a converging lens that has a focal length of 5.0 cm. A plane mirror is located 9.0 cm to the right of the lens. (a) Find the height and location of the image created by the lens. (b) After passing through the lens, the light encounters the mirror, and a second image is formed. Find the location and height of the image created by the mirror. (c) The mirror sends the light back toward the lens, and the lens creates another image. Find the location and height of this final image. (d) Determine which of these images are real and which are virtual. Justify your answer.

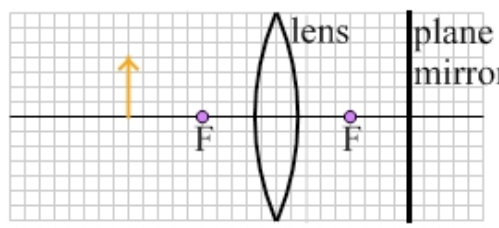


Figure 24.71: An object placed in front of a converging lens, with a plane mirror on the far side of the lens. For Exercise 65. Each square on the grid measures 1.0 cm × 1.0 cm.

66. Comment on following conversation, between two students discussing a situation in which they are trying to determine whether a particular lens is converging or diverging.

Jeremy: The first thing they tell us is that the image produced in the situation is virtual. Doesn't that mean the lens must be a diverging lens? That always gives virtual images.

Bridget: I don't think this tells us much, actually. Converging lenses can also give virtual images, if the object is farther from the lens than the focal point.

Jeremy: OK, so, then they say the image is smaller in size than the object. That doesn't tell us much either, right? Both lenses can produce images smaller than the object.

Student Worksheet: Lenses

One practical application of refraction is in lenses.

There are many parallels between how lenses work and how spherical mirrors work. For one thing, the same equations apply, which is rather surprising.

f is the focal length of the lens, (with an appropriate sign – see below).
The object distance d_o is the distance of the object from the lens.

The object distance d_i is the distance of the image from the lens.

The thin-lens equation: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ or, equivalently $d_i = \frac{d_o f}{d_o - f}$

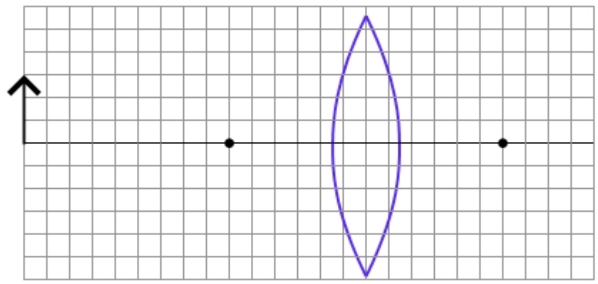
Sign convention: The basic rule on signs for lenses is that the object side of the lens is positive for object distance but negative for image distance, while the far side of the lens is positive for image distance. This is because the light goes through the lens. Lenses have two focal points, one on each side. Converging lenses (like converging mirrors) have positive focal lengths; diverging lenses (and mirrors) have negative focal lengths.

Magnification: The magnification tells us (as should the ray diagram) whether the lens makes an image that is smaller or larger than the object, and whether the image is upright or inverted. $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$ where h_i is the image height and h_o is the object height.

Example 1: An object is 15 units away from a convex (converging) lens that has a focal length of +6 units. Where is the image? What are its characteristics? Use the equations to help you.

Ray diagrams for lenses also work much like those for mirrors. The rays now have to obey Snell's Law, rather than the Law of Reflection, which makes it more challenging to figure out where the rays go. Again we have special rays that we draw because we know what they will do.

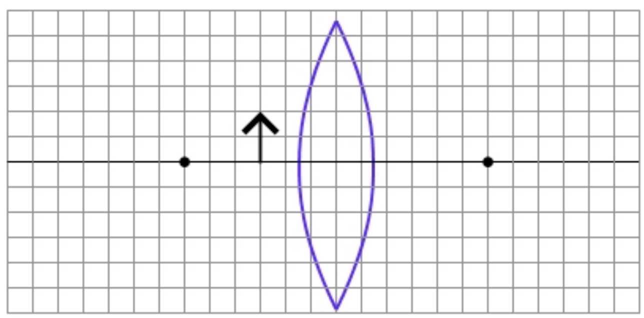
On the diagram draw the following special rays, all leaving the tip of the object.
1. Draw a straight line from the tip right through the center of the lens.
2. Draw a horizontal line from the tip to the lens, then it goes through the far focal point.
3. Draw a line from the tip through the near focal point to the lens, then horizontal.



Now draw several more rays from the tip to the lens, and show where they go.

Example 2: An object is 3 units away from a convex (converging) lens that has a focal length of +6 units. Where is the image? What are its characteristics? Use the equations to help you.

On the diagram draw the following special rays, all leaving the tip of the object.
1. Draw a straight line from the tip right through the center of the lens.
2. Draw a horizontal line from the tip to the lens, then it goes through the far focal point.
3. Draw a straight line from the left focal point that goes through the tip of the object to the lens, and changes direction to be horizontal on the far side of the lens.

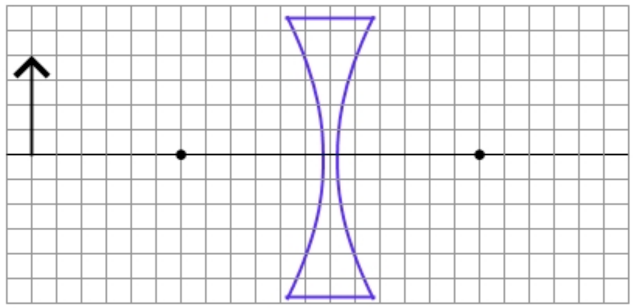


Now draw several more rays from the tip to the lens, and show where they go.

Example 3: An object is 12 units away from a concave (diverging) lens that has a focal length of –6 units. Where is the image? What are its characteristics? Use the equations to help you.

On the diagram draw the following special rays, all leaving the tip of the object.
1. Draw a straight line from the tip right through the center of the lens.
2. Draw a horizontal line from the tip to the lens, then it changes direction to go away from the focal point on the left.

3. Draw a straight line toward the right focal point that changes direction to be horizontal on the far side of the lens.



Now draw several more rays from the tip to the lens, and show where they go.

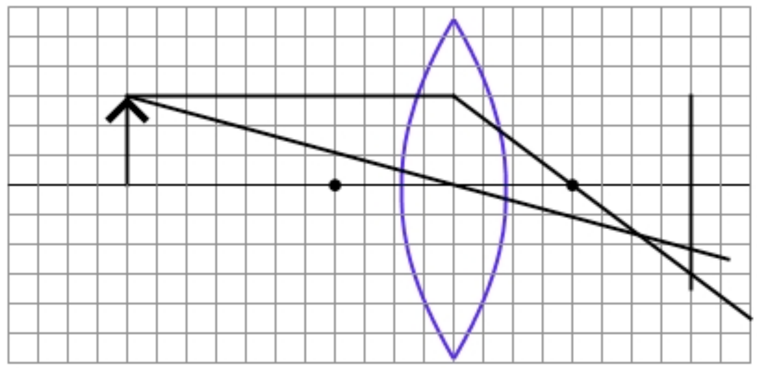
Student Worksheet: The Eye and the Camera

How does an eye work? How does a camera work?

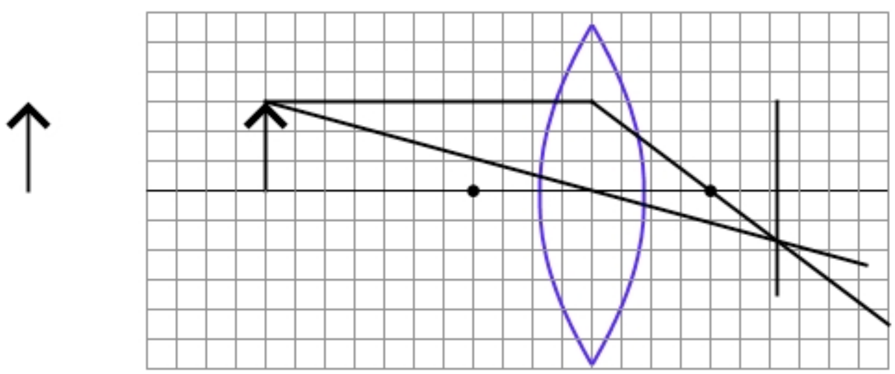
What similarities do you see between an eye and a camera?

What differences do you see?

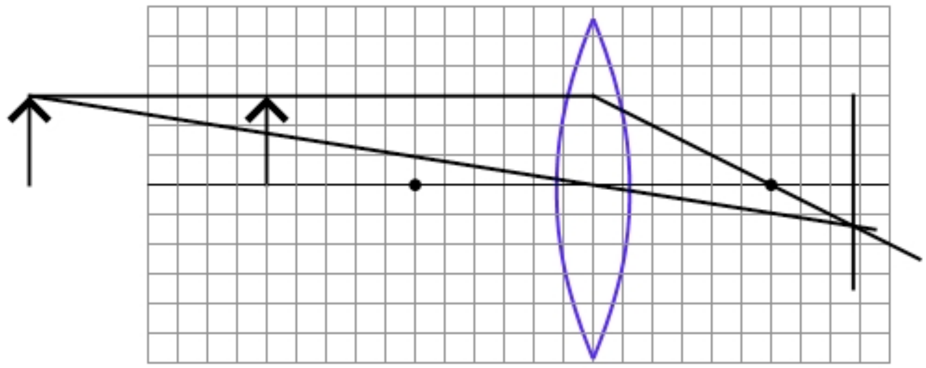
How do you focus? An object is a fixed distance from a screen on which you want to focus an image of the object. The lens you are using is creating an image somewhere other than the screen. What can you do to cause the image to be focused on the screen, without changing the distance from the object to the screen? Come up with as many methods as you can.



Are any of these methods used in the human eye, or in a camera?



The lens in this person's eye is a little too curved, so for an object that is far from the person, the image is created at a place that does not coincide with the retina. Where is the image? The person has no trouble seeing nearby objects, however. What is this problem called? How is it corrected? Sketch a ray diagram on the figure above to show how the rays go when the issue is corrected.



The lens in this person's eye is not curved enough, so for an object that is close to the person, the image is created at a place that does not coincide with the retina. Where is the image? The person has no trouble seeing distant objects, however. What is this problem called? How is it corrected?
Sketch a ray diagram on the figure above to show how the rays go when the issue is corrected.

Clicker Questions

These questions generally come with diagrams, or animations/simulations, or they accompany in-class demonstrations.

What happens when a parallel beam of light enters a rectangular glass block? Assuming the light exits the block along the side opposite to the side it entered, what path does the light follow when it emerges from the block?

1. The exact path it was following when it entered the block.
2. A path parallel to the original path, but displaced from it.
3. A path perpendicular to the original path.
4. None of the above.

A ray of light traveling in medium 1 encounters an interface between medium 1 and medium 2. Part of the light is refracted into medium 2, and part is reflected back into medium 1, as shown in the diagram.

If the refracted ray follows path D, not bending at all, what can you conclude?

1. $n_1 = n_2$
2. $n_2 > n_1$
3. $n_1 > n_2$
4. $n_1 > n_2$, and the angle of incidence in medium 1 is the critical angle for total internal reflection
5. $n_2 > n_1$, and the angle of incidence in medium 1 is the critical angle for total internal reflection
6. This is not actually possible

If the refracted ray follows path A, what can you conclude?

1. $n_1 = n_2$
2. $n_2 > n_1$
3. $n_1 > n_2$
4. $n_1 > n_2$, and the angle of incidence in medium 1 is the critical angle for total internal reflection
5. $n_2 > n_1$, and the angle of incidence in medium 1 is the critical angle for total internal reflection
6. This is not actually possible

If the refracted ray follows path C, what can you conclude?

1. $n_1 = n_2$

2. $n_2 > n_1$
3. $n_1 > n_2$
4. $n_1 > n_2$, and the angle of incidence in medium 1 is the critical angle for total internal reflection
5. $n_2 > n_1$, and the angle of incidence in medium 1 is the critical angle for total internal reflection
6. This is not actually possible

If the refracted ray follows path F, what can you conclude?

1. $n_1 = n_2$
2. $n_2 > n_1$
3. $n_1 > n_2$
4. $n_1 > n_2$, and the angle of incidence in medium 1 is the critical angle for total internal reflection
5. $n_2 > n_1$, and the angle of incidence in medium 1 is the critical angle for total internal reflection

This is not actually possible

A convex (converging) lens creates an image of a vertical light-bulb filament on a ground glass plate, which acts as a screen. What will happen to the image if the top half of the filament is covered up?

1. The top half of the image will vanish.
2. The bottom half of the image will vanish.
3. The entire image will vanish.
4. The entire image will remain, but will be dimmer.
5. Nothing will change.

What will happen to the image if, instead, the top half of the lens is covered up?

1. The top half of the image will vanish.
2. The bottom half of the image will vanish.
3. The entire image will vanish.
4. The entire image will remain, but will be dimmer.

5. Nothing will change.

An object is placed in front of a lens. The image formed by the lens is two times larger than the object.

What kind of lens is it?

1. convex (converging)
2. concave (diverging)
3. it could be either of the above

The focal length of the lens is 10 cm. We want to know where the object is but first consider this. How many solutions are there to the question "What is the object distance?"

1. 1
2. 2
3. more than 2

A person who is nearsighted can only create sharp images of close objects. Objects that are further away look fuzzy because the eye brings them in to focus at a point in front of the retina. To correct for this, a lens can be placed in front of the eye. What kind of lens is necessary?

1. A converging lens
2. A diverging lens

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Chapter 24: Additional Resources

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- Image Formation by Lenses
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Solutions

- Answers to Selected End of Chapter Problems
- Sample Question Solutions

Simulations

- Simulation: Refraction and the Minimization of Light Travel Time
- Simulation: Refraction, Time Minimization, and Snell's Law
- Simulation: Snell's Law

Additional Links

- *PhET* simulation: Geometric Optics