21-1 Waves

What is a wave? Put simply, a wave is a disturbance that carries energy from one place to another. Examples include waves on the surface of the ocean, sound waves that carry the sound of chirping birds to your ears on a spring morning, or the waves shown in Figure 21.1.

Figure 21.1: Waves caused by a drop of water hitting the water surface. Photo credit: PhotoDisc, Inc.

There are a number of ways to classify waves. One way is the following:

1. ***Mechanical waves.*** These include water waves, sound waves, and waves on strings, the kind of waves we will investigate in this chapter. Mechanical waves require a medium (such as water, air, or string) through which to travel. There is no net flow of mass through the medium, only energy.

2. ***Electromagnetic waves.*** Such waves include light, x-rays, microwaves, and radio waves. Electromagnetic waves do not need a medium through which to travel, and thus can travel through a vacuum. We will investigate electromagnetic waves in detail in Chapter 22.

3. ***Matter waves.*** These waves are associated with objects we often think of as particles, such as electrons and protons. Quantum physics, which we investigate in Chapter 27, tells us that everything, including ourselves, exhibits wave-particle duality, sometimes acting as a wave and sometimes as a particle.

For this chapter, we will confine ourselves to mechanical waves. Another way to classify waves is the following:

1. ***Transverse waves.*** In these waves, the particles of the medium oscillate in a direction transverse (perpendicular) to the direction the wave travels through the medium. A good example of this is a wave on a string, as shown in Figure 21.2. The various pieces of the string oscillate up and down, while the wave is traveling to the right.

2. ***Longitudinal waves.*** In these waves, the particles of the medium oscillate along the same direction in which the wave is traveling. A sound wave is a good example, in which air molecules oscillate back and forth along the direction the wave is traveling, as is shown in Figure 21.2. The regions of high density (corresponding to higher then average pressure) and low density (lower pressure) propagate to the right, while the air molecules themselves, on average, oscillate back and forth.

Wavelength and Period

To find the wavelength of a wave, we take a snapshot of the entire wave at one particular instant, as is shown in any of the five images of the string in Figure 21.2. The **wavelength** is the distance from, for instance, one peak to the next peak on the displacement versus position graph. Our symbol for wavelength is λ, the Greek letter lambda. The **period**, T, of the wave is the oscillation period for any particular part of the medium. If we focus on one piece of string, such as the piece colored red in Figure 21.2, and plot its displacement from equilibrium as a function of time, the period is the time between neighboring peaks on the displacement versus time graph.

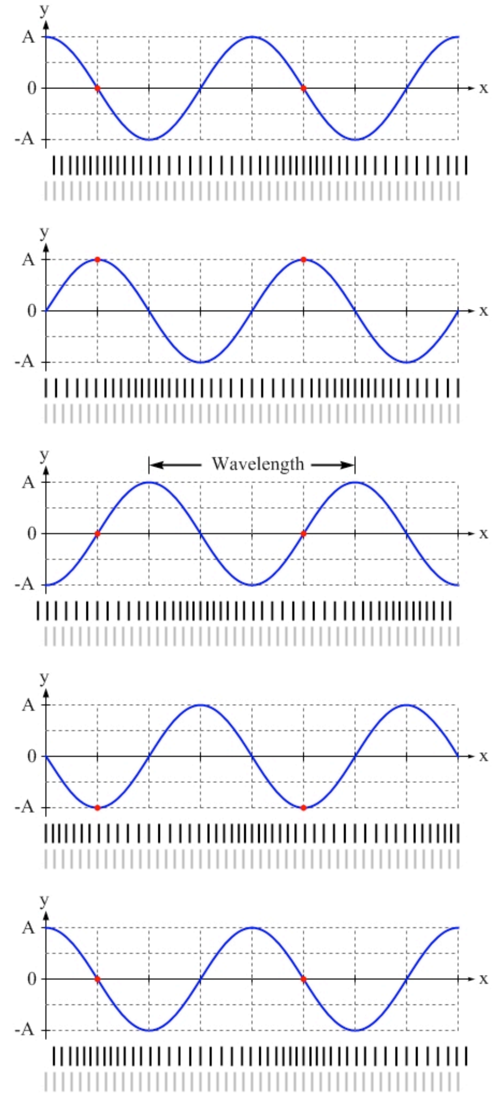


Figure 21.2: The figure shows fives pictures of a string, separated by equal time intervals. The string has a transverse traveling wave on it. Underneath each picture of the string is a representation of a longitudinal wave, such as a sound wave. The black lines represent the position of molecules of the medium as the wave passes, while the gray lines underneath represent the equilibrium position of these molecules. If time increases down the page, the waves are traveling to the right. If time increases up the page, the motion is to the left.

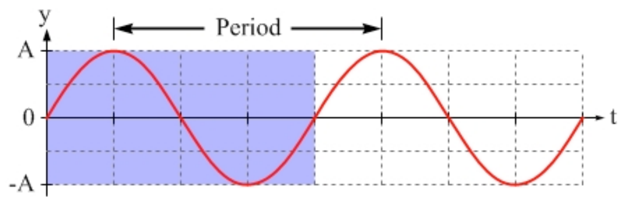


Figure 21.3: A plot of the displacement vs. time for the point on the string that is marked with a dot in Figure 21.2. The shaded region represents the time period covered by the five pictures in Figure 21.2.

Note that we are focusing on a simple kind of wave, a pure sine wave. More complex waves can be built up from sine waves of different wavelength, so our analysis can be generalized to more complicated waveforms.

The wave travels a distance of one wavelength in a time equal to one period. The wave speed is thus the distance over the time, $v = \lambda / T$. Instead of writing the equation in this form, however, we generally use the fact that the frequency, *f*, of the oscillation is the inverse of the period, $f = 1/T$. This leads to the equation in the box below.

In general, the connection between wave speed, frequency, and wavelength is:

$v = f\lambda$. (Equation 21.1: **Connecting speed, frequency, and wavelength**)

Equation 21.1, in the form it is presented above, gives the impression that the wave speed is determined by the frequency and wavelength. A better way to write the equation is as:

$$\lambda = \frac{v}{f},$$ (alternate form of Equation 21.1)

because the wave speed is set by the properties of the medium (such as the mass and tension of a string), and the frequency of the wave is the frequency of whatever is causing a particular part of the medium to oscillate. The wavelength is then determined by the combination of speed and frequency, as is given above in the alternate form of Equation 21.1.

Related End-of-Chapter Exercise: 42.

Essential Question 21.1: Which representation above, the graph of displacement versus position or the graph of displacement versus time, would you use to find the wave speed?

Answer to Essential Question 21.1: Both representations are needed. The wavelength is found from the graph of displacement versus position, while the period is found from the graph of displacement versus time. Both the wavelength and the period are needed to find the wave speed.

21-2 The Connection with Simple Harmonic Motion

Consider a single frequency transverse wave, like the one shown in Figure 21.4. There is clearly a connection between this wave and simple harmonic motion, because each part of the string experiences simple harmonic motion. Thus, for each part of the string we can use an equation like we used to describe simple harmonic motion, $y = A\cos(\omega t)$ or $y = A\sin(\omega t)$. These equations are good starting points, but they are not sufficient to describe what every point on the string is doing. For instance, at $t = 0$, the equation $y = A\cos(\omega t)$ gives $y = +A$, and only three pieces of the string, marked with dots, have $y = +A$.

Every point on the string does reach $y = +A$, but not at $t = 0$. For a given point, therefore, we can introduce something called a phase angle, θ, so the equation reflects the position (and the direction of the velocity) of the point at $t = 0$. Thus, each point has an equation of the form $y = A\cos(\omega t + \theta)$, with every point having a unique θ. Having a different equation to describe every point works, but it is cumbersome. Let's see if we can be more efficient in describing the wave mathematically.

First, consider a point on the string just to the right of the left-most point. A point just to the right of the left-most point does exactly what the left-most point does, just at a slightly later time. Thus, θ for that point is a small negative number, reflecting the small delay in the motion compared to the left-most point. Figure 21.5 shows graphs of the displacement versus time for two different sets of points, one set that is at $y = +A$ in the top picture, and the other set which is at $y = 0$, but moving in the positive y-direction, in the top picture. Note that the motion for the second set of points is delayed compared to the first, with a delay proportional to the distance between the points.

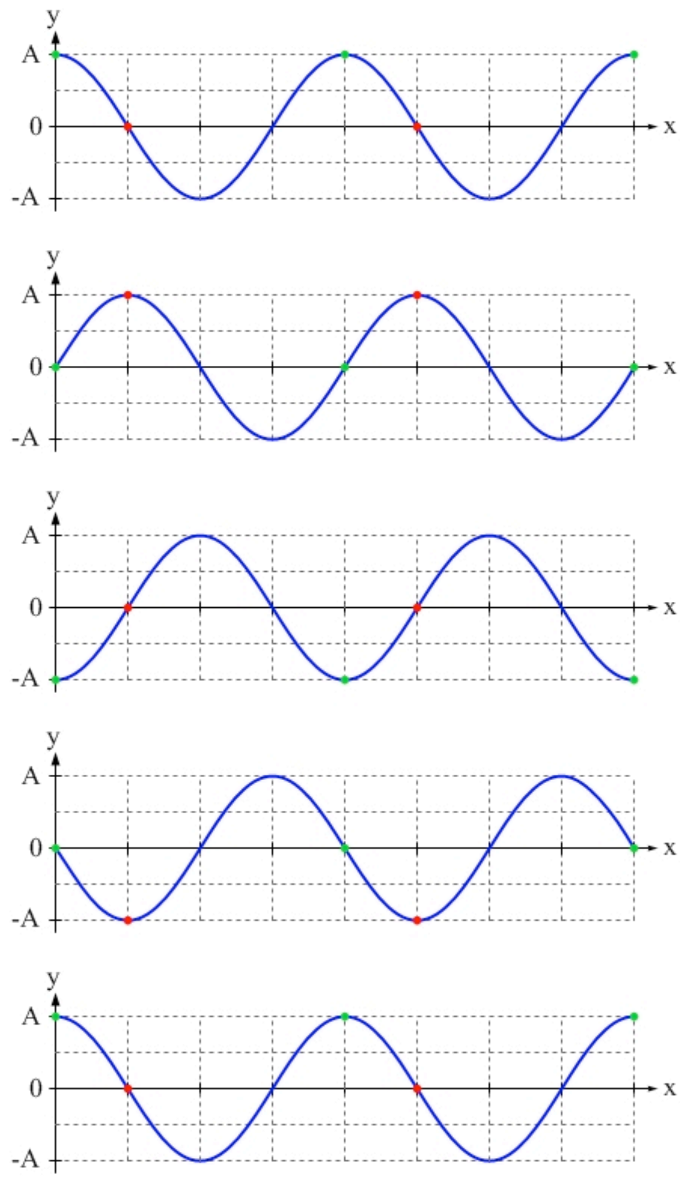


Figure 21.4: This figure, like Figure 21.2, shows fives pictures of a string, separated by equal time intervals. Time increases down the page, so the wave is traveling to the right.

Figure 21.5: Graphs of the displacement versus time for two sets of points, one set initially at $y = +A$, and the other set initially at $y = 0$ and moving in the $+y$ direction.

As x increases, the delay increases, and we find that the phase angle is, in fact, proportional to x, the distance of a point from the left- most point. Thus, we can say that, in the case of a wave traveling in the positive x-direction, $\theta = -kx$, where k is some constant.

If we can identify what the constant k is, we will be finished with our mathematical description. Let's focus now on the point exactly one wavelength to the right of the left-most point. These two points are in phase with one another, which means that whatever one of them does, the other does at the same time. Thus, the equation $y = A\cos(\omega t)$, for the left-most point (at $x = 0$), must agree with the equation for the second point, $y = A\cos(\omega t - k\lambda)$, at $x = \lambda$. Changing the value inside a cosine by a multiple of 2π produces the same result, and because this point is the first point to the right of $x = 0$ that is in phase with the point at $x = 0$, we must have $k\lambda = 2\pi$.

The constant k is known as the **wave number**, and is given by:

$$k = \frac{2\pi}{\lambda}.$$ (Equation 21.2: **the wave number**)

The wave number is, in some sense, the spatial equivalent of the angular frequency. The angular frequency is given by:

$$\omega = \frac{2\pi}{T}.$$ (Equation 21.3: **the angular frequency**)

Note that we now have a single equation that describes the wave. The equation tells us the displacement from equilibrium of each point in the medium at any value of t we might be interested in.

$$y = A\cos(\omega t \pm kx),$$ (Equation 21.4: **Equation of motion for a transverse wave**)

where the plus sign is used when the wave is traveling in the negative x-direction, and the minus sign is used when the wave is traveling in the positive x-direction.

Related End-of-Chapter Exercises: 13, 17, 18, 41.

Essential Question 21.2: In a particular case, the equation of motion of a transverse wave is:

$$y = (8.0 \text{ mm})\cos\left[(3\pi \text{ rad/s})t + (2\pi \text{ rad/m})x\right].$$

Determine the displacement of a point at $x = 2.0$ m at (a) $t = 0$, and (b) $t = 2.5$ s.

Answer to Essential Question 21.2: (a) When $t = 0$, the equation gives, at $x = 2.0$ m,
$y = (8.0\text{ mm})\cos\left[(4.0\pi\text{ rad})\right] = +8.0\text{ mm}$, because the cosine of an even multiple of pi is +1.0. Note that if you work this out on a calculator, your calculator needs to be in radians mode.
(b) If $t = 2.5$ s, the equation gives:
$y = (8.0\text{ mm})\cos\left[(7.5\pi\text{ rad}) + (4.0\pi\text{ rad})\right] = (8.0\text{ mm})\cos\left[(11.5\pi\text{ rad})\right] = 0$, because the cosine of $(n + 0.5)\pi$ is always 0. Again, keep your calculator in radians mode to get this answer from your calculator.

21-3 Frequency, Speed, and Wavelength

The speed of a wave depends on the medium the wave is traveling through. If the medium does not change as a wave travels, the wave speed is constant.

As we discussed in section 21-1, in one period, the wave travels one wavelength. Speed is distance over time, so $v = \lambda / T$. The frequency, f, is $1/T$, so the equation relating wave speed, frequency, and wavelength is $v = f\lambda$.

This equation (Equation 21.1), in this form, makes it look like speed is determined by frequency and wavelength, but this is not the case – the speed is determined by the medium. A good example is the speed of a wave on a stretched string. For a string with a tension F_T, a mass m, and a length L, the speed is given by:

$$v = \sqrt{\frac{F_T}{m/L}} = \sqrt{\frac{F_T}{\mu}},$$ (Eq. 21.5: **The speed of a wave on a string**)

where $\mu = m / L$ is the string's mass per unit length.

In general, then, the speed is determined by the medium, the frequency is determined by whatever is producing the wave (such as you, shaking the end of a string back and forth), and the wavelength is determined by Equation 21.1, through the combination of the speed and frequency.

An exception to this rule of thumb is a wave produced by a typical musical instrument. As we will discuss in more detail at the end of the chapter, when you play an instrument you excite a number of frequencies. The size of the instrument (such as the length of a guitar string) then determines the wavelengths of the particular frequencies that are favored. Thus, on a musical instrument, the length of the instrument determines the wavelength, the wave speed is again determined by the properties of the medium, and the combination of the wavelength and speed determines the frequency of the wave.

EXAMPLE 21.3 – Using the equation of motion for a transverse wave

The general equation for a wave traveling on a string is $y = A\cos(\omega t \pm kx)$. In a particular case, the equation is $y = (8.0\text{ cm})\cos\left[(60\text{ rad/s})t + (0.50\text{ rad/m})x\right]$. Determine:

(a) the wave's amplitude, wavelength, and frequency.
(b) the speed of the wave.
(c) the tension in the string, if the string has a mass per unit length of 0.048 kg/m.
(d) the direction of propagation of the wave.
(e) the maximum transverse speed of a point on the string.
(f) What is the displacement of a point at $x = 2.0$ m when $t = 1.0$ s?

SOLUTION

(a) The amplitude of the wave is the A in the equation, which is whatever is multiplying the cosine. In this particular case, the amplitude is 8.0 cm.

The value of k is whatever is multiplying the x, which is 0.50 rad/m. k is proportional to the inverse of the wavelength, with the wavelength given by $\lambda = 2\pi/(0.50 \text{ m}^{-1}) = 4\pi$ m. Note that we can put in or take out the unit of radians whenever we find it to be convenient.

The value of ω is whatever is multiplying the t, which is 60 rad/s. The frequency, f, is related to the angular frequency, ω, by a factor of 2π:

$$f = \frac{\omega}{2\pi} = \frac{60 \text{ rad/s}}{2\pi} = \frac{30}{\pi}\text{ Hz} = 9.5 \text{ Hz} .$$

(b) The speed of the wave can be found from the frequency and wavelength:

$$v = f\lambda = (30/\pi)\text{ Hz} \times (4\pi \text{ m}) = 120 \text{ m/s} .$$

(c) The tension in the string can be found by applying equation 21.5. Solving for tension gives:

$$F_T = v^2\mu = (120 \text{ m/s})^2 \times 0.048 \text{ kg/m} = 690 \text{ N} .$$

(d) In the equation describing the wave, the sign of the x-term is positive. This means that the more positive the x value, the sooner the wave reaches that point, so the wave is traveling in the negative x-direction.

(e) What does "maximum transverse speed" mean? It means the maximum y-direction speed of a point on the string. Any point can be used, because every point experiences the same motion, just at different times. To answer the question, remember that every point on the string is experiencing simple harmonic motion. Thus, this is really a harmonic motion question, not a wave question. Returning to what we learned in chapter 12, the maximum speed of a particle experiencing simple harmonic motion is:

$$v_{max} = A\omega = (8.0 \text{ cm}) \times 60 \text{ rad/s} = (0.08 \text{ m}) \times 60 \text{ rad/s} = 4.8 \text{ m/s} .$$

(f) We can enter the values of x and t right into the equation, giving:

$$\begin{aligned} y &= (8.0 \text{ cm})\cos\left[(60 \text{ rad/s})t + (0.50 \text{ rad/m})x\right] \\ &= (8.0 \text{ cm})\cos\left[(60 \text{ rad/s})(1.0 \text{ s}) + (0.50 \text{ rad/m})(2.0 \text{ m})\right] \\ &= (8.0 \text{ cm})\cos\left[(60 \text{ rad}) + (1.0 \text{ rad})\right] = (8.0 \text{ cm})\cos(61 \text{ rad}) = -2.1 \text{ cm}. \end{aligned}$$

Don't forget to put your calculator into radians mode when you do this calculation.

Related End-of-Chapter Exercises: 14, 15, 40.

Essential Question 21.3: Return to Example 21.3. If the wave's equation of motion was unchanged except for a doubling of the angular frequency, which of the answers in Example 21.3 would change, and how would they change?

Answer to Essential Question 21.3: Doubling the angular frequency, ω, causes the frequency to double in part (a). This, in turn, means that that wave speed must double, in part (b). In part (c), the tension is proportional to the square of the speed, so the tension is increased by a factor of 4. In part (e), the maximum transverse speed is proportional to ω, so the maximum transverse speed doubles. Finally, in part (f) we get a completely different value, $y = -0.39$ cm.

21-4 Sound and Sound Intensity

One way to produce a sound wave in air is to use a speaker. The surface of the speaker vibrates back and forth, creating areas of high and low density (corresponding to pressure a little higher than, and a little lower than, standard atmospheric pressure, respectively) in the region of air next to the speaker. These regions of high and low pressure (the sound wave) travel away from the speaker at the speed of sound. The air molecules, on average, just vibrate back and forth as the pressure wave travels through them. In fact, it is through the collisions of air molecules that the sound wave is propagated. Because air molecules are not coupled together, the sound wave travels through gas at a relatively low speed (for sound!) of around 340 m/s. As Table 21.1 shows, the speed of sound in air increases with temperature.

For other material, such as liquids or solids, in which there is more coupling between neighboring molecules, vibrations of the atoms and molecules (that is, sound waves) generally travel more quickly than they do in gases. This also is shown in Table 21.1.

Medium	Speed of sound
Air (0°C)	331 m/s
Air (20°C)	343 m/s
Helium	965 m/s
Water	1400 m/s
Steel	5940 m/s
Aluminum	6420 m/s

Table 21.1: Values of the speed of sound through various media.

Our ears can typically hear sounds with frequencies that lie between 20 Hz and 20 kHz, although the maximum frequency we are sensitive to tends to decrease with age (not to mention with prolonged exposure to high-intensity sound, such as loud music). We are typically most sensitive to sound waves that have frequencies near 2000 Hz, and considerably less sensitive to sounds at the extremes of our frequency range.

Other animals are sensitive to sounds outside of the human range. Elephants, for instance, communicate using sounds below 20 Hz. Because these sounds are not audible to humans, it took scientists quite a while to realize that elephants communicate with one another more than was first thought. Beyond the upper end of the human range, above 20 kHz, we classify sound as **ultrasound**. Dogs, bats, dolphins, and other animals can hear sounds in this range. Ultrasound also has important medical applications, such as in the imaging of a developing fetus in the womb. High-frequency sound waves traveling through the mother's body reflect differently from bone versus tissue, with the pattern of the reflected waves allowing an image to be formed.

Sound intensity

The intensity of a wave is defined to be its power per unit area: $I = P/A$.

For a source broadcasting uniformly in all directions, the wave spreads out like an inflating sphere, so the area in question is the surface area of a sphere.

$$I = \frac{P}{4\pi r^2}.$$ (Eq. 21.6: **Intensity for a source broadcasting uniformly in all directions**)

The sound intensity is proportional to the inverse square of the distance from the source. If the distance is doubled, for instance, the sound intensity decreases by a factor of four. Interestingly, a decrease in sound intensity by a factor of 4 is not perceived as such by the ear-brain system. The ear, in fact, responds logarithmically to sound intensity, and so we use a logarithmic scale for sound that is much like the Richter scale for earthquakes. Just as an earthquake measuring 7.0 on the Richter scale is 10 times more powerful than a quake measuring 6.0, and 100 times more powerful than an earthquake measuring 5.0, a 70 decibel (dB) sound has 10 times the power of a 60 dB sound, and 100 times the power of a 50 dB sound. Every 10 dB represents a change of one order of magnitude in intensity, no matter what the initial intensity is.

For the human ear, the smallest sound intensity that is audible has been determined to correspond to a sound intensity of about $I_0 = 1 \times 10^{-12}$ W/m^2. This value is known as the **threshold of hearing**. On the decibel scale, sounds are viewed in terms of how their intensity compares to the threshold of hearing.

$$\beta = (10 \text{ dB}) \log\left(\frac{I}{I_0}\right),$$ (Equation 21.7: **Absolute sound intensity level, in decibels**)

where the equation involves the log in base 10. An interesting reference point on the decibel scale is the **threshold of pain**, the most intense sound an average person can tolerate, which is 120 dB. Substituting 120 dB into equation 21.6, we find that, for the threshold of pain,

$$120 \text{ dB} = (10 \text{ dB}) \log\left(\frac{I}{I_0}\right), \text{ so } 12 = \log\left(\frac{I}{I_0}\right).$$

To solve the equation for I, the sound intensity corresponding to the threshold of pain, we do 10 to the power of each side of the equation. 10^x is the inverse function of log(x), so:

$$10^{12} = 10^{\log\left(\frac{I}{I_0}\right)} = \frac{I}{I_0} = \frac{I}{1 \times 10^{-12} \text{ W/m}^2}.$$

Thus, the intensity of the threshold of pain is 12 orders of magnitude larger than the threshold of hearing, or 1 W/m^2. The most amazing thing about this, however, is what this result tells us about the human ear. The human ear is an incredible instrument, allowing us to hear sounds covering 12 orders of magnitude – that's a factor of 1 trillion.

One convenient feature of the logarithmic scale is that an increase of X decibels corresponds to an increase by a particular factor in intensity, no matter where you start from. This is reflected in the following equation:

$$\Delta\beta = (10 \text{ dB}) \log\left(\frac{I_f}{I_i}\right).$$ (Equation 21.8: **Relative sound intensity level, in decibels**)

Related End-of-Chapter Exercises: 18 – 22, 39.

Essential Question 21.4: If a sound intensity level increases by 5 dB, by what factor does the intensity increase?

Answer to Essential Question 21.4: If a sound's intensity level increases by 5 dB, equation 21.7 tells us that: $0.5 = \log\left(\frac{I_f}{I_i}\right)$, which gives a ratio of final to initial intensity of $10^{0.5} = 3.2$. In other words, every 5 dB increase corresponds to increasing the sound intensity by a factor of 3.2.

21-5 The Doppler Effect for Sound

We have probably all had the experience of listening to the siren on an emergency vehicle as it approaches us, and hearing a shift in the frequency of the sound when the vehicle passes us. This shift in frequency is known as the Doppler effect, and it occurs whenever the wave source or the detector of the wave (your ear, for instance) is moving relative to the medium the wave is traveling in. Applications of the Doppler effect for sound include Doppler ultrasound, a diagnostic tool used to study blood flow in the heart. There is a related but slightly different Doppler effect for electromagnetic waves, which we will investigate in the next chapter, that has applications in astronomy as well as in police radar systems to measure the speed of a car.

EXPLORATION 21.5 – Understanding the Doppler effect

Let's explore the principles behind the Doppler effect. We will begin by looking at the situation of a stationary source of sound, and a moving observer.

Step 1 – ***Construct a diagram showing waves expanding spherically from a stationary source that is broadcasting sound waves of a single frequency. If you, the observer, remain stationary, you hear sound of the same frequency as that emitted by the source. Use your diagram to help you explain whether the frequency you hear when you move toward the source, or away from the source, is higher or lower than the frequency emitted by the source.***

We can represent the expanding waves as a set of concentric circles centered on the stationary source, as in Figure 21.6. This picture shows a snapshot of the waves at one instant in time, but remember that the waves are expanding outward from the source at the speed of sound. If you are stationary at position A, the waves wash over you at the same frequency as they were emitted. If you are at position A but moving toward the source, however, the frequency you observe increases, because you are moving toward the oncoming waves. Conversely, if you move away from the source (and you are traveling at a speed less than the speed of sound), you observe a lower frequency as you try to out-run the waves.

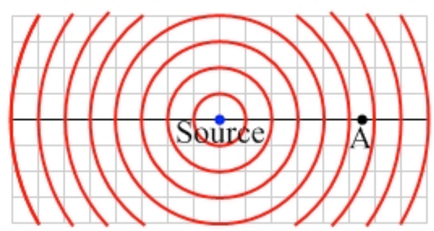


Figure 21.6: Waves emitted by a stationary source expand out away from the source, giving a pattern of concentric circles centered on the source. You, the observer, are at point A. If you are moving, the frequency of the waves you receive depends on both your speed and the direction of your motion.

Step 2 – ***Starting with the usual relationship connecting frequency, speed, and wavelength, $f = v / \lambda$, think about whether the observer moving toward or away from a stationary source effectively changes the wave speed or the wavelength. If the speed of sound is v and the observer's speed is v_o, write an equation for the frequency heard by the observer.*** As we can see from the pattern in Figure 21.6 above, the wavelength has not changed. What changes, when you move through the pattern of waves, is the speed of the waves with respect to you. When you move toward the source, the effective speed of the waves (the relative speed of the waves with respect to you) is $v + v_o$, while when you move away from the source the wave speed is effectively $v - v_o$. The frequency you observe, f', is thus the effective speed over the wavelength:

$$f' = \frac{v \pm v_o}{\lambda} = \frac{v}{\lambda}\left(\frac{v \pm v_o}{v}\right) = f\left(\frac{v \pm v_o}{v}\right), \quad \text{(Eq. 21.9: \textbf{Frequency for a moving observer})}$$

where f is the frequency emitted by the source, and where we use the + sign when the observer moves toward the source, and the – sign when the observer moves away from the source.

Step 3 – ***Construct a diagram showing waves expanding from a source that is moving to the right at half the speed of sound while broadcasting sound waves of a single frequency. Use your diagram to help you explain whether the frequency you hear when you are stationary is higher or lower than that emitted by the source, when the source is moving toward you and when the source is moving away from you.***

In this situation, the result is quite different from that in Figure 21.6, because each wave is centered on the position of the source at the instant the wave was emitted. Because the waves are emitted at different times, and the source is moving, we get the picture shown in Figure 21.7. To the left of the source, such as at point B, the waves are more spread out. Thus, when the source is moving away from the observer, the observed frequency is less than the emitted frequency. The reverse is true for a point to the right of the source: the waves are closer together than usual, so an observer in this region (such as at point A) observes a greater frequency than the emitted frequency.

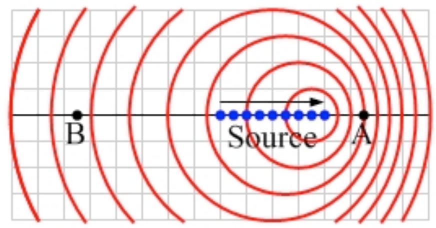


Figure 21.7: When a source of waves is moving relative to the medium, the wave pattern is asymmetric. An observer for which the source moves away observes a lower-frequency wave, while, when the source is moving toward the observer, a higher-frequency wave is observed. In the case shown, the source is moving to the right at half the wave speed.

Step 4 – ***Starting with the usual relationship connecting frequency, speed, and wavelength, $f = v / \lambda$, think about whether the source moving toward or away from a stationary observer effectively changes the wave speed or the wavelength. If the speed of sound is v and the source's speed is v_s, write an equation for the frequency heard by the observer.*** As we can see from the pattern in Figure 21.7, the movement of the source changes the wavelength. The waves still travel at the speed of sound, however. What changes, when you move through the pattern of waves, is the speed of the waves with respect to you. When the source moves toward the observer, the effective wavelength is $(v - v_s)/f$, while when the source moves away the wavelength is effectively $(v + v_s)/f$. The frequency you observe, f', is thus the speed over the effective wavelength:

$$f' = \frac{v}{\lambda'} = \frac{v}{v \mp v_s} f \;, \quad \text{(Eq. 21.10: \textbf{Frequency for a moving source})}$$

where f is the frequency emitted by the source. Use the – sign when the source moves toward the observer, and the + sign when the source moves away from the observer.

Key idea for the Doppler effect: Motion of a source of sound, or motion of an observer, can cause a shift in the observed frequency of a wave. **Related End-of-Chapter Exercises: 23, 24.**

Essential Question 21.5: Is the Doppler effect simply a relative velocity phenomenon? For instance, is the situation of an observer moving at speed v_1 toward a stationary source the same as a source moving at speed v_1 toward a stationary observer?

Answer to Essential Question 21.5: The Doppler effect for sound (and for all mechanical waves) is not a relative velocity phenomenon. The relative velocity of the source and observer is the same in these two situations, but the observed frequency is different in the two situations. One interesting example is when $v_I = v$, the wave speed. When the observer moves at speed v toward a stationary source, the observed frequency is twice the emitted frequency. When the source moves at a speed v toward a stationary observer, however, the observed frequency is infinite. We will investigate that situation further in the next section.

21-6 Sonic Booms, and the Doppler Effect in General

Essential Question 21.5 raises the question of what happens when a source of waves travels at the wave speed. We should also consider what happens when the source travels faster than the wave speed.

Let's begin by drawing a diagram like that in Figure 21.7, but with the source traveling to the right at the wave speed. In this special case, in Figure 21.8, because the source keeps up with the waves, the waves pile up at the source, leading to a large amplitude wave that moves with the source. This is known as a **sonic boom**, because a large amplitude corresponds to a loud sound. The observer at position A would hear the sonic boom when the source passed by.

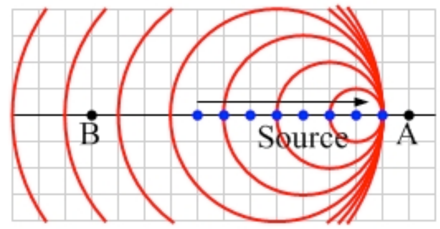


Figure 21.8: When the source moves at the wave speed, the waves pile up on one another at the source, creating a sonic boom.

Let's go further, and see what the picture looks like when the source travels faster than the waves. Figure 21.9 shows what happens when the source travels to the right at twice the wave speed. In this case, the waves pile up along lines that make an angle with the line of travel of the source. This pattern should look familiar to you, given that it looks like the waves left behind by a boat as it travels through water, as in the photograph in Figure 21.10. This tells us that the boat's speed is faster than the speed of the water waves.

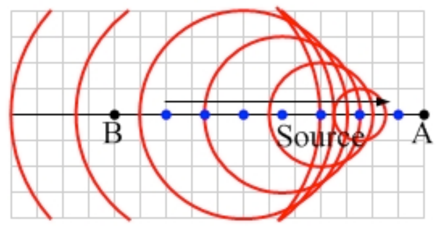


Figure 21.9: When the source moves faster than the waves, the waves create a wake pattern.

In section 21-5, we considered what happens when either the source moves or the observer moves, but not both. Let's now consider what happens in general, when both the source of a wave and the observer are moving with respect to the medium the waves are moving through. The general equation is simply a combination of the equations we derived in section 21-5 for the situations of a moving observer and a moving source.

Figure 21.10: A common example of the situation of a source of waves traveling faster through the medium than the waves themselves is in the wake created when a boat passes through water (here, the Avon Gorge in England). Photo credit: public-domain photo taken by Adrian Pingstone.

The Doppler effect: The Doppler effect describes the shift in frequency of a wave that occurs when the source of the waves, and/or the observer of the waves, moves with respect to the medium the waves are traveling through. The general equation for the observed frequency is:

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right), \qquad \text{(Equation 21.11: \textbf{The general Doppler equation})}$$

where f' is the frequency observed by the observer, f is the frequency of the waves emitted by the source, v is the speed of the wave through the medium, v_o is the speed of the observer, and v_s is the speed of the source. In the numerator, use the top (+) sign if the observer moves toward the source, and the bottom (–) sign if the observer moves away from the source. In the denominator, use the top (–) sign if the source moves toward the observer, and the bottom (+) sign if the source moves away from the observer.

EXAMPLE 21.6 – Catching a moth

A particular bat emits ultrasonic waves with a frequency of 56.0 kHz. The bat is flying at 16.00 m/s toward a moth, which is moving at 2.00 m/s away from the bat. The speed of sound is 340.00 m/s. (a) Assuming the moth could detect the waves, what frequency waves would it observe? (b) The waves reflect off the moth and are detected by the bat. What frequency are the waves detected by the bat?

SOLUTION

(a) Here, we use the general Doppler equation, where f = 56.0 kHz and v = 340 m/s. The observer is the moth, so v_o is 2.00 m/s, and we use the bottom sign (the minus sign) in the numerator because the moth is traveling away from the bat. The bat is the source, so v_s = 16.00 m/s, and we use the top sign (the minus sign) in the denominator because the bat is traveling toward the moth. This gives:

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right) = (56.0 \text{ kHz})\left(\frac{340.00 \text{ m/s} - 2.00 \text{ m/s}}{340.00 \text{ m/s} - 16.00 \text{ m/s}}\right) = (56.0 \text{ kHz}) \times 1.0432 = 58.42 \text{ kHz}$$

(b) Again, we use the general Doppler equation. This time, the moth acts as the source (because the moth reflects the waves back to the bat) and the bat is the observer. The frequency emitted by the moth is the frequency we found in part (a). Let's use f'' to denote the frequency of the waves detected by the bat, so f' = 58.42 kHz and v = 340 m/s. The observer is the bat, so v_o' is 16.00 m/s, and we use the top sign (the plus sign) in the numerator because the bat is traveling toward the moth. The moth is the source, so v_s' = 2.00 m/s, and we use the bottom sign (the plus sign) in the denominator because the moth is traveling away from the bat. This gives:

$$f'' = f'\left(\frac{v \pm v_o'}{v \mp v_s'}\right) = (58.42 \text{ kHz})\left(\frac{340.00 \text{ m/s} + 16.00 \text{ m/s}}{340.00 \text{ m/s} + 2.00 \text{ m/s}}\right) = (58.42 \text{ kHz}) \times 1.0409 = 60.8 \text{ kHz}$$

The bat can use the frequency of the detected wave to determine how fast, and in what direction, the moth is flying.

Related End-of-Chapter Exercises: 25 – 27, 46 – 48.

Essential Question 21.6: What happens when a source and observer have identical velocities? Is the observed frequency larger, smaller, or the same as the frequency emitted by the source?

Answer to Essential Question 21.6: When a source and an observer move in the same direction with the same speed, the observed frequency is the same as the frequency emitted by the source.

21-7 Superposition and Interference

What happens when two waves traveling through the same medium encounter one another? In general, we apply the principle of superposition to determine the net displacement of each point in the medium.

The principle of superposition: The net displacement of any point in a medium is the sum of the displacements at that point due to each of the individual waves.

Figure 21.11 shows what happens when two pulses moving in opposite directions along a stretched string meet one another. Both pulses displace the string upward as they travel, so when the peaks of the pulses coincide, the net displacement of the string at that point is equal to the sum of the amplitudes of the pulses. This is known as **constructive interference -** the displacements of the individual waves are in the same direction, and thus add together. An interesting implication of the principle of superposition is that the waves do not change one another's shape as they pass through one another. After passing through, they move away unchanged.

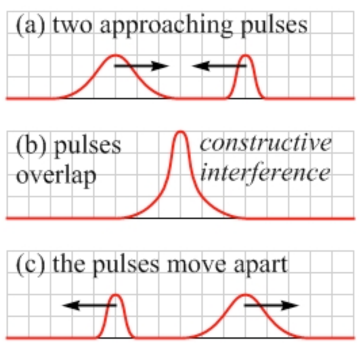


Figure 21.11: The successive images show two pulses moving in opposite directions along a string. At points where the pulses overlap, the net displacement of the string is the sum of the displacements due to the individual waves. In this case, (b) shows **constructive interference**, because the displacements of both pulses are in the same direction.

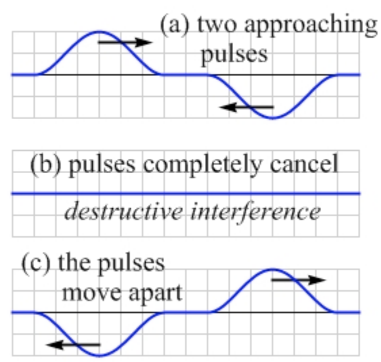


Figure 21.12: The successive images show two pulses, which are mirror images of one another, moving in opposite directions along a string. In this case, (b) shows completely **destructive interference**, with the pulses exactly canceling one another at the instant they overlap completely.

In Figure 21.12, we see what happens when two pulses that have opposite displacements meet one another while traveling along a stretched string. In this situation, because one pulse is a mirror image of the other, when the pulses coincide the net displacement of the string is zero everywhere, just for an instant. This is known as **destructive** interference, where the displacements of the individual waves are in opposite directions, and thus fully or partly cancel. Once again, after passing through one another, they move away as if they had never met.

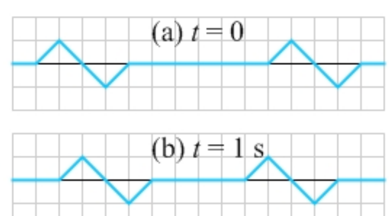
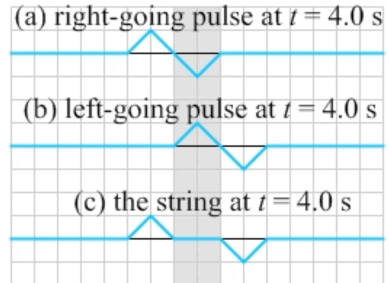


Figure 21.13: Two pulses, one traveling left and one traveling right, on a stretched string. The profile of the string is shown at (a) $t = 0$ and (b) $t = 1.0$ s.

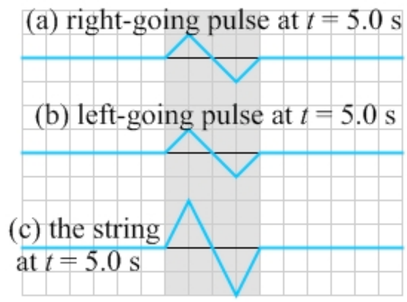
EXPLORATION 21.7 – A process for adding two pulses

Figure 21.13 shows two pulses traveling along a string. The string is shown at two separate times, $t = 0$, and $t = 1.0$ s. We want to know what the string looks like at $t = 4.0$ s, $t = 5.0$ s, and $t = 6.0$ s.



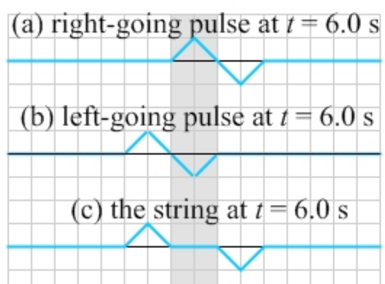
Step 1 – *Based on the two pictures in Figure 21.13, determine where each of the two pulses will be at t = 4.0 s. Sketch three diagrams, one above the other. First, sketch a diagram showing the position of the rightward-moving pulse. Second, sketch a diagram of the leftward-moving pulse. Use those two diagrams to determine where the two pulses overlap, and use superposition to draw the string as it looks with both pulses on it.* From Figure 21.13, we can see that the pulses travel along the string with a speed of 1 grid unit per second. After three more seconds have passed, the right-going pulse will be three units to the right, and the left-going pulse will be three units to the left, as shown in Figure 21.14. The pulses destructively interfere in the region of overlap, which is shaded in Figure 21.14.

Figure 21.14: The two separate pulses, in (a) and (b), and the profile of the string (c), at t = 4.0 s. The region where the pulses overlap is shaded.



Step 2 – *Repeat the process outlined in step 1 above, but at t = 5.0 s.* Because another second has passed, we slide each pulse over by one unit. At t = 5.0 s, the pulses completely overlap. Because they have identical profiles, the pulses constructively interfere, doubling the displacement at each point in the region of overlap, as shown in Figure 21.15.

Figure 21.15: The two separate pulses, in (a) and (b), and the profile of the string (c), at t = 5.0 s. The region where the pulses overlap is shaded.



Step 3 – *Repeat the process outlined in step 1 above, but at t = 6.0 s.* An additional second has passed, so we again slide each pulse over by one unit. At t =6.0 s, the net result is the same as at t = 4.0 s, with the two pulses simply swapping positions on the string, as shown in Figure 21.16.

Figure 21.16: The two separate pulses, in (a) and (b), and the profile of the string (c), at t = 6.0 s. The region where the pulses overlap is shaded.

Key idea: When two waves meet, apply the principle of superposition. A useful method is to sketch each of the waves separately, and then add the displacements to find the net displacement in the region where the waves overlap. **Related End-of-Chapter Exercises: 1 – 4.**

Interference in Two Dimensions

When two sources emit identical waves, an interesting pattern is created near the sources because of the interference that takes place. The type of interference that takes place at a point depends on the **path-length difference**: the difference between the distance from one source to the point and the distance from the second source to the point. When the sources emit identical waves, any point that is equidistant from the two sources (that is, having a path-length difference of zero), experiences constructive interference. These are not the only places where constructive interference occurs – any point at which the path-length difference is an integral number of wavelengths also experiences constructive interference. Destructive interference, on the other hand, occurs at points that are an integral number of wavelengths, plus half a wavelength, farther from one source than the other. We will discuss these ideas in more detail in chapter 24.

Essential Question 21.7: In the picture of the string in Figure 21.12(b), the string is completely flat. In Figure 21.12(c), the two pulses re-emerge from the flat string. How is this possible? For instance, where is the energy, in (b), necessary to re-form the two pulses?

Answer to Essential Question 21.7: How does the flat string pictured in Figure 21.12(b) differ from a regular flat string, from which no pulses would emerge? What is not obvious from the static image shown in Figure 21.12(b) is that the string, where the pulses overlap, is moving. Some sections of the string are moving down, while others are moving up, with the various parts of the string moving with velocities that are just right to re-create the two pulses properly. Thus, the energy needed to re-form the pulses is in the kinetic energy of various parts of the string.

21-8 Beats; and Reflections

When you listen to two sound waves of similar, but different, frequency, you generally hear the sound rising and falling in intensity, typically at the rate of a few cycles per second. This phenomenon is known as **beats,** and it is caused by interference between the two waves. Let's say the waves are initially in phase, with their peaks coinciding. The waves interfere constructively, producing a large-amplitude sound. Because the waves have different frequencies, however, they gradually drift out of phase. Eventually, the peak from one wave lines up with the trough (negative-displacement peak) in the other wave, leading to destructive interference and, when the interference is completely destructive, no sound. The larger the difference between the two frequencies, the faster the waves drift out of phase with one another. The phase difference continues to grow, but this eventually leads to peaks in the two waves lining up again. This cycle is demonstrated in Figure 21.17.

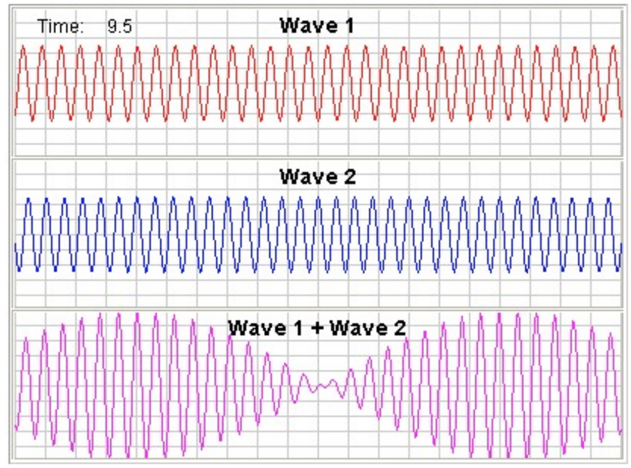


Figure 21.17: The phenomenon of beats is caused by interference between two waves that have different frequencies. The two individual waves are shown at the top and middle, while the superposition of the two waves is shown at the bottom. The rise and fall in the amplitude of the resultant wave is what we hear as beats.

The beat frequency, which is the frequency at which the intensity oscillates, is simply the difference between the frequencies of the two waves.

$$f_{beat} = f_{high} - f_{low} .$$ (Equation 21.12: **the beat frequency**)

String musicians can even tune their instrument using beats, by playing two strings at once and adjusting the tension in one string (which adjusts the frequency of the string). When the beats disappear, the frequencies of the two strings are equal.

Reflections

When a wave traveling along a string encounters the end of the string, the wave reflects. Exactly how the wave reflects depends on whether the end of the string is tied down or loose (or even something in between, such as tied to a spring, but we will consider only the two extremes).

On stringed instruments, for instance, the strings are fixed at the ends. The leading (right-most) edge of an upward going pulse, like that shown in Figure 21.18(a), propagates to the right along the string by each part of the string successively pulling up on the next part of the string. This propagation method works until the pulse reaches the end of the string, which is tied down. The part of the string next to the right end pulls up on the end, but the end does not move. Instead, by Newton's Third Law, the end exerts a downward force on the piece of the string next to it, leading to an inverted pulse traveling back along the string, as shown in part (e) of Figure 21.18.

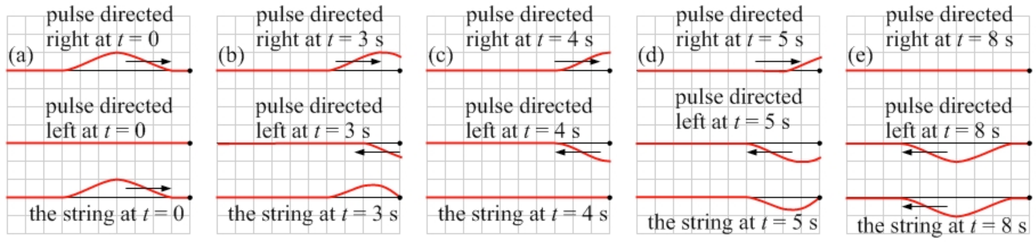


Figure 21.18: When a wave reflects from a fixed end, it reflects upside down.

Note that the string is completely flat in Figure 21.18 (c), halfway through the reflection of the pulse. This is caused by completely destructive interference taking place between the first half of the pulse, which has been inverted and is moving left, and the second half of the pulse, which is still upright and moving right. Figure 21.19 shows a way to visualize the reflection, as if a pulse directed right on the string is interfering with a mirror-image pulse, which is inverted, directed left on the string. Superposition can only work on the string itself, so we don't have to worry about any areas of overlap of the two pulses that are to the right of the end of the string.

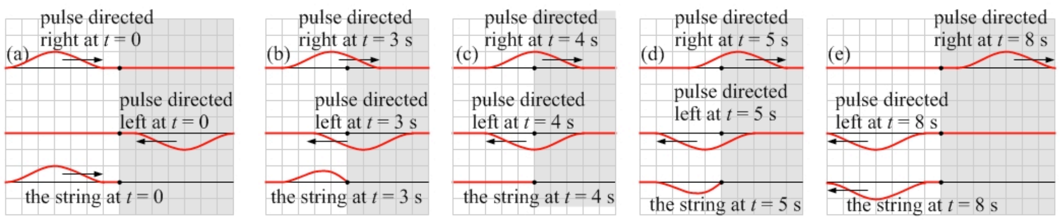


Figure 21.19: Reflection from a fixed end can be visualized as interference between a right-moving pulse, and an inverted copy of the pulse that is moving left. We are imagining the pulses existing to the right of the end of the string (in the shaded region), even though they cannot do so.

If the end of a string is not tied down, but is free to move, it is known as a **free end**. When a wave reflects from a free end, the end responds to the wave by moving, and the wave reflects without being inverted. Figure 21.20 shows the process for a pulse, which we can visualize as if a pulse directed right on the string is interfering with a mirror-image pulse directed left. Note that, in Figure 21.20(c), the end of the string is displaced by twice the amplitude of the pulse, because of constructive interference between the half of the pulse that has been reflected and is moving to the left, and the other half which is still moving to the right.

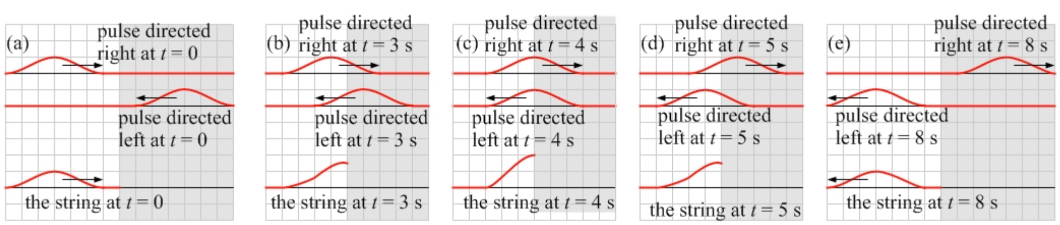


Figure 21.20: Reflection from a free end can be visualized as interference between a right-moving pulse, and an exact copy of the pulse that is moving left.

Reflection of waves: If a wave reflects from a fixed boundary of a medium, the reflected wave is inverted. If, instead, a wave reflects from a free boundary, such as the free end of a string, the reflected wave reflects without being inverted (that is, the reflected wave is upright).
Related End-of-Chapter Exercises: 9, 10, 53 – 55.

Essential Question 21.8: You hear a beat frequency of 6 Hz when you play two guitar strings simultaneously. If one string has a frequency of 330 Hz, what is the frequency of the other string?

Answer to Essential Question 21.8: Because the beat frequency is 6 Hz, we know that the two frequencies differ by 6 Hz. If one string is 330 Hz, the other string is either 336 Hz (6 Hz higher) or 324 Hz (6 Hz lower).

21-9 Standing Waves on Strings

In sections 21-9 and 21-10, we will discuss physics related to musical instruments, focusing on stringed instruments in this section and wind instruments in section 21-10.

Some stringed instruments (such as the harp) have strings of different lengths, while others (such as the guitar) use strings of the same length. We can apply the same principles to understand either kind of instrument. Consider a single string of a particular length that is fixed at both ends. The string is under some tension, so that when you pluck the string it vibrates and you hear a nice sound from the string, dominated by one particular frequency. How does that work?

When you pluck the string, you send waves of many different frequencies along the string, in both directions. Each time a wave reaches an end, the wave reflects so that is inverted. All of these reflected waves interfere with one another. For most waves, after multiple reflections the superposition leads to destructive interference. For certain special frequencies, for which an integral number of half-wavelengths fit exactly into the length of the string, the reflected waves interfere constructively, producing large-amplitude oscillations on the string at those frequencies.

These special frequencies produce **standing waves** on the string. Identical waves travel left and right on the string, and the superposition of such identical waves leads to a situation where the positions of zero displacement (the **nodes**) remain fixed, as do the positions of maximum displacement (the **anti-nodes**), so the wave appears to stand still. Figure 21.21 shows the left and right-moving waves on the string, and their superposition, which is the actual string profile, for the lowest-frequency standing wave on the string at various times.

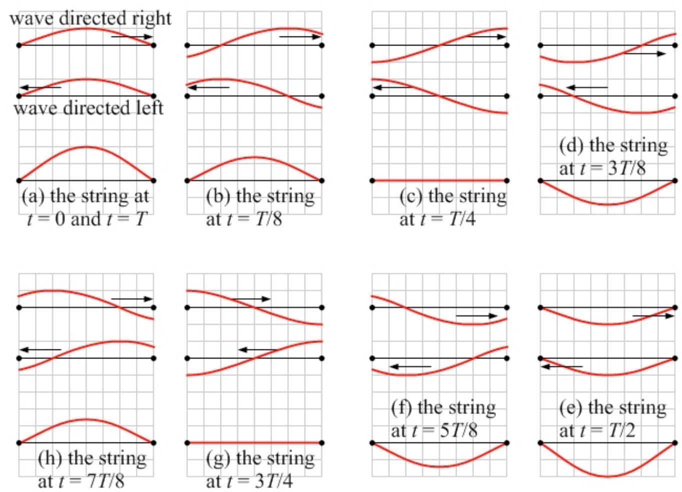


Figure 21.21: The string profile for the lowest-frequency standing wave (the **fundamental**) on the string at $t = 0$, and at regular time intervals after that, showing how the identical left and right-moving waves combine to form a standing wave. Go clockwise around the diagram to see what the string looks like as time goes by.

For a string fixed at both ends: The standing waves have a node (a point of zero displacement) at each end of the string. The various wavelengths that correspond to the special standing-wave frequencies are related to L, the length of the string, by:

$n\frac{\lambda_n}{2} = L$, so $\lambda_n = \frac{2L}{n}$, where n is an integer.

Using Equation 21.1, $v = f\lambda$, the particular frequencies that tend to be excited on a stretched string are:

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L},$$ (Eq. 21.13: **Standing-wave frequencies for a string fixed at both ends**)

The lowest-frequency standing wave on the string, corresponding to $n = 1$, is known as the ***fundamental***. The other frequencies, or ***harmonics***, are simply integer multiples of the fundamental. In general, when you pluck a string, the dominant sound is the fundamental, but the harmonics make the sound more pleasing than what a single-frequency note sounds like. Figure 21.22 shows the standing wave patterns for the fundamental and the two lowest harmonics.

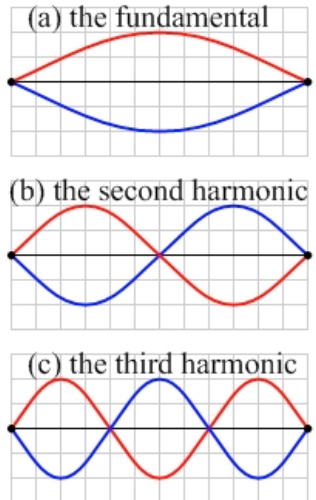


Figure 21.22: The standing wave patterns for the fundamental and the second and third harmonics, for a string fixed at both ends.

EXAMPLE 21.9 – Waves on a guitar string

A particular guitar string has a length of 72 cm and a mass of 6.0 grams.

(a) What is the wavelength of the fundamental on this string?

(b) If you want to tune that string so its fundamental frequency is 440 Hz (an A note), what should the speed of the wave be?

(c) When the string is tuned to 440 Hz, what is the string's tension?

(d) Somehow, you excite only the third harmonic, which has a frequency three times that of the fundamental. At $t = 0$, the profile of the string is shown in Figure 21.23, with the middle of the string at its maximum displacement from equilibrium. What is the oscillation period, T?

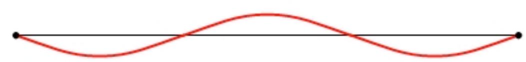


Figure 21.23: The string profile at $t = 0$ when the third harmonic has been excited on the string, with the middle of the string at its maximum displacement from equilibrium.

SOLUTION

(a) For the fundamental, exactly half a wavelength fits in the length of the string. Thus, the wavelength is twice the length of the string: $\lambda = 144$ cm $= 1.44$ m.

(b) Knowing the frequency and the wavelength, we can determine the wave speed: $v = f\lambda = (440 \text{ Hz})(1.44 \text{ m}) = 634$ m/s.

(c) Knowing the speed, we can use Equation 21.5, to find the tension in the string.

$$v = \sqrt{\frac{F_T}{(m/L)}}, \quad \text{so} \quad F_T = v^2 \times \frac{m}{L}.$$

In this case, we get: $F_T = v^2 \times \frac{m}{L} = (633.6 \text{ m/s})^2 \times \frac{0.006 \text{ kg}}{0.72 \text{ m}} = 3350 \text{ N}$.

(d) The fundamental frequency is 440 Hz, so the third harmonic has a frequency of 1320 Hz, three times that of the fundamental. The period is the inverse of the frequency, so:

$$T = \frac{1}{f} = \frac{1}{1320 \text{ Hz}} = 760 \ \mu\text{s}.$$

Related End-of-Chapter Exercises: 28, 29, 36, 59.

Essential Question 21.9: Return to the situation discussed in Example 21-9. Figure 21.23 shows the string profile at $t = 0$. Show the string profile at times of $t = T/4$, $T/2$, $3T/4$, and T.

Answer to Essential Question 21.9: Every half-period, the string profile is inverted, so at $t = T/2$ the string profile is inverted compared to what it is at $t = 0$, and after a full period ($t = T$), the profile is the same as that as $t = 0$. Halfway between these positions, the string profile is flat, as shown in Figure 21.24.

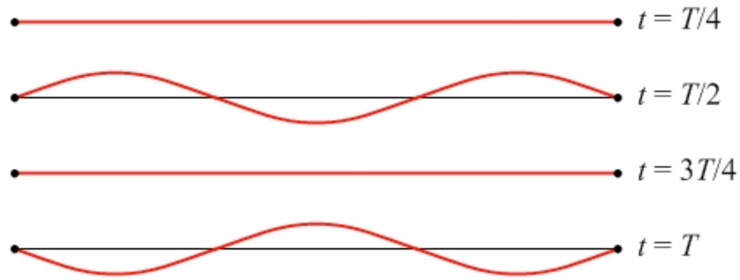


Figure 21.24: The string profile at (a) $t = T/4$, (b) $t = T/2$, (c) $t = 3T/4$, and (d) $t = T$, for the third harmonic.

21-10 Standing Waves in Pipes

Many musical instruments are made from pipes. Such instruments are known as wind instruments. A flute, for instance, is a single pipe in which the effective length can be changed by opening one of several holes in the pipe. In a trombone, the effective length is changed by sliding a tube in or out. In a pipe organ, in contrast, many different pipes, of fixed length, are used, with each pipe having a different fundamental frequency. The connection between all of these instruments is that the effective length of the tube determines the sound the pipe makes.

In contrast with a string instrument, in which a vibrating string sets up a sound wave in the air, the wave in a wind instrument is already a sound wave in a column of air, some of which escapes to make an audible sound. As with strings, however, standing waves produced by reflected waves determine the fundamental frequency of the sound wave produced by a particular pipe. Note that pipes can have both ends open, or have one end open and one end closed. For a sound wave, the open end of a pipe is like a free end, while the closed end of a pipe is like a fixed end. Thus, a pipe with only one end open sounds quite different from a pipe with both ends open, even if the tubes have the same length, because of the different standing waves that are produced by the different reflections in these pipes.

Because an open end acts like a free end for reflection, the standing waves for a pipe that is open at both ends have anti-nodes at each end of the pipe. We can satisfy this condition with standing waves in which an integral number of half-wavelengths fit in the pipe, as shown in parts (a) – (c) of Figure 21.25. This leads to the same equation for standing waves that we had in section 21-9, for the string fixed at both ends.

For a pipe open at both ends: The standing waves produced always have an anti-node at each end of the pipe. The frequencies that produce standing waves in such a pipe are:

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L}, \qquad \text{(Eq. 21.14: \textbf{Standing-wave frequencies for a pipe open at both ends})}$$

where n is an integer, and L is the effective length of the pipe.

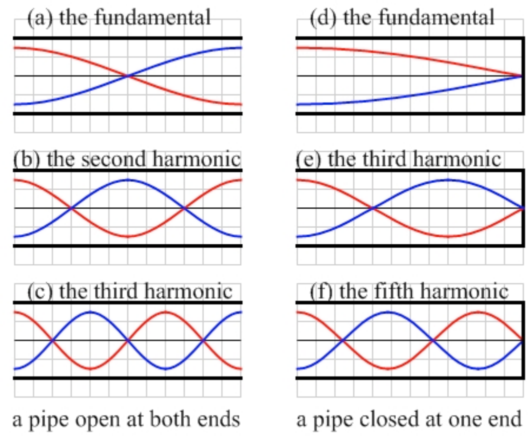
Because an open end acts like a free end, while a closed end acts like a fixed end, the standing waves for a pipe that is open at only one end have anti-nodes at the open end and nodes at the closed end. We can satisfy this condition with standing waves in which an odd integer number of quarter-wavelengths fit in the pipe, as shown in parts (d) – (f) of Figure 21.25. This leads to new equation for the standing-wave frequencies.

For a pipe open at one end only: The standing waves produced have an anti-node at the open end and a node at the closed end. The frequencies that produce standing waves in such a pipe are:

$$f_n = \frac{nv}{4L}, \qquad \text{(Eq. 21.15: \textbf{Standing-wave frequencies for a pipe open at one end})}$$

where n is an odd integer, and L is the effective length of the pipe.

Figure 21.25: (a) – (c) A representation of the standing waves in a pipe that is open at both ends, showing the fundamental (a), and the second (b) and third (c) harmonics. (d) – (f) A similar representation for a pipe that is closed at one end only, showing the fundamental (d), and the two lowest harmonics (e) and (f). For a pipe closed at one end only, the harmonics can only be odd integer multiples of the fundamental. Note that the waves in the pipe are sound waves, which are longitudinal waves. This representation shows the maximum displacement from equilibrium for the air molecules as a function of position along each of the pipes. The standing waves oscillate between the profile shown in red and the profile shown in blue.



EXAMPLE 21.10 – Waves in a pipe

A particular organ pipe has a length of 72 cm, and it is open at both ends. Assume that the speed of sound in air is 340 m/s.

(a) What is the wavelength of the fundamental in this pipe?

(b) What is the corresponding frequency of the fundamental?

(c) If one end of the pipe is now covered, what are the wavelength and frequency of the fundamental?

SOLUTION

(a) For the fundamental, exactly half a wavelength fits in the length of the pipe. Thus, the wavelength is twice the length of the pipe: $\lambda = 2 \times 72$ cm = 144 cm = 1.44 m.

(b) Knowing the speed of sound and the wavelength, we can determine the frequency:

$$f = \frac{v}{\lambda} = \frac{340 \text{ m/s}}{1.44 \text{ m}} = 236 \text{ Hz}.$$

(c) Covering one end of the pipe means the pipe is open at one end only, so now, for the fundamental, only one-quarter of a wavelength fits in the pipe rather than half a wavelength. This doubles the wavelength of the fundamental to 2.88 m. Doubling the wavelength reduces the frequency by a factor of two, so the new fundamental frequency is 118 Hz.

Related End-of-Chapter Exercises: 31, 33, 37, 38.

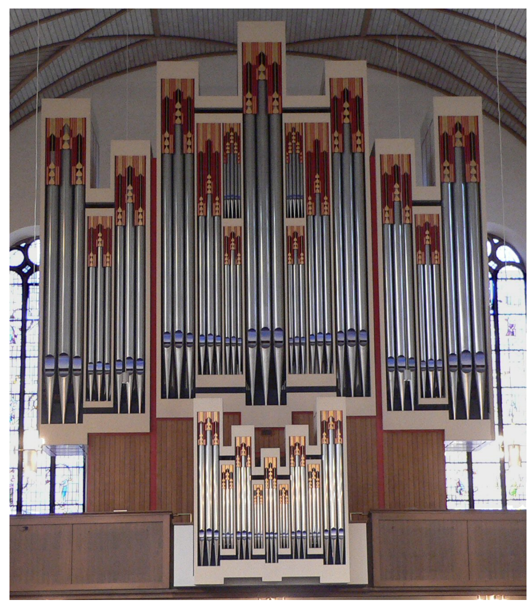


Figure 21.26: The photograph shows a pipe organ in Katharinenkirche, Frankfurt am Main, Germany. Each pipe has a unique frequency.
Photo credit: Wikimedia Commons.

Essential Question 21.10: Musical instruments made from pipes have a variety of pipes or one variable–length pipe. What happens to the fundamental frequency as the pipe length increases?

Answer to Essential Question 21.10: The longer the pipe, the longer the wavelength of the fundamental. Wavelength is inversely proportional to frequency, so the longer the pipe, the smaller the frequency.

Chapter Summary

Essential Idea: Waves.

A wave is a way to transfer energy from one place to another without needing a net flow of material. Waves are in integral part of the way we communicate, whether it be the signals that are picked up by our cell phones, and turned into recognizable speech by the phone's circuitry and speaker, or the light that brings the world to our eyes.

Types of Waves

In this chapter, we dealt with **mechanical waves**, which need a medium through which to travel. Such waves can be **transverse**, in which the particles of the medium oscillate in a direction perpendicular to the direction the wave travels, or **longitudinal**, in which the particles of the medium oscillate along the same direction as the direction the wave travels. A wave on a string is generally transverse, while sound waves are longitudinal.

The wave equation

In general, the relationship between wave speed, v, frequency, f, and wavelength, λ, is:

$v = f\lambda$. (Equation 21.1: **Connecting speed, frequency, and wavelength**)

Equation of motion for a single-frequency transverse wave

In general, the displacement of any point in the medium, at any instant in time, when a single-frequency transverse wave is propagating through the medium in the x-direction, is given by an equation of the form:

$y = A\cos(\omega t \pm kx)$, (Equation 21.4: **Equation of motion for a transverse wave**)

where the plus sign is used when the wave is traveling in the negative x-direction, and the minus sign is used when the wave is traveling in the positive x-direction.

The wave number, k, is related to the wavelength, λ, in the same way that the angular frequency, ω, is related to the period, T:

$k = \frac{2\pi}{\lambda}$. (Equation 21.2: **the wave number**)

$\omega = \frac{2\pi}{T}$. (Equation 21.3: **the angular frequency**)

Wave speed

In general, the wave speed is determined not by the frequency and wavelength, but by properties of the medium itself. For example, the speed of a wave on a string is determined by the tension in the string, F_T, and the mass per unit length, μ:

$v = \sqrt{\frac{F_T}{m/L}} = \sqrt{\frac{F_T}{\mu}}$, (Eq. 21.5: **The speed of a wave on a string**)

Sound Intensity

Intensity is the power per unit area: $I = P/A$. Because of the way the human ear responds to sound, we generally use a logarithmic scale to measure the intensity level of a sound:

$$\beta = (10 \text{ dB}) \log\left(\frac{I}{I_0}\right),$$ (Equation 21.7: **Absolute sound intensity level, in decibels**)

where the reference intensity $I_0 = 1 \times 10^{-12}$ W/m^2 is known as the threshold of hearing, and the log is in base 10.

The Doppler Effect

The Doppler effect describes the shift in frequency of a wave that occurs when the source of the waves, and/or the observer of the waves, moves with respect to the medium the waves are traveling through. If the source emits a frequency f, the frequency f' received by the observer is:

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right),$$ (Equation 21.11: **The general Doppler equation**)

where v is the speed of the wave through the medium, v_o is the speed of the observer, and v_s is the speed of the source. In the numerator, use the top (+) sign if the observer moves toward the source, and the bottom (–) sign if it moves away. In the denominator, use the top (–) sign if the source moves toward the observer, and the bottom (+) sign if it moves away.

Superposition and interference

When two or more waves overlap, we find the net effect by applying the **principle of superposition**: the net displacement of any point in a medium is the sum of the displacements at that point due to each of the individual waves. If the displacements of the individual waves are in the same direction at a point, we say that the waves experience **constructive interference**, leading to a large net displacement at that point. If the individual displacements are in opposite directions, **destructive interference** occurs, which means that the net displacement is small.

Beats

One example of superposition is when two waves of different frequencies interfere, leading to oscillations in the amplitude of the resultant wave. This is known as beats. The frequency at which the amplitude oscillates is the difference between the two frequencies.

$$f_{beat} = f_{high} - f_{low}.$$ (Equation 21.12: **the beat frequency**)

Standing waves

Standing waves are waves in which the **nodes** (points of zero displacement) and the **anti-nodes** (points of maximum displacement) remain at rest. Standing waves are generally produced by two identical waves traveling in opposite directions through a medium, and they describe the waves produced by string and wind instruments.

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L},$$ (**Standing-wave frequencies for strings and for pipes open at both ends**)

where L is the length of the string or pipe, v is the wave speed, and n is any integer. The lowest-frequency standing wave (for $n = 1$) is known as the **fundamental**, while the others are known as **harmonics**. Thus, harmonics are integer multiples of the fundamental.

$$f_n = \frac{nv}{4L},$$ (Eq. 21.15: **Standing-wave frequencies for a pipe open at one end only**)

where n is any odd integer.

End-of-Chapter Exercises

Exercises 1 – 12 are primarily conceptual questions designed to see whether you understand the main concepts of the chapter. For Exercises 1 – 4, the corresponding figure shows the profile of a string at $t = 0$ and at $t = 1.0$ s, as two pulses approach one another.

1. Two pulses travel toward one another, as shown in Figure 21.27. Sketch the profile of the string at (a) $t = 4.0$ s, (b) $t = 5.0$ s, and (c) $t = 6.0$ s.

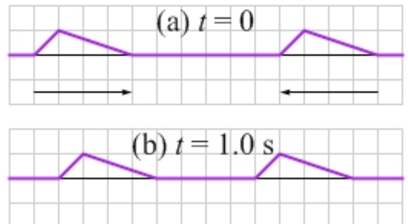


Figure 21.27: Two pulses approach each other along a string, for Exercise 1.

2. Two pulses travel toward one another, as shown in Figure 21.28. Sketch the profile of the string at (a) $t = 4.0$ s, (b) $t = 5.0$ s, and (c) $t = 6.0$ s.

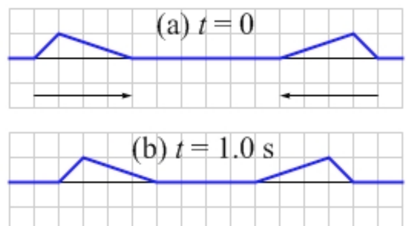


Figure 21.28: Two pulses approach each other along a string, for Exercise 2.

3. Two pulses travel toward one another, as shown in Figure 21.29. Sketch the profile of the string at (a) $t = 4.0$ s, (b) $t = 5.0$ s, and (c) $t = 6.0$ s.

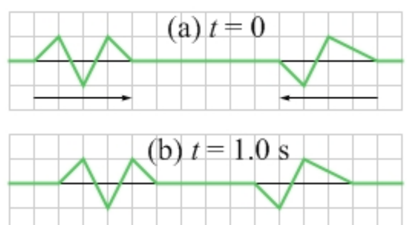


Figure 21.29: Two pulses approach each other along a string, for Exercise 3.

4. Two pulses travel toward one another, as shown in Figure 21.30. Sketch the profile of the string at (a) $t = 4.0$ s, (b) $t = 5.0$ s, and (c) $t = 6.0$ s.

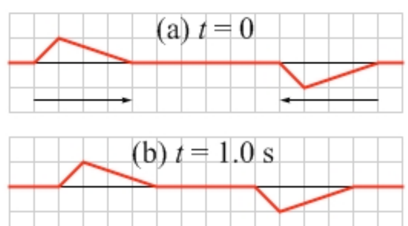


Figure 21.30: Two pulses approach each other along a string, for Exercise 4.

5. Two identical speakers, which are separated by a distance of 7.2 m, are pointed at one another. The speakers, which are in phase with one another, broadcast identical, single-frequency sound waves. There is one point on the line joining the speakers which always experiences constructive interference no matter what the frequency of the identical waves emitted by the speakers is. Where is this point? Explain why the interference is always constructive there.

6. Return to the situation discussed in Exercise 5. If the speed of sound is 340 m/s and the frequency of the waves emitted by each speaker is 170 Hz, find the location of all points along the line between the speakers at which the interference is (a) completely constructive, and (b) completely destructive.

7. Two pulses are traveling along a string, as shown in Figure 21.31. A particular point on the string is marked with a black dot. Plot the displacement of that point as a function of time, over the time interval $t = 0$ to $t = 8.0$ s.

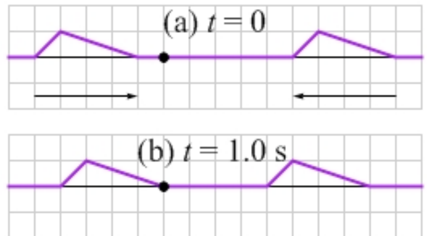


Figure 21.31: Two pulses travel along a string. A particular point on the string is marked with a black dot. For Exercise 7.

8. Two pulses are traveling along a string, as shown in Figure 21.32. A particular point on the string is marked with a black dot. Plot the displacement of that point as a function of time, over the time interval $t = 0$ to $t = 8.0$ s.

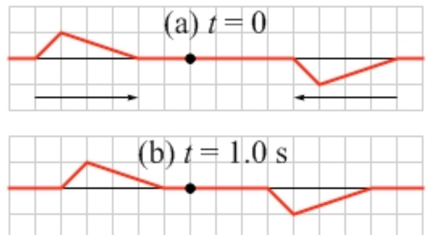


Figure 21.32: Two pulses travel along a string. A particular point on the string is marked with a black dot. For Exercise 8.

9. You have four tuning forks, with frequencies of 440 Hz, 445 Hz, 448 Hz, and 452 Hz. By using two tuning forks at a time, how many different beat frequencies can you produce, and what are the numerical values of these frequencies?

10. When you strike two tuning forks and listen to them both at the same time, you hear beats with a beat frequency of 6 Hz. If one tuning fork has a frequency of 512 Hz, what is the frequency of the other?

11. The profile of a string that supports a particular standing wave is shown at $t = 0$ in Figure 21.33. The string is fixed at both ends. At $t = 0$, the standing wave is at its maximum displacement from equilibrium. The standing wave is created by two identical traveling waves on the string, one moving to the right and the other to the left. (a) What is the amplitude of each of these traveling waves? (b) Sketch the profile of the string one-quarter of a period after $t = 0$. (c) Sketch the right-going and left-going waves one-quarter of a period after $t = 0$. Hint: the superposition of these two waves should give the profile in part (b).

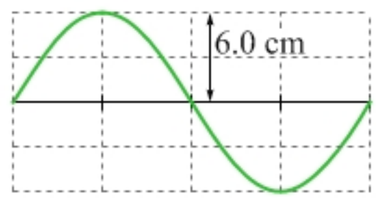


Figure 21.33: The profile of a string, at $t = 0$, that is fixed at both ends. The wave on the string is a standing wave, and the situation shown in the diagram shows the standing wave at its maximum displacement from equilibrium. For Exercise 11.

12. As you are walking along the street, a car blaring loud music passes you. As the car drives away from you, you recognize the music but you realize that it sounds funny. What is the problem?

Exercises 13 – 17 involve applying the equation of motion for a transverse wave.

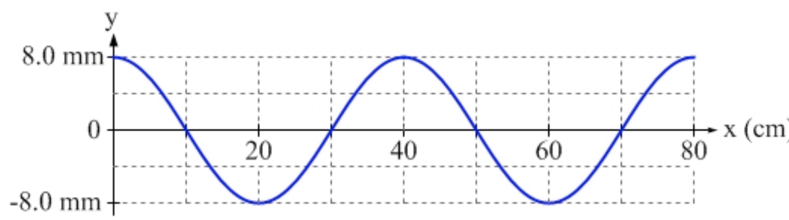
13. The equation of motion for a particular transverse wave is $y = (7.0 \text{ mm})\sin\left[(4\pi \text{ rad/s})t + (\pi \text{ m}^{-1})x\right]$. Determine the wave's (a) amplitude, (b) angular frequency, (c) frequency, (d) wavelength, and (e) velocity.

14. For a particular transverse wave that travels along a string that lies on the *x*-axis, the equation of motion is $y = (6.0 \text{ cm})\cos\left[(50 \text{ rad/s})t - (0.25 \text{ m}^{-1})x\right]$. Determine (a) the wave's amplitude, wavelength, and frequency, (b) the speed of the wave, (c) the tension in the string, if the string has a mass per unit length of 0.040 kg/m, (d) the direction of propagation of the wave, (e) the maximum transverse speed of a point on the string, (f) the displacement of a point at $x = 1.0$ m when $t = 2.0$ s.

15. The equation of motion for a particular wave traveling along a string along the *x*-axis is $y = A\cos\left[\omega t + (4.0 \text{ m}^{-1})x\right]$. The tension in the string is 34 N, and the string has a mass per unit length of 0.050 kg/m. The maximum transverse speed of a point on the string is 25 cm/s. Determine (a) the angular frequency, ω, and (b) the amplitude, A, of the wave.

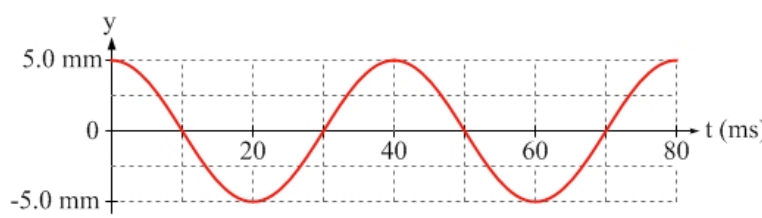
16. At a time of $t = 0$, the profile of part of a string is shown in Figure 21.34. The wave on the string is traveling in the $+x$ direction (to the right) at a speed of 20 cm/s. Write out the equation of motion for the wave.

Figure 21.34: The profile of part of a string at $t = 0$. The wave on the string is traveling to the right at 20 cm/s. For Exercise 16.



17. A graph of the motion of one point on a string (specifically, the point at $x = 0$), as a function of time is shown in Figure 21.35. The wave is traveling in the negative *x*-direction on a string that has a tension of 32 N, and with a mass per unit length of 60 grams per meter. Determine (a) the frequency of the wave, (b) the speed of the wave, (c) the wavelength, and (d) the expression for the wave's equation of motion.

Figure 21.35: A graph of the motion of one point on a string, at $x = 0$. The wave on the string is traveling in the negative *x*-direction. For Exercise 17.



Exercises 18 – 22 involve sound, sound intensity, and the decibel scale.

18. You are listening to the radio when one of your favorite songs comes on, so you turn up the volume. If you managed to increase the sound intensity by 15 dB, by what factor did the intensity of the sound, in W/m^2, increase?

19. You are working in a room in which the sound intensity is 75 dB. What is the corresponding intensity, in W/m^2?

20. When you apply the brakes on your car, they happen to squeak, emitting a 70 dB sound as observed by you sitting in the driver's seat of the car. When you sound the car's horn, however, you observe an 80 dB sound. As you are driving, a dog runs into the road in

front of your car, so you apply the brakes and sound the horn simultaneously. (Fortunately, the dog escapes unharmed.) Do you observe a 150 dB sound while you are stopping, with the brakes squeaking and the horn sounding together? Explain your answer, being as quantitative as possible.

21. When you stand 2.0 m away from a speaker that is emitting sound uniformly in all directions, the sound intensity you observe is 90 dB. What is the sound intensity at a distance of (a) 1.0 m from the speaker, and (b) 4.0 m from the speaker?

22. You are observing fishermen illegally catching fish by using a small explosive device to stun the fish. The explosion takes place near the surface of the water, so the sound of the explosion travels through both the air and the water. You record the sound of the explosion using two separate microphones, one in the air above the water and one below the water surface. (a) Which microphone picks up the sound first? (b) If the time delay between the sounds reaching the two microphones is 0.50 seconds, about how far are you from the fishermen?

Exercises 23 – 27 involve the Doppler effect. Assume the speed of sound is 340 m/s.

23. In a common classroom demonstration, a buzzer is turned on inside a soft football. The buzzer emits a tone of 256 Hz. (a) If the football is thrown directly at you at a speed of 12.0 m/s, what frequency do you hear? (b) Fortunately, you duck in time to have the ball pass over your head. What frequency do you observe as the ball moves away from you?

24. In another common classroom demonstration of the Doppler effect, the instructor whirls a buzzer, on the end of a string or electric cable, in a horizontal circle around their head. If the buzzer has a frequency of 500 Hz, the circle has a radius of 1.0 m, and the period of the buzzer's motion is 0.50 s, what are the maximum and minimum frequencies observed by the students in the classroom as they sit in their seats listening to the buzzer?

25. As you are riding your bicycle at 10.0 m/s north along a road, an ambulance traveling south approaches you. You observe the ambulance's siren to have a frequency of 352 Hz. However, the siren's frequency is actually 325 Hz, when the ambulance is at rest. (a) How fast is the ambulance traveling? (b) After the ambulance has passed you, what frequency do you observe for the siren?

26. Your car horn happens to have the unusual property of emitting a pure tone at a frequency of 440 Hz. You drive at 20 m/s toward a high wall, and sound the horn briefly. After a short time, you hear the echo of the sound, after it was reflected by the wall. What is the frequency of the echo?

27. A particular bat emits ultrasonic waves with a frequency of 68.0 kHz. The bat is flying at 12.00 m/s toward a moth, which is traveling at 3.00 m/s toward from the bat. The speed of sound is 340.00 m/s. (a) Assuming the moth could detect the waves, what frequency waves would it observe? (b) What frequency are the waves that reflect off the moth and are detected by the bat?

Exercises 28 – 32 involve standing waves.

28. A particular guitar string has a length of 75 cm, and a mass per unit length of 80 grams/meter. You hear a pure tone of 1320 Hz when a particular standing wave, represented by the sequence of images shown in Figure 21.24, is excited on the string. (a) What is the wavelength of this standing wave? (b) What is the speed of waves on this string? (c) What is the tension in the string? (d) What is the fundamental frequency of this string?

29. An Aeolian harp (named after Aeolus, the Greek god of the wind) consists of several strings fixed to a frame or a sounding box. The device is simply placed outside, and the strings are played randomly by the wind. You decide to make such a harp out of strings that all have a mass per unit length of 80 grams per meter, and that all have a tension of 50 N. (a) If you want one of the strings to have a fundamental frequency of 330 Hz, how long should you make it? (b) If you want another of the strings to have a fundamental frequency of 660 Hz (double that of the first string, and therefore exactly one octave higher up the scale), how long should it be?

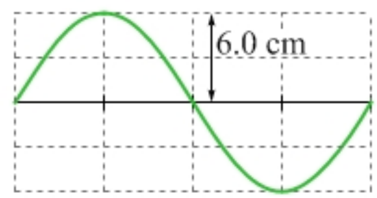


Figure 21.36: The profile of a particular standing wave on a string at $t = 0$, when the string is in one of its maximum displacement states, for Exercise 30.

30. The profile of a particular standing wave on a string is shown in Figure 21.36, showing the string at its maximum displacement from equilibrium at $t = 0$. The string has a length of 1.0 m, extending from $x = 0$ to $x = +1.0$ m. Over one period of oscillation for the standing wave, plot a graph of displacement as a function of time for the point at (a) $x = 0.25$ m, (b) $x = 0.50$ m, (c) $x = 0.65$ m.

31. As shown in Figure 21. 37, the height of an air column in a particular pipe is adjusted by changing the water level in the pipe. In a traditional experiment, a tuning fork is placed over the pipe, and the height of the air column is adjusted, by moving a reservoir of water up and down, until the pipe makes a loud sound, which is when the pipe's fundamental frequency matches the frequency of the tuning fork. If the speed of sound is 340 m/s, and an air column of 22.4 cm produces the loudest sound, what is the frequency of the tuning fork?

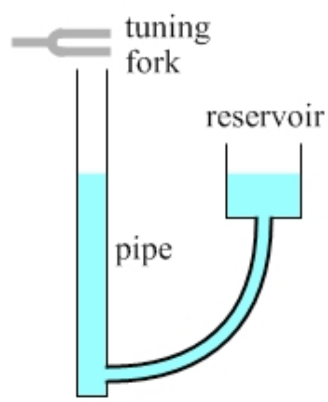


Figure 21.37: The height of the air column in this pipe can be adjusted by changing the water level in the pipe. When a tuning fork, which is emitting sound, is placed over the pipe, the pipe will emit a loud sound when the frequency of the tuning fork matches the fundamental frequency of the pipe. For Exercise 31.

32. A bloogle is a corrugated plastic tube, which is open at both ends, that emits a tone when you whirl it around your head. Generally, if you whirl it faster, the tube will emit a higher-frequency harmonic. You measure the various frequencies of a particular bloogle to be 420 Hz, 560 Hz, 700 Hz, and 840 Hz. (a) What is the fundamental frequency of this bloogle? (b) Estimate the length of the bloogle.

Exercises 33 – 38 involve applications of sound and waves.

33. Some cameras have automatic focusing systems that rely on ultrasonic emitters and detectors. You are trying to take a picture of your friends, who are 4.5 m from your camera. To focus correctly, the camera sends out a short ultrasonic pulse that reflects off your friends. If the speed of sound is 340 m/s, how much time passes between the emission of the pulse and the detection of the pulse by the camera?

34. One useful application of sound waves is a pair of noise canceling headphones. Such headphones have a microphone that picks up ambient noise (such as the noise of the engines inside a jet airplane). The wave representing the sound is then inverted and played through the speakers of the headphones into your ears. Explain, using principles of physics addressed in this chapter, how this works so that you hear a low-amplitude sound in the headphones.

35. One medical application of sound waves is in the use of ultrasound to see inside the womb to create an image of a fetus, as in the photograph shown in Figure 21.38. Do some research about this particular application of sound waves, and write two or three paragraphs describing how it works, and how it exploits the principles of physics discussed in this chapter.

Figure 21.38: An image of a fetus in the womb, at 24 weeks, obtained by ultrasonic imaging. Photo credit: Maciej Korzekwa, via iStockPhoto.com.

36. The frequencies of neighboring notes on a musical scale differ by a factor of $2^{1/12}$. A particular guitar string is tuned to sound an A note, of 440 Hz. The next highest note is $A^{\#}$ (A sharp). (a) What is the frequency of this particular note? (b) By changing the effective length of the string, by pressing the string down onto one of the frets on the guitar, you can get the string to sound $A^{\#}$ instead of A. If the string has a length L when it sounds A, what is the effective length of the string when it is sounding $A^{\#}$? (c) Explain why the spacing between frets on the guitar decreases as the effective length decreases.

37. You want to make a simple set of wind chimes out of metal pipes that are open at both ends. You would like to create a set of three pipes that sound a C-major chord, playing the notes C (264 Hz), E (330 Hz), and G (396 Hz). (a) What is the ratio of the lengths of the three pipes you should use to make your wind chimes? (b) Which pipe is the shortest, and what is its length? Assume the speed of sound is 340 m/s.

38. The human ear can be modeled, to a first approximation, as a pipe that is open at one end only. If the length of the ear canal is 25 mm in a typical person, and the speed of sound in air is 340 m/s, what is the ear's resonance frequency? (This is the frequency of the fundamental frequency of the pipe and, in theory, should correspond to the frequency of sound that a typical person is most sensitive to.)

General problems and conceptual questions

39. A track designed for running 100-meter races is 8 m wide. If the starter fires her starting pistol from one side of the track, near the runners, the runner next to her has an advantage over the runner in the lane on the other side of the track. (a) Approximately how much time passes between when the closest runner hears the sound of the starting gun and when the farthest runner hears the sound of the gun? (b) If the runners run at an average speed of 10 m/s, what distance does this time difference translate to? Note that in serious competitions, the starting gun is electronically connected to speakers attached to the starting blocks for each runner, so that the start is fair.

40. A single-frequency wave, with a wavelength of 25 cm, is traveling in the positive x-direction along a string, causing each particle in the string to oscillate in simple harmonic motion with a period of 0.20 s. If the maximum transverse speed of each particle is 20 cm/s, and the particle at $x = 0$ is at its maximum positive displacement from equilibrium at $t = 0$, determine: (a) the speed of the wave, (b) the amplitude of the wave, and (c) the equation of motion for the wave.

41. Figure 21.39 (a) shows a snapshot of a traveling wave at $t = 0$, while Figure 21.39 (b) is a graph of the displacement versus time for the point at $x = 0$. (a) Is the wave traveling in the positive or negative x-direction? Explain. (b) Write out the equation of motion for the wave. (c) Does the equation of motion change if the graph in Figure 21.39 (b) applies to the point at $x = 20$ cm, instead? If so, what is the equation of motion in that case?

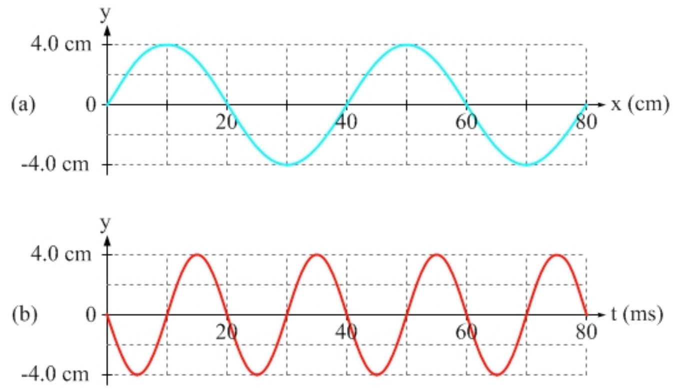


Figure 21.39: (a) A photograph of a wave on a string at $t = 0$. (b) A graph of the displacement versus time for the point at $x = 0$. For Exercises 41 and 42.

42. Figure 21.39 shows two representations of a traveling wave on a string. Figure 21.39 (a) shows a snapshot of a traveling wave at $t = 0$, while Figure 21.39 (b) is a graph of the displacement versus time for the point at $x = 0$. In each part below, state which representation you can use to find the answer, as well as giving the numerical value of the answer. (a) What is the wavelength? (b) What is the period? (c) What is the amplitude? (d) What is the speed of the wave?

43. Two trains are traveling along parallel tracks. Each train has a whistle that emits a tone of 333 Hz when the train is at rest. One train is traveling east at 5.00 m/s. The engineer in that train hears a beat frequency of 4.00 Hz when both train whistles are sounding. What is the velocity of the second train? Assume the speed of sound is 340 m/s. Summarize all the possible solutions to this exercise.

44. As shown in Figure 21.40, a child is swinging back and forth on a swing. The child is near a speaker that is broadcasting a pure (single-frequency) tone. The child is shown in five different positions during a swing. In which position will the child hear (a) the highest-frequency sound, and (b) the lowest-frequency sound? Briefly justify your answers.

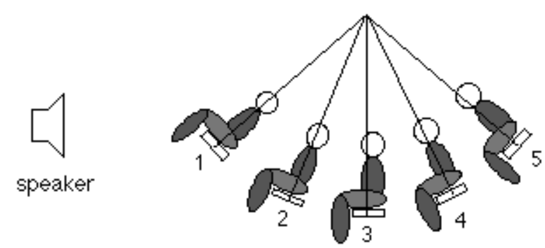


Figure 21.40: A child swinging back and forth on a string near a speaker that is broadcasting a pure tone, for Exercise 44.

45. The flow of blood through the heart can be studied with Doppler ultrasound. Ultrasonic waves are sent toward the heart, and by looking at the frequency of the waves that reflect from a particular spot in the heart, you can determine how fast blood is traveling in that region, and whether the blood is flowing toward or away from the ultrasound probe. An image is usually created from this data, with the colors of the various regions in the image reflecting the velocity of blood in those regions. If the probe sends out ultrasound with a frequency of 3.00 MHz, what is the frequency of waves that reflect back to the probe from an area of the heart (a) that is at rest? (b) where blood is traveling away from the probe with a speed that is 0.5% of the speed of sound in the medium? (c) where blood is traveling toward the probe at a speed of 0.7% of the speed of sound in the medium?

46. An ultrasonic sonar system emits ultrasonic waves that have a frequency of 600 kHz. The waves reflect from a plane that is moving at 50% of the speed of sound, directly toward the sonar system. (a) Find the frequency at which the waves reach the plane. (b) Find the frequency of the waves that are detected by the sonar system, after reflecting from the plane.

47. Repeat Exercise 46, but now have the plane moving directly away from the sonar system.

48. The pattern of sound waves emitted by a source traveling at constant velocity is shown in Figure 21.41. (a) In what direction is the source moving? (b) At what fraction of the speed of sound is the source traveling? If you are at rest, and the source is emitting waves that have a frequency of 480 Hz, what frequency do you observe if you are (c) at point A? (d) at point B?

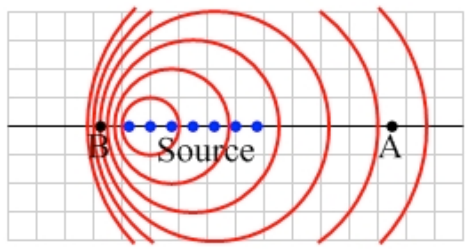


Figure 21.41: The pattern of circular waves emitted by a source that is traveling at a constant velocity. Each of the dots shows the position of the source when it emitted one of the wave peaks. For Exercise 48.

49. Repeat parts (a) – (c) of Exercise 48, but now base your answers on the pattern shown in Figure 21.42.

50. Do some research about what causes the loud sound when someone cracks a whip, and write a couple of paragraphs explaining the physics of whip-cracking.

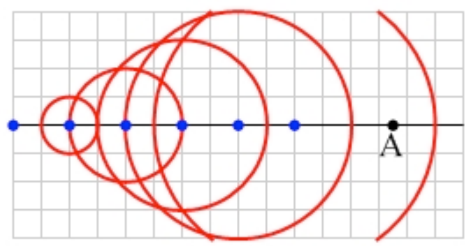


Figure 21.42: The pattern of circular waves emitted by a source that is traveling at a constant velocity. Each of the dots shows the position of the source when it emitted one of the wave peaks. For Exercise 49.

51. Two speakers, which are separated by a distance of 2.4 m, broadcast identical single-frequency sound waves. The speakers are in phase with one another. If you stand at a location that is 1.7 m farther from one speaker than the other, what are the lowest three frequencies at which (a) completely constructive interference occurs at your location, and (b) completely destructive interference occurs at your location?

52. Return to the situation described in Exercise 51. The speed of sound is 340 m/s, and the frequency of the waves emitted by the speakers is 340 Hz. You are initially right next to one of the speakers, and you then walk steadily away from it in a direction that is perpendicular to the line joining the two speakers. (a) At how many locations will you pass through a point at which completely constructive interference occurs? (b) How far are these locations from the speaker that marks your starting point?

53. In Figure 21.43, a pulse is traveling along a string toward the string's right end, which is a fixed end (shown as a dot on the right). Sketch the profile of the string at (a) $t = 4.0$ s, (b) $t = 6.0$ s, and (c) $t = 7.0$ s.

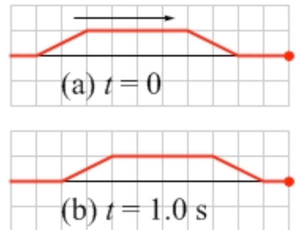


Figure 21.43: A pulse travels along a string toward the right end, which is either a fixed end (Exercise 53) or a free end (Exercise 54).

54. Repeat Exercise 53, but now the right end of the string is a free end instead of a fixed end.

55. In Figure 21.44, a pulse is traveling along a string toward the string's right end, which is a fixed end (shown as a dot on the right). A particular point on the string is shown as a black dot. Plot the displacement as a function of time for this point, over the time interval $t = 0$ to $t = 15.0$ s.

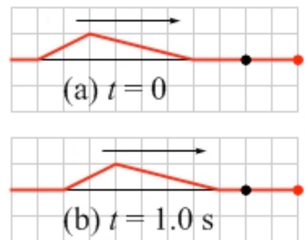


Figure 21.44: A pulse travels along a string toward the right end, which is either a fixed end (Exercise 55) or a free end (Exercises 56 and 57).

56. Repeat Exercise 55, if the right end of the string is a free end instead of a fixed end.

57. Repeat Exercise 56, except now plot the displacement as a function of time for the right end of the string, with the end being a free end.

58. Two stretched strings are placed next to one another, one with a length of 50 cm and the other with a length of 60 cm. The two strings have the same mass per unit length. You pluck the shorter one so that it vibrates at its fundamental frequency. You then adjust the tension in the longer string until it resonates with the first one. To resonate, the two strings must have the same frequency, so that vibrations on one string can cause the second string to vibrate. (a) What is the ratio of the speed of waves on the shorter string to the speed of waves on the longer string? (b) What is the ratio of the tension in the shorter string to the tension in the longer string?

59. A particular guitar string is under a tension of 38.5 N, and has a fundamental frequency of 320 Hz. If you want to tune the string so that it has a fundamental frequency of 330 Hz, to what value should you adjust the tension in the string?

60. A particular pipe that is open at both ends has a fundamental frequency of 442 Hz. When it, and a second pipe, have their fundamental frequencies excited simultaneously, a beat frequency of 8 Hz is observed. What is the ratio of the length of the first pipe to that of the second pipe if the second pipe is (a) also open at both ends, and (b) closed at one end.

61. As shown in Figure 21.45, a string passing over a pulley supports the weight of a 25 N block that hangs from the string. The other end of the string is fixed to a wall. The string has a mass per unit length of 75 grams per meter. The part of the string between the wall and the pulley is observed to oscillate with a fundamental frequency of 44 Hz. (a) What is the speed of waves on the string? (b) What is the distance, L, from the wall to the pulley? (c) If the weight hanging from the string is doubled, what will be the fundamental frequency of the part of the string between the wall and the pulley?

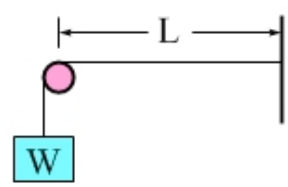


Figure 21.45: One end of a string is tied to a wall. The other end passes over a pulley, and supports a weight tied to the other end of the string. For Exercises 61 and 62.

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