20-1 Magnetic Flux

Let's begin by introducing the concept of flux. Flux means something quite different in physics than it does in everyday conversation. In physics, the flux through an area is simply a measure of the number of field lines passing through an area. In chapter 16, for instance, we could have defined an electric flux, a measure of the number of electric field lines passing through an area, in a way analogous to the following definition of magnetic flux. As we will see in section 20-2, magnetic flux turns out to play a crucial role in the generation of electricity.

> Magnetic flux is a measure of the number of magnetic field lines passing through an area. The symbol we use for flux is the Greek letter capital phi, Φ. The equation for magnetic flux is:
>
> $\Phi = BA\cos\theta$, (Equation 20.1: **Magnetic flux**)
>
> where θ is the angle between the magnetic field $\vec{B}$ and the area vector $\vec{A}$. The area vector has a magnitude equal to the area of a surface, and a direction perpendicular to the plane of the surface. The SI unit for magnetic flux is the weber (Wb). 1 Wb = 1 T m^2.

EXAMPLE 20.1 – Determining the magnetic flux

A rectangular piece of stiff paper measures 20 cm × 25 cm. You hold the piece of paper in a uniform magnetic field that has a magnitude of 4.0×10^{-3} T. For each situation below, sketch a diagram showing the magnetic field and the paper, and determine the magnitude of the magnetic flux through the paper, when the magnitude of the flux is (a) maximized, (b) minimized, and (c) halfway between its maximum and minimum value.

SOLUTION

(a) How should we hold the paper so that the largest number of field lines pass through it? As shown in Figure 20.1, we hold it so that the plane of the paper is perpendicular to the direction of the magnetic field. We can also understand this orientation by considering equation 20.1. To maximize the flux with an area vector of constant magnitude and a field of constant magnitude, we need to maximize the factor of $\cos\theta$. The factor of $\cos\theta$ reaches its maximum magnitude of 1 when θ , the angle between the area vector and the magnetic field, is either 0° or 180°. In other words, the area vector must be parallel to the magnetic field, which is the case when the plane of the paper is perpendicular to the magnetic field.

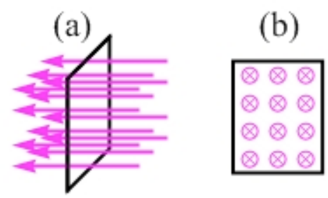


Figure 20.1: To maximize the magnetic flux through a flat area, orient the area so the plane of the area is perpendicular to the direction of the magnetic field. (a) shows a perspective view, while (b) shows the view looking along the field lines. In this case, the area vector is in the same direction as the field lines.

Because $\cos\theta$ has a magnitude of 1, the magnitude of the maximum flux equals the area multiplied by the magnetic field:

$$\Phi_{max} = AB = 0.20 \text{ m} \times 0.25 \text{ m} \times (4.0\times10^{-3} \text{ T}) = 2.0\times10^{-4} \text{ T m}^2 .$$

(b) The factor of $\cos\theta$ in equation 20.1 can be zero. Thus, the minimum magnitude of the magnetic flux is zero ($\Phi_{min} = 0$). How do we hold the paper so that there is no magnetic flux? As shown in Figure 20.2, if the plane of the paper is parallel to the magnetic field, no field lines pass through the paper and the magnetic flux is zero.

Figure 20.2: There is no flux when the plane of the area is parallel to the field. (a) shows a perspective view, while (b) shows the view looking along the field lines. In this case, the area vector is perpendicular to the field lines.

(c) Starting from the situation in Figure 20.1, tilting the loop by 60° (see Figure 20.3) gives a factor of $\cos\theta$ of ½, halfway between its maximum and minimum value. In this case the magnetic flux is $\Phi = 1.0\times10^{-4}\ \mathrm{T\ m^2}$, half its value from part (a).

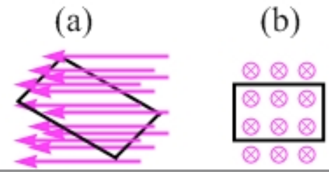


Figure 20.3: Tilting the loop from the orientation in Figure 20.1 reduces the flux. (a) shows a perspective view, while (b) shows the view along the field lines.

EXPLORATION 20.1 – Ranking situations based on flux

The four areas in Figure 20.4 are in a magnetic field. The field has a constant magnitude, and is directed into the page in the left half of the region and out of the page in the right half. Rank the areas based on the magnitude of the net flux passing through them, from largest to smallest. Note that, when we calculate net flux, field lines passing in one direction through an area cancel an equal number of field lines passing in the opposite direction through an area.

Region 1 is tied with region 4 for the largest area, but all the field lines in region 1 pass through in the same direction, giving region 1 the largest-magnitude flux. In contrast, the net flux through region 4 is zero because the flux through the right half of region 4 cancels the flux through the right side. The field lines in regions 2 and 3 all pass through in the same direction. Region 3 has an area of 3 boxes, while region 2 has an area of πr^2, where the radius is 1 unit, so region 2 has an area of π boxes. Because π is larger than 3, the magnetic flux through region 2 is larger than that through region 3. Thus, ranking by flux magnitude gives 1 > 2 > 3 > 4.

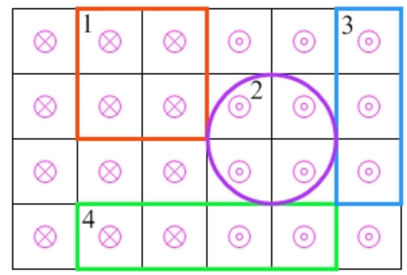


Figure 20.4: Four different regions in a magnetic field. The field has the same magnitude everywhere, but it is directed into the page in the left half of the field and out of the page in the right half.

Key idea for net flux: In calculating net flux, field lines passing one way through an area cancel an equal number of field lines passing in the opposite direction through that area.
Related End-of-Chapter Exercises: 1, 2, 4, 41.

An aside: Electric Flux and Gauss' Law

We did not mention electric flux when we talked about electric field, but we can define electric flux in an analogous way to magnetic field. Electric flux is a measure of the number of electric field lines passing through a surface. The equation for electric flux is:

$$\Phi_E = EA\cos\theta ,$$

where θ is the angle between the electric field $\vec{E}$ and the area vector $\vec{A}$.

There is a law called Gauss' Law, which says that the net electric flux passing through a closed surface is proportional to the net charge enclosed by that surface. Using the correct proportionality constant, Gauss' Law can be used to calculate electric fields in highly-symmetric situations. It is interesting to note that the analogous law in a magnetic situation, Gauss' Law for magnetism, is not nearly so useful. Because magnetic field lines are always continuous loops, the net magnetic flux passing through a closed surface is always zero – if a magnetic field line emerges from a surface, it must re-enter the surface at some other location, giving a net flux for that field line of zero, to ensure that the field line is a continuous loop.

Essential Question 20.1: Return to the situation described in Exploration 20.1, and shown in Figure 20.4. If we define out of the page as the positive direction for magnetic flux, rank the four areas by their net flux, from most positive to most negative.

Answer to Essential Question 20.1: In this case, the ranking is 2 > 3 > 4 > 1. Regions 2 and 3 have positive flux, because field lines directed out of the page pass through those regions. Region 4 still has a flux of zero, while region 1 has a negative flux because of the field lines passing through the region into the page.

20-2 Faraday's Law of Induction

Table 20.1 summarizes some experiments we do with a magnet and a loop of wire. The loop has a galvanometer in it, which is a sensitive current meter. When the needle on the meter is in the center there is no current in the loop. The direction and size of the needle's deflection reflects the direction and size of the current in the loop.

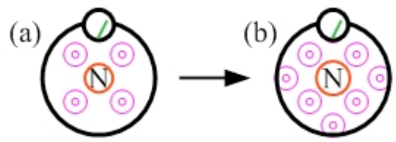
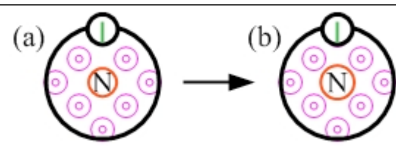
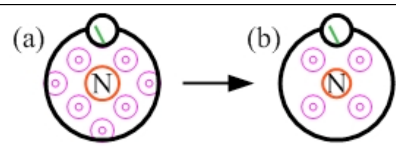
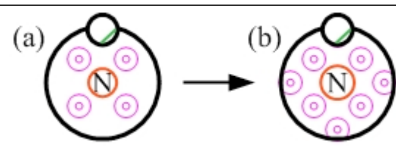
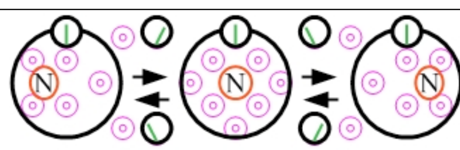
Experiment	Initial and final states	Meter reading
1. The north pole of the magnet is brought closer to the loop.	 **Figure 20.5**: The loop initially has a small flux (a), which increases as the magnet comes closer (b).	***While the magnet is moving closer***, the meter needle deflects to the right.
2. The north pole is held at rest close to the loop.	 **Figure 20.6**: The magnetic flux is large, but it is also constant the entire time.	The needle does not deflect at all.
3. The north pole of the magnet is moved away from the loop.	 **Figure 20.7**: The loop initially has a large flux (a), which decreases as the magnet is moved away (b).	***While the magnet is moving away***, the meter needle deflects to the left.
4. The north pole of the magnet is brought closer to the loop, but at a faster rate than it was in experiment 1.	 **Figure 20.8**: The same situation as Figure 20.5, but with less time between (a) and (b).	***While the magnet is moving closer***, the needle deflects farther to the right, but for less time, than in experiment 1.
5. The magnet is rotated back and forth in front of the loop.	 **Figure 20.9**: The flux oscillates from large to small and back again.	As the magnet oscillates, the needle oscillates back and forth at, in this case, double the frequency of the magnet.

Table 20.1: Various experiments involving a magnet and a wire loop connected to a current meter. The views in the figures are looking through the loop at the magnet.

Among the conclusions we can draw from these experiments are the following:

- A magnet interacting with a conducting loop can produce a current in the loop.
- A current arises only when the magnetic flux through the loop is changing.

The current is larger when the magnetic flux changes at a faster rate.

Exposing a loop or coil to a ***changing*** magnetic flux gives rise to a voltage, called an **induced emf.** Following Ohm's law, the induced emf gives rise to an **induced current** in the loop or coil. The emf induced by a changing magnetic flux in each turn of a coil is equal to the time rate of change of that flux. Thus, for a coil with N turns, the net induced emf is given by:

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}.$$ (Eq. 20.2: **Faraday's Law of Induction**)

The minus sign in equation 20.2 will be explained in section 20-3.

EXPLORATION 20.2 – Using graphs with Faraday's Law

A flat square conducting coil, consisting of 5 turns, measures 5.0 cm × 5.0 cm. The coil has a resistance of 3.0 Ω and, as shown in Figure 20.10, moves at a constant velocity of 10 cm/s to the right through a region of space in which a uniform magnetic field is confined to the 20 cm long region shown in the figure. The field is directed out of the page, with a magnitude of 3.0 T.

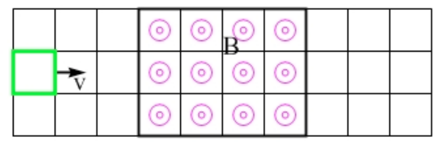


Figure 20.10: A flat conducting coil moves at constant velocity to the right through a region of space in which a uniform magnetic field is confined. The small squares on the diagram are 5.0 cm on each side.

Draw the coil's motion diagram, and a graph of the magnetic flux through the coil as a function of time. Define out of the page as the positive direction for flux. Finally, draw a graph of the emf induced in the coil as a function of time. The diagrams are in Figure 20.11. For the first 1.0 s there is no flux, because no field passes through the coil. The flux grows linearly with time during the half-second the coil moves into the field. The flux is constant, at $B \times A = 7.5\times10^{-3}\text{ T m}^2$, for the next 1.5 s, and then drops linearly to zero in the half-second the coil takes to leave the field.

Faraday's Law tells us that the induced emf is related to $\Delta\Phi/\Delta t$, which is the slope of the flux versus time graph. While the flux is increasing, the slope of the flux graph is constant with a value of $\Delta\Phi/\Delta t = (7.5\times10^{-3}\text{ T m}^2)/(0.50\text{ s}) = 1.5\times10^{-2}\text{ V}$. Multiplying by the factor of $-N$ from Faraday's Law, where N = 5 turns, gives an induced emf of -7.5×10^{-2} V. The induced emf drops to zero while the flux is constant, and then has a constant value of $+7.5 \times 10^{-2}$ V during the half-second period while the coil is leaving the magnetic field and the flux is decreasing.

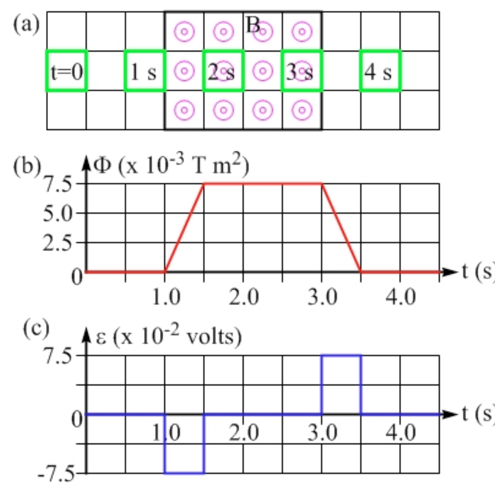


Figure 20.11: (a) A motion diagram showing the coil's position at 1-second intervals. Graphs, as a function of time, of (b) the magnetic flux through the coil and (c) the emf induced in the coil.

Key idea: The induced emf is proportional to the negative of the slope of the graph of flux as a function of time. **Related End-of-Chapter Exercises: 14 – 17, 23 – 26.**

Essential Question 20.2: What is the magnitude of the maximum current induced in the coil in the system described in Exploration 20.2?

Answer to Essential Question 20.2: To find the current of the largest magnitude, we apply Ohm's Law, using the induced emf of the largest magnitude. The resistance was given as 3.0 Ω in Exploration 20.2, so we get $I_{max} = \varepsilon_{max} / R = (0.075 \text{ V})/(3.0\ \Omega) = 0.025 \text{ A}$.

20-3 Lenz's Law and a Pictorial Method for Faraday's Law

Thus far we have discussed the fact that exposing a coil to a changing magnetic flux induces an emf in the coil. If there is a complete circuit, this emf gives rise to an induced current. In what direction is this induced current? The direction of the current also relates to the negative sign in Faraday's law. That negative sign is associated with a whole other law, Lenz's law.

Lenz's law: The emf induced by a changing magnetic flux tends to produce an induced current. The induced current produces a magnetic flux that acts to oppose the original change in flux.

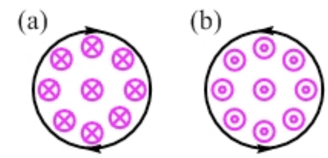


Figure 20.12: (a) A loop with a clockwise current gives rise to a magnetic field directed into the page within the loop. (b) The magnetic field within the loop is directed out of the page if the current is counterclockwise.

Let's go over a pictorial method for determining the direction of the induced current. As part of this method, recall from chapter 19 that, as shown in Figure 20.12, a current directed clockwise around a loop gives rise to a magnetic field directed into the page inside the loop, while a counterclockwise current produces a field directed out of the page in the loop. This can be confirmed with the right-hand rule. Curl the fingers on your right hand in the direction of the current, and your thumb points in the direction of the field inside the loop.

EXPLORATION 20.3 – A pictorial method for determining current direction

Step 1 – ***The loop in Figure 20.13 is moved from the Before position to the After position, closer to a long straight wire that carries current to the left. Sketch a diagram showing the magnetic field lines, produced by the current in the straight wire, that pass through the loop when the loop is in the Before position.***

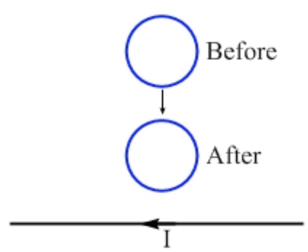


Figure 20.13: A conducting loop, above a long straight current-carrying wire, is moved from the Before position to the After position, closer to the wire.

Let's use the right-hand rule to find the direction of the field from the wire. Point the thumb on your right hand in the direction of the current. Then, curl your fingers on your right hand – they show the direction the field lines circle around the wire. This tells us that, above the wire, where the loop is, the field lines from the wire are directed into the page, as in Figure 20.14.

Figure 20.14: When the loop is in the Before position, the field from the long straight wire passes through the loop into the page.

Step 2 – ***Sketch a diagram showing the magnetic field lines, produced by the current in the straight wire, that pass through the loop when the loop is in the After position. If the flux changes from Before to After, show this on the diagram.*** The loop is above the wire in the After position, so the field is still directed into the page. However, the loop is now closer to the wire, where the field is larger, so we show more field lines on the diagram in Figure 20.15.

Figure 20.15: When the loop moves to the After position, coming closer to the wire where the field from the wire is stronger, we draw more field lines passing through the loop.

Step 3 – *Draw a "To Oppose" picture, with one field line representing the direction of the field the induced current in the loop creates to oppose the change in flux the loop experiences in moving from the Before position to the After position. Based on the direction of the field in the To Oppose picture, determine the direction of the induced current.* To oppose the increase in magnetic flux that occurs as the loop moves closer to the wire, the induced current in the loop creates a magnetic field in the opposite direction, out of the page. The pictorial method is qualitative, so we only need to draw one field line directed into the page in Figure 20.16.

Figure 20.16: To oppose the change in flux as the loop moves from Before to After, the loop creates a magnetic field out of the page with a counterclockwise induced current.

Using the right-hand rule, when the thumb is directed out of the page, the fingers curl counterclockwise, in the direction of the induced current.

Key ideas for the pictorial method: A pictorial method can be used to determine the direction of the induced current in a loop or coil that experiences a change in magnetic flux. The steps are:

1. Draw a Before picture, showing the field lines passing through before a change is made.
2. Draw an After picture, showing the field lines passing through after a change is made.
3. Draw a To Oppose picture, with a single field line to represent the direction of the field needed to oppose the change from the Before picture to the After picture. Then, apply the right-hand rule to find the direction of the induced current needed to produce this field.

Related End-of-Chapter Exercises: 18 – 20, 22.

EXAMPLE 20.3 – A quantitative analysis

Rank the four single-turn loops in Figure 20.17 based on the magnitude of their induced current, from largest to smallest. The loops are either moving into or out of a region of uniform magnetic field. The field is zero outside the region.

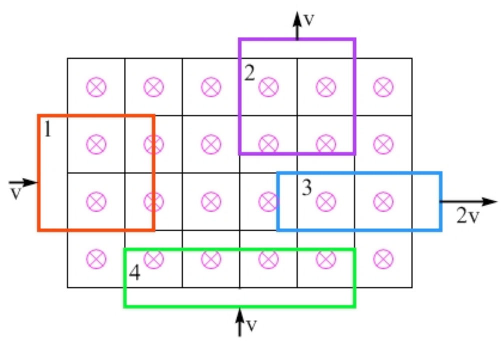


Figure 20.17: Four conducting loops, all of the same resistance, are moving with the velocities indicated either into (loops 1 and 4) or out of (loops 2 and 3) a region of uniform magnetic field.

SOLUTION

Combining Ohm's law with Faraday's law, we find that the magnitude of the current is given by:

$$I_{induced} = \frac{\varepsilon}{R} = \frac{1}{R}\left(\frac{\Delta(BA\cos\theta)}{\Delta t}\right)$$

$\cos\theta = 1$, and B does not change with time so: $I_{induced} = \frac{\varepsilon}{R} = \frac{B}{R}\left(\frac{\Delta A}{\Delta t}\right)$.

Writing the area in terms of the length L and width W: $I_{induced} = \frac{B}{R}\left(\frac{\Delta(LW)}{\Delta t}\right)$.

Defining L as the length of the loop perpendicular to the velocity, which is constant, then the magnitude of $\Delta W/\Delta t$ is the speed. The magnitude of the current is thus: $I_{induced} = BLv/R$.

The field and resistance are the same, so the induced current for the loops is proportional to the product of the speed multiplied by the length of the loop that is perpendicular to the velocity. This gives a ranking by current magnitude of 4 > 1 = 2 = 3. Loop 3 has half the length, perpendicular to its velocity, as loops 1 and 2, but makes up for that factor of two in its speed.

Essential Question 20.3: Find the direction of the induced current in the loops in Figure 20.17.

Answer to Essential Question 20.3: For loops 2 and 3, which are leaving the field, the magnetic flux into the page decreases as the fraction of the loop that is inside the field decreases. The pictorial method for loop 2 is shown in Figure 20.18. The To Oppose picture shows the field directed into the page, opposing the decrease in flux from Before to After, which requires an induced current directed clockwise. Similarly, loop 3 has a clockwise current.

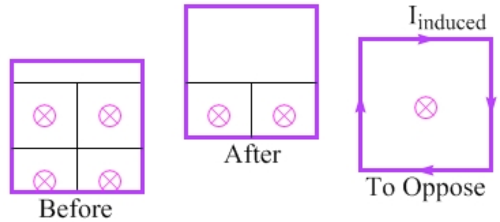


Figure 20.18: The pictorial method for loop 2, which tells us that loop 2 has an induced current that is directed clockwise.

For loops 1 and 4, which are entering the field, the Before and After pictures from Figure 20.18 switch positions. This gives a To Oppose picture with a magnetic field directed out of the page, requiring a counterclockwise current.

20-4 Motional emf

In each of the loops in Figure 20.17, the induced emf is associated with only one side of the rectangle, the side completely in the field, aligned perpendicular to the loop's velocity. Let's address this emf from another perspective.

EXPLORATION 20.4 – A metal rod moving through a magnetic field

As shown in Figure 20.19, a metal rod of length L is moving with a velocity $\vec{v}$ through a uniform magnetic field of magnitude B.

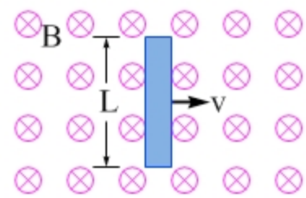


Figure 20.19: A metal rod moving through a magnetic field.

Step 1 – ***The rod has no net charge, but conduction electrons within the rod are free to move. First, assume that these electrons are moving through the field with the velocity of the rod. Apply the right-hand rule to determine the direction of the force these electrons experience.*** Hold the right hand above the page with the palm down and the fingers pointing to the right, in the direction of the velocity. When we curl the fingers they curl into the page, in the direction of the magnetic field. The thumb points up the page, showing the direction of the magnetic force on particles with positive charge. Electrons, which are negatively charged, experience a force in the opposite direction, down the page, leaving a net positive charge at the upper end of the rod. As shown in Figure 20.20, the moving rod acts like a battery.

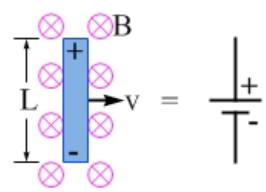


Figure 20.20: A conducting rod moving in a magnetic field acts like a battery, because of the separation of charge from the magnetic force acting on the rod's conduction electrons.

Step 2 –**Determine the effective emf of the rod. *As the rod becomes polarized, an electric field is set up in the rod. Show that the electric field gives rise to an electric force that is opposite to the magnetic force. Equate these two forces and, by treating the rod as a parallel-plate capacitor, determine the potential difference between the ends of the rod.*** The upper end of the rod is positive, so the electric field within the rod is directed down. An electron in the rod experiences an electric force that is opposite in direction to this electric field, because $\vec{F}_E = q\vec{E}$, and the charge on the electron is negative. Thus, electrons in the rod experience two forces, an electric force directed up and a magnetic force directed down. An equilibrium charge distribution is reached when these two forces balance, as shown in Figure 20.21.

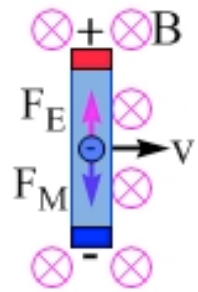


Figure 20.21: Equilibrium is reached when the electric force balances the magnetic force.

The magnetic force balances the electric force, so $qvB = qE$. The factors of q cancel, leaving $vB = E$. Treating the rod as a parallel-plate capacitor, the potential difference between the ends of the rod is $\Delta V = EL$. Solving for this potential difference, which we generally call a motional emf, ε, gives:

$$\Delta V = \varepsilon = -vBL. \qquad \text{(Equation 20.3: \textbf{Motional emf})}$$

The minus sign indicates that Lenz's law applies, and that the emf tends to produce a current that opposes any change in magnetic flux. Note that equation 20.3 applies when the rod, its velocity, and the direction of the magnetic field are all mutually perpendicular.

Key ideas for motional emf: A conductor moving with a velocity $\vec{v}$ through a magnetic field $\vec{B}$ has an induced emf across it given by $\varepsilon = -vBL$, where L is the length of the conductor that is perpendicular to both $\vec{v}$ and $\vec{B}$. The moving conductor thus acts like a battery.
Related End-of-Chapter Exercises: 9, 10, 28.

EXAMPLE 20.4 – Using a moving rod as a battery

Let's investigate what happens when we use the rod like a battery. We will connect the moving rod from Exploration 20.4 in a simple series circuit, by placing the rod on a pair of parallel conducting rails, as shown in Figure 20.22. The rod moves with negligible friction on the rails, which themselves have negligible resistance but are connected by resistor of resistance R. (a) If the rod moves to the right with a speed v in a field of magnitude B directed into the page, find an expression for the magnitude of the induced current. (b) Use the pictorial method from section 20-3 to find the direction of this current.

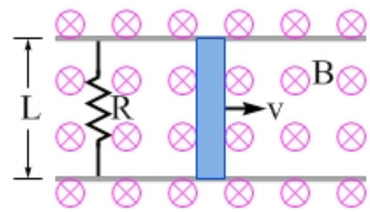


Figure 20.22: The system consisting of a conducting rod on frictionless rails. The rails are connected to one another through a resistor, and the rod moves through a magnetic field that is directed into the page.

SOLUTION

(a) In this situation we combine the equation for motional emf, $\varepsilon = -vBL$, with Ohm's law, $I = \Delta V / R$, to find that the induced current has a magnitude of:

$I = vBL / R$, matching the result from Example 20.3.

(b) As the rod moves to the right, the area of the conducting loop increases. Compared to the Before picture, the After picture shows the rod farther right, with more field lines passing through because the area of the loop has increased. As seen in Figure 20.23, the To Oppose picture shows field directed out of the page, opposing the extra into-the-page field lines in the After picture. To create a field out of the page, the induced current must be directed counterclockwise around the loop.

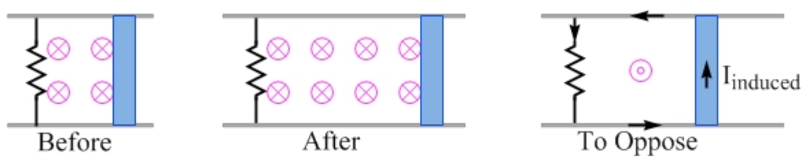


Figure 20.23: Applying the pictorial method to determine the direction of the induced current in the circuit.

Related End-of-Chapter Exercises: 29, 30, 53, and 55.

Essential Question 20.4: The moving rod in Example 20.4 experiences a magnetic force because of the interaction between the external magnetic field and the induced current passing through the rod. In what direction is this magnetic force? How is this direction consistent with Lenz's law?

Answer to Essential Question 20.4: The induced current is directed up through the rod, while the external magnetic field is directed into the page. Make sure you look at the external magnetic field, and not the field from the induced current! Applying the right-hand rule that goes with the expression for magnetic force, $F_M = ILB\sin\theta$, we find that the rod experiences a magnetic force directed to the left. Because the velocity of the rod is directed to the right, the magnetic force tends to slow the rod down, decreasing the time rate of change of flux. The tendency of the induced current to oppose the change in flux is completely consistent with Lenz's law.

20-5 Eddy Currents

Thus far, in this chapter, we have discussed induced currents set up in conducting loops and coils, and in moving rods connected in a circuit, in response to a changing magnetic flux. Something similar happens in solid pieces of conductor, in which a changing magnetic flux gives rise to what are called eddy currents. The meaning of the word "eddy" in this case is the same as its meaning when you refer to an eddy in a river, which is a swirl of water.

Eddy currents are swirling currents that are set up in conductors that are exposed to a changing magnetic flux. Consistent with Lenz's law, these swirling currents create their own magnetic field that tends to oppose the original change in flux.

Eddy currents can arise when there is relative motion between a conductor and a magnet. The relative motion tends to change the flux in the conductor, so by Lenz's law the eddy currents produce a magnetic field that acts to slow the motion. Magnetic interactions in ferromagnetic materials (like iron or steel) are often dominated by the strong attraction between the magnet and the ferromagnetic material. The effects of eddy currents are generally easier to observe in non-ferromagnetic materials like aluminum and copper. Such materials interact weakly with magnets that are at rest with respect to them, but the effects of eddy currents in such materials, which involve a changing magnetic flux, often produced by relative motion, can be dramatic.

Some examples of eddy currents include:

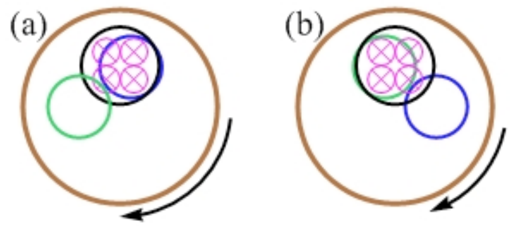
- **Train brakes.** Turning on an electromagnet to create a magnetic field that passes through part of a train wheel causes the wheel, and the train, to slow as different areas of the wheel move into and out of the field (see Exploration 20.5).
- **Magnetic damping on a balance.** Like the system in Figure 20.24, many triple-beam balances have an aluminum plate located near a magnet, to damp out (steadily reduce) the oscillations of the balance so it settles down quickly for a reading.
- **Dropping a magnet down an aluminum or copper tube.** Try this if you can. It is fascinating to look down the tube and see the magnet fall, almost in slow motion, as the eddy currents in the tube wall slow the magnet's fall.

Figure 20.24: A photograph of the two magnets (dark cylinders) used to damp out the oscillations of the aluminum plate that is part of an apparatus used in a Coulomb's law experiment. Photo credit: A. Duffy.

EXPLORATION 20.5 – Using eddy currents to stop a train

Figure 20.25 shows the wheel of a train. When the driver of the train applies the brakes, an electromagnet sets up a magnetic field, directed into the page, which passes through a circular region (in black) at the top of the wheel. Eddy currents are set up in a part of the wheel (in green) that is entering the field, as well as in a part of the wheel (in blue) that is leaving the field.

Figure 20.25: Two views of a train wheel exposed to a magnetic field at the top. The wheel is rotating clockwise. View (b) is at a later time than view (a), so the wheel is moving slower in (b).



Step 1 – ***Apply the pictorial method to the region of the wheel moving into the field, to determine the direction of the eddy currents induced in this region by the changing flux.*** This region is moving into the field, so the After picture shows more field lines passing through the region than the Before picture does (see Figure 20.26). The To Oppose picture shows a field line directed out of the page, opposing the change in flux. By the right-hand rule, the induced eddy currents in this region must swirl counterclockwise to create a field out of the page.

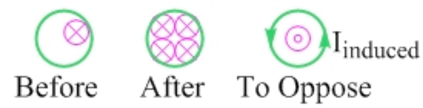


Figure 20.26: The pictorial method applied to the region that is entering the magnetic field.

Step 2 – ***Consider the interaction between the eddy currents in the region moving into the field and the external magnetic field. Does this interaction give rise to a torque on the wheel? If so, in what direction is the torque?*** Figure 20.27 shows a magnified view of the counterclockwise current interacting with the field. The right-hand part of the swirling current, directed up, is in the magnetic field. Applying the right-hand rule shows that there is a force directed to the left on this upward current. The left-hand part of the eddy current is outside the field, so there is no force on that part. The force to the left produces a counterclockwise torque on the wheel, relative to an axis through the wheel's center, acting to slow the wheel's clockwise motion.

Figure 20.27: The magnetic force acting on the eddy current in the region moving into the field produces a torque that acts to slow down the wheel.

Note that we will consider the region moving out of the field in Essential Question 20.5.

Key idea: Eddy currents can give rise to forces or torques that tend to slow relative motion between a conductor and a magnet. **Related End-of-Chapter Exercises: 56 and 57.**

Superconductors and the Meissner effect

One application of eddy currents is the levitation of a magnet above a superconductor, which is a material that has no electrical resistance. Superconductors generally exhibit zero resistance only at very low temperatures. The black disk at the bottom of Figure 20.28 is known as a high-temperature superconductor, because it has no resistance when it is cooled by liquid nitrogen, at a relatively high temperature (at least for superconductors!) of about 77 K (–196°C). When the magnet is brought close to the superconductor, the changing flux gives rise to eddy currents in the superconductor. The eddy currents, which can flow forever because there is no resistance, set up a magnetic field that exactly cancels the field from the magnet (this exclusion of magnetic field from the superconductor is known as the Meissner effect). The magnetic field from the eddy currents in the superconductor can support the magnet without the magnet and the superconductor being in contact.

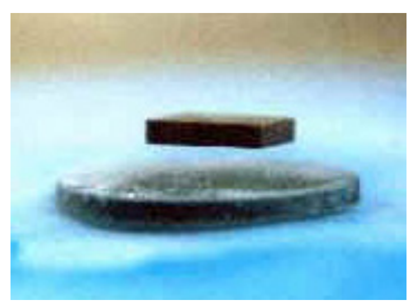


Figure 20.28: A magnet being levitated by the magnetic field produced by eddy currents in a superconducting disk, which is cooled by liquid nitrogen. Photograph from http://www.lbl.gov

Essential Question 20.5: Repeat steps 1 and 2 of Exploration 20.5 for the region of the wheel (see Figure 20.25) that is leaving the region of magnetic field.

Answer to Essential Question 20.5: For the region that is moving out of the field, the After picture shows fewer field lines passing through the region than the Before picture does (see Figure 20.29). The To Oppose picture shows a field line directed into the page, opposing the change in flux. By the right-hand rule, the induced eddy currents in this region must swirl clockwise to create a field into the page.

Figure 20.29: The pictorial method applied to the region that is leaving the magnetic field.

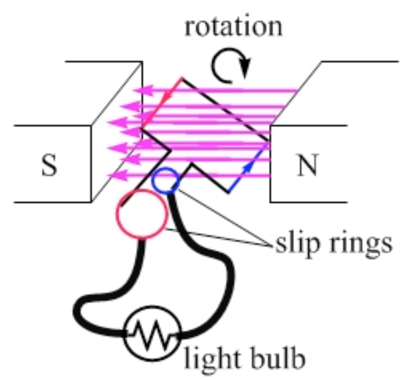
Figure 20.30 shows a magnified view of the clockwise current interacting with the field. The left-hand part of the swirling current, directed up, is in the magnetic field. Applying the right-hand rule shows that there is a force directed to the left on this upward current. The right-hand part of the eddy current is outside the field, so there is no force on that part. The force to the left produces a counterclockwise torque on the wheel, relative to an axis through the wheel's center, acting to slow the wheel's clockwise motion. Note that no matter which region we consider, one entering the field or one leaving the field, the torque always acts to slow the wheel.

Figure 20.30: The magnetic force acting on the eddy current in the region leaving the field produces a torque that acts to slow down the wheel.

20-6 Electric Generators

Faraday's Law, and the induced emf and induced current associated with it, is one of the most practical ideas in physics — it lies at the heart of most devices that generate electricity. The basic components that make up an electric generator are quite simple. All we need is a conducting loop exposed to a changing magnetic flux.

In section 19-6, we spent some time investigating a DC motor, which is a conducting loop in a magnetic field. When current is sent through the loop, the magnetic field exerts a torque on the loop that makes the loop spin, transforming electrical energy into mechanical energy. An electric generator does exactly the opposite, transforming mechanical energy into electrical energy. We can use the same device, a loop in a magnetic field, as a motor or a generator.



As the loop in Figure 20.31 rotates, the magnetic flux through the loop changes, giving rise to an induced emf. If we completed the circuit by connecting the ends of the loop directly to an electrical device, like a light bulb, the induced emf gives rise to an induced current that lights the bulb, but the wires would twist, either stopping the loop or breaking the wires. One solution to this problem is to have the wires coming from each side of the loop rub against the inside of fixed slip rings, which are then connected to the bulb. Each wire maintains electrical contact with one slip ring as the loop rotates.

Figure 20.31: The basic components of an electric generator are a conducting loop and a magnetic field. Rotating the loop changes the magnetic flux through the loop, giving rise to an induced emf.

Another solution to this problem, which is used in some electricity generating plants, is to keep the loop fixed and spin the magnets around the loop. This has the same effect as spinning the loop through the magnetic field.

EXPLORATION 20.6 – The form of the electric signal from a generator

Step 1 – ***The loop in Figure 20.31 rotates at a constant angular speed ω. Plot graphs of the flux through the loop, and the corresponding induced emf, as a function of time. Define the positive direction such that the flux at t = 0 is positive. Plot a second set of graphs to show what happens when the rotation rate doubles.*** As the loop rotates, the magnetic flux oscillates. We can understand the cosine dependence of the flux graph, shown at the top in Figure 20.32, by starting with the equation for magnetic flux, $\Phi = BA\cos\theta$. B, the magnetic field, and A, the area of the loop, are constant, but θ changes. Using a relationship from rotational kinematics, the angle between the field and the loop's area vector is given by $\theta = \omega t$. This gives the flux equation a form which shows the time dependence, and the cosine nature, explicitly: $\Phi(t) = BA\cos(\omega t)$.

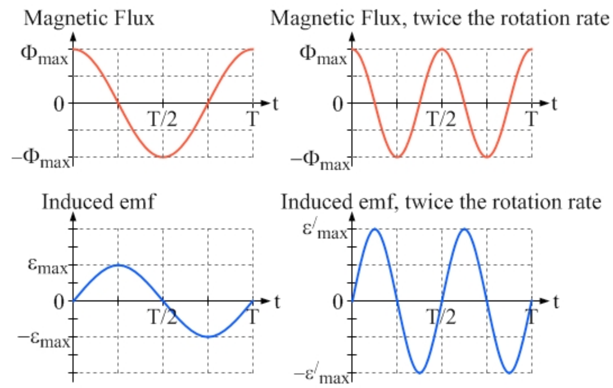


Figure 20.32: For the magnetic flux and induced emf graphs on the right, the rotation rate is twice as large as in the graphs on the left.

The induced emf is proportional to the negative of the slope of the flux versus time graph. Thus, the emf is zero when the flux graph reaches a positive or negative peak. When the flux is decreasing, the flux graph has a negative slope, so the induced emf is positive. The emf peaks when the flux passes through zero, and the flux graph has a large slope.

If the rotation rate increases, the maximum flux does not change but the flux changes in half the time, doubling the slope on the flux graph. This produces a corresponding doubling in the peak emf. Thus, the peak emf is proportional to the angular speed of the magnets.

Step 2 – ***Combine Faraday's Law with results from step 1 to find an expression for the emf induced in the loop as a function of time.*** If the loop has N turns, applying Faraday's Law gives:

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{\Delta\left[BA\cos(\omega t)\right]}{\Delta t}$$, where we have used the flux expression from step 1.

The field and area are constants, so they can be moved out front: $\varepsilon = -NBA\dfrac{\Delta\left[\cos(\omega t)\right]}{\Delta t}$.

Simplifying, we get: $\varepsilon = \omega NBA\,\sin(\omega t)$. This last step is easy to show using calculus. However, we saw both the sine dependence, and the fact that the induced emf is proportional to the angular speed, in step 1, so we can be confident in the result even without using calculus.

Key idea: As a function of time, the emf of an electric generator, in which either the loop or the magnetic field spins at a constant angular velocity $\vec{\omega}$, oscillates sinusoidally. In other words, the electric generator puts out alternating current. Expressed as an equation, the emf is:

$\varepsilon = \omega NBA\sin(\omega t) = \varepsilon_{max}\sin(\omega t)$. (Equation 20.4: **emf from an electric generator**)

The peak voltage, ε_{max}, is the product of ω, N (the number of turns in the loop), B (the strength of the magnetic field), and A (the loop area). **Related End-of-Chapter Exercises: 31 – 34.**

Essential Question 20.6: For generators in North America, which provide 60 Hz AC for a standard wall socket, what is ω , the magnitude of the angular velocity of the rotation?

Answer to Essential Question 20.6: In this case, we can use the relationship $\omega = 2\pi f$. If the frequency is 60 Hz, the angular speed of the magnets in a typical electricity generating facility is 120π , or 377, rad/s. In Europe, the frequency is 50 Hz, with a corresponding decrease in ω.

20-7 Transformers and the Transmission of Electricity

In this day and age, when we rely so heavily on electricity in our daily lives, it is hard to imagine a time when electricity was not so widely available. It was only in the late 1880's, not so long ago when measured against thousands of years of human history, that the "War of Currents" took place. The War of Currents was the battle between Thomas Edison and his General Electric company, who wanted direct current (DC) to be the standard for electricity distribution systems, and Nikola Tesla and George Westinghouse, the proponents of alternating current (AC) as the basis of such systems. The battle even included the electrocution of an elephant, in an Edison-led demonstration of the dangers of AC. The battle lasted a few years before the superiority of AC won the day, not to mention the contract for the large hydroelectric power plant at Niagara Falls.

What makes AC so superior to DC for electricity distribution? As we learned in section 20-6, it is very easy to generate AC. However, the big advantage of AC over DC is that it is extremely easy to change the voltage of AC, with very little loss of energy, while it is much more difficult, and less efficient, to do so for DC. With AC, this transformation of voltage is accomplished with a device called a transformer, which exploits Faraday's Law in its operation.

A transformer is quite a simple device (see Figure 20.33), typically consisting of two coils of wire wrapped around a core made of ferromagnetic material. The role of the core is to ensure that all the magnetic flux, generated by current passing through the primary coil, passes through the secondary coil. If this flux changes, because the current in the primary (*p*) coil changes, then an emf and a current are induced in the secondary (*s*). The two coils are exposed to the same changing flux, so we can relate them using Faraday's Law, which we re-arrange to read:

$$\frac{\Delta\Phi}{\Delta t} = -\frac{\varepsilon_p}{N_p} = -\frac{\varepsilon_s}{N_s} .$$

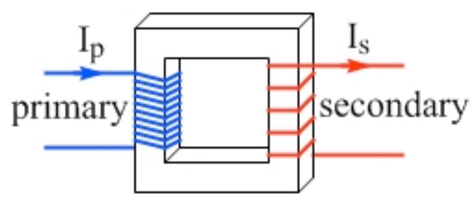


Figure 20.33: A transformer consists of two coils, the primary and the secondary, wrapped around a ferromagnetic core.

Recall from Chapter 18 that electrical power is the product of the potential difference and the current. In an ideal case there is no loss of energy, or power, in the transformer, so the power in the primary, $\varepsilon_p I_p$, is equal to the power in the secondary, $\varepsilon_s I_s$. The power relationship and the relationship from Faraday's Law, above, are combined into one equation below, where potential difference ΔV has replaced the emf ε .

If we assume that no energy is lost in the transformation process, we say that the transformer is ideal. For an ideal transformer, with alternating current in the primary, we have:

$$\frac{\Delta V_p}{\Delta V_s} = \frac{I_s}{I_p} = \frac{N_p}{N_s} .$$ (Equation 20.5: **Relations for an ideal transformer**)

The subscripts *p* and *s* stand for primary and secondary, respectively. ΔV represents the potential difference across the coil, *I* is the current, and *N* is the number of turns in the coil. Equation 20.5 is generally used to relate peak values of current or voltage in the primary to their corresponding peak values in the secondary, or to relate rms values in one coil to rms values in the other coil.

Note that transformers require a changing magnetic flux to generate a current in the secondary coil. Such a changing magnetic flux is provided by alternating current running through the primary, generating alternating current in the secondary.

What is a transformer good for? In general, transformers are used to change the voltage from a wall socket into a different voltage, which could be higher or lower, for use by a particular device. Some devices, such as microwave ovens and cathode ray tube televisions, require higher voltages than the 120 V rms that is provided by a wall socket in North America. Such devices have a **step-up transformer**, in which the secondary coil has a larger number of turns than the primary, to increase the voltage to the required level. Many devices also require a constant voltage, so the transformers also convert the alternating current to direct current.

Other devices, such as computers and clock radios, require a voltage such as 12 V, much less than the 120 V rms from a wall socket, to operate. The power cord in these devices is thus connected to a **step-down transformer**, in which the secondary coil has a smaller number of turns than the primary, to decrease the voltage. Note that, by energy (or power) conservation, decreasing the voltage is associated with an increase in current. In step-up transformers, on the other hand, the increase in voltage is accompanied by a corresponding decrease in current.

EXPLORATION 20.7 – Transformers in the electricity distribution system

A particular electricity generating facility generates electricity at the rate of 2.4 MW, with an rms current of 1000 A at an rms voltage of 2.4 kV.

Step 1 – ***If the electricity is sent through a cable (known as a transmission line) that has a resistance of 10 Ω on its way to New York City, determine the power lost during the transmission process*.** Here we can apply one version of the power equation from chapter 18, $P = I^2 R = (1000 \text{ A})^2 (10\ \Omega) = 10 \text{ MW}$. This is clearly ridiculous, because this power is more than 4 times larger than the power generated by the plant. The bottom line is that essentially all the electrical energy would be dissipated in the transmission line.

Step 2 – ***If, instead, the voltage is transformed to 240 kV before the electricity is sent along the transmission line, what is the power lost in the transmission process?*** Applying equation 20.5, increasing the voltage by a factor of 100 produces a corresponding decrease by a factor of 100, to 10 A, in the current. With 10 A of current in the transmission line, the power lost now is only $P = I^2 R = (10 \text{ A})^2 (10\ \Omega) = 1000 \text{ W}$. This is why power companies transmit electricity over long distances at high voltages, to minimize the current, thereby minimizing transmission losses.

Step 3 – ***The voltage is ultimately transformed back down to 120 V in New York City. What is the value of the rms current?*** Neglecting the power lost in step 2, stepping the voltage down by a factor of 2000, from 240 kV to 120 V, increases the current by this same factor of 2000, from 10 A to 20000 A, enough to meet the needs of a large number of residential customers.

Key ideas: Power companies make extensive use of both step-up and step-down transformers in the process of delivering electricity. **Related End-of-Chapter Exercises: 36 – 40, 60.**

Essential Question 20.7: A particular transformer has a primary coil with 100 turns and a secondary coil with 200 turns. If the primary coil has a constant potential difference of 20 V and a constant current of 5 A, what are the values of the potential difference across, and current in, the secondary? Assume the transformer is ideal.

Answer to Essential Question 20.7: Because the current in the secondary coil is constant, the magnetic flux through the secondary coil is constant. Because the flux through the secondary coil does not change, there is no induced emf in the secondary, and thus there is also no current. Transformers work very well for alternating current, but they do not work for direct current.

Chapter Summary

Essential Idea: Electromagnetic Induction.

One of the most practical applications of physics, the generation of electricity, relies on electromagnetic induction. Exposing a conducting loop to a changing magnetic flux (a change in the number of magnetic field lines passing through the loop) will induce an emf, or voltage, in the loop. In a complete circuit, this induced emf will gives rise to an induced current in the loop.

Magnetic Flux

Magnetic flux is a measure of the number of magnetic field lines passing through an area. The symbol we use for flux is the Greek letter capital phi, Φ.The equation for magnetic flux is:

$\Phi = BA\cos\theta$, (Equation 20.1: **Magnetic flux**)

where θ is the angle between the magnetic field $\vec{B}$ and the area vector $\vec{A}$.

Faraday's Law of Induction

Exposing a loop or coil to a ***changing*** magnetic flux gives rise to a voltage, called an **induced emf.** Following Ohm's law, the induced emf gives rise to an **induced current** in the loop or coil. The emf induced by a changing magnetic flux in each turn of a coil is equal to the time rate of change of that flux. Thus, for a coil with *N* turns, the net induced emf is given by:

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{\Delta(BA\cos\theta)}{\Delta t}.$$ (Eq. 20.2: **Faraday's Law of Induction**)

In many cases, graphing the magnetic flux as a function of time can be helpful because the induced emf is proportional to the negative of the slope of the graph of flux versus time.

Lenz's Law

Lenz's Law is associated with the minus sign in Faraday's Law. Lenz's Law states that the emf induced by a changing magnetic flux tends to produce an induced current. The induced current produces a magnetic flux that acts to oppose the original change in flux.

A pictorial method for applying Lenz's Law to determine the direction of induced current

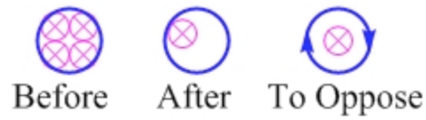
A pictorial method can be used to determine the direction of the induced current in a loop or coil that experiences a change in magnetic flux. The steps are:

1. Draw a Before picture, showing the field lines passing through before a change is made.
2. Draw an After picture, showing the field lines passing through after a change is made.
3. Draw a To Oppose picture, with a single field line to represent the direction of the field needed to oppose the change from the Before picture to the After picture. Then, apply the right-hand rule to find the direction of the induced current needed to produce this field.

Applying the pictorial method: an example

The magnetic field, directed into the page through a wire loop, is decreasing in magnitude. In which direction is the current induced in the loop?

Figure 20.29: The situation described here is similar to that of Figure 20.29. In the "To Oppose" picture, the field created by the induced current must also be directed into the page, to oppose the loss in field lines into the page the loop experiences.



Motional emf

A conductor moving with a velocity $\vec{v}$ through a magnetic field $\vec{B}$ has an induced emf, generally referred to as motional emf, across it given by:

$$\varepsilon = -vBL .$$ (Equation 20.3: **Motional emf**)

where L is the length of the conductor that is perpendicular to both $\vec{v}$ and $\vec{B}$. The moving conductor thus acts like a battery. Note that in many motional emf situations, and in other induced emf situations, Ohm's Law ($\varepsilon = IR$) is often used to determine the current resulting from the motional or induced emf.

Eddy currents

Eddy currents are swirling currents that are set up in conductors that are exposed to a changing magnetic flux. Consistent with Lenz's law, these swirling currents create their own magnetic field that tends to oppose the original change in flux. A practical application of eddy currents is in train brakes, in which braking forces arise from the interaction of the eddy currents in the train wheel and the magnetic field of an electromagnet. The field is turned on, and the eddy currents are set up, only when the operator of the train applies the brakes.

Electric generators

Alternating current can be generated very easily, simply by spinning a conducting loop at a constant rate in a uniform magnetic field. To avoid wires being twisted, in some cases the magnets producing the field are rotated around a conducting loop that is held fixed.

As a function of time, the emf of an electric generator, in which either the loop or the magnetic field spins at a constant angular velocity $\vec{\omega}$, oscillates sinusoidally. Expressed as an equation, the emf is:

$$\varepsilon = \omega NBA \sin(\omega t) = \varepsilon_{max} \sin(\omega t) .$$ (Equation 20.4: **emf from an electric generator**)

The peak voltage, ε_{max}, is the product of ω, N (the number of turns in the loop), B (the strength of the magnetic field), and A (the area of the loop).

Transformers

A transformer is a device for changing the voltage of an alternating current (AC) signal from one value to a higher or lower value. Transformers usually consist of two coils wrapped around a ferromagnetic core. An emf is induced in the secondary coil by a changing flux produced by the changing current in the primary coil. If no energy is lost in the transformation process, we say that the transformer is ideal.

$$\frac{\Delta V_p}{\Delta V_s} = \frac{I_s}{I_p} = \frac{N_p}{N_s} .$$ (Equation 20.5: **Relations for an ideal transformer)**

The subscripts p and s stand for primary and secondary, respectively. ΔV represents the potential difference across the coil, I is the current, and N is the number of turns in the coil. Equation 20.5 is generally used to relate peak values of current or voltage in the primary to their corresponding peak values in the secondary, or to relate rms values in one coil to rms values in the other coil.

End-of-Chapter Exercises

Exercises 1 – 12 are primarily conceptual questions designed to see whether you understand the main concepts of the chapter.

1. The four areas in Figure 20.34 are in a magnetic field. The field has a constant magnitude, but it is directed into the page on the left half of the figure and out of the page in the right half. (a) Rank the areas based on the magnitude of the net flux passing through them, from largest to smallest. (b) Defining out of the page to be the positive direction for magnetic flux, rank the areas based on the net flux, from most positive to most negative.

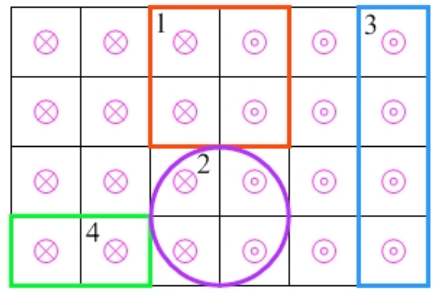


Figure 20.34: Four different regions in a magnetic field. The field is directed into the page in the left half of the figure and out of the page in the right half. For Exercises 1 and 2.

2. Repeat Exercise 1, but now the magnitude of the magnetic field in the left half of the figure is triple the magnitude of the field in the right half.

3. At a particular instant in time, there is no magnetic field passing through a conducting loop. (a) Could the emf induced in the loop be zero at that instant? Explain. (b) Could the emf induced in the loop be non-zero at that instant? Explain.

4. As shown in Figure 20.35, four identical loops are placed near a long straight wire that carries a current to the right. The wire is in the same plane as the loops. (a) Rank the loops based on the magnitude of the net flux passing through them, from largest to smallest. (b) Defining out of the page to be the positive direction for magnetic flux, rank the loops based on the net flux, from most positive to most negative.

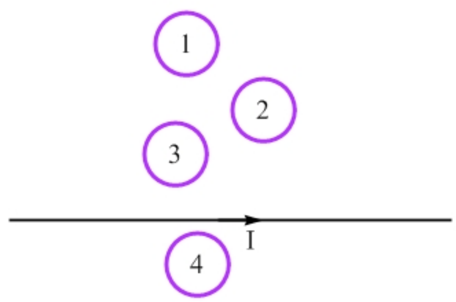


Figure 20.35: Four identical loops near a long straight wire carrying current to the right, for Exercise 4.

5. A long straight wire carrying current out of the page passes through the middle of a circular conducting loop that is in the plane of the page, as shown in Figure 20.36. In what direction is the induced current in the loop when the current in the long straight wire is increasing in magnitude?

Figure 20.36: A long straight wire passes through the center of a conducting loop. The current in the long straight wire is directed out of the page, and is increasing in magnitude. For Exercise 5.

6. Rank the four loops in Figure 20.37 based on (a) the magnitude of the induced current, from largest to smallest, and (b) the induced current, from largest clockwise to largest counterclockwise. The loops, which all have the same resistance, are moving with the velocities indicated either into (loops 1 and 4) or out of (loops 2 and 3) a region of magnetic field. The field has the same magnitude within the rectangular region, but is directed into the page in the left half of the region and out of the page in the right half.

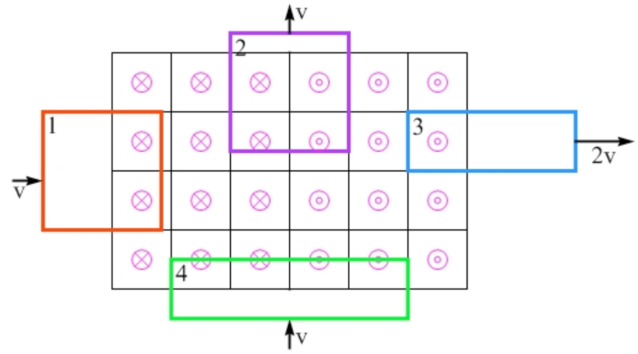


Figure 20.37: Four conducting loops, all of the same resistance, are moving with the velocities indicated either into (loops 1 and 4) or out of (loops 2 and 3) a region of magnetic field, for Exercise 6.

7. A square conducting loop located below a long straight current-carrying wire is moved away from the wire, as shown in Figure 20.38. While the loop is moving, the induced current in the loop is observed to be directed clockwise around the loop. In which direction is the current in the long straight wire?

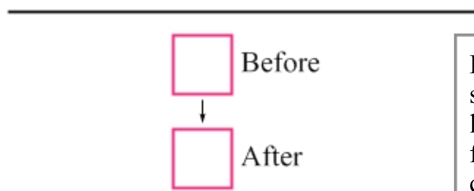


Figure 20.38: A square conducting loop is moved away from a long straight current-carrying wire, for Exercise 7.

8. A conducting loop is moved closer to a long straight wire, as shown in Figure 20.39. The current in the wire is constant, and directed out of the page, while the loop is in the plane of the page. While the loop is moving toward the wire, in what direction is the induced current in the loop? Explain.

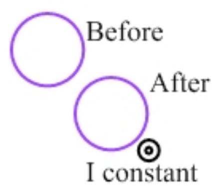


Figure 20.39: A conducting loop is moved closer to a long straight wire that carries a constant current directed out of the page, for Exercise 8.

9. Three conducting rods are moving with the velocities shown in Figure 20.40 through a region of uniform magnetic field that is directed into the page. You measure the motional emf of each bar with a voltmeter connected across the long dimension of each rod. For instance, for rods 1 and 3 you measure the potential difference between the right end and the left end of the rod. Rank the rods based on the magnitude of the potential difference you measure.

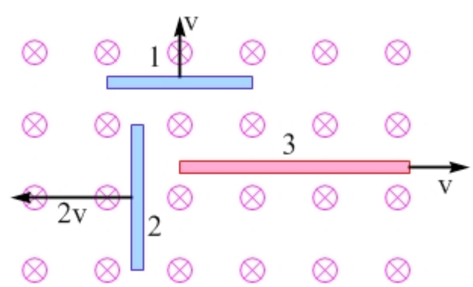


Figure 20.40: Three conducting rods are moving through a region of uniform magnetic field. The velocity and orientation of each rods is shown on the diagram. For Exercises 9 and 10.

10. Return to the situation shown in Figure 20.40 and described in Exercise 9. Which of the following is at a higher potential, considering the motional emf? (a) The left end of rod 1 or the right end of rod 1? (b) The upper end of rod 2 or the lower end of rod 2? (c) The left end of rod 3 or the right end of rod 3?

11. As shown in part (a) of Figure 20.41, a 12-volt battery is connected through a switch to the primary coil of a transformer. The secondary coil, which has fewer turns than the primary, is connected to a resistor. The switch is initially open. When the switch is closed, which of the following statements correctly describes the potential difference across the resistor? Explain your choice.
    - A constant potential difference of 12 V.
    - A non-zero constant potential difference that is smaller than 12 V.
    - A constant potential difference that is larger than 12 V.
    - A non-zero potential difference when the switch closes that quickly drops to zero.
    - The potential difference is zero the whole time.

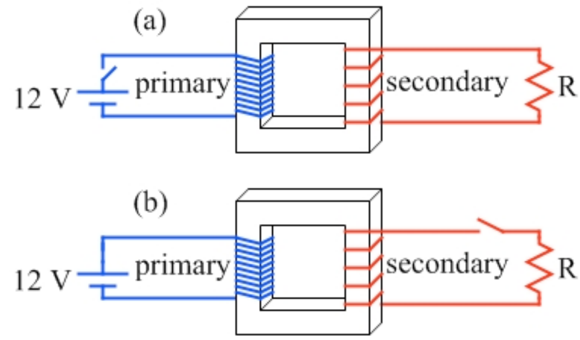


Figure 20.41: In (a), a 12-volt battery is connected to the primary coil of a transformer through a switch that is initially open. The secondary coil is connected to a resistor. In (b), the switch has been moved to the secondary side of the transformer.

12. Repeat Exercise 11, but use part (b) of Figure 20.41, in which the switch is moved to the secondary side of the transformer.

Exercises 13 – 17 deal with Faraday's Law.

13. A circular wire loop, with a single turn, has a radius of 12 cm. The loop is in a magnetic field that is decreasing at the rate of 0.20 T/s. At a particular instant in time, the field has a magnitude of 1.7 T. At that instant, determine the emf induced in the loop if (a) the field is directed perpendicular to the plane of the loop, (b) the field is directed parallel to the plane of the loop, (c) the angle between the field and the loop's area vector is 30°.

14. A coil of wire is connected to a galvanometer, as shown in Figure 20.42. The galvanometer needle deflects left when current is directed to the left through the coil; is vertical when there is no current, and deflects right when current is directed to the right. When a magnet, with its north pole closer to the magnet, is moved toward the right end of the coil, the galvanometer needle deflects to the left. In which direction will the needle deflect in the following cases? In all cases, the axis of the magnet coincides with the axis of the coil. (a) The magnet remains at rest inside the coil, with the south pole sticking out of the right end of the magnet. (b) The magnet, with its south pole closer to the magnet, is moved to the right away from the coil. (c) The magnet, with its south pole closer to the coil, remains at rest, while the coil is moved toward the magnet. The magnet is to the right of the coil at all times.

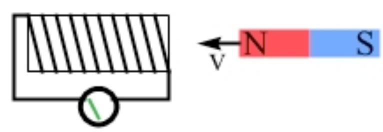
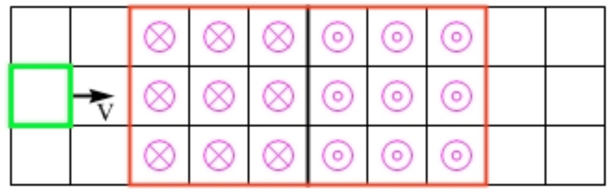


Figure 20.42: A coil, galvanometer, and magnet, for Exercise 14.

15. Figure 20.43 shows a square loop traveling at a constant velocity of 10 cm/s to the right through a region of magnetic field. The field is confined to the outlined rectangular region. The field is directed into the page in the left half of this region, and out of the page in the right half of this region. The field has the same magnitude at all points within the region. The small squares on the diagram measure 10 cm × 10 cm, and the loop is shown at $t = 0$. Rank the following times based on the magnitude of the emf induced in the loop as it moves through the field: $t_A = 0.5$ s; $t_B = 1.5$ s; $t_C = 4.5$ s; and $t_D = 6.0$ s.

Figure 20.43: A square loop moves through a magnetic field that is confined to the outlined rectangular region outlined. The magnetic field is directed into the page in the left half of this region, and out of the page in the right half. For Exercises 15 – 17.



16. Return to the situation described in Exercise 15, and shown in Figure 20.43. The loop is made from a single turn, and has a resistance of 6.0 Ω. The magnitude of the magnetic field at all locations in the region outlined in red is 3.0 T. (a) Sketch a motion diagram for the loop, showing its location at regular intervals. (b) Defining out of the page to be the positive direction for flux, draw a graph of the magnetic flux through the loop as a function of time. (c) Draw a graph of the emf induced in the loop as a function of time. (d) Find the maximum magnitude of the current induced in the loop as the loop moves through the magnetic field.

17. Return to the situation described in Exercises 15 and 16, and shown in Figure 20.43. If the loop's speed is increased by a factor of 3, do any of the following change as the loop moves through the field? Explain. (a) The maximum magnitude of the loop's magnetic flux. (b) The maximum magnitude of the induced emf. (c) The peak current in the loop.

Exercises 18 – 22 are designed to give you practice applying the pictorial method of solving problems involving Faraday's law.

18. As shown in Figure 20.44, a conducting loop is near a long straight wire that carries current to the right. In case 1, the current is increasing in magnitude. In case 2, the current is decreasing in magnitude. (a) If the magnitude of the induced current in the loop is the same in both cases, what does this tell us about the current in the two cases? (b) What is the direction of the induced current in the two cases? Justify your current directions by drawing the sets of three pictures associated with the pictorial method.

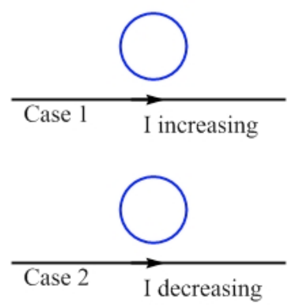


Figure 20.44: Two cases involving a conducting loop near a long straight wire, for Exercise 18.

19. Much like the situation shown in Figure 20.44, a conducting loop is placed above a long straight wire. The loop does not move with respect to the wire, while the current in the wire can be directed either left or right and may be increasing, decreasing, or constant. At a particular instant in time, the induced current in the loop is directed clockwise. (a) Could the current in the wire be directed to the right at this instant? Explain using the pictorial method. (b) Could the current in the wire be directed to the left at this instant? Explain using the pictorial method.

20. A square conducting loop is placed above a long straight wire that carries a constant current to the right. The loop can be moved along one of the three paths shown in Figure 20.45. (a) Rank the paths based on the magnitude of the current induced in the loop, assuming the speed of the loop is the same for each path. (b) Draw the set of three pictures associated with the pictorial method, and use the pictures to determine the direction of the current induced in the loop when it follows path 1. Repeat the process for (c) path 2, and (d) path 3.

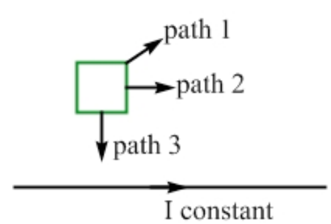


Figure 20.45: A square conducting loop is placed above a long straight wire that carries a constant current to the right. The loop can be moved along one of the three paths shown. For Exercise 20.

21. A conducting loop is placed exactly halfway between two parallel wires, each carrying a current to the right, as shown in Figure 20.46. At a particular instant in time, the currents in the wires have the same magnitude, but these currents may be increasing or decreasing. At this instant, we observe that there is a clockwise induced current in the loop. (a) Could one of the two currents be neither increasing or decreasing at this instant? Explain. (b) Could both currents be increasing in magnitude (possibly at different rates) at this instant? Explain. (c) Could one current be increasing while the other is decreasing at this instant? Explain.

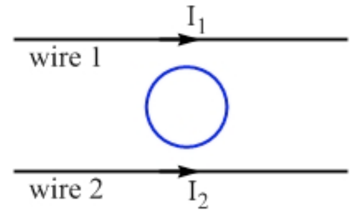


Figure 20.46: A conducting loop is located halfway between two wires that carry current to the right. At a particular instant, the currents in the two wires have the same magnitude. For Exercise 21.

22. Figure 20.47 shows a particular After picture, showing the field lines passing through a conducting loop After some change is made in the external field, along with six possible Before pictures and two possible To Oppose pictures. (a) For the To Oppose picture P, in which direction is the induced current in the loop? (b) For the To Oppose picture Q, in which direction is the induced current in the loop? (c) Which of the Before pictures could go with the After picture and the To Oppose picture P? Select all that apply. (d) Which of the Before pictures could go with the After picture and the To Oppose picture Q? Select all that apply.

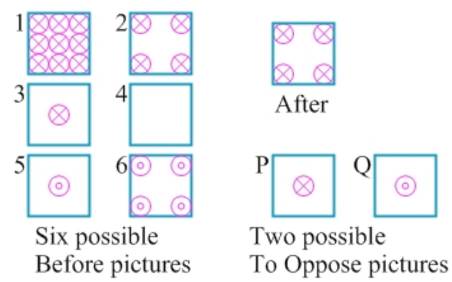


Figure 20.47: Six possible Before pictures, a particular After picture, and two possible To Oppose pictures, for the situation of a square conducting loop in a magnetic field. For Exercise 22.

Exercises 23 – 27 are designed to give you practice working with graphs of magnetic flux versus time in induced emf situations.

23. Consider the graph of magnetic flux through a conducting loop, as a function of time, shown in Figure 20.48. (a) At which time, $t = 10$ s, $t = 20$ s, or $t = 30$ s, does the induced emf have the largest magnitude? Explain. (b) Assuming the loop has a single turn, what is the magnitude of the induced emf at $t = 20$ s?

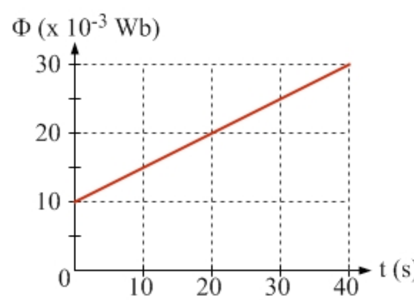


Figure 20.48: A graph of the magnetic flux passing through a particular conducting loop as a function of time, for Exercise 23.

24. The graph in Figure 20.49 shows the magnetic flux through a conducting loop as a function of time. The units of flux are webers, which are equivalent to tesla meters^2. The flux in the loop changes because the magnetic field passing through the loop changes. The loop itself is stationary. Compare the magnitude of the current induced in the loop at the following times. For each pair of times, state at which times the induced current has a larger magnitude, and explain your answer. (a) 5 s and 10 s. (b) 5 s and 18 s. (c) 5 s and 25 s. (d) 5 s and 40 s. (e) 25 s and 40 s.

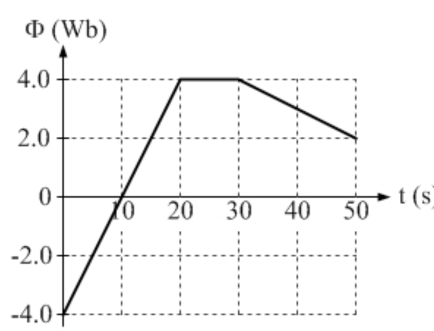


Figure 20.49: A graph showing the magnetic flux passing through a conducting loop as a function of time, for Exercises 24 – 26.

25. Return to the situation described in Exercise 24, and the graph in Figure 20.49. (a) If the conducting loop has a single turn, plot a graph of the emf induced in the loop as a function of time. Use the graph to answer the following questions. If the induced current at $t = 5$ s is directed clockwise, in what direction is the induced current in the loop at (b) $t = 10$ s? (c) $t = 15$ s? (d) $t = 40$ s?

26. Return to the situation described in Exercise 24, and the graph in Figure 20.49. The loop has a resistance of 0.10 Ω, an area of 20 m^2, and consists of a single turn. (a) What is the magnitude of the magnetic field passing through the loop at $t = 15$ s? Assume that the field is uniform and the direction of the field is perpendicular to the plane of the loop. (b) What is the magnitude of the current in the loop at $t = 15$ s?

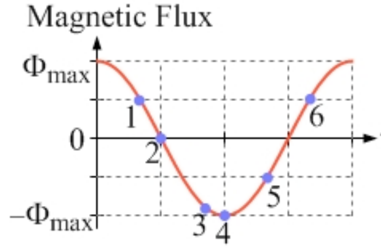


Figure 20.50: The graph shows the magnetic flux through a particular loop, which is at rest, as a function of time. Six points are labeled on the graph. For Exercise 27.

27. Figure 20.50 shows a graph of the magnetic flux, as a function of time, through a loop that is at rest. (a) If the induced current in the loop is directed clockwise at the instant corresponding to point 1, at which of the other five points is the induced current directed clockwise? (b) Compare the magnitude of the emf induced in the loop at point 3 and at point 4. (c) At which of the six points does the induced emf have its largest magnitude? Explain. (d) At which of the six points does the induced emf have its smallest magnitude? Explain.

Exercises 28 – 30 are designed to give you practice with motional emf situations.

28. A Boeing 747 has a wingspan of 64 m. (a) If the jet is flying north at a speed of 800 km/h in a region where the vertical component of the Earth's magnetic field is 4.0×10^{-5} T, directed down, what is the motional emf measured from wingtip to wingtip? (b) Which wingtip is at a higher potential?

29. As shown in Figure 20.51, you exert a constant force F to the right on a conducting rod that can move without friction along a pair of conducting rails. The rails are connected at the left end by a resistor and a switch that is initially open. There is a uniform magnetic field directed into the page. You do two experiments, starting with the rod at rest both times. For the first experiment the switch is open, and for the second experiment the switch is closed. If you apply the same force for the same amount of time in each experiment, in which experiment will the rod be moving faster? Explain your answer.

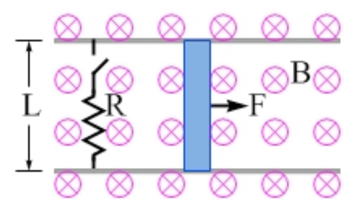


Figure 20.51: The conducting bar can move without friction on the conducting rails. The rails are connected at the left end by a resistor and a switch, which is initially open. For Exercises 29 and 30.

30. As shown in Figure 20.51, you exert a constant force F to the right on a conducting rod of length L that can move without friction along a pair of conducting rails. The rails are connected at the left end by a resistor of resistance R, and we can assume that the resistance of each rail and the rod is negligible in comparison to R. The switch, which is shown open in the diagram, is closed. There is a uniform magnetic field of magnitude B directed into the page, and the rod begins from rest. Both the rails and the magnetic field extend a long way to the right. (a) In terms of the variables specified here, determine the speed of the rod a long time after the rod begins to move. (b) What is the magnitude of the constant force required for the rod to reach a maximum speed of 2.0 m/s, if $L = 20$ cm, $B = 0.20$ T, and $R = 10$ Ω?

Exercises 31 – 35 deal with electric generators.

31. You have a hand-crank generator with a 100-turn coil, of area 0.020 m^2, which can spin through a uniform magnetic field that has a magnitude of 0.30 T. You can turn the crank at a maximum rate of 3 turns per second, but the hand crank is connected to the coil through a set of gears that makes the coil spin at a rate 8 times larger than the rate at which you turn the crank. What is the maximum emf you can expect to get out of this generator?

32. A particular electric generator consists of a single loop of wire rotating at constant angular velocity in a uniform magnetic field. Figure 20.52 shows three side views of the loop as the loop rotates clockwise, with the loop at various positions. Rank the three positions based on (a) the magnitude of the magnetic flux passing through the loop, and (b) the magnitude of the induced emf in the loop.

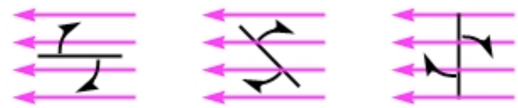


Figure 20.52: Three side views of a loop rotating in a uniform magnetic field. The plane of the loop is parallel to the magnetic field in case 1, at a 45° angle to the field in case 2, and perpendicular to the field in case 3. For Exercise 32.

33. At a particular location, the Earth's magnetic field has a magnitude of 5.0×10^{-5} T. At what angular frequency would you have to spin a 200-turn coil in this field to generate alternating current with a peak emf of 12 V, if each turn of the coil measures 5.0 cm × 5.0 cm?

34. You are designing an electric generator to mimic the alternating current put out by a wall socket in North America, which has a peak voltage of 170 V and a frequency of 60 Hz. The magnets you are using create a uniform magnetic field of 0.10 T. How many turns will you use in your coil, and what area will each of your turns have?

35. **Back emf**. In an electric motor, current flowing through a coil in a magnetic field gives rise to a torque that makes the coil spin. However, a coil spinning in a magnetic field acts as an electric generator, so the coil has an induced emf (known as back emf) that gives rise to an induced current. (a) Based on the principles of physics that we addressed in this chapter, would you expect the induced current to add to the original current in the coil, or subtract from it? Explain. (b) When a motor is first turned on, it takes a few seconds for the coil to reach its operating angular velocity. What happens to the net current in the motor during this start-up period? Is this consistent with a phenomenon you have probably observed, that when a device with a motor, such as an air conditioner, first starts up, the lights in your house dim for a couple of seconds? Note that the compressor in the air conditioner acts as a motor.

Exercises 36 – 40 involve transformers and the distribution of electricity.

36. The power transformer in a typical microwave provides 1000 W at an rms voltage of 2400 V on the secondary side. The primary is connected to a wall socket, which has an rms voltage of 120 V. The primary coil has 100 turns. Assuming the transformer is ideal, determine: (a) the number of turns in the secondary coil, (b) the rms current in the secondary, and (c) the rms current in the primary.

37. The AC adapter for a particular computer is plugged into the wall, so the primary side of the transformer in the adapter is connected to a 60 Hz signal with an rms voltage of 120 V and an rms current of 2.0 A. (a) The secondary of the transformer puts out

alternating current with an rms voltage of 20 V. Assuming the transformer is ideal, what is the rms current in the secondary? (b) After transforming the signal to 20 V AC, the adapter uses capacitors and diodes to turn the signal into direct current electricity at a voltage of 16 V and a current of 5 A. How much power, in this DC electricity, is provided to the computer by the adapter? (c) Use your answers to parts (a) and (b) to explain why, when you hold the adapter in your hand, the adapter feels rather warm.

38. A particular step-down transformer has its primary coil connected to the alternating current from a wall socket. Assuming the transformer is ideal, compare the following: (a) the power in the primary and the power in the secondary, (b) the rms voltage in the primary and the rms voltage in the secondary, (c) the rms current in the primary and the rms current in the secondary, (d) the frequency of the alternating current in the primary and secondary.

39. If you travel from one country to another, such as from the United States to England, you will find that an electronic device designed to plug into a North American wall socket will not plug into a wall socket in England. (a) There is a good reason for this. Explain why it is a good idea that European wall sockets do not directly accept North American plugs, and vice versa. (b) Travelers can purchase a travel adapter, which allows a North American device to be plugged into a European wall socket. In addition to matching the different plugs and sockets, a travel adapter is a transformer. What is the ratio of the number of turns in the two coils in such a travel adapter?

40. A typical pole-mounted transformer, the last step in the power distribution process before electricity is delivered to residential customers, has an rms voltage of 7200 volts on the primary coil. The secondary coil has an rms output voltage of 240 volts, which is delivered to one or more houses. (a) What is the ratio of the number in turns in the primary to the number in the secondary? (b) If the current on the primary side is 6.0 A, and each house requires an rms current of 50 A, how many houses can the transformer supply electricity to?

General problems and conceptual questions

41. A large glass window has an area of 6.0 m^2. The Earth's magnetic field in the region of the window has a magnitude of 5.0×10^{-5} T. (a) What is the maximum possible magnitude of the magnetic flux through the window from the Earth's field? (b) If the flux has a magnitude of 1.0×10^{-4} T m^2, what is the angle between the window's area vector and the Earth's magnetic field?

42. A particular flat conducting loop has an area of 5.0×10^{-3} m^2. The loop is at rest in a uniform magnetic field, directed perpendicular to the plane of the loop, which has a magnitude of 2.0 T. The field has a constant magnitude and direction. (a) Determine the magnitude of the magnetic flux passing through the loop. (b) Determine the magnitude of the emf induced in the loop in a 5.0-second interval.

43. Return to the situation shown in Figure 20.17, which we analyzed qualitatively in Essential Question 20.3. The magnetic field has a magnitude of 2.0 T, the value of v is 6.0 m/s, and the resistance of each single-turn loop is 3.0 Ω. If each small square in the picture measures 10 cm × 10 cm, determine the magnitude and direction of the induced current in (a) loop 1, (b) loop 2, (c) loop 3, and (d) loop 4.

44. A square loop consists of a single turn with a resistance of 4.0 Ω. The loop measures 10 cm × 10 cm, and has a uniform magnetic field passing through it that is directed out of the page. The loop contains a 12-volt battery, connected as shown in Figure 20.53. At the instant shown in the figure, there is no net current in the loop. At what rate is the magnetic field changing, and is the field increasing or decreasing in magnitude?

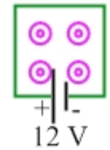


Figure 20.53: A square loop containing a 12-volt battery has no net current. For Exercise 44.

45. Figure 20.54 shows a cross-sectional view of a single-turn conducting loop, with a radius of 10 cm, inside a long current-carrying solenoid. The solenoid has a radius of 20 cm, and has 1000 turns per meter. The current in the solenoid is directed counterclockwise, and is increasing in magnitude. Draw a set of three pictures (Before, After, and To Oppose) to determine the direction of the induced current in the conducting loop.

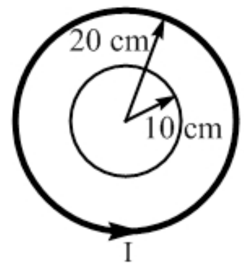


Figure 20.54: A conducting loop, with a radius of 10 cm, is placed at the center of a long current-carrying solenoid, which has a radius of 20 cm. For Exercises 45 and 46.

46. Return to the situation described in Exercise 45, and shown in Figure 20.54. If the current in the solenoid is increasing at the rate of 0.20 A/s, and the loop has a resistance of 4.0 Ω, determine the magnitude of the induced current in the conducting loop.

47. Figure 20.55 shows a graph of the emf induced in a loop as a function of time. (a) Plot a graph of the corresponding magnetic flux through the loop as a function of time. Your graph should be consistent with Figure 20.54 as well as Faraday's Law. (b) Is there only one possible flux graph for part (a), or are there multiple solutions possible? Explain.

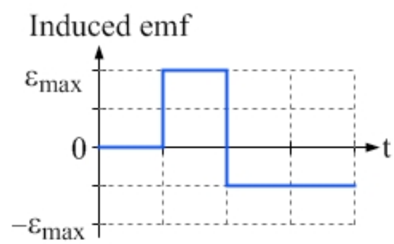


Figure 20.55: A graph of the emf induced in a loop as a function of time. For Exercise 47.

48. You probably have a credit or debit card with a magnetic stripe on the back. The magnetic stripe contains information that is stored in the pattern of magnetic fields on the stripe. When you put your card in a card reader, such as at a checkout counter, you need to slide the card through quickly. Using the principles of physics we have addressed in this chapter, explain how the card reader works, and why sliding the card through the reader slowly does not work.

49. There are many applications of electromagnetic induction, in addition to those we have examined in this chapter. Choose **one** of the following three subjects, and write a couple of paragraphs about how it works, highlighting in particular the role of electromagnetic induction. 1. An electric guitar. 2. A tape recorder. 3. A hard disk in a computer.

50. A biomedical application of electromagnetic induction is magnetoencephalography (MEG), which uses the tiny magnetic fields generated by currents in the brain to create an image of neural activity in the brain. Do some research about MEG, and write a couple of paragraphs describing it.

51. As shown in Figure 20.56, a conducting wire loop is placed below a long straight wire that is carrying a current to the right. The current in the wire is increasing in magnitude. (a) In which direction is the induced current in the loop? (b) The loop experiences a net force because of the interaction between the current in the long straight wire and the induced current. In what direction is this force? (c) If the loop moves in response to this net force, explain how this motion is consistent with Lenz's Law.

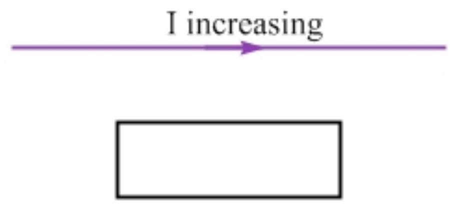


Figure 20.56: A conducting loop near a current-carrying long straight wire, for Exercise 51. The current in the wire is increasing.

52. There are some flashlights that do not require batteries. Instead, the flashlight has a magnet that moves back and forth through a coil when you shake it. The coil is connected to a capacitor which is in turn connected to a light-emitting diode (LED), which can glow brightly without requiring much current. Explain how such a flashlight works.

53. Figure 20.57 shows a conducting bar of length *L* that can move without friction on a pair of conducting rails. The rails are joined at the left by a battery of emf ε and a switch that is initially open. The bar, which is initially at rest, has a resistance *R*, and we will assume the resistance of all other parts of the circuit is negligible. The whole apparatus is in a uniform magnetic field, directed into the page, of magnitude *B*. Both the rails and the magnetic field extend far to the left and right. In terms of the variables specified above, determine the magnitude and direction of: (a) the current in the circuit immediately after the switch is closed, (b) the net force on the bar immediately after the switch is closed, (c) the current in the circuit a long time after the switch is closed, (d) the net force on the bar a long time after the switch is closed, and (e) the magnitude and direction of the velocity of the bar a long time after the switch is closed.

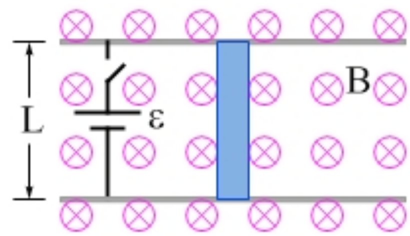


Figure 20.57: The bar, which is initially at rest, can slide without friction on the conducting rails. The rails are connected together on the left by a battery and a switch. For Exercises 53 and 54.

54. Repeat Exercise 53 (illustrated in Figure 20.57), but now use the values $L = 20$ cm, $\varepsilon = 12$ V, $R = 6.0\ \Omega$, and $B = 5.0 \times 10^{-2}$ T.

55. A conducting rod of mass *m* and length *L* can slide with no friction down a pair of vertical conducting rails, as shown in Figure 20.58. The rails are joined at the bottom by a light bulb of resistance *R*. The rails have stops near the bottom to prevent the rod from smashing the bulb. There is a uniform magnetic field of magnitude *B* directed out of the page. When the rod is released from rest, the force of gravity causes the rod to accelerate down the rails, but the rod soon reaches a terminal velocity (that is, it falls at constant speed). (a) In what direction is the induced current through the light bulb? (b) Sketch two free-body diagrams, one just after the bar is released and one when it has reached terminal velocity. (c) In terms of the variables specified here and *g*, the magnitude of the acceleration due to gravity, find an expression for the constant speed of the rod. (d) Describe what happens to the brightness of the light bulb as the rod accelerates from rest, and once the rod reaches terminal speed.

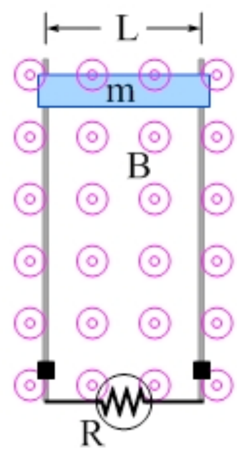


Figure 20.58: When a conducting rod of mass *m* is released from rest, it slides down the two vertical rails, for Exercise 55.

56. You can demonstrate the effect of eddy currents by dropping a magnet down a pipe made from copper or aluminum, as shown in Figure 20.59. In this case, the magnet is a bar magnet with its north pole at the lower end. Two sections of the tube are outlined, one near the top and one near the bottom. (a) Analyze the lower section of the tube to determine in which direction the eddy currents circulate in that section as the magnet approaches from above. What effect do these eddy currents have on the magnet? (b) Repeat for the upper section of the tube, to determine in which direction the eddy currents circulate in that section as the magnet moves farther away, and the effect of those currents on the magnet.

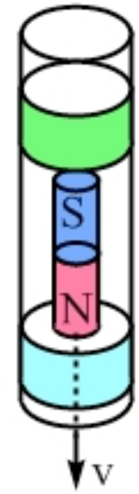


Figure 20.59: A pipe made of conducting, but non-ferromagnetic, material, through which a magnet has been dropped. For Exercise 56.

57. In Exploration 20.5, we explored how eddy currents can be used to bring a train to a stop. In that exploration, the magnetic field from the electromagnets was directed into the page. If the field was reversed, coming out of the page instead, would the train speed up or slow down? Explain.

58. The secondary of the transformer in a cathode ray tube television provides an rms voltage of 24 kV. The primary is connected to a wall socket, which has an rms voltage of 120 V. The primary coil has 100 turns, and the transformer is ideal. The television has a power rating of 75 W. (a) Find the number of turns in the secondary coil. (b) Find the rms current in the secondary, and (c) the rms current in the primary.

59. As shown in Figure 20.60, the ferromagnetic core of a transformer is generally made from several laminated sheets, which are electrically insulated from one another, rather than being made from a single piece of metal. This method of construction reduces the losses associated with eddy currents in the core itself. Explain how this works.

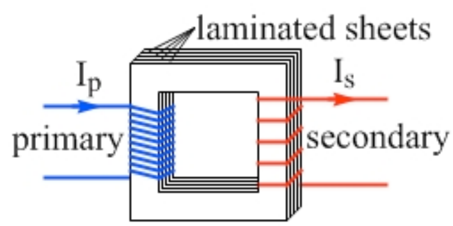


Figure 20.60: The core of a transformer is generally made from several laminated sheets, which are electrically insulated from one another, rather than being made from a single piece of metal. For Exercise 59.

60. A power company experiences losses of 20 kW when it transmits electricity along a particular transmission line at an rms voltage of 250 kV. What would the transmission losses be if the rms voltage was transformed to 500 kV instead?

61. Do some research about the War of Currents, the battle between the advocates of DC and the advocates of AC, which occurred during the late 1800's in the United States, and write a few paragraphs about it.

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Chapter 20: Additional Resources

Pre-session Movies on YouTube

- Magnetic Flux, and Faraday's Law
- Lenz's Law, and a Pictorial Approach
- Faraday's Law, and the Flux vs. Time Graph
- Motional Emf
- Electric Generators

Examples

- Sample Questions

Solutions

- Answers to Selected End of Chapter Problems
- Sample Question Solutions

Additional Links

- *PhET* simulation: Faraday's Electromagnetic Lab