18-1 Current, and Batteries

In Chapters 16 and 17, our main focus was on static charge. We turn now to look at situations involving flowing charge. Controlling that flow is the basis of many electric circuits.

Current is the rate at which charge flows. The symbol we use for current is *I*:

 $I = \frac{\Delta Q}{\Delta t}$

(Equation 18.1: Current, the rate of flow of charge)

The unit for current is the ampere (A). 1 A = 1 C/s.

The direction of current is the direction positive charges flow, a definition adopted by Benjamin Franklin before it was determined that in most cases the charges that flow in a circuit are electrons (negative charges). However, in a circuit positive charge flowing in one direction is equivalent to an equal amount of negative charge flowing at the same rate in the opposite direction.

In general, circuits in which charge flows in one direction are **direct current (DC)** circuits. In **alternating current (AC)** circuits (such as those in your house) the current direction continually reverses direction. Despite this difference, many of the concepts addressed in this chapter apply to both DC and AC circuits.

Batteries

What causes charge to flow? In general, charge flows from one point to another when there is a potential difference between the points. A battery can create such a potential difference. This potential difference gives rise to an electric field within the wires and the other elements of a circuit. Charged objects, such as electrons, in this field experience a force which can cause them to move. Note that, previously, we discussed how the electric field within a conductor is zero when the conductor is in static equilibrium. In this case, when we have a non-zero electric field within the conductor, we have more of a dynamic equilibrium – a steady flow of charge may be established as the charges respond to the non-zero field.

A battery can be thought of as an electron manufacturing and recycling system. It does not create electrons. Rather, a chemical reaction that liberates electrons takes place at the negative terminal of the battery. If the battery is connected in a circuit the electrons travel through the circuit, giving up energy along the way (such as to a light bulb a toaster element), to the positive terminal of the battery. At the positive terminal a different chemical reaction takes place that recycles the electrons, binding them into waste products. It is also important that charge flows within the battery between the positive and negative terminals – this charge is often positive ions.

Consider a lead-acid car battery as an example. The battery's positive terminal is made from lead dioxide, the negative terminal from lead, and both terminals are immersed in a solution of dilute sulphuric acid. The sulphuric acid contains water molecules as well as ions of H^+ and HSO_4^- . The chemical reaction that takes place at the negative terminal liberates two electrons:

 $Pb + HSO_4^- \Rightarrow PbSO_4 + H^+ + 2e^-$.

After traveling through the circuit, these electrons are recycled at the positive terminal:

$$PbO_2 + HSO_4^- + 3H^+ + 2e^- \Longrightarrow PbSO_4 + 2H_2O$$
.

To keep the system going, there must be a net flow of positive ions from the negative terminal to the positive terminal.

As in any manufacturing process, there are raw materials (the electrodes, and the acid solution), there is a product (the electrons), and there are waste products (the $PbSO_4$ and water).

A battery runs out when its raw materials are used up, or when enough waste products build up to inhibit the reactions. In a rechargeable battery, the battery is recharged by running the chemical reactions in the opposite direction, re-creating the electrodes and removing waste products.

Fuel cells use a similar process as batteries but, whereas a battery is a closed system in which its raw materials and waste products are sealed in a container, in a fuel cell everything is open so that raw materials can be continually fed into the system and waste products removed. A number of manufacturers are researching ways to run portable electronic devices, such as laptops and cell phones, from fuel cells instead of from batteries. The advantage offered by the fuel cell is that you could run the device for significantly longer than you could run it off a battery. Also, instead of plugging the device into an electrical outlet for a few hours to re-charge it, you could just take a few seconds to top it up with, say, methanol, and the device would be good to go again.

Figure 18.1 has three views of a circuit involving a battery, two wires, and a light bulb. Figure 18.1(a) shows conventional current, where the charges flowing are positive. Figure 18.1(b) shows the actual situation, showing that the charges flowing through the wires and the light-bulb filament are actually electrons, while positive H^+ ions flow within the battery itself. The two situations look different, but the light bulb would be equally bright in either case. Figure 18.1(c) shows the circuit diagram. The current *I* is in the direction of conventional current.



Figure 18.1: Three views of a battery-powered circuit. Figure (a) shows conventional current, in which the charge that flows is always positive. Figure (b) shows the actual situation, in which electrons flow through the circuit and positive ions flow within the battery. Figure (c) shows a circuit diagram for this circuit. R stands for resistor, which we cover in the next section. The arrow under the I shows the direction of the conventional current. The two parallel lines, the shorter one marked with a – and the longer one with a plus, represent the standard symbol for a battery in a circuit diagram. The + is the positive terminal, and the – is the negative terminal.

Every battery has an associated potential difference: for instance, a 9-volt battery provides a potential difference of around 9 volts. This is the potential difference between the battery terminals when there is no current, and is known as the battery emf, ε (emf stands for electromotive force, and you say emf as it is spelled, e-m-f).

Related End-of-Chapter Exercises: 13, 15.

Essential Question 18.1: Which is closer to the truth – a battery is a source of constant potential difference, or a battery is a source of constant current? Explain.

Answer to Essential Question 18.1: The statement that a battery is a source of constant potential difference (until it runs out, at least) is closer to the truth. As we will see in Section 18-2, the current provided by a battery depends on what the battery is connected to.

18-2 Resistance and Ohm's Law

For many objects we find that the current through them is proportional to the potential difference across them. Such objects are said to be ohmic, because they obey Ohm's Law.

The current I through an ohmic device is proportional to the potential difference ΔV across it:

 $\Delta V = IR \qquad (Equation 18.2:$ **Ohm's Law**)

R here is known as the electrical resistance. The smaller the resistance of an object the more current flows the object for a given potential difference. An object in a circuit that contributes a relatively large resistance to a circuit is known as a resistor.

Many resistors are simply wires of length L and cross-sectional area A. The electrical resistance of such an object is given by:

 $R = \frac{\rho L}{A}.$

(Equation 18.3: **Electrical resistance**)

The unit of resistance is the ohm (Ω). 1 Ω = 1 V/A.

 ρ in Equation 18.3 is the resistivity, a parameter that depends on the material the resistor is made from. Table 18.1 shows some resistivity values for different materials.

Material	Resistivity	Material	Resistivity	Material	Resistivity
Copper	$1.7 \times 10^{-8} \Omega$ m	Nichrome	$1.0 \times 10^{-6} \Omega$ m	Hard rubber	$1 \times 10^{13} \Omega$ m
Tungsten	$5.6 \times 10^{-8} \Omega m$	Silicon	640 Ω m	Teflon	$1.0 \times 10^{16} \ \Omega \ m$

Table 18.1: Resistivity values for various materials.

EXAMPLE 18.2 – Factors contributing to resistance

Three cylindrical resistors are made from the same material and have the same volume. The first resistor has a length L, a cross-sectional area A, and a resistance R. The second has a length L/2 and cross-sectional area 2A. The third has length 2L and cross-sectional area A/2.

- (a) Draw a picture of these three situations.
- (b) Rank the three cases in terms of the total resistance.
- (c) Determine the resistance, in terms of *R*, of the second and third resistors.

SOLUTION

(a) The three situations are shown in Figure 18.2.

(b) The ranking by resistance goes as 3 > 1 > 2. Resistance increases as length increases and crosssectional area decreases, so case 3, with the longest length and smallest cross-sectional area, has the largest resistance. Case 2 has the smallest resistance because it has the smallest length and the largest area.



Figure 18.2: The three situations of the resistors described in Example 18.2.

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(c) Applying Equation 18.2, we have the resistance in case 1 is $R = \rho L / A$. In case 2, applying Equation 18.2 gives

$$R_2 = \rho(L/2)/2A = \rho L/4A = R/4.$$

Doing a similar analysis in case 3 gives

$$R_3 = \rho (2L)/(A/2) = 4\rho L/A = 4R.$$

On a circuit diagram, a resistor is usually represented as shown in Figure 18.3(a), although if the resistor is a light bulb the symbol can look like that in Figure 18.3(b), which looks a lot like the actual light bulb.

Temperature dependence of resistance

An object's resistance also generally depends on its temperature. To understand this, consider a simple model of resistance. When there is no potential difference between the ends of a wire the conduction electrons move about at random, much like atoms in an ideal gas. With a potential difference, however, a net drift of electrons toward the higher potential end is superimposed on the random motion, as in Figure 18.4. The drift speed is of the order of millimeters/second, compared to the typical electron speeds of a thousand kilometers per second.

A potential difference between the ends of a wire creates an electric field within the wire. Conduction electrons respond to this field, accelerating along the wire. However, as electrons bump into atoms in the wire the collisions transfer energy from the electrons to the atoms. This is where the resistance of the wire comes from – the energy lost by electrons in these collisions.

At higher temperatures atoms vibrate more energetically, making it harder for electrons to move past – thus, resistance generally increases with temperature. In some materials, however, such as semiconductors, increasing the temperature frees more electrons to serve as conduction electrons and is thus associated with a decrease in resistance. We can use a simple model to describe the change in resistivity with temperature:

$\rho = \rho_i (1 + \alpha \Delta T)$, (Equation 18.4: **Temperature dependence of resistivity**)

where α is the temperature coefficient of resistivity, which depends on the material. Equation 18.4 is reminiscent of Equation 13.5, describing an object's change in length when it changes temperature. When a resistor changes temperature both the resistivity and the dimensions change, but the change in size is generally negligible compared to the change in the resistivity.

Related End-of-Chapter Exercises: 16 – 19.

Essential Question 18.2: In what direction is the electric field that gives rise to the drift velocity to the right, shown by the lighter path in Figure 18.4? Which end of the wire is at a higher potential?

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Figure 18.3: (a) A circuit diagram showing a battery, at left, connected to a resistor, such as a toaster element, at right. (b) A similar circuit, but this time the resistor is a light bulb.



Figure 18.4: With no potential difference a conduction electron follows the random motion illustrated by the darker path. With a potential difference, the electron follows the lighter path, drifting to the right in this case.

Answer to Essential Question 18.2: To cause the electron to drift to the right, as shown, the electric field must be directed to the left. Because the electron has a negative charge, the force it feels is opposite in direction to that of the electric field. Electric field points in the direction of decreasing potential, so the right end of the wire must be at a higher potential.

18-3 Circuit Analogies, and Kirchoff's Rules

Analogies can help us to understand circuits, because an analogous system helps us build a model of the system we are interested in. For instance, there are many parallels between fluid being pumped through a set of pipes and charge flowing around a circuit. There are also useful parallels we can draw between a circuit and a ski hill, in which skiers are taken to the top of the hill by a chair lift and then ski down via various trails. Let's investigate these in turn.



The pump in the fluid system is like the battery in the circuit; the water is like the charge; and the water wheel is like the light bulb in the circuit. The large pipes act like the wires, one pipe carrying water from the pump to the water wheel, and another carrying the water back to the pump, much as charge flows through one wire from the battery to the light bulb, and through a second wire back to the battery. The pressure difference in the fluid system is analogous to the potential difference across the resistor in the circuit.

EXPLORATION 18.3 – Analogies between a circuit and a ski hill

A basic ski hill consists of a chair lift, like that shown in Figure 18.6, that takes skiers up to the top of the hill, and a trail that skiers ski down to the bottom. A short and wide downward slope takes the skiers from the top of the lift to the top of the trail, and another takes the skiers from the bottom of the trail to the bottom of the lift.

Step 1 – Identify the aspects of the circuit that are analogous to the various aspects of the ski hill. In particular, identify what for the ski hill plays the role of the battery, the flowing charge, and the resistor. The chair lift is like the battery in the circuit, while the skiers are like the charges. The chair lift raises the gravitational potential energy of the skiers, and the skiers dissipate all that energy as they ski down the trail (which is like the resistor in the circuit) to the bottom. Similarly, the battery raises the electric potential energy of the charges, and that energy is dissipated as the charges flow through the resistor.



Figure 18.6: A chair lift taking skiers up a hill. Photo credit: PhotoDisc/Getty Images.

Step 2 – Does the analogy have limitations? Identify at least one difference between the ski hill and the electric circuit. What happens when the chair lift / battery is turned off? On the ski hill the skiers keep skiing down to the bottom, but in the circuit if a switch is opened the net flow of charge stops. This difference stems from the fact that with the ski hill the potential difference is imposed by something external to the system, the Earth's gravity, while in the circuit the battery provides the potential difference. Another difference is that in the circuit the charges are identical and obey basic laws of physics, while on a ski hill the skiers are not identical, and make choices regarding when to stop for lunch or to enjoy the view, and which route to take down the hill.

Step 3 - Let's use our ski hill analogy to understand two basic rules about circuits, which we will make use of throughout the rest of the chapter. Figure 18.7 shows one ski trail dividing into two, trails A and B. The same thing happens in a circuit, with one path dividing into paths A and B. *What is the relationship between N, the total number of skiers, and N_A and N_B, the number of skiers choosing trails A and B, respectively? What is the analogous relationship between the current I in the top path in the circuit, and the currents I_A and I_B in paths A and B?*

We do not lose or gain skiers, so some skiers choose trail A and the rest choose B, giving $N = N_A + N_B$. The skiers

come back together when the trails re-join, and N skiers continue down the trail. Similarly, a certain number of charges flow through the top path in the circuit, and the charges take either path A or path B through the circuit before re-combining. The rate of flow of charge is the current, so we can say that $I = I_A + I_B$. This is the **junction rule**.

Step 4 - Figure 18.8 shows various points on a ski hill and in a circuit. For a complete loop, say from point 3 back to point 3, if we add up the changes in gravitational potential as we move around the loop, what will we get? If we do the analogous process for the circuit, adding up the electric potential differences as we move around a complete loop, what will we get?

In both cases we get zero. There is as much up as down on the ski hill, so when we return to the starting point the net change in potential is zero. The same applies to the circuit. This is the **loop rule**.

Figure 18.7: (a) One trail splits temporarily into two on the ski hill; (b) one path splits into two in the circuit.

Figure 18.8: Equivalent points marked on a ski hill and on a circuit.

Key Ideas for Analogies: Analogies, particularly the gravitation-based ski-hill analogy, can give us considerable insight into circuits. In this case we used the analogy to come up with what are known as Kirchoff's Rules. The Junction Rule – *the total current entering a junction is equal to the total current leaving a junction*. The Loop Rule – *the sum of all the potential differences for a closed loop is zero*. Related End-of-Chapter Exercises: 1 and 2.

Essential Question 18.3: In Figure 18.7, let's say the resistance of path A is larger than that of path B. Which current is larger? What is the blanket statement summarizing this idea?

Answer to Essential Question 18.3: If path A has a larger resistance then path B we would expect the current in path A to be smaller than that in path B, much as we might expect more skiers to choose trail B if it is an easier trail than trail A. The blanket statement about this is – current prefers the path of least resistance.

18-4 Power, the Cost of Electricity, and AC Circuits

A typical light bulb has two numbers on it. One is the power, in watts, and the other is the voltage, which is typically 120 V in North America, matching something about the voltage you obtain from a typical household electrical outlet. With these numbers you can determine the current through the bulb and the resistance of the bulb. Let's understand how this is done.

A change in electrical potential energy is given by the equation $\Delta U = q \Delta V$. Power is the time rate of change of energy. If we divide electrical potential energy by time we get:

$$P = \frac{\Delta U}{t} = \frac{q \,\Delta V}{t} = \frac{q}{t} \Delta V = I \,\Delta V \;.$$

Using Ohm's Law, $\Delta V = IR$, we can write the power equation in three ways:

$$P = I \Delta V = I^2 R = \frac{(\Delta V)^2}{R}.$$
 (Equation 18.5: Electrical power)

EXAMPLE 18.4A – Calculating resistance

- (a) What is the resistance of a bulb stamped with "100 W, 120 V?"
- (b) What is the resistance of a bulb stamped with "40 W, 120 V?"
- (c) If you pick up such a light bulb and measure its resistance when the bulb is not lit, the measured value is much less than what you calculate in parts (a) or (b). Why is this?

SOLUTION

(a) We can re-arrange one form of the power equation to solve for resistance of the 100 W bulb:

$$R = \frac{(\Delta V)^2}{P} = \frac{120 \text{ volts} \times 120 \text{ volts}}{100 \text{ W}} = \frac{120 \text{ volts} \times 120 \text{ volts}}{(10 \times 10) \text{ W}} = 12 \times 12 \Omega = 144 \Omega.$$

(b) The resistance of the 40 W bulb can be found in a similar way:

$$R = \frac{(\Delta V)^2}{P} = \frac{120 \text{ volts} \times 120 \text{ volts}}{40 \text{ W}} = (120 \times 3.0) \Omega = 360 \Omega.$$

(c) What we calculated above is the resistance of a bulb when the bulb has 120 volts across it. This is when it glows brightly because the filament is a few thousand kelvin. If you measure the resistance when the bulb is off, with the filament at room temperature, the resistance is much less because of the temperature dependence of resistance. As we discussed earlier, resistance generally increases as temperature increases. This is what is going on with the bulbs.

Related End-of-Chapter Exercises: 20, 28.

The Cost of Electricity

On your electric bill you are charged about 20 cents for every kilowatt-hour of electricity you used in a month. What kind of unit is the kilowatt-hour?

 $1 \text{ kW} - \text{hour} = 1000 \text{ W} \times 3600 \text{ s} = 3.6 \times 10^6 \text{ J}.$

The kilowatt-hour is a unit of energy. Note that you are charged only about 20 cents for each 3.6 million joules of energy delivered to your residence. To find the total cost associated with running a particular device, you do the following multiplication:

(Number of hours it is on)×(power rating in kW)×(cost per kW-hr) (Eq. 18.6: Electricity cost)

The power rating of a device is generally stamped on it. Clock radios and televisions usually show this on the back or the bottom, either on a sticker or printed directly on the device.

EXAMPLE 18.4B – The cost of watching television (the electrical cost, that is)

In a typical American household the television set is on for three hours a day. If the power company charges 20 cents for each kW-hr, what is the daily cost of having the TV on for that length of time if the TV's power rating is 300 W?

SOLUTION

First, let's convert the TV's power rating to kW by dividing by 1000, to get 0.30 kW. Then let's simply apply equation 18.6. The daily cost is:

 $(3 \text{ hr}) \times (0.30 \text{ kW}) \times (20 \text{ cents/kW-hr}) = 18 \text{ cents}.$

This is amazingly cheap, particularly compared to what it costs to go to the movies.

Related End-of-Chapter Exercises: 48 – 50.

AC Circuits

In many situations we can treat an electrical outlet, in North America. as acting like a 120-volt battery (in 170 V Europe it would be like a 220-volt 120 V battery). In reality, however, the voltage 0 signal in North America is a sine wave that oscillates between +170 volts and -170 volts with a frequency of 60 Hz (60 -170 V cycles/s), as in Figure 18.9. For a sine wave it turns out that the root mean square value of the voltage is the peak value divided by $\sqrt{2}$. This is where the 120 V comes from, it is the root-mean-square value of the voltage signal.

When connected to a wall socket, an incandescent light bulb flickers, but it does so at a rate much faster than is observable to us. The average power dissipated in the light bulb, however, is the root-mean-square voltage multiplied by the root-mean-square current.

Essential Question 18.4: When connected to a wall socket, an incandescent light bulb flickers at a frequency that is not 60 Hz. At what frequency does it flicker? Why?

Answer to Essential Question 18.4: The bulb flickers at 120 Hz. It is dim at the start of the cycle in Figure 18.9, bright when the voltage goes positive, dim when the voltage passes through zero, and bright again when the voltage goes negative. Thus, for each cycle of the voltage, the bulb goes through two cycles, so its frequency is twice that of the voltage. Note that the bulb does not go completely off when the voltage passes through zero because the filament takes time to cool.

18-5 Resistors in Series

In many circuits, resistors are placed in series, as in Figure 18.10(b), so that the charge flows through the resistors in sequence. Let's start with a battery connected to a single 10 Ω resistor, which is made from a wire of a particular resistivity and cross-sectional area. If we break the resistor into two pieces, one 40% as long as the original and one 60% as long, the pieces have resistances of 4 Ω and 6 Ω , respectively, because resistance is directly proportional to length.

Figure 18.10: A single 10 Ω resistor can be thought of as two resistors, whose resistances add to 10 Ω , placed in series with one another.

If the 4 Ω and 6 Ω resistors are connected by a wire of

negligible resistance, the battery sees no difference between the single 10 Ω resistor and the 4 Ω and 6 Ω resistors that are connected in series – the battery is still trying to force charge to flow through a total resistance of 10 Ω . We can generalize by splitting the original resistor into more than two pieces, but we always end up with the same total resistance.

Thus, we can say that when we have N resistors connected in series, the equivalent resistance of the set of resistors is:

 $R_{eq} = R_1 + R_2 + \ldots + R_N$. (Eq. 18.7: Equivalent resistance of resistors in series)

EXPLORATION 18.5A – Finding the current through one resistor

The emf of the battery in Figure 18.10(b) is 20 volts.

Step 1 – How does the current through the 4 Ω resistor in Figure 18.10(b) compare to that through the 6 Ω resistor? Explain why. The current is the same at all points in a series circuit. All the charge that flows through the 4 Ω resistor keeps going to flow through the 6 Ω resistor. The rate of flow is also the same, because charge cannot pile up anywhere in the circuit. Because current is the rate of flow of charge this means the current is the same through both resistors.

Step 2 – What is the current through the 4 Ω resistor in Figure 18.10(b)? It is tempting to apply Ohm's Law directly to the 4 Ω resistor, with a potential difference of 20 volts. Resist this temptation, because the potential difference across that resistor is something less than 20 volts! The most straightforward way to proceed is to first find the equivalent resistance of the circuit, which is 4 Ω + 6 Ω = 10 Ω in this case, and apply Ohm's Law to find the current in the circuit:

$$I_{circuit} = \frac{\Delta V}{R_{eq}} = \frac{20 \text{ volts}}{10 \Omega} = 2.0 \text{ A}$$

The current is the same at all points in a series circuit, so this is the current through each resistor.

Step 3 – What is the potential difference across each of the two resistors in Figure 18.10(b)? What is the sum of the potential differences? Why do we get this result? Now we can apply Ohm's Law to the individual resistors, to find that:

 $\Delta V_{4\Omega} = IR = (2.0 \text{ A})(4.0 \Omega) = 8.0 \text{ V}$ and $\Delta V_{6\Omega} = IR = (2.0 \text{ A})(6.0 \Omega) = 12 \text{ V}.$

These add together to equal 20 volts, the battery voltage. This is to be expected from Kirchoff's loop rule. In a series circuit, the loop rule tells us that the sum of the potential differences across the resistors equals the battery voltage.

Key ideas for series circuits: Components in a series circuit have equal currents passing through
them. The sum of the potential differences across the resistors in a series circuit is equal to the
battery voltage.Related End-of-Chapter Exercises: 25 and 46.

EXAMPLE 18.5B – Which bulb is brighter?

A standard 40 W light bulb is connected in series with a standard 100 W light bulb and an electrical outlet, which we can treat as a 120 V battery, as shown in Figure 18.11. (a) Provide a qualitative analysis to show which bulb is brighter. (b) Analyze the circuit quantitatively to estimate the power dissipated in each bulb. Hint: we calculated the resistance of these bulbs in Section 18.4.

100 W light bulb and a 40 W light bulb connected in series.

SOLUTION

(a) It's easy to think of justifications for all three possible answers. An argument in favor of the 100 W bulb being brighter is that such a bulb is brighter than a 40 W bulb when used at home. We could also argue that the bulbs are equally bright because they are in series, and therefore have the same current. The correct answer, however, is that the bulb marked as 40 W is brighter in this case. The bulbs do have equal currents, but the brightness is determined by the power. In this case let's use the equation $P = I^2 R$. Because the current is the same, the bulb with higher resistance has more power dissipated in it and is brighter. We showed in Section 18.4 that the 40 W bulb has a larger resistance. Note that bulbs are designed to be placed in parallel, so it should not be too surprising that they behave in unexpected ways when they are in series.

(b) Let's use the resistances we calculated in Section 18.4, 144 Ω for the 100 W bulb and 360 Ω for the 100 W bulb. The equivalent resistance of the two bulbs in series the sum of these values, about 500 Ω . Knowing the equivalent resistance enables us to find the current in the circuit, from Ohm's Law: $I = \Delta V / R_{eq} = (120 \text{ V})/(500 \Omega) = 0.24 \text{ A}$. The power dissipated in each bulb is:

$$P_{40W} = I^2 R = (0.24 \text{ A})^2 (360 \Omega) = 21 \text{ W}$$
 and $P_{100W} = I^2 R = (0.24 \text{ A})^2 (144 \Omega) = 8.3 \text{ W}$

The bulb marked "40 W" dissipates more power, and is thus brighter.

Essential Question 18.5: In the situation above, the bulbs will not actually have the resistances we used in the calculation. Why not? Will the bulbs actually dissipate more power or less power?

Answer to Essential Question 18.5: Each bulb has a potential difference less than the 120 V the bulb is designed for, so the bulbs are dimmer than usual. This reduces the filament temperature, lowering its resistance, which decreases the equivalent resistance of the circuit. This increases the current in the circuit, increasing the power dissipated in each bulb because $P = I^2 R$. The decrease in *R* is offset by the increase in *I*, and the extra factor of *I* gives a net increase.

18-6 Resistors in Parallel

When charge has more than one path to choose from between two points in a circuit, we say that those paths are in parallel with one another. Interestingly, adding a resistor to a circuit can actually decrease the resistance of that circuit if the resistor is placed in parallel with another resistor in the circuit. Such is the case in Figure 18.12(b), where the circuit has a lower net resistance than the circuit in Figure 18.12(a).

Figure 18.12: In (b), a 6.0Ω resistor is placed in parallel with the 4.0Ω resistor. (c) The charge now has a larger effective area to flow through, so the resistance of the circuit has been reduced.

If the resistors in Figure 18.12(b) have the same length and resistivity, then they must have different cross-sectional areas. The two resistors can be replaced by one equivalent resistor with the same length and resistivity, and with a cross-sectional area equal to the sum of the cross-sectional areas of the original resistors, as in Figure 18.12(c). Resistance is inversely proportional to area, so adding the second resistor in parallel actually decreases the resistance of the circuit.

If we have N resistors connected in parallel, their equivalent resistance is given by:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}.$$
 (Eq. 18.8: Equivalent resistance of resistors in parallel)

EXPLORATION 18.6 – Current in a parallel circuit

The emf of the battery in Figure 18.12(b) is 12 volts.

Step 1 – How does the current through the 4.0 Ω resistor in Figure 18.12(b) compare to that through the 6.0 Ω resistor? Explain why. When the charge has two paths to choose from, as it does in this circuit, more of the charge passes through the lower resistance path. Thus, the current through the 4.0 Ω resistor is larger than that through the 6.0 Ω resistor.

Step 2 – Find the equivalent resistance of the circuit in Figure 18.12(b) and use it to find the current through the battery in that circuit.

Applying equation 18.8 gives: $\frac{1}{R_{eq}} = \frac{1}{4.0 \Omega} + \frac{1}{6.0 \Omega} = \frac{3}{12 \Omega} + \frac{2}{12 \Omega} = \frac{5}{12 \Omega}.$

Inverting this to find the equivalent resistance gives $R_{eq} = (12/5)\Omega = 2.4 \Omega$. In other words, the battery acts as if it is connected to a single 2.4 Ω resistor, and thus the current in the circuit, and through the battery, is $I = \varepsilon / R_{eq} = (12 \text{ V})/2.4 \Omega = 5.0 \text{ A}$.

Step 3 – What is the current through each of the two resistors in Figure 18.12(b)? What is the sum of the currents? Why do we get this result? Each resistor in the circuit is directly connected to the battery, and thus each resistor has 12 V across it. Applying Ohm's Law to each resistor:

$$I_{4\Omega} = \frac{\Delta V}{R} = \frac{12 \text{ V}}{4.0 \Omega} = 3.0 \text{ A}$$
 and $I_{6\Omega} = \frac{\Delta V}{R} = \frac{12 \text{ V}}{6.0 \Omega} = 2.0 \text{ A}$.

These sum to 5.0 A, the current through the battery. This is consistent with Kirchoff's junction rule, which says the current entering a junction equals the current leaving the junction.

Key ideas for parallel circuits: Components in parallel with one another have the same potential difference across them. Current splits between parallel paths, with more current passing through the path with lower resistance. Related End-of-Chapter Exercises: 21 and 24.

In a calculation like that in Step 2, a common error is to forget to invert when applying Equation 18.8, stating the answer incorrectly as $R_{eq} = (5/12)\Omega = 0.42 \Omega$. Checking units can

prevent this error. Also, a rule of thumb is that the equivalent resistance of two resistors in parallel is between half the smaller resistance and the smaller resistance. In Step 2 above, in which the smaller resistance is 4.0 Ω , the equivalent resistance must be between 2.0 Ω and 4.0 Ω . The smaller the value of the larger resistance the smaller the equivalent resistance.

EXAMPLE 18.6 – Splitting the current

The current entering a particular section of a circuit is *I*. As shown in Figure 18.13, the current divides between two parallel paths, a current I_1 that passes through a resistor of resistance R_1 , and a current I_2 that passes through a resistor of resistance R_2 . What fraction of the current passes through each resistor? In other words, express I_1 and I_2 in terms of I, R_1 , and R_2 .

SOLUTION

We have two relationships we can use to find the answer. One is the junction rule, which tells us that $I = I_1 + I_2$, which we can reFigure 18.13: A section of a circuit in which two resistors are in parallel.

arrange to get $I - I_1 = I_2$. The second relationship is the idea that the

potential difference across the two resistors is the same, because they are in parallel.

$$\Delta V = I_1 R_1 = I_2 R_2$$
. Using $I - I_1 = I_2$ gives $I_1 R_1 = (I - I_1) R_2 = I R_2 - I_1 R_2$.

Solving for I_1 , and using that expression to solve for I_2 , gives:

$$I_1 = \frac{R_2}{R_1 + R_2} I$$
 and $I_2 = \frac{R_1}{R_1 + R_2} I$.

This is consistent with the idea that current prefers the path of least resistance, and tells us exactly how current splits between two branches that are in parallel with one another.

Essential Question 18.6: Two resistors, with resistances of 10Ω and 30Ω , are in parallel with one another (and only one another) in a circuit. What fraction of the current entering this part of the circuit passes through each resistor?

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$$R_1$$

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Answer to Essential Question 18.6: The fact that one resistance is three times the other means that three times as much current passes through the smaller resistor as through the larger resistor. Thus $\frac{3}{4}$ of the current passes through the 10Ω resistor and $\frac{1}{4}$ passes through the 30Ω resistor.

18-7 Series-Parallel Combination Circuits

In many circuits some resistors are in series while others are in parallel. In such seriesparallel combination circuits we often want to know the current through, and/or the potential difference across, each resistor. Let's explore one method for doing this. This method can be used if the circuit has one battery, or when multiple batteries can be replaced by a single battery.

EXPLORATION 18.7 – The contraction/expansion method of circuit analysis

Four resistors are connected in a circuit with a battery with an emf of 18 V, as shown in Figure 18.14. The resistors have resistances $R_1 = 4.0 \Omega$, $R_2 = 5.0 \Omega$, $R_3 = 7.0 \Omega$, and $R_4 = 6.0 \Omega$. Our goal is to find the current through each resistor.

Step 1 – Label the currents at various points in the circuit. This can help determine which resistors are in series and which are in parallel. This is done in Figure 18.15(a). The current passing through the battery is labeled I. This current splits, with a current I_1 through resistor R_1 , and a current I_2 through resistor R_2 . The current I_2 goes on to pass through R_3 , so R_2 and R_3 are in series with one another. The two currents re-combine at the top right of the circuit, giving a net current of I directed from right to left through resistor R_4 and back to the battery.

Figure 18.14: A series-parallel combination circuit with one battery and four resistors.

Step 2 – Identify two resistors that are either in series or in parallel with one another, and replace them by a resistor of

equivalent resistance. Resistors R_2 and R_3 are in series, so they can be replaced by their equivalent resistance of 12.0 Ω (resistances add in series), as shown in Figure 18.15(b). Is this the only place we could start in this circuit? For instance, are resistors R_1 and R_2 in parallel? To be in parallel, both ends of the resistors must be directly connected by a wire, with nothing in between. The left ends of resistors R_1 and R_2 are directly connected, but the right ends are not, with resistor R_3 in between. In fact, the only place to start in this circuit is with R_2 and R_3 .

Step 3 – Continue the process of replacing two resistors by an equivalent resistor until the circuit is reduced to one equivalent resistor. In the next step, shown in Figure 18.15(c), the two resistors in parallel, R_1 and R_{23} , are replaced by their equivalent resistance of 3.0 Ω . Finally, in Figure 18.15(d), the two resistors in series are replaced by their equivalent resistance, 9.0 Ω .

Step 4 – *Apply Ohm's Law to find the total current in the circuit.* With only one resistor we know both its resistance and the potential difference across it, so we can apply Ohm's Law:

$$I = \frac{\varepsilon}{R_{eq}} = \frac{18 \text{ V}}{9.0 \Omega} = 2.0 \text{ A}$$

Step 5 – Label the potential at various points in the single-resistor circuit. Choose a point as a reference. Here we choose the negative terminal of the battery to be V = 0. The other side of the battery is therefore +18 volts. Wires have negligible resistance, so $\Delta V = IR = 0$ across each wire. Thus, all points along the wire leading from the negative terminal of the battery have V = 0, while all points along the wire leading from the positive terminal have V = +18 V. See Figure 18.15(e).

Figure 18.15: The various steps in the contraction/expansion method.

(a) Labeling the current at various points can help identify which resistors are in series and which are in parallel.

(b) R_2 and R_3 are in series, and can be replaced by one equivalent resistor, R_{23} .

(c) - (d) Resistors are replaced a pair at a time to find the circuit's equivalent resistance.

(e) Apply Ohm's Law to find the total current.

(f - h) Expansion reverses the steps. At each expansion step, we find the current and potential difference for each resistor.

Step 6 – Expand the circuit back from one resistor to two. Find the current through,

and potential difference across, both resistors. Expansion reverses the steps of the contraction. In Figure 18.15(f), we replace the 9.0 Ω resistor by the 3.0 Ω and 6.0 Ω resistors, in series, it came from. When a resistor is split into two in series, the current (2.0 A in this case) through all three resistors is the same. We can now use Ohm's law to find the potential difference. For the two resistors, we get $\Delta V_{3\Omega} = IR = 6.0$ V and $\Delta V_{6\Omega} = IR = 12$ V. Thus, the wire connecting the 3.0 Ω and 6.0 Ω resistors is at a potential of V = +12 V. This is consistent with the loop rule, and the fact that the direction of the current through a resistor is the direction of decreasing potential.

Step 7 – *Expand the circuit back from two resistors to three.* The 3.0 Ω resistor is replaced by the 4.0 Ω and 12.0 Ω resistors, in parallel, which it came from, as shown in Figure 18.15(g). When one resistor is split into two in parallel, the potential difference across all three resistors is the same. That is 6.0 V in this case. We can then apply Ohm's Law to find the current in each resistor, giving $I_1 = \Delta V / R = 6 \text{ V}/4 \Omega = 1.5 \text{ A}$ and $I_2 = \Delta V / R = 6 \text{ V}/12 \Omega = 0.5 \text{ A}$. These add to the 2.0 A through their equivalent resistor, as we expect from the junction rule.

Step 8 – Continue the expansion process, at each step finding the current through, and potential difference across, each resistor. In this circuit there is one more step. This is shown in Figure 18.15(h), where the 12.0 Ω resistor is split into the original 5.0 Ω and 7.0 Ω resistors. These resistors have the same current, 0.5 A, as the 12.0 Ω resistor, and their potential differences can be found from Ohm's Law and sum to the 6.0 V across the 12.0 Ω resistor.

Key ideas for the contraction/expansion method: One way to analyze a circuit is to contract the circuit to one equivalent resistor and then expand it back. In each step in the contraction two resistors that are either in series or in parallel are replaced by one resistor of equivalent resistance. In the expansion when one resistor is expanded to two in series all three resistors have the same current, while when one resistor is expanded to two in parallel all three resistors have the same potential difference across them. **Related End-of-Chapter Exercises: 27, 32, 33, 36.**

Essential Question 18.7: Check the answer above by comparing the power associated with the battery to the total power dissipated in the resistors. Why should these values be the same?

Answer to Essential Question 18.7: The power provided to the circuit by the battery can be found from $P = \varepsilon I = (18 \text{ V}) \times (2.0 \text{ A}) = 36 \text{ W}$. The equation $P = I^2 R$ gives the power dissipated in

each resistor: $P_1 = I_1^2 R_1 = (1.5 \text{ A})^2 (4.0 \Omega) = 9.0 \text{ W}$; $P_2 = I_2^2 R_2 = (0.5 \text{ A})^2 (5.0 \Omega) = 1.25 \text{ W}$;

 $P_3 = I_2^2 R_3 = (0.5 \text{ A})^2 (7.0 \Omega) = 1.75 \text{ W}$; and $P_4 = I^2 R_4 = (2.0 \text{ A})^2 (6.0 \Omega) = 24 \text{ W}$, for a total of

36 W. Thus, the power input to the circuit by the battery equals the power dissipated in the resistors, which we expect because of conservation of energy.

18-8 An Example Problem; and Meters

Let's now explore a situation that involves many of the concepts from the last few sections, and allows us to discuss the role of a switch in a circuit.

EXPLORATION 18.8 – Three bulbs and two switches

Three identical light bulbs, A, B, and C, are placed in the circuit shown in Figure 18.16 along with two switches, 1 and 2, and a battery with an emf of 120 V (like a standard electrical outlet).

Figure 18.16: A circuit with one battery, two switches, and three identical light bulbs. The switches are initially open.

Step 1 – Are any bulbs on when the switches are both

open? If so, which bulbs are on? If not, explain why not. For a bulb to glow a current must pass through it. For there to be a current there must be a complete circuit, a conducting path for charges to flow through from one terminal of the battery to the other. With switch 1 open there is not a complete circuit, so all the bulbs are off.

Step 2 – *Kirchoff's loop rule is true even when the switches are open. How is this possible?*

This is possible because the potential difference across switch 1 is equal to the battery emf. If we define the wire connecting the negative terminal of the battery to the left side of switch 1 to be at V = 0, all other parts of the circuit, including the right side of the switch, are at a potential of

V = +120 V. There are no potential differences across the bulbs because there is no current.

Step 3 – *Complete these sentences. An open switch has a resistance of* _____. *A closed switch has a resistance of* _____. We generally treat an open switch as having a resistance of infinity. A closed switch acts like a wire, so we assume it has a resistance of zero.

Step 4 – *Rank the bulbs based on their brightness when switch 1 is closed and switch 2 is open. What is the potential difference across each bulb?* Bulb C is off – because switch 2 is open there is no current in that part of the circuit. Thus, the circuit has bulbs A and B in series with one another and the battery. Because bulbs A and B are identical, and have the same current through them, they are equally bright. The ranking is A=B>C. Bulbs A and B share the emf of the battery, with a potential difference of 60 V across each bulb. Bulb C has no potential difference across it.

Step 5 – What happens to the brightness of each bulb when switch 2 is closed (so both switches are closed)? What is the potential difference across each bulb? With switch 2 closed, charge flows through bulb C, so C comes on and is brighter than before. Bulbs B and C are in parallel, and have equal resistance, so half the current passes through B and half through C. Bulb B got all the current before switch 2 was closed, so you might think that bulb B is now obviously dimmer. However, closing switch 2 decreases the overall resistance of the circuit, increasing the current. So, bulb B only gets half the current, but the total current increases – which effect dominates?

Because all the current passes through bulb A, increasing the 120 V current in the circuit increases both the 0 V brightness of, and the potential difference across, bulb A. By the loop rule, increasing A's potential difference means B's potential difference decreases, so

Figure 18.17: Labeling the potential at various points helps us understand what happens to the bulbs when switch 2 is closed.

B's current and brightness must also be less. To summarize, A and C get brighter, while B gets dimmer. B and C are now equally bright, and A is the brightest of all. Assuming the bulbs have the same resistance, A has 80 V across it, while B and C each have 40 V across them, as shown in Figure 18.17.

Key Ideas for Switches: We can treat an open switch as having infinite resistance, and a closedswitch as having no resistance.Related End-of-Chapter Exercises: 5 - 7.

Ammeters Measure Current

A meter that measures current is known as an **ammeter**. Should an ammeter be wired in series or parallel? Should the ammeter have a small resistance or a large resistance? Does adding an ammeter to the circuit increase or decrease the current through the resistor of interest?

Circuit elements that are in series have the same current passing through them. Thus, to measure the current through a resistor an ammeter should be placed in series with that resistor, as in Figure 18.18. Adding the ammeter, which has some resistance, increases the equivalent resistance of the circuit and thus reduces the current in the circuit. The resistance of the ammeter should be as small as possible to minimize the effect of adding the ammeter to the circuit.

Voltmeters Measure Potential Difference

A meter that measures potential difference is known as a **voltmeter**. Should a voltmeter be wired in series or parallel? Should it have a small or a large resistance? How does adding a voltmeter to a circuit affect the circuit?

Circuit elements in parallel have the same potential difference across them. Thus, to measure the potential difference across a resistor a voltmeter should be placed in parallel with that resistor, as in Figure 18.19. Connecting the voltmeter, which has some resistance, in parallel decreases the resistance of the circuit, increasing the current. The resistance of the voltmeter should be as large as possible to minimize the effect of adding the voltmeter.

Related End-of-Chapter Exercises: 47, 60.

Essential Question 18.8: Can you add a 5.0 Ω resistor to the circuit in Figure 18.20 so that some current passes through it while the current through original resistors is unchanged? Explain.

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that's resistor R_2 .

Figure 18.20: The circuit for Essential Question 18.8.

Figure 18.18: An ammeter, represented by an A inside a circle, is used to measure the current through whatever is in series with it. In this case, that's everything in the circuit.

Figure 18.19: A voltmeter,

represented by a V inside a

circle, is used to measure the

potential difference of whatever is in parallel with it. In this case,

> 3.0 Ω 6.0 Ω

Answer to Essential Question 18.8: Yes, it can be done by placing the 5.0 Ω resistor in parallel with the original set of resistors. This is how circuits work in your house; turning on a lamp, for instance, does not affect a TV, even when the lamp and the TV are plugged into the same outlet.

18-9 Multi-loop Circuits

In many circuits with more than one battery the batteries cannot be replaced by a single battery. In such cases we rely on Kirchoff's Rules, the loop rule and the junction rule.

EXPLORATION 18.9 – Analyzing a multi-loop circuit

Solve for the current through each of the resistors in the circuit shown in Figure 18.22. The emf's of the three batteries are: $\varepsilon_1 = 2.0 \text{ V}$; $\varepsilon_2 = 15 \text{ V}$; and $\varepsilon_3 = 12 \text{ V}$. The resistances of the four resistors are $R_1 = 1.0 \Omega$; $R_2 = 5.0 \Omega$; $R_3 = 2.0 \Omega$; $R_4 = 3.0 \Omega$.

Figure 18.21: Connecting the new resistor across the battery does not change the current elsewhere in the circuit.

Figure 18.22: A multi-loop circuit with three batteries and four resistors.

Step 1 – Choose a direction for the current, and label the current, in each branch of the

circuit. A branch is a path from one junction to another in a circuit. Figure 18.23 shows the two junctions (points at which more than two current paths come together) as circles, with the three different branches running from one junction to the other. Within each branch, everything is in

series, so each branch has its own current, labeled I_1, I_2 , and I_3 . In many cases, such as this one,

we do not know for certain which way the current goes in a particular branch. We simply choose a direction for each current, and if we are incorrect we will get a minus sign for that current when we solve for it.

Step 2 – Based on the directions chosen for each current, put a + sign at the higher-potential end of each resistor and a - sign at the lower-potential end. This step is optional, but can make it easier to set up the loop-rule equations correctly in step 3. Current is directed from the high-potential end of a resistor to the low-potential end, giving the signs in Figure 18.23.

Step 3 – Apply the loop rule to one complete loop in the circuit. This means applying the equation $\sum \Delta V = 0$. ΔV for a battery is the battery emf, which is positive if we go from the – terminal to the + terminal and negative if we go the other way. ΔV for a

resistor is IR for that resistor, positive if we go from – to + (with the current) and negative if we go the other way (against the current). Let's start with the inside loop on the left, going clockwise around the loop starting from the lower-left corner. Keeping track of the signs carefully, we get:

+2.0 V –
$$(1.0 \Omega)I_1$$
 + $(5.0 \Omega)I_2$ – 15 V – $(3.0 \Omega)I_1$ = 0

This simplifies to $-13 \text{ V} - (4.0 \Omega)I_1 + (5.0 \Omega)I_2 = 0$. [Equation 1]

Figure 18.23: The three currents in the circuit are labeled, and directions are chosen for them. The resistors are labeled with high-potential (+) and low-potential (–) ends based on the direction chosen for current; the current direction through a resistor is in the direction of decreasing potential.

Step 4 – Keep applying the loop rule to complete loops. Each new equation should involve a branch not involved in any previous equations. Stop writing down loop equations when you have involved every branch at least once. For this particular circuit, we only have one more

branch (the one on the right) to involve, so we just need one more loop equation. We can either write a loop equation for the inside loop on the right or for all the way around the outside. Let's try the inside loop on the right. Starting at the lower right and going counter-clockwise, we get:

+12 V – $(2.0 \Omega)I_3$ + $(5.0 \Omega)I_2$ –15 V = 0. This simplifies to –3 V + $(5.0 \Omega)I_2$ – $(2.0 \Omega)I_3$ = 0. [Equation 2]

We could still go around the outside to obtain a third loop equation; however, because that third equation can be obtained from Equations 1 and 2 above it does not give us any new information, and thus does not give us the third independent equation we need.

Step 5 – *Apply the junction rule to come up with additional equations relating the variables.* The junction rule says that the sum of the current entering a junction must equal the sum of the current leaving the junction. In this circuit both junctions give the same equation, so no matter which junction we choose we get:

$$I_1 + I_2 + I_3 = 0$$
. [Equation 3]

This actually tells us that at least one of the currents must be directed opposite to the way we guessed, because we cannot have three currents coming into a junction and no current directed away (or vice versa), so we expect at least one current to have a minus sign in the end.

Step 6 – *Solve the equations to find the currents in the circuit.* This is now an exercise in algebra, solving three equations in three unknowns. Here is one way to do it. Noting that I_2 appears in both Equation 1 and Equation 2, solve Equation 3 for I_2 and substitute it into both Equations 1 and 2. From Equation 3 we get $I_2 = -I_1 - I_3$.

Equation 1 becomes: $-13 \text{ V} - (9.0 \Omega) I_1 - (5.0 \Omega) I_3 = 0$.	[Equation 4]
Equation 2 becomes: $-3 \text{ V} - (5.0 \Omega) I_1 - (7.0 \Omega) I_3 = 0$.	[Equation 5]

Multiply Equation 4 by a factor of +7: $-91 \text{ V} - (63 \Omega)I_1 - (35 \Omega)I_3 = 0$. [Equation 6] Multiply Equation 5 by a factor of -5: $+15 \text{ V} + (25 \Omega)I_1 + (35 \Omega)I_3 = 0$. [Equation 7]

Add Equations 6 and 7: $-76 \text{ V} - (38 \Omega)I_1 = 0$, which gives $I_1 = -2.0 \text{ A}$. Substituting this into Equation 4 (or Equation 5) gives $I_3 = +1.0 \text{ A}$. Using Equation 3 then gives $I_2 = +1.0 \text{ A}$.

Key idea for a multi-loop circuit: In a circuit with multiple batteries we can use Kirchoff's loop rule and Kirchoff's junction rule to solve for any unknown parameters. The loop rule is actually a statement of conservation of energy applied to circuits, while the junction rule is a statement of conservation of charge. **Related End-of-Chapter Exercises: 37 – 41, 61, 62.**

Essential Question 18.9: Re-draw Figure 18.23 with the current in each branch labeled correctly. Also, label the potential at all points in the circuit, if the lower junction is at a potential of V = 0. What is the potential difference between the two junctions?

Figure 18.24: The solution to the circuit in Exploration 18.9, with the correct currents and with the potential labeled at various points.

across the 5.0 Ω resistor is IR = 5.0 V. As we move through the resistor in the same direction as the current

current in each branch, and the potential at various points relative to V = 0 at the lower junction. Labeling potential is like doing the loop rule. Starting at the lower junction and

moving up the middle branch, the potential stays at V = 0until we reach the 15-volt battery. Crossing the battery

from the – terminal to the + terminal raises the potential

by the battery emf to +15 V. The potential difference

the potential decreases, reaching +15 V - 5 V = +10 V at the upper end of that resistor, and at the upper junction. Thus, the upper junction has a potential 10 V higher than the lower junction.

Labeling the potential is a way to check the answer for the currents. Going from the lower junction to the upper junction via any branch gives the same answer for the potential of the upper junction. If the answer depended on the path, we would know something was wrong.

18-10 RC Circuits

In some circuits the current changes as time goes by. An example of this is an RC circuit, involving a resistor (R) and a capacitor (C).

EXPLORATION 18.10 – RC Circuits

In the RC circuit in Figure 18.25, the resistor and capacitor are in series with one another. There is also a battery of emf ε , and a switch that is initially in the "discharge" position. The capacitor is initially uncharged, so there is no current in the circuit.

Step 1 – What are the general equations for the potential difference across a resistor, and the potential difference across a capacitor? The potential difference across a resistor is given by Ohm's law,

 $\Delta V_R = IR$, while the potential difference across a capacitor is

given by $\Delta V_C = Q/C$.

Figure 18.25: An RC circuit with a battery, resistor, capacitor, and switch.

Step 2 – Use the loop rule to find the potential difference across the resistor, and across the capacitor, immediately after the switch is moved to the "charge" position. The capacitor is uncharged, so its potential difference is zero. The closed switch has no potential difference, so by the loop rule the potential difference across the resistor equals the emf of the battery.

Step 3 – What happens to the potential difference across the capacitor, the potential difference across the resistor, and the current in the circuit as time goes by? In this circuit, the battery pumps charge from one plate of the capacitor to the other. Because the charge is pumped through the resistor the rate of flow of charge is limited. As time goes by the charge on the capacitor increases, as does the potential difference across the capacitor (being proportional to the charge on the capacitor). By the loop rule, the potential difference across the resistor, and the current in the circuit, decreases as time goes by. Because the current (the rate of flow of charge) decreases, the rate at which the potential difference across the capacitor rises also decreases, slowing the rate at which the current decreases. This gives rise to the exponential relationships reflected in Figure 18.26, and characterized by the product of resistance and capacitance, which has units of time.

 $\tau = RC$. (Equation 18.9: Time constant for a series RC circuit)

Figure 18.26: Plots of the current, resistor voltage, and capacitor voltage, as a function of time as the capacitor charges.

Step 4 – If we wanted the capacitor voltage to increase more quickly, could we change the resistance? If so, how? Could we accomplish this by changing the capacitance? If so, how? To change the capacitor voltage more quickly we could change the resistance or the capacitance. Decreasing the resistance increases the current, so charge flows to the capacitor more quickly. Decreasing the capacitance gives a larger potential difference across the capacitor with the same amount of charge, so that also works. This is consistent with the definition of the time constant, the product *RC*, which is a measure of how quickly the current, and potential differences, change in the circuit. Decreasing the time constant means that these quantities change more quickly.

Step 5 – When the switch has been in the "Charge" position for a long time, the circuit approaches a steady state, in which the current and the resistor voltage both approach zero, and the capacitor voltage approaches the battery emf. If the switch is now moved to the "Discharge" position, what happens to the potential difference across the capacitor, the potential difference across the resistor, and the current in the circuit as time goes by? Now the capacitor discharges through the resistor, so the current is in the opposite direction as it is when the capacitor is charging. The magnitude of the current decreases as time goes by because the potential difference across the resistor, which is the negative of the capacitor voltage by of the loop rule, decreases as time goes by. This gives rise to the relationships shown in Figure 18.27.

Figure 18.27: Plots of the current, resistor voltage, and capacitor voltage, as a function of time as the capacitor discharges.

Key ideas for RC Circuits: The current in an RC circuit changes as time goes by. In general, the current decreases exponentially with time. The expressions for the potential differences across the resistor and capacitor also involve negative exponentials of a quantity proportional to time, but the loop rule is satisfied at all times. **Related End-of-Chapter Exercises: 11, 12, 63, 64.**

Essential Question 18.10: A particular RC circuit is connected like that in Figure 18.25. The battery emf is 12 V, and the resistor has a resistance of 47 Ω . When the switch is placed in the "Charge" position, it takes 2.5 ms for the capacitor voltage to increase from 0 V to 3.0 V. What is the capacitance?

Answer to Essential Question 18.10: Let's first determine the time constant, using Equation 18.11, and replacing the factor *RC* by τ . This gives:

 $(3.0 \text{ V}) = (12 \text{ V})(1 - e^{-(2.5 \text{ ms})/\tau}).$

Divide both sides by 12 V: $0.25 = 1 - e^{-(2.5 \text{ ms})/\tau}$.

Re-arrange, then take the natural log of both sides: $\ln(0.75) = -\frac{2.5 \text{ ms}}{\tau}$.

Solving for the time constant gives: $\tau = -\frac{2.5 \text{ ms}}{\ln(0.75)} = 8.7 \text{ ms}.$

Multiplying ohms by farads gives seconds, so $C = \frac{\tau}{R} = \frac{8.69 \text{ ms}}{47 \Omega} = 180 \,\mu C$.

Chapter Summary

Essential Idea: Direct Current Circuits.

Electric circuits are essential to our daily lives, being part of electronic devices like cell phones and iPods. In this chapter we explored the principles of how basic circuits work.

Electric Current

Current is the rate at which charge flows. The symbol we use for current is *I*:

 $I = \frac{\Delta Q}{\Delta t}$ (Equation 18.1: Current, the rate of flow of charge) The unit for current is the ampere (A). 1 A = 1 C/s.

The direction of current is the direction positive charges flow.

Ohm's Law and Resistance (Equation 18.2: Ohm's Law) $\Delta V = IR$

The electrical resistance of a wire of length L and cross-sectional area A is: (Equation 18.3: Electrical resistance)

$$R = \frac{\rho L}{A}$$
.

The unit of resistance is the ohm (Ω). 1 Ω = 1 V/A.

The resistivity ρ depends on the material the resistor is made from.

Resistors in Series and Parallel

If N resistors are connected in series, their equivalent resistance is given by:

 $R_{eq} = R_1 + R_2 + \ldots + R_N \, .$ (Eq. 18.7: Equivalent resistance of resistors in series)

If N resistors connected in parallel, their equivalent resistance is given by:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}.$$
 (Eq. 18.8: Equivalent resistance of resistors in parallel)

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Electric Power, and the Cost of Electricity

 $P = I \Delta V = I^2 R = \frac{(\Delta V)^2}{R}.$ (Equation 18.5: Electrical power)

The cost for operating a particular electrical device can be determined from:

(Number of hours it is on)×(power rating in kW)×(cost per kW-hr) (Eq. 18.6: Electricity cost)

The contraction/expansion method for analyzing series-parallel combination circuits

When a circuit has one battery with an emf ε , and resistors that have series and parallel connections, the current through, and potential difference across, each resistor can be found by:

- 1. Identifying two resistors that are either in series or in parallel, and replacing them by their equivalent resistance. Repeat this contraction until one resistor is left.
- 2. Determining the total current in the circuit by applying Ohm's Law: $I = \varepsilon / R_{ea}$.
- 3. Gradually reversing the steps in the contraction. When one resistor is expanded to two in series all three resistors have the same current, while when one resistor is expanded to two in parallel all three resistors have the same potential difference across them.

Applying Kirchoff's Rules to analyze Multi-loop Circuits

Kirchoff's Rules can also be applied to analyze a circuit. The loop rule is a statement of conservation of energy, while the junction rule is a statement of conservation of charge.

Loop rule: the sum of all the potential differences around a closed loop is zero.

Junction rule: the total current entering a junction equals the total current leaving the junction.

To analyze a multi-loop circuit (a circuit with multiple batteries connected in a way that they can not be replaced by a single equivalent battery) we use the following steps:

- 1. Label the currents in the various branches of the circuit.
- 2. Choose a loop and apply the loop rule to obtain an equation involving one or more of the unknowns (often, these are the currents). Repeat until each branch has been used at least once. Successive equations must involve a branch not involved in previous equations.
- 3. Apply the junction rule to obtain one or more equations relating the currents.
- 4. Solve the system of equations for the unknowns.

RC Circuits

In a series RC circuit, the current, and the potential differences across the resistor and the capacitor, change with time. When **the capacitor is being charged** by the battery:

$$I = \frac{\varepsilon}{R} e^{-t/RC} \quad \text{(Eq. 18.10: Current)} \qquad \Delta V_C = \varepsilon \left(1 - e^{-t/RC}\right) \text{(Eq. 18.11: Capacitor voltage)}$$
$$\Delta V_R = \varepsilon \ e^{-t/RC} \quad \text{(Eq. 18.12: Resistor voltage)}$$

When the battery is removed and the capacitor is discharging:

$$I = -\frac{\Delta V_{C,\max}}{R} e^{-t/RC} \quad \text{(Eq. 18.13)} \qquad \Delta V_C = \Delta V_{C,\max} e^{-t/RC} \quad \text{(Eq. 18.14: Capacitor voltage)}$$
$$\Delta V_R = -\Delta V_{C,\max} e^{-t/RC} \quad \text{(Eq. 18.15: Resistor voltage)}$$

End-of-Chapter Exercises

Exercises 1 – 12 are primarily conceptual questions designed to see whether you understand the main concepts of the chapter.

- You are responsible for running the lift at a ski hill, and you can adjust the speed of the lift so there is a smooth flow of skiers up and down the hill. The ski patrol notifies you that one of the trails down the hill (which is in parallel with other trails) needs to be closed temporarily because of an accident. Should you adjust the speed at which the lift carries skiers up the hill? Justify your answer based on an analogy with a circuit.
- 2. Many people think that current gets used up in a circuit, and that in a circuit like that in Figure 18.28, the current through bulb A is different from that through bulb B. (a) Is that true? (b) Use an analogy, such as the fluid analogy or the ski-hill analogy from Section 18.3, to justify your answer to part (a). (c) What gets used up in a circuit?
- 3. You have two resistors. You connect one resistor to a variable power supply (like a battery with a variable emf) and measure the current through it as a function of the potential difference across it. You repeat the process with the second resistor. You then repeat the process with the two resistors connected in parallel, measuring the total current in the circuit as a function of the potential difference across the parallel combination, and do it all a fourth time with the two resistors in series. (a) Which graph in Figure 18.29 goes with which process? (b) What is the resistance of each of the two resistors?
- 4. Graph c in Figure 18.29 shows the current through a resistor as a function of the voltage across it. (a) Is the resistor an ohmic device? How do you know? (b) What is the resistance of the resistor?
- 5. Four identical light bulbs are arranged in the circuit in Figure 18.30, connected to a 20-volt battery. There are also two switches in the circuit. Switch *a*, in series with bulb A, is initially closed, while switch *b*, in parallel with bulb B, is initially open. Assume that the resistance of each bulb is the same no matter how much current passes through it at all times, that the switches have negligible resistance when they are closed, and that the battery is ideal (no internal resistance). If switch *b* is then closed (and switch *a* remains closed) what happens to the brightness of (a) bulb A? (b) bulb B? (c) bulb C? (d) bulb D?

Figure 18.28: A circuit with two light bulbs, A and B, and a battery, for Exercise 2.

Figure 18.29: Graphs of current vs. potential difference for the situations described in Exercises 3 and 4.

- 6. Return to the system described in Exercise 5 and shown in Figure 18.30. In what position should the two switches be to maximize the brightness of (a) bulb A? (b) bulb B? (c) bulb C? (d) bulb D?
- 7. The four bulbs in Figure 18.30 are identical. Rank the bulbs based on their brightness, from brightest to dimmest, if (a) both switches are closed, (b) switch a is closed and switch b is open, (c) switch a is open and switch b is closed, (d) both switches are open.
- 8. As shown in Figure 18.31, three resistors are connected in a circuit with a 12-volt battery. The resistance of resistor C is neither zero nor infinite, but its exact value is unknown. Resistor A has a resistance of 4.0Ω . Resistor B has a resistance of 3.0Ω . (a) Can you say which resistor has the most current through it, or does that depend on the resistance of resistor C? If so, which resistor has the most current through it? (b) Is it possible to rank the resistors based on the potential difference across them? If so, rank them from largest to smallest.
- 9. Return to the situation shown in Figure 18.31, and described in Exercise 8. If the resistance of resistor C is increased, what happens to the current through (a) resistor A? (b) resistor B? (c) resistor C? (d) the battery? If the current through the battery is 2.0 A, what is the resistance of resistor C?
- 10. Consider the circuit from Essential Question 18.8, which is redrawn in Figure 18.32. (a) Is it possible to add a 5.0 Ω resistor to the circuit so that the current through at least one of the original resistors increases? (b) If not, explain why not. If so, determine how many ways there are to do this, draw a circuit diagram for each and, in each diagram, circle the resistor(s) with the increased current.
- 11. Consider the RC circuit shown in Figure 18.33. The battery has an emf of 18 V; and resistor A has a resistance of 3.0 ohms. At t = 0 the capacitor is uncharged. At some time later, when the potential difference across the capacitor is 3.0 V, the switch is closed. This brings resistor B, with a resistance of 6.0 ohms, into the circuit. Immediately after the switch is closed, determine: (a) the potential difference across resistor B, (b) the current through resistor A, (c) and the current through resistor B. (d) Describe the effect of closing the switch on the capacitor. In particular, does the rate at which it is charging change? (e) What is going on in the circuit a long time after the switch is closed?

Figure 18.31: A circuit with three resistors and a 12-volt battery, for Exercises 8 and 9.

Figure 18.33: An RC circuit consisting of a battery, resistor, and capacitor, along with a second resistor that can be added to the circuit by closing a switch. For Exercise 11.

12. A series RC circuit consists of a resistor, a capacitor that is initially uncharged, and a battery. Two graphs are shown for this situation in Figure 18.34. The lower graph is for the initial situation. The upper graph is for a second trial in which either the resistance or the capacitance has been changed. (a) If the graph shows the potential difference across the resistor as a function of time, as the capacitor charges, what was changed before the second trial, the resistance or the capacitance? Explain, and state whether the resistance or capacitance was increased or decreased. (b) Repeat part (a), but now the graph shows the current in the circuit as a function of time, as the capacitor charges, instead.

Figure 18.34: The graph shows either the current or the resistor voltage, as a function of time, for a series RC circuit in which the capacitor charges. For Exercise 12.

Exercises 13 – 19 deal with current, batteries, and resistance.

- 13. At a particular point, there is a current *I* associated with *N* protons flowing past the point in a time *T*. Which of the following changes, done individually (not sequentially) corresponds to a doubling of the current? Justify each answer. (a) The velocity of the protons is doubled, so N protons flow past in a time of T/2. (b) The number of protons flowing past in the time *T* is doubled. (c) Electrons are added to the system so that *N* protons and *N* electrons, all flowing the same way, pass by in a time *T*. (d) The protons are replaced by positive ions that each have a charge of +2e, flowing past at the same rate the protons were.
- 14. A current of 2.0 A is directed to the right past a particular point in a circuit. (a) If the charges flowing are electrons, in what direction are the electrons flowing? (b) Assuming all the electrons flow past in the same direction, how many electrons flow past every second? (c) Do all the electrons really flow past in the same direction? If not, what does the answer to part (b) represent?
- 15. A particular battery is rated at 2800 milliamp-hours. (a) What kind of unit is the milliamp-hour? (b) If this battery is powering a digital camera, which uses a current of 600 mA, how long will the battery last?
- 16. The wire connecting a wall switch to an overhead light is 2.4 m long. When the light is turned on there is a net flow of electrons in one direction along the wire; in other words, there is a current in the wire. This net flow is actually rather slow. In this case let's say the average speed of the electrons is 0.20 mm/s (this is known as the drift speed). (a) Based on this, what is the average time it takes an electron at the switch to reach the light bulb filament when the switch is closed (and assuming the electrons flow in that direction!)? (b) In your experience, how long does it take the bulb to come on when you close the switch? (c) How do you reconcile the answers from parts (a) and (b)?
- 17. A particular wire has a circular cross-section and a resistance *R*. The wire is then "drawn out" stretched until it is four times longer but has the same volume as before. (a) Has its resistance increased, decreased, or stayed the same? (b) What is the resistance of the wire now, in terms of *R*?
- 18. A nichrome wire has a length of 2.5 m. How long should a copper wire be to have the same resistance as the nichrome wire? The wires have the same cross-sectional area.

19. We can construct a resistor with a resistance that does not change with temperature by joining a material with a positive temperature coefficient to a material with a negative temperature coefficient, if we choose the parameters correctly. For instance, let's say we want to make a resistor with a total resistance of 100 ohms. We will do this by joining together, end-to-end, a length of copper wire (copper has a temperature coefficient of +0.0068/°C) and a length of carbon wire (carbon has a resistivity of 3.0 x 10⁻⁴ Ω m and a temperature coefficient of -0.00050/°C). If each wire has a circular cross-section with a radius of 1.25 mm, and we want the resistance to be 100 ohms even if the temperature fluctuates, how long should the two pieces of wire be?

Exercises 20 – 25 deal with resistors in series or resistors in parallel.

- 20. **Application: a three-way light bulb**. One way to make a three-way light bulb (a bulb that shines with three different brightnesses, depending on the position of a switch) is to allow connections to the bulb filament so that when the switch is in position 1 the 120 V from the wall socket is connected across the entire filament; in position 2 the 120 V is connected across 40% of the length of the filament; and in position 3 the 120 V is connected across the remaining 60% of the length of the filament. Assume the filament has a uniform cross-section, and that the resistance of the filament is independent of the filament's temperature. (a) Rank the switch positions based on the brightness of the bulb, from brightest to dimmest. (b) If the bulb dissipates 150 W when the switch is in position 2, how much power is dissipated in the other two switch positions?
- 21. Application: another three-way light bulb. A second way to make a three-way light bulb is to have two different filaments. In switch position 1, the wall socket voltage is across one filament, which dissipates 50 W. In position 2, the voltage is across the second filament, which dissipates 100 W. In position 3, the voltage is across both filaments in parallel. (a) What is the resistance of the 50-W filament? (b) What is the resistance of the 100-W filament? (c) What is the equivalent resistance of the two filaments when they are in parallel? (d) What is the power dissipated when the switch is in position 3?
- 22. You have three identical resistors, one battery, and a number of wires. (a) Show how you can connect the resistors and the battery in a circuit so that the currents through the three resistors are equal (and non-zero!). Is there more than one way to accomplish this? (b) Show how you can connect the resistors and the battery in a circuit so that the current through one of the resistors is twice as large as that through each of the other two resistors. Is there more than one way to accomplish this?
- 23. You have five identical resistors. You connect one or more of the resistors in some way between point A and point B; one or more in some way between point B and point C; and one or more in some way between point C and point D. There are no other connections between the points, which are all along a straight line in alphabetical order. You then measure the resistance between points A and B to be 5 Ω , between points B and C to be 10 Ω , and between points C and C to be 20 Ω . (a) Find the resistance of each resistor. (b) Sketch a diagram showing how the resistors are connected.
- 24. You have two identical resistors that are in parallel with one another. When a third identical resistor is wired in parallel with the first two, the equivalent resistance of the set of resistors changes by 6.0Ω . (a) Does the equivalent resistance increase or decrease? (b) What is the resistance of one of these resistors?

25. You have N identical resistors connected in series with a 12-volt battery. The current through each resistor is 0.50 A. When one more resistor, identical to the others, is added to the circuit, in series, the current through each resistor drops to 0.40 A. (a) What is N? (b) What is the resistance of each resistor?

Exercises 26 – 36 involve series-parallel combination circuits.

- 26. You have three resistors, with resistances of 23 Ω , 47 Ω , and 100 Ω . By using one, two, or all three, list the different equivalent resistance values can you create with these resistors.
- 27. As shown in Figure 18.35, four resistors are connected in a circuit with a 20-volt battery. (a) Identify two resistors that are either in series or in parallel, and replace them by a single equivalent resistor. Re-draw the circuit. (b) Repeat part (a) twice more until the circuit has been reduced to a single equivalent resistor. (c) Apply Ohm's Law to find the total circuit current. (d) Reverse the order of the steps in the contraction, and expand the circuit back to its original configuration. At each step in the expansion, draw the circuit diagram and label the current through each resistor. Also, label the potential at various points in the circuit, relative to V = 0 at the negative terminal of the battery.
- 28. Four resistors are connected to a battery that has an emf of ε , as shown in Figure 18.36. The battery has a current *I* passing through it, and provides a total power *P* to the circuit. State whether each of the following statements is true or false, and explain your answer. (a) The sum of the currents through the four resistors is equal to *I*. (b) The sum of the potential differences across the four resistors is equal to ε . (c) The sum of the power dissipated in the four resistors is equal to *P*.
- 29. Four resistors are connected to a battery that has an emf of ε , as shown in Figure 18.36. (a) What is the equivalent resistance of the circuit, in terms of *R*? (b) What fraction of *I*, the current through the battery, passes through the 5*R* resistor? (c) Rank the resistors based on the current through them, from largest to smallest. (d) Rank the resistors based on the potential difference across them, from largest to smallest.
- 30. Figure 18.37 shows five identical light bulbs in a circuit with a battery. Rank the bulbs based on their brightness, from brightest to dimmest.
- 31. Figure 18.37 shows five identical light bulbs in a circuit with a 120-volt battery. Each bulb dissipates 150 W of power if it has a potential difference of 120 V across it. Assume that the resistance of each bulb stays the same no matter what the current is through the bulb. (a) What is the

Figure 18.35: Four resistors are connected to a battery that has an emf of 20 V. For Exercise 27.

Figure 18.36: Four resistors are connected to a battery that has an emf of ϵ . For Exercises 28 and 29.

Figure 18.37: Five identical light bulbs are placed in a circuit with a battery. For Exercises 30 and 31.

resistance of one of these bulbs? (b) Is the total power dissipated in this circuit more than or less than 150 W? Justify your answer without explicitly calculating the power. (c) Find the power dissipated in each bulb, and the total power dissipated in the circuit.

- 32. Five resistors are wired in a circuit with a single battery, as shown in Figure 18.38. (a) Is resistor A connected in series with resistor B, in parallel with resistor B, or neither? Explain. (b) Rank the resistors based on the current through them, from largest to smallest.
- 33. Five resistors are wired in a circuit with a single battery, as shown in Figure 18.38. (a) In terms of R, what is the equivalent resistance of the circuit? (b) If the current through resistor D is I_D , find the current through each of the other resistors, in terms of I_D . (c) Rank the resistors based on the potential difference across them, from largest to smallest.
- 34. Three resistors are connected in an equilateral triangle, as shown in Figure 18.39. A battery is connected to this set of resistors so that its positive terminal is connected by a wire to one of the lettered points, and the negative terminal is connected to another of the lettered points. Consider the following three cases:

Case 1: Positive terminal connected to a; negative terminal to b.

Case 2: Positive terminal connected to a; negative terminal to c.

- Case 3: Positive terminal connected to b; negative terminal to c. Find the equivalent resistance of the circuit in (a) case 1, (b) case 2, and (c) case 3.
- 35. Four resistors are connected together in the configuration shown in Figure 18.40. A battery is connected to this set of resistors so that its positive terminal is connected by a wire to one of the lettered points, and the negative terminal is connected to another of the lettered points. Consider the following three cases:

Case 1: Positive terminal connected to a; negative terminal to b.

- Case 2: Positive terminal connected to a; negative terminal to c.
- Case 3: Positive terminal connected to b; negative terminal to c. Rank these cases based on (a) their equivalent resistance; (b) the magnitude of the current through the 4 Ω resistor; (c) the magnitude of the potential difference across the 8 Ω resistor.
- 36. Return to the situation described in Exercise 35, and shown in Figure 18.40. In which of the cases is (a) R₂ in parallel with R₃? (b) R₁ in series with R₄? (c) R₁ in series with the combination of R₂ and R₃? Explain your answer.

Figure 18.38: Five resistors are wired in a circuit with a single battery. For Exercises 32 and 33.

Figure 18.39: Three resistors connected in an equilateral triangle, for Exercise 34.

Figure 18.40: Four resistors, and three possible locations the terminal of a battery can be connected to. For Exercises 35 and 36.

Exercises 37 – 43 deal with Kirchoff's Rules and multi-loop circuits.

- 37. If the current through the 56 Ω resistor in Figure 18.41 is 95 milliamps, what is the emf of the battery?
- 38. The four resistors from Exercise 37 are connected in a different circuit with a different battery, of unknown emf, as shown in Figure 18.42. If the potential difference across the 56 Ω resistor is 3.2 V, what is the emf of this battery?
- 39. Three batteries are connected in a circuit with three resistors, as shown in Figure 18.43. The currents in two of the branches are known and marked correctly on the diagram. (a) Determine the magnitude and direction of the current through the 2-volt battery. (b) Find the emf of the battery in the middle branch of the circuit. (c) Find the value of the resistance *R* of the resistor at the bottom left. (d) What is the potential difference, $V_a V_b$, between points *a* and *b* in the circuit?
- 40. The multi-loop circuit in Figure 18.44 is made up of three resistors and two batteries. The various currents in the circuit are labeled. (a) Write out the junction rule at one junction in the circuit. (b) Show that applying the junction rule at the other junction results in the same equation. (c) Apply the loop rule to the inside loop on the left to obtain a loop equation. (d) Apply the loop rule to the inside loop on the right to obtain another loop equation. (e) Apply the loop rule to the outside loop to obtain a third loop equation. (f) Show that one of your loop equations can be obtained by either adding or subtracting the other two loop equations.
- 41. Return to the situation described in Exercise 40 and shown in Figure 18.44. The two batteries have emf's of $\varepsilon_1 = 8.0$ V

and $\varepsilon_3 = 12.0$ V. If the resistors have resistances of

 $R_1 = 4.0 \Omega$; $R_2 = 6.0 \Omega$; and $R_3 = 8.0 \Omega$, determine the three currents in the circuit.

42. Three 3.00 Ω resistors, three batteries, an ammeter, a voltmeter, and one unknown resistor *R* are connected in the circuit in Figure 18.45. Assume that the ammeter has negligible resistance and that the voltmeter has infinite resistance. The batteries have emf's of: $\varepsilon_1 =$

20.0 volts; $\varepsilon_2 = 5.00$ volts; and $\varepsilon_3 = 10.0$ volts. The ammeter reads +0.650 A, the positive

Figure 18.41: Four resistors connected in a circuit with a battery of unknown emf, for Exercise 37.

Figure 18.42: Four resistors connected in a circuit with a battery of unknown emf, for Exercise 38.

Figure 18.43: A multi-loop circuit, for Exercise 39.

Figure 18.44: A multi-loop circuit consisting of three resistors and two batteries. For Exercises 40 and 41.

Figure 18.45: A multi-loop circuit consisting of three batteries and four resistors, as well as one ammeter and one voltmeter. For Exercise 42.

value indicating that the current direction shown for I_3 on the diagram is correct. The directions shown for the other two currents may or may not be correct. For parts (a) and (b), include a minus sign if the current is in the direction opposite to that shown on the diagram. (a) What is the value of I_2 ? (b) What is the value of I_1 ? (c) What is the resistance of the unknown resistor R? (d) What is the reading on the voltmeter? The voltmeter gives a positive value if the potential at its positive terminal is higher than the potential at its negative terminal.

43. A particular multi-loop circuit consists of three batteries and three resistors, as shown in Figure 18.46. The three currents in the circuit are labeled, but their directions are not necessarily correct. (a) Apply the junction rule to obtain an equation relating the three currents. (b) Based on your junction equation, would you expect one or more of the currents in the circuit to be directed opposite to the direction shown on the diagram? Explain.

44. A particular multi-loop circuit consists of three batteries and three resistors, as shown in Figure 18.46. The battery emf's are $\varepsilon_1 = 10$ V; $\varepsilon_2 = 2$ V, and $\varepsilon_3 = 6$ V. (a) Solve **Figure 18.46**: A particular multi-loop circuit consists of three batteries and three resistors. For Exercises 43 – 45.

for the magnitude and direction of the three unknown currents. Note that you should not have to do lots of algebra to solve this problem. (b) What is the potential difference between points A and B in the circuit? Which point is at the higher potential?

45. Return to the situation described in Exercise 44, and shown in Figure 18.46. By adjusting the emf of the second battery (that is, by adjusting ε_2) you can set $I_2 = 0$. (a) What is ε_2

in this situation? (b) What is the potential difference between points A and B in this situation? Which point is at the higher potential?

Exercises 46 and 47 deal with the internal resistance of a battery.

- 46. A real battery can be modeled as an emf in series with a small internal resistor of resistance r. The circuit on the left in Figure 18.47 shows one such battery connected in a circuit with a voltmeter and a resistor. When R, the resistance of the other resistor in the circuit, is very large the voltmeter reads 6.3 V. When R is 5.0 Ω the voltmeter reads 5.8 V. What is the battery's internal resistance r?
- 47. As shown in the circuit on the left in Figure 18.47, a resistor *R* with a resistance of 9.80 Ω is connected to a battery with an emf of $\varepsilon = 6.00$ volts and an internal resistance of

Figure 18.47: The circuit at left shows one battery connected in a circuit with a voltmeter and a resistor, while in the circuit at right a second battery, with a different emf, has been added. For Exercises 46 - 47.

 $r = 1.00 \ \Omega$. (a) What is the current through resistor R? (b) The voltmeter measures the terminal voltage on the battery. Assuming the voltmeter has an infinite resistance, what is the terminal voltage? (c) As shown in the figure on the right, a second battery with an unknown emf ε_2 and an internal resistance of $r = 1.00 \ \Omega$ is placed in parallel with the

first battery. The terminal voltage measured by the voltmeter is now 5.00 V. What is the current through resistor R now? (d) What is the emf of the second battery?

General problems and conceptual questions

- 48. A hair dryer is rated at 1850 W. How much does it cost you to use the hair dryer for 10 minutes every day if you are charged 20 cents for every kW-h?
- 49. A particular 100-W incandescent light bulb costs \$0.50, and can be left on for 1000 hours before burning out. A compact fluorescent light bulb of equal brightness, but with a power rating of 20 W, costs \$4.00, and lasts for 5000 hours. Over the lifetime of the bulb, what is the total energy used by (a) the incandescent bulb? (b) the fluorescent bulb? For providing 5000 hours of light, compare the total energy used by, and total cost, of (c) five 100-W bulbs, and (d) one fluorescent bulb. Assume you are charged 20 cents for every kW-h.
- 50. Rank the following based on the daily energy cost associated with them. A: a clock radio rated at 5 W (5 watts) that is on 24 hours a day. B: a 300-W television that is on for 3 hours a day. C: a 1000-W microwave oven that is in use for 20 minutes a day. D: a 90-W computer that is on for 8 hours a day.
- 51. Four resistors are connected to a battery, as shown in Figure 18.48. If the potential difference across through the 38 Ω resistor is 2.5 V, what is the emf of the battery?

Figure 18.48: Four resistors connected in a circuit with a battery of unknown emf, for Exercise 51.

52. The four resistors from Exercise 51 are connected in a different circuit with a different battery, of unknown emf, as shown in Figure 18.49. If the current through the 38 Ω resistor is 84 milliamps, what is the current through the 47 Ω resistor?

Figure 18.49: Four resistors connected in a circuit with a battery of unknown emf, for Exercise 52.

53. Three resistors are connected in a circuit with a 9-volt battery, as shown in Figure 18.50. Resistor 1 has a resistance of 9 Ω and resistor 3 has a resistance of 6 Ω . The resistance of resistor 2 is some value between 4 Ω and 20 Ω . (a) Can you tell which resistor has the largest current through it, or does that depend on the value of resistor 2? Explain. (b) If the resistance of resistor 1? Explain.

Figure 18.50: A circuit consisting of three resistors and a 9-volt battery. For Exercises 53 – 54.

54. Three resistors are connected in a circuit with a 9-volt battery, as shown in Figure 18.50. Resistor 1 has a resistance of 9 Ω and resistor 3 has a resistance of 6 Ω . The resistance of resistor 2 is 12 Ω . (a) What is the equivalent resistance of the circuit? (b) What is the value of the current through resistor 3?

55. Consider the circuit from Essential Question 18.8, which is re-drawn in Figure 18.51. (a) How would you add a 5.0 Ω resistor to the circuit so that the current through the 3.0 Ω resistor is maximized?
(b) By how much does the current through the 3.0 Ω resistor increase in that case?

- 56. Figure 18.52 shows a circuit consisting of five resistors and one battery. (a) Find the equivalent resistance of the circuit, in terms of R. (b) Which resistor has the largest potential difference across it? Briefly justify your answer. (c) Rank the resistors based on the current passing through them, from largest to smallest.
- 57. Figure 18.52 shows a circuit consisting of five resistors and one battery. The battery has an emf of 2.0 V, and $R = 2.0 \Omega$. Find the current through each resistor.

Figure 18.52: A circuit consisting of five resistors and a battery, for Exercises 56 and 57.

- 58. Return to the Answer to Essential Question 18.9. Note that the current through the 2.0-volt battery is passing through the battery from the positive terminal to the negative terminal. The current through the other two batteries is directed from the negative terminal to the positive terminal. (a) What is happening to the 2.0-volt battery in this situation? (b) Find the total power input to the circuit, and the total power dissipated in the circuit. Should we expect these values to be the same?
- 59. Five resistors are connected in a circuit with a battery, as shown in Figure 18.53. (a) Which resistor has the largest current passing through it? Explain. Now compare just the 8 Ω resistor and the 10 Ω resistor. Of these two resistors, which has the largest (b) current through it? (c) potential difference across it? Justify your answers.
- 60. Five resistors are connected in a circuit with a battery, as shown in Figure 18.53. (a) Re-draw the circuit diagram, adding an ammeter to measure the current through the 5 Ω resistor, and a voltmeter to measure the potential difference across the 5 Ω resistor. (b) If the battery has an emf of 20 V, what is the reading on the ammeter? (c) What is the reading on the voltmeter?

Figure 18.53: A circuit consisting of five resistors and a battery. For Exercises 59 and 60.

- 61. Consider the circuit shown in Figure 18.54. (a) Starting at the negative terminal of the 16-V battery, and going clockwise around the loop, write out a loop equation for the inside loop on the left by applying the loop rule. (b) What is the effect of using a different starting point? Write out a loop equation for the same loop, starting at the bottom left corner of the circuit, and going clockwise around the loop. Is this equation equivalent to the equation from (a)? Explain. (c) What is the effect of going counter-clockwise around the loop instead? Write out a loop equation for the same loop, starting at the same place as in (a) but going counter-clockwise. Is this equation equivalent to the equation from (a)? Explain.
- 62. Consider the circuit shown in Figure 18.54. Find (a) *I*₁, (b) *I*₂, and (c) *I*₃.
- 63. A series RC circuit consists of a 12-volt battery, a resistor, capacitor, and a switch, all connected in series. The switch is open and the capacitor is initially uncharged. The switch is closed at t = 0 and the capacitor begins to charge. At t = 3.0 s, the potential difference across the capacitor is 2.0 V. (a) What is the potential difference across the capacitor at t = 6.0 s? (b) At what time is the potential difference across the capacitor equal to 6.0 V? (c) At what time is it equal to 12 V?
- 64. Consider the RC circuit shown in Figure 18.55. The battery has an emf of 12 V; and the two resistors each have a resistance of 3.0 ohms. At t = 0, the capacitor is charged, with the top plate positive and the capacitor voltage is equal to 12 V. The capacitance of the capacitor is 1.0 F. (a) Assuming the switch remains open, find the current in the circuit at t = 5.0 s. The switch is then closed. Determine the current through each of the two resistors (b) immediately after the switch is closed, (c) when the capacitor voltage is 9.0 V, and (d) a long time after the switch is closed.
- 65. Comment on the statements made by three students who are discussing an issue related to DC circuits. The question that they are discussing is whether it is possible to add a resistor to a circuit and increase the current passing through the single battery in the circuit. There is already a non-zero current in the circuit before the resistor is added.

Terry: I don't think that adding a resistor to the circuit makes any difference to the current. The current comes from the battery, and if we don't change the battery then the current stays the same.

Sarah: I disagree with that. If we connect the resistor in parallel with another resistor in the circuit, then the equivalent resistance of the circuit is going to go down, and the current through the battery is going to go up.

Andy: No way! I disagree with Terry, too, but I think that if you add a resistor to the circuit that you have to be increasing the total resistance, not decreasing it. That's going to decrease the current, not increase it.

Figure 18.55: An RC circuit consisting of a battery, resistor, and capacitor, along with a second resistor that can be added to the circuit by closing a switch. For Exercise 64.

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Chapter 18: Additional Resources

Pre-session Movies on YouTube

- <u>Current, Batteries, and Resistance</u>
- Resistors in Parallel, Resistors in Series
- <u>Series-Parallel Combination Circuits</u>
- <u>Worksheet for the Series-Parallel Combination Circuits Video</u>
- <u>Kirchoff's Rules</u>

Examples

• <u>Sample Questions</u>

Solutions

- <u>Answers to Selected End of Chapter Problems</u>
- <u>Sample Question Solutions</u>

Simulations

- Simulation: RC Circuit
- <u>Worksheet for the RC Circuit Simulation</u>

Additional Links

- PhET simulation: Ohm's Law
- *PhET* simulation: Battery-Resistor Circuit
- PhET simulation: Circuit Construction Kit (DC Only)

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