17-1 Electric Potential Energy

Whenever charged objects interact with one another, there is an energy associated with that interaction. In general, we have two special cases to consider. The first is the energy associated with a charged object in a uniform electric field, and the second is the energy associated with the interaction between point charges. These are analogous to the two situations we examined earlier for gravity.

> The potential energy associated with the interaction between one object with a charge q and a second object with a charge Q that is a distance r away is given by:
>
> $$U_E = \frac{kqQ}{r}$$ (Eq. 17.1: **Potential energy for the interaction between two charges**)
>
> where $k = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$ is a constant. If the charges are of opposite signs, then the potential energy is negative – this indicates an attraction. If the charges have the same sign, the potential energy is positive, indicating a repulsion.

Once again, we can see the parallel with gravity, for which the equivalent expression is:

$$U_G = -\frac{GmM}{r}$$ (Eq. 8.4: **Potential energy for the interaction between two masses**)

where $G = 6.67 \times 10^{-11} \text{N m}^2/\text{kg}^2$ is the universal gravitational constant. For the interaction between charges k takes the place of G, and the values of the charges q and Q take the place of the values of the masses, m and M.

Case 1 – A charged object in a uniform electric field

This situation is directly analogous to the situation of an object with mass in a uniform gravitational field. When we raise a ball of mass m through a height h in a uniform gravitational field g directed down, the change in potential energy is $\Delta U_G = mgh$. Note that we take h to be the component of the displacement parallel to, and opposite in direction to, the field. If the ball experiences a displacement $\Delta\vec{r}$ then an equivalent equation is $\Delta U_G = -m\vec{g} \bullet \Delta\vec{r} = -mg\Delta r\cos\theta$, where θ is the angle between the gravitational field and the displacement.

The equivalent expression for a charge object in a uniform electric field is:

$$\Delta U_E = -q\vec{E} \bullet \Delta\vec{r} = -qE\Delta r\cos\theta,$$ (Eq. 17.2: **Change in potential energy in a uniform field**)

where θ is the angle between the electric field $\vec{E}$ and the displacement $\Delta\vec{r}$.

We are free to define the zero level of potential energy, but only in a uniform field.

Related End-of-Chapter Exercise: 6.

Case 2 – The electric potential energy of a set of point charges

In this situation, we look at the interaction between pairs of charges. We use equation 17.1 to calculate the energy of each pair of interacting objects, and then simply add up all these numbers because potential energy is a scalar. Note that we are not free to define the zero level. The zero is defined by equation 17.1, in fact, because the potential energy goes to zero as the distance between the charges approaches infinity.

Compare Exploration 17.1 to Exploration 8.4

EXPLORATION 17.1 – Calculate the total potential energy in a system

Three charged balls, of charge $-q$, $+2q$, and $-3q$, are placed in a line, as shown in Figure 17.1. What is the total electric potential energy of this system?

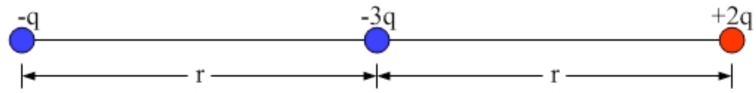


Figure 17.1: Three equally spaced balls placed in a line.

To determine the total potential energy of the system, consider the number of interacting pairs. In this case, there are three ways to pair up the objects, so there are three terms to add together to find the total potential energy. Because energy is a scalar, we do not have to worry about direction. Using a subscript of 1 for the ball of charge $-q$, 2 for the ball of charge $+2q$, and 3 for the ball of charge $-3q$, we get:

$$U_{Total} = U_{13} + U_{23} + U_{12} = \frac{k(-q)(-3q)}{r} + \frac{k(+2q)(-3q)}{r} + \frac{k(-q)(+2q)}{2r} = -\frac{4kq^2}{r}.$$

When a system has a negative total energy (including the total kinetic energy, of which there is none in this situation), that is indicative of a bound system. In general, there is a greater degree of attraction in the system than repulsion.

Key ideas for electric potential energy: Potential energy is a scalar. The total electric potential energy of a system of objects can be found by adding up the energy associated with each interacting pair of objects. **Related End-of-Chapter Exercises: 4, 42, 46.**

Work – an equivalent approach

Consider again the system shown in Figure 17.1. If the three charged balls start off infinitely far away from their final positions, and infinitely far from one another, how much work do we have to do to assemble the balls into the configuration shown in Figure 17.1? Assume that, aside from their interactions with us, the balls interact only with one another, electrostatically.

Pick one ball to bring into position first. Let's start with the ball with the $-3q$ charge. Because the other charged balls are still infinitely far away, it takes no work to bring the first ball into position. There are no other interactions to worry about.

Now, let's bring the ball with the $-q$ charge into position. The potential energy changes from 0, when the two balls are infinitely far away, to $+3kq^2/r$, when those two balls are in their final positions. This potential energy comes from work we do – we do $+3kq^2/r$ worth of work.

Finally, bring the ball with the $+2q$ charge into position. The potential energy associated with this ball changes from 0, when that ball is at infinity, to $-2kq^2/(2r) + (-6kq^2)/r = -7kq^2/r$, when the ball with the $+2q$ charge is in its final position.

Adding the two individual work values to find the total work to assemble the system gives $+3kq^2/r - 7kq^2/r = -4kq^2/r$, the same result we got for the potential energy of the system. The work done in assembling the system is equal to the system's potential energy.

Essential Question 17.1: Return to Exploration 17.1. If we replace the ball of charge $+2q$ by a ball of charge $-2q$, does the potential energy of the system increase, decrease, or stay the same?

Answer to Essential Question 17.1: The potential energy of the system would increase and, in fact, would become $+10kq^2/r$. This is because the balls would all have charge of the same sign, and, overall, the particles of the system would repel one another.

17-2 Example Problems Involving Potential Energy

EXAMPLE 17.2A – Stopping an electron

An electron with a speed of 1.2×10^5 m/s enters a region where there is a uniform electric field with E = 500 N/C, as shown in Figure 17.2.

(a) Show that gravitational influences can be neglected in this situation.

(b) If, after traveling for some distance d in the field, the electron comes to rest for an instant, in which direction is the field?

(c) Calculate d.

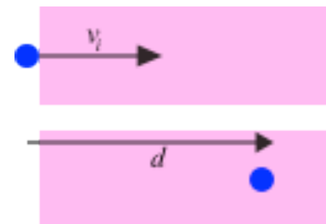


Figure 17.2: The top diagram shows the electron entering the electric field. The bottom diagram shows the point where the electron comes instantaneously to rest.

SOLUTION

(a) Let's assume the electron is at the surface of the Earth, where the acceleration due to gravity is the familiar g = 9.8 m/s^2. What is the acceleration associated with the electric field? The magnitude of the force applied to the electron by the field is $F = qE$. Dividing by the mass gives the magnitude of the acceleration of the electron:

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{(1.60\times10^{-19}\text{ C})(500\text{ N/C})}{9.11\times10^{-31}\text{ kg}} = 8.78\times10^{13}\text{ N/kg}.$$

This is 13 orders of magnitude larger than g, which justifies neglecting gravity.

(b) To stop the electron, the force on the electron from the field must be directed opposite to the electron's initial velocity. Because the electron has a negative charge, the field and force are in opposite directions. Thus, the electric field must be in the same direction as the electron's initial velocity.

(c) Let's apply energy conservation here, using the usual energy equation:
$U_i + K_i + W_{nc} = U_f + K_f$.

In this case, there are no non-conservative forces acting, and the final state is the point at which the electron comes instantaneously to rest. Thus, the final kinetic energy is zero. In addition, we can define the initial electric potential energy to be zero. This gives $K_i = U_f$.

The final potential energy is given by: $U_f = U_i + \Delta U = 0 + \left(-(-e)Ed\right) = +eEd$.

Putting everything together gives: $\frac{1}{2}mv_i^2 = +eEd$.

Solving for the distance gives: $d = \frac{mv_i^2}{2eE} = \frac{(9.11\times10^{-31}\text{ kg})(1.2\times10^5\text{ m/s})^2}{2(1.60\times10^{-19}\text{ C})(500\text{ N/C})} = 8.2\times10^{-5}\text{ m}$.

Compare the following Example to Example 8.4.

EXAMPLE 17.2B – Applying conservation ideas

Two identical balls, of mass 1.0 kg and charge +2.0 μC, are initially separated by 2.0 m in a region of space in which they interact only with one another. When they are released from rest, they accelerate away from one another. When they are 4.0 m apart, how fast are they going?

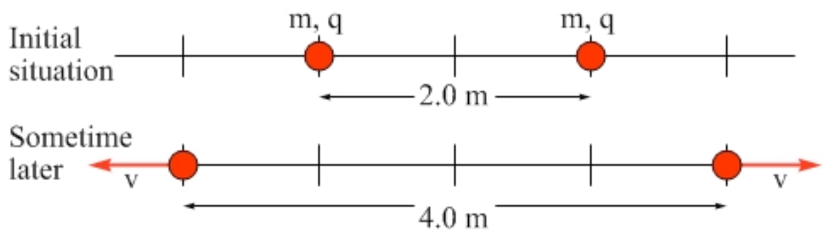


Figure 17.3: The balls are initially at rest. Repulsion from their like charges causes them to accelerate away from one another.

SOLUTION

Figure 17.3 shows the balls at the beginning and when they are separated by 4.0 m. Analyzing forces, we find that the force on each ball decreases as the distance between the balls increases. This makes it difficult to apply a force analysis. Energy conservation is a simpler approach. Our energy equation is:

$$U_i + K_i + W_{nc} = U_f + K_f \, .$$

In this case, no non-conservative forces act and, in the initial state, the kinetic energy is zero because both objects are at rest. This gives $U_i = U_f + K_f$. The final kinetic energy represents the kinetic energy of the system, the sum of the kinetic energies of the two objects.

Do we need to account for gravity here? In this situation we do not, which we can tell by calculating that the gravitational potential energy is much less than the electric potential energy. The change in potential energy is really what is important, but that is of the same order of magnitude as the potential energy.

$$U_{iG} = -\frac{Gmm}{r} = -\frac{\left(6.67\times10^{-11}\text{ N m}^2/\text{kg}^2\right)(1.0\text{ kg})^2}{\text{r}} \qquad U_{iE} = \frac{kqq}{r} = -\frac{\left(8.99\times10^{9}\text{ N m}^2/\text{C}^2\right)(2.0\times10^{-6}\text{ C})^2}{\text{r}}$$

Let's solve this generally, using a mass of m and a final speed of v for each ball.

The energy equation becomes: $\dfrac{kqq}{2.0\text{ m}} = \dfrac{kqq}{4.0\text{ m}} + \dfrac{1}{2}mv^2 + \dfrac{1}{2}mv^2$.

Simplifying: $\dfrac{kq^2}{4.0\text{ m}} = mv^2$.

Solving for v gives:

$$v = \sqrt{\frac{kq^2}{m(4.0\text{ m})}} = \sqrt{\frac{\left(8.99\times10^{9}\text{ N m}^2/\text{C}^2\right)\left(2.0\times10^{-6}\text{ C}^2\right)}{(1.0\text{ kg})(4.0\text{ m})}} = 9.5\times10^{-2}\text{ m/s} \, .$$

Related End-of-Chapter Exercises: Problems 1, 2, 3, 32, 33.

Essential Question 17.2: How would we solve Example 17.2B if the balls had different masses? To be specific, let's say the ball on the left has a mass of 1.0 kg while the ball on the right has a mass of 3.0 kg. What are the speeds of the two balls in that case?

Answer to Essential Question 17.2: Because there is no net external force, the system's momentum is conserved. There is no initial momentum. For the net momentum to remain zero, the two momenta must always be equal-and-opposite. Defining right to be positive, and using 1 as a subscript for the ball on the left and 2 for the other ball, momentum conservation gives:

$0 = -mv_1 + 3mv_2$, which we can simplify to $v_1 = 3v_2$.

The energy conservation equation is: $+\frac{kq^2}{2.0 \text{ m}} = m(3v_2)^2 + 3mv_2^2 = 12mv_2^2$

This gives $v_2 = \sqrt{\frac{kq^2}{m(24.0 \text{ m})}}$, and $v_1 = 3v_2 = 3\sqrt{\frac{kq^2}{m(24.0 \text{ m})}}$.

Using m = 1.0 kg, we get $v_2 = 3.9 \times 10^{-2}$ m/s and $v_1 = 12 \times 10^{-2}$ m/s.

17-3 Electric Potential

Gravitational potential, V_g, is related to gravitational potential energy in the same way that field is related to force, by a factor of the mass. Like field, gravitational potential is a way of measuring how an object with mass, or a set of objects with mass, influences the region around it.

The electric potential, V, at a particular point can be defined in terms of the electric potential energy, U, associated with an object of charge q being placed at that point:

$$V = \frac{U}{q}, \quad \text{or} \quad U = qV. \qquad \text{(Eq. 17.3: \textbf{Connecting electric potential and potential energy})}$$

The unit for electric potential is the volt (V). 1 V = 1 J/C.

A special case is the electric potential from a point charge with a charge Q:

$$V = \frac{kQ}{r}, \qquad \text{(Equation 17.4: \textbf{Electric potential from a point charge})}$$

where r is the distance from the charge to the point in space where we are finding the potential. Electric potential is a scalar, so it has a sign but not a direction.

We can visualize potential by drawing **equipotentials**, lines or surfaces that connect points of the same potential. On a 2-D picture like Figure 17.4 equipotentials are lines; in a three-dimensional world we have equipotential surfaces. Around a point charge, for instance, the equipotential surfaces are spheres centered on the charge. As Figure 17.4 suggests, we can define a gravitational potential analogous to electric potential.

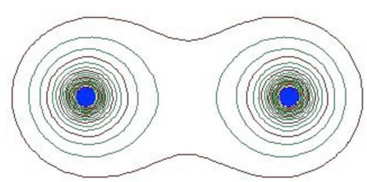
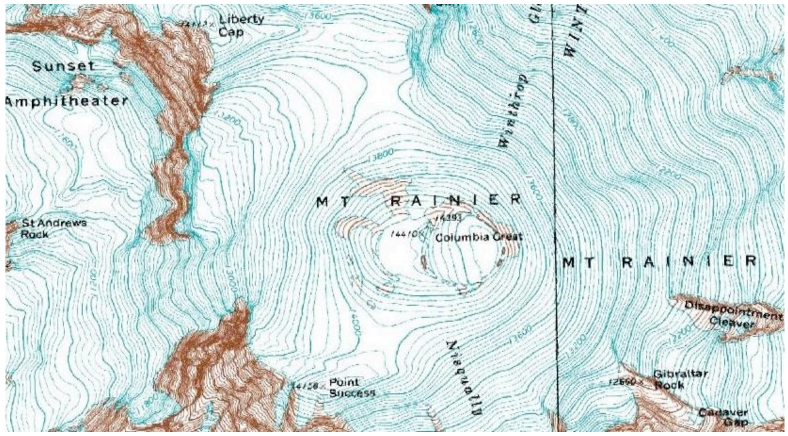


Figure 17.4: This diagram could represent the electric potential near two like charges, or the gravitational potential near two hills.

This is a little abstract, but you have probably seen gravitational equipotential lines, or contour lines, on a topographical map, as in Figure 17.5. Contour lines connect points at the same height – such points have the same gravitational potential. Where the lines are close together the terrain is steep; where the lines are far apart the terrain is flatter. If you want to go for an easy hike you have two choices. You can stick to flat terrain where the contour lines are far apart, or you can walk along a trail along a contour line, where the height is constant.

Figure 17.5: Contour lines on a topographical map, showing the terrain around the summit of Mt. Rainier, in Washington State. Photo credit: NASA/USGS.



Moving along a contour line requires no work, while moving from one contour line to another does involve work, because the gravitational potential energy changes. Equipotential lines give us the same information. Moving a charged object at constant speed from one place to another along an equipotential requires no work; but moving from one equipotential to another does involve work. Note that equipotential lines are always perpendicular to field lines, and the field points in the direction of decreasing potential.

Potential in a Uniform Field

In a uniform field, we can define the zero for potential to be anywhere we find to be convenient. The value of the potential at a particular point is not what is important. What is of primary importance is how the potential changes from one point to another. In a uniform electric field we define the potential difference (the difference in potential between two points) as:

$$\Delta V = -E\,\Delta r\cos\theta \,, \qquad \text{(Equation 17.5: \textbf{Potential difference in a uniform field})}$$

where θ is the angle between the electric field $\vec{E}$ and the displacement $\Delta\vec{r}$.

Electric potential has units of volts, or J/C. We see that the electric potential difference between points A and B in Figure 17.6 is +20 J/C (starting at A and going to B), and between points C and B (starting at C and going to B) it is also +20 J/C. If we move an object that has a charge of +2 mC from A to B, or from C to B, what is the change in electric potential energy? This equals the work we do to move the object, assuming its kinetic energy remains the same.

$$\Delta U = q\,\Delta V = q\left(V_B - V_A\right) = 2\text{ mC}\left(+10\text{ J/C} - \left(-10\text{ J/C}\right)\right) = 2\text{ mC}\times\left(+20\text{ J/C}\right) = +40\text{ mJ}\,.$$

Moving the 2 mC object from C to B also produces a +40 mJ change in potential energy; in moving it from A to C there is no change in potential energy (and no work), and moving it from B to either A or C changes the potential energy by –40 mJ.

If we change objects, using an object with a charge of +5 mC, for instance, the change in potential energy is easy to find. It is simply the new charge multiplied by the change in potential.

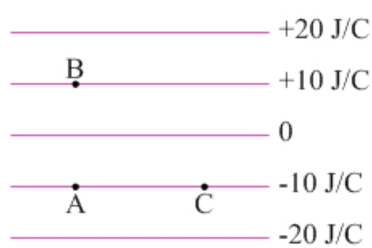


Figure 17.6: Equally spaced equipotentials are a hallmark of a uniform field. Field points in the direction of decreasing potential, so the field is directed down in this case.

Related End-of-Chapter Exercises: Problems 13, 14, 36, 43.

Essential Question 17.3: If we moved an object with a charge of –2 mC from point A to point B in Figure 17.6, is the change in potential energy still +40 mJ? Explain.

Answer to Essential Question 17.3: No. If we re-do the calculation $\Delta U = q\Delta V$ we find that the change in potential energy is –40 mJ. Because potential energy is a scalar, –40 mJ is quite different from +40 mJ. We have to force a positively charged object to move to a region of higher potential (and higher potential energy). A negatively charged object already feels a force toward higher potential, and moving in that direction lowers the potential energy, so we have to pull back on it to cause it to stop at the point of higher potential.

17-4 Electric Potential for a Point Charge

A point charge is generally an object like a small charged ball. Such objects contribute to both the electric field and the electric potential in the space around them. The contribution of an object of charge Q to the electric potential at a point a distance r away is given by:

$$V = \frac{kQ}{r}.$$ (Equation 17.4: **Electric potential from a point charge**)

Interestingly, the potential outside a charged sphere that has a spherically symmetric charge distribution is also given by Equation 17.6, where Q is the total charge on the sphere and r is the distance from the center of the sphere to a point outside the sphere. In a two-dimensional diagram of this situation, such as that in Figure 17.7, the equipotential lines are circles centered on the object. Note that the potential difference (which is more important than the value of the potential at a point) between one equipotential line and the next is a constant value, but the lines get further apart as you move away from the object because the field decreases - the farther you are from the object, the smaller its influence.

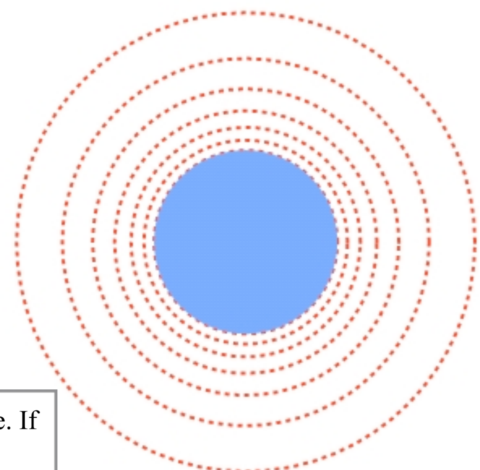


Figure 17.7: Equipotentials near a charged conducting sphere. If the equipotential at the surface of the sphere connects all the points where the potential is –10 J/C, the equipotentials at gradually increasing distances from the sphere connect all points of potential -9 J/C, -8 J/C, -7 J/C, -6 J/C, -5 J/C, and -4 J/C, respectively. The field points in the direction of decreasing potential, toward the center of the sphere.

EXAMPLE 17.4 – Calculating the electric potential

Two balls are on the x-axis, a ball of charge $+q$ at $x = 0$ and a ball of charge $+9q$ at $x = +4a$.

(a) Find the electric potential at: (i) the point halfway between them, and (ii) $x = +a$, where the net electric field is zero.

(b) Graph the electric potential at points along the axis between $x = -4a$ and $x = +8a$.

SOLUTION

As usual, let's begin with a diagram of the situation. This is shown in Figure 17.8.

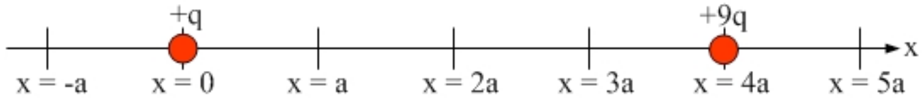


Figure 17.8: The position of the two charged balls on the x-axis.

To solve part (a) we use superposition, the idea that the potential at any point is simply the sum of the potential at that point from each of the objects.

(a)(i) The point halfway between the objects is $x = +2a$. Remembering that the r in the equation represents the distance from an object with mass to the point we're considering, which also happens to be $2a$ for both objects in this case, we get:

$$V_{net} = V_{+q} + V_{+9q} = +\frac{kq}{2a} + \frac{k(9q)}{2a} = +\frac{10kq}{2a} = +\frac{5kq}{a}.$$

(a)(ii) The point $x = +a$ is a distance a from the object of charge $+q$ and $3a$ from the object of charge $+9q$. The net potential is again the sum of the two individual contributions:

$$V_{net} = V_{+q} + V_{+9q} = +\frac{kq}{a} + \frac{k(9q)}{3a} = +\frac{4kq}{a}.$$

(b) To plot a graph of the potential as a function of position, we should have a general expression for the net potential. This is:

$$V_{net} = V_{+q} + V_{+9q} = +\frac{kq}{|x|} + \frac{k(9q)}{|4a - x|}.$$

The graph of this function between $x = -4a$ and $x = +8a$ is shown in Figure 17.9.

Related End-of-Chapter Exercises: 10-12, 56, 57.

There is a connection between the electric potential and the electric field that the graph in Figure 17.9 can help us to understand. The electric field is zero at $x = +a$. Does anything special happen at $x = +a$ on the potential graph? It turns out that, at $x = +a$, the graph reaches a local minimum, so the slope of the potential-vs.-position graph is zero at that point.

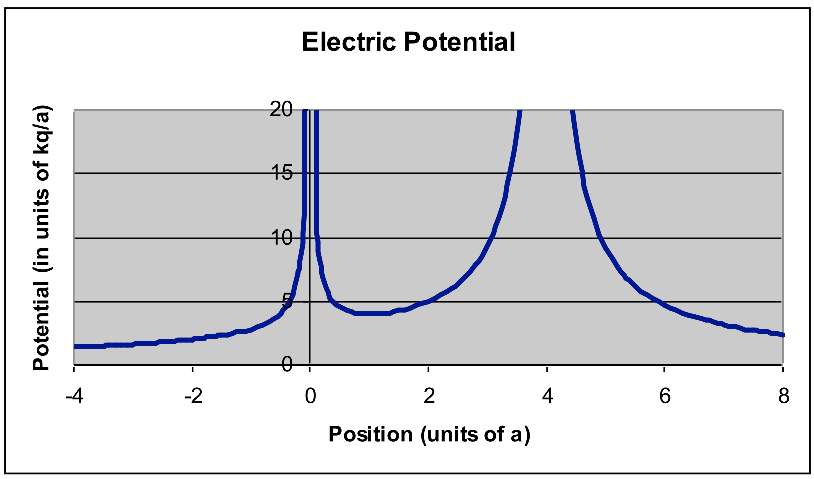


Figure 17.9: Graph of the electric potential along the axis near the balls. Note how the potential approaches positive infinity as you approach either of the balls.

For $0 < x < +a$, the electric field along the axis points in the positive x direction while the slope of the potential is a negative value. Just the opposite happens in the range $+a < x < +4a$. The electric field is actually the negative of the slope of the potential-vs.-position graph. So, the potential graph is rather powerful, telling us at a glance something about where the electric field is strongest and in what direction it is, while at the same time giving us information about how potential energy would change if we placed another object with charge at a particular location.

Essential Question 17.4: What do we have to account for in determining net electric field that we do not have to account for in determining the net electric potential?

Answer to Essential Question 17.4: A fundamental difference between field and potential is that field is a vector while potential is a scalar. To find the net field we need to account for direction, adding the various fields as vectors. Finding potential simply involves adding numbers.

17-5 Working with Force, Field, Potential Energy, and Potential

Let's consider two important special cases. The first involves a charged object in a uniform electric field, while the second involves the electric field from point charges.

EXAMPLE 17.5A – Charged particles at the corners of a square

Consider the four situations shown in Figure 17.10, involving four charged particles of charge $+q$ or $-q$ placed so that there is one charge at each corner of the square.

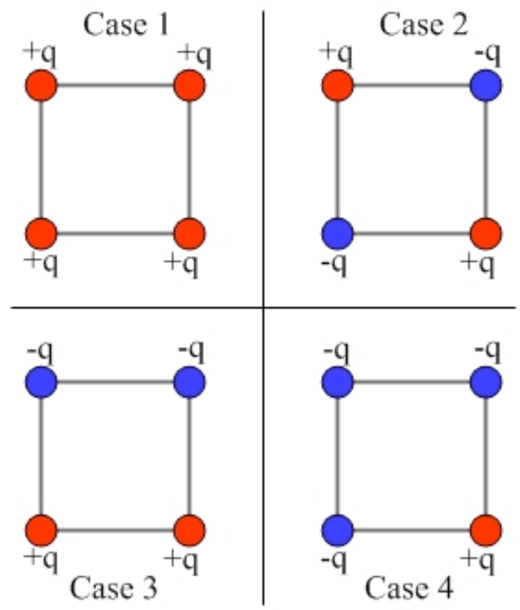


Figure 17.10: Four different arrangements of equal-magnitude point charges placed so that there is one charged particle at each corner of a square.

(a) In which case does the charge at the top right corner of the square experience the net force of the largest magnitude?
(b) Is the net electric field at the center of the square zero in any of the configurations? If so, which?
(c) Is the net electric potential at the center of the square zero in any of the configurations? If so, which?
(d) Is the electric potential energy of the system equal to zero in any of the configurations? If so, in which configuration(s)?

SOLUTION

(a) The charged particle at the top right corner experiences the largest magnitude net force in case 1, where all the forces have a component directed away from the center. In the other cases the particle at the top right experiences a mix of attractive and repulsive forces, which in this case causes some partial cancellation of the forces.

(b) The net electric field is zero at the center of the square in cases 1 and 2. If we pair up the charges that are diagonally across from one another, in cases 1 and 2 the fields within each pair cancel, giving no net field at the center.

(c) Each positive charge makes a positive contribution to the potential, while each negative charge makes a negative contribution. To produce a zero net potential at the center in this situation we need two positive charges and two negative charges, which is true in case 2 and case 3. The potential at the center of the square is positive in case 1 and negative in case 4.

(d) To find the total potential energy, we can work out the energy associated with each interacting pair of charges (with four charges there are six such interactions) by applying Equation 17.1, and add up these six terms to find the total potential energy. When we do this in case 4, we find that the total potential energy is zero. The total potential energy is non-zero in all other cases. If the square measures $L \times L$, the addition of the six terms gives:

$$U_{net,case4} = \frac{k(-q)(+q)}{L} + \frac{k(-q)(-q)}{L} + \frac{k(+q)(-q)}{L} + \frac{k(-q)(-q)}{L} + \frac{k(-q)(-q)}{\sqrt{2}\,L} + \frac{k(-q)(+q)}{\sqrt{2}\,L} = 0$$

Related End-of-Chapter Exercises for Example 17.5A: 19 - 23.

EXAMPLE 17.5B – Asking questions

Two charged particles, one with a charge of $+q$ and the other with a charge of $-3q$, are placed on the x-axis at $x = 0$ and $x = +4a$, respectively. This is shown in Figure 17.11. Give an example of a question pertaining to this situation that involves:

(a) Force; (b) Electric field; (c) Electric field, with a follow-up question on force; (d) Electric potential energy; (e) Electric potential; (f) Electric potential, with a follow-up question on potential energy.

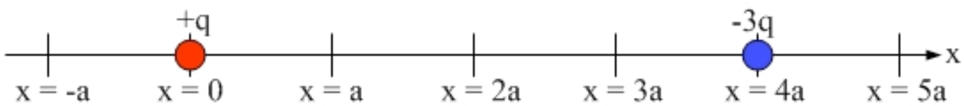


Figure 17.11: Two charged particles are separated by a distance of $4a$ on the x-axis.

SOLUTION

(a) Because force involves an interaction between objects, and we only have two objects in the system, there are a limited number of questions we could ask involving force unless we add another charge to the system. One is: "What is the magnitude and direction of the force exerted on the $+q$ charge by the $-3q$ charge?"

(b) "What is the magnitude and direction of the net electric field at $x = +5a$?" Because the two charged particles create an electric field at all points around them, we can ask an infinite number of questions involving field – we have an infinite number of points to choose from.

(c) "Consider the point $x = +2a$, $y = +2a$. (i) What is the magnitude and direction of the net electric field at that point? (ii) If a third charged particle with a charge of $+2q$ is placed at that point, what is the magnitude and direction of the force it experiences because of the two charges?"

(d) The pattern for (a) – (c) can be repeated for potential energy and potential. "What is the electric potential energy associated with this system of two charges?"

(e) "What is the net electric potential at x = +5a due to the two charges?" As with field, we can ask about the potential at any point.

(f) "Consider the point $x = +2a$, $y = +2a$. (i) What is the net electric potential at that point? (ii) If a third charged particle with a charge of $+2q$ is placed at that point, what is the change in potential energy for the system?"

Related End-of-Chapter Exercises: 16 - 18.

Essential Question 17.5: Return to the situation described in Example 17.5A. In which configurations, could you bring a fifth charged particle from infinitely far away and place it at the center of the square without doing any net work? In which configurations would you do negative net work in bringing a fifth charged particle from infinitely far away to the center of the square?

Answer to Essential Question 17.5: The net work you do to bring a charge Q from infinity to the center is $W_{You} = Q\Delta V = Q(V_f - V_i)$. V_i , the potential at infinity, is zero, so $W_{You} = QV_f$. This is zero if V_f , the potential at the center of the square from the four charges, is zero, which is true in cases 2 and 3. V_f is positive in case 1, so your net work in that case is negative if Q is negative. In case 4 V_f is negative, so your net work is negative if Q is positive. Negative work by you means that the fifth charge experiences a net attraction, and would accelerate unless you held it back.

17-6 Capacitors and Dielectrics

A capacitor is a device for storing charge. One example is a parallel-plate capacitor, consisting of two identical metal plates, each of area *A*, placed parallel to one another and separated by a distance *d*. The space between the plates is sometimes filled with insulating material. If it is we say that the capacitor contains a **dielectric**.

One way to charge a capacitor is to connect it to a battery, connecting one wire from the positive terminal of the battery to one plate of the capacitor, and a second wire from the negative terminal of the battery to the other plate. A diagram of this is shown in Figure 17.12, along with a circuit diagram. In the circuit diagram a capacitor is shown as two parallel lines of equal length, looking very much like a parallel-plate capacitor, and a battery is represented by two parallel lines of different length. The longer line on the battery symbol represents the positive terminal.

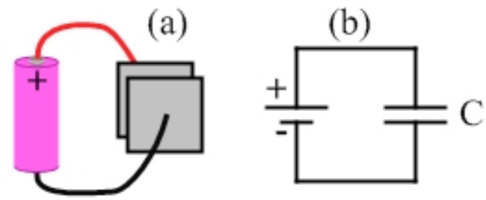


Figure 17.12: (a) A diagram, and (b) a circuit diagram, showing a battery connected to a capacitor (C) by two wires.

It is useful to think of the battery as a charge pump. The battery pumps electrons from the plate attached to the positive terminal, leaving that plate with a charge of +*Q*, to the plate attached to the negative terminal, giving that plate a charge of –*Q*. The capacitor then stores a charge *Q*. The battery pumps charge until the potential difference, ΔV , across the capacitor equals the potential difference across the battery. Another word for potential difference is voltage - the capacitor is charged when its voltage equals the battery voltage.

The charge Q stored in a capacitor is proportional to the capacitor's potential difference, ΔV :

$Q = C\,\Delta V$. (Eq. 17.6: **Charge on a capacitor is proportional to its voltage**)

where C is the capacitance of the capacitor, a measure of how much charge is stored for a particular voltage. The capacitance depends on the geometry of the capacitor, as well as on what, if anything, is between the plates of the capacitor. For a parallel-plate capacitor made of two plates of equal area *A*, separated by a distance *d*, the capacitance is given by:

$C = \frac{\kappa \varepsilon_0 A}{d}$, (Eq. 17.7: **Capacitance of a parallel-plate capacitor**)

where κ is the dielectric constant of the material between the capacitor plates, and

$\varepsilon_0 = \frac{1}{4\pi k} = 8.85\times10^{-12}\ \mathrm{C^2/\left(N\ m^2\right)}$ is **the permittivity of free space**.

The MKS unit for capacitance is the farad (F). 1 F = 1 C/V.

A charged capacitor, with a charge $+Q$ on one plate and $-Q$ on the other, has an electric field directed from the positive plate to the negative plate. The field is uniform if the distance d between the plates is much smaller than the dimensions of the plate itself. In this case the magnitude of the field, E, is connected to, ΔV, the potential difference across the capacitor by:

$$E = \frac{|\Delta V|}{d}.$$ (Eq. 17.8: **Magnitude of the electric field in a parallel-plate capacitor**)

Dielectrics.

A material in the space between capacitor plates is known as a **dielectric**. Usually this is an insulating material, because filling the space with a conductor would discharge the capacitor. It is interesting to review what happens with a conductor, however. As in Figure 17.13, when a conductor is placed in an external electric field the conduction electrons re-distribute themselves, giving an induced electric field E_{ind} inside the conductor equal in magnitude but opposite in direction to the external field. This gives a net field of zero inside the conductor.

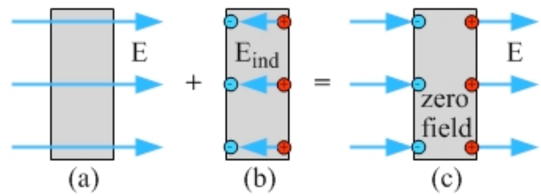


Figure 17.13: Conduction electrons distribute themselves so the field is zero inside a conductor.

The process is similar in an insulator. If the molecules in the insulator are unpolarized, the electric field polarizes them by shifting the electron clouds around the nuclei in the direction opposite to the external field; if the molecules are polarized to begin with, and randomly aligned as in Figure 17.14, the molecules experience a torque from the field that tends to align them with the field. In either case the positive and negative charges cancel one another in the bulk of the insulator, but there is a net positive charge along one face of the insulator and a net negative charge along the other face. This gives rise to an induced electric field E_{ind} inside the conductor that is opposite in direction to the external field. The induced field has a smaller magnitude than the external field, leading to a partial cancellation of fields inside the insulator, as in Figure 17.15.

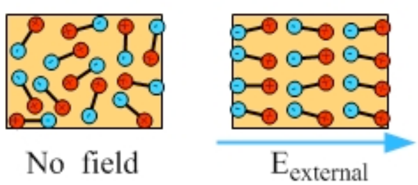


Figure 17.14: Polar molecules align with the field, when placed in an electric field.

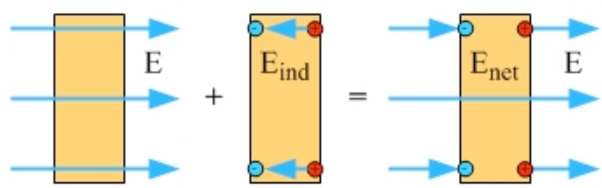


Figure 17.15: The induced field partly cancels the external field in the dielectric.

Related End-of-Chapter Exercises: 31, 61.

The dielectric constant of an insulator is the ratio of the external field to the net field inside the insulator:

$$\kappa = \frac{\vec{E}_0}{\vec{E}_{net}} \geq 1.$$ (Eq. 17.9: **The dielectric constant**)

Selected dielectric constants are shown in Table 17.1.

Material	Dielectric constant
Vacuum	1.00000
Dry air	1.00056
Polystyrene	2.6
Mylar	3.1
Paper	3.6
Water	80

Table 17.1: Dielectric constants of various materials.

Essential Question 17.7: What is the dielectric constant of a conductor?

Answer to Essential Question 17.7: The net field inside a conductor is zero. Using $E_{net} = 0$ in equation 17.9 gives κ = infinity. Thus, conductors have infinite dielectric constants.

17-7 Energy in a Capacitor, and Capacitor Examples

When a capacitor stores charge, creating an electric field between the plates, it also stores energy. The energy is stored in the electric field itself. The energy density (the energy per unit volume in the field) is proportional to the square of the magnitude of the field:

$$\text{Energy density} = \frac{1}{2}\kappa\varepsilon_0 E^2 . \qquad \text{(Eq. 17.10: \textbf{Energy per unit volume in an electric field})}$$

U, the total potential energy stored in the capacitor is thus the energy density multiplied by the volume of the region between the capacitor plates, $A \times d$. The energy can be written as:

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}Q\Delta V = \frac{Q^2}{2C} . \qquad \text{(Equation 17.11: \textbf{Energy stored in a capacitor})}$$

Let's now explore the two main ways we set up capacitor problems.

EXPLORATION 17.7A – The capacitor stays connected to the battery

A parallel-plate capacitor is connected to a battery that has a voltage of V_0. The capacitor has an initial capacitance of C_0, with air filling the space between the plates. The capacitor stores a charge Q_0, has an electric field of magnitude of E_0, and stores an energy U_0. With the battery connected to the capacitor, the spacing between the plates is doubled, and then a dielectric with a dielectric constant 5 times that of air is inserted, completely filling the space between the plates.

Step 1 – ***Do any of the parameters given above stay constant during this process?*** Yes, the potential difference is constant. Because the capacitor is connected to the battery at all times, the potential difference across the capacitor matches the battery voltage.

Step 2 – ***Complete Table 17.2 to show the values of the various parameters after each step.***

Situation	Potential difference	Capacitance	Charge	Electric field	Energy
Initially	V_0	C_0	Q_0	E_0	U_0
Spacing is doubled					
Dielectric inserted					

Table 17.2: Fill in the table to show what happens to the various parameters in this situation.

The completed table is Table 17.3. The potential difference is constant, so we start with that column. We then use Equation 17.7 to find the capacitance. Doubling the distance between the plates reduces the capacitance by a factor of 2; increasing the dielectric constant by a factor of 5 then increases C by a factor of 5, for an overall increase by a factor of 5/2 = 2.5.

Knowing ΔV and C, we can use Equation 17.6, $Q = C\Delta V$, to find the charge on the capacitor. Because the potential difference is constant the charge changes in proportion to the capacitance. Similarly, Equation 17.8, $E = |\Delta V|/d$, tells us what happens to the magnitude of the electric field, and the form of equation 17.11 that says the energy is proportional to $C(\Delta V)^2$ tells us what happens to the energy. It is interesting that inserting the dielectric has no effect on the electric field – this is because the battery increases the charge by a factor of 5 to keep the potential difference, and therefore the field, the same when the dielectric is inserted.

Situation	Potential difference	Capacitance	Charge	Electric field	Energy
Initially	V_0	C_0	Q_0	E_0	U_0
Spacing is doubled	V_0	$0.5\,C_0$	$0.5\,Q_0$	$0.5\,E_0$	$0.5\,U_0$
Dielectric inserted	V_0	$2.5\,C_0$	$2.5\,Q_0$	$0.5\,E_0$	$2.5\,U_0$

Table 17.3: Keeping track of the various parameters as changes are made to the capacitor.

Key idea: When a capacitor remains connected to a battery the capacitor voltage is constant – it equals the battery voltage. If changes are made we first determine how the capacitance changes, and then use the various equations to determine what happens to other parameters.

EXPLORATION 17.7B – The battery is disconnected from the capacitor before the changes

Let's return our parallel-plate capacitor to the same initial state as in the previous Exploration. The wires connecting the capacitor to the battery are then removed. After this the spacing between the plates is doubled, and then a dielectric with a dielectric constant 5 times that of air is inserted into the capacitor, completely filling the space between the plates.

Step 1 – *What stays constant during this process?* The charge is constant. With the capacitor disconnected from the battery, the charge is stranded on the plates.

Step 2 – *Complete Table 17.2 to show the values of the various parameters after each step.* The completed table is Table 17.4. The charge is constant, so we start with that column. Applying Equation 17.7, we can then determine what happens to the capacitance. Doubling the distance between the plates reduces the capacitance by a factor of 2; increasing the dielectric constant by a factor of 5 then increases C by a factor of 5, for an overall increase by a factor of 5/2 = 2.5.

Knowing Q and C, we can use Equation 17.6, $Q = C\Delta V$, to find the potential difference across the capacitor. Note that ΔV is inversely proportional to the capacitance. Equation 17.8, $E = |\Delta V| / d$, tells us what happens to the electric field, and the form of equation 17.11 that says the energy is proportional to $Q\,\Delta V$ tells us what happens to the energy. In this case, inserting the dielectric decreases the field, as we would expect from our previous discussion of dielectrics.

Situation	Potential difference	Capacitance	Charge	Electric field	Energy
Initially	V_0	C_0	Q_0	E_0	U_0
Spacing is doubled	$2\,V_0$	$0.5\,C_0$	Q_0	E_0	$2\,U_0$
Dielectric inserted	$0.4\,V_0$	$2.5\,C_0$	Q_0	$0.2\,E_0$	$0.4\,U_0$

Table 17.4: Keeping track of the various parameters as changes are made to the capacitor.

Key idea: When a capacitor is not connected to anything the charge on the capacitor remains constant. If changes are made we first determine how the capacitance changes, and then use the various equations to determine what happens to other parameters.

Related End-of-Chapter Exercises: 25 – 30.

Essential Question 17.7: In Exploration 17.7B, the energy stored by the capacitor doubles when the spacing between the plates doubles. Where does this extra energy come from?

Answer to Essential Question 17.7: The capacitor plates, being oppositely charged, attract one another. Positive work is required to pull the plates farther apart. The energy associated with that process is the extra energy stored by the capacitor.

Chapter Summary

Essential Idea: Electric Potential Energy and Electric Potential.

In this chapter we continued looking at parallels between how charged particles interact and how objects with mass interact. As with gravitational situations, conservation of energy can be applied to many situations involving charged particles. We also went beyond what we had done with gravity, defining electric potential. The analogy still holds – we can define a gravitational potential for objects with mass that is much like electric potential for charged objects.

Electric Potential Energy

The potential energy associated with the interaction between one object with a charge q and a second object with a charge Q that is a distance r away is given by:

$$U_E = \frac{kqQ}{r}$$ (Eq. 17.1: **Potential energy for the interaction between two charges**)

where $k = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$ is a constant. If the charges are of opposite signs then the potential energy is negative – this indicates an attraction. If the charges have the same sign the potential energy is positive, indicating a repulsion.

Potential energy is a scalar. The total electric potential energy of a system of charged objects can be found by adding up the energy associated with each interacting pair of objects. For a charged particle in a uniform electric field we use the change in potential energy:

$$\Delta U_E = -q\vec{E} \bullet \Delta\vec{r} = -qE\Delta r\cos\theta ,$$ (Eq. 17.2: **Change in potential energy in a uniform field**)

where θ is the angle between the electric field $\vec{E}$ and the displacement $\Delta\vec{r}$.

In a uniform field, we are free to define the zero level of potential energy. With equation 17.1, however, the potential energy is zero when the charges are separated by an infinite distance.

Electric Potential

Electric potential helps us understand how a charged object, or a set of charged objects, affects the region around it. The electric potential, V, at a particular point can be defined in terms of the electric potential energy, U, associated with an object of charge q placed at that point:

$$V = \frac{U}{q}, \quad \text{or} \quad U = qV .$$ (Eq. 17.3: **Connecting electric potential and potential energy**)

The unit for electric potential is the volt (V). 1V = 1 J/C.

A special case is the electric potential from a point charge with a charge Q:

$$V = \frac{kQ}{r},$$ (Equation 17.4: **Electric potential from a point charge**)

where r is the distance from the charge to the point in space where we are finding the potential. Electric potential is a scalar, so it has a sign but not a direction.

Equipotentials connect points of the same potential. Equipotentials are perpendicular to field lines, because field points in the direction of decreasing potential. In a uniform field, or for a small displacement in a non-uniform field, we can connect the potential difference to the field:

$$\Delta V = -E\,\Delta r \cos\theta \,, \qquad \text{(Equation 17.5: \textbf{Potential difference in a uniform field})}$$

where θ is the angle between the electric field $\vec{E}$ and the displacement $\Delta\vec{r}$.

Working with Force, Field, Potential Energy, and Potential

Keep two points in mind when working with force, field, potential energy, and potential:

- Force and field are vectors, while potential energy and potential are scalars.
- Forces and potential energies arise from interactions between charges, requiring at least two charges. Field and potential exist with a single charged object.

Capacitors and Dielectrics

The charge Q stored in a capacitor is proportional to the capacitor's potential difference, ΔV:

$$Q = C\,\Delta V \,, \qquad \text{(Eq. 17.6: \textbf{Charge on a capacitor is proportional to its voltage})}$$

where C is the capacitance of the capacitor, a measure of how much charge is stored for a particular voltage. The capacitance depends on the geometry of the capacitor, as well as on what, if anything, is between the plates of the capacitor. For a parallel-plate capacitor made of two plates of equal area A, separated by a distance d, the capacitance is given by:

$$C = \frac{\kappa\,\varepsilon_0\,A}{d} \,, \qquad \text{(Eq. 17.7: \textbf{Capacitance of a parallel-plate capacitor})}$$

where κ is the dielectric constant of the material between the capacitor plates, and

$\varepsilon_0 = \dfrac{1}{4\pi k} = 8.85\times10^{-12}\ \text{C}^2/\left(\text{N m}^2\right)$ is **the permittivity of free space**.

The MKS unit for capacitance is the farad (F). 1 F = 1 C/V.

The dielectric constant of an insulator is the ratio of the external field to the net field inside:

$$\kappa = \frac{\vec{E}_0}{\vec{E}_{net}} \geq 1 \,. \qquad \text{(Eq. 17.9: \textbf{The dielectric constant})}$$

$$E = \frac{|\Delta V|}{d} \,. \qquad \text{(Eq. 17.8: \textbf{Magnitude of the electric field in a parallel-plate capacitor})}$$

$$U = \frac{1}{2}C\left(\Delta V\right)^2 = \frac{1}{2}Q\,\Delta V = \frac{Q^2}{2C} \,. \qquad \text{(Equation 17.11: \textbf{Energy stored in a capacitor})}$$

The energy is stored in the electric field. The energy density (the energy per unit volume) of an electric field is:

$$\text{Energy density} = \frac{1}{2}\kappa\varepsilon_0 E^2 \,. \qquad \text{(Eq. 17.10: \textbf{Energy per unit volume in an electric field})}$$

When a capacitor is connected to a battery the capacitor voltage (i.e., potential difference) is equal to the battery voltage. When a capacitor is charged and then all connections to the capacitor are removed, however, the charge on the capacitor is constant.

End-of-Chapter Exercises

Exercises 1 – 12 are primarily conceptual questions designed to see whether you understand the main concepts of the chapter.

1. (a) If the electric field at a particular point is zero, does that imply that the electric potential is also zero at that point? If so, explain why. If not, give an example involving two or more point charges where the electric field is zero at a point but the electric potential is not. (b) If the electric potential at a particular point is zero, does that imply that the electric field is also zero at that point? If so, explain why. If not, give an example involving two or more point charges where the electric potential is zero at a point but the electric field is non-zero there.

2. For interactions between point charges, the electric potential energy is defined to be zero when the charges are separated by an infinite distance. Is it possible for a collection of two or more point charges to have an electric potential energy of zero when the charges are finite distances from one another? If not, explain why not. If so, give an example in which this happens.

3. An electric dipole consists of two charged particles, one with a charge of $+Q$ and the other with a charge of $-Q$, separated by some distance. In a particular dipole, the positive charge is located on the x-axis at $x = +10$ cm, and the negative charge is located on the x-axis at $x = -10$ cm. Assume non-conservative forces, and gravitational interactions, are negligible for this situation. (a) How much net work do you need to do to bring a third particle, with a charge of $+2Q$, from very far away to the origin if you bring the charge toward the origin along the positive y-axis? Justify your answer. (b) How much net work do you need to do if you bring the third particle to the origin by a more circuitous route, via a path that takes it quite close to the particle of charge $+Q$? Justify your answer.

4. (a) Sketch field lines that represent a uniform field directed down. Add some equipotential lines to your sketch. (b) Make a second diagram that shows a second electric field with twice the magnitude and the same direction as the first field. Add equipotential lines to this diagram so that the potential difference between the lines is the same as that in your first diagram.

5. Figure 17.16 shows electric field lines and equipotentials in three different regions. Comment on each diagram, stating either that it is physically possible or, if not, what is wrong with the situation shown.

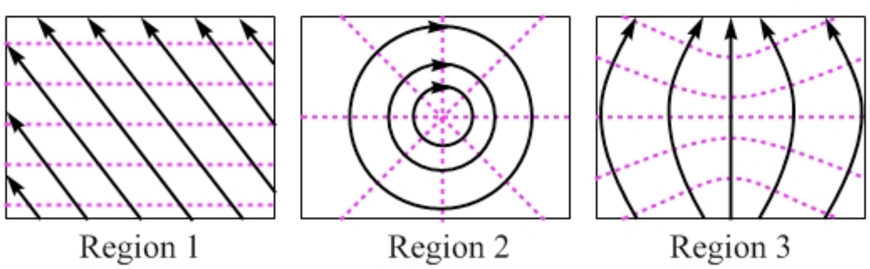


Figure 17.16: State whether the field lines (black arrows) and equipotential lines (dashed lines) are physically correct. For Exercise 5.

6. Two charged balls are placed on the x-axis, as shown in Figure 17.17. The first ball has a charge $+q$ and is located at the origin, while the second ball has a charge $+3q$ and is located at $x = +4a$. A third charged ball is now brought in and placed nearby. Your goal is to determine the sign of the charge on the third ball, and to narrow down its location. (a) The force on the third ball is in the $+x$ direction, with no component in any other direction. What, if anything, does this tell you about the third ball? (b) When the third ball is added to the system, the potential at $x = 2a$ decreases. What, if anything, do you know about the third ball now? (c) When the third ball is added to the system, the force on the ball of charge $+q$ increases, while the force on the ball of charge $+3q$ decreases. Both these forces maintain their original directions. What, if anything, do you know about the third ball now?

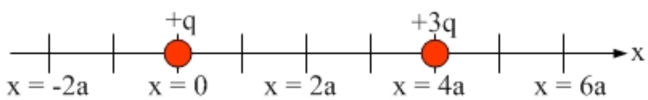


Figure 17.17: Two charged balls on the x-axis. For Exercise 6.

7. Four charged balls, with charges of $+4q$, $-5q$, $-6q$, and $-q$, are placed on either the x or y axes as shown in Figure 17.18. Each charge is the same distance from the origin. You will now remove one of the charged balls from the system. Which charged ball should you remove to cause (a) the largest decrease in the magnitude of the electric field at the origin? (b) the largest increase in the potential at the origin? (c) the largest increase in the potential energy of the system?

8. Return to the system of four charged balls described in Exercise 7 and shown in Figure 17.18. The system is now changed by reversing the sign of each of the charges. Describe what effect, if any, that change has on the answers to Exercise 7.

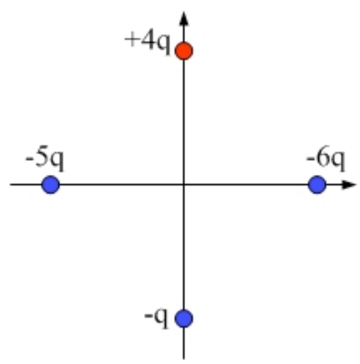


Figure 17.18: Four charged balls are placed on either the x or y axis so that each ball is the same distance from the origin. For Exercises 7 and 8.

9. A parallel-plate capacitor is connected to a battery that has a voltage of V_0. The capacitor has an initial capacitance of C_0, with a dielectric of dielectric constant three times larger than that of air filling the space between the plates. The capacitor stores a charge Q_0, has an electric field of magnitude of E_0, and stores an energy U_0. With the battery still connected to the capacitor, the dielectric is removed, leaving air between the plates. The distance between the plates is then decreased by a factor of 2. Make a table to show the potential difference, capacitance, charge, magnitude of the field inside the capacitor, and energy stored by the capacitor, in terms of their initial values, after each change is made.

10. Repeat Exercise 9 but, this time, the wires connecting the capacitor to the battery are removed before the changes are made.

11. Return to the situation described in Exploration 17.7B. When the dielectric is inserted, the energy stored by the capacitor is reduced by a factor of 5. If the dielectric is removed, the energy returns to its original value. (a) Where does the energy go when the dielectric is inserted? (b) Where does the energy come from when the dielectric is removed?

12. A parallel-plate capacitor with air between the plates is connected to a 12-volt battery. (a) What is the potential difference across the capacitor? (b) Without bringing in additional batteries, is it possible to increase the potential difference across the capacitor to 36 V, by making changes such as removing the battery and/or changing the separation between the plates? If so, describe how you could do it. If not, explain why not.

Exercises 13 – 19 deal with electric potential energy.

13. In the Bohr model of the hydrogen atom, the electron in the ground state is a distance of 5.3×10^{-11} m from the proton in the nucleus. Assuming the proton remains at rest, what is the escape speed of the electron? In other words, if the electron was given an initial velocity directed away from the proton, what is the minimum speed it has to have to completely escape from the proton? Assume nothing but the proton influences the electron.

14. Two charged objects are released from rest when they are a particular distance apart. As they accelerate away from one another, their velocities are always equal in magnitude and opposite in direction. Assuming that the only thing acting on one object is the other object, are the following statements true or false? Justify each answer. Statement 1 – The two objects have charges of the same sign. Statement 2 – The two objects have charges of the same magnitude. Statement 3 – The two objects have equal mass.

15. Return to the situation described in Exercise 14. Let's say that the objects are identical small balls with the same mass and charge. If each ball has a mass of 25 grams, a charge of 5.0×10^{-6} C, and is originally separated by 8.0 cm, how fast is each ball moving when the separation between the balls has doubled?

16. Three balls, with charges of $+4q$, $-2q$, and $-q$, are equally spaced along a line. The spacing between the balls is r. We can arrange the balls in three different ways, as shown in Figure 17.19. In each case the balls are in an isolated region of space very far from anything else. (a) Rank the arrangements according to their potential energy, from most positive to most negative. See if you can do this without explicitly calculating the potential energy in each case. (b) Verify your answer to part (a) by calculating the potential energy in each case.

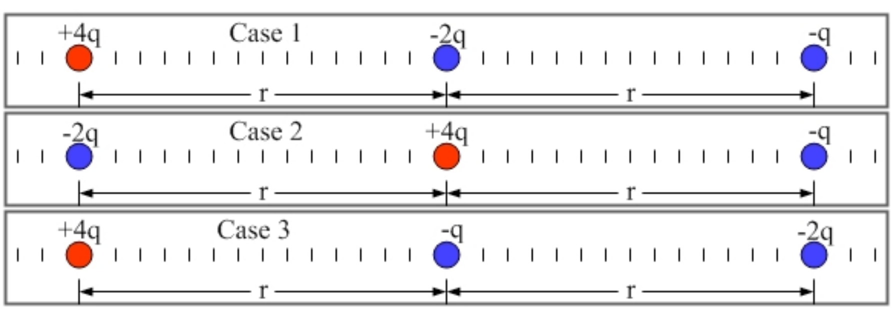


Figure 17.19: Three different arrangements of three balls of charge $+4q$, $-2q$, and $-q$ placed on a line with a distance r between neighboring balls. For Exercises 16 and 17.

17. Consider the three arrangements of charged balls shown in Figure 17.19. When assembling these arrangements, the middle ball is the last one added to the system in each case. How much work do you have to do, against the forces applied by the other two balls, to bring the middle ball from infinity to the point halfway between the other balls in (a) Case 1? (b) Case 2?, (c) Case 3?

18. An electron and a proton are each released from rest in a uniform electric field that has a magnitude of 500 N/C. The energy and speed of each particle is measured after it has moved through a distance of 25 cm. Assume the particles do not influence one another, but are influenced only by the electric field. Without doing any calculations, determine which particle (a) has more kinetic energy, (b) has a higher speed, (c) takes more time to cover 25 cm. Justify your answers. Now calculate the kinetic energy, speed, and elapsed time for (d) the electron, and (e) the proton.

19. In an electron beam in a cathode ray tube television, the electrons are accelerated from rest through a potential difference of 15 kV on their way to the screen. What is the speed of the electrons?

Exercises 20 – 26 deal with electric potential.

20. A ball of with a charge of $+6q$ is placed on the x-axis at $x = -2a$. There is a second ball of unknown charge at $x = +a$. If the net electric potential at the origin due to the two balls is $+\frac{2kq}{a}$, what is the charge of the second ball? Find all possible solutions.

21. A single point charge is located at an unknown point on the x-axis. There are no other charged objects nearby. You measure the electric potential at $x = +2.0$ m to be –200 volts, while the potential at $x = +5.0$ m is –400 volts. What is the sign and magnitude of the point charge, and where is it located? State all possible answers.

22. A single point charge is placed on the x-axis at $x = -2.0$ m. If the electric potential at $x = 0$ because of this charge is –500 volts, determine the sign and magnitude of the charge of the point charge.

23. Two charged balls are placed on the x-axis, as shown in Figure 17.20. The first ball has a charge $+q$ and is located at the origin, while the second ball has a charge $-3q$ and is located at $x = +4a$. Your goal is to find all the points on the axis, a finite distance from the charges, where the net electric potential due to these two balls is zero. Start qualitatively. Provide a justification for whether or not there are any such points (a) to the left of the ball of charge $+q$, (b) between the balls, and (c) to the right of the ball of charge $-3q$. (d) Find the locations of all such points.

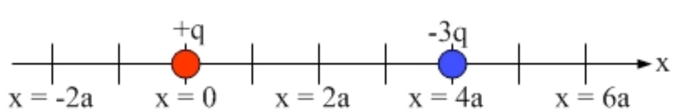


Figure 17.20: Two charged balls on the x-axis. For Exercises 23 and 24.

24. Return to the situation described in Exercise 23 and shown in Figure 17.20. (a) If we reverse the sign of the charge on both the balls, do the answers to Exercise 23 change? If so, describe how. (b) If we reverse the sign of the charge on just one of the balls, do the answers to the previous exercise change? If so, describe how.

25. The potential difference between two points, A and B, in a uniform electric field has a magnitude of 30 volts. Assume gravity is negligible in this situation. When an electron is released from rest at point A, it passes through B a short time later. (a) If the potential at point A is +50 volts, what is the potential at point B?

26. (a) Make a sketch on a piece of paper showing three points, A, B, and C. Point B is located a distance of 20 cm from, and directly above, point A, while point C is located a distance of 20 cm, and horizontally to the right, from point A. The points are in a uniform electric field. All you know about its direction is that it is in the plane of the piece of paper. (b) You measure the electric potential at point A to be +40 volts, while the potential at point B is +50 volts. What, if anything, does this tell you about the magnitude and/or the direction of the electric field? (c) You then measure the electric potential at point C to be +50 volts. What, if anything, can you say about the magnitude and/or the direction of the electric field now? (d) If possible, sketch the field lines and show the +30 V, +40 V, +50 V, and +60 V equipotentials, and find the magnitude of the electric field.

Exercises 27 – 32 involve force, field, potential, and potential energy.

27. Two charged balls are placed on the x-axis, as shown in Figure 17.21. The first ball has a charge $+q$ and is located at the origin, while the second ball has a charge $+3q$ and is located at $x = +4a$. (a) Which of the balls experiences a larger magnitude force because of its interaction with the other ball? Why? (b) Determine the magnitude and direction of the force experienced by the ball of charge $+q$. (c) Calculate the magnitude and direction of the net electric field at $x = +2a$. (d) Determine the electric potential energy of this system. (e) Calculate the electric potential at $x = +2a$, relative to $V = 0$ at infinity.

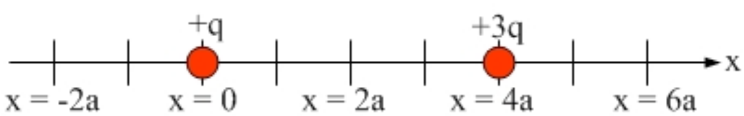


Figure 17.21: Two charged balls on the x-axis. For Exercises 27 and 28.

28. Two charged balls are placed on the x-axis, as shown in Figure 17.21. The first ball has a charge $+q$ and is located at the origin, while the second ball has a charge $+3q$ and is located at $x = +4a$. (a) Determine the location of all points on the x-axis, a finite distance from the balls, where the net electric field is zero. (b) Determine the location of all points on the x-axis, a finite distance from the balls, where the total electric potential is zero, relative to $V = 0$ at infinity.

29. Three charged balls, with charges of $+q$, $–3q$, and $+2q$, are placed at the corners of a square that measures L on each side, as shown in Figure 17.22. (a) What is the magnitude and direction of the force experienced by the ball at the top right corner? (b) What is the magnitude and direction of the net electric field at the lower left corner? (c) What is the electric potential energy of this set of charged objects? (d) What is the electric potential at the lower left corner?

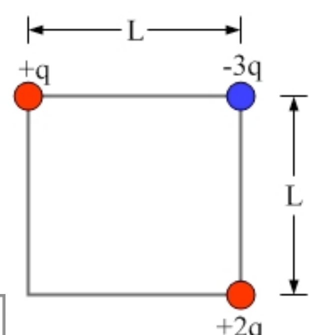


Figure 17.22: Three charged balls, with charges of $+q$, $–3q$, and $+2q$, are placed at the corners of a square that measures L on each side. For Exercise 29.

30. Four charged balls, with charges of $+4q$, $-5q$, $-6q$, and $-q$, are placed on either the x or y axes as shown in Figure 17.23. Each ball is a distance d from the origin. (a) Calculate the magnitude and direction of the force exerted on the ball of charge $+4q$ by the other three balls. (b) Calculate the magnitude and direction of the net electric field at the origin. (c) Calculate the electric potential energy of this set of charged balls. (d) Calculate the electric potential at the origin, relative to $V = 0$ at infinity.

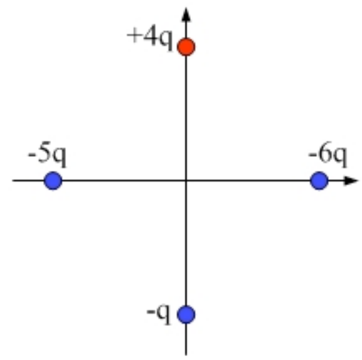


Figure 17.23: Four charged balls are placed on either the x or y axis so that each ball is the same distance from the origin. For Exercises 30 and 31.

31. Four charged balls, with charges of $+4q$, $-5q$, $-6q$, and $-q$, are placed on either the x or y axes as shown in Figure 17.23. Each ball is a distance d from the origin. (a) If you remove one ball from the system, which ball requires you to do the largest positive work to remove it? (b) Calculate the value of the work required to remove that ball.

32. A point object with a charge of $+Q$ is placed on the x-axis at $x = -a$. A second point object of unknown charge is placed at an unknown location on either the x-axis or the y-axis. The potential energy associated with the point charges is $U = +2kQ^2/a$; the electric potential at the origin due to the charges is $V = +4kQ/a$; and the net electric field at the origin due to the charges points in the negative x direction. (a) What is the sign of the charge on the second object? Justify your answer. (b) Qualitatively, what, if anything, can you say about the location of the second object? Explain. (c) Determine the sign and magnitude of the charge on the second object, and determine its location. (d) What is the magnitude of the net electric field at the origin due to the two charges?

Exercises 33 – 35 deal with capacitors.

33. A parallel-plate capacitor with air between the plates is connected to a battery that has a voltage of V_0. The capacitor has an initial capacitance of C_0. The capacitor stores a charge Q_0, has an electric field of magnitude of E_0, and stores an energy U_0. The following steps are then carried out. Step 1, the distance between the plates is doubled; step 2, the wires connecting the capacitor to the battery are removed; step 3, a dielectric with a dielectric constant 4 times that of air is inserted, completely filling the space between the plates; step 4, the capacitor is re-connected to the battery. Make a table to show the potential difference, capacitance, charge, magnitude of the field inside the capacitor, and energy stored by the capacitor, in terms of their initial values, after each step.

34. A parallel-plate capacitor with air between the plates is connected to a battery that has a voltage of V_0. The capacitor has an initial capacitance of C_0. The capacitor stores a charge Q_0, has an electric field of magnitude of E_0, and stores an energy U_0. The following steps are then carried out. Step 1, a second capacitor, identical to the first but initially uncharged, is placed so it is touching the first capacitor, effectively doubling the area of the capacitor plates; step 2, the wires connecting the capacitor to the battery are removed; step 3, the distance between the plates is doubled; step 4, the capacitor is re-connected to the battery. Make a table to show the potential difference, capacitance, charge, magnitude of the field inside the capacitor, and energy stored by the capacitor, in terms of their initial values, after each step.

35. The membrane of a living cell can be treated as a parallel-plate capacitor, with a plate separation of 10 nm and a dielectric constant of 5. The area of the plates is approximately $5\times10^{-9}\ \text{m}^2$. How much energy is stored in the cell membrane? Assume that the potential difference across the membrane has a magnitude of 100 millivolts.

General problems and conceptual questions

36. Two small charged balls are released from rest when they are 6.0 cm apart. One ball has a mass of 50 grams and a charge of $5.0\times10^{-5}\ \text{C}$, while the second ball has three times the mass and twice the charge as the first ball. Assuming the balls are influenced only by one another, how fast is each ball moving when they are very far apart?

37. Three identical balls, each with a mass of 75 grams and a charge of $8.0\times10^{-4}\ \text{C}$, are arranged so there is one ball at each corner of an equilateral triangle. Each side of the triangle measures 25 cm. (a) Assuming the balls are influenced only by one another, describe what will happen when the balls are released from rest. (b) When the balls are 50 cm from one another, how fast are they moving? (c) How fast are they moving when they are very far from one another?

38. A charged particle is given an initial speed of 80 cm/s in a uniform electric field. The initial velocity is the same direction as that of the field. Assume gravity can be neglected in this situation. After covering a distance of 30 cm, the particle has a velocity of 40 cm/s, directed in the same direction as its initial velocity. (a) What is the sign of the particle's charge? (b) How much time did the particle take to cover 30 cm? (c) What is the additional distance covered by the particle before it comes instantaneously to rest?

39. The electric potential in a particular region is due solely to a nearby point charge. You find that the potential at a location 50 cm from the charge is 60 volts higher than the potential at a location only 10 cm from the charge. (a) Is this possible? Explain. (b) If it is possible, determine the sign and magnitude of the charge on the point charge.

40. A ball of charge $+2q$ is placed on the x-axis at $x = -2a$. A second ball of charge $-q$ is placed nearby so that the net electric potential at the origin because of the two balls is $-\frac{2kq}{a}$. Where is the second ball?

41. Three small balls, with charges of $+q$, $-2q$, and $+3q$, can be placed on the x-axis in three different configurations, as shown in Figure 17.24. In each case one charge is at $x = -a$, one is at $x = +a$, and the third is at $x = +2a$. Rank the configurations based on (a) the magnitude of the force experienced by the ball of charge $-2q$, (b) the net electric field at $x = 0$, (c) the potential energy of the configuration, (d) the total electric potential at $x = 0$.

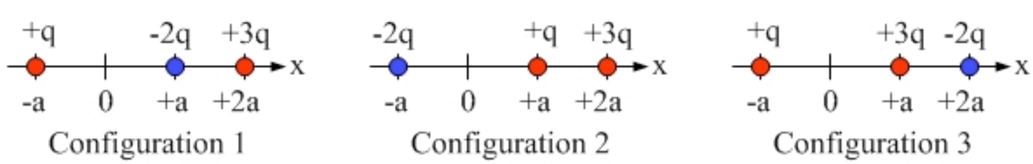


Figure 17.24: Three different configurations of three charged balls, for Exercises 41 and 42.

42. Return to the system described in Exercise 41 and shown in Figure 17.24. Calculate the potential energy in (a) configuration 1, (b) configuration 2, and (c) configuration 3. (d) Is it possible to reverse the locations of two of the charged balls in any configuration without affecting the potential energy of the configuration? Explain.

43. A particle with a mass of 24 grams and a charge of $+3.0\times10^{-5}$ C has a speed of 0.75 m/s when it passes through a point at which the potential is +1200 volts. What is the particle's speed when it passes through a second point at which the potential is –1200 volts? Assume that the only force acting on the particle comes from the electric field.

44. The electric potential a distance *r* from the center of a charged sphere with a spherically symmetric charge distribution is the same as that from a point charge that has the same total charge as the sphere, as long as the point is outside the sphere. Consider a conducting sphere with a net charge of +30 μC and a radius of 4.0 cm. Assume that the system is in electrostatic equilibrium, and that there are no other objects nearby. (a) What is the electric potential at a point 10 cm from the center of the sphere? (b) What is the electric potential at the surface of the sphere? (c) What is the electric field at the center of the sphere? (d) What is the electric potential at the center of the sphere?

45. Consider the three cases shown in Figure 17.25. Rank these cases, from most positive to most negative, based on the (a) electric potential at the origin; (b) electric potential energy of the set of charges.

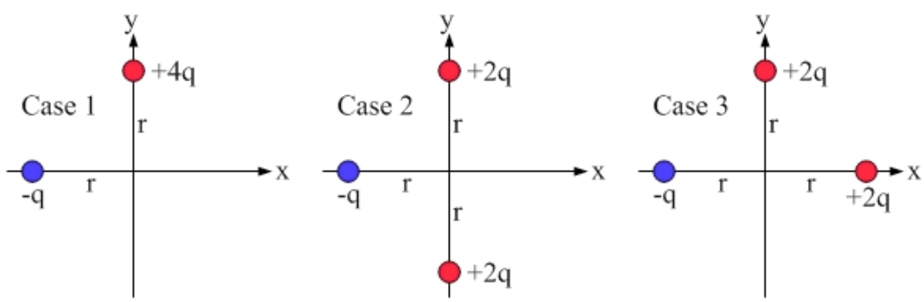


Figure 17.25: Three different configurations of charged objects, for Exercises 45 – 47.

46. Consider the three cases shown in Figure 17.25. Calculate the electric potential energy of the set of charges in (a) case1; (b) case 2; (c) case 3.

47. Consider the three cases shown in Figure 17.25. Determine the electric potential at the origin in (a) case1; (b) case 2; (c) case 3.

48. Four small charged balls are arranged at the corners of a square that measures L on each side, as shown in Figure 17.26 (a) Find the electric potential energy associated with this set of balls. (b) What is the electric potential at the center of the square, relative to $V = 0$ at infinity? (c) If you doubled the length of each side of the square, so neighboring charges were separated by a distance of $2L$ instead, what would happen to your answer to part (a)?

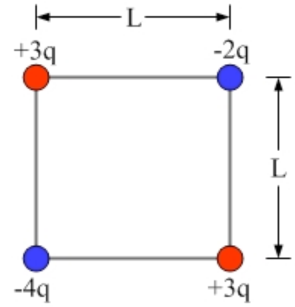


Figure 17.26: Four charged balls at the corners of a square, for Exercises 48 – 50.

49. Four small charged balls are arranged at the corners of a square that measures L on each side, as shown in Figure 17.26. How much work would you have to do to remove one of the balls with a charge of $+3q$ from the system?

50. Four small charged balls are arranged at the corners of a square that measures L on each side, as shown in Figure 17.26. If you could place a fifth ball, having any charge you wish, at the center of the square, could you give the system a total electric potential energy of zero? If not, explain why not. If so, determine the sign and magnitude of the fifth ball.

51. Three charged balls are placed in a line, as shown in Figure 17.27. Ball 1 has an unknown charge and sign, and is a distance $2r$ to the left of ball 2. Ball 2 has a charge of $+Q$. Ball 3 has an unknown non-zero charge and sign, and is a distance r to the right of ball 2. Ball 3 was the last ball brought into the system, and it required zero net work to bring ball 3 from very far away to the location shown. (a) Is there enough information here to find the sign of the charge on ball 1? If so, what is the sign? (b) Can we find the magnitude of the charge on ball 1? If so, what is it? (c) Can we find the sign of the charge on ball 3? If so, what is the sign? (d) Can we find the magnitude of the charge on ball 3? If so, what is it?

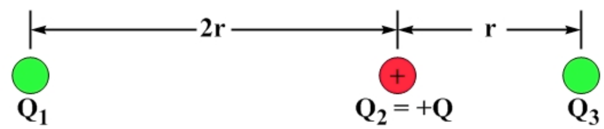


Figure 17.27: Three charges in a line. Only the sign and magnitude of charge 2 are known, although we also know that it took no net work to bring charge 3 into the system from far away. For Exercise 51.

52. A single point charge is located at an unknown point on the x-axis. There are no other objects nearby. You measure the electric potential at $x = +2.0$ m to be +600 volts, while the potential at $x = +5.0$ m is +150 volts. What is the sign and magnitude of the point charge? State all possible answers.

53. A point charge with a charge of +5.0 μC is located at the origin. A second point charge is located at $x = +2.0$ m, with a charge of –9.0 μC. (a) Analyze the situation qualitatively to determine approximate locations of any points along the straight line that passes through both charges where the net electric potential due to these two point charges is zero. (b) Determine the location of all such points that are a finite distance from the charges.

54. Repeat Exercise 53, except now the second point charge has a charge of +18.0 μC.

55. Return to the situation described in Exercise 53. Are there any points a finite distance from the charges at which the net electric potential due to the two charges is zero, that are off the line passing through the charges? If so, explain how you would locate them.

56. Two point charges are placed at different locations on the x-axis. The graph of the electric potential as a function of position on the axis is shown in Figure 17.28. The position is given in units of a. Note that the electric potential is zero at $x = +a$. (a) Where are the two charges located? (b) If one of the charges has a charge of $+3q$, what is the charge of the other point charge? (c) Is the electric field equal to zero at any point on the x-axis in the region shown on the graph, $-4a \le x \le +4a$? Justify your answer by referring to the graph.

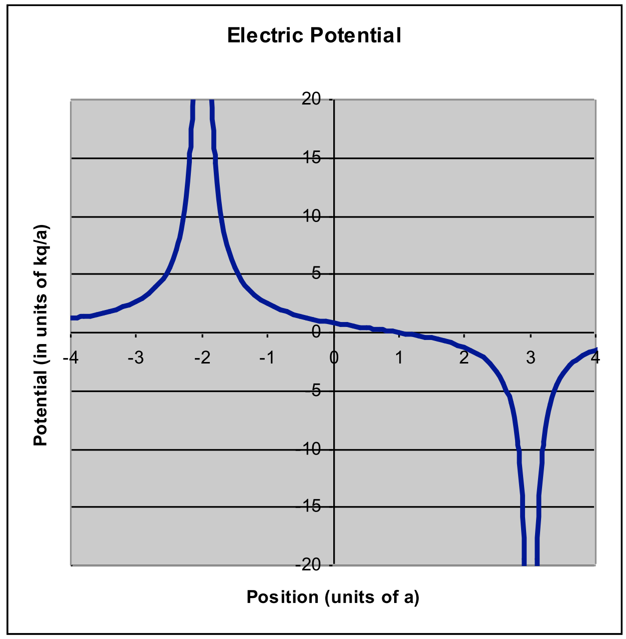


Figure 17.28: The graph of the electric potential along the x-axis in the region $-4a \le x \le +4a$. The potential is due to two point charges that are located on the x-axis. The electric potential is zero at $x = +a$. For Exercise 56.

57. Two point charges are placed at different locations on the x-axis. The graph of the electric potential as a function of position on the axis is shown in Figure 17.29. The position is given in units of a. (a) Where are the two charges located? (b) If one of the charges has a charge of $-2q$, what is the charge of the other point charge? (c) Is the electric field equal to zero at any point on the x-axis in the region shown on the graph, $-4a \le x \le +4a$? Justify your answer by referring to the graph.

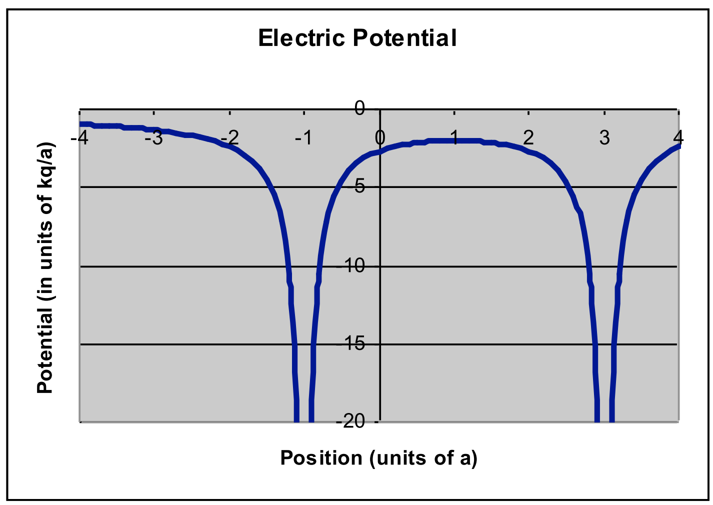


Figure 17.29: The graph of the electric potential along the x-axis in the region $-4a \le x \le +4a$. The potential is due to two point charges that are located on the x-axis. For Exercise 57.

58. A parallel-plate capacitor that has air between the plates is initially charged by being connected to a battery. With the battery still connected, one of the following changes is made. Change 1: the distance between the plates is tripled. Change 2: a dielectric with a dielectric constant 2 times that of air is inserted, completely filling the space between the plates. Change 3: another identical, but initially uncharged, capacitor is placed next to the first so they are touching, effectively doubling the area of each plate. Rank each of these changes, from largest to smallest, based on (a) the final potential difference across the capacitor, (b) the final charge stored on the capacitor, (c) the magnitude of the final electric field with the capacitor, (d) the final energy stored by the capacitor.

59. Repeat Exercise 58, but now the wires connecting the capacitor to the battery are removed before the changes take place.

60. (a) For a parallel-plate capacitor made up of two plates of area A separated by a distance d, what is the volume of the space between the plates? (b) Combine Equation 17.10, for the energy density, with Equations 17.7 and 17.8 to derive one form of Equation 17.11, for the energy stored in a capacitor. (c) Bring in one additional relationship to show how the other forms of Equation 17.11 follow from the expression you obtained in (b).

61. Comment on this part of a conversation between two of your classmates.

Paul: So, let's say we have a charged parallel-plate capacitor that has nothing between the plates. If I then insert a dielectric between the plates, the electric field in the capacitor is reduced, right?

Mary: I don't think so. Sometimes that's true but not always.

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