16-1 Electric Charge

In previous chapters, we have been concerned with several properties of objects, such as mass, momentum, energy, and angular momentum. These last three properties are associated with the object's motion, but the first property, mass, we often view as being an inherent property of the object itself. We often think of charge in a similar way, particularly the charge of an electron. Larger objects, such as ourselves, generally acquire a charge when they either lose electrons or acquire some extra electrons (we do this when we scuff our feet across a carpet, for instance). Table 16.1 shows the masses and charges of three basic constituents of atoms.

Particle	**Mass (kg)**	**Charge**
Electron	9.11×10^{-31} kg	$-e=-1.602\times10^{-19}$ C
Proton	1.672×10^{-27} kg	$+e=+1.602\times10^{-19}$ C
Neutron	1.674×10^{-27} kg	0

Table 16.1: The masses and charges of the electron, proton, and neutron, the basic building blocks of the atom.

EXPLORATION 16.1 – Experimenting with charge

One way to charge an object is to rub it with a cloth made from a different material. In this Exploration, we will investigate what happens when we do this with various combinations of materials. Such investigations go back as far as the ancient Greeks.

Step 1 – ***For this experiment, we need a piece of silk, two glass rods, and one piece of string. Suspend one of the glass rods from a string tied around the middle of the rod so that the rod is balanced. Rub one end of the rod with the silk. Rub one end of a second glass rod with the silk, and then bring the rubbed end close to, but not touching, the rubbed end of the rod that is suspended from the string. What do you observe?***
What you should observe in this case is that the end of the suspended rod moves away from the other rod – the suspended rod is repelled by the second rod. By Newton's third law, we know that the rods must be repelling one another with equal-and-opposite forces.

Step 2 – ***For this experiment, we need a piece of fur, two rubber rods, and one piece of string. Suspend one of the rubber rods from a string tied around the middle of the rod so that the rod is balanced. Rub one end of the rod with the fur. Rub one end of a second rubber rod with the fur, and then bring the rubbed end close to, but not touching, the rubbed end of the rod that is suspended from the string. What do you observe?***
Again, what you should observe in this case is that the end of the suspended rod moves away from the other rod – the suspended rod is repelled by the second rod. By Newton's third law, we know that the rods must be repelling one another with equal-and-opposite forces.

Step 3 – ***Now, bring the rubbed end of a glass rod (rubbed with silk) close to, but not touching, the rubbed end of the rubber rod (rubbed with fur) that is suspended from the string. What do you observe?***
In this situation, what you should observe is that the rods attract one another.

Step 4 – ***Repeat the experiments with a number of other types of rod material rubbed with different materials. What do you observe?***
In general, you should observe that all rubbed rods tend to act either like a glass rod rubbed with silk, or like a rubber rod rubbed with fur.

Step 5 – ***Can you explain these observations using a simple model involving charge? If so, describe the features of the model.***

The model we use to explain the observations with the rods is to first say that rubbing one material with a different material generally causes a transfer of charge from one material to the other. All glass rods rubbed with silk, for instance, should acquire charge of the same sign. To account for the observation that identical charged rods repel one another, our model states that like charges repel. We also build into the model that unlike charges attract, explaining why a glass rod rubbed with silk will attract a rubber rod rubbed with fur – a rubber rod rubbed with fur must acquire charge of the opposite sign to a glass rod rubbed with silk. Our model also accounts for two types of charge, which we call positive and negative.

Key ideas about interacting rubbed rods: The observations we make with the charged rods enable us to construct a basic model of charge. In this model, there are two types of charge, positive and negative. However, both types of charge can be obtained from the transfer of electrons, which have a negative charge. Rubbing a glass rod with silk generally transfers electrons from the glass to the silk, leaving the glass with a positive charge. Rubbing a rubber rod with fur generally transfers electrons from the fur to the rubber, giving the rubber a negative charge.
Related End of Chapter Exercise: 40.

Acquiring Charge

Everyday objects contain large numbers of electrons (negative charges) and protons (positive charges). In many instances the number of electrons is the same as the number of protons, so the object has no net charge. It is quite easy to give an object a net charge, however.

As we have learned, one way to charge an object is to rub it with a different material. For instance, rubbing a glass rod with silk transfers electrons from the glass to the silk, leaving the glass rod with a positive charge and giving the silk a negative charge. How effective this process is, and which material ends up with the negative charge, depends on where the two materials fit in the triboelectric series, shown in Table 16.2. "Tribos" is a Greek word meaning "rubbing", so triboelectricity is all about giving objects net electric charges by rubbing. Many centuries ago, the Greeks themselves did experiments with charge, rubbing amber with wool. It is no coincidence that the Greek word for amber is "electron."

Rubbing promotes charge transfer, but all that is necessary is to bring the two materials into contact, causing chemical bonds (which involve electrons) to form between them. Upon separation the atoms in one material tend to keep some of the electrons while atoms in the other material tend to give them up. In general, the farther apart the materials in the triboelectric series, the more charge is transferred, with the material farther down the list acquiring electrons and ending up with a negative charge.

MOST POSITIVE
Leather
Rabbit's fur
Glass
Nylon
Wool
Silk
Paper
Cotton **NEUTRAL**
Amber
Polystyrene
Rubber balloon
Hard rubber
Saran wrap
Polyethylene
Vinyl (PVC) **MOST NEGATIVE**

Table 16.2: The triboelectric series. When one material is brought into contact with another and then separated, some electrons can be transferred from one to the other. The material further down the list generally becomes negative.

Essential Question 16.1: 1 coulomb (1 C) represents a large amount of charge. If –1.0 C worth of electrons is transferred from a glass rod to a piece of silk, how many electrons are involved? By how much does the mass of each object change? *(The answer is at the top of the next page.)*

Answer to Essential Question 16.1: Dividing the total charge by the charge on one electron gives the number of electrons involved:

$$n = \frac{q}{-e} = \frac{-1.0 \text{ C}}{-1.602 \times 10^{-19} \text{ C/electron}} = 6.2 \times 10^{18} \text{ electrons}.$$

Because electrons are transferred from the glass rod to the silk, the mass of the glass rod decreases while the mass of the silk increases. Multiplying the number of electrons transferred by the mass of each electron (see Table 16.1) gives the total mass involved, which is very small:

$$m = \left(6.242 \times 10^{18} \text{ electrons}\right) \times \left(9.11 \times 10^{-31} \text{ kg/electron}\right) = 5.7 \times 10^{-12} \text{ kg}.$$

16-2 Extending our Model of Charge

Key facts about charge: keep the following in mind when dealing with electric charge.

- The symbol for charge is q or Q. The MKS unit for electric charge is the coulomb (C), although we will also use units of e, the magnitude of the charge on the electron.
- Unlike mass, which is always positive, charges can be either positive (+) or negative (–).
- Like charges (both + or both –) repel one another; unlike charges (a + and a –) attract.
- Charge is quantized – it can only be particular values. Charging an object generally involves a transfer of electrons, so the charge on an object is an integer multiple of e.
- Charge is conserved. This is another of the fundamental conservation laws of physics, that the net charge of a closed system remains constant. See Example 16.2 for an application.

Conductors and Insulators

When an electric appliance, such as a television or refrigerator, is on, electric charges flow through the wires connecting the appliance to a wall socket, and through the wires within the appliance itself. Even so, it is generally safe to touch the cable connecting the appliance to the wall socket as long as the metal wires inside that cable are completely covered by rubber. This exploits the different material properties of metal and rubber, specifically the differences in their conductivity. Metals (which we classify as conductors) generally have conductivities that are orders of magnitude larger than the conductivities of materials like rubber and plastic – those materials we call insulators. The major difference between these two classes of materials is that, in an insulator, each electron is closely tied to its molecule, while some fraction of the electrons in a conductor (these are known as the conduction electrons) are free to move around.

Charge is Quantized

When something is quantized it can not take on just any value – only particular values are possible. An example is money, which is quantized in units of pennies (in the USA and Canada, at least). It is possible to have \$1.27, the equivalent of 127 pennies, but it is not possible to have 2/7 of a dollar. For something to be quantized does not necessarily mean that its allowed values are integer multiples of its smallest unit, but that is how things work with money and charge.

For now, we can say that the smallest unit of charge is $e = 1.602 \times 10^{-19} \text{ C}$, the magnitude of the charge on the electron and the proton. Expressing charge quantization as an equation:

$q = ne$, (Eq. 16.1: **Charge is quantized, and comes in integer multiples of *e***)

where n is any positive or negative integer.

Charging by Induction

An uncharged conducting object like a metal sphere can be charged by rubbing it with a charged rod, acquiring charge of the same sign as that of the rod. However, it can also be charged without touching it with a charged rod, in the process known as charging by induction:

1. ***Bring a charged insulating rod close to the conductor, without allowing the rod to touch the conductor.*** Bringing the charged rod close causes conduction electrons in the conductor to move in response to the presence of the charge. The electrons move toward the rod, if the rod is positive, or (as shown in Figure 16.1), they move away from the rod if the rod is negative. The conductor is now polarized, but it still has no net charge.

2. ***Ground the conductor.*** A ground is a large object, like the Earth, that can accept or give up electrons without being affected. We can ground the conductor by connecting a wire from the conductor to a metal pipe. This allows electrons to be transferred from ground to the conductor, if the rod is positive, or from the conductor to ground if the rod is negative. The conductor now has an excess charge with a sign opposite to that of the charge on the rod.

3. ***Remove the ground connection.*** This strands the transferred charge.

4. ***Remove the charged rod.*** The charge on the conductor redistributes itself, but the conductor keeps the net charge it has by the end of step 2.

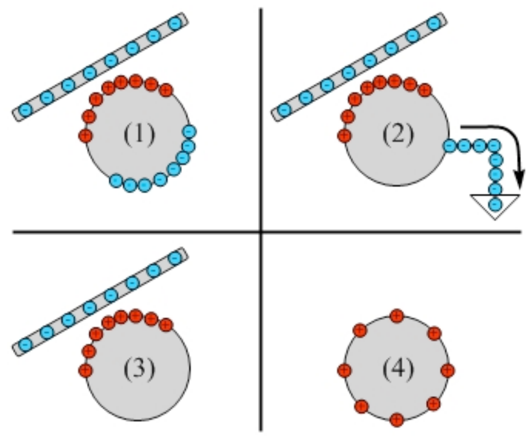

Figure 16.1: The process of charging by induction.

In the charging by induction process, the conducting object ends up with a net charge of the opposite sign as the charge on the rod. Note that steps 1 and 2 in the process can be reversed – the charge is only transferred after both these steps have been done. However, steps 3 and 4 must be done in the order above. Removing the charged rod before removing the ground connection allows charge transferred between the conductor and ground to return to where it started. The presence of the rod keeps the net charge on the conductor until the ground connection is removed.

EXAMPLE 16.2 – Two spheres touch

Two identical conducting spheres sit on separate insulating stands. Sphere A has a net positive charge of $+8Q$, while sphere B has a net negative charge of $-2Q$. The spheres are touched together briefly and then separated again. How much charge is on each sphere now?

SOLUTION

Each sphere has a charge of $+3Q$. The net charge in the system is $+6Q$, so, if each sphere ends up with half of this, we satisfy the law of Conservation of Charge and also get the symmetry we expect because the spheres are identical. Even though sphere B has a net charge of $-2Q$, it transfers $-5Q$ to sphere A. The extra $-3Q$ is made up of some of sphere B's conduction electrons.

Related End-of-Chapter Exercises for this section: 2 and 3.

Essential Question 16.2: In Example 16.2, the spheres are conducting. Would we get the same result if the spheres were both made of rubber, an insulating material? Explain.

Answer to Essential Question 16.2: No. Charge does not flow on an insulator, so touching charged rubber spheres together could transfer a small amount of charge between the spheres, but most of the excess charge would stay where it was. This is the big difference between conductors and insulators – charge flows easily in a conductor but does not flow through an insulator.

16-3 Coulomb's Law

To qualitatively measure the charge on an object, we can use an electroscope (see Figure 16.2). When the electroscope is charged (such as by rubbing it with a charged rod), the charge distributes itself over the entire electroscope because the electroscope is made from conducting material. Like charges repel, so the arm of the electroscope swings out, as in the electroscope on the right. The larger the charge, the more the arm swings out. To get a more quantitative measure of charge than we can get with an electroscope, we use Coulomb's law.

Figure 16.2: An uncharged electroscope, on the left, and a charged electroscope, on the right. Photo courtesy A. Duffy.

Point charges are charged objects that are so small that all the charge is effectively at one point. The force between point charges of charge q and Q, separated by a distance r, is given by:

$$\vec{F}_E = \frac{kqQ}{r^2}\hat{r}$$ (Eq. 16.2: **Coulomb's Law for the force between point charges**)

where $k = 8.99\times10^9 \text{ N m}^2/\text{C}^2$ is a constant. The unit vector $\hat{r}$ tells us the force is directed along the line joining the charges. If the charges have opposite signs, qQ is negative and the force is attractive, directed toward the object applying the force. If the charges have the same sign, qQ is positive and the force is repulsive, directed away from the object applying the force.

Comparing Coulomb's law to Newton's law of universal gravitation, which gives the force between two objects with mass, we see that they have the same form:

$$\vec{F}_G = -\frac{GmM}{r^2}\hat{r}$$ (Equation 8.1: **Newton's Law of Universal Gravitation**)

where $G = 6.67\times10^{-11}\text{N m}^2/\text{kg}^2$ is the universal gravitational constant. For Coulomb's law, k takes the place of G, and the charges q and Q take the place of the masses, m and M. Note that objects with mass always attract one another, while charged objects can attract or repel.

EXAMPLE 16.3 – Inside the hydrogen atom

A hydrogen atom contains an electron and a proton. In the Bohr model of the atom, the electron follows a circular orbit around the proton. Determine the ratio of the magnitudes of the electrostatic force to the gravitational force between the proton and electron.

SOLUTION

Note that we don't need to know the radius, because the radius cancels out in the ratio.

$$\frac{F_E}{F_G} = \frac{k|qQ|/r^2}{GmM/r^2} = \frac{k|qQ|}{GmM} = \frac{\left(9.0\times10^9 \text{ N m}^2/\text{C}^2\right)\times\left(1.6\times10^{-19}\text{ C}\right)\times\left(1.6\times10^{-19}\text{ C}\right)}{\left(6.67\times10^{-11}\text{ N m}^2/\text{kg}^2\right)\times\left(9.11\times10^{-31}\text{ kg}\right)\times\left(1.67\times10^{-27}\text{ kg}\right)} = 2.3\times10^{39}$$

The gravitational interaction is negligible, being 39 orders of magnitude smaller!

EXPLORATION 16.3 – The principle of superposition

Three balls, with charges of $+q$, $-2q$, and $-3q$, are equally spaced along a line. The spacing between the balls is r. We can arrange the balls in three different ways, as shown in Figure 16.3. In each case, the balls are in an isolated region of space very far from anything else.

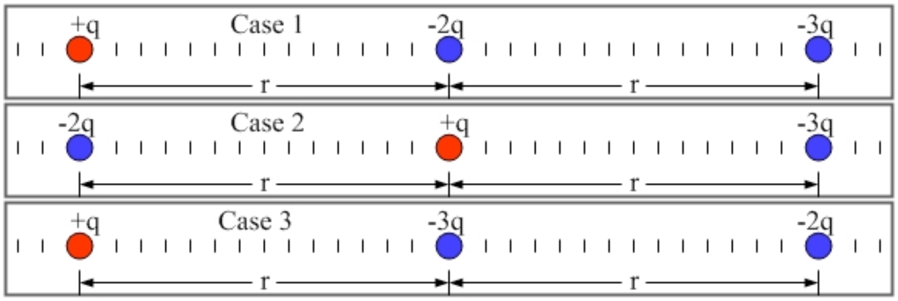

Figure 16.3: Three different arrangements of three balls of charge $+q$, $-2q$, and $-3q$ placed on a line with a distance r between neighboring balls. Each ball experiences two electrostatic forces, one from each of the other balls. We can neglect any other interactions.

Step 1 – ***Consider Case 1. Is the electrostatic force that the ball of charge +q exerts on the ball of charge –3q affected by the fact that the ball of charge –2q lies between the other two balls?*** No, we can apply the principle of superposition – the force from the interaction between any pair of charged objects is unaffected by the presence of any other charged object in the vicinity.

Step 2 – ***In which case does the ball of charge –2q experience the largest-magnitude net force? Argue qualitatively.*** Let's attach arrows to the ball of charge $-2q$, as in Figure 16.4, to represent the two forces it experiences in each case. The length of each arrow is proportional to the force.

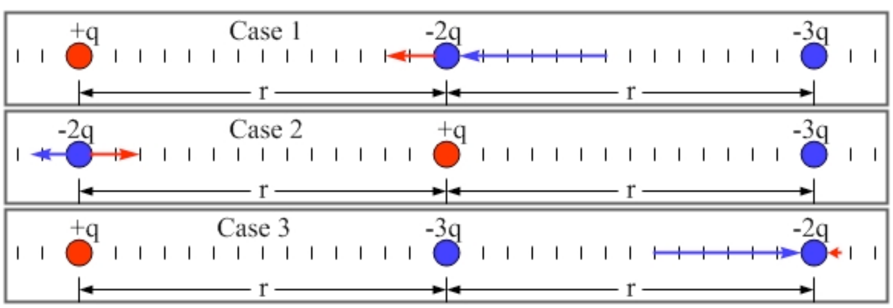

Figure 16.4: Attaching force vectors to the ball of charge $-2q$, which is attracted to the $+q$ ball and repelled by the $-3q$ ball. The length of each vector is drawn in units of kq^2/r^2.

The net force is largest in case 1. In cases 2 and 3, the forces partly cancel, while only in case 1 are the directions of the two forces acting on the ball of charge $-2q$ the same, and the net force case 1 is clearly larger than it is in the other two cases.

Key ideas about adding electrostatic forces: The net force acting on an object can be found using the principle of superposition, remembering that each individual force is unaffected by the presence of other forces. **Related End of Chapter Exercises: 28, 29.**

Essential Question 16.3: An object with a charge of $+5Q$ is placed a distance r away from an object with a charge of $+2Q$. Which object exerts a larger electrostatic force on the other?

Answer to Essential Question 16.3: Newton's third law tells us that the electrostatic forces the two objects exert on one another are equal in magnitude (and opposite in direction). This follows from Coulomb's law, because whether we look at the force exerted by the first object or the second object the factors going into the equation are the same in both cases.

16-4 Applying the Principle of Superposition

EXPLORATION 16.4 – Three objects in a line

Let's return again to the situation of three different arrangements of three balls that we looked at in Exploration 16.3. The balls, with charges of $+q$, $-2q$, and $-3q$, are equally spaced along a line. The spacing between the balls is r. In each case, the balls are in an isolated region of space very far from anything else.

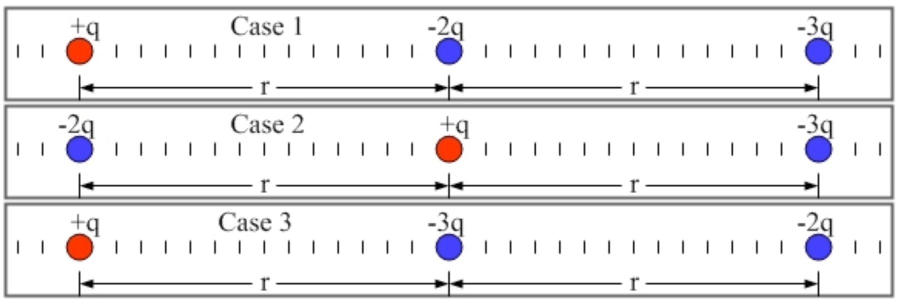

Figure 16.5: Three different arrangements of three balls of charge $+q$, $-2q$, and $-3q$ placed on a line with a distance r between neighboring balls.

Step 1 – ***Calculate the force experienced by the ball of charge –2q in each case.***

To do this, we will make extensive use of Coulomb's law. Let's define right to be the positive direction, and use the notation $\vec{F}_{21}$ for the force that the ball of charge $-2q$ experiences from the ball of charge $+q$. The + and – signs in the equation come from the direction of the force, not the signs on the charges. In each case:

$$\vec{F}_{2,net} = \vec{F}_{21} + \vec{F}_{23}$$

$$\text{Case 1: } \vec{F}_{2,net} = \vec{F}_{21} + \vec{F}_{23} = -\frac{kq(2q)}{r^2} - \frac{k(2q)(3q)}{r^2} = -\frac{2kq^2}{r^2} - \frac{6kq^2}{r^2} = -\frac{8kq^2}{r^2}$$

$$\text{Case 2: } \vec{F}_{2,net} = \vec{F}_{21} + \vec{F}_{23} = +\frac{kq(2q)}{r^2} - \frac{k(2q)(3q)}{(2r)^2} = +\frac{2kq^2}{r^2} - \frac{3kq^2}{2r^2} = +\frac{kq^2}{2r^2}$$

$$\text{Case 3: } \vec{F}_{2,net} = \vec{F}_{21} + \vec{F}_{23} = -\frac{kq(2q)}{(2r)^2} + \frac{k(2q)(3q)}{r^2} = -\frac{kq^2}{2r^2} + \frac{6kq^2}{r^2} = +\frac{11kq^2}{2r^2}$$

The ball of charge $-2q$ does experience the largest-magnitude net force in case 1.

Key ideas about adding electrostatic forces: Again, we see that the net force acting on an object can be found using the principle of superposition, remembering that each individual force is unaffected by the presence of other forces. In addition, + and – signs should be based on the direction of the force, rather than the signs of the charges.
Related End of Chapter Exercises: 30, 31.

EXAMPLE 16.4 – A two-dimensional situation

Compare this example to Example 8.1. Three balls, with charges of $+q$, $-2q$, and $+3q$, are placed at the corners of a square measuring L on each side, as shown in Figure 16.6. Assume this set of three balls is not interacting with anything else in the universe, and assume that gravitational interactions are negligible. What is the magnitude and direction of the net electrostatic force on the ball of charge $+q$?

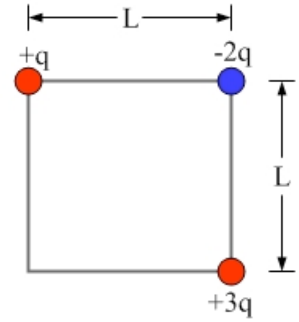

Figure 16.6: Three charged balls placed at the corners of a square.

SOLUTION

Let's attach force vectors (see Figure 16.7) to the ball of charge $+q$, which is attracted to the $-2q$ ball and repelled by the $+3q$ ball. The length of each vector is proportional to the magnitude of the force it represents.

We can find the two individual forces acting on the ball of charge $+q$ using Coulomb's law. Let's define +x to the right and +y up.

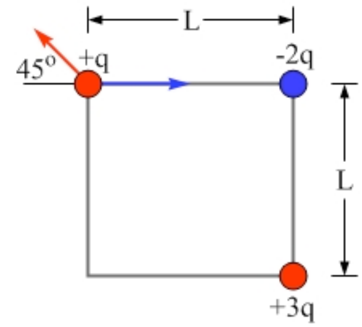

Figure 16.7: Attaching force vectors to the ball of charge $+q$.

From the ball of charge $-2q$: $\vec{F}_{21} = \frac{kq(2q)}{L^2}$ to the right.

From the ball of charge $+3q$: $\vec{F}_{31} = \frac{kq(3q)}{L^2 + L^2}$ at 45° above the $-x$-axis.

Finding the net force is a vector addition problem.

In the x-direction, we get:

$$\vec{F}_{1x} = \vec{F}_{21x} + \vec{F}_{31x} = +\frac{2kq^2}{L^2} - \frac{3kq^2}{2L^2}\cos 45^\circ = \left(2 - \frac{3}{2\sqrt{2}}\right)\frac{kq^2}{L^2}.$$

Note that the signs on each term come not from the signs on the charges, but from comparing the direction of the forces to the directions we chose to be positive above.

In the y-direction, we get: $\vec{F}_{1y} = \vec{F}_{21y} + \vec{F}_{31y} = 0 + \frac{3kq^2}{2L^2}\sin 45^\circ = \left(+\frac{3}{2\sqrt{2}}\right)\frac{kq^2}{L^2}$.

The Pythagorean theorem gives the magnitude of the net force on the ball of charge $+q$:

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2} = \sqrt{\left(4 - \frac{6}{\sqrt{2}} + \frac{9}{8} + \frac{9}{8}\right)}\frac{kq^2}{L^2} = 1.42\frac{kq^2}{L^2}.$$

The angle is given by: $\tan\theta = \frac{F_{1y}}{F_{1x}} = \frac{\frac{3}{2\sqrt{2}}}{\frac{4\sqrt{2}-3}{2\sqrt{2}}} = \frac{3}{4\sqrt{2}-3}$.

So, the angle is 48.5° above the $+x$-axis.

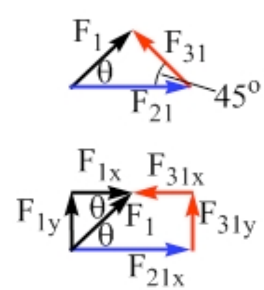

Figure 16.8: The triangle representing the vector addition problem above.

Related End-of-Chapter Exercises: 4, 5, 16, 17, 41, 49, 50, 53.

Essential Question 16.4: In Exploration 16.4, on the previous page, which ball experiences the largest-magnitude net force in (i) Case 1, (ii) Case 2, and (iii) Case 3?

Answer to Essential Question 16.4: The object experiencing the largest-magnitude net force is the ball of charge $-2q$ in case 1, and the ball of charge $-3q$ in cases 2 and 3.

16-5 The Electric Field

There are important parallels between the electric field $\vec{E}$ and the gravitational field $\vec{g}$, so many that you may find it helpful to review Section 8-3.

The electric field, $\vec{E}$, at a particular point can be defined in terms of the electric force, $\vec{F}_E$, that an object of charge q would experience if it were placed at that point: $\vec{E} = \frac{\vec{F}_E}{q}$, or $\vec{F}_E = q\vec{E}$. (Eq. 16.3: **Connecting electric field and electric force**)   The units for electric field are N/C.
A special case is the electric field from a point charge with a charge Q: $\vec{E} = \frac{kQ}{r^2}\hat{r}$, (Equation 16.4: **Electric field from a point charge**)   where r is the distance from the charge to the point in space where we are finding the field. The magnitude of the field is kQ/r^2. The electric field points away from a positive charge and toward a negative charge.

One way to think about an electric field is the following: it is a measure of how a charged object, or a set of charged objects, influences the space around it.

Visualizing the electric field

It can be useful to draw a picture that represents the electric field near an object, or a set of objects, so we can see at a glance what the field in the region is like. In general there are two ways to do this, by using either field lines or field vectors. The field-line representation is shown in Figure 16.9. Figure 16.9(a) represents a uniform electric field directed down, while Figure 16.9 (b) represents the electric field near a negative point charge. Figure 16.9(c) shows the field from an electric dipole, which consists of two objects with equal-and-opposite charge.

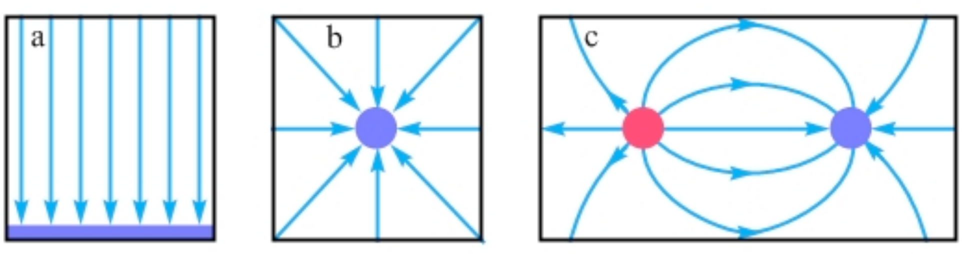

Figure 16.9: Field-line diagrams for various situations. Diagram *a* represents a uniform electric field directed down, while diagram b represents the electric field near a negative point charge. Diagram c shows the electric field near an electric dipole, which is a pair of charges of equal magnitude and opposite sign separated by some distance.

Question: How is the direction of the field at a particular point shown on a field-line diagram? What indicates the relative strength of the field at a point on the field-line diagram?

Answer: As with gravitational field lines, each field line has a direction marked on it with an arrow that shows the direction of the electric field at all points along the field line. The relative strength of the field is indicated by the density of the field lines (that is, by how close the lines are). The more lines there are in a given area, the larger the field.

A second method of representing a field is to use field vectors. A field vector diagram reinforces the idea that every point in space has an electric field associated with it, because a grid made up of equally spaced dots is superimposed on the picture and a vector is attached to each of these grid points. All the vectors are the same length. The situations represented by the field-line patterns in Figure 16.9 are now re-drawn in Figure 16.10 using the field-vector representation.

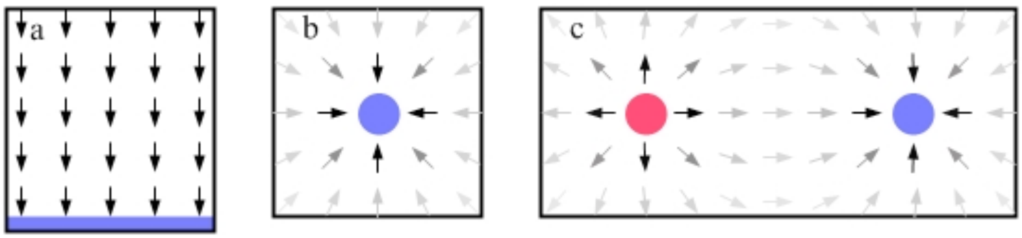

Figure 16.10: Field-vector diagrams for various situations. In figure *a,* the field is uniform and directed down. Figure *b* represents the non-uniform but symmetric field found near a negative point charge, while figure *c* shows field vectors for an electric dipole.

Question: How is the direction of the field at a particular point shown on a field-vector diagram? What indicates the relative strength of the field at a point on the field-vector diagram?

Answer: The direction of the field at a particular point is represented by the direction of the field vector at that point (or the ones near it if the point does not correspond exactly to the location of a field vector). The relative strength of the field is indicated by the darkness of the arrow. The larger the magnitude of the field the darker the arrow.

Related End of Chapter Exercises: 25, 26, 58.

We often use a test charge to sample the electric field near a charge or a set of charges. A test charge is a positive charge of such a small magnitude that it has negligible impact on the field it is sampling. Based on the relationship $\vec{F}_E = q\vec{E}$, the force on a positive test charge is in the direction of the electric field at the point where the test charge is, and the size of the force is proportional to the magnitude of the field at that point.

Essential Question 16.5: Figure 16.11 shows the force a positive test charge feels when it is placed at the location shown, near two charged balls. The ball on the left has a positive charge $+Q$, while the ball on the right has an unknown sign and magnitude. Based on the force experienced by the test charge, what is the sign of the charge on the ball on the right? How does the magnitude of that charge compare to Q?

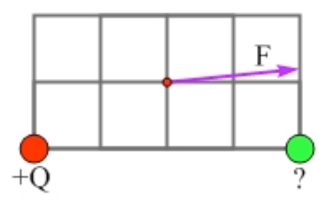

Figure 16.11: The arrow shows the force experienced by a test charge when it is placed at the position shown near two charged balls.

Answer to Essential Question 16.5: The $+Q$ charge exerts a force on the test charge that points directly away from the $+Q$ charge. The unknown charge shifts the direction of the force on the test charge towards the unknown charge, so the unknown charge must be negative. If the unknown charge was $-Q$, by symmetry the test charge would experience a net force directed right, with no vertical component. Decreasing the magnitude of the unknown charge gives the net force an upward component; so the magnitude of the unknown charge is less than Q.

16-6 Electric Field: Special Cases

Let's consider two important special cases. The first involves a charged object in a uniform electric field, while the second involves the electric field from point charges.

EXPLORATION 16.6A – Motion of a charge in a uniform electric field

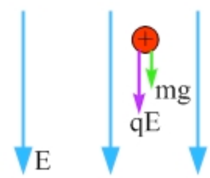

Figure 16.12: Free-body diagram for a proton in a uniform electric field directed down.

Step 1 – ***Sketch the free-body diagram for a proton placed in a uniform electric field of magnitude E = 250 N/C that is directed straight down. Then apply Newton's Second Law to find the acceleration of the proton. How important is gravity in this situation?*** The free-body diagram in Figure 16.12 shows the two forces applied to the proton, the downward force of gravity and the downward force applied by the electric field.

Taking down to be positive, applying $\sum \vec{F} = m\vec{a}$, gives: $+qE + mg = m\vec{a}$.

The values of the mass and charge of the proton are given in Table 16.1. Solving for the acceleration gives:

$$\vec{a} = +\frac{qE}{m} + g = +\frac{(1.60\times10^{-19}\ \text{C})(250\ \text{N/C})}{(1.67\times10^{-27}\ \text{kg})} + 9.8\ \text{m/s}^2 = +2.40\times10^{10}\ \text{m/s}^2 .$$

Note that gravity is negligible in this case, because g is orders of magnitude less than qE/m.

Step 2 – ***How far has the proton traveled 25 µs after being released from rest in this field?*** The electric field is uniform, so the acceleration is constant. We can apply the constant acceleration equations from one-dimensional motion (Chapter 2) or projectile motion (Chapter 4).

From Eq. 2.11: $\vec{x} = \vec{x}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2 = 0 + 0 + \frac{1}{2}(2.395\times10^{10}\ \text{m/s}^2)(25\times10^{-6}\ \text{s})^2 = 0.75\ \text{m}$.

Key idea: The acceleration of a charged particle in a uniform electric field is constant, so the constant-acceleration equations from Chapters 2 and 4 apply. The scale of the accelerations and times are different but the physics is the same. **Related End-of-Chapter Exercises: 27 and 28.**

EXAMPLE 16.6A – Where is the electric field equal to zero?

Two point charges, with charges of $q_1 = +2Q$ and $q_2 = -5Q$, are separated by a distance of 3.0 m, as shown in Figure 16.13. Determine all locations on the line passing through the two charges where their individual electric fields combine to give a net electric field of zero.

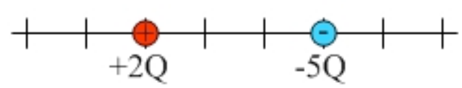

Figure 16.13: Two point charges separated by 3.0 m.

(a) For the net field to be zero, what condition(s) must the individual fields satisfy?

(b) Using qualitative arguments, can the electric field be zero at:

I. a point on the line, to the left of the $+2Q$ charge?

II. a point on the line, in between the charges?

III. a point on the line, to the right of the $-5Q$ charge?

(c) Calculate the locations of all points near the charges where the net electric field is zero.

SOLUTION

(a) The two individual fields add as vectors, so they must cancel one another for the net field to be zero. The two fields must be of equal magnitude and point in opposite directions.

(b) Figure 16.14 shows three representative points on the line through the charges. In region I, to the left of the $+2Q$ charge, the two fields are in opposite directions. There is only one point at which the fields cancel in that region because close to the $+2Q$ charge the field from that charge dominates, and far from the charges the field from the $-5Q$ charge dominates because that charge is larger. At some point in between the fields exactly balance. In region II, between the charges, there is no such point because both fields are directed right, and can not cancel.

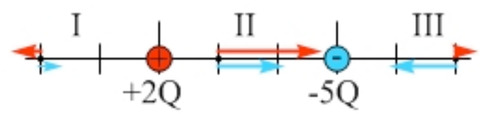

Figure 16.14: Choosing a point in each region allows us to do a qualitative analysis to determine where the net field is zero. At each point we draw two field vectors, one from each charge.

We also see from Figure 16.14 that at a point to the right of the $-5Q$ charge the two fields point in opposite directions. However, because such points are closer to the $-5Q$ charge (the larger-magnitude charge), the field from the $-5Q$ charge is always larger in magnitude than the field from the $+2Q$ charge. Thus, there are no points in region III where the net field is zero.

(c) Thus, we can conclude that there is only one point at which the net electric field is zero. Let's say this point is a distance x to the left of the $+2Q$ charge. Equating the magnitude of the field from one charge at that point to the magnitude of the field from the second charge gives:

$$\frac{k|q_1|}{x^2} = \frac{k|q_2|}{(x+3.0\text{ m})^2}.$$

Canceling factors of k leads to: $\dfrac{2Q}{x^2} = \dfrac{5Q}{(x+3.0\text{ m})^2}$.

Canceling factors of Q and re-arranging gives: $\dfrac{(x+3.0\text{ m})^2}{x^2} = \dfrac{5}{2}$.

Taking the square root of both sides gives: $\dfrac{x+3.0\text{ m}}{x} = \pm\sqrt{2.5}$.

Using the + sign gives, when we solve for x: $x = \dfrac{3.0\text{ m}}{\sqrt{2.5}-1} = 5.16\text{ m}$.

Thus, the electric field is zero at the point 5.16 m to the left of the $+2Q$ charge.

Related End-of-Chapter Exercises: 57, 59, and 60.

Essential Question 16.6: The method in the previous example gives two solutions. What is the second solution in this case and what is its physical meaning?

Answer to Essential Question 16.6: We can find the other solution by using the – sign. This gives: $x' = \frac{3.0 \text{ m}}{-\sqrt{2.5}-1} = -1.16 \text{ m}$. This is a subtle point, but because x was defined as the distance of the point to the left of the origin, the negative answer gives us a point 1.16 m to the right of the origin, between the point charges. This is actually the other point on the line joining the charges where the fields from the two charges have the same magnitude. However, between the charges those two fields have the same direction (both pointing right), so they add rather than canceling.

16-7 Electric Field Near Conductors

At equilibrium, the conduction electrons in a conductor move about randomly, somewhat like atoms of ideal gas, but there is no net flow of charge in any direction. If there is a change in the external electric field the conductor is exposed to, however, the conduction electrons respond by redistributing themselves, very quickly coming to a new equilibrium distribution. At equilibrium a number of conditions apply:

1. There is no electric field inside the solid part of the conductor.
2. The electric field at the surface of the conductor is perpendicular to the surface.
3. If the conductor is charged, excess charge lies only at the surface of the conductor.
4. Charge density is highest, and electric field is strongest, on pointy parts of a conductor.

Let's investigate each of these conditions in more detail.

At equilibrium, *E* = 0 within solid parts of a conductor.

If electric field penetrates into a conductor, conduction electrons immediately respond to the field. Because $\vec{F} = q\vec{E}$, and electrons are negative, electrons feel a force that is opposite to the field. As shown in Figure 16.15, there is a net movement of electrons to the region where the field enters the conductor. The field lines end at the electrons at the surface, so *E* = 0 within the conductor. This redistribution of electrons leaves positive charge at the other side of the conductor, so field lines start up again there and go away from the conductor.

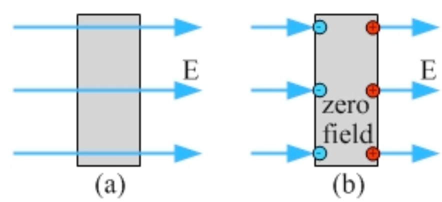

Figure 16.15: Conduction electrons in a conductor quickly redistribute themselves until the field is zero inside the conductor.

At equilibrium, electric field lines are perpendicular to the surface of a conductor.

If the electric field lines end at the surface of a conductor but are not perpendicular to the surface, as in Figure 16.16(a), the charges at the surface feel a force from the field. As Figure 16.16(b) shows, the component of the force parallel to the surface ($F_{\parallel}$) causes the charges to flow along the surface, carrying the field lines with them. The charges are in equilibrium when the electric field lines are perpendicular to the surface, as in Figure 16.16(c).

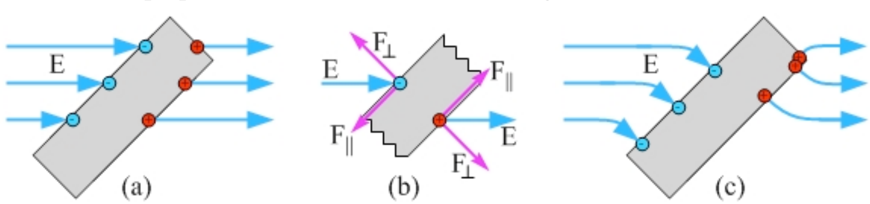

Figure 16.16: If electric field lines are not perpendicular to the surface of a conductor, the charges at the surface redistribute themselves until the field lines are perpendicular.

Electrons at the surface still feel a force component perpendicular to the surface that is trying to remove the electrons from the conductor. In most cases this will not happen because the conductor is surrounded by insulating material (such as air), but if the field is strong enough electrons will jump off the surface. When this happens there is a spark from the conductor.

At equilibrium, any excess charge lies only at the surface of a conductor.

This statement is a consequence of the fact that at equilibrium $E = 0$ within the conductor. If there was excess charge in the bulk of the conductor field lines would either start there, if it was positive, or end there, if it was negative. This non-zero field inside the conductor would cause the charge to move to the surface to bring the field to zero within the conductor.

Charge tends to accumulate on pointy parts of a conductor.

Figure 16.17 shows three different situations. In Figure 16.17(a) a metal sphere has a net positive charge. At equilibrium the excess charge is distributed uniformly over the surface of the sphere. Moving any of the charges around results in forces that act on these charges, driving them back to the equilibrium distribution. In contrast, Figure 16.17(b) shows excess charge (negative in this case, but our analysis is equally valid for positive charge) distributed evenly along a conducting rod. The charge at the center experiences no net force from the other charges, but the other charges experience net forces that push them toward the ends of the rod: each charge on the right experiences a net force pushing it further right, while each charge on the left experiences a net force pushing it further left. The equilibrium situation for the rod is more like that shown in Figure 16.17(c), where there is a much larger charge density at the ends than in the middle.

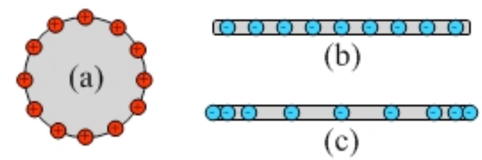

Figure 16.17: In (a), excess positive charge is uniformly distributed over the surface of a metal sphere. The charge is at equilibrium because there are no forces acting on the charges to move them around the sphere. In (b), however, uniformly distributing charge along the length of a conducting rod results in net forces on the charges that shift them toward the ends of the rod, as in (c).

This helps us to understand how a lightning rod works. Lightning occurs when charge builds up, increasing the local electric field to a large enough value that charge can travel between a cloud and, say, your house. Without a lightning rod this can take a long time, requiring a lot of charge, so that when the discharge finally happens it can involve a great deal of energy and cause significant damage. With a sharply-pointed lightning rod, attached to ground, on your house, however, charge and field builds up quickly at the tip of the lightning rod. This causes a slow and steady drain of charge from the cloud to the rod and then the ground, much safer than one sudden large discharge. The lightning rod was invented by Benjamin Franklin.

Related End-of-Chapter Exercises: 62 – 64.

Essential Question 16.7: A point charge with a charge of $+Q$ is placed at the center of a hollow thick-walled metal sphere. The sphere itself has no net charge on it. Which of the three pictures in Figure 16.18 correctly shows the equilibrium charge distribution on the metal sphere?

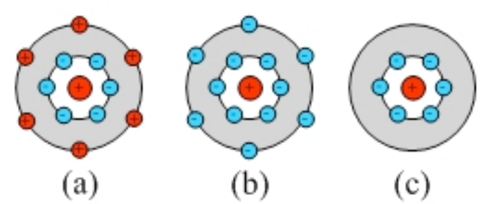

Figure 16.18: Three possible equilibrium situations for when a charge of $+Q$ is placed at the center of a hollow thick-walled metal sphere that has no net charge. In (a) the sphere has a total charge $+Q$ on its outer surface and $-Q$ on its inner surface; in (b) the sphere has a charge $-Q$ on both its inner and outer surfaces, and in (c) the sphere has a charge of $-Q$ on its inner surface.

Answer to Essential Question 16.7: Figure 16.18(a) shows the correct result. Enough conduction electrons in the sphere are attracted to the inner surface that the field lines from the center $+Q$ charge do not penetrate into the sphere. This takes $-Q$ worth of charge. That leaves a net positive charge $+Q$ on the outer surface, as shown. In addition, (a) is the only picture that shows no net charge on the sphere itself.

Chapter Summary

Essential Idea: Electric Charge.

In many situations the interaction between objects that have a net electric charge, or the behavior of a charged object in a uniform electric field, is analogous to that of objects with mass that interact with each other via the force of gravity, or that are in a uniform gravitational field.

Key Facts about Charge

- The symbol for charge is q or Q. The MKS unit for electric charge is the Coulomb (C), although we will also use units of e, the magnitude of the charge on the electron.
- Unlike mass, which is always positive, charges can be either positive (+) or negative (–).
- Like charges (i.e, both + or both –) repel one another; unlike charges (a + and a –) attract.
- Charge is quantized – it can only be particular values. Since charging an object generally involves a transfer of electrons, the charge on an object is an integer multiple of e.
- Charge is conserved. This is another of the fundamental conservation laws of physics, that the net charge of a closed system remains constant.
- Most materials fall into one of two categories. Charge can flow easily through **conductors**, while it does not readily flow through **insulators**. Metals are good conductors, while plastic and rubber are examples of good insulators.
- When an insulating rod of one material is rubbed by another material, electrons can be transferred from one to the other. Which material ends up with a negative charge, and how much charge is transferred, depends on where the two materials lie in the **triboelectric series** (see Table 16.1).

Coulomb's Law for the Force Between Charged Objects

The force that an object with a charge q exerts on a second object with a charge Q that is a distance r away is given by Coulomb's Law:

$$\vec{F}_E = \frac{kqQ}{r^2}\hat{r} \qquad \text{(Equation 16.2)}$$

where $k = 8.99\times10^9 \text{ N m}^2/\text{C}^2$ is a constant. The unit vector $\hat{r}$ indicates that the force is directed along the line joining the two charged objects. If the charges are of opposite signs then the product qQ is negative and the force is attractive, directed back toward the object applying the force. If the charges have the same sign then qQ is positive and the force is repulsive, directed away from the object applying the force.

The principle of superposition applies – the net force on a charged object is the vector sum of the individual forces acting on that object.

Electric Field

The electric field, $\vec{E}$, at a particular point can be defined in terms of the electric force, $\vec{F}_E$, that an object of charge q would experience if it were placed at that point:

$$\vec{E} = \frac{\vec{F}_E}{q}, \quad \text{or} \quad \vec{F}_E = q\vec{E} \qquad \text{(Eq. 16.3: \textbf{Connecting electric field and electric force})}$$

The units for electric field are N/C. An object with a positive charge experiences a force in the direction of the field, while an object with a negative charge experiences a force opposite in direction to the electric field.

Visualizing the Electric Field

We can visualize the electric field by drawing field lines or field vectors. Field lines are continuous lines that start on positive charges, or come from infinity, and that end on negative charges, or go off to infinity. The direction of the field at a point is tangent to the field line passing through that point. The magnitude of the field increases as the density of the field lines increases. In a field vector pattern equal-length arrows are drawn at equally spaced points. The direction of an arrow shows the direction of the field at that point, and the darkness of an arrow indicates the strength of the field at that point.

A Charged Object in a Uniform Electric Field

A uniform field has the same magnitude and direction at all points. A charged object in a uniform electric field experiences a constant acceleration that is given by $\vec{a} = \vec{F} / m = q\vec{E} / m$. Because the acceleration is constant we can apply the constant-acceleration equations from Chapter 2 (for one-dimensional motion) or from Chapter 4 (for projectile motion in two dimensions) to determine equations of motion for the object.

Electric Field from a Point Charge

Electric fields are set up by charges. One special case is the electric field from a point charge, which is a charged object so small that it can be considered to be a point. If the point charge has a charge Q the magnitude of the electric field a distance r away from it is given by:

$$E = \frac{kQ}{r^2}. \qquad \text{(Equation 16.4: \textbf{Electric field from a point charge})}$$

Electric field is a vector. The electric field points away from a positive charge and toward a negative charge. The net electric field from multiple point charges can be found by superposition, adding the fields from each charge as vectors.

Electric Field near a Conductor, at Electrostatic Equilibrium

When a conducting object has a net charge, and/or when the conducting object is placed in a region in which there is an electric field, a number of conditions apply when electrostatic equilibrium is reached. Electrostatic equilibrium is defined as there being no net flow of charge within or on the conducting object.

1. There is no electric field inside the solid part of the conductor.
2. The electric field at the surface of the conductor is perpendicular to the surface.
3. If the conductor is charged, excess charge lies only at the surface of the conductor.
4. Charge density is highest, and electric field is strongest, on pointy parts of a conductor.

End-of-Chapter Exercises

Exercises 1 – 14 are mainly conceptual questions designed to see if you have understood the main concepts of the chapter. Treat all charged balls as point charges.

1. While you are solving a physics problem, you calculate that the charge on a particular object has a value of $+2.5\times10^{-22}$ C . Can this be correct? Choose the one correct statement about this from the set of three options below. Note that *e* represents the magnitude of the charge on the electron.

A – Yes, this answer could be correct.
B – No, this answer cannot be correct because the charge represents a small fraction of *e*.
C – No, this answer cannot be correct. The value has a magnitude larger than *e*, but it does not represent an integer multiple of *e*.

2. You have three identical metal spheres that have different initial net charges. Sphere A has a net charge of $+5Q$; sphere B has a net charge of $-3Q$; and sphere C has a net charge of $+6Q$. You first touch sphere B to sphere A, and then separate them; you then touch sphere A to sphere C, and then separate them; and finally you touch sphere C to sphere B, and then separate them. (a) Assuming no charge is transferred to you, what is the total combined charge on the three spheres at the end of the process? (b) What is the charge on each one of the spheres at the end of the process?

3. Consider again the system of three charged metal spheres in Exercise 2. You can set their initial charges to be whatever you wish, but you touch them together as described in the previous problem. (a) Is it possible for each sphere to end up with the same non-zero net charge? If so, give an example. (b) Is it possible for each sphere to end up with a different amount of charge? If so, give an example. (c) Is it possible for the sign of the charge on one sphere to be opposite to the charge on the other two spheres, at the end of the process? If so, give an example.

4. A small charged ball with a charge of $+5Q$ is located at a distance of 2.0 m from a charged ball with a charge of $+Q$. Which ball exerts a larger-magnitude force on the other? Justify your answer.

5. Ball A is charged, and so is ball B. The two balls are separated by a distance of *d*, and they can be treated as point charges. Which of the following changes, done individually, would cause the force that ball B exerts on ball A to double? If the change does not cause a doubling of the force state explicitly what effect the change has. (a) Double the charge on ball B. (b) Double the charge on ball A. (c) Double the charge on both balls. (d) Decrease the separation between the balls to *d*/2.

6. The electric field in the region shown in Figure 16.19 is produced by a single point charge, but the location of that point charge is unknown. At the point ($x = 1$, $y = 1$), we know that the electric field is directed to the right. (a) If this is all we knew about the field, what could we say about the location and sign of the point charge? (b) We also know that, at the point ($x = 3$, $y = 3$), the electric field is directed down. With this additional information, what can we say about the location and sign of the point charge?

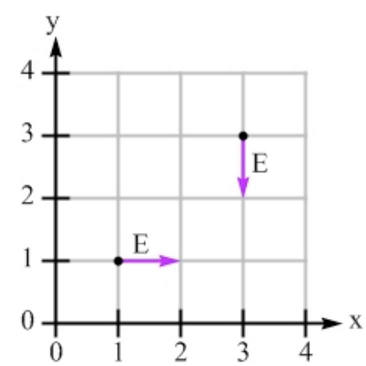

Figure 16.19: The electric field in this region is produced by a single point charge. The location of the point charge is not shown. For Exercise 6.

7. In a particular uniform electric field, the electric field lines are directed to the right. You draw a diagram to reflect this field, showing a number of equally spaced parallel arrows, separated by 1 cm, that are directed to the right. What would be the spacing between the arrows on your field-line diagram if the field was reduced in magnitude by a factor of 2?

8. You want to sketch a field-line pattern for a situation involving two point charges separated by some distance. One charge has a magnitude of $+3Q$ while the other has a magnitude of $-Q$. (a) If you draw 15 lines emerging from the $+3Q$ charge, how many should you draw ending on the $-Q$ charge? (b) Where do the remaining lines go? (c) At a point quite far from the two charges, the electric field looks like the electric field from a single point charge. What is the charge of this single point charge?

9. As shown in Figure 16.20, a positive test charge experiences a net force directed right when it is placed exactly halfway between a ball of charge $+Q$ and a second ball of unknown charge. (a) What is the direction of the electric field at the point where the test charge is? (b) What, if anything, can you conclude about the sign and/or magnitude of the charge on the second ball?

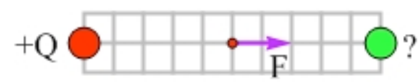

Figure 16.20: A positive test charge is located halfway between a ball of charge $+Q$ and a second ball of unknown charge. For Exercise 9.

10. As shown in Figure 16.21, a positive test charge experiences a net force directed right when the test charge is placed twice as far from a ball of unknown charge as it is from a ball of charge $+Q$. (a) What is the direction of the electric field at the point where the test charge is? (b) What, if anything, can you conclude about the sign and/or magnitude of the charge on the second ball?

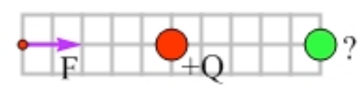

Figure 16.21: A ball of charge $+Q$ is located halfway between a positive test charge and a second ball of unknown charge. For Exercise 10.

11. Figure 16.23 shows three charged balls, which are equally spaced along a line. Each ball has a non-zero charge, but the signs of the charges on balls 1 and 3 are not shown. Ball 2 has a negative charge. The figure also shows the net force acting on each ball, because of its interaction with the other two balls. Assume that the only forces acting are electrostatic forces. (a) Ball 1 experiences no net force. What, if anything, does this tell us about the sign and magnitude of the charge on ball 3? Explain. (b) The net force on ball 2 is directed to the left. What, if anything, does this tell us about the sign and magnitude of the charge on ball 1? Explain. (c) Rank the balls, from largest to smallest, based on the magnitude of the charge on them.

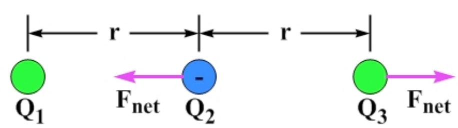

Figure 16.22: Three charged balls are equally spaced along a line. The sign of the charge on ball 2 is negative, but the signs of the charges on the other two balls are not shown. The net force on each ball is also shown – ball 1 experiences no net force due to its electrostatic interaction with the other two balls. For Exercise 11.

12. Three balls, with charges of $+q$, $+2q$, and $+3q$, are arranged so there is one ball at each corner of an equilateral triangle. Rank the balls based on the magnitude of the net electrostatic force they experience, from largest to smallest.

13. Five balls, two of charge $+q$ and three of charge $-2q$, are arranged as shown in Figure 16.23. What is the magnitude and direction of the net electrostatic force on the ball of charge $+q$ that is located at the origin?

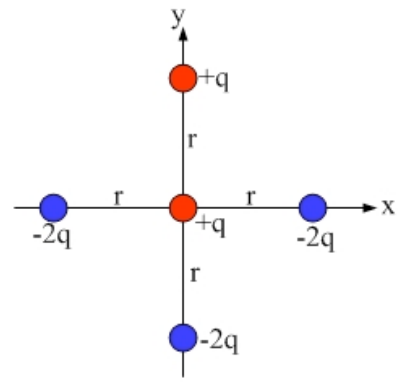

Figure 16.23: An arrangement of five charged balls, for Exercise 13.

14. Three charged balls are placed so that each is at a different corner of a square, as shown in Figure 16.24. Balls 1 and 3 both have positive charges, but the sign of the charge on ball 2 is not shown. The figure also shows the net force acting on each of the balls – the only forces that matter here are those associated with the interactions between the charges. (a) What, if anything, does the direction of the net force acting on ball 2 tell us about the sign of the charge on ball 2? Explain. (b) What, if anything, does the direction of the net force acting on ball 2 tell us about how the magnitude of the charge on ball 1 compares to the magnitude of the charge on ball 3? Explain. (c) What, if anything, does the direction of the net force acting on ball 3 tell us about how the magnitude of the charge on ball 1 compares to the magnitude of the charge on ball 2? Explain. (d) Rank the balls, from largest to smallest, based on the magnitude of the charge on them.

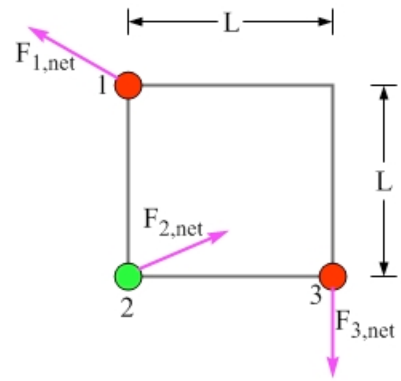

Figure 16.24: An arrangement of three charged balls, for Exercise 14. The arrows represent the net electrostatic force acting on each ball.

Exercises 15 – 20 deal with Coulomb's Law. Treat all charged balls as point charges.

15. Two point charges, one with a charge of $+Q$ and the other with a charge of $+16Q$, are placed on the x-axis. The $+Q$ charge is located at $x = +6a$, and experiences a force of magnitude kQ^2/a^2 because of its electrostatic interaction with the second charge. Where is the second point charge located? State all possible solutions.

16. Two small identical conducting balls have different amounts of charge on them. Initially, when they are separated by 75 cm, one ball exerts an attractive force of 1.50 N on the second ball. The balls are then touched together briefly, and then again separated by 75 cm. Now, both balls have a positive charge, and the force that one ball exerts on the other is a repulsive force of 1.10 N. What was the charge on the two balls originally?

17. Two charged balls are placed on the x-axis, as shown in Figure 16.25. The first ball has a charge $+q$ and is located at the origin, while the second ball has a charge $-4q$ and is located at $x = +4a$. A third ball, with a charge of $+2q$, is then brought in and placed somewhere on the x-axis. Assume that each ball is influenced only by the other two balls, and neglect gravitational interactions. (a) Could the third ball be placed so that all three balls simultaneously experience no net force due to the other two? (b) Could the third ball be placed so that at least one of the three balls experiences no net force due to the other two? Briefly justify your answers.

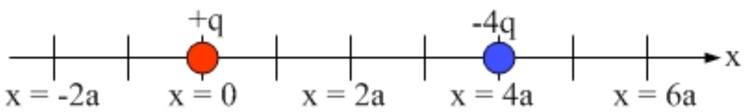

Figure 16.25: Two charged balls on the *x*-axis. For Exercises 17 – 19.

18. Return to the situation described in Exercise 17, and find all the possible locations where the third ball could be placed so that at least one of the three balls experiences no net force due to the other two.

19. Two charged balls are placed on the *x*-axis, as shown in Figure 16.25. The first ball has a charge $+q$ and is located at the origin, while the second ball has a charge $-4q$ and is located at $x = +4a$. Could a third ball, with an appropriate charge, be brought in and placed somewhere on the *x*-axis so that all three balls simultaneously experience no net force due to the other two? If so, find the charge and location of the third ball.

20. Three balls, each with the same magnitude charge, are arranged so there is one ball at each corner of an equilateral triangle. Each side of the triangle is exactly 1 meter long. (a) If each ball experiences a net force of 8.00×10^{-6} N because of the other two balls, what is the charge of each ball? (b) Must the sign of the charge on each ball be the same, or could the charge on one ball be opposite to that of the charge on the other two balls? Explain.

Exercises 21 – 31 deal with electric field. Treat all charged balls as point charges.

21. An electron with an initial velocity of 1500 m/s directed straight up is in a uniform electric field of 200 N/C that is also directed straight up. (a) The electron is near the surface of the Earth. Is it reasonable to neglect the influence of gravity in this situation? Justify your answer. (b) How long does it take for the electron to come instantaneously to rest? (c) How far does the electron travel in this time?

22. An electron with an initial velocity of 7.5×10^{5} m/s directed horizontally is in a uniform electric field of 400 N/C that is directed straight up. The electron starts 2.0 m above a flat floor. (a) How long does it take the electron to reach the floor? (b) How far does the electron travel horizontally in this time? (c) What is the speed of the electron as it runs into the floor?

23. A single point charge is located at an unknown point on the *x*-axis. There are no other charged objects nearby. You measure the electric field at the origin to be 600 N/C in the positive *x*-direction, while the electric field on the *x*-axis at $x = +4.0$ m is 5400 N/C in the negative *x*-direction. What is the sign and magnitude of the point charge, and where is it located?

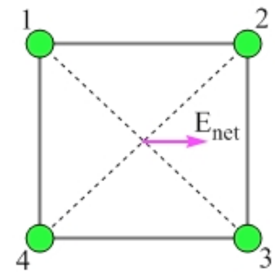

Figure 16.26: Four point charges are placed so that there is one charge at each corner of a square. The charges all have the same magnitude, but they may have different signs. The net electric field at the center of the square, due to these four charges, is directed right. For Exercise 24.

24. The net electric field at the center of a square is directed to the right, as shown in Figure 16.26. This net field is the vector sum of electric fields from four point charges, which are located so that there is one point charge at each corner of the square. The charges have identical magnitudes, but may be positive or negative. Which are positive and which are negative? Is there more than one possible answer?

25. A ball of with a charge of $+6q$ is placed on the x-axis at $x = -2a$. There is a second ball of unknown charge at $x = +a$. If the net electric field at the origin due to the two balls has a magnitude of $\frac{kq}{a^2}$, what is the charge of the second ball? Find all possible solutions.

26. Repeat the previous problem, but now the net electric field at the origin has a magnitude of $\frac{3kq}{a^2}$.

27. A ball with a charge of $-2q$ is placed on the x-axis at $x = -a$. There is a second ball with a charge of $+q$ that is placed on the x-axis at an unknown location. If the net electric field at the origin due to the two balls has a magnitude of $\frac{6kq}{a^2}$, what is the location of the second ball? Find all possible solutions.

28. Three balls, with charges of $+4q$, $-2q$, and $-q$, are equally spaced along a line. The spacing between the balls is r. We can arrange the balls in three different ways, as shown in Figure 16.27. In each case the balls are in an isolated region of space very far from anything else. (a) In which case does the ball with the charge of $+4q$ experience a larger-magnitude net force? Give a qualitative argument. (b) Calculate the magnitude and direction of the net force experienced by the $+4q$ charge in each case.

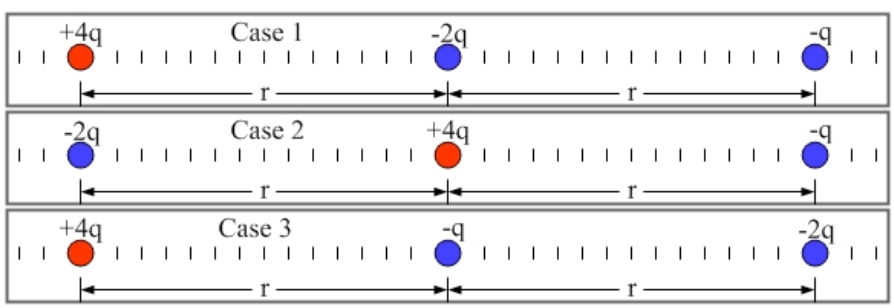

Figure 16.27: Three different arrangements of three balls of charge $+4q$, $-2q$, and $-q$ placed on a line with a distance r between neighboring balls. For Exercises 28 – 31.

29. Return to the situation described in Exercise 28, and shown in Figure 16.27. (a) Duplicate the diagram, and then draw in two force vectors on each ball in each case, to represent the force each ball experiences due to the other two balls. Figure 16.4 shows an example of this process. (b) Rank the three cases, from largest to smallest, based on the magnitude of the net force exerted on the ball in the middle of the set of three balls.

30. Return to the situation described in Exercise 28, and shown in Figure 16.27. Which ball experiences the largest-magnitude net force in (a) Case 1, (b) Case 2, and (c) Case 3? Calculate the magnitude and direction of the force applied to the ball that is experiencing the largest magnitude force in (d) Case 1, (e) Case 2, and (f) Case 3.

31. Consider the situation shown in case 3 in Figure 16.27. Each charged ball experiences a net force because of the other two balls – if you did the previous problem you would have calculated the three different net forces already. (a) If you add these three net forces as vectors, what do you get? Why? (b) Would you get the same result in all similar situations, including cases 1 and 2 in Figure 16.27? Why or why not?

Exercises 32 – 36 deal with test charges. Treat all charged balls as point charges.

32. As shown in Figure 16.28 (a), a positive test charge placed exactly halfway between a ball of charge $+Q$ and a second ball of unknown charge experiences a net force directed right. When the second ball is removed from the situation, as in Figure 16.28 (b), the force experienced by the test charge increases by a factor of 3/2. What is the sign and magnitude of the charge on the second ball in Figure 16.28 (a)?

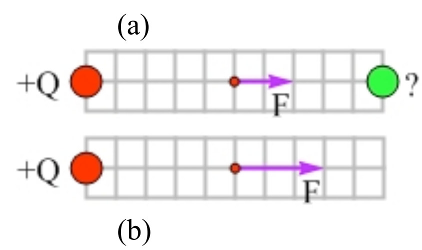

Figure 16.28: When the second ball shown in (a) is removed, as in (b), the force on the test charge increases by a factor of 3/2. For Exercise 32.

33. Figure 16.29 shows the net force experienced by a positive test charge located at the center of the diagram. The force comes from two nearby charged balls, one with a charge of $+Q$ and one with an unknown charge. (a) What is the sign of the charge on the second ball? (b) Is the magnitude of the charge on the second ball more than, less than, or equal to Q? (c) Find the sign and magnitude of the charge on the second ball.

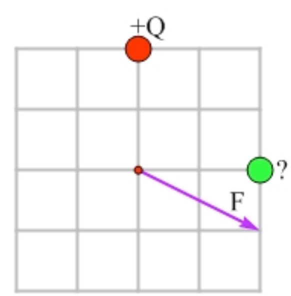

Figure 16.29: The two charged balls produce a net force directed down and to the right, as shown, on the test charge at the center of the diagram. For Exercise 33.

34. Figure 16.30 shows the net force experienced by a positive test charge located at the center of the diagram. The force comes from two nearby charged balls, one with a charge of $+Q$ and one with an unknown charge. (a) What is the sign of the charge on the second ball? (b) Is the magnitude of the charge on the second ball more than, less than, or equal to Q? (c) Find the sign and magnitude of the charge on the second ball.

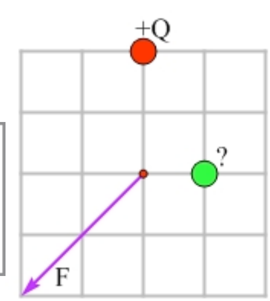

Figure 16.30: The two charged balls produce a net force at a 45° angle directed down and to the left, as shown, on the test charge at the center of the diagram. For Exercise 34.

35. Figure 16.31 shows the net force experienced by a positive test charge located at the center of the diagram. The force comes from two nearby charged balls, one with a charge of $+Q$ and one with an unknown charge. What is the sign and magnitude of the charge on the second ball?

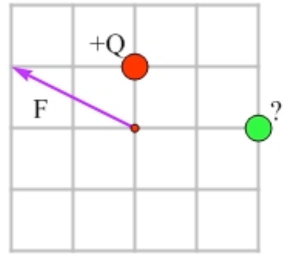

Figure 16.31: The two charged balls produce a net force directed up and to the left, as shown, on the test charge at the center of the diagram. For Exercise 35.

36. Two identical test charges are located at different positions, as shown in Figure 16.32. The test charges experience forces of the same magnitude, and in the directions shown. Could these forces be produced by a single nearby point charge? If so, state where that point charge would be and what you know about it. If not, explain why not.

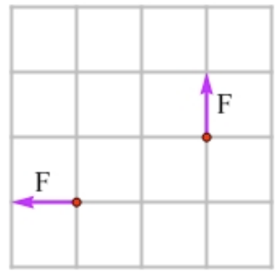

Figure 16.32: Two identical positive test charges experience the forces shown in the diagram. For Exercise 36.

General problems and conceptual questions. Treat all charged balls as point charges.

37. Benjamin Franklin made significant scientific contributions to our understanding of electric charge. Do some research on these contributions, and write two or three paragraphs describing them.

38. The SI unit of charge, the coulomb, is named after Charles-Augustin de Coulomb. Do some research on Coulomb (the scientist) and write a short biographical sketch of him.

39. A photocopier relies on the basic principles of charge. Do some research on how a photocopier works, and write a step-by-step explanation of the photocopying process.

40. At the laundromat, you put your silk pajamas in the dryer with your woolen sweater. When you take them out again, you find they are stuck together, because of static cling. Explain why this happens.

41. What is the speed of an electron in the ground state of a hydrogen atom? See Example 16.3A for relevant data, and use the Bohr model of the hydrogen atom, in which the electron follows a circular orbit around the proton.

42. Two small identical conducting balls have different amounts of charge on them. When they are first separated by 35 cm, one ball exerts a force with a magnitude of 4.50 N on the second ball. The balls are then touched together briefly, and then again separated by 35 cm. Now the force that one ball exerts on the other has a magnitude of 7.50 N. What was the charge on the two balls originally? Is there more than one possible solution?

43. Two identical balls each have a charge of -2.5×10^{-6} C. The balls hang from identical strings that are at 8.0° from the vertical because of the repulsive force between the charged balls. The balls are separated by a distance of 10 cm, as shown in Figure 16.33. What is the mass of each ball?

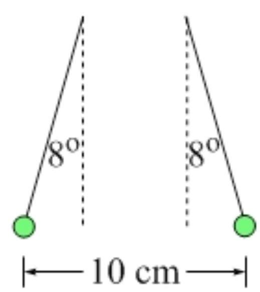

Figure 16.33: Two identical charged balls hang from strings, for Exercise 43.

44. Three balls, with charges of $+q$, $-2q$, and $+3q$, are arranged so there is one ball at each corner of an equilateral triangle. Each side of the triangle is exactly 2 meters long. (a) Find the magnitude of the net electrostatic force acting on the ball of charge $+3q$. (b) What is the magnitude of the electric field at the center of the triangle?

45. (a) Referring to Figure 16.34, which of the two balls of charge $+q$ experiences the largest net electrostatic force? Justify your answer. (b) What is the magnitude of the net electrostatic force experienced by the three different balls of charge $-2q$?

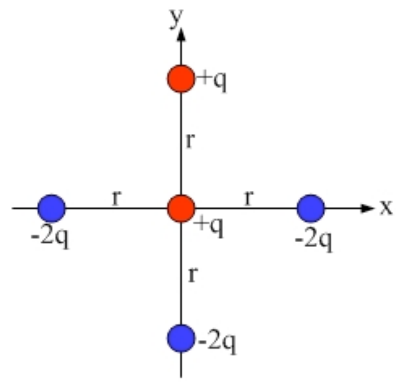

Figure 16.34: An arrangement of five charged balls, for Exercise 45.

46. A ball of charge $+3q$ is placed on the x-axis at $x = -a$. There is a second ball with an unknown charge that is placed on the x-axis at an unknown location. If the electrostatic force the second ball exerts on the first ball has a magnitude of $\frac{kq^2}{2a^2}$ and the net electric field at $x = 0$ due to these balls is $\frac{69kq}{25a^2}$ in the positive x direction, what is the charge and location of the second ball? Find all possible solutions.

47. Consider the three cases shown in Figure16.35. Rank these cases, from largest to smallest, based on the (a) magnitude of the electrostatic force experienced by the ball of charge $-q$; (b) magnitude of the electric field at the origin.

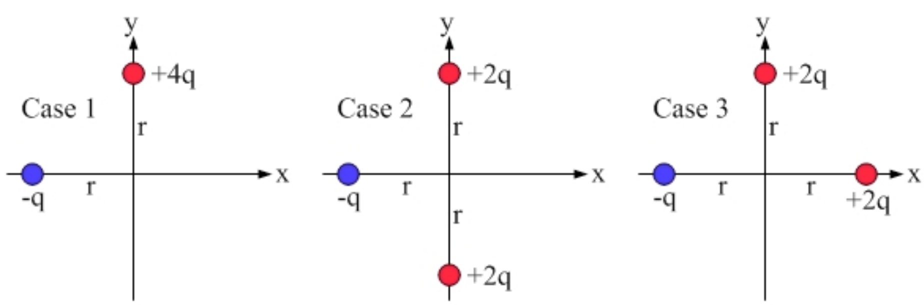

Figure 16.35: Three different configurations of charged objects, for Exercises 47 – 49.

48. Consider the three cases shown in Figure 16.35. Determine the magnitude and direction of the electrostatic force experienced by the ball of charge $-q$ in (a) case1; (b) case 2; (c) case 3.

49. Consider the three cases shown in Figure 16.35. Determine the magnitude and direction of the electric field at the origin in (a) case1; (b) case 2; (c) case 3.

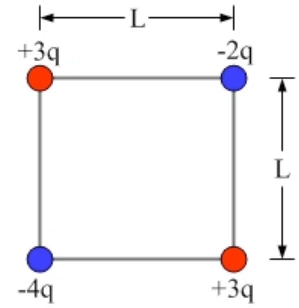

Figure 16.36: Four charged balls at the corners of a square, for Exercises 50 – 53.

50. Four small charged balls are arranged at the corners of a square that measures L on each side, as shown in Figure 16.36. (a) Which ball experiences the largest-magnitude force due to the other three balls? (b) What is the direction of the net force acting on the ball with the charge of $-4q$? (c) If you doubled the length of each side of the square, so neighboring charges were separated by a distance of $2L$ instead, what would happen to the magnitude of the force experienced by each charge?

51. Four small charged balls are arranged at the corners of a square that measures L on each side, as shown in Figure 16.36. To what value can you adjust the charge on the ball of charge $+3q$ in the lower right corner so that the ball of charge $+3q$ in the upper left corner experiences (a) a net force that points directly toward the ball of charge $-4q$, or (b) a net force that points directly away from the ball of charge $-2q$? (c) Determine the magnitude of the net force (in terms of k, q, and L) acting on the $+3q$ charge in the upper left corner in these two situations.

52. Four small charged balls are arranged at the corners of a square that measures L on each side, as shown in Figure 16.36. (a) If you adjust the charge on the ball with the $-4q$ charge at the lower left, could you bring the net force acting on the ball with the $-2q$ charge to zero? (b) If so, calculate the sign and magnitude of the charge on the ball in the lower left corner that would be required. If not, explain why not.

53. Four small charged balls are arranged at the corners of a square that measures L on each side, as shown in Figure 16.36. (a) Calculate the magnitude and direction of the electric field at the center of the square. (b) Could you change the amount of charge on one of the balls to produce a net electric field at the center that is directed horizontally to the left? If so, which ball would you change the charge of and what would you change it to? If not, explain why not.

54. A ball with a weight of 10 N hangs down from a string that will break if its tension is greater than or equal to 25 N. The ball has a charge of $+5.0 \times 10^{-6}$ C. You want to break the string by introducing a uniform electric field. What is the magnitude and direction of the minimum electric field required to cause the string to break?

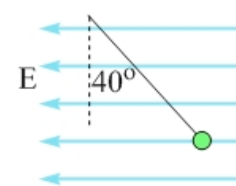

Figure 16.37: The equilibrium position of a ball in a uniform electric field directed left, for Exercise 55.

55. A small charged ball with a weight of 10 N hangs from a string. When the ball is placed in a uniform electric field of 800 V/m directed left, the string makes a 40° angle with the vertical, as shown in Figure 16.37. What is the sign and magnitude of the charge on the ball?

56. Return to Example 16.3B, in which we calculated the magnitude and direction of the net force exerted on the $-q$ charge by the other two charges. Now determine the magnitude and direction of the net force exerted on (a) the $+2q$ charge, and (b) the $-3q$ charge.

57. Return to the situation of Example 16.3B. Determine the magnitude and direction of the net electric field at (a) the center of the square, and (b) the unoccupied corner.

58. Three charged balls are placed in a line, as shown in Figure 16.38. Ball 1, which has an unknown charge and sign, is a distance $2r$ to the left of charge 2. Ball 2 is positive, with a charge of $+Q$. Ball 3 has an unknown non-zero charge and sign, and is a distance r to the right of ball 2. Ball 3 feels no net electrostatic force because of the other two balls. (a) Is there enough information given here to find the sign of the charge on ball 1? If so, what is the sign? (b) Can we find the magnitude of the charge on ball 1? If so, what is it? (c) Can we find the sign of the charge on ball 3? If so, what is the sign? (d) Can we find the magnitude of the charge on ball 3? If so, what is it?

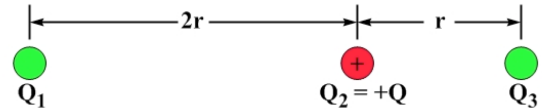

Figure 16.38: Three charges in a line. Only the sign and magnitude of charge 2 are known, although we also know that charge 3 is in equilibrium. For Exercise 58.

59. A single point charge is located at an unknown point on the x-axis. There are no other charged objects nearby. You measure the electric field at $x = +2.0$ m to be 6000 N/C, directed in the $+x$ direction, while the field at $x = +5.0$ m has a magnitude of 1500 N/C. What is the sign and magnitude of the point charge? State all possible answers.

60. In Example 16.6A we found one point on the line passing through two unequal charges at which the net electric field is zero. Are there any such points that are off this line, a finite distance from the charges? Use one or more diagrams to support your answer.

61. Consider the field-line diagram shown in Figure 16.39. The arrows show field lines emerging from charge 1, on the left, and ending at charge 2, on the right. (a) What is the sign of each of these charges? (b) If the charge on charge 1 has a magnitude of 10 μC, what is the charge on charge 2?

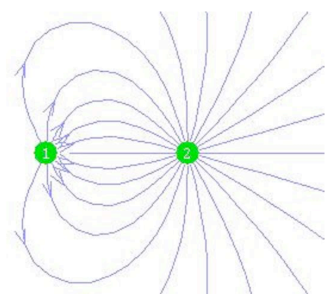

Figure 16.39: A field-line pattern near two charged objects, for Exercise 61.

62. One point charge is located at the origin, and has a charge of +5.0 μC. A second point charge is located at x = +2.0 m, and has a charge of –9.0 μC. (a) Analyze the situation qualitatively to determine approximate locations of any points where the net electric field due to these two point charges is zero. (b) Determine the location of all such points.

63. Repeat Exercise 62, except now the second point charge has a charge of +18.0 μC.

64. A charge of unknown sign and magnitude is located halfway between a small ball with a charge of $+Q$ and a positive test charge. The test charge experiences a net force directed right, as shown in Figure 16.40. (a) What, if anything, can you conclude about the sign and/or magnitude of the unknown charge? (b) If you moved the test charge to the point halfway between the $+Q$ charge and the unknown charge, in which direction would the force be on the test charge? (c) You return the test charge to the position shown in Figure 16.41. You then observe that when you shift the position of the unknown charge a little to the right, the force experienced by the test charge decreases in magnitude a little. What, if anything, can you conclude about the sign and/or magnitude of the unknown charge?

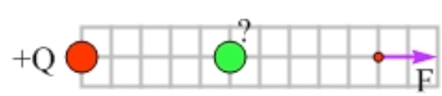

Figure 16.40: A charge of unknown sign and magnitude is located halfway between a small ball of charge $+Q$ and a positive test charge. For Exercise 64.

65. One of the safest places to be in a lightning storm is inside a car, as long as the car is made of metal. Even if lightning strikes the car you should be safe inside. Explain why this is.

66. (a) Sketch a field-line pattern for a situation in which there is a uniform electric field directed straight down. (b) Re-draw the pattern for when a neutral metal sphere is placed into the field. Draw the sphere large enough that it covers a region on the diagram where at least 5 field lines pass through on your original diagram.

67. Figure 16.41 shows possible equilibrium distributions of charge on a hollow, thick-walled metal sphere that has a net charge of $-Q$. Which is most correct, assuming there are no other charged objects in the vicinity?

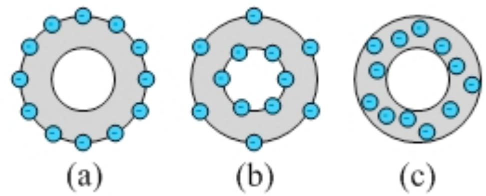

Figure 16.41: Possible equilibrium distributions of charge on a hollow, thick-walled metal sphere that has a net charge of $-Q$. In (a) the charge is spread uniformly over the outer surface; in (b) half the charge is distributed over the outer surface and half is on the inner surface; and in (c) the charge is randomly distributed throughout the bulk of the sphere. For Exercise 67.

68. Three students are having a conversation. Explain what you think is correct about what they say, and what you think is incorrect. In particular, how would you respond to Brenda's questions in the last statement?

Brenda: So, the question says that we have two objects, one with a +5Q charge and the other with a +Q charge, and it asks us for which one exerts a larger magnitude force on the other. Well, that's the +5Q object, right – it exerts 5 times as much force as the other one.

Paul: Let's think about Coulomb's law – it does say that when you increase one of the charges that the force goes up.

Lauren: Thinking about Coulomb's law makes sense, except that, in Coulomb's law, the force is proportional to the product of the two charges. You can apply it to each charge, and you get the same answer. So, I think the forces are the same.

Paul: That's consistent with Newton's third law, too – the objects have to exert equal and opposite forces on one another. That sounds right.

Brenda: That just doesn't make sense to me – shouldn't the bigger charge exert more force? I kind of got Newton's third law when we were talking about colliding carts a few months ago, but how can it can apply for things that don't even touch each other, like these little charges?

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- Simulation: Electric Field in One Dimension
- Worksheet for Electric Field in One Dimension Simulation
- Simulation: Electric Force in Two Dimensions
- Simulation: Electric Field in Two Dimensions
- Worksheet for Electric Field in Two Dimensions Simulation

Additional Links

- *PhET* simulation: John Travoltage
- *PhET* simulation: Balloons and Static Electricity
- *PhET* simulation: Electric Field Hockey