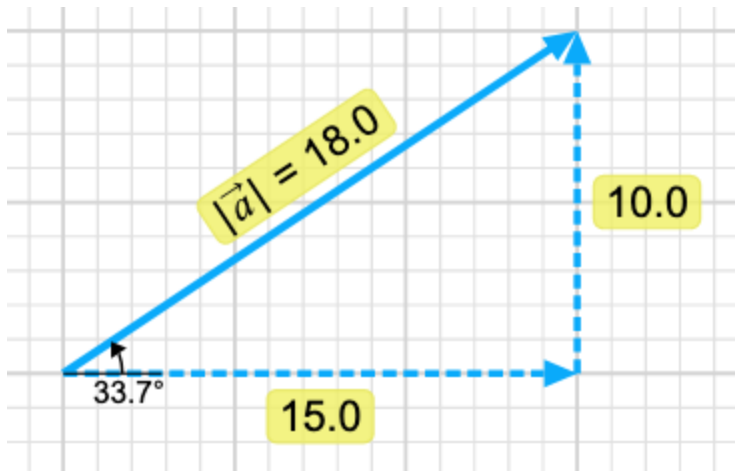


# PY105 Math Review Packet

**Vectors:** <https://phet.colorado.edu/en/simulation/vector-addition>. Vectors are very important in PY105. Here's a good interactive tool. The 2D version is a good place to start. Click and drag vector 'a' onto the grid, you can then drag to resize or rotate. In the top right you can show the values of each component (or leg) of the vector, the magnitude (length) of the vector, as well as the angle the vector makes with the x axis.



1.) The vector **a** to the left has two components,  $a_x$  and  $a_y$ . What are they?

2.) How would you calculate the magnitude  $|a|$  (hypotenuse) of this vector, if you only knew the components? (Your answer should be a little more than 18, the

simulation is only precise to 1 decimal place)

3.) How do the trig functions Sin(theta), Cos(theta), Tan(theta) relate to the components ( $a_x$ ,  $a_y$ ) and magnitude  $|a|$  of this vector?

4.) Drag another vector, **b**, onto the grid. Shape it any way you like. Click on the sum checkbox in the top right, this will show you the resulting vector when you add **a** and **b** together. How would you calculate the magnitude of this new vector? Can you just add the magnitude of **a** and **b** individually? (Bonus: There is a special scenario where you can, what is it?)

## PY105 – Math Review Packet

### 1. Solving equations using algebra

It's like a game, but with pencil and paper. All you have to do is follow the rules of the game.

While you learned most algebra schools in middle and high school, it's helpful to have some review of some basic rules.

Rule: Whatever you do on the left, you must do the same on the right.

$$x - 4 = 7 \Rightarrow x - 4 + 4 = 7 + 4$$

Goal: Move the letters and numbers so that  $x$  only appears on the left.

Example from textbook, solve for  $v$ :

$$4v^2 - 7 = 3 - v^2$$

Remember the goal is to move  $v$  to the left:

$$\begin{aligned}4v^2 - 7 + v^2 &= 3 - v^2 + v^2 \\5v^2 - 7 &= 3 \\5v^2 - 7 + 7 &= 3 + 7 \\5v^2 &= 10 \\5v^2 / 5 &= 10 / 5 \\v^2 &= 2 \\\sqrt{v^2} &= 2 \\v = \sqrt{2} \text{ or } v &= -\sqrt{2}\end{aligned}$$

### 2. Quadratic Equation

If you obtain an equation with this form:  $2x^2 + 4x + 9 = 0$ , this is the quadratic equation.

General form of the quadratic equation is:  $ax^2 + bx + c = 0$ , which can often be disguised by more terms, and you can simplify it to this form.

To solve for  $x$ , use the quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Although sometimes you can “see” the solution directly:

$$x^2 - 8x + 15 = 0 \Rightarrow (x - 3)(x - 5) \Rightarrow x = 3 \text{ or } x = 5$$

### 3. Units

How fast am I driving? I am driving at constant speed. After 2 hours I covered 80 miles. How fast am I driving? (a)  $v=40$ , (b)  $v=64.4$ , (c) 17.9, (d) none

Answer is (d). They are all wrong. The units are missing. These numbers have no meaning without their units.

All these numbers are actually correct, but without the units, you cannot actually tell:

(a) 40 miles/hour, (b) 64.4 km/hour, (c) 17.9 m/s

Why are units important in STEM? Example: The Mars Climate Orbiter was launched by NASA on December 11, 1998. Communication with the satellite was lost on September 23, 1999. Part of the software was using American units (feet, pounds, etc.) and other parts were using International units (meters, newtons, etc.). Moving 80 miles is not the same as moving 80 kilometers! Computer software produced output of pound-seconds instead of the standard SI units of Newton-seconds. Result: spacecraft passed too close to the planet, causing it to pass through the upper atmosphere and disintegrate.

In science we prefer to use SI units (Système International). SI Units: meters / kilograms / seconds / etc., instead of units we are used to in everyday use in US like feet / miles / pounds

Using SI makes calculations easier and prevents errors!

- First convert, e.g., km/hr to meter/sec, feet to meters, etc. - see next page!
- Do your calculations, the final result is guaranteed to have the right units!
- Convert your answer back to whatever units you prefer; or are asked in a question, etc.

#### SI Units – Prefixes

Instead of meters, one can also work with:

- micrometers  $1 \mu\text{m} = 10^{-6}\text{m}$
- millimeters  $1 \text{mm} = 0.001\text{m} (=10^{-3}\text{m})$
- centimeters  $1 \text{cm} = 0.01\text{m} (=10^{-2}\text{m})$
- kilometers  $1 \text{km} = 1000\text{m} (=10^3\text{m})$
- milliseconds  $1 \text{ms} = 0.001\text{s} (=10^{-3}\text{s})$

Be extra careful with kilogram (standard unit is kg – not the gram!!)

- gram  $1 \text{g} = 0.001 \text{kg} (=10^{-3}\text{g})$

## 5. Dimensional Analysis

A correct answer always uses the correct units.

Distance  $x$  covered after  $t$  seconds at speed  $v$ :  $x = vt$

because: [meters] = [meter/sec]  $\times$  [sec]

This cannot be right:  $x = v/t$

because: [meters]  $\neq$  [meter/sec] / [sec] = [meter/sec<sup>2</sup>]

Practice examples:

1. 250 g  $\rightarrow$  kg
2. 3 hours  $\rightarrow$  seconds
3. 7.8 cm<sup>3</sup>  $\rightarrow$  m<sup>3</sup>

## 5. Significant Figures

Accuracy is indicated by the number of meaningful digits or significant figures.

To determine the number of significant figures in a number use the following rules:

1. Non-zero digits are always significant
  - Examples: 457 cm (3 significant figures); 0.25 g (2 significant figures)
2. Any zeros between two significant digits are significant
  - Examples: 1005 kg (4 significant figures); 1.03 cm (2 significant figures)
3. Zeros to the left of the first nonzero digits in a number are not significant, because they only indicate placement of the decimal point
  - Examples: 0.02 g (1 significant figure); 0.0025 cm (two significant figures)
4. When a number ends in zeros that are not to the right of a decimal point, the zeros are *not necessarily* significant
  - Example: 56,000 mm can have 2, 3, 4 or 5 significant figures. If you think about it, it doesn't make much sense to measure mm precise to 5 significant figures. So, to make clear how many significant figures (accuracy) we use scientific notation  $5.6 \times 10^3$  mm (2 significant figures), or  $5.60 \times 10^3$  mm (3 significant figures)
5. A final zero or trailing zeros in the decimal portion **ONLY** are significant
  - Example: .500 or .632000 the zeros are significant whereas .006 or .000968 the zeros are NOT significant

For addition and subtraction:

1. Count the number of significant figures in the decimal portion **ONLY** of each number in the problem
2. Add or subtract in the normal fashion
3. Your final answer may have no more significant figures **to the right of the decimal** than the **LEAST** number of significant figures in any number in the problem. Or, our answer can only be as precise as the least precise number we are adding.
  - Example:  $1.26 + 102.3 = 103.56$ . 1.26 has 3 significant figures. 102.3 has 4. But we don't say the answer has 3 (which would be 104). Instead, the least precise number is 102.3 which is accurate to the tenths place. So, our final answer also needs to be accurate to the tenths place, which is 103.6 (we round up).

For multiplication and division use the following rule:

1. The **LEAST** number of significant figures in any number of the problem determines the number of significant figures in the answer. (You are now looking at **the entire number**, not just the decimal portion)
  - Example:  $5.26 \times 6.1 = ?$  5.26 has 3 significant figures, 6.1 has 2 significant figures. Calculator shows 32.086. But the result can only have 2 significant figures. So the answer is 32

## 5. Uncertainty

Let's say we measure something with our ruler (which has the smallest unit of measurement in mm). If we measure 3 mm, it's incorrect to say 3.00 mm. Instead, we say 3 mm +/- some degree or error (which is the uncertainty) which describes the accuracy we are able to measure.

For example, in the pre-test, we had the sides of a small rectangular box as  $1.30 \pm 0.03$  cm;  $2.15 \pm 0.05$  cm and  $3.2 \pm 0.2$  cm. The volume is  $8.94$  cm<sup>3</sup>.

The problem asked for the uncertainty.

To calculate uncertainty, we figure out % uncertainty for each side:

$$\% \text{ uncertainty side 1} = (0.03/1.30) * 100 = 2.31\%$$

$$\% \text{ uncertainty side 2} = (0.05/2.15) * 100 = 2.33\%$$

$$\% \text{ uncertainty side 3} = (0.2/3.2) * 100 = 6.25\%$$

The percent uncertainty in the volume is the sum of uncertainties in the three sides.

Adding these up, you get 10.9%.

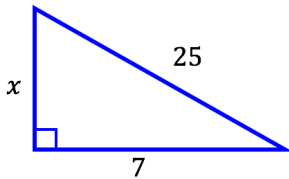
$$\text{Uncertainty} = (\% \text{ uncertainty} / 100) (\text{measurement}) = (10.9\%/100)(8.94 \text{ cm}^3) = 0.975 \text{ cm}^3$$

The problem said to report uncertainty to 1 significant figure, so we round the answer to 1 cm<sup>3</sup>

## 6. Triangles

Are key because they are the entry point to understanding vectors (which are important when we learn about displacement, forces, etc.). Some basic knowledge of trigonometry will help solve vector and physics problems.

Simplest triangle is a right angle triangle:

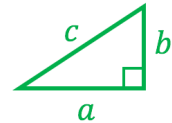


Use Pythagorean theorem:  $a^2 + b^2 = c^2$

or in this case:  $7^2 + x^2 = 25^2$

Solve:  $x^2 = 25^2 - 7^2 = 625 - 49 = 576$

$$x = 24$$



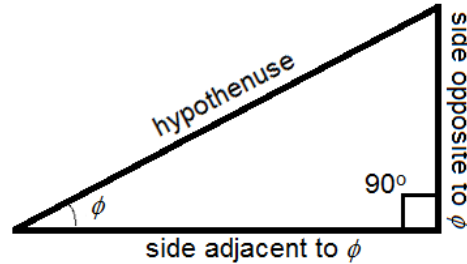
# Trigonometry

Used for: To calculate angles, directions, etc.

Prior knowledge: -

## Sine and cosine & the right triangle

The functions  $\sin(x)$ ,  $\cos(x)$ ,  $\tan(x)$  can be defined using a right triangle (one corner is exactly  $90^\circ$ ):



Remember your SOH CAH TOA:

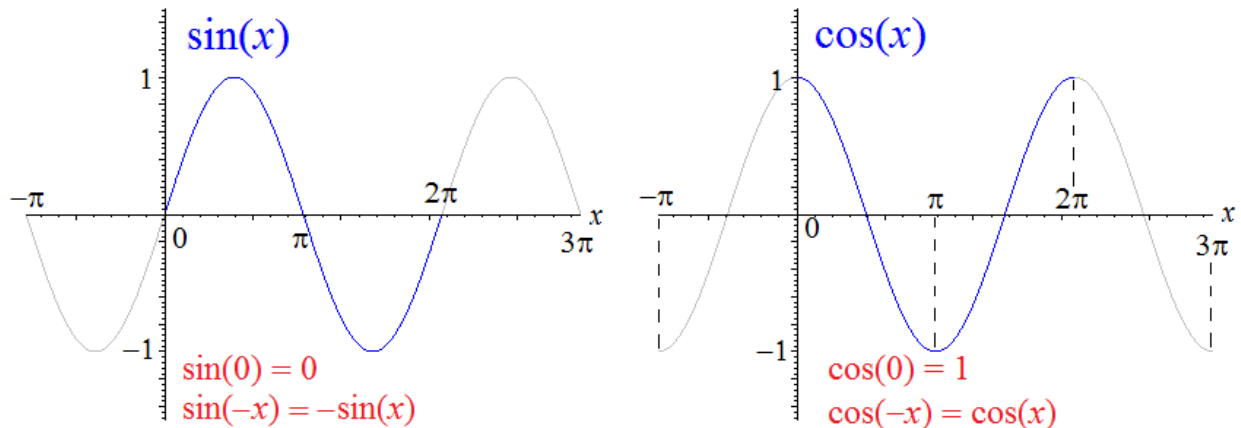
$$\text{SOH: } \sin \phi = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\text{CAH: } \cos \phi = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\text{TOA: } \tan \phi = \frac{\text{Opposite}}{\text{Adjacent}}$$

## Graphs of $\sin x$ and $\cos x$

Make sure to memorize the graphs of  $\sin x$  and  $\cos x$ . They repeat every  $360^\circ (= 2\pi)$ :



From these graphs you can also see that the cosine is the same as sine shifted by  $90^\circ (= \pi/2)$ :

$$\cos(x) = \sin\left(x + \frac{1}{2}\pi\right) \quad \sin(x) = \cos\left(x - \frac{1}{2}\pi\right)$$

This is easy to check by plugging in e.g.  $x=0$ :  $\cos(0) = \sin\left(\frac{1}{2}\pi\right) = 1$  and  $\sin(0) = \cos\left(-\frac{1}{2}\pi\right) = 0$



### Table of values for sin x and cos x

In general you need a calculator to calculate  $\sin(x)$  or  $\cos(x)$  but some angles appear quite frequently and their  $\sin x$  and  $\cos x$  values are easy to memorize:

radians:	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	$\pi$
degrees:	0°	30°	45°	60°	90°	120°	135°	150°	180°
sin x	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
cos x	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	-1

Notice an easy way to remember this table:

sin x	$\frac{1}{2}\sqrt{0}$	$\frac{1}{2}\sqrt{1}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{4}$				0
cos x	$\frac{1}{2}\sqrt{4}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{1}$	$\frac{1}{2}\sqrt{0}$				-1

If you remember the graphs of  $\sin(x)$  and  $\cos(x)$ , and their symmetry, then together with the values in this table, it's much easier to work with  $\sin(x)$  and  $\cos(x)$ !

### Trigonometry identities

The most useful one is:

$$\sin^2 x + \cos^2 x = 1$$

### Tangent, cosecant, secant, cotangent

Some combinations of  $\sin x$  and  $\cos x$  appears frequently, and have a special name:

$$\text{tangent: } \tan x = \frac{\sin x}{\cos x} \qquad \text{cotangent: } \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$\text{secant: } \sec x = \frac{1}{\cos x} \qquad \text{cosecant: } \csc x = \frac{1}{\sin x}$$

### Inverse trigonometric functions

How would you solve something like  $0.25 = \cos(x)$ ? Or more generally:  $y = \cos(x)$ ?

The answer is to use the "inverse" of the cosine, which on your calculator is labeled as  $\cos^{-1}$ :

$$x = \cos^{-1}(0.25) \quad \text{or more generally: } x = \cos^{-1}(y)$$

BE CAREFUL: the function  $\cos^{-1}(y)$  is not the same as  $1/\cos(y)$ ! Because the notation  $\cos^{-1}(y)$  can be confusing, it's better to use  $x = \arccos(y)$  which is a different name for exactly the same function. The inverse of  $\sin(x)$  and  $\tan(x)$  are  $\arcsin(y)$  and  $\arctan(y)$ , respectively.