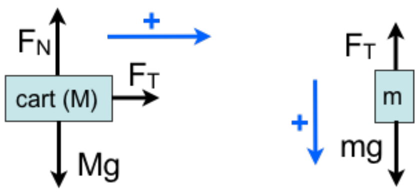
PY105 (Fall 2021) – Motion with Constant Acceleration
"Thinking about the lab"

1. The free body diagrams for the zero-friction case



Choose consistent positive directions (positive is to the right, and positive is down). The cart and mass have to move in the same direction. If the cart moves to the right, the mass moves down. This is the same scenario as Worksheet 8, Part 5 and similar to Homework 4, Problem 4: Block and Ball.

Apply Newton's Second Law to both free body diagrams: $\sum \vec{F} = m\vec{a}$

For the cart: $F_T = Ma$ (eqn 1)
For the hanging mass: $mg - F_T = ma$
Substituting eqn 1: $mg - Ma = ma$

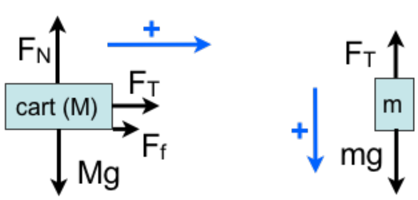
We can rewrite this as: $mg = Ma - ma$ (eqn 2)

Note: eqn 2 is the exact same result as if we looked at the system only and with the mg as the only force driving the motion (neglecting the internal tension forces): $\sum \vec{F}_{system} = (M + m)\vec{a}$

Rewriting eqn 2, we get:

$$a_{theoretical} = \frac{mg}{M+m} \quad \text{(eqn 3)}$$

2. The free body diagrams with friction and cart moving away from the pulley



Since the cart is now moving to the left (away from the pulley), the mass must move up, and the friction force always acts to oppose the direction of motion. Keeping our convention of positive directions, both the mass and the cart now move in a negative direction. The acceleration vector is in the same direction as before (to the right, and down), so during this time interval the cart (and block) is slowing down.

Like before, apply Newton's Second Law to both free body diagrams: $\sum \vec{F} = m\vec{a}$

For the cart: $F_T + F_f = Ma$
$F_T = Ma - F_f$ (eqn 4)

For the hanging mass: $mg - F_T = ma$
Substituting eqn 4: $mg - Ma - F_f = ma$

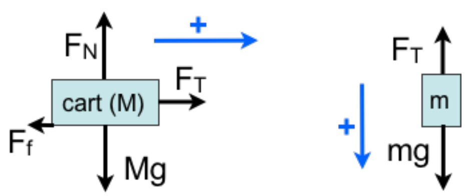
We can rewrite this as: $mg + F_f = Ma + ma$ (eqn 5)

From eqn 5, solving for a (block moving up, cart moving left): $a_{up} = \frac{mg + F_f}{M+m}$ (eqn 6)

You have learned that $F_f = \mu_k F_N$. So eqn 6 becomes: $a_{up} = \frac{mg + \mu_k F_N}{M+m}$ (eqn 7)

For the cart, we know $F_N = Mg$. So eqn 7 becomes: $a_{up} = \frac{mg + \mu_k Mg}{M+m}$ (eqn 8)

3. The free body diagrams with friction and cart moving toward the pulley



Since the cart is now moving to the <u>right</u> (toward the pulley), the mass must move down, and the friction force always acts to oppose the direction of motion. Keeping our convention of positive directions, both the mass and the cart now move in a positive direction. The acceleration vector is in the same direction as before (to the right, and down), so during this time interval the cart (and block) as a system is speeding up.

Like before, apply Newton's Second Law to both free body diagrams: $\sum \vec{F} = m\vec{a}$

For the cart: $F_T - F_f = Ma$

$F_T = Ma + F_f$ (eqn 9)

For the hanging mass: $mg - F_T = ma$

Substituting eqn 9: $mg - Ma - F_f = ma$

We can rewrite this as: $mg - F_f = Ma + ma$ (eqn 10)

If the cart is moving toward the pulley, acceleration of the mass is down. From eqn 10, solving for a:

$a_{down} = \frac{mg - F_f}{M+m} = \frac{mg - \mu_k F_N}{M+m} = \frac{mg - \mu_k Mg}{M+m}$ (eqn 11)

4. Calculating the coefficient of friction from the data you collect in the lab

If we average the results (for which you will have data) for a_{up} (eqn 8) and a_{down} (eqn 11), which simply means (eqn 8 + eqn 11) all divided by 2, we get the same result as eqn 3, the non-friction case: $a_{theoretical} = \frac{mg}{M+m}$

We can then find the effect of friction by subtracting the non-friction case from the friction case:

$$a_{up} - a_{theoretical} = \frac{\mu_k Mg}{M+m}$$

Finally,

$$\mu_k = \frac{M+m}{Mg}\left(a_{up} - a_{theoretical}\right)$$