11-1 Applying Newton's Second Law for Rotation

Let's learn how to apply Newton's second law for rotation to systems in which the angular acceleration $\vec{\alpha}$ is non-zero. The analysis of such systems is known as **rotational dynamics**.

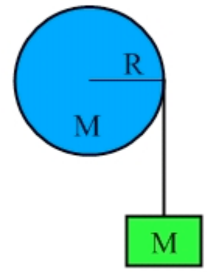


Figure 11.1: A diagram of the pulley and block system. The block hangs down from a string wrapped around the outside of the pulley.

EXPLORATION 11.1 – A mass and a pulley

A pulley with a mass *M* and a radius *R* is mounted on a frictionless horizontal axle passing through the center of the pulley. A block with a mass *M* hangs down from a string that is wrapped around the outside of the pulley. Assume that the pulley is a uniform solid disk. The goal of this Exploration is to determine the acceleration of the block when the system is released from rest. Before we do anything else, we should draw a diagram of the situation based on the description above. The diagram is shown in Figure 11.1.

Step 1 – ***Draw a free-body diagram of the block after the system is released from rest.*** The block accelerates down when the system is released, which means that the block must have a net force acting on it that is directed down. There are only two forces acting on the block, an upward force of tension and a downward force of gravity. Thus, for the net force to be directed down, the force of tension must have a smaller magnitude than the force of gravity. Note that a common mistake is to assume that the force of tension is equal to the force of gravity acting on the block. Thinking about Newton's second law and how it applies to the block helps us to avoid making this error.

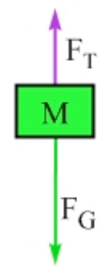


Figure 11.2: The free-body diagram of the block.

Step 2 – ***Draw a free-body diagram of the pulley.*** A complete free-body diagram of the pulley, shown in Figure 11.3 (a), reflects that fact that the center-of-mass of the pulley remains at rest, so the net force must be zero. There is still a non-zero net torque, about an axis through the center of the pulley and perpendicular to the page, that gives rise to an angular acceleration. Generally, when we sum torques about an axis through the center, we draw a rotational free-body diagram, as in Figure 11.3 (b), including only forces that produce a torque. In this case, the only force producing a torque about the center of the pulley is the force of tension. As we discussed in Step 1, above, the force of tension is not equal to the weight of the block when the block has a non-zero acceleration.

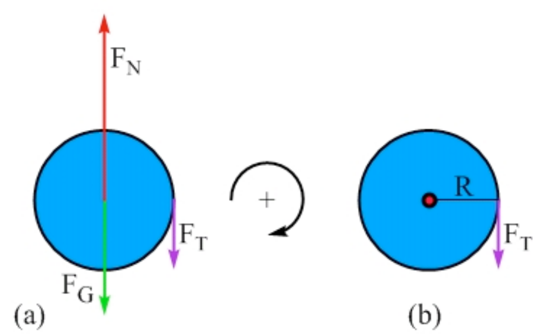


Figure 11.3: The full free-body diagram of the pulley, in (a), and the rotational free-body diagram in (b), showing the only force acting on the pulley that produces a torque about an axis perpendicular to the page through the center of the pulley.

Step 3 – ***Apply Newton's Second Law to the block.*** The block accelerates down, so let's define down to be the positive direction. Newton's second law is $\sum \vec{F} = M\vec{a}$. Using the free-body diagram in Figure 11.2 to evaluate the left-hand side gives:

$$+Mg - F_T = +Ma \, .$$

Step 4 – ***Apply Newton's second law for rotation to the pulley.*** Here, let's define clockwise to be positive for rotation, both because this is the direction of the angular acceleration and because the pulley going clockwise is consistent with the block moving down, in the direction we defined as positive for the block's motion.

Newton's second law for rotation is $\sum\vec{\tau} = I\vec{\alpha}$. Using the free-body diagram in 11.4(b) to evaluate the left-hand side gives: $+R\,F_T \sin(90°) = +I\alpha$.

In the description of the problem we were told, "the pulley is a uniform solid disk." This tells us what to use for the rotational inertia, *I*, on the right-hand side of the equation. Looking up the expression for the rotational inertia of a solid disk in Figure 10.22, we get $I = MR^2/2$. Inserting this into the equation above, and using the fact that sin(90˚) = 1, gives:

$$+R\,F_T = +\frac{1}{2}MR^2\alpha .$$

Canceling a factor of *R* gives: $+F_T = +\frac{1}{2}MR\alpha$.

Step 5 – ***Put the resulting equations together to solve for the block's acceleration.*** Looking at the force equation, $+Mg - F_T = +Ma$, and the equation we obtained from summing torques, $+F_T = +MR\alpha/2$, we have only two equations but three unknowns, F_T, *a*, and α. We could put the two equations together to eliminate the force of tension but then we'd be stuck.

Fortunately, we have another connection we can exploit, which is $\alpha = a/R$. The justification here is as follows. As the block accelerates down, every point on the string moves with the same magnitude acceleration as the block. We assume the string does not slip on the pulley, so the outer edge of the pulley (the part in contact with the string), moves with the string. Thus, the tangential acceleration of a point on the outer edge of the pulley is equal in magnitude to the acceleration of the string, which equals the magnitude of the block's acceleration. Finally, we connect the magnitude of the tangential acceleration of the outer edge of the pulley to the magnitude of the pulley's angular acceleration using $\alpha = a_T/R$. Putting everything together boils down to $\alpha = a/R$, which we can substitute into the equation that came from summing torques:

$$+F_T = +\frac{1}{2}MR\alpha = +\frac{1}{2}MR\frac{a}{R} = +\frac{1}{2}Ma .$$

Using this result in the force equation gives: $+Mg - F_T = +Ma$.

Substituting in for the force of tension gives: $+Mg - \frac{1}{2}Ma = +Ma$.

Thus, $a = 2g/3$, with the acceleration being directed down.

Key ideas: Applying Newton's second law for rotation helps us analyze situations that are purely rotational. Problems that involve both rotation and straight-line motion, as is the case in Exploration 11.1, can be analyzed by combining a torque analysis with a force analysis. **Related End-of-Chapter Exercises: 1, 13.**

Essential Question 11.1: If the pulley in Exploration 11.1 is changed from a uniform solid disk to a uniform solid sphere of the same mass and radius as the disk, how does that affect the block's acceleration? How does that affect the tension in the string?

Answer to Essential Question 11.1: The effect of the change would be to decrease the rotational inertia of the pulley, because the rotational inertia of a solid sphere is $0.4MR^2$ compared with $0.5MR^2$ for the disk. The smaller the rotational inertia of the pulley, the less the pulley holds back the block, so the block's acceleration would increase. On the other hand, the force of tension would decrease. This is most easily seen by analyzing the block. If the block's acceleration increases, the net force on the block must increase. The force of gravity acting on the block remains constant, so the only way to increase the net force acting down on the block is to decrease the upward force of tension.

11-2 A General Method, and Rolling without Slipping

Let's begin by summarizing a general method for analyzing situations involving Newton's second law for rotation, such as the situation in Exploration 1.1. We will then explore rolling. We will tie together the two themes of this section in sections 11-3 and 11-4.

A General Method for Solving a Newton's Second Law for Rotation Problem

These problems generally involve both forces and torques.

1. Draw a diagram of the situation.
2. Draw a free-body diagram showing all the forces acting on the object.
3. Choose a rotational coordinate system. Pick an appropriate axis to take torques about, and then apply Newton's second law for rotation ($\sum \vec{\tau} = I\vec{\alpha}$) to obtain a torque equation.
4. Choose an appropriate x-y coordinate system for forces. Apply Newton's second law ($\sum \vec{F} = m\vec{a}$) to obtain one or more force equations. The positive directions for the rotational and x-y coordinate systems should be consistent with one another.
5. Combine the resulting equations to solve the problem.

Rolling without Slipping

Let's now examine a rolling wheel, which could be a bicycle wheel or a wheel on a car, truck, or bus. We will focus on a special kind of rolling, called **rolling without slipping**, in which the object rolls across a surface without slipping on that surface. This is actually what most rolling situations are, although our analysis would not apply to situations such as you spinning your car wheels on an icy road. Let's consider various aspects of rolling without slipping.

When we dealt with projectile motion in Chapter 4, we generally split the motion into two components, which were usually horizontal and vertical. To help understand rolling, we will follow a similar process. Rolling can be viewed as a combination, or superposition, of purely translational motion (moving a wheel from one place to another with no rotation) and purely rotational motion (only rotation with no movement of the center of the wheel). In the special case of rolling without slipping, there is a special connection between the translational component of the motion and the rotational component. Let's explore that connection.

EXPLORATION 11.2 – Rolling, rolling, rolling

We have a wheel of radius R that we will roll across a horizontal floor so that the wheel makes exactly one revolution. The wheel rolls without slipping on the floor.

Step 1 – ***Consider the rotational part of the motion only (focus on the fact that the wheel spins around exactly once). What distance does a point on the outer edge of the wheel travel because of this spinning motion?***

Because we're ignoring the rotational motion, the distance traveled by a point on the outer edge of the wheel because of the spin is equal to the circumference of the wheel itself. This is a distance of $2\pi R$. See the top diagram in Figure 11.4.

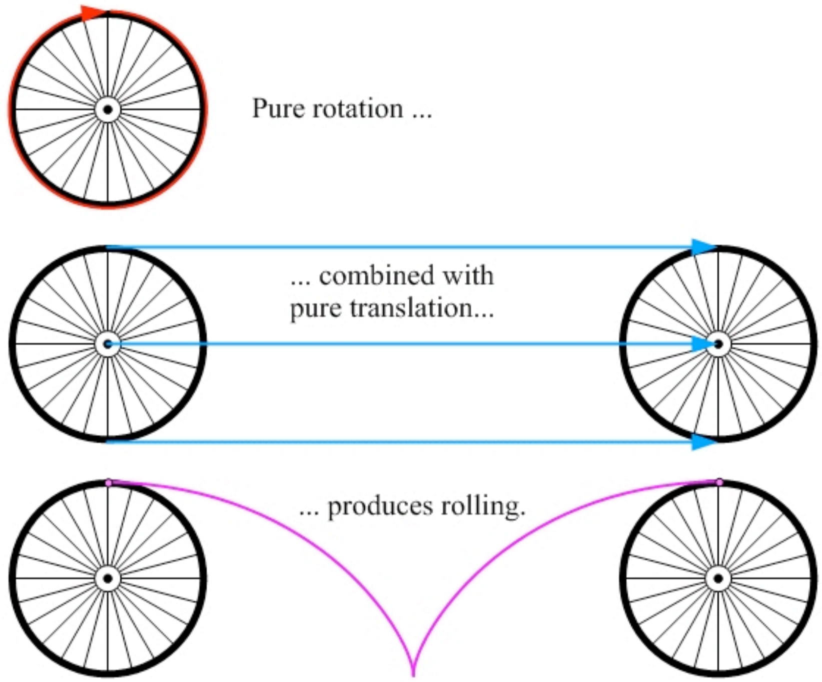


Figure 11.4: A pictorial representation of how the rotational component and the translational component of the motion combine to produce the interesting shape of the path traced out by a point on the outer edge of the wheel that is rolling without slipping. This shape is known as a cycloid.

Step 2 – ***Now consider the translational part of the motion only (i.e., ignore the fact that the wheel is spinning, and imagine that we simply drag the wheel a particular distance without allowing the wheel to rotate). What is the distance that any point on the wheel moves if we drag the wheel a distance equal to that it would move if we rolled it so it rolled through exactly one revolution?*** To determine what this distance is, imagine that we placed some double-sided tape around the wheel before we rolled it, and that the tape sticks to the floor. This is shown in the middle diagram in Figure 11.4. Rolling the wheel through one revolution lays down all the tape on the floor, covering a distance that is again equal to the circumference of the wheel. Thus, focusing on the translational distance only, the translational distance moved by every point on the wheel as the wheel rolls through one revolution is $2\pi R$.

Step 3 – ***Assuming the rolling is done at constant speed, compare the speed of a point on the outer rim, associated only with the wheel's rotation, to the translational speed of the wheel's center of mass.*** We can find these speeds by dividing the appropriate distances by the time during which the motion takes place. Because the distances associated with the two components of the motion are equal, and the time of the motion is the same for the two components, these two speeds are equal.

Key ideas for rolling: Rolling can be considered to be a superposition of a pure translational motion and a pure rotational motion. In the special case of rolling without slipping, the distance moved by a point on the outer edge of a wheel associated with the rotational component is equal to the translational distance of the wheel. The speed of a point on the outer edge because of the rotational component is also equal to the translational speed of the wheel.
Related End-of-Chapter Exercises: 4, 17.

Essential Question 11.2: Different points on a wheel that is rolling without slipping have different speeds. Considering one particular instant, which point on the wheel is moving slowest? Which point is moving the fastest?

Answer to Essential Question 11.2: As we will investigate in more detail in section 11-3, when a wheel rolls without slipping, the point at the bottom of the wheel has the smallest speed (the speed there is zero, in fact), while the point at the top of the wheel is moving fastest.

11-3 Further Investigations of Rolling

Let's continue our analysis of rolling, starting by thinking about the velocity of various points on a wheel that rolls without slipping. We will then go on to investigate rolling spools.

EXPLORATION 11.3 – Determining velocity

Let's turn now from thinking about speeds to thinking about velocities. Consider a wheel rolling without slipping with a constant translational velocity $\vec{v}$, directed to the right, across a level surface. For each point below, determine the point's net velocity by combining, as vectors, the point's translational velocity (the velocity associated with the translational component of the motion) with its velocity because of the rotational component of the motion.

Step 1 – ***Find the net velocity of the center of the wheel.***

Our axis of rotation passes through the center of the wheel. The center of the wheel therefore has no rotational velocity (because $v_{rot} = r\omega$, and r = 0). Thus, the net velocity of the center of the wheel is its translational velocity, $\vec{v}$. Note that every point on the wheel has the same translational velocity. Two equal vectors are shown at the center of the wheel in Figure 11.5. One represents the translational velocity at that point, and the other represents the net velocity at that point.

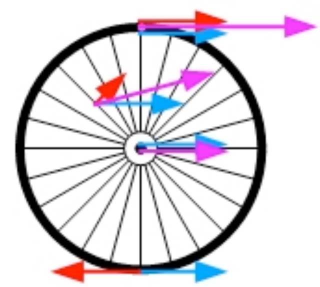


Figure 11.5: The translational (equal vectors all directed right), rotational (tangent to the circle), and net velocities of various points on the wheel. The net velocity at a point is a vector sum of the translational and rotational velocities.

Step 2 – ***Find the net velocity of the point at the very top of the wheel.***

Here, we use the fact that the rotational speed is equal to the translational speed, so we are adding two velocities of equal magnitude. At the top of the wheel, the velocities also point in the same direction, so the net velocity is $2\vec{v}$, as shown in Figure 11.5.

Step 3 – ***Find the net velocity of the point at the very bottom of the wheel.***

At the bottom of the wheel, the rotational velocity exactly cancels the translational velocity, because the vectors point in opposite directions and have equal magnitudes. The net velocity of that point is zero – the point is instantaneously at rest! This is a special condition that is characteristic of rolling without slipping. No slipping implies no relative motion between the surfaces in contact, which means the point at the bottom of the wheel that is in contact with the road surface is at rest.

Figure 11.5 also shows the net velocity at another point on the wheel, a point above and to the left of the center. As with all points, the translational velocity is a vector directed to the right. The velocity associated with the pure rotation is tangent to the circle that passes through the point (and centered at the center of the wheel) - this has a magnitude of $v/2$, because the point is halfway between the center and the rim. The net velocity is the longest of the three vectors at that point, the vector sum of the translational and rotational velocities.

Key ideas for rolling: The net velocity of a point on a rolling wheel can be found by adding, as vectors, the point's translational velocity and its rotational velocity. In the special case of a wheel rolling without slipping with a translational velocity $\vec{v}$, the net velocity of the center of the wheel is $\vec{v}$; while that of the point at the top of the wheel is $2\vec{v}$. A point on the outer edge of the wheel actually comes instantaneously to rest when it reaches the bottom of the wheel.
Related End-of-Chapter Exercises: 5, 6.

EXAMPLE 11.3 – Unrolling a ribbon from a spool

A long ribbon is wrapped around the outer edge of a spool. You pull horizontally on the end of the ribbon so the ribbon starts to unwind from the spool as the spool rolls without slipping across a level surface.

(a) When you have moved the end of the ribbon through a horizontal distance L, how far has the spool moved?

(b) Does your answer change if the ribbon is instead wrapped around the spool's axle, which has a radius equal to half the radius of the spool? If so, how does the answer change?

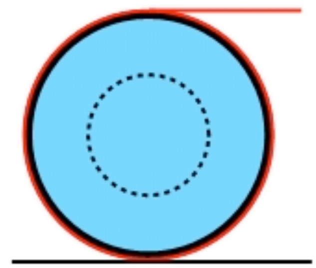


Figure 11.6: A spool is rolling without slipping to the right because you are pulling, to the right, on the red ribbon that is wrapped around the spool.

SOLUTION

(a) A diagram of the situation is shown in Figure 11.6. Once again, we can think of the spool's rolling motion as a combination of its translational motion and its rotational motion. We can thus say that the end of the ribbon moves because (a) the spool has a translational motion, and (b) the spool is rotating. The speed of the ribbon matches the speed of the top of the spool, because there is no slipping between the ribbon and the spool. Recalling the result from Exploration 11.3, the top of the spool has a velocity twice that of the center of the spool. Putting these facts together means that the center of the spool has a velocity half that of the end of the ribbon at any instant, and so the spool covers a distance of $L/2$, half the distance covered by the end of the ribbon.

(b) What if the ribbon is wrapped around the spool's axle and you move the end of the ribbon through a distance L? The answer changes because the rotational contribution to the net velocity changes. As shown in Figure 11.7, the ribbon now comes off the axle at the top of the axle, at a point halfway between the edge and the center of the spool. The net velocity at that point on the spool is 1.5 times the velocity of the center of the spool: the translational velocity is equal to the velocity of the center, while the rotational velocity is half that of the center, because at a radius of R/2 we have:

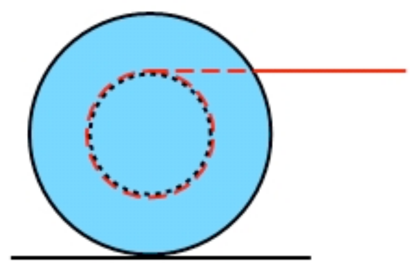


Figure 11.7: The ribbon is wrapped around the axle of the spool, which has a radius half that of the spool. The ribbon comes off the axle at the top.

$$v_{rot} = \frac{R}{2}\omega = \frac{1}{2}R\omega = \frac{1}{2}v_{trans}\,.$$

Putting it another way, the velocity of the center of the spool is now two-thirds of the velocity of the end of the ribbon. If the end of the ribbon travels a distance L, the spool translates through a distance of $2L/3$.

Related End-of-Chapter Exercises: 18, 19.

Essential Question 11.3: In a situation similar to that in Figure 11.7, you pull to the right on a ribbon wrapped around the axle of a spool. This time, however, the ribbon is wound so it comes away from the spool underneath the axle, as shown in Figure 11.8. When you pull to the right on the ribbon, the spool rolls without slipping. In which direction does it roll? Sketch a free-body diagram of the spool to help you think about this.

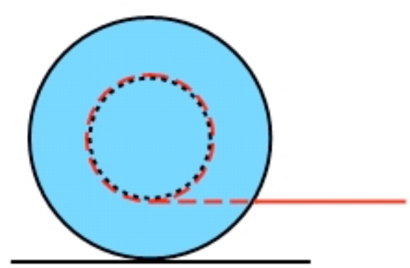


Figure 11.8: A ribbon is wrapped around the axle of the spool so the ribbon comes off the axle below the axle.

Answer to Essential Question 11.3: Many people focus on the counterclockwise torque, relative to an axis perpendicular to the page that passes through the center of the spool, exerted by the force of tension and conclude that the spool rolls to the left. Before jumping to conclusions, however, draw the free-body diagram (after drawing your own, see Figure 11.9). As usual there is a downward force of gravity and an upward normal force. Horizontally there is a force of tension, directed right, exerted by the ribbon. With no friction, the force of tension would cause the spool to move right and spin counterclockwise, so the bottom of the spool would move right with respect to the horizontal surface. Friction must therefore be directed left to oppose this, and, because we know the spool rolls without slipping, the force of friction must be static friction.

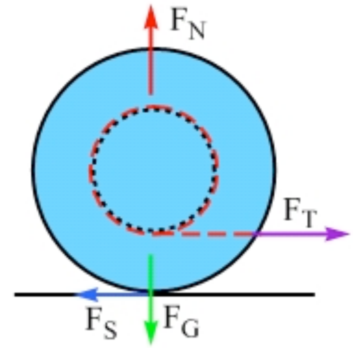


Figure 11.9: The free-body diagram of the spool.

Now we have the complete free-body diagram, we can see that the answer to the question is not obvious. There is one force left and one force right – which is larger? Relative to an axis through the center, there is one torque clockwise and one counterclockwise – which is larger? A quick way to get the answer is to consider an axis perpendicular to the page, passing through the point where the spool makes contact with the horizontal surface. Relative to this axis, three of the four forces give no torque, and the torque from the tension in the string is in a clockwise direction. Clockwise rotation of the spool, relative to the point where the spool touches the surface, is consistent with the spool rolling without slipping to the right. This is opposite to what you would conclude by focusing only on the torque about the center from the force of tension. The spool rolls to the right.

11-4 Combining Rolling and Newton's Second Law for Rotation

Let's now look at how we can combine torque ideas with rolling-without-slipping concepts.

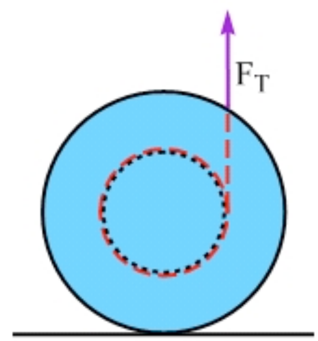


Figure 11.10: An upward force is exerted on the string wrapped around the axle of the spool.

EXPLORATION 11.4 – A vertical force but a horizontal motion

A spool of mass M has a string wrapped around its axle. The radius of the axle is half that of the spool. An upward force of magnitude F_T is exerted on the end of the string, as shown in Figure 11.10. This causes the spool, which is initially at rest, to roll without slipping as it accelerates across the level surface.

In which direction does the spool roll? Which horizontal force is responsible for the spool's horizontal acceleration? Let's begin by drawing a free-body diagram of the spool. Figure 11.11 shows a partial free-body diagram, showing only the vertical forces acting on the spool. There is a downward force of gravity acting on the spool, and an upward force of tension applied by the string (note that F_T must be less than or equal to Mg, so the spool has no vertical acceleration). There is also an upward normal force, required to balance the vertical forces.

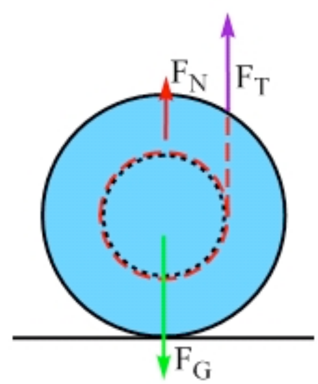


Figure 11.11: A partial free-body diagram of the spool, showing the vertical forces acting on it.

Is there a horizontal force? If there is, what could it be? Let's go back and think about what is interacting with the spool. The force of gravity accounts for the interaction between the Earth and the spool, and the force of tension accounts for the interaction between the string and the spool. The only interaction left is the interaction between the surface and the spool. The surface exerts a contact force on the spool. Remember that we generally split the contact force into components, the normal force (which we have accounted for) and the force of friction (which we have not).

If there is a horizontal force acting, it can only be a force of friction. Do we need friction in this situation? Consider what would happen if the free-body diagram shown in Figure 11.11 was complete, and there was no friction. Taking an axis perpendicular to the page through the center of the spool, the tension force would give rise to a counterclockwise torque. Because the net force acting on the spool would be zero, however, the spool would simply spin counterclockwise without moving. This is inconsistent with the rolling-without-slipping motion we are told is occurring. There must be a force of friction acting on the spool to cause the horizontal motion.

Note that, without friction, the bottom of the spool rotates to the right relative to the surface. The force of friction must therefore be directed to the left, acting to oppose the relative motion that would occur without friction. Because the force of friction is the only horizontal force acting on the spool, the spool accelerates to the left.

To gain another perspective on this situation, we can follow the procedure discussed in the Answer to Essential Question 11.3, and consider the sum of the torques about the contact point (the point where the spool makes contact with the ground). Both the normal force and the force of gravity pass through the contact point, so they don't give rise to any torques about the contact point. If there is a force of friction, whether it is directed to the right or the left it would also pass through the contact point, giving rise to no torque about that point. Thus, the only force that produces a torque about the contact point is the tension force. Relative to the contact point, this torque is directed counter-clockwise, which is consistent with rolling without slipping to the left. Starting from rest, rolling to the left requires a horizontal force directed to the left, which can only be a friction force.

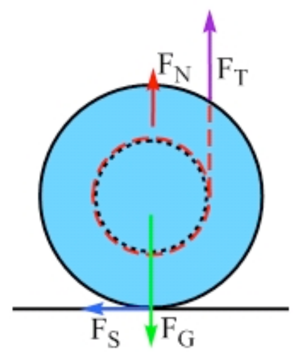
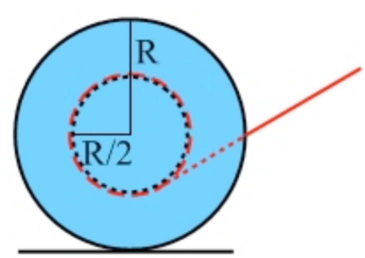


Figure 11.12: The complete free-body diagram of the spool.

Is the force of friction kinetic friction or static friction? Because the spool is rolling without slipping, and the bottom of the spool is instantaneously at rest relative to the surface it is in contact with, the force of friction is the static force of friction. This may sound counter-intuitive, since there is relative motion between the spool as a whole and the surface, but it is very similar to the walking (without slipping) situation that we thought about in Chapter 5. When walking, as long as our shoes do not slip on the floor, a force of static friction acts in the direction of motion. The same thing happens here - in this case, the force of static friction is the only horizontal force acting on the spool, so it is the force accelerating the wheel horizontally. The complete free-body diagram for the rolling-without-slipping situation is shown in Figure 11.12.

Key ideas for rolling without slipping: Rolling without slipping often involves a force of friction, which must be a static force of friction. The static force of friction is often (although not always) in the direction of motion.
Related End-of-Chapter Exercises: 3, 50.



Essential Question 11.4: In the situation shown in Figure 11.13, you pull on the end of a ribbon wrapped around the axle of a spool. Your force is exerted in the direction shown. If the spool rolls without slipping, in which direction does the spool roll?

Figure 11.13: A ribbon is wrapped around the axle of the spool so the ribbon comes off the axle in the direction shown.

Answer to Essential Question 11.4: Once again, it is simplest to take torques about an axis perpendicular to the page, passing through the point at which the spool touches the ground. The only force giving rise to a torque about this point is the tension in the ribbon, which gives a clockwise torque. If the spool rotates clockwise with respect to its bottom point, the motion of the spool is to the right.

11-5 Analyzing the Motion of a Spool

EXPLORATION 11.5 – Continuing the analysis of the rolling spool

Let's return to the situation described in Exploration 11.4, and focus in particular on the free-body diagram in Figure 11.12. Our goal is to determine the magnitude of the spool's acceleration in terms of F_T and M. The spool consists of two disks, each of mass $M/3$ and radius R, connected by an axle of mass $M/3$ and radius $R/2$.

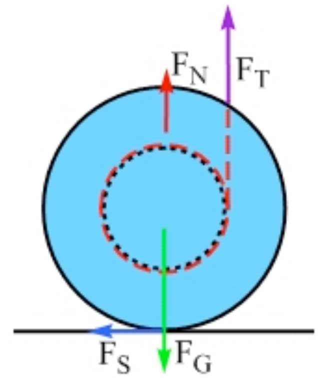


Figure 11.12: The complete free-body diagram of the spool.

Step 1 – ***Apply Newton's Second Law for the horizontal forces.***

The spool accelerates left, so let's define left to be the positive direction.

$$\sum \vec{F}_x = M\,\vec{a}_x .$$

Because the acceleration is entirely in the x-direction, we can replace $\vec{a}_x$ by $\vec{a}$.

Evaluating the left-hand side of this expression with the aid of Figure 11.12 gives:

$$+F_s = +M\,a .$$

Step 2 – ***Find the expression for the spool's rotational inertia about an axis perpendicular to the page passing through the center of the spool.***

Why are we doing this? Well, we'll need to apply Newton's second law for rotation to solve this problem, and that involves the spool's rotational inertia. To find the spool's rotational inertia, we can use the expression for the rotational inertia of a solid disk or cylinder ($I = ½ mr^2$) about the center . Let's apply this equation to the three pieces of the spool and add them together to find the net rotational inertia.

Each of the two disks contributes $\frac{1}{2}\left(\frac{M}{3}\right)R^2 = \frac{1}{6}MR^2$ to the rotational inertia.

The axle contributes $\frac{1}{2}\left(\frac{M}{3}\right)\left(\frac{R}{2}\right)^2 = \frac{1}{24}MR^2$.

The total rotational inertia is $I = \frac{1}{6}MR^2 + \frac{1}{6}MR^2 + \frac{1}{24}MR^2 = \frac{9}{24}MR^2 = \frac{3}{8}MR^2$.

Step 3 – ***Apply Newton's Second Law for Rotation to obtain a connection between the force of friction and the upward force $\vec{F}_T$ applied to the string.***

Taking torques about an axis perpendicular to the page and passing through the center of the spool is a good way to do this, because the force of gravity and the normal force pass through this axis and therefore give no torque about that axis. Since the spool has a counterclockwise angular acceleration let's take counterclockwise to be positive for torques. Applying Newton's second law for rotation gives:

$$\sum \vec{\tau} = I\vec{\alpha} .$$

Referring to Figure 11.12, and using the equation $\vec{\tau} = r\,F\sin\theta$, we have:

$$+\frac{R}{2}F_T\sin(90°) - R\,F_S\sin(90°) = +I\alpha \ .$$

Recognizing that $\sin(90°) = 1$, and substituting the expression for the spool's rotational inertia we found above, gives:

$$+\frac{R}{2}F_T - R\,F_S = +\frac{3}{8}MR^2\alpha \ .$$

Canceling a factor of *R* gives: $+\frac{1}{2}F_T - F_S = +\frac{3}{8}MR\alpha$.

Step 4 – ***What is the connection between the spool's acceleration and its angular acceleration?***
For rolling without slipping, the connection between the acceleration and the angular acceleration is $a = R\alpha$, although it is always a good idea to check whether the positive direction for the straight-line motion is consistent with the positive direction for rotation. In our case they are consistent, since we chose them based on the motion. If we had reversed one of the positive directions, however, we would have had a negative sign in the equation.

Step 5 – ***Combine the results above to determine the spool's acceleration in terms of F_T and M.***
Let's first substitute $a = R\alpha$ into our final expression from step 3, to get:

$$+\frac{1}{2}F_T - F_S = +\frac{3}{8}Ma \ .$$

In step 1, we determined that $F_s = M\,a$, so we get:

$$+\frac{1}{2}F_T - Ma = +\frac{3}{8}Ma \ ;$$

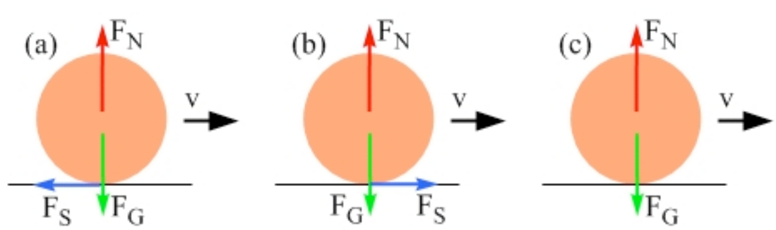
$$+\frac{1}{2}F_T = +\frac{11}{8}Ma \ ;$$

$$\vec{a} = \frac{4\,F_T}{11M} \text{ ,directed to the left.}$$

Key idea: Solving a rolling-without-slipping problem often involves analyzing the rotational motion, analyzing the one-dimensional motion, and combining the analyses.
Related End-of-Chapter Exercises: 51, 53.

Essential Question 11.5: Consider a hard ball that is rolling without slipping across a smooth level surface. If the ball maintains a constant velocity, in what direction is the static force of friction acting on the ball? Consider the three-possible free-body diagrams for the ball in Figure 11.14 below, and state which free-body diagram is appropriate for this situation.

Figure 11.14: Possible free-body diagrams for a ball rolling, without slipping, at constant velocity to the right across a horizontal surface.



Answer to Essential Question 11.5: No friction force can act on the ball, so the correct free-body diagram is that shown in Figure 11.14 (c). A force of friction in the direction of motion would increase the ball's translational speed, and the counterclockwise torque from the force of friction would decrease the ball's angular speed. A force of static friction directed opposite to the ball's velocity would decrease the translational speed while increasing the rotational speed. The ball rolls horizontally at constant velocity only if no friction force acts.

11-6 Angular Momentum

By now, we have looked at enough analogies between straight-line motion and rotational motion that we can simply take a straight-line motion equation, replace the straight-line motion variables by their rotational counterparts, and write down the equivalent rotational equation. We could also derive the rotational equations following a derivation parallel to the one we used for the straight-line motion equation, but the end result would be the same.

Let's try this for angular momentum. In Chapter 6, we used the following expression for the linear momentum, $\vec{p}$, of an object of mass *m* moving with velocity $\vec{v}$: $\vec{p} = m\vec{v}$.

Using the symbol $\vec{L}$ to represent angular momentum, we can come up with the equivalent expression for angular momentum by replacing mass *m* by its rotational equivalent, rotational inertia *I*, and velocity $\vec{v}$ by its rotational equivalent $\vec{\omega}$:

$$\vec{L} = I\vec{\omega} . \qquad \text{(Equation 11.1: \textbf{Angular momentum})}$$

We made a number of statements about momentum in Chapter 6. Equivalent statements apply to angular momentum, including:

- Angular momentum is a vector, pointing in the direction of angular velocity.
- The angular momentum of a system can be changed by applying a net torque.
- If no net torque acts on a system, its angular momentum is conserved.

Let's explore this idea of angular momentum conservation.

EXPLORATION 11.6 – Jumping on the merry-go-round

A little red-haired girl named Sarah, with mass *m*, runs toward a playground merry-go-round, which is initially at rest, and jumps on at its edge. Sarah's velocity $\vec{v}$ is tangent to the circular merry-go-round. Sarah and the merry-go-round then spin together with a constant angular velocity $\vec{\omega}_f$. The merry-go-round has a mass *M*, a radius *R*, and has the form of a uniform solid disk. Assume that Sarah's "radius" is small compared to *R*. The goal of this Exploration is to determine an expression for $\vec{\omega}_f$. We can treat this as a collision.

Step 1 – ***Sketch two diagrams, one showing Sarah running toward the merry-go-round and the other showing Sarah and the merry-go-round rotating together after Sarah has jumped on. Imagine that you're looking down on the situation from above.*** These two diagrams are shown in Figure 11.15.

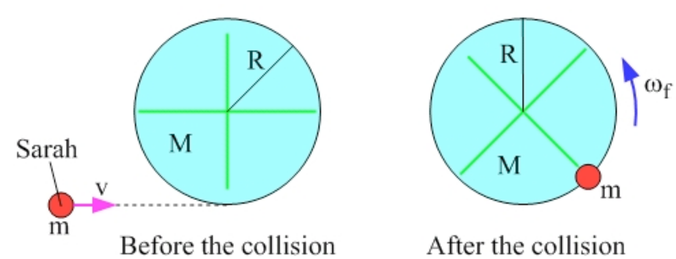


Figure 11.15: On the left is the situation before the collision, as Sarah runs toward the merry-go-round, while on the right is the situation after the collision, with Sarah and the merry-go-round rotating together with a constant angular velocity.

Step 2 – ***What kind of momentum does the Sarah/merry-go-round system have, if any, before Sarah jumps on the merry-go-round? What about after Sarah jumps on?*** After the collision, when the system is rotating, the system clearly has a non-zero angular momentum. Before the collision, however, it is not obvious that the system has any angular momentum, because nothing is rotating. Sarah certainly has a linear momentum, however, because she has a non-zero velocity.

Step 3 – ***Convert Sarah's linear momentum before the collision to an angular momentum, using a method modeled on the way we convert a force to a torque.*** Although there is no rotation before the collision, we can say that the system has an angular momentum with respect to an axis perpendicular to the page that passes through the center of the merry-go-round. Consider how we get torque from force, where the magnitude of the torque is given by $\tau = r\,F\sin\phi$. Angular momentum is found from linear momentum in a similar fashion, with its magnitude given by:

$L = r\,p\sin\phi = r\,(mv)\sin\phi$, (Eq. 11.2: **Connecting angular momentum to linear momentum**)

where ϕ is the angle between the line we measure distance along and the line of the linear momentum.

Relative to the axis through the center of the merry-go-round, the angular momentum is: $\vec{L}_i = R\,mv\sin(90°) = R\,mv$, in a counterclockwise direction.

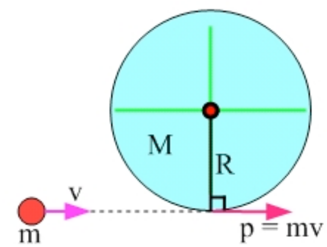


Figure 11.16: The lever-arm method to determine Sarah's angular momentum, with respect to an axis passing through the center of the merry-go-round.

Step 4 – ***Apply angular momentum conservation to express $\vec{\omega}_f$, the angular velocity of the system after the collision, in terms of the variables above.*** Angular momentum is conserved because there are no external torques acting on the Sarah/merry-go-round system, relative to a vertical axis passing through the center of the turntable. We will justify this further in section 11-7. Thus, we can say: Angular momentum before the collision = angular momentum afterwards.

The angular momentum afterwards is $\vec{L}_f = I\,\vec{\omega}_f$. The system's rotational inertia after the collision is the sum of the rotational inertias of Sarah, and the ½ MR^2 of the merry-go-round. Sarah's "radius" is small compared to *R,* so we treat Sarah as a point, assuming that all her mass is the same distance, R , from the center of the turntable. Sarah's rotational inertia is thus mR^2 .

Thus, the rotational inertia of the system after the collision is $I = \frac{1}{2}MR^2 + mR^2$.

Taking counterclockwise to be positive, angular momentum conservation gives: $\vec{L}_i = \vec{L}_f$.

$$+R\,mv = I\vec{\omega}_f = \left(\frac{1}{2}MR^2 + mR^2\right)\vec{\omega}_f .$$

Solving for the final angular velocity of the system gives:

$$\vec{\omega}_f = +\frac{mv}{\frac{1}{2}MR + mR} \quad \text{or,} \quad \vec{\omega}_f = \frac{mv}{\frac{1}{2}MR + mR} \text{ directed counterclockwise.}$$

Key ideas: Linear momentum converts to angular momentum in the same way force converts to torque. Also, we apply momentum conservation ideas to rotational collisions in the same way we analyze collisions in one and two dimensions. **Related End-of-Chapter Exercises: 32, 34, 59.**

Essential Question 11.6: Is it possible for Sarah, with the same initial speed, to jump onto the merry-go-round at the same point, but not make it spin? If so, how could she do this?

Answer to Essential Question 11.6: One way for Sarah to jump onto the merry-go-round, without causing the merry-go-round to spin, is for Sarah to direct her velocity at the center of the merry-go-round, instead of tangent to it. If Sarah ran directly toward the center of the merry-go-round she would have no angular momentum before the collision and there would be no reason for the system to spin after the collision.

11-7 Considering Conservation, and Rotational Kinetic Energy

In step 4 of Exploration 11.6, we stated that the angular momentum of the system consisting of Sarah and the merry-go-round was conserved, because no external torques were acting on the system. Let's justify that statement. We do not have to concern ourselves with vertical forces, such as the force of gravity or the normal force applied to the merry-go-round by the ground, because vertical forces give no torque about a vertical axis of rotation. We also do not have to concern ourselves with the force that Sarah exerts on the merry-go-round, or the equal-and-opposite force the merry-go-round exerts on Sarah, because the system we're considering consists of the combination of Sarah and the merry-go-round, so those are internal forces and cancel one another. Still, let's examine those forces a little.

Individual free-body diagrams for Sarah and the merry-go-round when Sarah first jumps on the merry-go-round are shown in Figure 11.17. Through some combination of friction between her shoes and the merry-go-round, and a contact force between her hands and any handholds on the merry-go-round, there is a force component that acts to the left on Sarah from the merry-go-round (this reduces her speed), and an equal-and-opposite force component that acts to the right on the turntable by Sarah (providing the torque that gives the merry-go-round an angular acceleration). However, the turntable does not accelerate to the right. This is because there is a horizontal force applied on the turntable by whatever the turntable's axis is connected to, which we can consider to be the Earth. As shown in Figure 11.17, the Sarah/merry-go-round system has a net external force acting on it at this point, which is why the *linear* momentum of the system is not conserved. However, this net external force gives rise to no torque about an axis through the center of the merry-go-round, because the force passes through that axis. Because there is no net external torque acting on the system, the system's angular momentum is conserved.

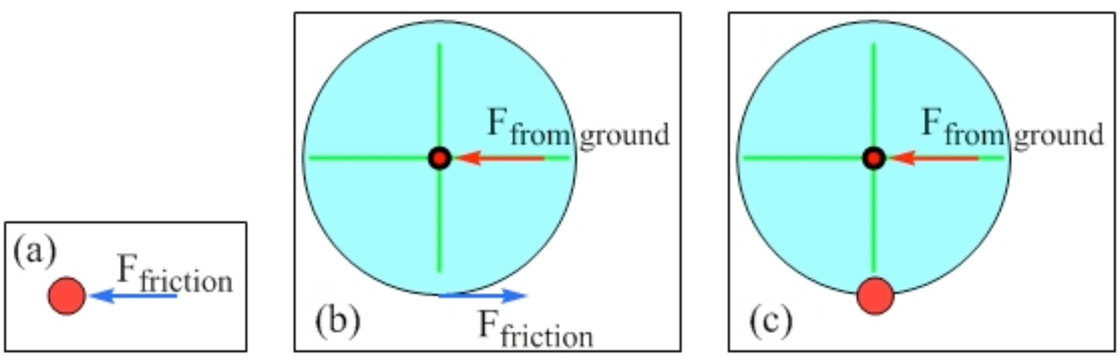


Figure 11.17: Free-body diagrams for Sarah, the merry-go-round, and the system consisting of Sarah and the merry-go-round together, when Sarah initially makes contact with the merry-go-round. Vertical forces are ignored in this overhead view.

Rotational Kinetic Energy

Let's now move from the rotational equivalent of linear momentum to the rotational equivalent of translational kinetic energy. The equation we used previously for kinetic energy is $K = ½ mv^2$. We can find the equivalent expression for kinetic energy in a rotational setting by replacing mass m by rotational inertia I, and speed v by angular speed ω. The kinetic energy of a purely rotating object is thus given by:

$$K = \frac{1}{2} I\omega^2 . \qquad \text{(Equation 11.3: \textbf{Rotational kinetic energy})}$$

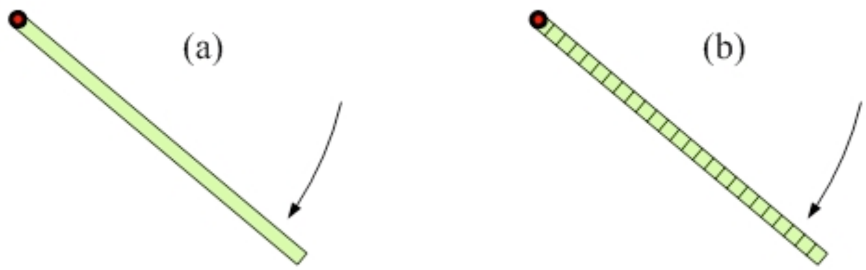


Figure 11.18: (a) A rod that has been released from rest when it was horizontal is now moving. We can find its kinetic energy by breaking the rod into small pieces, as shown in (b), finding the kinetic energy of each piece, and adding these kinetic energies together to find the net kinetic energy.

Let's make sure our substituting-the-equivalent-rotational-variables method of arriving at rotational equations makes sense. Consider, for instance, a uniform rod that can rotate about an axis through one end. If we hold the rod horizontal and then release it from rest, the rod swings down. What is the rod's kinetic energy at a particular instant, say at the instant shown in Figure 11.18 (a)? One thing we could do is, as shown in Figure 11.18 (b), break the rod into small pieces of mass m_i, determine the speed v_i of each piece, find the kinetic energy $\frac{1}{2} m_i v_i^2$ of each piece, and then add up all these kinetic energies to find the total kinetic energy:

$$K = \sum \frac{1}{2} m_i v_i^2 .$$

Because the speed of each piece is different, while the angular speed of each piece is the same, let's write the sum in terms of the rod's angular speed instead:

$$K = \sum \frac{1}{2} m_i (r_i \omega)^2 .$$

If we bring the constants of ½ and ω^2 out in front of the sum, our expression becomes $K = \frac{1}{2}\omega^2 \sum m_i r_i^2$, which we can write as $K = \frac{1}{2} I\omega^2$, because the definition of rotational inertia is $I = \sum m_i r_i^2$. This expression for the kinetic energy agrees with what we came up with above (and it works for any rotating object, not just a rod!).

Essential Question 11.7: In Chapter 7 we used names such as "elastic collision" and "inelastic collision" to classify various collisions. Under what category would the Sarah/merry-go-round collision described in the previous Exploration fall?

Answer to Essential Question 11.7: Because Sarah and the merry-go-round stick together and move as one after the collision, the collision is completely inelastic.

11-8 Racing Shapes

Let's make use of the expression for rotational kinetic energy we derived in section 11-7, and apply it to analyze the motion of an object that rolls without slipping down a slope. The analysis can be done in terms of energy conservation (as we will do), or in terms of thinking about forces and torques and applying Newton's second law and Newton's second law for rotation. The analysis in those terms can be found on the accompanying web site.

EXPLORATION 11.8 – Racing shapes

You have various shapes, including a few different solid spheres, a few rings, and a few uniform disks and cylinders. The objects have various masses and radii. When you race the objects by releasing them from rest two at a time, they roll without slipping down an incline of constant angle. Our goal is to determine which object reaches the bottom of the incline in the shortest time. Let's analyze this for a generic object of mass *M*, radius *R*, and rotational inertia, about an axis through the center of mass, of cMR^2.

Step 1 – ***Sketch a free-body diagram for the object as it rolls without slipping down the ramp.*** A diagram and a free-body diagram is shown in Figure 11.19. The Earth applies a downward force of gravity to the object, while the incline applies a contact force. We split the contact force into two forces, a normal force perpendicular to the incline and a force of friction directed up the slope. This is a static force of friction, because the object does not slip as it rolls. The force of static friction is directed up the slope, not because the motion of the object is down the slope, but because the object has a clockwise angular acceleration (its angular velocity is clockwise and increasing as it rolls down). Taking an axis through the center of the object, the static force of friction is the only force that can provide the torque associated with this angular acceleration – the other two forces pass through the center of the object and thus give no torque about that axis.

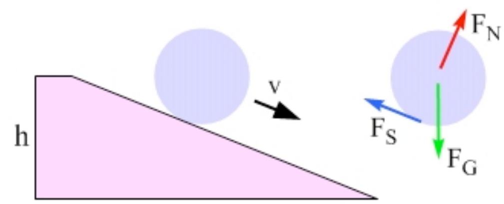


Figure 11.19: The diagram and free-body diagram of an object as it rolls without slipping down a ramp. A force of friction directed up the ramp provides the clockwise torque associated with the object's clockwise angular acceleration. The force of friction is static because the object does not slip as it rolls.

Step 2 – ***Let's analyze this in terms of energy conservation, using the same conservation of energy equation we used in previous chapters. Start by eliminating the terms that are zero in the equation.*** Recall that the energy conservation equation is: $K_i + U_i + W_{nc} = K_f + U_f$. The object is released from rest, so the initial kinetic energy K_i is zero. We can also define the bottom of the incline to be the zero level for gravitational potential energy, so the final potential energy is $U_f = 0$. We also have no work being done by non-conservative forces. This may seem somewhat counter-intuitive at first, because static friction acts on each object as it rolls down the hill, but it is kinetic friction that is associated with a loss of mechanical energy. Static friction, because it involves no relative motion (and therefore no displacement to use in the work equation), does not produce a loss of mechanical energy.

The conservation of energy equation can thus be written: $U_i = K_f$.

Let's say that each object starts from a height *h* above the bottom of the incline. Because the zero for potential energy is at the bottom, the initial gravitational potential energy can be written as: $U_i = Mgh$. Our energy conservation term can thus be written $Mgh = K_f$.

Step 3 – ***Split the kinetic energy term into two pieces, one representing the translational kinetic energy and one representing the rotational kinetic energy. Express the rotational kinetic energy in terms of M and v_f (the speed at the bottom of the incline) and solve for v_f.*** First, let's think about why considering two types of kinetic energy is appropriate. When an object's center-of-mass is moving, the object has translational kinetic energy $KE_{trans} = \frac{1}{2}Mv^2$. When an object is only rotating, it has a rotational kinetic energy $KE_{rot} = \frac{1}{2}I\omega^2$. A rolling object, however, is both translating as well as rotating, and thus it has both these forms of kinetic energy.
Our energy equation now becomes: $Mgh = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2$.

Let's make two substitutions to rewrite the rotational kinetic energy term. First, we can use our expression for rotational inertia, $I = cMR^2$. Then, we use the relationship between speed and angular speed that applies to rolling without slipping,: $\omega = v/R$. Our energy equation is now:

$$Mgh = \frac{1}{2}Mv_f^2 + \frac{1}{2}cMR^2\frac{v_f^2}{R^2}.$$

Note that all factors of mass *M* and radius *R* cancel, leaving: $gh = \frac{1}{2}v_f^2 + \frac{1}{2}cv_f^2$.

Solving for v_f, the object's speed at the bottom of the incline, gives: $v_f = \sqrt{\frac{2gh}{1+c}}$.

This result is consistent with the $v_f = \sqrt{2gh}$ result we obtained in previous chapters (for the speed of a ball dropped from rest through a height *h*, for instance), giving us some confidence that the answer is correct.

So, which object wins the race? The winner is the object with the highest speed at the bottom, which requires the smallest value of *c.* Recall that *c* is the numerical factor in the moment of inertia, $I = cMR^2$. For the various shapes we were racing we have *c* = 2/5 for solid spheres; *c* = 1/2 for uniform disks and cylinders; and $c = 1$ for rings. Thus, in the rolling races, a solid sphere beats any disk (or cylinder) and any ring, while any disk or cylinder beats any ring.

Key ideas: We can apply energy conservation in an analysis of rotating, or rolling, objects, just as we did in previous situations. Our energy conservation equation from Chapter 7 needs no modification. All we have to do is to use the expression for the kinetic energy of rotating objects: $KE_{rot} = \frac{1}{2}I\omega^2$. **Related End-of-Chapter Exercises: 7, 8, 10.**

Essential Question 11.8: In Exploration 11.8, we determined that, in the races of rolling objects, a solid sphere would beat a disk or cylinder, which would beat a ring. What if we raced two of the same kind of object against one another (such as a sphere versus a sphere)? Which object would win? The object with the larger mass, smaller mass, larger radius, or smaller radius?

Answer to Essential Question 11.8: Review the analysis in step 3 of Exploration 11.8. Both the mass and radius cancel out of the energy conservation equation. This tells us, surprisingly, that the mass and radius are irrelevant. In other words, all uniform solid spheres roll the same, all uniform solid disks (or cylinders) roll the same, and all rings roll the same – all the races involving two of the same kind of object end in a tie.

11-9 Rotational Impulse and Rotational Work

Let's continue our method of determining rotational equations from their straight-line motion counterparts by writing down expressions for rotational impulse and rotational work. In Chapter 6, the impulse relationship we came up with was: $\Delta\vec{p} = \vec{F}_{net}\,\Delta t$. In words, this equation tells us that the change in momentum an object experiences is equal to the product of the net force applied to the object multiplied by the time interval over which it is applied. Transforming this to a rotational setting, an object's change in angular momentum is equal to the net torque it experiences multiplied by the time interval over which that net torque is applied:

$\Delta\vec{L} = \vec{\tau}_{net}\,\Delta t$. (Equation 11.4: **Rotational impulse**)

Similarly, we can consider the concept of work in a rotational setting. For straight-line motion, if we meld the work equation with the work-energy theorem we get:

$\Delta K = W_{net} = \vec{F}_{net} \bullet \Delta\vec{r} = F_{net}\,\Delta r \cos\phi$. (Equation 6.8: **Work-kinetic energy theorem**)

In chapter 6, we used the variable θ to represent the angle between the net force $\vec{F}_{net}$ and the displacement $\Delta\vec{r}$. We'll use ϕ here instead because in this chapter we're using θ to represent the angular position of a rotating object.

To find the expression for work in a rotational setting, start with equation 6.8. Replace force $\vec{F}$ by its rotational equivalent, $\vec{\tau}$, and replace displacement $\Delta\vec{r}$ by its rotational equivalent $\Delta\vec{\theta}$. This gives:

$\Delta K = W_{net} = \vec{\tau}_{net} \bullet \Delta\vec{\theta} = \tau_{net}\,\Delta\theta\,\cos\phi$. (Equation 11.5: **Rotational work**)

If the dot product notation confuses you, feel free to ignore it! Because we'll deal only with rotation about one axis (rotation in one dimension), we can make Equation 11.5 simpler:

$\Delta K = W_{net} = \pm\tau_{net}\,\Delta\theta$. (Eq. 11.6: **Work-kinetic energy theorem, for rotation**)

We use the plus sign when the torque is in the same direction as the angular displacement, and the minus sign when the torque is opposite to the direction of the angular displacement.

EXAMPLE 11.9 – Comparing the motions

Note – compare this example to Example 6.3. The methods of analysis in that example and this one are virtually identical. Two objects, *A* and *B*, are initially at rest. The objects have the same mass and radius. Object *A* is a uniform solid disk, while object *B* is a bicycle wheel that can, for this purpose, be considered to be a ring. Each object rotates with no friction about an axis through its center, perpendicular to the plane of the disk/wheel. Identical net torques are then applied to the objects by pulling on strings wrapped around their outer rims. Each net torque is removed once the object it is applied to has accelerated through one complete rotation.

(a) After the net torques are removed which object has more kinetic energy?
(b) After the net torques are removed which object has more speed?
(c) After the net torques are removed, which object has more momentum?

SOLUTION

(a) A diagram of this situation is shown in Figure 11.20. Because the objects start from rest, the angular displacement of each is in the same direction as the net torque (clockwise, in the case shown in Figure 11.20). Because the objects experience equal torques and equal angular displacements the work done on the objects is the same, by Equation 11.6. This means the change in kinetic energy is the same for each, and, because they both start with no kinetic energy, their final kinetic energies are equal.

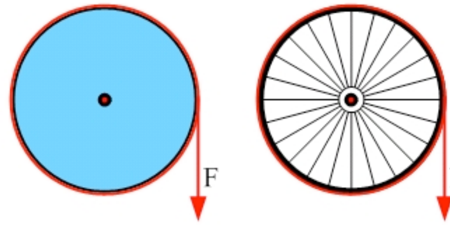


Figure 11.20: Diagrams of the disk and wheel. Each object starts from rest and rotates about an axis perpendicular to the page passing through the center of the object. The force exerted on the string wrapped around the object is removed once the object has accelerated through exactly one revolution.

(b) Unlike Example 6.3, in which the objects had different masses, these objects have the same mass *M* and the same radius *R*. This is a rotational situation, however, so what matters is how their rotational inertias compare. Object *A*, a uniform solid disk rotating about an axis through its center, has a rotational inertia of $I_A = \frac{1}{2}MR^2$. Object B, which we are treating as a ring, has a rotational inertia of $I_B = MR^2$. Thus the relationship between the rotational inertias is $I_A = \frac{1}{2}I_B$. If the objects have the same kinetic energy but *B* has a larger rotational inertia then *A* must have a larger angular speed. Setting the final kinetic energies equal, $K_A = K_B$, gives:

$$\frac{1}{2}I_A\omega_A^2 = \frac{1}{2}I_B\omega_B^2 .$$

Canceling factors of $\frac{1}{2}$ gives: $I_A\omega_A^2 = I_B\omega_B^2$

Bringing in the relationship between the rotational inertias gives: $\frac{1}{2}I_B\omega_A^2 = I_B\omega_B^2$.

This gives $\omega_A = \sqrt{2}\,\omega_B$, so object *A* has a larger angular speed than object *B*.

(c) One way to find the angular momenta is as follows:

$$\vec{L}_A = I_A\vec{\omega}_A = \frac{1}{2}I_B\vec{\omega}_A = \frac{1}{2}I_B\left(\sqrt{2}\,\vec{\omega}_B\right) = \frac{1}{\sqrt{2}}I_B\vec{\omega}_B = \frac{1}{\sqrt{2}}\vec{L}_B .$$

Thus, object *B*, the wheel, has a larger angular momentum than object *A*, the disk. As in Example 6.3, we can understand this result conceptually. The change in angular momentum is the net torque multiplied by the time over which the net torque acts. Both objects experience identical torques, but because *B* has a larger rotational inertia, *B* takes more time to spin through one revolution than *A* does. Because the torque is applied to *B* for a longer time, *B*'s change in angular momentum, and final angular momentum, has a larger magnitude than *A*'s.

Related End-of-Chapter Exercises: 22, 23.

Essential Question 11.9: Return to the situation described in Example 11.9, but now object *B* is replaced by object *C*, a bicycle wheel of the same mass as object A but with a different radius. Once again, we can treat the bicycle wheel as a ring. The situation described in Example 11.9 is repeated, but this time objects *A* and *C* end up with the same rotational kinetic energy and the same angular momentum. How is this possible? Be as quantitative about your answer as you can.

Answer to Essential Question 11.9: If the two objects have the same kinetic energy and angular momentum, they must have the same rotational inertia. This allows us to solve for the radius of object C:

$$I_A = I_C \,; \qquad \frac{1}{2} M\, R_A^2 = M\, R_C^2 \,; \qquad R_C = \frac{1}{\sqrt{2}} R_A \,.$$

Chapter Summary

Essential Idea

Concepts that we found to be powerful for analyzing motion in previous chapters, such as Newton's Second Law, Conservation of Momentum, and Conservation of Energy, are equally powerful for analyzing motion in a rotational setting.

A General Method for Solving a Problem Involving Newton's Second Law for Rotation

1. Draw a diagram of the situation.
2. Draw a free-body diagram showing all the forces acting on the object.
3. Choose a rotational coordinate system. Pick an appropriate axis to take torques about, and apply Newton's Second Law for Rotation ($\sum \vec{\tau} = I\vec{\alpha}$) to obtain a torque equation.
4. Choose an appropriate *x*-*y* coordinate system for forces. Apply Newton's Second Law ($\sum \vec{F} = m\vec{a}$) to obtain one or more force equations. The positive directions for the rotational and *x*-*y* coordinate systems should be consistent with one another.
5. Combine the resulting equations to solve the problem.

Rolling

It can be very helpful to look at rolling as a combination of purely translational motion and purely rotational motion.

Angular Momentum

Angular momentum is a vector, pointing in the direction of angular velocity. The angular momentum of a system can be changed by applying a net torque. If no net torque acts on a system its angular momentum is conserved.

Straight-line motion concept	**Analogous rotational motion concept**	**Connection**
Newton's Second Law, $\sum \vec{F} = m\vec{a}$	Second Law for Rotation, $\sum \vec{\tau} = I\vec{\alpha}$	Same form
Momentum: $\vec{p} = m\vec{v}$	Angular momentum: $\vec{L} = I\,\vec{\omega}$	$L = r\,p\sin\theta$
Translational kinetic energy: $K = \frac{1}{2}mv^2$	Rotational kinetic energy: $K = \frac{1}{2}I\omega^2$	Same form
Impulse: $\vec{F}\,\Delta t = \Delta\vec{p}$	Rotational impulse: $\vec{\tau}\,\Delta t = \Delta\vec{L}$	Same form
Work: $\Delta K = W_{net} = F_{net}\,\Delta r\cos\phi$	Work: $\Delta K = W_{net} = \tau_{net}\,\Delta\theta\,\cos\phi = \pm\tau_{net}\,\Delta\theta$	Same form

Table 11.1: The equations we use in rotational situations are completely analogous to those we use in analyzing motion in one, two, or three dimensions.

End-of-Chapter Exercises

Exercises 1 – 12 are conceptual questions that are designed to see if you have understood the main concepts of the chapter.

1. Figure 11.20 shows four different cases involving a uniform rod of length L and mass M is subjected to two forces of equal magnitude. The rod is free to rotate about an axis that either passes through one end of the rod, as in (a) and (b), or passes through the middle of the rod, as in (c) and (d). The axis is marked by the red and black circle, and is perpendicular to the page in each case. This is an overhead view, and we can neglect any effect of the force of gravity acting on the rod. Rank these four situations based on the magnitude of their angular acceleration, from largest to smallest.

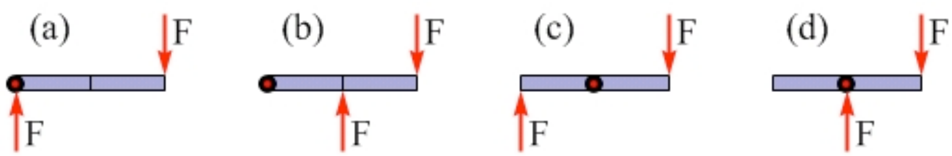


Figure 11.20: Four situations involving a uniform rod that can rotate about an axis being subjected to two forces of equal magnitude. For Exercise 1.

2. A pulley has a mass M, a radius R, and is in the form of a uniform solid disk. The pulley can rotate without friction about a horizontal axis through its center. As shown in Figure 11.21, the string wrapped around the outside edge of the pulley is subjected to an 8.0 N force in case 1, while, in case 2, a block with a weight of 8.0 N hangs down from the string. In which case is the angular acceleration of the pulley larger? Briefly justify your answer.

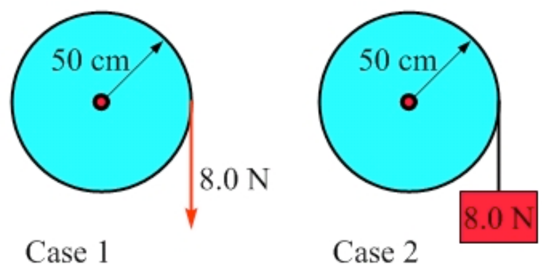


Figure 11.21: A frictionless pulley with a string wrapped around its outer edge. The string is subjected to an 8.0 N force in case 1, while in case 2 a block with a weight of 8.0 N hangs from the end of the string. For Exercise 2.

3. Consider again the spool shown in Figure 11.22, which we examined in Essential Question 11.2. In Essential Question 11.2, the spool rolled without slipping when a force to the right was exerted on the end of the string, but in this case let's say there is no friction between the spool and the horizontal surface. (a) Does the spool move? If so, which way does it move? (b) Does the spool rotate? If so, which way does it rotate?

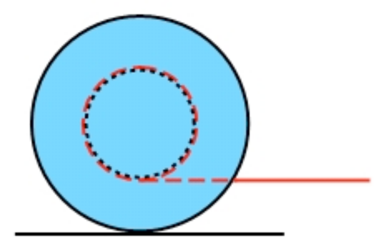


Figure 11.22: A spool with a ribbon wrapped around its axle. A force directed to the right is applied to the end of the ribbon. For Exercise 3.

4. In Exploration 11.2, we looked at how rolling motion can be viewed as a superposition of two simpler motions, pure translation and pure rotation. Have we done this before, broken down a more complicated motion into two simpler motions? If so, in what sort of situation? Comment on the similarities and differences between what we did previously and what we're doing in this chapter, for rolling.

5. A uniform solid cylinder rolls without slipping at constant velocity across a horizontal surface. In Exploration 11.3, we looked at how the net velocity of any point on such a rolling object can be determined. Is there any point on the cylinder with a net velocity directed in exactly the opposite direction as the cylinder's translational velocity? Briefly justify your answer.

6. You take a photograph of a bicycle race. Later, when you get home and look at the photo, you notice that some parts of each bicycle wheel in your photo are blurred, while others are not, or not as badly blurred. Compare the sharpness of the center of a wheel, the top of a wheel, and the bottom of a wheel. (a) Which point do you expect to be the most blurred? Why? (b) Which point do you expect to be in the sharpest focus? Why?

7. You have a race between two objects that have the same mass and radius by rolling them, without slipping, up a ramp. One object is a uniform solid sphere while the other is a ring. (a) Sketch a free-body diagram for one of the objects as it rolls without slipping up the ramp. (b) If the two objects have the same velocity at the bottom of the ramp, which object rolls farther up the ramp before turning around? Briefly justify your answer.

8. Repeat part (b) of the previous exercise, but this time the objects have the same total kinetic energy at the bottom of the ramp.

9. A figure skater is whirling around with her arms held out from her body. (a) What happens to her angular speed when she pulls her arms in close to her body? Why? (b) What happens to the skater's kinetic energy in this process? Explain your answer.

10. A solid cylinder is released from rest at the top of the ramp, and the cylinder rolls without slipping down the ramp. Defining the zero for gravitational potential energy to be at the level of the bottom of the ramp, the cylinder has a gravitational potential energy of 24 J when it is released. Draw a set of energy bar graphs to represent the cylinder's gravitational potential energy, translational kinetic energy, rotational kinetic energy, and total mechanical energy when the cylinder is (a) at the top of the ramp; (b) halfway down the ramp; and (c) at the bottom of the ramp.

11. Repeat Exercise 10, replacing the solid cylinder by a ring.

12. You have a race between a uniform solid sphere and a basketball, by releasing both objects from rest at the top of an incline. Which object reaches the bottom of the ramp first, assuming they both roll without slipping? Justify your answer.

Exercises 13 – 16 are designed to give you practice solving typical Newton's second law for rotation problems. For each exercise start with the following steps: (a) draw a diagram; (b) draw one or more free-body diagrams, as appropriate; (c) choose an appropriate rotational coordinate system and apply Newton's second law for rotation; (d) apply Newton's second law.

13. A block with a mass of 500 g is at rest on a frictionless table. A horizontal string tied to the block passes over a pulley mounted on the edge of the table, and the end of the string hangs down vertically below the pulley. The pulley is a uniform solid disk with a mass of 2.0 kg and a radius of 20 cm that rotates with no friction about an axis through its center. You then exert a constant force of 4.0 N down on the end of the string. The goal is to determine the acceleration of the block. Use g = 10 m/s^2. Parts (a) – (d) as described above. (e) Find the block's acceleration.

14. Repeat the previous exercise, but now there is some friction between the block and the table. The coefficient of static friction is $\mu_S = 0.40$, while the coefficient of kinetic friction is $\mu_K = 0.30$.

15. A 2.0-m long board is placed with one end on the floor and the other resting on a box that has a height of 30 cm. A uniform solid sphere is released from rest from the higher end of this board and rolls without slipping to the lower end. The goal is to determine how long the sphere takes to move from one end of the board to the other. Parts (a) – (d) as described above. (e) Combine your equations to determine the sphere's acceleration. (f) How long does it take the sphere to reach the lower end of the board?

16. A uniform solid cylinder with a mass of 2.0 kg rests on its side on a horizontal surface. A ribbon is wrapped around the outside of the cylinder with the end of the ribbon coming away from the cylinder horizontally from its highest point. When you exert a constant force of 4.0 N on the cylinder, the cylinder rolls without slipping in the direction of the force. The goal of this exercise is to determine the cylinder's acceleration. Parts (a) – (d) as described above. (e) Find the magnitude and direction of the force of friction exerted on the cylinder. (f) Find the cylinder's acceleration.

Exercises 17 – 21 deal with rolling situations.

17. A particular wheel has a radius of 50 cm. It rolls without slipping exactly half a rotation. (a) What is the translational distance moved by the wheel? (b) Considering motion due to the wheel's rotation only, what is the distance traveled by a point on the outer edge of the wheel? (c) Consider now the magnitude of the total displacement experienced by the point on the outer edge of the wheel that started at the top of the wheel. Is this equal to the sum of your two answers from (a) and (b)? Explain why or why not. (d) Work out the magnitude of the displacement of the point referred to in (c).

18. One end of a 2.0-m long board rests on a cylinder that has been placed on its side on a horizontal surface. You hold the other end of the board so the board is horizontal. When you walk forward, the cylinder rolls so the board does not slip on the cylinder, and the cylinder also rolls without slipping across the floor. When you move forward 1.0 m, how far does the cylinder move?

19. A particular point on a wheel is halfway between the center and the outer edge. When the point is at the same distance from the ground as the center of the wheel, the point's speed is 25.2 m/s. If the wheel has a radius of 35.0 cm and is rolling without slipping, find the translational speed of the wheel.

20. A solid sphere is released from rest and rolls without slipping down a ramp inclined at 12° to the horizontal. What is the sphere's speed when it is 1.0 m (measuring vertically) below the level it started?

21. Return to the situation described in Exercise 20, but this time the incline is changed to 6°. The sphere again rolls without slipping starting from rest. Comparing the sphere's speed in both cases when it is 1.0 m (measuring vertically) below its starting point, in which case is the sphere moving faster? Rather then doing another calculation, see if you can come up with a conceptual argument to justify your answer.

Exercises 22 – 31 are modeled after similar exercises in previous chapters. Note the similarities between how we analyze rotational situations and how we analyze straight-line motion situations.

22. Two uniform solid disks, *A* and *B*, are initially at rest. The mass of disk *B* is two times larger than that of disk *A*. Identical net torques are then applied to the two disks, giving them each an angular acceleration as they rotate about their centers. Each net torque is removed once the object it is applied to has rotated through two revolutions. After both net torques are removed, how do: (a) the kinetic energies compare? (b) the angular speeds compare? (c) the angular momenta compare? (Compare this to Exercise 9 in Chapter 6.)

23. Repeat Exercise 22, assuming that both net torques are removed after the same amount of time instead. (Compare this to Exercise 10 in Chapter 6.)

24. Two identical grinding wheels of mass *m* and radius *r* are spinning about their centers. Wheel A has an initial angular speed of ω, while wheel B has an initial angular speed of 2ω. Both wheels are being used to sharpen tools. As shown in Figure 11.23, in both cases the tool is being pressed against the wheel with a force *F* directed toward the center of the wheel, and the coefficient of kinetic friction between the wheel and the tool is μ_K. The tool does not move from the position shown in the diagram. (a) If it takes wheel A a time *T* to come to a stop, how long does it take for wheel B to come to a stop? (b) Find an expression for *T* in terms of the variables specified in the exercise. (c) If wheel A rotates through an angle θ before coming to rest, through what angle does wheel B rotate before coming to rest? (d) Find an expression for θ in terms of the variables specified in the exercise. (Compare this to Exercise 52 in Chapter 6.)

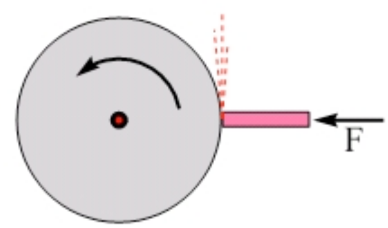


Figure 11.23: Sharpening a tool by holding it against a grinding wheel, for Exercises 24 – 26.

25. Return to the situation described in Exercise 24. How would *T*, the stopping time for wheel A, change if (a) *m* was doubled? (b) ω was doubled? (c) μ_K was doubled? (Compare this to Exercise 53 in Chapter 6.)

26. Return to the situation described in Exercise 24. How would θ, the angle wheel A rotates through before stopping, change if (a) *m* was doubled? (b) ω was doubled? (c) μ_K was doubled? (Compare this to Exercise 54 in Chapter 6.)

27. You pick up a bicycle wheel, with a mass of 800 grams and a radius of 40 cm, and spin it so the wheel rotates about its center. Assume that the mass of the wheel is concentrated in the rim. The initial angular speed is 5.0 rad/s, but after 10 s the angular speed is 3.0 rad/s. The goal here is to determine the magnitude of the frictional torque acting to slow the wheel, assuming it to be constant. (a) Sketch a diagram of the situation. (b) Choose a positive direction, and show this on the diagram. (c) Draw a free-body diagram of the wheel, focusing on the torque(s) acting on the wheel. (d) Write an expression for the net torque acting on the wheel. (e) Write an expression representing the wheel's change in angular momentum over the 10-second period. (f) Use the equation $\vec{\tau}\,\Delta t = \Delta\vec{L}$ to relate the expressions you wrote down in parts (d) and (e). (g) Solve for the frictional torque acting on the wheel. (Compare this to Exercise 23 in Chapter 6.)

28. At a time $t = 0$, a bicycle wheel with a mass of 4.00 kg has an angular velocity of 5.00 rad/s directed clockwise. For the next 8.00 seconds it then experiences a net torque, as shown in the graph in Figure 11.24 (taking clockwise to be positive). The wheel rotates about its center, and we can treat the wheel as a ring with a radius of $\frac{1}{\sqrt{2}}$ m. (a) Sketch a graph of the wheel's angular momentum as a function of time. (b) What is the cart's maximum angular speed during the 8.00-second interval the varying torque is being applied? At what time does the cart reach this maximum speed? (c) What is the cart's minimum angular speed during the 8.00-second interval the varying torque is being applied? At what time does the cart reach this minimum speed? (Compare this to Exercise 24 in Chapter 6.)

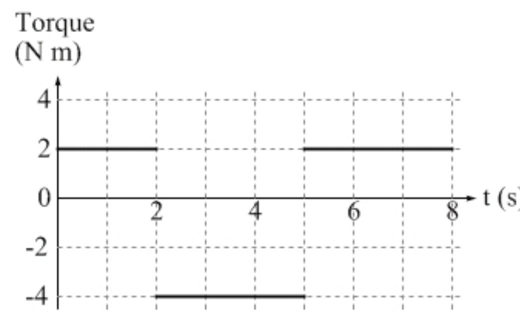


Figure 11.24: A plot of the torque applied to a bicycle wheel as a function of time, for Exercise 28.

29. Two blocks are connected by a string that passes over a frictionless pulley, as shown in Figure 11.25. Block A, with a mass $m_A = 2.0$ kg, rests on a ramp measuring 3.0 m vertically and 4.0 m horizontally. Block B hangs vertically below the pulley. The pulley has a mass of 1.0 kg, and can be treated as a uniform solid disk that rotates about its center. Note that you can solve this exercise entirely using forces and the constant-acceleration equations, but see if you can apply energy ideas instead. Use $g = 10$ m/s^2. When the system is released from rest, block A accelerates up the slope and block B accelerates straight down. When block B has fallen through a height $h = 2.0$ m, its speed is $v = 6.0$ m/s. (a) At any instant in time, how does the speed of block A compare to that of block B? (b) Assuming there is no friction acting on block A, what is the mass of block B? (Compare this to Exercise 44 in Chapter 7.)

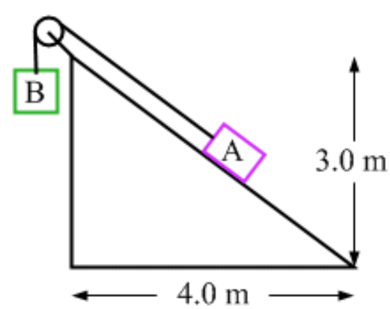


Figure 11.25: Two blocks connected by a string passing over a pulley, for Exercises 29 and 30.

30. Repeat Exercise 29, this time accounting for friction. If the coefficient of kinetic friction for the block A – ramp interaction is 0.625, what is the mass of block B? (Compare this to Exercise 45 in Chapter 7.)

31. A uniform solid sphere of mass m is released from rest at a height h above the base of a loop-the-loop track, as shown in Figure 11.26. The loop has a radius R. What is the minimum value of h necessary for the sphere to make it all the way around the loop without losing contact with the track? Express your answer in terms of R, and assume that the sphere's radius is much smaller than the loop's. (Compare this to Exercise 49 in Chapter 7.)

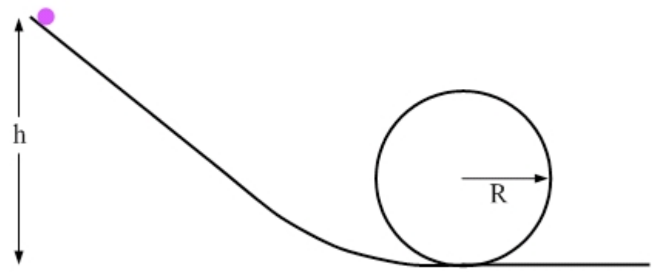


Figure 11.26: A solid sphere released from rest from a height h above the bottom of a loop-the-loop track, for Exercise 31.

Exercises 32 – 34 deal with angular momentum conservation.

32. A beetle with a mass of 20 g is initially at rest on the outer edge of a horizontal turntable that is also initially at rest. The turntable, which is free to rotate with no friction about an axis through its center, has a mass of 80 g and can be treated as a uniform disk. The beetle then starts to walk around the edge of the turntable, traveling at an angular velocity of 0.060 rad/s clockwise **with respect to the turntable**. (a) Qualitatively, what does the turntable do while the beetle is walking? Why? (b) With respect to you, motionless as you watch the beetle and turntable, what is the angular velocity of the beetle? What is the angular velocity of the turntable? (c) If a mark was placed on the turntable at the beetle's starting point, how long does it take the beetle to reach the mark? (d) Upon reaching the mark, the beetle stops. What does the turntable do? Why?

33. A bullet with a mass of 12 g is fired at a wooden rod that hangs vertically down from a pivot point that passes through the upper end of the rod. The bullet embeds itself in the lower end of the rod and the rod/bullet system swings up, reaching a maximum angular displacement of 60˚ from the vertical. The rod has a mass of 300 g, a length of 1.2 m, and we can assume the rod rotates without friction about the pivot point. What is the bullet's speed when it hits the rod? Assume the bullet is traveling horizontally when it hits the rod, and use $g = 10$ m/s^2.

34. A particular horizontal turntable can be modeled as a uniform disk with a mass of 200 g and a radius of 20 cm that rotates without friction about a vertical axis passing through its center. The angular speed of the turntable is 2.0 rad/s. A ball of clay, with a mass of 40 g, is dropped from a height of 35 cm above the turntable. It hits the turntable at a distance of 15 cm from the middle, and sticks where it hits. Assuming the turntable is firmly supported by its axle so it remains horizontal at all times, find the final angular speed of the turntable-clay system.

Use conservation of energy to solve Exercises 35 – 38. For each exercise begin by (a) writing down the energy conservation equation and choosing a zero level for gravitational potential energy; (b) identifying the terms that are zero and eliminating them; (c) writing out expressions for the remaining terms, remembering to account for both translational kinetic energy and rotational kinetic energy.

35. A uniform solid disk is released from rest at the top of a ramp, and rolls without slipping down the ramp. The goal of the exercise is to determine the disk's speed when it reaches a level 50 cm below (measured vertically) its starting point. Parts (a) – (c) as described above. (d) What is that speed?

36. The pulley shown in Figure 11.27 has a mass $M = 2.0$ kg and radius $R = 50$ cm, and can be treated as a uniform solid disk that can rotate about its center. The block (which has a mass of 800 g) hanging from the string wrapped around the pulley is then released from rest. The goal of the exercise is to determine the speed of the block when it has dropped 1.0 m. Parts (a) – (c) as described above. (d) What is the block's speed after dropping through 1.0 m?

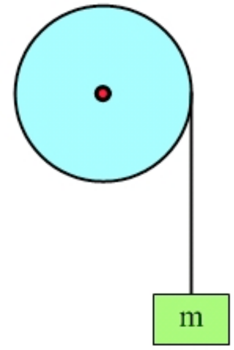


Figure 11.27: A block and a pulley, for Exercise 36.

37. A uniform solid sphere with a mass $M = 1.0$ kg and radius $R = 40$ cm is mounted on a frictionless vertical axle that passes through the center of the sphere. The sphere is initially at rest. You then pull on a string wrapped around the sphere's equator, exerting a constant force of 5.0 N. The string unwraps from the sphere when you have moved the end of the string through a distance of 2.0 m. The goal of the exercise is to determine the resulting angular speed of the sphere. Parts (a) – (c) as described above. (d) What is the resulting angular speed?

38. A uniform solid sphere with a mass of $M = 1.6$ kg and radius $R = 20$ cm is rolling without slipping on a horizontal surface at a constant speed of 2.1 m/s. It then encounters a ramp inclined at an angle of 10° with the horizontal, and proceeds to roll without slipping up the ramp. The goal of this exercise is to determine the distance the sphere rolls up the ramp (measured along the ramp) before it turns around. Parts (a) – (c) as described above. (d) How far does the sphere roll up the ramp? (e) Which of the values given in this exercise did you not need to find the solution?

General Problems and Conceptual Questions.

39. Figure 11.28 shows four different cases involving a uniform rod of length L and mass M is subjected to two forces of equal magnitude. The rod is free to rotate about an axis that either passes through one end of the rod, as in (a) and (b), or passes through the middle of the rod, as in (c) and (d). The axis is marked by the red and black circle, and is perpendicular to the page in each case. This is an overhead view, and we can neglect any effect of the force of gravity acting on the rod. If the rod has a length of 1.0 m, a mass of 3.0 kg, and each force has a magnitude of 5.0 N, determine the magnitude and direction of the angular acceleration of the rod in (a) Case (a); (b) Case (b); (c) Case (c); (d) Case (d).

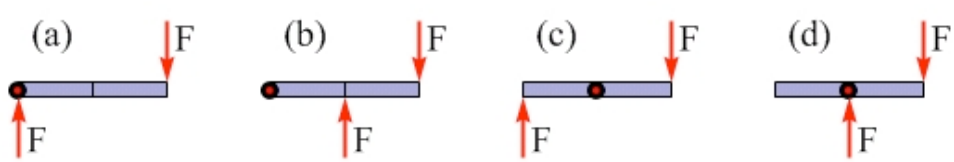


Figure 11.28: Four situations involving a uniform rod that can rotate about an axis being subjected to two forces of equal magnitude. For Exercise 39.

40. A pulley has a mass M, a radius R, and is in the form of a uniform solid disk. The pulley can rotate without friction about a horizontal axis through its center. As shown in Figure 11.29, the string wrapped around the outside edge of the pulley is subjected to an 8.0 N force in case 1, while in case 2 a block with a weight of 8.0 N hangs down from the string. If M = 2.0 kg and R = 50 cm, calculate the angular acceleration of the pulley in (a) case 1; (b) case 2.

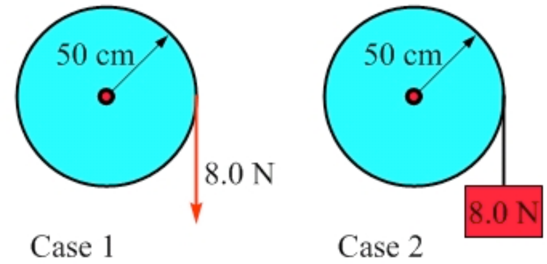


Figure 11.29: A frictionless pulley with a string wrapped around its outer edge. The string is subjected to an 8.0 N force in case 1, while in case 2 a block with a weight of 8.0 N hangs from the end of the string. For Exercise 40.

41. Atwood's machine is a system consisting of two blocks that have different masses connected by a string that passes over a frictionless pulley, as shown in Figure 11.30. The pulley has a mass m_p. Compare the tension in the part of the string just above the block with the larger mass, M, to that in the part of the string just above the block with the smaller mass, m, in the following cases: (a) You hold on to the smaller-mass block to keep the system at rest; (b) The system is released from rest; (c) You hold on to the smaller-mass block and pull down so the blocks move with constant velocity. Justify your answers in each case.

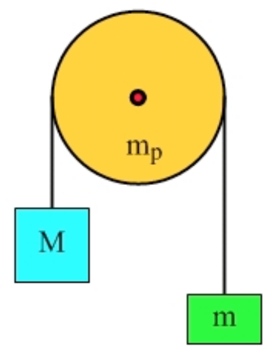


Figure 11.30: Atwood's machine – a device consisting of two objects connected by a string that passes over a pulley. For Exercises 41 – 43.

42. Atwood's machine is a system consisting of two objects connected by a string that passes over a frictionless pulley, as shown in Figure 11.30. In Chapter 5, we neglected the effect of the pulley, but now we know how to account for the pulley's impact on the system. (a) If the two objects have masses of M and m, with $M > m$, and the pulley is in the shape of a uniform solid disk and has a mass m_p, derive an expression for the acceleration of either block, in terms of the given masses and g. (b) What does the expression reduce to in the limit where the mass of the pulley approaches zero? (c) How does accounting for the fact that the pulley has a non-zero mass affect the magnitude of the acceleration of a block?

43. Consider again the Atwood's machine described in Exercise 42 and pictured in Figure 11.30. (a) If M = 500 g, m = 300 g, and the pulley mass is m_p = 400 g, what is the magnitude of the acceleration of one of the blocks? (b) If the system is released from rest, what is the angular velocity of the pulley 2.0 seconds after the motion begins? The pulley has a radius of 10 cm.

44. A particular double-pulley consists of a small pulley of radius 20 cm mounted on a large pulley of radius 50 cm, as shown in Figure 11.31. A block of mass 2.0 kg hangs from a string wrapped around the large pulley. To keep the system at rest, what mass block should be hung from the small pulley?

45. A particular double pulley consists of a small pulley of radius 20 cm mounted on a large pulley of radius 50 cm, as shown in Figure 11.31. The pulleys rotate together, rather than independently. A block of mass 2.0 kg hangs from a string wrapped around the large pulley, while a second block of mass 2.0 kg hangs from the small pulley. Each pulley has a mass of 1.0 kg and is in the form of a uniform solid disk. Use $g = 9.8$ m/s^2. (a) What is the acceleration of the block attached to the large pulley? (b) What is the acceleration of the block attached to the small pulley?

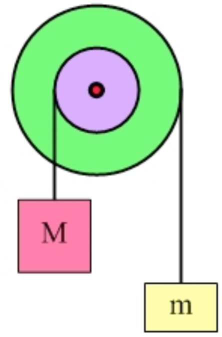


Figure 11.31: A double-pulley system, with a 2.0 kg block hung from the string wrapped around the large pulley and a second block hung from the string wrapped around the small pulley. For Exercises 44 – 46.

46. A pulley consists of a small uniform disk of radius 0.50 m mounted on a larger uniform disk of radius 1.0 m. Each disk has a mass of 1.0 kg. The pulley can rotate without friction about an axis through its center. As shown in Figure 11.31, a block with mass m = 1.0 kg hangs down from the larger disk while a block of mass M hangs down from the smaller disk. If the angular acceleration of the system has a magnitude of 1.0 rad/s^2 what is the value of M? Consider all possible answers, and use $g = 10$ m/s^2.

47. A yo-yo consists of two identical disks, each with a mass of 40 g and a radius of 4.0 cm, joined by a small cylindrical axle with negligible mass and a radius of 1.0 cm. When the yo-yo is released it essentially rolls without slipping down the string wrapped around the axle. If the end of the string (the one you would hold) remains fixed in place, determine the acceleration of the yo-yo. Use $g = 9.8$ m/s^2.

48. As shown in Figure 11.32, blocks A and B are connected by a massless string that passes over the outer edge of a pulley that is a uniform solid disk. The mass of block A is equal to that of block B; the mass of the pulley, coincidentally, is also the same as that of block A. When the system is released from rest it experiences a constant (and non-zero) acceleration. There is no friction between block A and the surface. Use $g = 10.0$ m/s^2. (a) What is the acceleration of the system? (b) The two parts of the string have different tensions. In which part of the string is the tension larger, between block A and the pulley or between the pulley and block B? Briefly justify your answer. (c) If the tension in one part of the string is 3.00 N larger than the tension in the other part, what are the values of the tensions in the two parts of the string?

49. Repeat Exercise 48, but this time there is some friction between block A and the surface, with a coefficient of kinetic friction of 0.500.

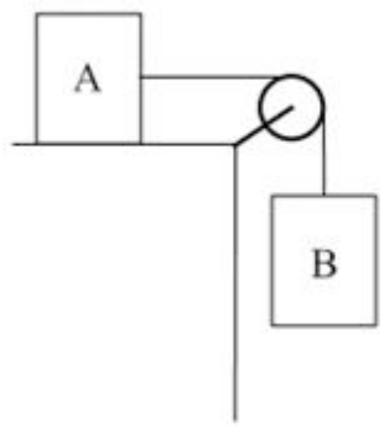


Figure 11.32: Two blocks connected by a string passing over a pulley, for Exercises 48 and 49.

50. A spool has a string wrapped around its axle, with the string coming away from the underside of the spool. The spool is on a ramp inclined at 20˚ with the horizontal, as shown in Figure 11.33. There is no friction between the spool and the ramp. Assuming you can exert as much or as little force on the end of the string as you wish (always directed up the slope) which of the following situations are possible? If so, explain how the situation could be achieved; if not, explain why not. (a) The spool remains completely motionless. (b) The spool rotates about its center but does not move up or down the ramp. (c) The spool has no rotation but moves down the ramp.

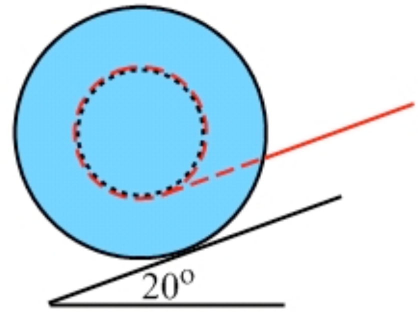


Figure 11.33: A spool on a ramp inclined at 20˚ with the horizontal. The string wrapped around the spool's axle comes away from the spool on its underside. Any force exerted on the end of the string is exerted up the slope. For Exercises 50 and 51.

51. Return to the situation described in Exercise 50 and shown in Figure 11.33, but now there is friction between the spool and the ramp. Is it possible for the spool to remain completely motionless now? If so, explain how. If not, explain why not.

52. Consider the spool shown in Figure 11.34. The spool has a radius R, while the spool's axle has a radius of $R/2$. There is some friction between the spool and the horizontal surface it is on, so that when a modest tension is exerted on the string the spool may roll one way or the other without slipping. It turns out that when the angle θ between the string and the vertical is larger than some critical value θ_C, the spool rolls without slipping one way; when $\theta < \theta_C$, the spool rolls the other way, and when $\theta = \theta_C$, the spool remains at rest. (a) Find θ_C. (b) Which way does the spool roll when $\theta < \theta_C$?

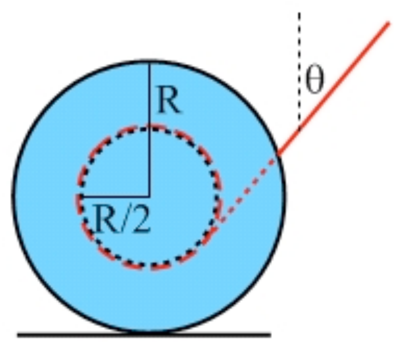


Figure 11.34: A spool on a horizontal surface, for Exercise 52.

53. A spool consists of two disks, each of radius R and mass M, connected by a cylindrical axle of radius $R/2$ and mass M. When an upward force of magnitude F is exerted on a string wrapped around the axle, the spool will roll without slipping as long as F is not too large. (a) What is the spool's rotational inertia, in terms of M and R, about an axis through its center? (b) If the coefficient of static friction between the spool and the horizontal surface it is on is 0.50, what is the maximum value F can be, in terms of M and g, for the spool to roll without slipping?

54. A solid sphere rolls without slipping when it is released from rest at the top of a ramp that is inclined at 30˚ with respect to the horizontal, but, if the angle exceeds 30˚, the sphere slips as it rolls. Calculate the coefficient of static friction between the sphere and the incline.

55. While fixing your bicycle, you remove the front wheel from the frame. A bicycle wheel can be approximated as a ring, with all the mass of the wheel concentrated on the wheel's outer edge. The wheel has a mass M, a radius R, and it is initially spinning at a particular angular velocity. There is a constant frictional torque that is causing the wheel to slow down, however. You also have a uniform solid disk of the same mass and radius as the bicycle wheel. It also has the same initial angular velocity and the same frictional torque as the wheel. Which of these objects will spin for the longer time? Justify your answer.

56. As shown in Figure 11.35, a bowling ball of mass M and radius R = 20.0 cm is released with an initial translational velocity of $\vec{v}_0$ = 14.0 m/s to the right and an initial angular velocity of $\vec{\omega}_0 = 0$. The bowling ball can be treated as a uniform solid sphere. The coefficient of kinetic friction between the ball and the surface is $\mu_K = 0.200$. The force of kinetic friction causes a linear acceleration, as well as a torque that causes the ball to spin. The ball slides along the horizontal surface for some time, and then rolls without slipping at constant velocity after that. **Use g = 10 m/s²**. (a) Draw the free-body diagram of the ball showing all the forces acting on it while it is sliding. (b) What is the acceleration of the ball while it is sliding? (c) What is the angular acceleration of the ball while it is sliding? (d) How far does the ball travel while it is sliding? (e) What is the constant speed of the ball when it rolls without slipping?

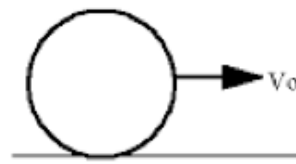


Figure 11.35: The initial state of a bowling ball, for Exercise 56.

57. Figure 11.36 shows the side view of a meter stick that can rotate without friction about an axis passing through one end. Pennies (of negligible mass in comparison to the mass of the meter stick) have been placed on the meter stick at regular intervals. When the meter stick is released from rest, it rotates about the axis. Some of the pennies remain in contact with the meter stick while some lose contact with it. (a) Which pennies do you expect to lose contact with the meter stick, the ones close to the axis or the ones farther from it? (b) Determine the initial acceleration of the right end of the meter stick, and of the center of the meter stick, to help justify your conclusion in (a).

Figure 11.36: A side view of a meter stick, with pennies resting on it at regular intervals, that is initially held horizontal. The meter stick can rotate without friction about an axis through the left end. For Exercises 57 and 58.

58. Return to the situation described in Exercise 57 and shown in Figure 11.36. Assuming the axis is at the 0-cm mark of the meter stick, determine the point on the meter stick beyond which the pennies will lose contact with the meter stick when the system is released from rest.

59. A particularly large playground merry-go-round is essentially a uniform solid disk of mass 4*M* and radius *R* that can rotate with no friction about a central axis. You, with a mass *M*, are a distance of *R*/2 from the center of the merry-go-round, rotating together with it at an angular velocity of 2.4 rad/s clockwise (when viewed from above). You then walk to the outside of the merry-go-round so you are a distance *R* from the center, still rotating with the merry-go-round. Consider you and the merry-go-round to be one system. (a) When you walk to the outside of the merry-go-round, does the angular momentum of the system increase, decrease, or stay the same? Why? (b) Does the kinetic energy of the system increase, decrease, or stay the same? Why? (c) If you started running around the outer edge of the merry-go-round, at what angular velocity would you have to run to make the merry-go-round alone come to a complete stop? Specify the magnitude and direction.

60. Consider the following situations. For each, state whether you would apply energy methods, torque/rotational kinematics methods, or either to solve the exercise. You don't need to solve the exercise. (a) Find the final speed of a uniform solid sphere that rolls without slipping down a ramp inclined at 8.0˚ with the horizontal, if the sphere is released from rest and the vertical component of its displacement is 1.0 m. (b) Find the time it takes the sphere in (a) to reach the bottom of the ramp. (c) Find the number of rotations the sphere in (a) makes as it rolls down the ramp, if the sphere's radius is 15 cm.

61. Return to Exercise 60, and this time solve each part.

62. The planet Earth orbits the Sun in an orbit that is roughly circular. Assuming the orbit is exactly circular, which of the following is conserved as the Earth travels along its orbit? (a) Its momentum? (b) Its angular momentum, relative to an axis passing through the center, and perpendicular to the plane, of the orbit? (c) Its translational kinetic energy? (d) Its gravitational potential energy? (e) Its total mechanical energy? For any that are not conserved, explain why.

63. A typical comet orbit ranges from relatively close to the Sun to many times farther than the Sun. Which of the following is conserved as the comet travels along its orbit? (a) Its angular momentum, relative to an axis passing through the Sun, and perpendicular to the plane, of the orbit? (b) Its translational kinetic energy? (c) Its gravitational potential energy? (d) Its total mechanical energy? For any that are not conserved, explain why.

64. Two of your classmates, Alex and Shaun, are carrying on a conversation about a physics problem. Comment on each of their statements.

    Alex: In this situation, we have to draw the free-body diagram for a sphere that is rolling without slipping up an incline. OK, so, there's a force of gravity acting down, and a normal force perpendicular to the surface. Is there a friction force?

    Shaun: Don't we need another force directed up the incline, in the direction of motion?

    Alex: I don't think so. I think we just need to add a kinetic friction force down the incline, opposing the motion.

    Shaun: Wait a second – isn't it static friction? Isn't it always static friction when something rolls without slipping?

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