9-1 The Buoyant Force

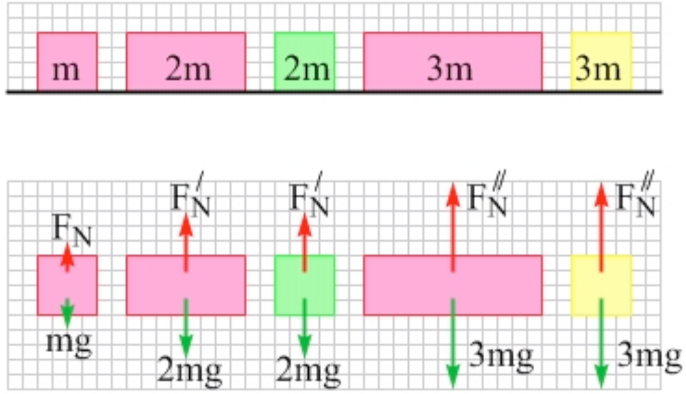


Figure 9.1: A diagram of the blocks we will place in a beaker of water, and the free-body diagram for each block as it sits on a table.

We should begin by defining what a fluid is. Many people think of a fluid as a liquid, but **a fluid is anything that can flow**. By this definition, a fluid can be a liquid or a gas. Flowing fluids can be rather complicated, so let's start with static fluids – fluids that are at rest.

Let's consider some experiments involving various blocks that float in a container of water. The blocks are represented in Figure 9.1, which shows how the masses of the blocks compare, and also shows the free-body diagrams of the blocks as they sit in equilibrium on a table. Starting from the left, the first, second, and fourth blocks are all made from the same material. The other two blocks are both made from different material.

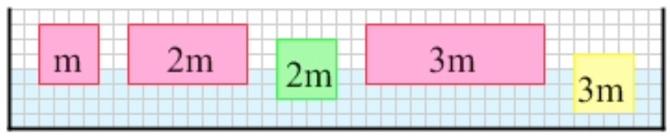
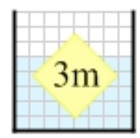


Figure 9.2: A diagram of the blocks floating in the beaker of water.

Our first goal is to look at the similarities between the normal force (a force arising from contact between solid objects) and the force arising from the interaction between an object and a fluid that the object is completely or partly submerged in. Figure 9.2 illustrates how the blocks float when they are placed in the container of water. We have taken some liberties here, because in reality some of the blocks would tilt 45˚ and float as shown in Figure 9.3. Neglecting this rotation simplifies the analysis without affecting the conclusions.

Figure 9.3: We will ignore the fact that blocks that are submerged more than 50% tend to float rotated by 45˚ from the way they are drawn in Figure 9.2. Neglecting this fact will simplify the analysis without affecting the conclusions.



EXPLORATION 9.1 – Free-body diagrams for floating objects

Sketch the free-body diagram of the blocks in Figure 9.2 as they float in the container of water. Note that each block is in equilibrium – what does that imply about the net force acting on each block? Because each block is in equilibrium, the net force acting on each block must be zero.

What forces act on each block? As usual there is a downward force of gravity. Because each block is in equilibrium, however, the net force acting on each block is zero. For now, let's keep things simple and show, on each block, one upward force that balances the force of gravity. The free-body diagrams are shown in Figure 9.4. Note that there is no normal force, because the blocks are not in contact with a solid object. Instead, they are supported by the fluid. We call the upward force applied by a fluid to an object in that fluid **the buoyant force**, which we symbolize as $\vec{F}_B$.

Because the objects are only in contact with the fluid, the fluid must be applying the upward buoyant force to each block. Compare the free-body diagrams in Figure 9.1, when the blocks are in equilibrium on the table, with the free-body diagrams in Figure 9.4, when the blocks

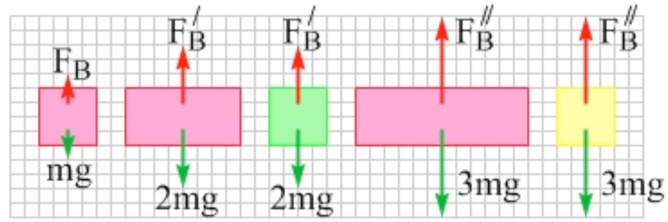


Figure 9.4: Free-body diagrams for the blocks floating in equilibrium in the beaker of water. F_B represents the buoyant force, an upward force applied on each block by the fluid.

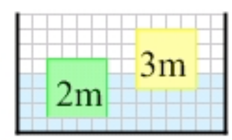
are in equilibrium while floating in the fluid. For a floating object, at least, there are a lot of similarities between the buoyant force exerted by a fluid and the normal force exerted by a solid surface.

Examine Figures 9.2 and 9.4 closely. Even though the two blocks of mass 2*m* are immersed to different levels in the fluid, they displace the same volume of fluid, so they experience equal buoyant forces. The 3*m* blocks displace 50% more volume than do the blocks of mass 2*m*, and they experience a buoyant force that is 50% larger. The block of mass *m*, on the other hand, displaces half the volume of fluid that the blocks of mass 2*m* do, and experiences a buoyant force that is half as large. We can conclude that ***the buoyant force exerted on an object by a fluid is proportional to V_{disp}, the volume of fluid displaced by that object***. We can express this as an equation (where $\propto$ means "is proportional to"),

$$F_B \propto V_{disp} .$$ (Eq. 9.1: **Buoyant force is proportional to volume of fluid displaced**)

Key idea about the buoyant force: An object in a fluid experiences a net upward force we call the buoyant force, $\vec{F}_B$. The magnitude of the buoyant force is proportional to the volume of fluid displaced by the object. **Related End-of-Chapter Exercise: 2.**

The conclusion above is supported by the fact that if we push a block farther down into the water and let go, the block bobs up. The buoyant force increases when we push the block down because the volume of fluid displaced increases, so, when we let go, the block experiences a net upward force. Conversely, when a block is raised, it displaces less fluid, reducing the buoyant force and giving rise to a net downward force when we let go. Figure 9.5 shows these situations and the corresponding free-body diagrams.



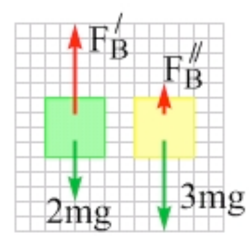


Figure 9.5: In this case, the blocks are not at equilibrium. The block on the left has been pushed down into the water and released. Because it displaces more water than it does at equilibrium, the buoyant force applied to it by the water is larger than the force of gravity applied to it by the Earth and it experiences a net upward force. The reverse is true for the block on the right, which has been lifted up and released. Displacing less water causes the buoyant force to decrease, giving rise to a net downward force.

Figure 9.6: To be able to float, this large ship needs to displace a very large volume of fluid. This large volume of fluid is displaced by the part of the ship that is below the water surface, and which, therefore, is not visible to us in this photograph. Photo credit: by Peter Griffin, from http://www.publicdomainpictures.net..

Essential Question 9.1: Two objects float in equilibrium in the same fluid. Object A displaces more fluid than object B. Which object has a larger mass?

Answer to Essential Question 9.1: Object A. When an object floats in equilibrium, the buoyant force exactly balances the force of gravity. Object A displaces more fluid, so it experiences a larger buoyant force. This must be because object A weighs more than object B.

9-2 Using Force Methods with Fluids

EXAMPLE 9.2 – A block on a string

A block of weight $mg = 45$ N has part of its volume submerged in a beaker of water. The block is partially supported by a string of fixed length that is tied to a support above the beaker. When 80% of the block's volume is submerged, the tension in the string is 5.0 N.

(a) What is the magnitude of the buoyant force acting on the block?

(b) Water is steadily removed from the beaker, causing the block to become less submerged. The string breaks when its tension exceeds 35 N. What percent of the block's volume is submerged at the moment the string breaks?

(c) After the string breaks, the block comes to a new equilibrium position in the beaker. At equilibrium, what percent of the block's volume is submerged?

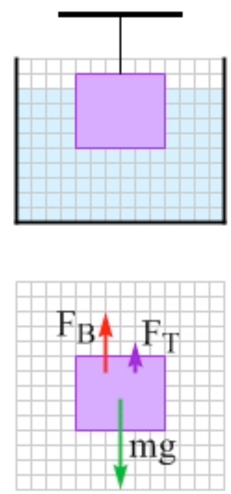


Figure 9.7: A diagram and a free-body diagram for the 45 N block floating in the beaker of water while partly supported by a string.

SOLUTION

As usual, we should begin with a diagram of the situation. A free-body diagram is also very helpful. These are shown in Figure 9.7.

(a) On the block's free-body diagram, we draw a downward force of gravity, applied by the Earth. We also draw an upward force of tension (applied by the string), and, because the block displaces some fluid, an upward buoyant force (applied by the fluid). The block is in equilibrium, so there must be no net force acting on the block.

Taking up to be positive, applying Newton's Second Law gives:
$\sum \vec{F} = 0$.

Evaluating the left-hand side with the aid of the free-body diagram gives:
$+F_T + F_B - mg = 0$.

Solving for the buoyant force gives: $F_B = mg - F_T = +45 \text{ N} - 5.0 \text{ N} = +40 \text{ N}$.

(b) As shown in Figure 9.8, removing water from the beaker causes the block to displace less fluid, so the magnitude of the buoyant force decreases. The magnitude of the tension increases to compensate for this. Applying Newton's Second Law again gives us essentially the same equation as in part (a). We can use this to find the new buoyant force, $F_B{}'$. Just before the string breaks we have:

$F_B{}' = mg - F_T' = +45 \text{ N} - 35 \text{ N} = +10 \text{ N}$.

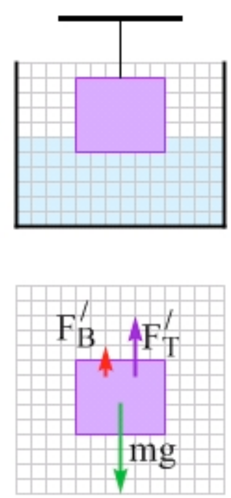


Figure 9.8: A diagram and free-body diagram of the situation just before the string breaks.

Now, we can apply the idea that the buoyant force is proportional to the volume of fluid displaced. If a buoyant force of 40 N corresponds to a displaced volume equal to 80% of the block's volume, a buoyant force of 10 N (1/4 of the original force) must correspond to a displaced volume equal to 20% of the block's volume (1/4 of the original displaced volume).

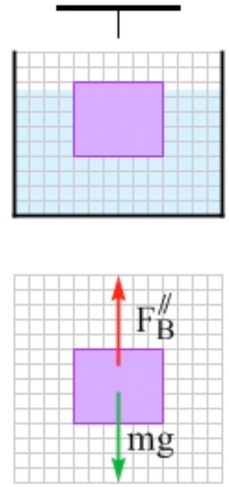


Figure 9.9: A diagram and free-body diagram for the situation after the string breaks, when the block has come to a new equilibrium position in the beaker.

(c) After the string breaks and the block comes to a new equilibrium position, we have a simpler free-body diagram, as shown in Figure 9.9. The buoyant force now, F_B'', applied to the block by the fluid, must balance the force of gravity applied to the block by the Earth. This comes from applying Newton's Second Law:

$$\sum \vec{F} = 0 .$$

Taking up to be positive, evaluating the left-hand side with the aid of the free-body diagram gives:

$$F_B'' - mg = 0 \text{, so } F_B'' = mg = 45 \text{ N} .$$

Using the same logic as in (b), if a buoyant force of 40 N corresponds to a displaced volume equal to 80% of the block's volume, a buoyant force of 45 N must correspond to a displaced volume equal to 90% of the block's volume.

Related End-of-Chapter Exercises: 21, 36.

Let's now extend our analysis to objects that sink. First, hang a block from a spring scale (a device that measures force) to measure the force of gravity acting on the block. With the block hanging from the spring scale, the scale reads 10 N, so there is a 10 N force of gravity acting on the block. A diagram and two free-body diagrams (one for the spring scale and one for the block) are shown in Figure 9.10.

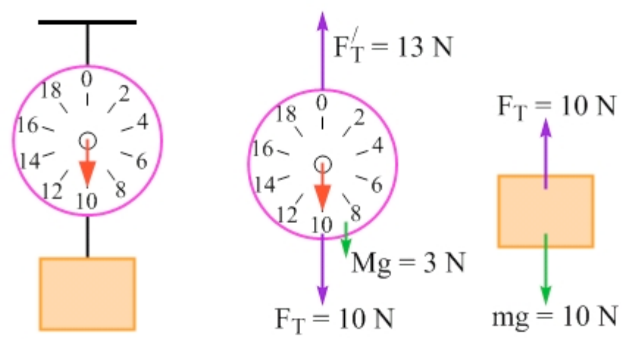


Figure 9.10: A diagram showing a block hanging from a spring scale, as well as free-body diagrams for the spring scale (which itself has a force of gravity of 3 N acting on it) and the block.

Question: With the block still suspended from the spring scale, let's dip the block into a beaker of water until it is exactly half submerged. Make a prediction. As we lower the block into the water, will the reading on the spring scale increase, decrease, or stay the same? Briefly justify your prediction.

Answer: The reading on the spring scale should decrease. This is because the spring scale no longer has to support the entire weight of the block. The more the block is submerged, the larger the buoyant force, and the smaller the spring-scale reading.

Essential Question 9.2: The spring scale reads 10 N when the block is out of the water. Let's say it reads 6.0 N when exactly 50% of the block's volume is below the water surface. What will the scale read when the entire block is below the water surface? Why?

Answer to Essential Question 9.2: We can apply the idea that the buoyant force acting on an object is proportional to the volume of fluid displaced by that object. When the block is half submerged, the buoyant force is 4.0 N up because the buoyant force and the spring scale, which exerts a force of 6.0 N up, must balance the downward 10 N force of gravity acting on the block. When the block is completely submerged, it displaces twice as much fluid, doubling the buoyant force to 8.0 N up. The spring scale only has to apply 2.0 N of force up on the block to make the forces balance. Diagrams and free-body diagrams for these situations are shown in Figure 9.11.

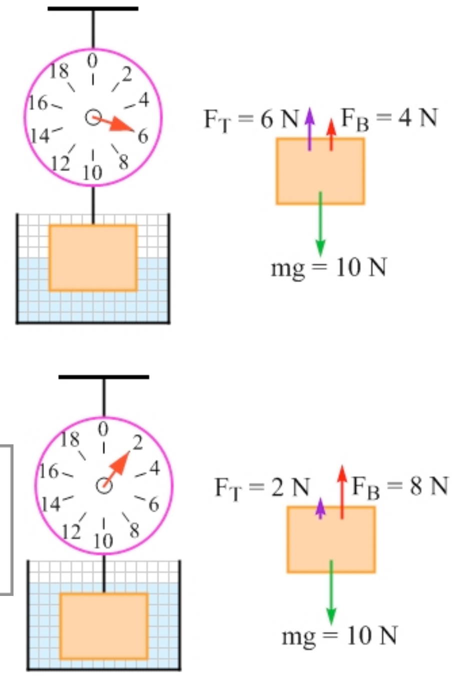


Figure 9.11: The top diagrams show the situation and free-body diagram for a block suspended from a spring scale when the block is half submerged in water. The bottom diagrams are similar, except that the block is completely submerged.

9-3 Archimedes' Principle

EXPLORATION 9.3 – What does the buoyant force depend on?

We know that the buoyant force acting on an object is proportional to the volume of fluid displaced by the object. What else does it depend on? Let's experiment to figure this out. We'll use a special beaker with a spout, as shown in Figure 9.12. In each case, we will fill the beaker to a level just below the spout, so that when we add a block to the beaker any fluid displaced by the block will flow down the spout into a second catch beaker. The catch beaker sits on a scale, so we can measure the weight of the displaced fluid. The fluid in the beaker will be either water or a second liquid, so we can figure out whether the fluid in the beaker makes any difference.

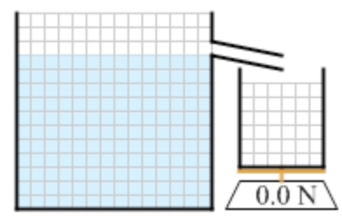


Figure 9.12: The beaker with the spout, and the catch beaker sitting on the scale. The scale is tared so it will read directly the weight of fluid in the catch beaker.

The blocks we will work have equal volumes but different masses. The weights of the blocks are 8 N, 16 N, and 24 N. Before we add a block to the beaker, we will make sure the beaker is filled to just below the level of the spout, and that the catch beaker is empty. If a block sinks in the fluid, we will hang it from a spring scale before completely submerging the block, so we can find the buoyant force from the difference between the force of gravity acting on the block and the reading on the spring scale. Also, the scale under the catch beaker is tared, which means that with the empty catch beaker sitting on it the scale reads zero and will read directly the weight of any fluid in the catch beaker.

The results of the experiments with water are shown in Figure 9.13, along with the corresponding free-body diagrams. In every case, ***the magnitude of the buoyant force acting on the block is equal to the weight of the fluid displaced by the block.***

Does this only work with water? Let's try it with the second fluid. The results are shown in Figure 9.13. Here we notice some differences - the 8 N block still floats buts displaces twice the volume of fluid it did in the water; the 16 N block now sinks; and the 24 N block still sinks but has half the buoyant force it had when it was in the water. Once again, however, the magnitude of the buoyant force on the block is equal to the weight of fluid displaced by the block.

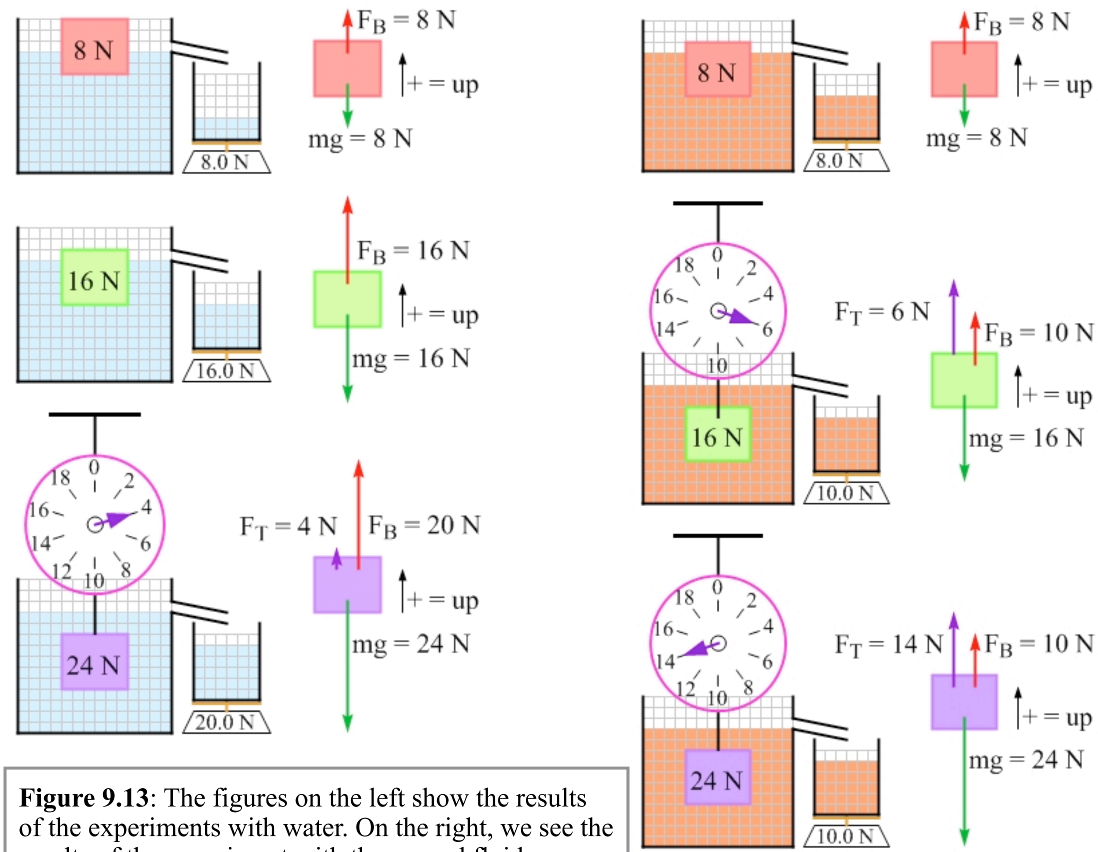


Figure 9.13: The figures on the left show the results of the experiments with water. On the right, we see the results of the experiment with the second fluid.

With the second fluid, we see that the buoyant force the fluid exerts on an object is still proportional to the volume of fluid displaced. However, we can also conclude that displacing a particular volume of water gives a different buoyant force than displacing exactly the same amount of the other fluid. Some property of the fluid is involved here.

To determine which property of the fluid is associated with the buoyant force, let's focus on the fact that the buoyant force is equal to the weight of the fluid displaced by the object:

$$F_B = m_{disp} g .$$

If we bring in mass density, for which we use the symbol ρ , we can write this equation in terms of the volume of fluid displaced. The relationship between mass, density, and volume is:

$$m = \rho V . \qquad \text{(Equation 9.2: \textbf{Mass density})}$$

Using this relationship in the equation for buoyant force gives:

$$F_B = m_{disp} g = \rho_{fluid} V_{disp} g . \qquad \text{(Equation 9.3: \textbf{Archimedes' Principle})}$$

Key Idea regarding Archimedes' Principle: The magnitude of the buoyant force exerted on an object by a fluid is equal to the weight of the fluid displaced by the object. This is known as Archimedes' principle. **Related End-of-Chapter Exercises: 4, 7.**

Essential Question 9.3: How does the mass density of the second fluid in Exploration 9.3 compare to the mass density of water?

Answer to Essential Question 9.3: The second fluid has a density that is half that of water. We can see that because a particular volume of water has a mass that is twice as much as the mass of an equal volume of the second fluid.

9-4 Solving Buoyancy Problems

Archimedes was a Greek scientist who, legend has it, discovered the concept while taking a bath, whereupon he leapt out and ran naked through the streets shouting "Eureka!" Archimedes was thinking about this because the king at the time wanted Archimedes to come up with some way to make sure that the king's crown was made out of solid gold, and was not gold mixed with silver. Archimedes realized that he could use his principle to determine the density of the crown, and he could then compare it to the known density of gold.

Using Equation 9.3, we can now explain the results of the block-and-two-fluid experiment above. The differences we observe between when we place the blocks in water and when we place them in the second fluid can all be explained in terms of the difference between the density of water and the density of the second fluid. In fact, to explain the results of Exploration 9.2 the second fluid must have half the density of water. The 10-N block, for instance, floats in both fluids and therefore the buoyant force is the same in both cases, exactly equal-and-opposite to the 10 N force of gravity acting on the block. Because the density of the second fluid is half the density of water, the block needs to displace twice the volume of fluid in the second fluid to achieve the same buoyant force. The 30-N block, on the other hand, displaces the same amount of fluid in each case. However, it experiences twice the buoyant force from the water as it does from the second fluid because of the factor of two difference in the densities.

What happens with the 20-N block is particularly interesting, because it floats in water and yet sinks in the second fluid. This raises the question, what determines whether an object floats or sinks when it is placed in a fluid?

EXPLORATION 9.4 – Float or sink?

How can we tell whether an object will float or sink in a particular fluid? As we have considered before, when an object floats in a fluid the upward buoyant force exactly balances the downward force of gravity. This gives: $F_B = mg$.

Using Archimedes' principle, we can write the left-hand side as: $\rho_{fluid} V_{disp}\, g = mg$.

The factors of g cancel (this tells us that it doesn't matter which planet we're on, or where on the planet we are), giving: $\rho_{fluid} V_{disp} = m$.

If we write the right-hand side in terms of the density of the object, we get, for a floating object:

$$\rho_{fluid} V_{disp} = \rho_{object} V_{object} .$$

Re-arranging this equation leads to the interesting result (that applies for floating objects only):

$$\frac{\rho_{object}}{\rho_{fluid}} = \frac{V_{disp}}{V_{object}} . \qquad \text{(Equation 9.4: \textbf{For floating objects})}$$

Equation 9.4 answers the question of what determines whether an object floats or sinks in a fluid – the density. ***If an object is less dense than the fluid it is in then it floats***. An object that is less dense than the fluid it is in floats because the object displaces a volume of fluid smaller than its own volume – in other words, the object floats with part of it sticking out above the surface of the fluid. On the other hand, an object more dense than the fluid it is in must displace a volume of fluid larger than its own volume in order to float. This is certainly not possible for the solid blocks we have considered above. Thus, we can conclude that ***an object with a density larger than the density of the fluid it is in will sink in that fluid***.

Key Ideas: Whether an object floats or sinks in a fluid depends on its density. An object with a density less than that of a fluid floats in that fluid, while an object with a larger density than that of a fluid will tend to sink in that fluid. **Related End-of-Chapter Exercises: 1, 13.**

Equation 9.4 tells us that we can determine the density of a floating object by observing what fraction of its volume is submerged. For instance, if an object is 30% submerged in a fluid its density is 30% of the density of the fluid. Table 9.1 shows the density of various materials.

Material	Density (kg/m^3)	Material	Density (kg/m^3)
Interstellar space	10^{-20}	Planet Earth (average)	5500
Air (at 1 atmosphere)	1.2	Iron	7900
Water (at 4˚C)	1000	Mercury (the metal)	13600
Sun (average)	1400	Black hole	10^{+19}

Table 9.1 The density of various materials.

What about an object that has the same density as the fluid it is in? This is known as **neutral buoyancy**, because the upward buoyant force on the object balances the downward force of gravity on the object when the object is 100% submerged. Because the net force acting on the object is zero it is in equilibrium at any of the positions shown in Figure 9.14. This is true as long as the fluid density does not change with depth, which is something of an idealization. Again we are using a model, with an assumption of the model being that a fluid is incompressible – its density is constant.

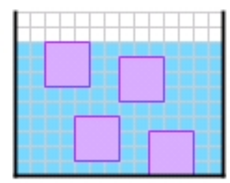


Figure 9.14: A neutrally buoyant object (an object with the same density as the surrounding fluid) will be at equilibrium at any of the positions shown. All other objects will either float at the surface, or sink to the bottom.

The general method for solving a typical buoyancy problem is based on the method we used in chapter 3 for solving a problem involving Newton's Laws. Now, we include Archimedes' principle. In general buoyancy problems are 1-dimensional, involving vertical forces, so that simplifies the method a little.

A General Method for Solving a Buoyancy Problem

1. Draw a diagram of the situation.
2. Draw one or more free-body diagrams, with each free-body diagram showing all the forces acting on an object as well as an appropriate coordinate system.
3. Apply Newton's Second Law to each free-body diagram.
4. If necessary, bring in Archimedes' principle, $F_B = \rho_{fluid} V_{disp}\, g$.
5. Put the resulting equations together and solve.

Essential Question 9.4: Let's say the four objects shown in Figure 9.14 have densities larger than that of the fluid. Can any of the objects be at equilibrium at the positions shown? Explain.

Answer to Essential Question 9.4: Objects that are denser than the fluid they are in tend to sink to the bottom of the container. One object in Figure 9.14 already rests at the bottom, so it is in equilibrium. For the three higher objects, the force of gravity, acting down, is larger than the buoyant force that acts up. These three objects are not at equilibrium, and will sink to the bottom.

9-5 An Example Buoyancy Problem

EXAMPLE 9.5 – Applying the general method

Let's now consider an object that sinks to the bottom of a beaker of liquid. The object is a block with a weight of 20 N, when weighed in air. The beaker it is to be placed in contains some water, as well as a waterproof scale that rests on the bottom of the beaker. This scale is tared to read zero, and let's assume the scale is unaffected by any changes in the level of the water above it. The beaker itself rests on a second scale that reads 50 N, the combined weight of the beaker, the water, and the scale inside the beaker. When the 20-N block is placed in the beaker, it sinks to the bottom and comes to rest on the scale in the beaker, which now reads 5.0 N. This is known as the **apparent weight** of the block. Let's assume $g = 10$ m/s^2 to simplify the calculations.

(a) What is the magnitude and direction of the buoyant force applied on the block by the water?
(b) With the block now completely immersed in the water, what is the reading on the scale under the beaker?
(c) What is the block's density and volume?

SOLUTION

Let's begin with the first two steps in the general method, by drawing a diagram of the situation and a free-body diagram of the block. These are shown in Figure 9.15, where up is taken to be the positive direction. Note that three forces act on the block, one of which is the downward force of gravity. The 5.0 N reading on the scale is the magnitude of the downward normal force applied by the block on the scale. By Newton's Third Law, the scale applies an upward 5.0 N normal force on the block. The third force acting on the block is the upward buoyant force applied on it by the water.

(a) The block is in equilibrium (at rest with no acceleration), so we can apply Newton's Second Law to determine the buoyant force acting on the block.

$$\Sigma\vec{F} = m\vec{a} = 0 .$$

Looking at the free-body diagram to evaluate the left-hand side gives:

$$+F_B + F_N - mg = 0 .$$

Solving for the buoyant force gives:

$$F_B = mg - F_N = 20\text{ N} - 5.0\text{ N} = 15\text{ N} \text{, directed up.}$$

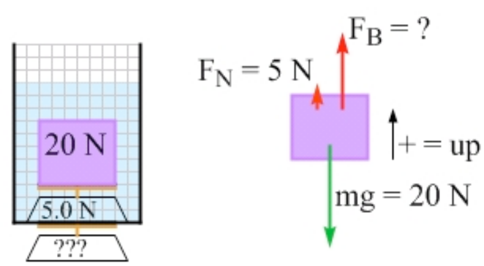


Figure 9.15: A diagram and free-body diagram for the block resting on the scale inside the beaker of fluid.

(b) What is the reading on the scale under the beaker? The scale under the beaker supports everything on top of it, so with the block inside the beaker the scale under the beaker reads 70 N. This comes from adding the full 20-N weight of the block to the original 50 N, from the beaker, water, and scale inside the beaker.

Doesn't the water support 15 N of the block's weight, via the buoyant force? Yes, it does. However, if the water exerts a force of 15 N up on the block, then by Newton's third law the block exerts a 15 N force down on the water. The water passes this force along to the beaker, which passes it along to the scale under the beaker. Similarly, the block exerts a 5.0-N normal force down on the scale inside the beaker, and the scale passes this force along to the beaker, which passes it along to the scale under the beaker. Now matter how you look at it, adding a 20-N block to the beaker ends up increasing the reading on the scale under the beaker by 20 N.

(c) Let's derive a general equation that tells us how the density of a submerged object is related to its weight *mg* and apparent weight W_{app}. The apparent weight is numerically equal to the normal force experienced by the submerged object.

In part (a), we used Newton's Second Law to arrive at an expression for the buoyant force acting on our submerged object, obtaining: $F_B = mg - F_N$.

Writing this in terms of the apparent weight gives: $F_B = mg - W_{app}$.

For a submerged object, the apparent weight is less than the actual weight. If we call f the ratio of the apparent weight to the actual weight, $f = W_{app} / mg$, we can write the previous equation, using $W_{app} = f\, mg$, as:

$$F_B = mg - f\, mg = (1 - f)mg .$$

Now, use Archimedes' principle to transform the left-hand side of the equation:

$$\rho_{fluid} V_{disp}\, g = (1 - f)mg .$$

Finally, write the object's mass in terms of its density: $\rho_{fluid} V_{disp}\, g = (1 - f)\rho_{object} V_{object}\, g$.

The volume of fluid displaced by an object that is completely submerged is equal to its own volume, so we can cancel the factors of volume as well as the factors of *g*, leaving:

$$\rho_{fluid} = (1 - f)\rho_{object} .$$

Solving for the density of the object, we can write the equation in various ways:

$$\rho_{object} = \frac{\rho_{fluid}}{1 - f} = \frac{\rho_{fluid}}{1 - \frac{W_{app}}{mg}} = \frac{mg\,\rho_{fluid}}{mg - W_{app}} .$$

That applies generally to a completely submerged object. In our case, where we have $f = 1/4$, we find that the density of the block is:

$$\rho_{block} = \frac{4}{3}\rho_{water} = 1330\ \text{kg/m}^3 .$$

Related End-of-Chapter Exercises: 9, 18.

Essential Question 9.5: It is possible for small metal objects, such as sewing needles or Japanese yen, to float on the surface of water, if carefully placed there. Can we explain this in terms of the buoyant force?

Answer to Essential Question 9.5: No. These metal objects are denser than water, so we expect them to sink in the water (which they will if they are not placed carefully at the surface). They are held up by the surface tension of the water. Surface tension is beyond the scope of this book but it is similar to how a gymnast is supported by a trampoline – the water surface can act like a stretchy membrane that can support an object that is not too massive.

9-6 Pressure

Where does the buoyant force come from? What is responsible, for instance, for the small upward buoyant force exerted on us by the air when we are surrounded by air? Let's use a model in which the fluid is considered to be made up of a large number of fast-moving particles that collide elastically with one another and with anything immersed in it. For simplicity, let's examine the effect of these collisions on a block of height h and area A that is suspended from a light string, as shown in Figure 9.16.

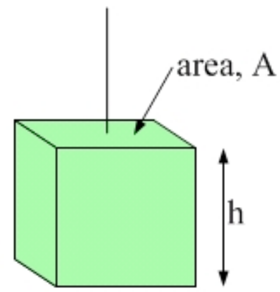


Figure 9.16: A block of height h and area A supported by a light string.

Consider a collision involving an air molecule bouncing off the left side of the block, as in Figure 9.17. Assuming the block remains at rest during the collision (the block's mass is much larger than that of the air molecule), then, because the collision is elastic, the magnitude of the molecule's momentum remains the same and only the direction changes: the component of the molecule's momentum that is directed right before the collision is directed left after the collision. All other momentum components remain the same. The block exerts a force to the left on the molecule during the collision, so the block experiences an equal-and-opposite force to the right.

There are a many molecules bouncing off the left side of the block, producing a sizable force to the right on the block. The block does not accelerate to the right, however, because there is also a large number of molecules bouncing off the right side of the block, producing a force to the left on the block. Averaged over time, the rightward and leftward forces balance. Similarly, the forces on the front and back surfaces cancel one another.

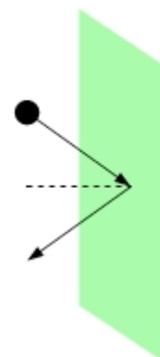


Figure 9.17: A magnified view of a molecule bouncing off the left side of the block.

If all the forces cancel out, how do these collisions give rise to the buoyant force? Consider the top and bottom surfaces of the cube. Because the buoyant force exerted on the cube by the air is directed vertically up, the upward force on the block associated with air molecules bouncing off the block's bottom surface must be larger than the downward force on the block from air molecules bouncing off the block's top surface. Expressing this as an equation, and taking up to be positive, we get:

$$+F_{bottom} - F_{top} = +F_B = +\rho_{fluid} V_{disp} g \, .$$

The volume of air displaced by the block is the block's entire volume, which is its area multiplied by its height: $V_{disp} = Ah$. Substituting $V_{disp} = Ah$ into the expression above gives:

$$+F_{bottom} - F_{top} = +F_B = +\rho_{fluid} A h g \, .$$

This is the origin of the buoyant force – the net upward force applied to the block by molecules bouncing off the block's bottom surface is larger in magnitude than the net downward force applied by molecules bouncing off the block's upper surface. This is a gravitational effect – the buoyant force is proportional to g. One way to think about this is that if the molecules at the block's top surface have a particular average kinetic energy, to conserve energy those at the block's bottom surface should have a larger kinetic energy because their gravitational potential energy is less. Thus, molecules bouncing off the bottom surface are more energetic, and they impart a larger average force to the block than the molecules at the top surface.

Dividing both sides of the previous equation by the area A gives:

$$+\frac{F_{bottom}}{A}-\frac{F_{top}}{A}=+\rho_{fluid}\,h\,g\,. \qquad \text{(Equation 9.5)}$$

The name for the quantity of force per unit area is **pressure**.

$$\text{Pressure}=\frac{\text{Force}}{\text{Area}} \quad \text{or} \quad P=\frac{F}{A}. \qquad \text{(Equation 9.6: \textbf{Pressure})}$$

The MKS unit for pressure is the pascal (Pa). $1\ \text{Pa}=1\ \text{N/m}^2$.

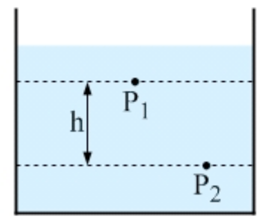


Figure 9.18: The pressure difference between two points is proportional to the vertical distance between them. Pressure increases with depth in a static fluid.

Using the symbol P for pressure, we can write Equation 9.5 as:

$$P_{bottom}-P_{top}=\rho_{fluid}\,h\,g\,.$$

We can write this equation in a general way, so that it relates the pressures of any two points, points 1 and 2, in a static fluid, where point 2 is a vertical distance h below the level of point 1. This gives:

$$P_2=P_1+\rho g h\,. \qquad \text{(Equation 9.7: \textbf{Pressure in a static fluid})}$$

As represented by Figure 9.18, only the vertical level of the point matters. Any horizontal displacement in moving from point 1 to point 2 is irrelevant.

EXPLORATION 9.6 – Pressure in the L

Consider the L-shaped container in Figure 9.19. Rank points A, B, and C in terms of their pressure, from largest to smallest.

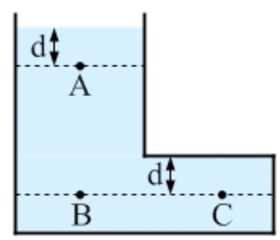


Figure 9.19: A container shaped like an L that is filled with fluid and open at the top.

Because pressure in a static fluid depends only on vertical position, points B and C have equal pressures, and the pressure at that level in the fluid is higher than that at point A. The fact that there is a column of water of height d immediately above both points A and C is irrelevant. The fact that C is farthest from the opening is also irrelevant. Only the vertical position of the points matters.

Key ideas: In a static fluid the pressure at any point is determined by that point's vertical position. All points at the same level have the same pressure, and points lower down have higher pressure than points higher up.
Related End-of-Chapter Exercises: 10, 51.

Essential Question 9.6: Water is placed in a U-shaped tube, as shown in Figure 9.20. The tube's left arm is open to the atmosphere, but the tube's right arm is sealed with a rubber stopper. Rank points A, B, and C based on their pressure, from largest to smallest.

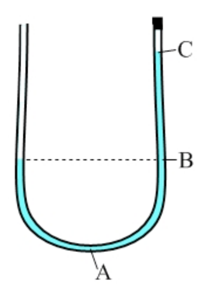


Figure 9.20: A U-shaped water-filled tube that is sealed at the top right by a rubber stopper.

Answer to Essential Question 9.6: A>B>C. Point A, being the lowest of the three points, has the highest pressure. Point C, being the highest of the three points, has the lowest pressure.

9-7 Atmospheric Pressure

At sea level on Earth, standard atmospheric pressure is 101.3 kPa, or about 1.0×10^5 Pa, a substantial value. Atmospheric pressure is associated with the air molecules above sea level. Air is not very dense, but the atmosphere extends upward a long way so the cumulative effect is large. The reason we, and most things, don't collapse under atmospheric pressure is that in almost all situations there is pressure on both sides of an interface, so the forces balance. If you can create a pressure difference, however, you can get some interesting things to happen. This is how suction cups work, for instance – by removing air from one side the air pressure on the outside of the suction cup gives rise to a force that keeps the suction cup attached to a surface. It's also fairly easy to use atmospheric pressure to crush a soda can (see end-of-chapter Exercise 6).

In many situations what matters is the **gauge pressure**, which is the difference between the total pressure and atmospheric pressure. The total pressure is generally referred to as the **absolute pressure**. For instance, the absolute pressure at the surface of a lake near sea level is 1 atmosphere (1 atm), so the gauge pressure there would be 0. The gauge pressure 10 meters below the surface of a lake is about 1 atmosphere (1 atm), taking the density of water to be 1000 kg/m^3, because:

$$\rho\, g\, h \approx 1000\,\frac{\text{kg}}{\text{m}^3}\times 10\,\frac{\text{m}}{\text{s}^2}\times 10\text{ m} = 1\times 10^5\,\frac{\text{kg m}}{\text{s}^2}\,\frac{1}{\text{m}^2} = 1\times 10^5\,\frac{\text{N}}{\text{m}^2} = 1\times 10^5\text{ Pa}\,.$$

The absolute pressure 10 m below the surface is about 2 atm. This is particularly relevant for divers, who must keep in mind that every 10 m of depth in water is associated with an additional 1 atmosphere worth of pressure.

EXAMPLE 9.7 – Under pressure

A plastic box is in the shape of a cube measuring 20 cm on each side. The box is completely filled with water and remains at rest on a flat surface. The box is open to the atmosphere at the top. Assume atmospheric pressure is 1.0×10^5 Pa and use g = 10 m/s^2.

(a) What is the gauge pressure at the bottom of the box?
(b) What is the absolute pressure at the bottom of the box?
(c) What is the force associated with this absolute pressure?
(d) What is the force associated with the absolute pressure acting on the inside surface of one side of the box?
(e) What is the net force associated with pressure acting on one side of the box?
(f) What is the net force acting on one side of the box?

SOLUTION

As usual let's begin with a diagram of the situation, shown in Figure 9.21.

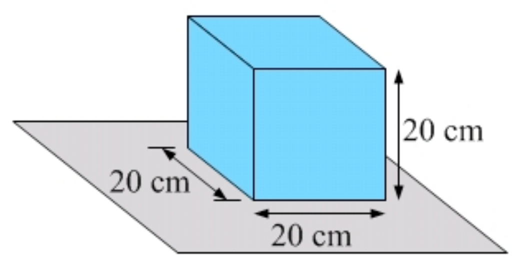


Figure 9.21: A box in the shape of a cube that is open at the top and filled with water.

(a) Because the pressure at the top surface is atmospheric pressure, the gauge pressure at the bottom is simply the pressure difference between the top of the box and the bottom. Applying Equation 9.7, regarding the pressure difference between two points in a static fluid, we get the gauge pressure at the bottom:

$$\Delta P = \rho\, g\, h = 1000 \frac{\text{kg}}{\text{m}^3} \times 10 \frac{\text{m}}{\text{s}^2} \times 0.20 \text{ m} = 2000 \text{ Pa} .$$

(b) The absolute pressure at the bottom is the gauge pressure plus atmospheric pressure. This gives: $P_{bottom} = P_{atm} + P_{gauge} = 1.0 \times 10^5 \text{ Pa} + 2000 \text{ Pa} = 1.02 \times 10^5 \text{ Pa}$. Stating this to three significant figures would violate the rules about significant figures when adding, so we should really round this off to $1.0 \times 10^5 \text{ Pa}$.

(c) To find the force from the pressure we use Equation 9.6, re-arranged to read $\text{Force} = \text{Pressure} \times \text{Area}$. This gives a force of $F_{bottom} = \left(1.0 \times 10^5 \text{ Pa}\right)\left(0.2 \text{ m}\right)^2 = 4000 \text{ N}$, directed down at the bottom of the box.

(d) Finding the force associated with the side of the box is a little harder than finding it at the bottom, because the pressure increases with depth in the fluid. In other words, the pressure is different at points on the side that are at different depths. Because the pressure increases linearly with depth, however, we can take the average pressure to be the pressure halfway down the side of the box. The gauge pressure at a point inside the box that is halfway down the side is:

$$P_{gauge} = \rho g h = 1000 \frac{\text{kg}}{\text{m}^3} \times 10 \frac{\text{m}}{\text{s}^2} \times 0.1 \text{ m} = 1000 \text{ Pa}.$$

To find the force associated with the pressure we use absolute pressure, so we get: $F_{side} = \left(P_{atm} + P_{gauge}\right) \times \text{Area} = \left(1.0 \times 10^5 \text{ Pa} + 1000 \text{ Pa}\right) \times \left(0.2 \text{ m}\right)^2 = 4040 \text{ N}$, which we should round off to 4000 N directed out from the center of the box.

(e) In part (d) we were concerned with the fluid pressure applying an outward force on one side of the box. Now we need to account for the air outside the box exerting an inward force on the same side of the box. This force is simply atmospheric pressure multiplied by the area, and is thus 4000 N directed inward. The net force associated with pressure is thus the combination of the 4040 N force directed out and the 4000 N force directed in, and is thus 40 N directed out. The same result can be obtained from $F_{pressure} = P_{gauge} \times \text{Area}$.

(f) Because the box and all its sides remain at rest, the net force on any one side must be zero, so this 40 N outward force associated with the gauge pressure of the water must be balanced by forces applied to one side by the rest of the box.

Related End-of-Chapter Exercises: 27, 28.

Essential Question 9.7: In Example 9.7, we accounted for the change in water pressure with depth, but we did not account for the increase in air pressure with depth, which could affect our calculation of the inward force exerted by the air on a side of the box. Explain why we can neglect this change in air pressure.

Answer to Essential Question 9.7: The increase in pressure with depth is proportional to the product of the density multiplied by the vertical distance. Because the density of the water is on the order of 1000 times larger than that of air, we can neglect this effect for the air.

9-8 Fluid Dynamics

Let's turn now from analyzing fluids at rest to analyzing fluids in motion. The study of flowing fluids is known as fluid dynamics. Fluid dynamics can be rather complex, so we will make some simplifying assumptions. These include:

1. The flow is steady – flow patterns are maintained without turbulence.
2. The fluid is incompressible – its density is constant.
3. The fluid is non-viscous – there is no resistance to the flow.
4. The flow is irrotational – there are no swirls or eddies.

Under these assumptions we get what is called streamline flow, indicated by the blue streamlines in the pipe shown in Figure 9.22.

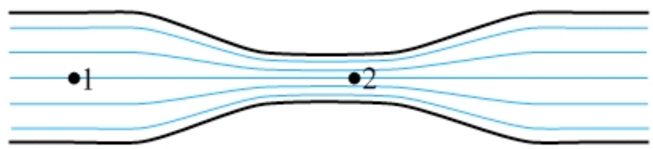


Figure 9.22: Streamline flow through a pipe.

Continuity

There are two main equations we will apply to analyze flowing fluids. The first of these is called the continuity equation, which comes from the fact that when an incompressible fluid flows through a tube of varying cross-section, the rate at which mass flows past any point in the tube is constant. If the flow rate varied, fluid would build up at points where the flow rate is low.

The mass flow rate is the total mass flowing past a point in a given time interval divided by that time interval. At a point where the flow is in the x-direction with a speed v and the tube has a cross-sectional area A, the mass flow rate is given by:

$$\text{mass flow rate} = \frac{\Delta m}{\Delta t} = \frac{\rho\ \Delta V}{\Delta t} = \frac{\rho\ A\,\Delta x}{\Delta t} = \rho\ A\,v\,.$$

Consider the streamline flow pattern in Figure 9.22. The mass flow rate is the same at two different points, 1 and 2, in the tube, so:

$$\rho_1\,A_1\,v_1 = \rho_2\,A_2\,v_2\,.$$

One of our assumptions is that the density is the same at all points, so we can reduce the preceding equation to:

$$A_1\,v_1 = A_2\,v_2\,.$$ (Equation 9.8: **The Continuity Equation**)

The main implication of the continuity equation is that the speed of the fluid increases as the cross-sectional area of the tube decreases, and vice versa. The streamlines in Figure 9.22 show the change in speed that corresponds to a change in area. Where the streamlines are farther apart, such as at point 1, the flow speed is less. Where the streamlines are close together, such as at point 2, the speed is higher.

The second equation we will apply to flowing fluids is an energy conservation equation, transformed to be particularly useful for fluids. Let's start by writing out our energy conservation equation from chapter 6:

$$U_1 + K_1 + W_{nc} = U_2 + K_2 \,.$$

The potential energy we're talking about here is gravitational potential energy, in the form $U = m\,g\,y$, and we can write the kinetic energy as $K = (1/2)mv^2$. The energy equation can thus be written as:

$$mgy_1 + \frac{1}{2}mv_1^2 + W_{nc} = mgy_2 + \frac{1}{2}mv_2^2 \,.$$

Let's apply this equation to a fluid flowing through a pipe. Figure 9.23 shows the two points we are considering, and two cylindrical regions of fluid are highlighted, one at each point. The cylindrical regions have equal volumes.

In the case of a flowing fluid, the work done by non-conservative forces is related to forces that arise because of pressure differences. We can write the W_{nc} term as:

$$W_{nc} = F_{nc}\,\Delta x = F_1\,\Delta x_1 - F_2\,\Delta x_2 = P_1 A_1\,\Delta x_1 - P_2 A_2\,\Delta x_2 \,.$$

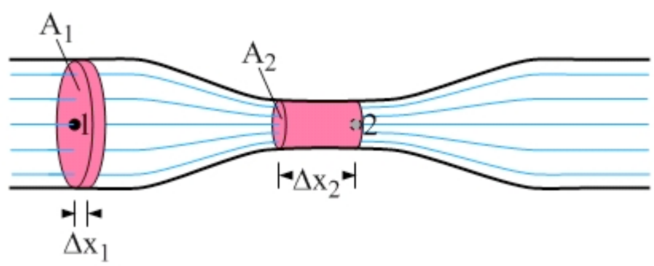


Figure 9.23: The two cylindrical regions, one at point 1 and one at point 2, have the same volume.

Substituting this expression for W_{nc} into the energy conservation relationship gives:

$$mgy_1 + \frac{1}{2}mv_1^2 + P_1 A_1 \Delta x_1 = mgy_2 + \frac{1}{2}mv_2^2 + P_2 A_1 \Delta x_1 \,.$$

The *m* here represents the mass of the fluid in one of the cylindrical regions in Figure 9.23 (the cylindrical regions have equal masses because of their equal volumes).

Let's simplify the equation by dividing both sides by the volume *V* of one of the cylindrical regions ($V = A_1\,\Delta x_1 = A_2\,\Delta x_2$). Because mass/volume = density, we get:

$$\rho g y_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \frac{1}{2}\rho v_2^2 + P_2 \,.$$ (Equation 9.9: **Bernoulli's Equation**)

Bernoulli's equation represents conservation of energy applied to fluids, although each term has units of energy density (energy per unit volume).

Related End-of-Chapter Exercises: 31, 56.

Essential Question 9.8: Consider a special case of Bernoulli's equation, when the fluid is at rest. Which of the equations we have examined previously in this chapter is equivalent to Bernoulli's equation with the two terms involving speed set to zero?

Answer to Essential Question 9.8: Eliminating the speed terms means that we can write Bernoulli's equation as: $\rho g y_1 + P_1 = \rho g y_2 + P_2$. Re-arranging this equation to solve for the pressure at point 2 gives: $P_2 = P_1 + \rho g y_1 - \rho g y_2 = P_1 + \rho g (y_1 - y_2)$. This equation is equivalent to Equation 9.7, the equation for pressure in a static fluid.

9-9 Examples Involving Bernoulli's Equation

EXPLORATION 9.9 – Pressure inside a pipe
Step 1 - ***Make a prediction. In the pipe shown in Figure 9.24, is the pressure higher at point 2, where the fluid flows fastest, or at point 1? The fluid in the pipe flows from left to right.***

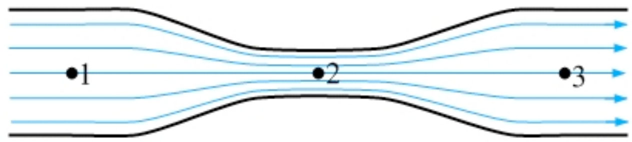


Figure 9.24: Fluid flowing through a pipe from left to right.

Many people predict that the pressure is higher at point 2, where the fluid is moving faster.

Step 2 - ***Apply the continuity equation, and Bernoulli's equation, to rank points 1, 2, and 3 according to pressure, from largest to smallest.*** Let's see if the common prediction, that the pressure is highest at point 2, is correct. First, apply the continuity equation: $A_1 v_1 = A_2 v_2 = A_3 v_3$. Looking at the tube, we know that $A_1 = A_3 > A_2$, which tells us that $v_2 > v_1 = v_3$.

Now, let's apply Bernoulli's Equation. Comparing points 1 and 2, we start with:

$$\rho g y_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \frac{1}{2}\rho v_2^2 + P_2 .$$

The vertical positions of these two points are equal so the $\rho g y$ terms cancel out:

$$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2 .$$

Let's re-write this as: $P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$.

The continuity equation told us that $v_2 > v_1$, so the right-hand side of the above equation is positive. This means the left-hand side must also be positive, implying that $P_1 > P_2$. Thus, the pressure at point 2, where the fluid speed is highest, is less than the pressure at point 1. For points at the same height, higher speed corresponds to lower pressure. We can make sense of this by considering a parcel of fluid that moves from point 1 to point 2. Because this parcel of fluid speeds up as it travels from point 1 to point 2, there must be a net force acting on it that is directed right. This force must come from a difference in pressure between points 1 and 2. For the force to be directed right, the pressure must be larger on the left, at point 1.

We can also use Bernoulli's equation to show that the pressure at point 3 is equal to that at point 1. Thus we can conclude that $P_1 = P_3 > P_2$.

Key idea for an enclosed fluid: In general, in an enclosed fluid the pressure decreases as the speed of the fluid flow increases. **Related End-of-Chapter Exercises: 52, 53.**

EXAMPLE 9.9 – How fast?

A Styrofoam cylinder, filled with water, sits on a table. You then poke a small hole through the side of the cylinder, 20 cm below the top of the water surface. What is the speed of the fluid emerging from the hole?

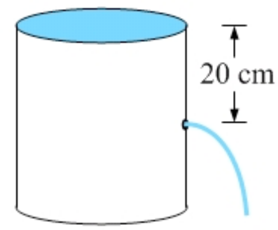


Figure 9.25: The Styrofoam container of water, with a small hole 20 cm from the top.

SOLUTION

As usual, begin by drawing a diagram of the situation, as shown in Figure 9.25.

We're going to apply Bernoulli's equation, which means identifying two points that we can relate via the equation. Point 2 is outside the container where the hole is, because that is the place where we're trying to find the speed. Point 1 needs to be somewhere inside the container. Any point inside will do, although the most sensible places are either at the top of the container, where we know the pressure, or inside the container at the level of the hole. Let's choose a point at the very top, and apply Bernoulli's equation:

$$\rho g y_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \frac{1}{2}\rho v_2^2 + P_2 .$$

First, we should recognize that, because both of our points are exposed to the atmosphere, we have $P_1 = P_2 = P_{atm}$. The pressure terms cancel in the equation, leaving:

$$\rho g y_1 + \frac{1}{2}\rho v_1^2 = \rho g y_2 + \frac{1}{2}\rho v_2^2 .$$

We can cancel factors of density. We are also free to define a zero for our y positions to be anywhere we find convenient. If we say $y = 0$ at the level of the hole we get $y_1 = +20 \text{ cm}$ and $y_2 = 0$, so the equation reduces to:

$$g y_1 + \frac{1}{2} v_1^2 = \frac{1}{2} v_2^2 .$$

If we knew the fluid speed at point 1 we could solve for the speed at point 2. This is a good time to bring in the continuity equation, which relates the speeds at the two points: $A_1 v_1 = A_2 v_2$. In this case we can say that, because A_1, the cross-sectional area of the cylinder, is so much larger than A_2, the cross-sectional area of the hole, then v_1 is much smaller than v_2. Thus, the $(1/2)v_1^2$ term is negligible compared to the $(1/2)v_2^2$ term. Our equation thus reduces to:

$$g y_1 = \frac{1}{2} v_2^2 .$$

Solving for the speed at which the fluid emerges from the hole gives:

$$v_2 = \sqrt{2 g y_1} = \sqrt{2 \times 9.8 \frac{\text{m}}{\text{s}^2} \times 0.20 \text{ m}} = 2.0 \text{ m/s} .$$

Related End-of-Chapter Exercises: 27, 28.

Essential Question 9.9: If you dropped an object from rest, what would its speed be after it had fallen through a distance of 20 cm? How does this compare to the result of the Example 9.9, where we found the speed of water emerging from a hole 20 cm below the water surface?

Answer to Essential Question 9.9: These two situations appear to be different, but the answers are the same. In both cases the speed is given by an equation of the form $v = \sqrt{2gh}$. The equations, and the speeds, are the same because, in both cases, we can apply conservation of energy, with gravitational potential energy being transformed to kinetic energy.

9-10 Viscosity and Surface Tension

Until this point, we have made a number of simplifying assumptions regarding the behavior of fluids, including assuming no viscosity (no resistance to flow). For any real fluid flowing through a pipe, there is some viscosity. In the case of air or water, the viscosity is generally low, but for applications such as blood flowing through blood vessels in the human body, the resistance to flow is an important factor.

Viscosity (a measure of a fluid's resistance to flow) is generally measured by means of a pair of horizontal parallel plates of area A with fluid filling the space between them. The bottom plate is at rest, as is the fluid right next to it, while the top plate has a horizontal speed v, with the fluid next to it moving with the same velocity as that plate. This situation is known as Couette flow. Viscosity arises because neighboring layers of fluid have different speeds, and there is some frictional resistance between the layers as they move past one another. In general, the speed of the fluid increases linearly as you move from the fixed plate to the moving plate. In a Newtonian fluid, the viscosity (η) is a constant equal to the force required to keep the moving plate moving with constant velocity v , multiplied by the length of the flow and divided by both the speed and the plate area. There are several classes of non-Newtonian fluids, in which the viscosity changes as the speed changes. Those are interesting, but beyond the scope of this book.

To give you some feel for typical numbers, the viscosity of water at 20˚C is 0.001002 Pa s, while that of motor oil is about 0.250 Pa s.

Unlike the somewhat artificial situation of Couette flow, in a typical application fluid is flowing through a pipe in which the pipe is stationary. In this case, the fluid next to the pipe wall is at rest, and the fluid at the center of the pipe is moving fastest. If we use Q to denote the volume flow rate ($Q = Av$), then the volume flow rate of a viscous fluid is given by:

$$Q = \frac{\pi R^4 \Delta P}{8 \eta L} .$$ (Eq. 9.10: **The Hagen-Poiseuille equation**)

The equation is named for the German physicist and hydraulic engineer Gottlif Hagen and the French physician Jean Marie Louis Poiseuille, who independently arrived at the equation experimentally in 1839 and 1838, respectively.

The equation pertains to a fluid flowing through a pipe with a radius R and a length L, with a pressure difference of ΔP between the ends of the pipe. The viscosity is denoted by η, the Greek letter eta.

Surface Tension

In general, we can't walk on water (at least not the liquid variety), and we also can't float a quarter on water. However, certain insects (water striders, for instance) can support themselves perfectly well on the surface of a pond, and it is possible to float a Japanese yen, or a metal paper clip, in a glass of water. Based on what we learned earlier, the yen coin or the paper clip should sink - they are each made from material that is denser than water - but they float because of surface tension. In essence, for these light objects, the water surface acts as an elastic membrane, much like the surface of a trampoline does for us.

The elastic nature of the water surface comes from the attraction the water molecules have for one another. A water strider will dent the surface, but (unlike us) will not break the surface. Similarly, the paper clip floating on the water surface dents the surface, much like we do when standing on a trampoline. It can break the surface and sink to the bottom - it can take a few tried to get it to float at the surface, but if you place it gently and carefully on the water surface, the force of gravity acting on it can be balanced by the force associated with surface tension. Adding some liquid soap to the water reduces its surface tension, so you can make the floating paper clip sink just by adding a few drops of soap. In addition to a paper clip, you can also float a Japanese yen coin, as shown in Figure 9.26.

Figure 9.26: A Japanese yen coin, made from aluminum, which is supported by the surface tension at the surface of the water in a glass of water. Note how the water surface is indented, with the surface acting much like an elastic membrane. Photo credit: A. Duffy.

You have probably noticed that water often tends to form drops. This is because there is energy associated with the surface, so surfaces generally take on the smallest area possible, to minimize that surface energy. A sphere is the shape that minimizes the surface area, for a given volume. A similar effect is seen with soap films, which can take on interesting shapes when a frame is drawn out of soapy water - the shape of the film tends to minimize the film's surface area.

Surface tension and the lungs

Surface tension is actually quite important for our breathing. First, think about blowing up a balloon. You have to work to fill the balloon, but if you don't hold the neck of the balloon then the balloon will simply deflate by itself, because of the balloon's surface tension. Our lungs have a very large number of tiny balloons, essentially - these are called alveoli. We use muscles to breathe in, inflating the alveoli. However, just like the balloon, the alveoli deflate all by themselves, to minimize surface tension. That's a key part of the breathing process.

The alveoli have a mucus coating on their walls, which acts as a ***surfactant*** (a material that reduces surface tension, like dish soap does when you're doing dishes). Unlike dish soap, however, which has a fixed surface tension, the mucus has a surface tension that increases with the size of the alveoli. This is important for a number of reasons. Having a low surface tension when the alveoli are small prevents surface tension from collapsing the alveoli, and surface tension increasing as the alveoli expand prevents the alveoli from getting too large. This also explains why premature babies often have breathing issues - their lungs do not have the mucus coating the alveoli to reduce the surface tension, making it difficult to inflate the alveoli.

Essential Question 9.10: This question relates to viscosity. Let's say that Fred, who eats french fries on a daily basis, gradually experiences a narrowing of his blood vessels, because of plaque buildup in the vessel walls. All other things being equal, what would be the new flow rate through a blood vessel that experienced a 5% decrease in radius, compared to the original flow rate? In actuality, the blood pressure can change so that the flow rate does not drop quite so significantly. Would you expect the blood pressure to increase or decrease?

Answer to Essential Question 9.10: The new radius is 95% of the original radius. Taking a factor of 0.95 to the fourth power gives approximately 0.81, so the new flow rate would only be 81% of the original flow rate. The flow rate would not drop as much if the pressure difference between the ends of the blood vessel increased. This would generally be accomplished by increasing the blood pressure (which can lead to health issues, of course).

9-11 Drag and the Ultracentrifuge

When an object is falling through a viscous fluid, a drag force acts on it. Unlike the kinetic friction force we looked at earlier in the book, which has a magnitude that is independent of speed, the viscous drag force is generally proportional to the speed, and opposite in direction to the velocity. This is known as Stokes' drag, with the drag force on a spherical particle of radius r moving at speed v through a fluid of viscosity η being:

$$F_d = -6\pi\eta r v. \qquad \text{(Equation 9.11: \textbf{Stokes' drag force for a spherical particle})}$$

When an object falls through the fluid, it will reach a terminal velocity when the drag force plus the buoyant force is equal and opposite to the force of gravity. In general, the smaller the object, the smaller the magnitude of the terminal velocity. Very small objects fall very slowly. If your goal is to separate particles from the fluid the particles are in, this can be a problem - it can take a long time for the particles to settle out at the bottom.

This is where an ultracentrifuge comes in. The job of the ultracentrifuge is to spin the fluid very quickly in a circular path. In that case, the effect is just like increasing the value of g by a large factor. Effectively, as far as the particles are concerned, they are in a gravitational field with a strength given by the centripetal acceleration, $\omega^2 r$. Spinning very quickly gives very large values of the angular speed (ω), leading to very large "effective gravity" that separates out the particles quickly and efficiently. Let's consider an example.

EXAMPLE 9.11 – Analyzing a blood sample

You get a blood sample drawn while you're seeing your doctor, and the sample is sent to the lab for analysis. A key part of the analysis involves running the sample (contained in a cylindrical tube) through an ultracentrifuge to separate out the components, which have different densities (the red blood cells being most dense, at 1125 kg/m^3, and the plasma being least dense, at 1025 kg/m^3). The average density of blood is about 1060 kg/m^3. What is the purpose of an ultracentrifuge, which, say, has a rotation rate of 5000 rpm and an acceleration 5000 times larger than the acceleration due to gravity? Why don't they just stand the tube of blood up vertically to let gravity separate it? Do a quantitative analysis, using the following values. The mass of a red blood cell is about 27×10^{-15} kg, the viscosity of blood is about 3.5×10^{-3} Pa s, and we will model the cell as a sphere of radius 3.5×10^{-6} m .

SOLUTION

We'll start by determining what gravity can do by itself. A red blood cell in a vertical tube of blood will reach a terminal velocity (v_t) when the drag force plus the buoyant force is equal and opposite to the force of gravity.

$$mg = \rho_{fluid} V g + 6\pi\eta r v_t .$$

We can replace the volume of the cell by $V = m / \rho_{cell}$, which gives

$$mg = \frac{\rho_{fluid}}{\rho_{cell}} mg + 6\pi\eta r v_t .$$

This re-arranges to $v_t = \left(1 - \frac{\rho_{fluid}}{\rho_{cell}}\right)\frac{mg}{6\pi\eta r}$, solving for the terminal speed of a blood cell.

Now, we'll substitute the relevant values into our equation (recognizing that our model has some limitations, such as that red blood cells are not spheres, and issues with the fluid density not being constant).

$$v_t = \left(1 - \frac{\rho_{fluid}}{\rho_{cell}}\right)\frac{mg}{6\pi\eta r} = \left(1 - \frac{1060 \text{ kg/m}^3}{1125 \text{ kg/m}^3}\right)\frac{\left(27\times10^{-15} \text{ kg}\right)\left(9.8 \text{ N/kg}\right)}{6\pi\left(3.5\times10^{-3} \text{ Pa s}\right)\left(3.5\times10^{-6} \text{ m}\right)} = 6.6\times10^{-8} \text{ m/s}.$$

This is very slow, of course. If we needed to wait until the red blood cell traveled a distance of 3.3 cm through the tube of blood, say, we would have to wait for 500000 s, which is approximately 6 days.

In our ultracentrifuge, where the acceleration is 5000 *g*, what is the difference? We use the same equation we derived above for the terminal speed, but we replace the factor of *g* by 5000 *g*. That increases the terminal velocity by a factor of 5000, and reduces the time it takes the blood cell to travel 3.3 cm by a factor of 5000, so the time would be 100 s instead of 500000 s (about 2 minutes instead of 6 days). Despite the simplifying assumptions in our model, this is in keeping with the recommendations of ultracentrifuge manufacturers, that a spin time of five minutes at 5000 *g* is appropriate. Our analysis gives a value of the same order of magnitude.

Where does the 5000 *g* come from? This goes back to uniform circular motion, in which the acceleration is given by:

$$a_c = \frac{v^2}{R} = \omega^2 R.$$

R here is the radius of the circular path traveled by a blood cell inside the centrifuge (not to be confused with the *r* used above, for the radius of the blood cell itself). Expressing the angular speed in rad/s, and the acceleration in meters per second, we could solve for *R*, for instance.

Related End-of-Chapter Exercises: 67 - 71.

Essential Question 9.11: A manufacturer has two different centrifuges, one that spins at 5000 rpm, and another that spins at 10000 rpm. Everything else is the same. The manufacturer makes the following claim about the faster centrifuge - "It costs three times as much, but it is also three times as efficient - you can run samples through the faster centrifuge in one third the time!" Based on our analysis above, the factor of three seems somewhat surprising - what would we expect the difference to be between the two models? Accounting for the fact that it takes some time for the centrifuge to reach its maximum angular speed, and to slow down to a stop at the end of a run, might this factor of three actually be plausible?

Answer to Essential Question 9.11: If we double the angular velocity, the acceleration will increase by a factor of four (the acceleration goes as the square of the angular speed). That causes a corresponding decrease by a factor of 4 in the time to separate components of a sample - the manufacturer may be understating the case! However, the spin up and slow down time, which is probably longer for the faster centrifuge, means that the device is not consistently a factor of four better. That would decrease the ratio, although maybe not all the way down to a factor of three.

Chapter Summary

Essential Idea for Fluids

Even though situations involving fluids look quite different from those we examined in earlier chapters, we can apply the same methods we applied earlier. Forces are very useful for understanding situations in which an object floats or sinks in a static fluid, while energy-conservation ideas can help us to analyze situations involving moving fluids.

The Buoyant Force

An object in a fluid experiences a net upward force we call the buoyant force, $\vec{F}_B$. The magnitude of the buoyant force is proportional to the volume of fluid displaced by the object.

$F_B \propto V_{disp}$. (Equation 9.1: **Buoyant force**)

Mass Density

Much of what happens with fluids involves mass density (often referred to simply as density). For instance, an object with a mass density larger than the mass density of the fluid it is in generally sinks in that fluid, while an object with a lower mass density than a fluid floats in the fluid. Using the symbol ρ for mass density, the relationship between mass, density, and volume is:

$m = \rho V$, or $\rho = \dfrac{m}{V}$ (Eq. 9.2: **Connection between mass and mass density**)

Archimedes' Principle

The magnitude of the buoyant force exerted on an object by a fluid is equal to the weight of the fluid displaced by the object. This is known as Archimedes' principle:

$F_B = m_{disp} g = \rho_{fluid} V_{disp} g$. (Equation 9.3: **Archimedes' Principle**)

A General Method for Solving a Buoyancy Problem

1. Draw a diagram of the situation.
2. Draw one or more free-body diagrams, with each free-body diagram showing all the forces acting on an object as well as an appropriate coordinate system.
3. Apply Newton's Second Law to each free-body diagram.
4. If necessary, bring in Archimedes' principle, $F_B = \rho_{fluid} V_{disp} g$.
5. Put the resulting equations together and solve.

Pressure

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \quad \text{or} \quad P = \frac{F}{A}. \qquad \text{(Equation 9.6: \textbf{Pressure})}$$

The MKS unit of pressure is the pascal (Pa). $1\ \text{Pa} = 1\ \text{N/m}^2$.

Standard atmospheric pressure is 101.3 kPa, or about 1.0×10^5 Pa.

Pressure in a static fluid

$$P_2 = P_1 + \rho g h \quad . \qquad \text{(Equation 9.7: \textbf{Pressure in a static fluid})}$$

Fluid dynamics

$$A_1 v_1 = A_2 v_2 . \qquad \text{(Equation 9.8: \textbf{The Continuity Equation})}$$

$$\rho g y_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \frac{1}{2}\rho v_2^2 + P_2 . \qquad \text{(Equation 9.9: \textbf{Bernoulli's Equation})}$$

Bernoulli's Equation comes from applying energy-conservation ideas to fluids.

Viscosity, and the drag force

Viscosity is a measure of a fluid's resistance to flow, and arises because of friction between neighboring layers of fluid that are moving with different velocities.

If we use Q to denote the volume flow rate ($Q = Av$), then the volume flow rate of a viscous fluid is given by:

$$Q = \frac{\pi R^4 \Delta P}{8 \eta L} . \qquad \text{(Eq. 9.10: \textbf{The Hagen-Poiseuille equation})}$$

The equation pertains to a fluid flowing through a pipe with a radius R and a length L, with a pressure difference of ΔP between the ends of the pipe. The viscosity is denoted by η, the Greek letter eta.

The drag force on a spherical particle of radius r moving at speed v through a fluid of viscosity η is:

$$F_d = -6\pi\eta r v . \qquad \text{(Equation 9.11: \textbf{Stokes' drag force for a spherical particle})}$$

End-of-Chapter Exercises

Exercises 1 – 12 are primarily conceptual questions that are designed to see if you have understood the main concepts of the chapter.

1. A particular block floats with 30% of its volume submerged in water, but with only 20% of its volume submerged in a second fluid. Which fluid exerts a larger buoyant force on the block? Briefly justify your answer.

2. A solid brick and a solid wooden block have exactly the same dimensions. When they are placed in a bucket of water, the brick sinks to the bottom but the block floats. Which object experiences a larger buoyant force?

3. An aluminum ball and a steel ball are exactly the same size, but the aluminum ball has less mass. Both balls sink to the bottom of a glass of water. (a) Briefly explain why the aluminum ball has less mass. (b) Which ball displaces more water in the glass?

4. When a block suspended from a spring scale is half submerged in Fluid A, the spring scale reads 8 N. When the block is half submerged in Fluid B, the spring scale reads 6 N. Which fluid has a higher density? Explain.

5. (a) Explain the physics behind a drinking straw. How is it possible to use a straw to drink through? (b) If you fill the straw partly with fluid and then seal the upper end of the straw with your thumb, you can get the column of fluid to remain in the straw. How does that work?

6. To crush a soda can with atmospheric pressure, start by placing a small amount of water into an empty soda can. Carefully heat the can until steam comes out of the can. It's important to let this process continue long enough for water vapor to drive most of the air out of the can. If you then quickly remove the can from the heat source and invert it into a bowl of cold water so that the opening to the can is under water, the can should almost instantly collapse. Do this all very carefully to make sure you don't burn yourself. Explain why the can collapses.

7. Three cubes of identical volume but different density are placed in a container of fluid. The blocks are in equilibrium when they are in the positions shown in Figure 9.27. If the strings connected to blocks A and C were cut, those blocks would not be in equilibrium. Rank the cubes based on the magnitude of (a) their densities; (b) the buoyant forces they experience.

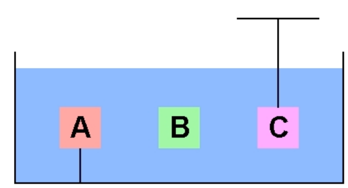
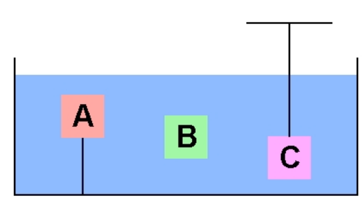


Figure 9.27: Three cubes of identical volume but different density at equilibrium in a container of fluid. For Exercise 7.

8. The two strings in Figure 9.27 are now lengthened, giving the situation shown in Figure 9.28. The cubes still have identical volumes but different densities. Rank the cubes based on the magnitude of (a) the force applied by the fluid on the top surface of the cube; (b) the force applied by the fluid on the bottom surface of the cube; (c) the buoyant force acting on the cube.

Figure 9.28: The same three cubes of identical volume but different density at equilibrium in a container of fluid as in Figure 9.27, but with longer strings. For Exercise 8.



9. A beaker of water is placed on a scale. If you dip your little finger into the water, making sure that you don't touch the beaker itself, does the scale reading increase, decrease, or stay the same? Explain.

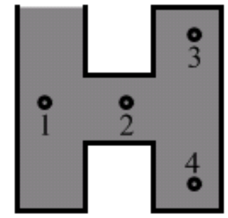


Figure 9.29: Four points are labeled in an H-shaped container of fluid, that is open to the atmosphere only at the top left. For Exercise 10.

10. Four points are labeled in a container shaped like the letter H, as shown in Figure 9.29. The container is filled with fluid, and open to the atmosphere only at the top left of the container. Rank the points based on their pressure, from largest to smallest. Use only > and/or = signs in your ranking (e.g., 2>1=3>4).

11. If you are careful when you lie down and get up again, it does not hurt to lie on a bed of nails. Supporting your weight on a single nail is a different story, however. Explain why you can lie comfortably on a bed of nails, but not with your weight supported by the point of a single nail.

12. Water is placed in a U-shaped tube, as shown in Figure 9.30. The tube is open to the atmosphere at the top left, but the tube is sealed with a rubber stopper at the top right. Can the water in the tube remain as shown, or must the level on the right drop and the level on the left rise? Explain.

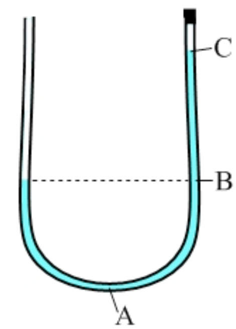


Figure 9.30: Fluid in a U-shaped tube that is open to the atmosphere on the left and sealed on the right. For Exercise 12.

Exercises 13 – 16 deal with buoyancy and/or density.

13. A particular block floats with 30% of its volume submerged in water, but with only 20% of its volume submerged in a second fluid. Taking the density of water to be 1000 kg/m^3, determine the density of (a) the block; (b) the second fluid.

14. You put some water into a glass and carefully mark the level of the top of the water. You then pour some of the water into one section of an ice cube tray and place the tray in the freezer to form a single ice cube. (a) Keeping in mind that water expands in volume by about 10% when it freezes, what will happen when the ice cube is placed back in the glass? Will the top of the water be higher, lower, or the same as it was before? (b) As the ice melts, will the water level rise, fall, or stay the same? Briefly explain your answers.

15. You have a glass of water with one or more ice cubes in it. Test your friends by asking them what will happen to the ice when you pour some oil on top of the water and ice. The density of the oil must be less than that of ice so the ice does not float to the top of the oil. Will the ice cube(s) float higher or lower in the water when the oil is poured on top, or will the level be unchanged? How will you explain the result to your friends?

16. When a block suspended from a spring scale is half submerged in Fluid A, the spring scale reads 8 N. When the block is half submerged in Fluid B, the spring scale reads 6 N. If the density of one fluid is twice as large as the density of the other, determine the buoyant force acting on the block when the block is half submerged in (a) Fluid A; (b) Fluid B. (c) What is the weight of the block?

Exercises 17 – 26 are designed to give you some practice with applying the general method of solving a typical buoyancy problem. For each exercise begin with the following parts: (a) Draw a diagram of the situation. (b) Sketch one of more free-body diagrams, including appropriate coordinate systems for each. (c) Apply Newton's Second Law.

17. A wooden block with a weight of 8.0 N floats with 60% of its volume submerged in oil. Parts (a) – (c) as described above. (d) What is the magnitude and direction of the buoyant force exerted on the block by the oil?

18. A metal ball with a weight of 12.0 N hangs from a string tied to a spring scale. When the ball is half-submerged in a particular fluid, the spring scale reads 7.0 N. The goal of this exercise is to find the buoyant force exerted on the block by the fluid when the fluid is both half-submerged and completely submerged, and to find the reading on the spring scale when the ball is completely submerged. Parts (a) – (c) as described above, making sure that you draw two sets of diagrams, one when the ball is half submerged and one when the ball is completely submerged. Find the buoyant force exerted on the ball by the fluid when the ball is (d) half submerged; (e) completely submerged. (f) What is the reading on the spring scale when the ball is completely submerged?

19. A basketball floats in a large tub of water with $1/11^{th}$ of its volume submerged. The mass of the basketball is 500 grams. The goal of this exercise is to find the radius of the ball, assuming the water has a density of 1000 kg/m^3. Parts (a) – (c) as described above. (d) What is the volume of water displaced by the ball? (e) What is the volume of the ball? (f) What is the equation for the volume of a sphere? (g) What is the radius of the basketball? (h) Did you use a particular value for g, the acceleration due to gravity? If so, what was it? Comment on the effect, if any, of you using a different value of g for the calculation.

20. Consider again the situation described in the previous exercise, with the basketball floating in the tub of water. Use $g = 9.80$ m/s^2. The goal of this exercise is to determine the force you need to apply to the ball to hold it below the surface of the water. Parts (a) – (c) as described above. You should have sketched diagrams for the floating ball in the previous exercise, so now draw a set of diagrams for the ball when you are holding it completely submerged below the water. What is the buoyant force applied to the ball by the water when the ball is (d) floating? (e) completely submerged? (f) What is the force you need to apply to the ball to hold it under the water?

21. A low-density block with a weight of 10 N is placed in a beaker of water and tied to the bottom of the beaker by a vertical string of fixed length. When the block is 25% submerged, the tension in the string is 15 N. The string will break if its tension exceeds 65 N. As water is steadily added to the beaker, the block becomes more and more submerged. Parts (a) – (c) as described above. (d) What fraction of the block is submerged at the instant the string breaks? (e) After the string breaks and the block comes to a new equilibrium position in the beaker, what fraction of the block's volume is submerged?

22. A large hot-air balloon has a mass of 300 kg, including the shell of the balloon, the basket, and the passengers, but not including the air inside the balloon itself. The goal of the exercise is to determine the volume of the balloon, assuming the air inside the balloon has a density that is 90% of the density of the air outside, that the density of the air outside the balloon is 1.30 kg/m^3, and that the balloon is floating in equilibrium above the ground. Parts (a) – (c) as described above. (d) What is the volume of the balloon?

23. You are designing a pair of Styrofoam (density 1.30 kg/m^3) shoes that you can wear to walk on water. Your mass is 50 kg, and you want the shoes to be 30% submerged in the water. Parts (a) – (c) as described above. (d) What volume of Styrofoam do you need for each shoe? (e) Estimate the volume of a typical shoebox, and compare the volume of one of the Styrofoam shoes to the volume of a shoebox.

24. You, Archimedes, suspect that the king's crown is not solid gold but is instead gold-plated lead. To test your theory, you weigh the crown, and find it to weigh 60.0 N and to have an apparent weight of 56.2 N when it is completely submerged in water. Parts (a) – (c) as described above. (d) What is the average density of the crown? (e) Is it solid gold? If not, find what fraction (by weight) is gold and what fraction is lead.

25. A square raft at the local beach is made from five 2.0-meter wooden boards that have a square cross-section measuring 40 cm x 40 cm. The goal of the exercise is to determine the largest number of children that can stand on the raft without the raft being completely submerged if the boards have a density of 500 kg/m^3. Assume that each child has a mass of 35 kg. Parts (a) – (c) as described above. (d) How many children can stand on the raft without the raft being completely submerged, assuming each child is completely out of the water?

26. After heavy rains have stopped, you go out in a boat to check the level of the water in a reservoir behind a dam (your boat is in the reservoir). You notice that the water is dangerously close to spilling over the top of the dam, and when you look up at the sky you see more dark clouds approaching. Your boat has a very heavy anchor in it, so the goal of the exercise is to determine how throwing the anchor overboard would affect the level of the water in the reservoir (assuming the boat displaces a reasonable fraction of the water in the reservoir). Parts (a) – (c) as described above, where you should draw two sets of diagrams, one when the anchor is in the boat and the other when the anchor is resting at the bottom of the reservoir. (d) Using your diagrams to help you, determine whether the water level in the reservoir rises, falls, or stays the same when you toss the anchor overboard.

Exercises 27 – 30 deal with pressure.

27. A cylindrical barrel is completely full of water and sealed at the top except for a narrow tube extending vertically through the lid. The barrel has a diameter of 80 cm, while the tube has a diameter of 1 cm. You can actually cause the lid to pop off by pouring a relatively small amount of water into the tube. To what height do you need to add water to the tube to get the lid to pop off the barrel? The lid pops off when the vector sum of the force of the atmosphere pushing down on the top of the lid and the force of the water pushing up on the bottom of the lid is 250 N up.

28. Before the negative environmental impact of mercury was fully understood, many barometers utilized a column of mercury to measure atmospheric pressure. This was first done by Evangelista Torricelli in 1643. A design for a simple barometer is shown in Figure 9.31, where there is negligible pressure at the top of the inverted column. (a) What is the height of a column of mercury that produces a pressure at its base equal to standard atmospheric pressure? (b) If water was used instead, what is the height of the column of water required? (c) Does this help explain why mercury was chosen as the working fluid in many barometers? Are there any other advantages mercury offers over water in this application?

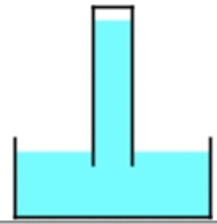


Figure 9.31: A simple liquid barometer, with an inverted column of fluid in a reservoir of that same fluid. The reservoir is open to the atmosphere, but the column is not. For Exercises 28 and 29.

29. Consider again the liquid barometer described in Exercise 28 and shown in Figure 9.31. Because of an approaching storm system, the local atmospheric pressure drops from 101.3 kPa to 99.7 kPa. (a) Does this cause the liquid to rise or fall in the tube? Determine the change in height of the column of liquid if the liquid is (b) mercury; (c) water.

30. In 2002, Tanya Streeter of the USA set a world record of 160 m for the deepest dive without breathing assistance (such as SCUBA gear). At that depth, what is the absolute pressure?

Exercises 31 – 35 address issues in fluid dynamics.

31. If you turn on a water faucet so that the water flows smoothly, you should observe that the cross-sectional area of the water stream decreases as the stream drops. (a) Explain why the water stream narrows. At a particular point, the flow speed is 10 cm/s and the stream has a cross-sectional area of 2.0 cm^2. At a point 20 cm below this point, determine (b) the flow speed, and (c) the cross-sectional area of the stream.

32. Take a bottle of water, filled to a depth of 25 cm, and carefully poke a small hole in the bottom of the bottle with a nail. (a) When you remove the cap from the bottle, what is the speed of the water emerging from the hole? (b) When you screw the cap back on the bottle, the water should stop coming out of the hole. Explain why.

33. A cylinder of height H sits on the floor. The cylinder is completely full of water, but a stream of water is emerging horizontally from the side of the cylinder at a distance h from the top. In terms of H, h, and g, determine: (a) the speed with which the water is emerging from the cylinder; (b) the time it takes the water to travel from the hole to the floor; (c) the horizontal distance traveled by the water as it falls.

34. Consider again the cylinder described in Exercise 33, but this time let's say there are three holes in the side of the cylinder. The holes are at distances of $H/4$, $H/2$, and $3H/4$ from the top of the cylinder. (a) Make a prediction – which stream of water travels furthest horizontally before reaching the floor? What do you base your prediction on? (b) Check your prediction using $H = 1.0$ m. Calculate the horizontal distance traveled, before reaching the floor, by the water from each hole.

35. While washing your hands at a sink, you determine that the water emerges from the faucet, which has a diameter of 1.0 cm, with a speed of 1.8 m/s. If the water comes from a pump located 1.5 m below the faucet, what is the absolute pressure at the pump if the pipe leading from the pump has a diameter of (a) 1.0 cm; (b) 8.0 cm?

General problems and conceptual questions

36. Consider the situation shown in Figure 9.32. (a) In figure A, what is the tension in the string? In figure B, what is the (b) buoyant force on the ball? (c) scale reading? In figure C, what is the (d) buoyant force on the ball? (e) tension in the string? (f) scale reading? In figure D, what is the (g) buoyant force on the ball? (h) scale reading?

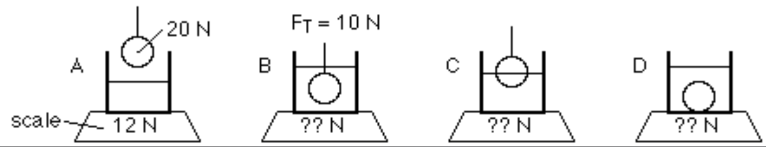


Figure 9.32: In figure A, a 20 N ball is supported by a string. It hangs over a beaker of fluid that sits on a scale. The scale reading is 12 N. In figure B the ball is completely submerged in the fluid. In figure C the ball is exactly half submerged. In figure D the string has been cut and the ball rests on the bottom of the beaker.

37. As shown in Figure 9.33, a wooden cube measuring 20.0 cm on each side floats in water with 80.0% of its volume submerged. Suspended by a string below the wooden cube is a metal cube. The metal cube measures 10.0 cm on each side and has a specific gravity of 5.00. (a) Which cube has a larger buoyant force acting on it? (b) Taking the density of water to be 1000 kg/m^3, what is the density of the wooden cube? (c) What is the tension in the string between the cubes? Assume the string itself has negligible mass and volume. (d) The pair of blocks is now placed in a different liquid. When the blocks are at equilibrium in this new liquid, the buoyant force acting on the wooden cube is exactly the same as the buoyant force acting on the metal cube. What is the density of this new liquid?

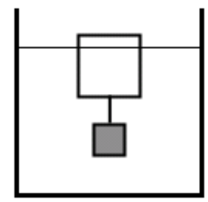


Figure 9.33: A metal cube suspended from a wooden cube, for Exercises 37 – 39.

38. Consider the situation shown in Figure 9.33, in which a wooden cube measuring 20.0 cm on each side floats in a fluid with 80.0% of its volume submerged. Suspended by a string below the wooden cube is a metal cube. The metal cube measures 10.0 cm on each side. If the wooden cube has a density of 800 kg/m^3 and the metal cube has a density of 1600 kg/m^3 what is the density of the fluid?

39. Consider the situation described in Exercise 38. (a) Describe qualitatively what will happen if the string is cut. (b) What is the magnitude and direction of the acceleration of each block immediately after the string is cut? (c) After a long time, where will the blocks be?

40. Three cubes of identical volume but different density are placed in a container of fluid. The blocks are in equilibrium when they are in the positions shown in Figure 9.34. If the strings connected to blocks A and C were cut, those blocks would not be in equilibrium. If the densities of the cubes have a 1:2:3 ratio and the magnitude of the tension in the string attached to block A is F_T, what is the magnitude of the tension in the string attached to block C? Express your answer in terms of F_T.

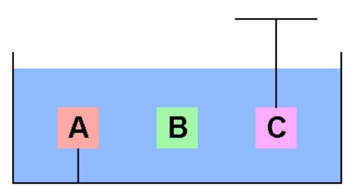


Figure 9.34: Three cubes of identical volume but different density at equilibrium in a container of fluid. For Exercise 40.

41. A toy balloon, which has a mass of 3.5 g before it is inflated, is filled with helium (with a density of 0.18 kg/m^3) to a volume of 8000 cm^3. What is the minimum mass that should be hung from the balloon to prevent it from rising up into the air?

42. Who was Archimedes? When did he live, and what else is he known for aside from buoyancy? Do some background reading and write a couple of paragraphs about him.

43. Use equation 9.7 to estimate the height of the atmosphere. Do you expect this to be a reasonably accurate measure of the height? Would your calculated value represent a lower limit or an upper limit of the true height of the atmosphere? Explain.

44. A popular demonstration about atmospheric pressure is called the Magdeburg hemispheres. Two hemispheres are held together while air is pumped out from between them. As long as there is a good seal between the hemispheres, it is extremely hard to separate them. (a) Explain why. (b) If the hemispheres are 20 cm in diameter, determine the force required to separate them, assuming all the air is evacuated from inside. This demonstration was first done by Otto von Guericke in the German town of Magdeburg around 1656, where two teams of horses tried unsuccessfully to pull the hemispheres apart.

45. Standard atmospheric pressure, which is 1 atm or 101.3 kPa, can be quoted in many different units. State atmospheric pressure in three other units, at least two of which are not SI units.

46. Pour your favorite carbonated beverage into a tall glass and watch the bubbles rise. What should happen to the size of the bubbles as they rise? Why? Can you observe the bubbles changing size as they rise?

47. A brick with a density of 4200 kg/m^3 measures 8 cm by 15 cm by 30 cm. It can be placed with any of its six faces against the floor. (a) Find the maximum and minimum values of the normal force exerted by the floor on the block when the block is resting on the floor in its various orientations. (b) Find the maximum and minimum values of the pressure associated with the block resting on the floor in its various orientations. Neglect any contribution from atmospheric pressure.

48. What are "the bends", in reference to deep-sea diving? Write a paragraph or two on what causes the bends, how to prevent them, and how to treat a diver who has the bends.

49. Mountain climbers, and people who live at high altitudes, have difficulty making a good cup of tea or coffee, despite the fact that they follow the usual procedure of heating water until it boils, and bringing the tea or coffee together with the water. What is the problem?

50. As an engineer at a mine, you are in charge of pumping water out of a flooded mine shaft that extends 20 m down from the surface. You consider two different configurations, one in which the pump is placed at the surface and it essentially sucks water out of the shaft in the same manner that a drinking straw works, and another in which the pump is placed at the bottom of the shaft and pumps water up to the surface. Assuming the pump is fully submersible (i.e., that it will work when completely submerged in water at the bottom of the shaft), which of these configurations is more appropriate for this situation? Why?

51. Water is placed in a U-shaped tube, as shown in Figure 9.35. The tube is open to the atmosphere at the top left, but the tube is sealed with a rubber stopper at the top right. Point A is 20 cm below point B, and point C is 30 cm above point B. Determine the gauge pressure at (a) point A; (b) point B; (c) point C.

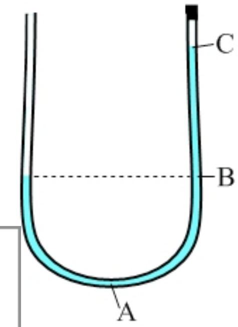


Figure 9.35: Fluid in a U-shaped tube that is open to the atmosphere on the left and sealed on the right. For Exercise 51.

52. A flexible tube can be used as a simple siphon to transfer fluid from one container to a lower container. This is shown in Figure 9.36. The fluid has a density of 800 kg/m^3. If the tube has a cross-sectional area that is much smaller than the cross-sectional area of the higher container, what is the speed of the fluid at (a) point Z? (b) point Y? (c) What is the absolute pressure at point Y? See the dimensions given in Figure 9.35, and take atmospheric pressure to be 101.3 kPa.

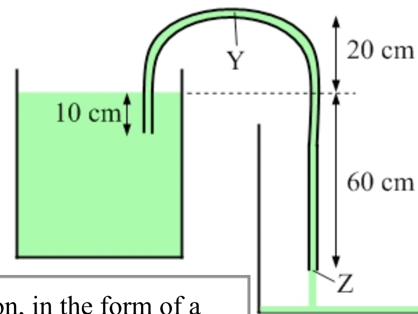


Figure 9.36: A siphon, in the form of a flexible tube, being used to transfer fluid from the container on the left to the container on the right.

53. A venturi tube is a tube with a constriction in it. Pressure in a venturi tube can be measured by attaching a U-shaped fluid-filled device to the venturi tube as shown in Figure 9.37. (a) If the fluid in the U is water, and there is a 10 cm difference between the water levels on the two sides, what is the pressure difference between points 1 and 2 in the venturi tube? (b) The venturi tube has air flowing through it. If the cross-sectional area of the venturi tube is 6 times larger at point 1 than it is at point 2, what is the air speed at point 2?

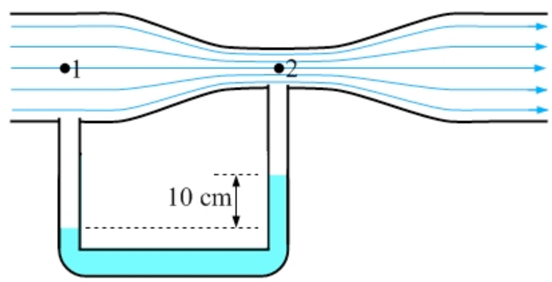


Figure 9.37: A venturi tube with a water-filled U-tube to measure pressure, for Exercise 53.

54. Your town supplies water with the aid of a tall water tower that is on top of the highest hill in town. The top surface of the water in the tank is open to the atmosphere. (a) Explain why it makes sense to use such a water tower in the water-distribution system. (b) Would you expect the water pressure to be highest in homes at higher elevations or at lower elevations in the town, all other factors being equal?

55. Consider the water tower described in the previous exercise. Opening a valve at the base of the tower allows water to flow out a pipe that bends up, projecting water straight up into the air. If the water reaches a maximum height of 8.5 m above the base of the tower, to what depth is the water tower filled with water?

56. As you wash your car, you are using an ordinary garden hose to spray water over the car. You notice that, if you cover most of the open end of the hose with your hand, the water sprays out with a higher speed. Explain how this works.

57. About once every 30 minutes, a geyser known as Old Faceful projects water 15 m straight up into the air. (a) What is the speed of the water when it emerges from the ground? (b) Assuming the water travels to the surface through a narrow crack that extends 10 m below the surface, and that the water comes from a chamber with a large cross-sectional area, what is the pressure in the chamber?

58. A drinking fountain projects water at a 45˚ angle with respect to the horizontal. The water reaches a maximum height of 10 cm above the opening of the fountain. With what speed was it projected into the air?

59. While taking a shower, you notice that the shower head is made up of 40 small round openings, each with a radius of 1.5 mm. You also determine that it takes 8.0 s for the shower to completely fill a 1-liter container you hold in the water stream. The water in your house is pumped by a pump in the basement, 7.2 m below the level of the shower head. If the pump maintains an absolute pressure of 1.5 atm., what is the cross-sectional area of the pipe connected to the pump?

60. There were many famous members of the Bernoulli family, and they did not all get along. Do some background reading on the Bernoullis and write a paragraph or two about them, making sure that you identify the member of the family credited with the Bernoulli equation we used in this chapter.

61. Three students are having a conversation. Explain what you think is correct about what they say, and what you think is incorrect.

Jenna: OK, so here's the question. "A block floats 60% submerged in Fluid A, and only 40% submerged in Fluid B. Which fluid applies a larger buoyant force to the block?" That should be easy, right? The buoyant force is proportional to the volume displaced, so Fluid A exerts the larger force.

Jaime: I think they give the same buoyant force. In both cases, the fluid has to support the full weight of the block. So, the buoyant force equals mg in both cases.

Jenna: Is it the full mg, though? Or is it 60% of mg in the first case and 40% in the second?

Michael: What if we think about densities? Fluid B has a bigger density, and the buoyant force is proportional to the fluid density, so it should really be B that exerts the bigger force.

Jaime: That's actually why it works out to be equal. The buoyant force depends on both the volume displaced and the density, so B has less volume displaced but more density, and it balances out.

62. In Essential Question 9.10, we analyzed what would happen to the flow rate if the radius of a blood vessel decreased by 5%. For that same situation, if the heart adjusted to maintain the original flow rate, by what factor would the pressure difference between the ends of the blood vessel increase?

63. By what factor would the cross-sectional area of a blood vessel have to change (all other factors being unchanged) for the flow rate to be reduced by 50%?

64. If the radius of a blood vessel drops to 80% of its original radius because of the buildup of plaque, and the body responds by increasing the pressure difference across the blood vessel by 10%, what will have happened to the flow rate?

65. A patient in the hospital is receiving a saline solution through a needle in their arm. The needle is 3.0 cm long, horizontal, with one end in a vein in the arm, and the other end attached to a wide tube that extends down from the bag of solution, which hangs from a pole so that the fluid level is 90 cm above the needle. The inner radius of the needle is 0.20 mm. The top of the fluid is exposed to the atmosphere, and the flow rate of the fluid (which has a density of 1025 kg/m^3 and a viscosity of 0.0010 Pa s) through the needle is 0.30 L/h. What is the average gauge pressure inside the vein where the needle is?

66. At 20˚C, honey has a viscosity about 4000 times larger than that of water. For a particular tube, 400 mL/s of water will flow through for a particular pressure difference. (a) If the same tube, with the same pressure difference, is used for honey, what will the flow rate be? (b) If you want the flow rate for honey to be the same as that for water, with the same pressure difference, by what factor should you increase the radius of the tube?

67. In Example 9.11, we discussed an ultracentrifuge with an angular speed of 5000 rpm, producing a centripetal acceleration of 5000 *g*. (a) What is the angular speed, in rad/s? (b) What is the average radius of the circle through which the sample spins, to produce 5000 *g?*

68. A particular ultracentrifuge operates at two different angular speeds, one twice the other, and has two different places where sample tubes can be loaded into the device, one twice as far from the center as the other. If the smallest centripetal acceleration you can obtain with this centrifuge is 2000 *g*, what are the other values you can obtain for the centripetal acceleration?

69. Section 9.11, which discussed the terminal speed of an object falling through a viscous fluid, included the following statement: "In general, the smaller the object, the smaller the magnitude of the terminal velocity." In Example 9.11, we also derived an equation for the terminal speed, which was:

$$v_t = \left(1 - \frac{\rho_{fluid}}{\rho_{object}}\right)\frac{mg}{6\pi\eta r}$$

Note the factor of *r* in the denominator on the right side of the equation. You have two balls, made from identical material, so they have the same density, but one has twice the radius of the other. The smaller ball has a terminal speed of 4.0 mm/s in a particular viscous fluid. Predict the terminal speed of the larger ball in this same fluid. Is your prediction consistent with the statement that smaller objects have smaller terminal speeds? Explain why or why not.

70. Find a clear plastic shampoo bottle, almost full of shampoo. Make sure the bottle is tightly capped! Shake the bottle to get some air bubbles, with a good mixture of bubble sizes. When you invert the bottle, what do you observe? Which bubbles rise fastest? Is this consistent with what we learned about objects moving through a viscous fluid, or not?

71. You drop a steel ball bearing, with a radius of 2.0 mm, into a beaker of honey. Note that honey has a viscosity of 4.0 Pa s and a density of 1360 kg/m^3, and steel has a density of 7800 kg/m^3. (a) What is the terminal speed of the ball bearing? (b) Aluminum has a density of 2700 kg/m^3. What radius should an aluminum ball have to have the same terminal speed in honey that the steel ball has?

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Examples

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Solutions

- Answers to Selected End of Chapter Problems
- Sample Question Solutions

Additional Links

- *PhET* simulation: Balloons and Buoyancy