7-1 The Law of Conservation of Energy

In Chapter 6, we looked at the work-energy relationship: $W_{net} = \Delta K$. If we break the net work up into two pieces, W_{con} , the work done by conservative forces (such as gravity), and W_{nc} , the work done by non-conservative forces (such as tension or friction), then we can write the equation as: $W_{con} + W_{nc} = \Delta K$. Recall that the work done by a conservative force does not depend on the path taken between points. The work done by non-conservative forces is path dependent, however.

As we did in Chapter 6 with the work done by gravity, the work done by any conservative force can be expressed in terms of potential energy, using $W_{con} = -\Delta U$. We can now write the work-energy equation as

$-\Delta U + W_{nc} = \Delta K$. (Equation A).

Let's use *i* to denote the initial state and *f* to denote the final state. The change in a quantity is its final value minus its initial value, so we can use $\Delta K = K_f - K_i$ and $\Delta U = U_f - U_i$. Substituting these expressions into Equation A gives $-(U_f - U_i) + W_{nc} = K_f - K_i$. With a bit of re-arranging to make everything positive, we get Equation 7.1, below.

Equation 7.1 expresses a basic statement of the **Law of Conservation of Energy**: "Energy can neither be created nor destroyed, it can only be changed from one form to another."

$K_i + U_i + W_{nc} = K_f + U_f$ (Equation 7.1: **Conservation of energy**)

The law of conservation of energy is so important that we will use it in Chapters 8, 9, and 10, as well as in many chapters after that. With equation 7.1, we have the only equation we need to solve virtually any energy problem. Let's discuss its five different components.

K_i and K_f are the initial and final values of the kinetic energy, respectively.

U_i and U_f are the initial and final values of the potential energy, respectively.

W_{nc} is the work done by non-conservative forces (such as by the force of friction).

The conservation of energy equation is very flexible. So far, we have discussed one form of kinetic energy, the translational kinetic energy given by $K = (1/2)mv^2$. When we get to Chapter 11, we will be able to build rotational kinetic energy into energy conservation without needing to modify equation 7.1. Similarly, no change in the equation will be necessary when we define a general form of gravitational potential energy in Chapter 8, and define spring potential energy in Chapter 12. It will only be necessary to expand our definitions of potential and kinetic energy.

Mechanical energy is the sum of the potential and the kinetic energies. If no net work is done by non-conservative forces (if $W_{nc} = 0$), then mechanical energy is conserved. This is the **principle of the conservation of mechanical energy**.

EXAMPLE 7.1 – A frictional best-seller

A popular book, with a mass of 1.2 kg, is pushed across a table. The book has an initial speed of 2.0 m/s, and it comes to rest after sliding through a distance of 0.80 m.

(a) What is the work done by friction in this situation?
(b) What is the average force of friction acting on the book as it slides?

SOLUTION

(a) As usual, we should draw a diagram of the situation and a free-body diagram. These diagrams are shown in Figure 7.1. Three forces act on the book as it slides. The normal force is directed up, at 90° to the displacement, so the normal force does no work. The effect of the force of gravity is accounted for via the potential energy terms in equation 7.1, but the gravitational potential energy does not change, because the book does not move up or down. The only force that affects the energy is the force of friction. The work done by friction is accounted for by the W_{nc} term in the conservation of energy equation.

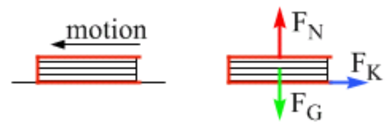

Figure 7.1: A diagram of the sliding book and a free-body diagram showing the forces acting on it as it slides.

So, the five-term conservation of energy equation, $K_i + U_i + W_{nc} = K_f + U_f$, can be reduced to two terms, because $U_i = U_f$ and the final kinetic energy $K_f = 0$. We are left with:

$$K_i + W_{nc} = 0 \qquad \text{so, } W_{nc} = -K_i = -\frac{1}{2}mv_i^2 = -\frac{1}{2}(1.2 \text{ kg})(2.0 \text{ m/s})^2 = -2.4 \text{ J}.$$

The work done by the non-conservative force, which is kinetic friction in this case, is negative because the force of friction is opposite in direction to the displacement.

(b) To find the force of kinetic friction, F_K, use the definition of work. In this case, we get: $W_{nc} = F_K \,\Delta r \cos\theta = -F_K \,\Delta r$.

Solving for the force of friction gives $F_K = -\dfrac{W_{nc}}{\Delta r} = -\dfrac{-2.4 \text{ J}}{0.80 \text{ m}} = 3.0 \text{ N}$.

Related End-of-Chapter Exercises: 37, 40, 42.

A general method for solving a problem involving energy conservation

This general method can be applied to a wide variety of situations.

1. Draw a diagram of the situation. Usually, we use energy to relate a system at one point, or instant in time, to the system at a different point, or a different instant.
2. Write out equation 7.1, $K_i + U_i + W_{nc} = K_f + U_f$.
3. Choose a level to be the zero for gravitational potential energy. Defining the zero level so that either U_i or U_f (or both) is zero is often best.
4. Identify the terms in the equation that are zero.
5. Take the remaining terms and solve.

Essential Question 7.1: Did we have to solve Example 7.1 using energy ideas, or could we have used forces and Newton's second law instead?

Answer to Essential Question 7.1: In the case of the book sliding on the table, we can apply either an energy analysis or a force analysis. Let's now compare these different methods.

7-2 Comparing the Energy and Force Approaches

In Example 7.1, there is no real advantage in using an energy analysis over a force analysis. In some cases, however, the energy approach is much easier than the force approach.

EXPLORATION 7.2A – Which ball has the higher speed?

Three identical balls are launched with equal speeds *v* from a height *h* above level ground. Ball A is launched horizontally, while the initial velocity of ball B is at 30º above the horizontal, and the initial velocity of ball C is at 45º below the horizontal. Rank the three balls, based on their speeds when they reach the ground, from largest to smallest. Neglect air resistance.

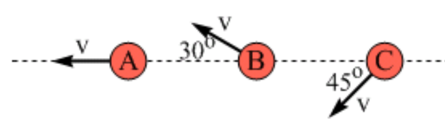

Figure 7.2: A diagram showing the directions of the initial velocities of the three balls in Exploration 7.2A.

Step 1 – ***Sketch a diagram of the situation.*** See Figure 7.2.

Step 2 – ***Briefly describe how to solve this problem using methods applied in earlier chapters.*** Consider the projectile-motion analysis we applied in chapter 4. For each ball, we would break the initial velocity into components, determine the *y*-component of the final velocity using one of the constant-acceleration equations, and then find the magnitude of the final velocity by using the Pythagorean theorem. We would have to go through the process three times, once for each ball.

Step 3 – ***Instead, solve the problem using an energy approach.*** Our starting point for energy is always the conservation of energy equation, $K_i + U_i + W_{nc} = K_f + U_f$. There is no air resistance, so $W_{nc} = 0$. If we define the zero for gravitational potential energy as the ground level, then $U_f = 0$, and $U_i = mgh$, where *m* is the mass of a ball (each ball has the same mass). Substituting this expression into the conservation of energy equation gives: $K_i + mgh = K_f$.

Both terms on the left are the same for all three balls. The balls have the same initial kinetic energy and they experience the same change in potential energy. Thus, all three balls have identical final kinetic energies. Because $K_f = (1/2)mv_f^2$, and the balls have equal masses, the final speeds are equal. Based on one energy analysis that works for all three balls, instead of three separate projectile-motion analyses, the ranking of the balls based on final speed is A=B=C.

Step 4 – ***Would the answer be different if the balls had unequal masses?*** Starting from $K_i + mgh = K_f$, and using the definition of kinetic energy, we can show that mass is irrelevant:

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 .$$

Factors of mass cancel, giving: $v_f = \sqrt{2gh + v_i^2}$.

So, the balls have the same final speed even if their masses are different.

Key idea for solving problems: We now have two powerful ways of analyzing physical situations. We can either apply force ideas, or apply energy ideas. In certain situations the energy approach is simpler than the force approach. **Related End-of-Chapter Exercises: 6, 38.**

EXPLORATION 7.2B – Which cart wins the race?

As shown in Figure 7.3, two identical carts have a race on separate tracks. Cart A's track follows a straight path sloping down, while cart B's track dips down below A's just after the start and rises up to meet A's again just before the finish line. If the carts are released at the same time, which cart reaches the end of the track first? Make a prediction, and justify your answer.

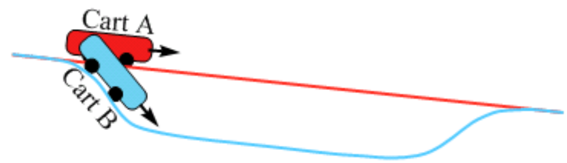

Figure 7.3: A race between two identical carts. Cart A's track has a constant slope, while cart B's track dips down below A's before rising up to meet A's again at the end.

After considering the three balls of Exploration 7.2A, it is tempting to predict that the race will end in a tie. An energy analysis, for instance, follows the same logic as that in Exploration 7.2A, except that the analysis in this case is simplified by the initial kinetic energy being zero. Once again, energy tells us that the carts should arrive at the finish line with the same speed. Having the same speed does not mean that they arrive at the same time, however.

Another popular answer is that A wins the race because B travels farther. In actuality, for most tracks, cart B wins the race. Cart A gradually picks up speed as it loses potential energy. In contrast, cart B immediately drops below A, transforming potential energy into kinetic energy, and reaching a speed larger than that of cart A. The carts then travel along parallel paths, with B always moving faster than A. Even while B slows as it climbs the hill near the end, it is traveling faster than A. The larger distance B travels is more than made up for by B's larger average speed.

Key idea about energy and time: Energy can be a powerful concept, but energy generally gives us no direct information about time. **Related End-of-Chapter Exercises: 4, 35.**

Let's compare and contrast the energy approach with the force approach. Energy can be a very effective tool, because in many cases we only have to consider the initial and final states and we don't have to worry about how the system gets from one state to the other. On the other hand, energy tells us nothing about the time it takes to get from one state to another. In Exploration 7.2B, for instance, the three balls reach the ground at different times, but we would have to use forces, and the constant-acceleration equations, to find those times. Energy is also a scalar, so it tells us nothing about direction. Energy is perfect, however, for connecting speed and position.

If we want to learn about time, or about the direction of a vector, analyzing forces is a better approach. So far, though, we are limited to applying force concepts to situations in which the net force is constant, when we can apply the constant-acceleration equations. We will go beyond this in Chapter 8 but, at the level of physics we are concerned with in this book, we will always be limited in how far we can go with forces. A good example of the limitations of force is shown in Figure 7.4, where an object slides from point A to point B along various paths. If the object comes down path 1, the straight line connecting A and B, we can use forces or energy to analyze the motion, even if friction acts as the object slides. If the object slides along a path other than path 1, then we can't get far with forces. The difficulty is that the force approach is ***path dependent*** – the forces applied to the object depend on the path, and the forces change if the object moves along a different path.

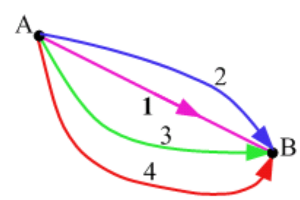

Figure 7.4: Various paths for an object to slide along in traveling from A to B.

Essential Question 7.2: Could we use energy to analyze the motion along paths 2, 3, or 4 in Figure 7.4?

Answer to Essential Question 7.2: If there is no friction, the energy analysis is path independent, so we treat all paths the same. If friction does act on the object, however, even energy is hard to apply, because then the work done by friction depends on the path.

7-3 Energy Bar Graphs: Visualizing Energy Transfer

Energy conservation is a powerful tool. To make it easier for us to use this tool, it can be useful to use a visual aid to keep track of the various types of energy. One way to visualize how energy in a system is transformed from one type of energy to another is to use energy bar graphs. Consider the following example.

EXAMPLE 7.3A – Learning to use energy bar graphs

Consider a ball that you throw straight up into the air, and neglect air resistance. Define the gravitational potential energy to be zero at the point from which you release it. Draw energy bar graphs to show how the ball's mechanical energy is divided between kinetic energy and gravitational potential energy at the following points: (a) the point you release it; (b) the point halfway between the release point and the maximum-height point, on the way up; (c) the maximum-height point; (d) the point halfway between the release point and the maximum height, on the way down; and (e) the release point, on the way down.

SOLUTION

The five sets of bar graphs are shown in Figure 7.5. The vertical position of the ball is indicated above each set of graphs, making it clearer why the second and fourth sets of graphs are the same, and why the first and last sets are the same. As the ball rises, its kinetic energy is transformed into potential energy, reaching 100% potential at the maximum height point. As the ball falls, the potential energy transforms back to kinetic energy.

Related End-of-Chapter Exercises: 21, 22.

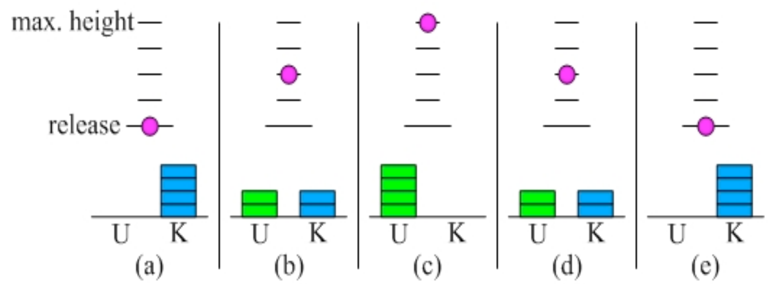

Figure 7.5: Energy bar graphs for a ball going straight up and down. The ball's corresponding vertical position is shown above each of the bar

In this case, the total mechanical energy is conserved. Let's do another example to see how bar graphs are used to depict a situation in which energy is not conserved.

EXAMPLE 7.3B – Extending the use of energy bar graphs

A string is tied to a block that has a mass of 2.0 kg. As shown in Figure 7.6, the string passes over a pulley, and you hang onto the end of the string to prevent the block from moving. Initially, the block is 1.0 m above the ground. You then pull down on the string so the block accelerates upward at a constant rate of 4.0 m/s^2. Use g = 10 m/s^2, and define the zero for gravitational potential energy to be at ground level. Draw bar graphs to show the block's gravitational potential energy, kinetic energy, total mechanical energy, and the work you have done on the block (a) at the instant the block starts to move, and (b) 0.50 s after the block starts to move.

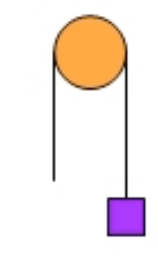

Figure 7.6: The system of the string, pulley, and block. You hold the string.

SOLUTION

(a) Let's define up to be the positive y-direction. Initially, the block is at rest and you have not done any work, so the energy is all gravitational potential energy. This relation is written $U_i = mgy_i = (2.0 \text{ kg}) \times (10 \text{ m/s}^2) \times (1.0 \text{ m}) = 20 \text{ J}$. Because there is no kinetic energy, the block's total mechanical energy is 20 J. The bar graphs for the initial energies are shown in Figure 7.7.

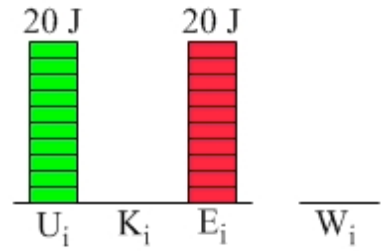

Figure 7.7: Energy bar graphs showing the energy at the start of the motion. Each small rectangle represents an energy of 2 J. E is the total mechanical energy.

(b) To find the block's gravitational potential energy at $t = 0.50$ s, we must determine how far off the ground the block is at that time. Because the acceleration is constant, we can use a constant-acceleration equation:

$$\vec{y}_{t=0.5} = \vec{y}_i + \vec{v}_{iy} t + \frac{1}{2}\vec{a}_y t^2 = +1.0 \text{ m} + 0 + \frac{1}{2}\left(+4.0 \text{ m/s}^2\right) \times (0.50 \text{ s})^2 = +1.5 \text{ m} .$$

This y-position corresponds to a gravitational potential energy of:

$$U_{t=0.5} = mgy_{t=0.5} = (2.0 \text{ kg}) \times (10 \text{ m/s}^2) \times (+1.5 \text{ m}) = +30 \text{ J} .$$

To find the kinetic energy at $t = 0.50$ s, let's find the speed of the block. We can use a constant-acceleration equation to find the block's velocity at $t = 0.50$ s and then take the magnitude to get the speed.

$$\vec{v}_{y,t=0.5} = \vec{v}_{iy} + \vec{a}_y t = 0 + \left(+4.0 \text{ m/s}^2\right) \times (0.50 \text{ s}) = +2.0 \text{ m/s} .$$

The kinetic energy is then

$$K_{t=0.5} = \frac{1}{2}mv^2 = \frac{1}{2}(2.0 \text{ kg}) \times (2.0 \text{ m/s})^2 = 4.0 \text{ J} .$$

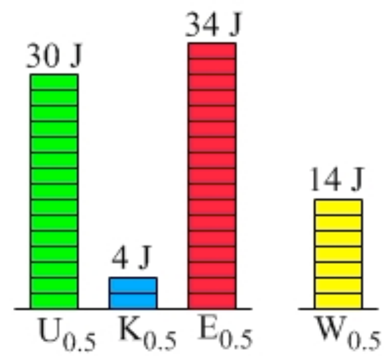

Figure 7.8: Energy bar graphs showing the energy of the block, and the work (W) you have done on the block up to that point, at $t = 0.50$ s.

The total mechanical energy is the sum of the potential energy and the kinetic energy, for a total of 34 J. This is 14 J larger than the initial mechanical energy, meaning that you must have done 14 J of work on the block. All the energies are represented by the bar graphs in Figure 7.8.

To summarize this section, the visualization technique of drawing energy bar graphs can be applied to systems in which mechanical energy is conserved, or even to systems when it is not conserved. Drawing these bar graphs is a good way to keep track of the different types of energy in a system.

Related End-of-Chapter Exercises: 23, 24.

Essential Question 7.3: In Example 7.3B, the bar graphs representing the various energies and the work are all either positive or zero. Could any of these be negative in some circumstances?

Answer to Essential Question 7.3: All of them can be negative except for the kinetic energy, which can't be negative because $K = (1/2)mv^2$ and neither the square of the speed nor the mass can be negative. What is key is how the energies change, not what the values of the energies are.

7-4 Momentum and Collisions

Let's extend our understanding of momentum by analyzing a **collision**, which is an event in which two objects interact. As we learned in Chapter 6, Newton's third law tells us that, when no net external force acts on a system, the total momentum of the system is conserved. The momenta of the individual objects can change, but the total momentum of the system does not.

Generally, when we analyze a collision, we look at the situation immediately before the collision and compare it to the situation immediately after the collision. What happens during the collision itself can be interesting, and complicated. Fortunately, by using momentum we don't have to worry about such complications. The usual starting point in analyzing a collision is to write down a conservation of momentum equation reflecting the following relation:

Total momentum before the collision = total momentum after the collision.

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$ (Equation 7.2: **Momentum conservation**)

where the subscripts *i* and *f* stand for initial and final, and the two colliding objects are denoted by 1 and 2.

EXPLORATION 7.4 – Two carts collide…again

Two identical carts experience a collision on a horizontal track. Before the collision, cart 1 is moving at speed *v* to the right, directly toward cart 2, which is at rest. Immediately after the collision, cart 2 is moving with a velocity of $v/2$ to the right.

Step 1 - *What is the velocity of cart 1 immediately after the collision?* Let's begin, as usual, by drawing a diagram of the situation in Figure 7.9, showing the carts before and after the collision.

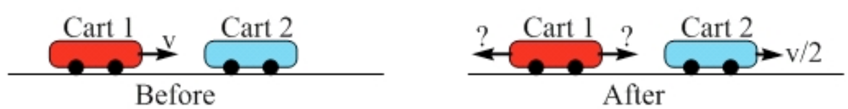

Figure 7.9: Two carts, immediately before and immediately after their collision.

The first step in applying equation 7.2 is to remember that momentum is a vector. Let's define right as the positive *x*-direction. We can say that each cart has a mass *m*, and we are given that $\vec{v}_{1i} = +v$, $\vec{v}_{2i} = 0$, and $\vec{v}_{2f} = +v/2$. Substituting all these terms into the conservation of momentum equation gives:

$$+mv = m\vec{v}_{1f} + m\frac{v}{2}$$

Dividing out a factor of *m* and solving for the velocity of cart 1 after the collision gives:

$$\vec{v}_{1f} = +\frac{v}{2}.$$

The two carts have the same velocity, and thus move together, after the collision. We could arrange this special case by attaching Velcro to both carts so they stick together. When the objects move together afterwards, we say that the collision is ***completely inelastic***.

Step 2 - ***Is kinetic energy conserved in this collision?*** Kinetic energy does not have to be conserved in a collision, although in certain special cases it is. Let's see what happens to the kinetic energy in this case.

Before the collision: $K_i = \frac{1}{2}mv_{1i}^2 + \frac{1}{2}mv_{2i}^2 = \frac{1}{2}mv^2$.

After the collision: $K_f = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2 = \frac{K_i}{2}$.

In this case, only 50% of the kinetic energy from before the collision is in the system as kinetic energy after the collision. The total energy has to be conserved, but in this case, some of the kinetic energy of cart 1 before the collision is transformed to other forms of energy (such as thermal energy, which is energy associated with the motion of molecules, and sound energy) in the collision process.

Step 3 - ***What is the velocity of the system's center of mass before the collision?*** By dividing both sides of Equation 6.4, for the position of the center of mass, by a time interval, Δt, and using the definition of velocity, we can obtain an equation for the velocity of the center of mass:

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$ (Equation 7.3: **Velocity of the center of mass**)

The m's represent the masses of the various pieces of the object or system. The terms in the numerator on the right represent the momenta of the individual parts of the system, so the equation really says that the total momentum of the system is the vector sum of the momenta of its parts, which seems sensible.

Applying the equation to the two-cart system before the collision gives:

$$\vec{v}_{CM,i} = \frac{+mv + m \times 0}{m+m} = +\frac{v}{2}.$$

This result makes sense because the center of mass is halfway between the carts, so the center of mass covers half the distance as cart 1 does in the same time.

Step 4 - ***What is the velocity of the system's center of mass after the collision?*** Applying equation 7.3 to the system after the collision gives:

$$\vec{v}_{CM,f} = \frac{+m\frac{v}{2} + m\frac{v}{2}}{m+m} = +\frac{v}{2}.$$

It should come as no surprise that the velocity of the center of mass after the collision is the same as the velocity of the center of mass before the collision. Rather, this result is expected as a consequence of momentum conservation. In short, the center of mass does not even register that a collision has taken place.

Key ideas: In a collision, in general, the system's momentum is conserved while the system's kinetic energy is not necessarily conserved. In addition, in general, the motion of the system's center of mass is unaffected by the collision. **Related End-of-Chapter Exercises: 25 – 28.**

Essential Question 7.4: Under what condition is the momentum of a system conserved in a collision?

Answer to Essential Question 7.4: For momentum to be conserved, either no net force is acting on the system, or the net force must act over such a small time interval that it has a negligible effect on the momentum of the system.

7-5 Classifying Collisions

If we attach Velcro to our colliding carts, and the carts stick together after the collision, as in Exploration 7.4, the collision is **completely inelastic**. If we remove the Velcro, so the carts do not stick together, we can set up a collision with the same initial conditions (cart 1 moving toward cart 2, which is stationary) and get a variety of outcomes. We generally classify these outcomes into four categories, depending on what happens to the kinetic energy in the collision.

We can also define a parameter k called the ***elasticity***. Elasticity is the ratio of the relative velocity of the two colliding objects after the collision to the negative of their relative velocity before the collision. By this definition, the elasticity should always be positive:

$$k = \frac{\vec{v}_{2f} - \vec{v}_{1f}}{\vec{v}_{1i} - \vec{v}_{2i}} \qquad \text{(Equation 7.4: \textbf{Elasticity})}$$

The four categories of collisions can also be defined in terms of the elasticity.

Type of Collision	Kinetic Energy	Elasticity	Example
Super-elastic	$K_f > K_i$	$k > 1$	Carts are initially stationary, then pushed apart by a spring-loaded piston, as in Exploration 6.4. An explosion.
Elastic	$K_f = K_i$	$k = 1$	Carts with repelling magnets.
Inelastic	$K_f < K_i$	$k < 1$	Describes most collisions, such as two cars that make contact when colliding but that don't stick together.
Completely inelastic	$K_f < K_i$, and the objects stick together	$k = 0$	Carts with Velcro, as in Exploration 7-3, or chewing gum hitting the sidewalk.

Table 7.1: Collisions can be classified in terms of what happens to the kinetic energy or in terms of the elasticity. Note that, in an elastic collision, the fact that $k = 1$ can be obtained by combining the momentum conservation equation with the conservation of kinetic energy equation.

EXAMPLE 7.5 – An assist from gravity

Sending a space probe from Earth to another planet requires a great deal of energy. In many cases, a significant fraction of the probe's kinetic energy can be provided by a third planet, through a process known as a ***gravitational assist***. For instance, the Cassini-Huygens space probe launched on October 15, 1997, used four gravitational assists, two from Venus, one from Earth, and one from Jupiter, to speed it on its more than 1 billion km trip to Saturn, arriving there on July 1, 2004. We can treat a gravitational assist as an elastic collision, because the long-range interaction of the probe and the planet provides no mechanism for a loss of mechanical energy.

Figure 7.10: The Cassini-Huygens space probe while it was being assembled. The desk and chair at the lower left give a sense of the scale. Photo courtesy NASA/JPL-Caltech.

A space probe with a speed v is approaching Venus, which is traveling at a velocity V in the opposite direction. The probe's trajectory around the planet reverses the direction of the probe's velocity. (a) How fast does the probe travel away from Venus? (b) If $v = 1 \times 10^5$ m/s, and $V = 3.5 \times 10^5$ m/s, what is the ratio of the probe's final kinetic energy to its initial kinetic energy?

SOLUTION

Let's begin with a diagram of the situation, shown in Figure 7.11. Although we will analyze this situation as a collision, the objects do not make contact with one another.

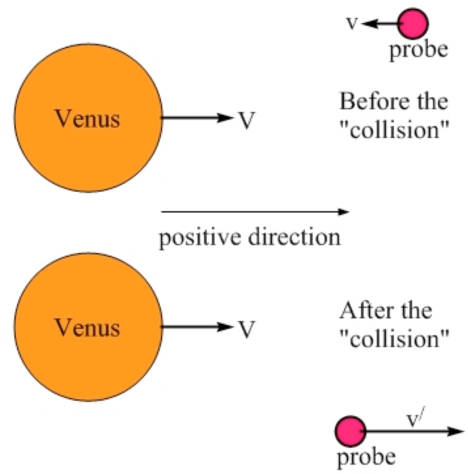

Figure 7.11: Before and After situations for the interaction between Venus and the space probe.

(a) The probe's speed depends on how far away it is from Venus. Because no distances were given, let's assume the probe has speed v when it is so far from Venus that the gravitational pull of Venus is negligible. We will work out the final velocity under the same assumption. This is an elastic collision. We could apply conservation of momentum and conservation of energy, but we were not given any masses and the resulting equations can be challenging to combine to find the final velocity of the probe. Let's try working with the elasticity k instead.

Because this collision is elastic, $k = 1$. The elasticity is the ratio of the final relative speed to the initial relative speed, so those two relative speeds must be equal for k to equal 1. Also, we can reasonably assume that the probe's mass is much smaller than the planet's mass and that the planet's motion is unaffected by its interaction with the probe. Thus, the planet's velocity is V in the direction shown both before and after the collision.

Defining the direction of the planet's velocity as the positive direction, plugging everything into the elasticity equation gives:

$$1 = \frac{\vec{v}_{2f} - \vec{v}_{1f}}{\vec{v}_{1i} - \vec{v}_{2i}} = \frac{V - v_{1f}}{-v - V} \quad ,$$

where both the numerator and denominator are negative.

Solving for the final speed of the probe gives $v_{1f} = 2V + v$.

(b) Substituting in the numbers gives a final speed of 8×10^5 m/s, an increase by a factor of 8 in speed. Kinetic energy is proportional to the square of the speed, so the probe's kinetic energy is increased by a factor of 64. A more formal way to show this relation is the following:

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}mv_f^2}{\frac{1}{2}mv_i^2} = \frac{v_f^2}{v_i^2} = \frac{\left(8x10^5 \text{ m/s}\right)^2}{\left(1x10^5 \text{ m/s}\right)^2} = 64 \, .$$

The probe gains an enormous amount of energy, and it does so without requiring massive amounts of fuel to be burned. This is why probes to the outer planets are often first sent toward Venus, because the large increase in speed more than makes up for the extra distance traveled.

Related End-of-Chapter Exercises: 11, 54, 55.

Essential Question 7.5: In our analysis in example 7.5, we assumed that the planet's speed is constant. Is this absolutely correct? Is it a reasonable assumption?

Answer to Essential Question 7.5: To conserve energy of the planet-probe system, the planet's speed must decrease when the probe's speed increases. Because the planet's mass is so much larger than the probe's, this decrease in speed is negligible. Thus, the assumption is reasonable.

7-6 Collisions in Two Dimensions

Momentum conservation also applies in two and three dimensions. The standard approach to a two-dimensional (or even three-dimensional) problem is to break the momentum into components and conserve momentum in both the *x* and *y* directions separately. For colliding objects, the conservation of momentum equation in the *x*-direction, for instance, is:

$\vec{p}_{1ix} + \vec{p}_{2ix} = \vec{p}_{1fx} + \vec{p}_{2fx}$. (Eq. 7.5: **Conserving momentum in the x-direction**)

This can be written in an equivalent form:

$m_1\vec{v}_{1ix} + m_2\vec{v}_{2ix} = m_1\vec{v}_{1fx} + m_2\vec{v}_{2fx}$ (Eq. 7.6: **Momentum conservation, *x*-direction**)

Similar equations apply in the *y*-direction.

EXPLORATION 7.6 – A two-dimensional collision

An object of mass *m,* moving in the +*x*-direction with a velocity of 5.0 m/s, collides with an object of mass 2*m*. Before the collision, the second object has a velocity given by $\vec{v}_{2i} = -3.0 \text{ m/s } \hat{x} + 4.0 \text{ m/s } \hat{y}$, while, after the collision, its velocity is 3.0 m/s in the +*y*-direction. What is the velocity of the first object after the collision?

Step 1 – ***Draw a diagram of the situation.*** This is shown in Figure 7.12.

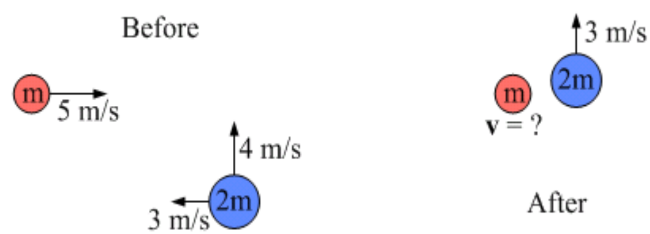

Figure 7.12: A diagram of the objects before and after they collide.

Step 2 - *Set up a table showing the momentum components of each object before and after the collision.* Organizing components into a table helps us keep the *x*-direction information separate from the *y*-direction information. We can combine the components into one vector at the end.

	***x*-direction**	***y*-direction**
Before the collision	$\vec{p}_{1ix} = m\vec{v}_{1ix} = +(5 \text{ m/s})m$	$\vec{p}_{1iy} = m\vec{v}_{1iy} = 0$
	$\vec{p}_{2ix} = 2m\vec{v}_{2ix} = -(6 \text{ m/s})m$	$\vec{p}_{2iy} = m\vec{v}_{2iy} = +(8 \text{ m/s})m$
After the collision	$\vec{p}_{1fx} = m\vec{v}_{1fx} = ?$	$\vec{p}_{1fy} = m\vec{v}_{1fy} = ?$
	$\vec{p}_{2fx} = 2m\vec{v}_{2fx} = 0$	$\vec{p}_{2fy} = 2m\vec{v}_{2fy} = +(6 \text{ m/s})m$

Table 7.2: Organizing the collision data in a table helps to keep the *x*-direction information separate from the *y*-direction information, and doing so can also help us solve the problem.

Step 3 – ***Apply conservation of momentum in the x-direction, and find the x-component of the first object's final velocity.*** Applying momentum conservation in the *x*-direction involves writing down the equation $\vec{p}_{1ix} + \vec{p}_{2ix} = \vec{p}_{1fx} + \vec{p}_{2fx}$.

This gives $\vec{p}_{1fx} = \vec{p}_{1ix} + \vec{p}_{2ix} - \vec{p}_{2fx} = +(5 \text{ m/s})m - (6 \text{ m/s})\, m - 0 = -(1 \text{ m/s})m$.

To get the velocity component in the x-direction we just divide by the mass, m.

$$\vec{v}_{1fx} = \frac{\vec{p}_{1fx}}{m} = \frac{-(1\ \text{m/s})m}{m} = -1\ \text{m/s}.$$

Step 4 – ***Use a similar process in the y-direction to find the y-component of the first object's final velocity.*** Applying momentum conservation in the y-direction involves writing down the equation $\vec{p}_{1iy} + \vec{p}_{2iy} = \vec{p}_{1fy} + \vec{p}_{2fy}$.

This equation gives $\vec{p}_{1fy} = \vec{p}_{1iy} + \vec{p}_{2iy} - \vec{p}_{2fy} = 0 + (8\ \text{m/s})m - (6\ \text{m/s})m = +(2\ \text{m/s})m$.

To get the velocity component in the y-direction, we divide by the mass, m.

$$\vec{v}_{1fy} = \frac{\vec{p}_{1fy}}{m} = \frac{+(2\ \text{m/s})m}{m} = +2\ \text{m/s}.$$

Step 5 – ***Combine the x and y components to find the first object's final speed. Also, write down an expression for the first object's final velocity.*** We can use the Pythagorean theorem to find the final speed of the first object:

$$v_{1f} = \sqrt{v_{1fx}^2 + v_{1fy}^2} = \sqrt{1^2 + 2^2} = \sqrt{5}\ \text{m/s}.$$

The velocity can be written in terms of components as $\vec{v}_{1f} = -1\ \text{m/s}\,\hat{x} + 2\ \text{m/s}\,\hat{y}$. The first ball's final velocity is shown in Figure 7.13.

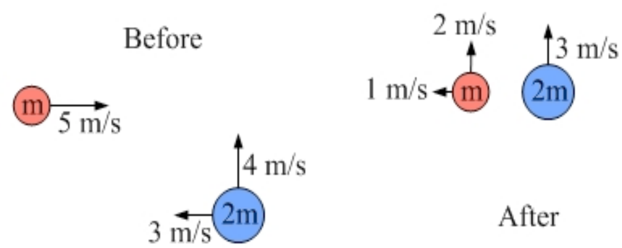

Figure 7.13: The situation Before and After the collision.

Key idea for momentum problems: We can solve a momentum problem in two dimensions with a strategy based on the independence of x and y, breaking a two-dimensional problem into two independent one-dimensional problems. **Related End-of-Chapter Exercises: 29, 57.**

Now that we've looked at a few examples, let's summarize a general method for solving a problem in which there is a collision.

A General Method for Solving a Problem That Involves a Collision

1. Draw a diagram of the situation, showing the velocity of the objects immediately before and immediately after the collision.
2. In a two-dimensional situation, set up a table showing the components of the momentum before and after the collision for each object.
3. Use momentum conservation: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$. (Apply this twice, once for each direction, in a two-dimensional situation.) Account for the fact that momentum is a vector by using appropriate + and – signs.
4. If you need an additional relationship (such as in the case of an elastic collision), use the elasticity relationship or write an energy-conservation equation.

Essential Question 7.6: The strategy outlined above, which we applied in Exploration 7.6, relies on breaking vectors into components. Is there another method that we could use to solve the problem without using components?

Answer to Essential Question 7.6: A whole-vector approach, not splitting the velocity and momentum vectors into components, would also work (see End-of-Chapter Exercise 58).

7-7 Combining Energy and Momentum

To analyze some situations, we apply both energy conservation and momentum conservation in the same problem. The trick is to know when to apply energy conservation (and when not to!) and when to apply momentum conservation. Consider the following Exploration.

EXPLORATION 7.7 – Bringing the concepts together

Two balls hang from strings of the same length. Ball A, with a mass of 4.0 kg, is swung back to a point 0.80 m above its equilibrium position. Ball A is released from rest and swings down and hits ball B. After the collision, ball A rebounds to a height of 0.20 m above its equilibrium position and ball B swings up to a height of 0.050 m. Let's use $g = 10 \text{ m/s}^2$ to simplify the calculations.

Step 1 – ***Sketch a diagram of the situation.*** This is shown in Figure 7.14.

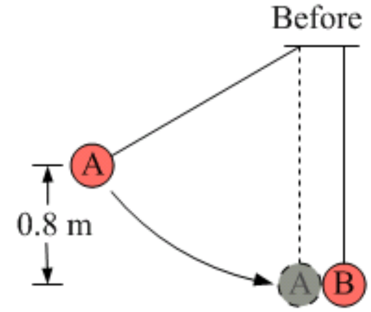

Figure 7.14: A diagram of the two balls on strings. Ball A is swung back until it is 0.80 m higher than its equilibrium point and released from rest.

Step 2 – ***Our goal is to find the mass of ball B. Can we find the mass by setting the initial gravitational potential energy of ball A equal to the sum of the final potential energy of ball A and the final potential energy of ball B? Explain why or why not.*** The answer to the question is no. We can use energy conservation to help solve the problem, but setting the mechanical energy before the collision equal to the mechanical energy after the collision is assuming too much. The balls make contact in the collision, so it is likely that some of the mechanical energy is transformed to thermal energy, for instance.

Step 3 – ***Apply energy conservation to find the speed of ball A just before the collision.*** The gravitational potential energy of ball A is transformed into kinetic energy just before the collision. We will neglect the work done by air resistance, so we can apply energy conservation before the collision. Let's start with the conservation of energy equation: $K_i + U_i + W_{nc} = K_f + U_f$.

For ball A's swing before the collision, we know that the initial kinetic energy is zero. We are assuming that non-conservative forces do no work. We can also define the zero level for gravitational potential energy to be the lowest point in the swing, just before A hits B, so $U_f = 0$. The five-term equation reduces to:

$$U_i = K_f ;$$

$$mgh = \frac{1}{2}mv_f^2 ;$$

So, $v_f = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.8} = \sqrt{16} = 4.0 \text{ m/s}$.

Step 4 – ***Apply conservation of energy again to find the speed of ball A just after the collision.*** We could try applying conservation of momentum here, but there are too many unknowns. Instead, we can follow the conservation of energy method we used above. Note that we will not state that the kinetic energy immediately before the collision is equal to the kinetic energy after the collision, because that is not true. We can apply energy conservation, however, if we confine ourselves to the mechanical energy before the collision (as in step 3) or to the mechanical energy after the collision (this step). If we focus on the upswing, we have the kinetic energy of ball A, immediately after the collision, being transformed into gravitational potential energy. The conservation of energy equation reduces to:

$$K_i = U_f \, ;$$

$$\frac{1}{2} m v_i^2 = mgh \, ;$$

So, $v_i = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.2} = \sqrt{4.0} = 2.0 \text{ m/s}$.

Let's be clear on what we have calculated in parts 3 and 4, because the notation can be confusing. We are analyzing the motion in three separate parts. The first part of ball A's motion is the downswing, which we analyzed in step 3. The third part is the upswing, which we analyzed in step 4. The second part is the collision, which we still have to analyze. The velocity of ball A immediately before the collision, at the end of the downswing, is $\vec{v}_{Ai} = 4.0 \text{ m/s}$ to the right, while A's velocity just after the collision, at the start of the upswing, is $\vec{v}_{Af} = 2.0 \text{ m/s}$ to the left. These are the values we will use in the conservation of momentum equation in step 5.

Step 5 – ***First, apply energy conservation to find the speed of ball B after the collision. Then, apply momentum conservation to find the mass of ball B.*** We still have to find the velocity of ball B, after the collision, before we use the conservation of momentum equation to find ball B's mass. We can find B's speed immediately after the collision by following the same process we used for ball A in step 3. We get:

$$K_i = U_f$$

$$\frac{1}{2} m v_i^2 = mgh$$

So, $v_{Bf} = \sqrt{2gh_B} = \sqrt{2 \times (10 \text{ m/s}^2) \times 0.050 \text{ m}} = \sqrt{1.0 \text{ m}^2/\text{s}^2} = 1.0 \text{ m/s}$.

The velocity of ball B immediately after the collision is $\vec{v}_{Bf} = 1.0 \text{ m/s}$ to the right.

Now, we can write out a conservation of momentum equation to solve for the mass of ball B. It is critical to account for the fact that momentum is a vector. In this case, we account for the vector nature of momentum by using a minus sign for the velocity of ball A after the collision to reflect that it is moving to the left, when we chose right to be the positive direction. This gives: $m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf}$, where $\vec{v}_{Bi} = 0$. Solving for the mass of ball B gives:

$$m_B = \frac{m_A \vec{v}_{Ai} - m_A \vec{v}_{Af}}{\vec{v}_{Bf}} = \frac{(4.0 \text{ kg}) \times (+4.0 \text{ m/s}) - (4.0 \text{ kg}) \times (-2.0 \text{ m/s})}{+1.0 \text{ m/s}} = 24 \text{ kg} \, .$$

Key idea: In some situations, we can apply conservation of energy and conservation of momentum ideas together. In general, we apply conservation of momentum to connect the situation before the collision to the situation after the collision. We use energy conservation to learn something about the situation before the collision and/or the situation afterwards.
Related End-of-Chapter Exercises: 30 – 32.

Essential Question 7.7: Is the collision in Exploration 7.7 super-elastic, elastic, inelastic, or completely inelastic? Justify your answer in two different ways.

Answer to Essential Question 7.7: The balls don't stick together, so we know the collision is not completely inelastic. One way to classify the collision is to find the elasticity, *k* (see equation 7.4).

$$k = \frac{\vec{v}_{Bf} - \vec{v}_{Af}}{\vec{v}_{Ai} - \vec{v}_{Bi}} = \frac{1.0 \text{ m/s} - (-2.0 \text{ m/s})}{4.0 \text{ m/s} - 0} = 0.75 .$$

The fact that *k* is less than 1 means the collision is inelastic. We can confirm this result by looking at the kinetic energy before and after the collision.

$$K_i = \frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2 = \frac{1}{2} \times (4.0 \text{ kg}) \times (4.0 \text{ m/s})^2 + 0 = 32 \text{ J} .$$

$$K_f = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2 = \frac{1}{2} \times (4.0 \text{ kg}) \times (2.0 \text{ m/s})^2 + \frac{1}{2} \times (24.0 \text{ kg}) \times (1.0 \text{ m/s})^2 ;$$

$$K_f = 8.0 \text{ J} + 12 \text{ J} = 20 \text{ J} .$$

The kinetic energy in the system after the collision is less than it is before the collision, so we have an inelastic collision.

Chapter Summary

Essential Idea about Conservation Laws

Many physical situations can be analyzed using forces, which we learned about in previous chapters, and/or by applying the fundamental concepts of conservation of momentum and conservation of energy, which we learned about in this chapter.

Comparing the Energy and Force Methods

The primary methods we use to analyze situations are to use forces and Newton's Laws, or to use energy conservation. Let's compare these two methods.

- The energy approach can be very effective, because we often just have to deal with the initial and final states and we don't have to account for the path taken by the system in going from one state to another, as we do with the force approach.
- The energy approach, by itself, does not give us any information about time, such as about how long it takes a system to move from one state to another. If you need to know about time, use a force analysis.
- Energy is a scalar. Thus energy, by itself, tells us nothing about direction. Force is a vector, and this it can give us information about direction.
- If W_{nc}, the work done by non-conservative forces, is zero, then the total ***mechanical energy*** (the sum of the kinetic and potential energies) is conserved.

A General Method for Solving a Problem Involving Energy Conservation

1. Draw a diagram of the situation. Usually, we use energy to relate a system at one point, or instant in time, to the system at a different point, or a different instant.
2. Apply energy conservation: $K_i + U_i + W_{nc} = K_f + U_f$. (Eq. 7.1)
3. Choose a level to be the zero for gravitational potential energy. Setting the zero level so that either U_i or U_f (or both) is zero is often best.
4. Identify the terms in the equation that are zero.
5. Take the remaining terms and solve.

Collisions and Momentum Conservation

In general, the momentum of a system is conserved in a collision, but the system's kinetic energy is often not conserved in a collision. In fact, one of the two ways in which we classify collisions is based on how the kinetic energy before the collision compares to that afterwards. The second way collisions can be classified is in terms of the ***elasticity***, *k*, which is the ratio of the relative speed of the colliding objects after the collision to their relative speed after the collision:

$$k = \frac{\vec{v}_{2f} - \vec{v}_{1f}}{\vec{v}_{1i} - \vec{v}_{2i}} \quad . \qquad \text{(Equation 7.4: } \textbf{Elasticity}\text{)}$$

This equation is particularly useful when the collision is elastic and the relative velocity of the objects has the same magnitude before and after the collision.

The four collision categories are:

Type of Collision	Kinetic Energy	Elasticity
Super-elastic	$K_f > K_i$	$k > 1$
Elastic	$K_f = K_i$	$k = 1$
Inelastic	$K_f < K_i$	$k < 1$
Completely inelastic	$K_f < K_i$, and the objects stick together	$k = 0$

A General Method for Solving a Problem Involving a Collision

1. Draw a diagram of the situation, showing the velocity of the objects immediately before and immediately after the collision.
2. In a two-dimensional situation, set up a table showing the components of the momentum before and after the collision for each object.
3. Use momentum conservation: $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$. (Eq. 7.2)

   Apply equation 7.2 twice, once for each direction, in a two-dimensional situation). Account for the fact that momentum is a vector with + and – signs.
4. If you require an additional relationship (such as in the case of an elastic collision) use the elasticity relationship or write an energy-conservation equation.

End-of-Chapter Exercises

Several of these exercises can be answered without a calculator, if you use g = 10 m/s^2.

Exercises 1 – 12 are conceptual questions designed to see whether you understand the main concepts of the chapter.

1. Why is it more tiring to walk for an hour up a hill than it is to walk for an hour on level ground?

2. (a) Is it possible for the gravitational potential energy of a system to be negative? (b) Is it possible for the kinetic energy of a system to be negative? (c) Can the total mechanical energy of a system be negative?

3. Given the right (or wrong, depending on your perspective) conditions, a mudslide or avalanche can occur, in which a section of earth or snow that has been at rest slides down a steep slope, reaching impressive speeds. Where does all the kinetic energy that the mud or snow has at the bottom of the slope come from?

4. Three identical blocks (see Figure 7.15) are released simultaneously from rest from the same height *h* above the floor. Block A falls straight down, while blocks B and C slide down frictionless ramps. B's ramp is steeper than C's. (a) Rank the blocks according to their speed, from largest to smallest, when they reach the floor. (b) Rank the blocks according to the time it takes them to reach the floor, from greatest to least. (c) If the two ramps are not frictionless, and the coefficient of friction between the block and ramp is identical for the ramps, do any of your rankings above change? If so, how?

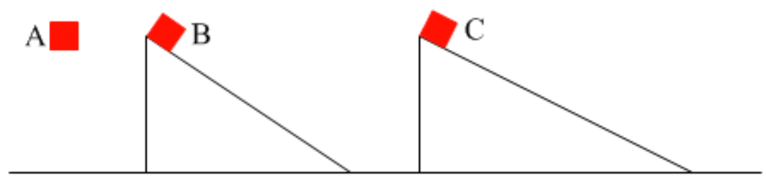

Figure 7.15: Three identical blocks are simultaneously released from rest from the same height above the floor, for Exercise 4.

5. You are on a diving platform 3.0 m above the surface of a swimming pool. Compare the speed you have when you hit the water if you: A, drop almost straight down from rest; B, run horizontally at 4.0 m/s off the platform; C, leap almost straight up, with an initial speed of 4.0 m/s, from the end of the platform.

6. Consider the following situations. For each, state whether or not you would apply energy methods, force/projectile motion methods, or either to solve the exercise. You don't have to solve the exercise, but you can if you wish. (a) Find the maximum height reached by a ball fired straight up from level ground with a speed of 8.0 m/s. (b) Find the maximum height reached by a ball launched from level ground at a 45˚ angle above the horizontal if its launch speed is 8.0 m/s. (c) Find the time taken by the ball in part (b) to reach maximum height. (d) Determine which of the balls, the one in (a) or the one in (b), returns to ground level with the higher speed. (e) Determine the horizontal distance traveled by the ball in (b) before it returns to ground level.

7. You drop a large rock on an empty soda can, crushing the can. (a) Is mechanical energy conserved in this process? Explain. (b) Is energy conserved in this process? Explain.

8. A block of mass m is released from rest at a height h above the base of a frictionless loop-the-loop track, as shown in Figure 7.16. The loop has a radius R. In this situation, $h = 3R$, and, defining the block's gravitational potential energy to be zero at point a, the block's gravitational potential energy at point b is twice the size of the block's kinetic energy at point b. Sketch energy bar graphs showing the block's gravitational potential energy, kinetic energy, and total mechanical energy at (a) the starting point; (b) point a; (c) point b.

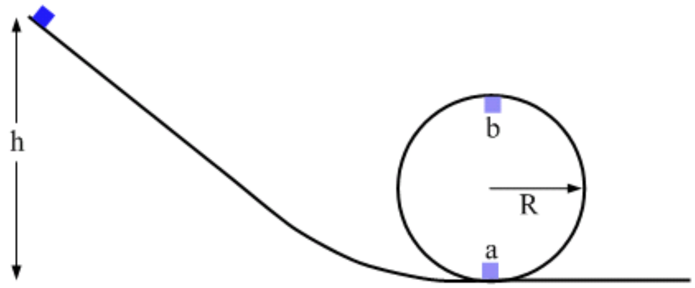

Figure 7.16: A block released from rest from a height h above the bottom of a loop-the-loop track, for Exercise 8.

9. Two boxes, A and B, are released simultaneously from rest from the top of ramps that have the same shape. Box A slides without friction down its ramp, while a kinetic friction force acts on box B as it slides down its ramp. The two boxes have the same mass. For the two boxes, plot the following as a function of time: (a) the kinetic energy; (b) the gravitational potential energy, taking the bottom of the ramp to be zero; (c) the total mechanical energy. There are no numbers here, so just show the general trend on each graph.

10. Repeat Exercise 9, but now plot the graphs as a function of distance traveled along the ramp instead of as a function of time.

11. A block is sliding along a frictionless horizontal surface with a speed v when it encounters a spring. The spring compresses, bringing the spring momentarily to rest, and then the spring returns to its original length, reversing the direction of the block's motion. If the block moves away from the spring at speed v, how can we explain what the spring has done in terms of conservation of energy? Note: this is a preview of how we will handle energy conservation for springs in Chapter 12. Hint: is there a parallel between what the spring does to the block and what the force of gravity does to the block if we toss the block straight up in the air?

12. Comment on the applicability of conservation of energy, conservation of mechanical energy, and momentum conservation in each of the following situations. (a) A car accelerates from rest. (b) In six months, the Earth goes halfway around the Sun. (c) Two football players collide and come to rest on the ground. (d) A diver leaps from a cliff and plunges toward the ocean below.

Exercises 13 – 16 deal with various aspects of the same situation.

13. A ball with a mass of 200 g is tied to a light string with a length of 2.4 m. The end of the string is tied to a hook, and the ball hangs motionless below the hook. Keeping the string taut, you move the ball back and up until it is a vertical distance of 1.25 m above its equilibrium point. You then release the ball from rest. (a) What is the highest speed the ball achieves in its subsequent motion? (b) Where does the ball achieve this maximum speed? (c) What is the maximum height reached by the ball in its subsequent motion? (d) Of the three numerical values stated in this exercise, which one(s) do you actually require to solve the problem?

14. Take a ball with a mass of 200 g and drop it from rest. (a) When the ball has fallen a distance of 1.25 m, how fast is it going? (b) How does this speed compare to the maximum speed of the identical ball in Exercise 13? Briefly explain this result. (c) Which ball takes longer to drop through a distance of 1.25 m? Justify your answer.

15. Consider the ball in Exercise 13. (a) Is it reasonable to assume that the work done by non-conservative forces is negligible over the time during which the ball swings down through the equilibrium position and up to its maximum height point on the other side? Why or why not? (b) If we watch this ball for a long time, it will eventually stop and hang motionless below the hook. Explain, in terms of energy conservation, why the ball eventually comes to rest. (c) In part (b), how much work is done by resistive forces in bringing the ball to rest?

16. Consider the ball in Exercise 13. Assuming that mechanical energy is conserved (that friction and air resistance are negligible), graph the ball's potential energy, kinetic energy, and total energy as a function of height above the equilibrium position. Take the zero of potential energy to be the equilibrium position.

Exercises 17 – 20 are designed to give you some practice in applying the general method of solving a problem involving energy conservation. For each exercise, begin with the following: (a) Sketch a diagram of the situation, showing the system in at least two states that you will relate by using energy conservation. (b) Write out equation 7.1, and define a zero level for gravitational potential energy. It is usually most convenient to define a zero level so that the initial and/or final gravitational potential energy terms are zero. (c) Identify which, if any of the terms in the equation equal zero, and explain why they are zero.

17. You drop your keys, releasing them from rest from a height of 1.2 m above the floor. The goal of this exercise is to use energy conservation to determine the speed of the keys just before they reach the floor. Assume $g = 9.8 \text{ m/s}^2$. Parts (a) – (c) as above. (d) Use the remaining terms in the equation to find the speed of the keys before impact.

18. During a tennis match, you mis-hit the ball, making the ball go straight up in the air. The ball, which has a mass of 57 g, reaches a maximum height of 7.0 m above the point at which you hit it, and the ball's velocity just before you hit it was 12 m/s directed horizontally. The goal of the exercise is to determine how much work your racket did on the ball. Parts (a) – (c) as described above. (d) Determine the work the racket did on the ball. (e) Would your answer to part (d) change if the initial velocity was not horizontal but had the same magnitude?

19. You and your bike have a combined mass of 65 kg. Starting from rest, you pedal to the top of a hill, arriving there with a speed of 6.0 m/s. The net work done on you and the bike by non-conservative forces during the ride is 1.5×10^4 J. The goal of the exercise is to determine the height difference between your starting point and the top of the hill. Parts (a) – (c) as described above. (d) Determine the height difference between your start and end points.

20. A block slides back and forth, inside a frictionless hemispherical bowl. The block's speed is 20 cm/s when it is halfway (vertically) between the lowest point in the bowl and the point where it reaches its maximum height. The goal of the exercise is to determine the maximum height of the block, relative to the bottom of the bowl. Parts (a) – (c) as described above. (d) Determine the block's maximum height.

Exercises 21 – 25 involve energy bar graphs.

21. You throw a ball to your friend, launching it at an angle of 45° from the horizontal. Neglect air resistance, define the zero of gravitational potential energy to be the height from which you release the ball, and assume your friend catches the ball at the same height from which you released it. Draw a set of energy bar graphs, showing the ball's gravitational potential energy and the kinetic energy, for each of the following points: (a) the launch point; (b) the point at which the ball is halfway, vertically, between the launch point and the maximum height; (c) the point where it reaches maximum height.

22. You are on your bicycle at the top of an incline that has a constant slope. You release your brakes and coast down the incline with constant acceleration, taking a time T to reach the bottom. Neglecting all resistive forces, and taking the zero of gravitational potential energy to be at the bottom of the incline, sketch a set of energy bar graphs, showing your gravitational potential energy and kinetic energy for the following points: (a) your starting point (b) at a time of $T/2$ after you start to coast (c) halfway down the incline (d) at the bottom of the incline.

23. Repeat Exercise 22, but this time make it more realistic by accounting for a resistive force. The bar graphs should show your gravitational potential energy, kinetic energy, and total mechanical energy, with a separate bar graph for the work done by the resistive force. Assume the resistive force is constant, and that it causes your kinetic energy at the bottom of the incline to be half of what it would be if the resistive force were not present. If the total time it takes you to come down the incline is now T', in part (b) the energy bar graphs should represent the energies at a time of $T'/2$ after you start to coast.

24. You show three of your friends a set of energy bar graphs. The bar graphs represent the energy, at the release point, of a ball hanging down from a string that you have pulled up and back and released from rest, so it swings with a pendulum motion. These bar graphs are the "Initial" set in Figure 7.17. You ask your three friends to draw the bar graphs representing the ball's energy as it passes through the lowest point in its swing. Margot draws the set of bar graphs shown at the upper right, Jean the set on the lower left, and Wei the set on the lower right. (a) Are the sets of bar graphs, drawn by your friends, consistent with the idea of energy conservation? Justify your answer. (b) Which (if any) of your friends has the right answer? (b) If Jean has the right answer, from what height above the lowest point was the ball released? Assume each of the small rectangles making up the bar graphs represents 1 J, that $g = 10$ m/s^2, and that the ball's mass is 1.0 kg.

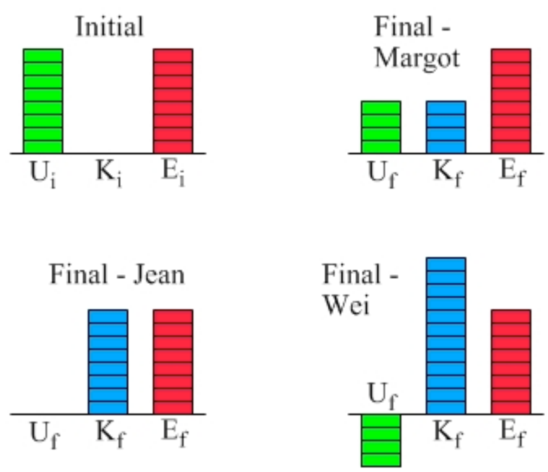

Figure 7.17: Energy bar graphs, for Exercise 24.

Exercises 25 – 29 are designed to give you some practice in applying the general method for solving a problem that involves a collision. For each exercise, start with the following parts: (a) Draw a diagram showing the objects immediately before and immediately after the collision. (b) Apply equation 7.2, the momentum-conservation equation. Choose a positive direction, and account for the fact that momentum is a vector with appropriate + and – signs.

25. A car with a mass of 2000 kg is traveling at a speed of 50 km/h on an icy road when it collides with a stationary truck. The two vehicles stick together after the collision, and their speed after the collision is 10 km/h. The goal of this exercise is to find the mass of the truck. Parts (a) – (b) as described above. (c) Solve for the mass of the truck.

26. Repeat Exercise 25, except that, in this case, the truck is moving at 20 km/h in the opposite direction of the car before the collision, and, after the collision, the two vehicles move together at 10 km/h in the same direction the truck was traveling initially.

27. Two identical air-hockey pucks experience a one-dimensional elastic collision on a frictionless air-hockey table. Before the collision, puck A is moving at a velocity of v to the right, while puck B has a velocity of $2v$ to the left. The goal of the exercise is to determine the velocity of each puck after the collision. Parts (a) – (b) as described above. (c) Use the elasticity relationship to get a second connection between the two final velocities. (d) Find the two final velocities.

28. Repeat Exercise 27, except that in this case puck B has a mass twice as large as the mass of puck A.

29. While shooting pool, you propel the cue ball at a speed of 1.0 m/s. It collides with the 8-ball (initially stationary), propelling the 8-ball into a corner pocket. The cue ball is deflected by 42° from its original path by the collision, and it moves away from the collision with a speed of 0.70 m/s. The goal of this exercise is to determine the magnitude and direction of the 8-ball's velocity after the collision. The cue ball has a little more mass than the 8-ball, but assume for this exercise that the masses are equal. Parts (a) – (b) as described above. For part (b), set up a table to keep track of the x and y components of the momenta of the two balls before and after the collision. (c) Use the information in the table to determine the velocity of the 8-ball after the collision.

Exercises 30 – 32 involve combining energy conservation and momentum conservation.

30. As shown in Figure 7.18, a wooden ball with a mass of 250 g swings back and forth on a string, pendulum style, reaching a maximum speed of 4.00 m/s when it passes through its equilibrium position. Use $g = 10.0$ m/s^2. (a) What is the maximum height above the equilibrium position reached by the ball in its motion? (b) At one instant, when the ball is at its equilibrium position and moving left at 4.00 m/s, it is struck by a bullet with a mass of 10.0 g. Before the collision, the bullet has a velocity of 300 m/s to the right. The bullet passes through the ball and emerges with a velocity of 100 m/s to the right. What is the magnitude and direction of the ball's velocity immediately after the collision? Neglect any change in mass for the ball.

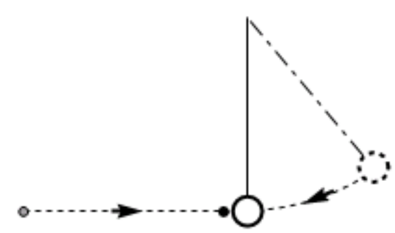

Figure 7.18: A bullet colliding with a ball on a string, for Exercise 30.

31. A pendulum, consisting of a ball of mass m on a light string of length 1.0 m, is swung back to a 45° angle and released from rest. The ball swings down and, at its lowest point, collides with a block of mass $2m$ that is on a frictionless horizontal surface. After the collision, the block slides 1.0 m across the frictionless surface and an additional 0.50 m across a horizontal surface where the coefficient of friction between the block and the surface is 0.10. (a) What is the block's speed after the collision? (b) What is the velocity of the ball after the collision? (c) Is the ball-block collision elastic, inelastic, or completely inelastic? Justify your answer. Use g = 10 m/s^2 to simplify the calculations.

32. Two balls hang from strings of the same length. Ball A, with a mass m, is swung back to a height h above its equilibrium position. Ball A is released from rest and swings down and hits ball B, which has a mass of $3m$. Assuming that all collisions between the balls are elastic, describe the subsequent motion of the two balls.

General Problems and Conceptual Questions

33. A Boeing 747 has a mass of about 3 x 10^5 kg, a cruising speed of 965 km/h, and cruises at an altitude of about 10 km. (a) Assuming the plane starts from rest at an airport at sea level, how much energy is required to reach its cruising height and altitude? Neglect air resistance in this calculation. (b) Comment on the validity of neglecting air resistance.

34. One way to estimate your power is to time yourself as you run up a flight of stairs. (a) In terms of simplifying the analysis, should you start from rest at the bottom of the stairs or should you give yourself a running start and try to keep your speed as constant as possible? (b) Which of the following distance(s) is/are most important for the power calculation, the magnitude of the straight-line displacement along the staircase or the vertical or horizontal components of this displacement? (c) Find a staircase and a stopwatch and estimate your average power.

35. A toy car rolls along a track. Starting from rest, the car drops gradually to a level 2.0 m below its starting point and then gradually rises to a level 1.0 m below its starting point, where it is traveling at a speed v_f. The goal of the exercise is to find v_f. Assume that mechanical energy is conserved, and use g = 9.8 m/s^2. (a) Should you first use energy conservation to relate the initial point to the lowest point, and then apply energy conservation to relate the lowest point to the final point, or can you relate the initial point directly to the final point using energy conservation? Justify your answer. (b) Find v_f.

36. Ball A is released from rest at a height h above the floor and has a speed v when it reaches the floor. (a) If ball B, which has half the mass of ball A, is released from rest at a height of $4h$ above the floor, what is its speed when it reaches the floor? Neglect air resistance. (b) What if ball B had double the mass of A instead?

37. A box with a mass of 2.0 kg slides at a constant speed of 3.0 m/s down a ramp. The ramp is in the shape of a 3-4-5 triangle, as shown in Figure 7.19. (a) Does friction act on the box? Briefly justify your answer. (b) If you decide that friction does act on the box, calculate the coefficient of kinetic friction between the box and the ramp. (c) The mass and speed of the box are given, but could you solve this exercise without them? Briefly explain.

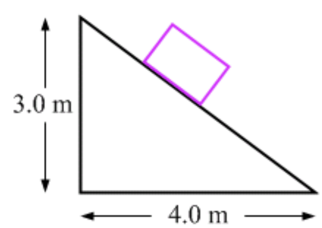

Figure 7.19: A box sliding on an incline, for Exercise 37.

38. A ball is launched with an initial velocity of 28.3 m/s, at a 45˚ angle, from the top of a cliff that is 10.0 m above the water below. Use g = 10.0 m/s^2 to simplify the calculations. (a) What is the ball's speed when it hits the water? (b) What is the ball's speed when it reaches its maximum height? (c) What is the maximum height (measured from the water) reached by the ball in its flight? Note: you could answer these questions using projectile motion methods, but try using an energy conservation approach instead.

39. You drop a 50-gram Styrofoam ball from rest. After falling 80 cm, the ball hits the ground with a speed of 3.0 m/s. Use g = 10 m/s^2. (a) With what speed would the ball have hit the ground if there had been no air resistance? (b) How much work did air resistance do on the ball during its fall? (c) Is your answer to (b) positive, negative, or zero? Explain.

40. As shown in Figure 7.20, two frictionless ramps are joined by a rough horizontal section that is 4.0 m long. A block is placed at a height of 124 cm up the ramp on the left and released from rest, reaching a maximum height of 108 cm on the ramp on the right before sliding back down again. (a) How far up the ramp on the left does the block get in its subsequent motion? (b) What is the coefficient of kinetic friction between the block and the rough surface? (c) At what location does the block eventually come to a permanent stop?

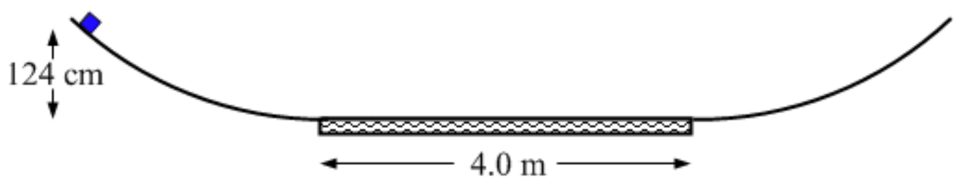

Figure 7.20: A block released from rest 124 cm above the bottom of a track. The curved parts of the track are frictionless, while there is some friction between the track and the block on the 4.0-meter long horizontal section of the track. For Exercises 40 – 42.

41. Consider again the situation described in Exercise 40. If you took this apparatus to the Moon, where the acceleration due to gravity is one-sixth of what it is on Earth, and released the block from rest from the same point, what (if anything) would change about the motion?

42. Consider again the situation described in Exercise 40. Now, a different block is released from the point shown, 124 cm above the flat part of the track. This block does not reach the other side at all, but instead it stops somewhere in the rough section of the track. (a) What could be different about this block compared to the block in exercise 35? (b) What, if anything, can you say about the coefficient of kinetic friction between this block and the rough surface based on the information given here?

43. Two ramps have the same length, height, and angle of incline. One of the ramps is frictionless, while for the second ramp the coefficient of kinetic friction between the ramp and a particular block is $\mu_K = 0.25$. You release the block from rest at the top of the frictionless ramp, and when it reaches the bottom of the incline its kinetic energy is a particular value K_1. When you repeat the process with the second ramp, you find that the block's kinetic energy at the bottom of the ramp is 80% of K_1. At what angle with respect to the horizontal are the ramps inclined?

44. Two blocks are connected by a string that passes over a massless, frictionless pulley, as shown in Figure 7.21. Block A, with a mass $m_A = 2.0$ kg, rests on a ramp measuring 3.0 m vertically and 4.0 m horizontally. Block B hangs vertically below the pulley. Note that you can solve this exercise entirely using forces and the constant-acceleration equations, but see if you can apply energy ideas instead. Use $g = 10$ m/s^2. When the system is released from rest, block A accelerates up the slope and block B accelerates straight down. When block B has fallen through a height $h = 2.0$ m, its speed is $v = 6.0$ m/s. (a) At any instant in time, how does the speed of block A compare to that of block B? (b) Assuming that no friction is acting on block A, what is the mass of block B?

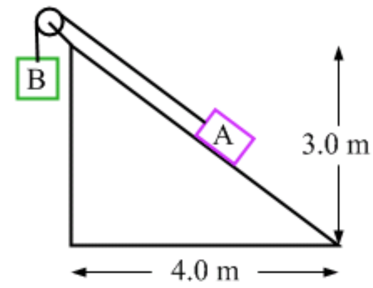

Figure 7.21: Two blocks connected by a string passing over a pulley, for Exercises 44 and 45.

45. Repeat Exercise 44, this time accounting for friction. If the coefficient of kinetic friction for the block A – ramp interaction is 0.625, what is the mass of block B?

46. Tarzan, with a mass of 80 kg, wants to swing across a ravine on a vine, but the cliff on the far side of the ravine is 1.0 m higher than the cliff where Tarzan is now and 2.0 m higher than Tarzan's lowest point in his swing. Use $g = 10$ m/s^2 to simplify the calculations. (a) If Tarzan wants to reach the cliff on the far side, how much kinetic energy must he have when he jumps off the cliff where he starts? (b) How fast is Tarzan going at the bottom of his swing? (c) If Tarzan swings along a circular arc of radius 10 m, what is the tension in the vine when Tarzan reaches the lowest point in his swing?

47. A block of mass m is released from rest at a height h above the base of a frictionless loop-the-loop track, as shown in Figure 7.22. The loop has a radius R. When the block is at point b, at the top of the loop, the normal force exerted on the block by the track is equal to mg. (a) What is h, in terms of R? (b) What is the normal force acting on the block at point a, at the bottom of the loop?

48. Consider again the situation described in Exercise 47, and shown in Figure 7.22. What is the block's speed at (a) point a (b) point b? Your answers should be given in terms of m, g, and/or R only.

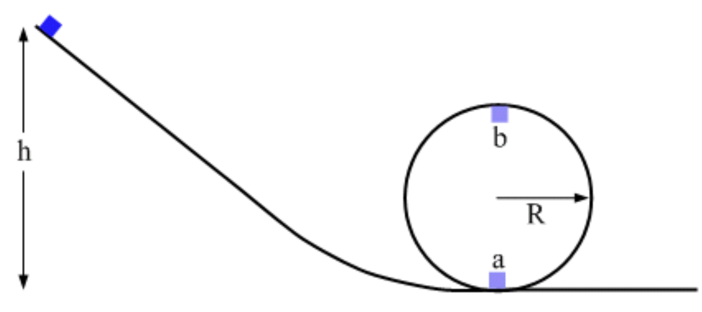

Figure 7.22: A block released from rest from a height h above the bottom of a loop-the-loop track, for Exercises 47 – 49.

49. A block of mass m is released from rest, at a height h above the base of a frictionless loop-the-loop track, as shown in Figure 7.22. The loop has a radius R. What is the minimum value of h necessary for the block to make it all the way around the loop without losing contact with the track? Express your answer in terms of R.

50. On an incline, set up a race between a low-friction block that slides easily down the incline and a ball that rolls down the incline. A good approximation of a low-friction block is a toy car, or a wheeled cart, with low-friction bearings in its wheels. (a) Predict the winner of the race if you release both objects from rest. Run the race to check your prediction. (b) If we assume that mechanical energy is conserved for both objects over the course of the race, how can you explain the result? Note: this is a preview of how we will handle energy conservation for rolling objects in chapter 11.

51. Two air-hockey pucks collide on a frictionless air-hockey table, as shown in Figure 7.23. Before the collision puck A, with a mass of m, is traveling at 20 m/s to the right, while puck B, with a mass of $4m$, is stationary. After the collision puck A is traveling to the left at 4.0 m/s. (a) What is the velocity of puck B after the collision? (b) Is this collision super-elastic, elastic, or inelastic? Justify your answer.

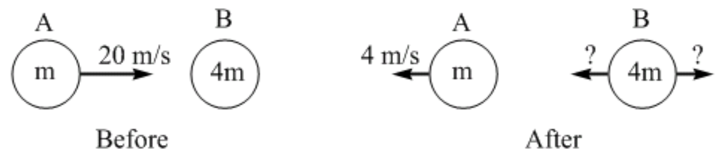

Figure 7.23: Two air-hockey pucks just before and just after they collide, for Exercise 51.

52. Two identical carts experience a collision on a horizontal track. Immediately before the collision, cart 1 is moving at speed v to the right, directly toward cart 2, which is moving at speed v to the left. If the collision is completely inelastic then: (a) What is the velocity of cart 1 immediately after the collision? (b) Is kinetic energy, or momentum, conserved in this collision? (c) What is the velocity of the system's center of mass before the collision? (d) What is the velocity of the system's center of mass after the collision?

53. Two carts experience a collision on a horizontal track. Immediately before the collision, cart 1 is moving at speed v to the right, directly toward cart 2, which is moving at speed v to the left. If cart 2's mass is three times larger than cart 1's mass, and the collision is completely inelastic, what is the velocity of cart 1 immediately after the collision?

54. A one-dimensional collision takes place between object 1, which has a mass m_1 and a velocity $\vec{v}_{1i}$ that is directed toward object 2, which has mass m_2 and is initially stationary.

(a) If the collision is completely inelastic, what is the velocity of the two objects immediately after the collision? (b) If the collision is completely elastic, what are the velocities of the two objects after the collision? Hint: for part (b) make use of the elasticity, k, defined in equation 7.4. Making use of the result of part (b), (c) under what condition is object 1 stationary after the collision? (d) Under what condition does object 1 reverse its direction because of the collision?

55. A one-dimensional elastic collision between an object of mass m and velocity $+\vec{v}$, and a second object of mass $3m$ and velocity $-\vec{v}$, is a special case. (a) Find the velocities of the two objects after the collision to see why. Note that you can arrange such a collision by placing a baseball or tennis ball on top of a basketball and letting the balls fall straight down from rest. (b) Assuming the masses of the basketball and baseball are in the special 3:1 ratio, that all collisions are elastic, and that the balls are dropped from a height h above the floor, how high up should the baseball go after the collision?

56. Two cars of the same mass collide at an intersection. Just before the collision, one car is traveling east at 30 km/h and the other car is traveling south at 40 km/h. If the collision is completely inelastic, so that the two cars move as one object after the collision, what is the speed of the cars immediately after the collision?

57. Because you are an accident reconstruction expert, working with the local police department, you are called to the scene of an accident at a local parking lot. The speed limit posted in the parking lot is 20 miles per hour. Although nobody was hurt in the accident, the police officer in charge would like to determine whether or not anyone was at fault, for insurance purposes. When you reconstruct the accident, you find that the cars, an Acura MDX and a Volkswagen Jetta, were approaching one another at a 90˚ angle. After the collision, the cars locked together and slid for 3.3 m, traveling along a path at 45˚ to their original paths, before coming to rest. You also determine that the Acura has a mass of 2000 kg, the Jetta's mass is 1500 kg, and the coefficient of kinetic friction for the car tires sliding on the dry pavement is somewhere between 0.75 and 0.85. (a) Which car was traveling faster before the collision? (b) Should either one of the drivers be given a speeding ticket and be determined to be at fault for the accident? Justify your answer.

58. A wooden block with a mass of 200 g rests on two supports. A piece of sticky chewing gum with a mass of 50 g is fired straight up at the block, colliding with the block when the gum's speed is 10 m/s. The gum sticks to the block, and we want to find the maximum height reached by the block and gum in its subsequent motion. (a) To solve for this maximum height, should we set the gum's kinetic energy before the collision equal to the gravitational potential energy of the gum-block system after the collision? Why or why not? (b) What is the maximum height reached by the gum-block system?

59. Re-do Exploration 7.6, but solve it another way, using a whole-vector approach by adding vectors graphically. First, add the momentum of the first object before the collision to that of the second object before the collision. That resultant vector is the total momentum before the collision, and because momentum is conserved, it is also the total momentum after the collision. Using this fact and the known momentum of the second object after the collision, you should be able to use the cosine law to find the momentum of the first object after the collision. Does the result match what we found using the component method in Exploration 7.6?

60. You release a rubber ball, from rest, at a point 1.00 m above the floor, and you observe that the ball bounces back to a height of 87.0 cm. (a) What is the net impulse experienced by the ball, which has a mass of 50.0 g, while it is in contact with the floor? (b) What is the elasticity, k, characterizing the collision between the ball and the floor? (c) Assuming the elasticity is the same for each collision, how many times will the ball bounce off the floor before losing half its mechanical energy?

61. Two different collisions take place in a large level parking lot, which is otherwise empty of vehicles. In collision A, a car with mass M traveling at a speed of v_i , runs into a stationary truck of mass $4M$. In collision B, a truck of mass $4M$, traveling at the same speed v_i , runs into a stationary car of mass M. In both collisions, the two vehicles stick together and the combined object skids to a halt because of friction. Assume that the force of friction is constant and the same for both collisions. (a) What is the speed of the combined object immediately after (i) collision A? (ii) collision B? (b) If, in collision A, the combined object slides for a time T and a distance D after the collision, for how long and through what distance does the combined object slide in collision B?

62. Comment on the statements made by two students who are working together to solve the following problem, and state the answer to the problem. A cart with a mass of 2.0 kg has a velocity of 4.0 m/s in the positive *x*-direction. The first cart collides with a second cart, which is identical to the first and has a velocity of 2.0 m/s in the negative *x*-direction. After the collision, the first cart has a velocity of 1.0 m/s in the positive *x*-direction. What is the velocity of the second cart after the collision?

Martha: This is pretty easy. We can use momentum conservation, and we don't even have to worry about the masses, because the masses are the same. So, we have a total of 4 plus 2 equals 6 meters per second before the collision, so we must have a total of 6 meters per second afterwards, too. The first cart has 1 meter per second afterwards, so the second cart must have 5 meters per second afterwards.

George: But, what direction is it going afterwards? We need to give the velocity, so is it in the plus x-direction or the minus x-direction?

Martha: It can't be minus x, because that would mean the two carts would pass through each other. It must bounce back, and go in the plus x-direction.

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