6-1 Rewriting Newton's Second Law

In this chapter, we will begin by taking a look at two ideas that we are familiar with from previous chapters. Let's see what happens when we combine Newton's second law, $\vec{F}_{net} = m\vec{a}$, with the definition of acceleration, $\vec{a} = \Delta\vec{v} / \Delta t$. This gives $\vec{F}_{net} = m\Delta\vec{v} / \Delta t$. Let's be a bit creative and write this relationship in a form more as Newton himself originally did:

$$\vec{F}_{net} = \frac{\Delta(m\vec{v})}{\Delta t}$$

(Equation 6.1: **General form of Newton's Second Law**)

Equation 6.1 is more general than Newton's second law stated in the form $\vec{F}_{net} = m\vec{a}$, because equation 6.1 allows us to work with systems (such as rockets) where the mass changes. $\vec{F}_{net} = m\vec{a}$ applies only to systems where the mass is constant, although so many systems have constant mass that we find this form of the equation to be very useful.

The general form of Newton's second law connects the net force on an object with the rate of change of the quantity $m\vec{v}$. This quantity has a name, which you may already be familiar with.

An object's **momentum** is the product of its mass and its velocity. Momentum is a vector, pointing in the direction of the velocity. The symbol we use to represent momentum is $\vec{p}$.

$$\vec{p} = m\vec{v}\,.$$

(Equation 6.2: **Momentum**)

Equation 6.1 can be re-arranged to read: $\vec{F}_{net}\,\Delta t = \Delta(m\,\vec{v}) = \Delta\vec{p}$.

Thus, to change an object's momentum, all we have to do is to apply a net force for a particular time interval. To produce a larger change in momentum, we can apply a larger net force or apply the same net force over a longer time interval. The product of the net force and the time interval over which the force is applied is such an important quantity that we should name it, too.

The product of the net force and the time interval over which the force is applied is called **impulse**. An impulse produces a change in momentum. An impulse is a vector.

$$\vec{F}_{net}\,\Delta t = \Delta(m\,\vec{v}) = \Delta\vec{p}\,.$$

(Equation 6.3: **Impulse**)

EXPLORATION 6.1 – Hitting the boards

Just before hitting the boards of an ice rink, a hockey puck is sliding along the ice at a constant velocity. The components of the velocity are 3.0 m/s in the direction perpendicular to the boards and 4.0 m/s parallel to the boards. After bouncing off the boards, the puck's velocity component perpendicular to the boards is 2.0 m/s and the component parallel to the boards is unchanged. The puck's mass is 160 g.

Step 1 - ***What is the impulse applied to the puck by the boards?*** Let's sketch a diagram (see Figure 6.1) to help visualize what is going on. Impulse is the product of the net force and the time--but we don't know the net force so we can't get impulse that way. Impulse is also equal to the change in momentum, as we can see from equation 6.3, so let's figure out that change.

Remember that momentum, impulse, and velocity are all vectors. Let's choose a coordinate system with the positive x-direction to the right and the positive y-direction up. It is important to notice that there has been no change in the puck's velocity in the y-direction, so there is no change in momentum, no impulse, and no net force in that direction. We can focus on the x-direction to answer the question.

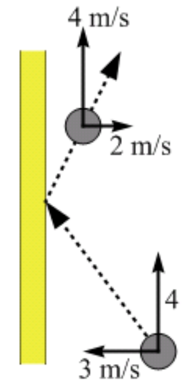

Figure 6.1: The puck's velocity components before and after it collides with the boards.

The puck's mass is constant, so the puck's momentum changes because there is a change in velocity. What is the magnitude of the change in the puck's velocity in the x-direction, perpendicular to the boards? It's tempting to say 1.0 m/s, but it is actually 5.0 m/s. That result comes from:

$$\Delta\vec{v}_x = \vec{v}_x - \vec{v}_{ix} = +2.0 \text{ m/s} - (-3.0 \text{ m/s}) = +5.0 \text{ m/s} .$$

Knowing the change in velocity, we can find the impulse, which is in the $+x$-direction:

$$\Delta\vec{p}_x = m\,\Delta\vec{v}_x = 0.16 \text{ kg} \times (+5.0 \text{ m/s}) = +0.80 \text{ kg m/s} .$$

Step 2 - ***If the puck is in contact with the boards for 0.050 s, what is the average force applied to the puck by the boards?*** The force varies over the 0.050 s the puck is in contact with the boards, but we can get the average force from:

$$\vec{F}_x = \frac{\Delta\vec{p}_x}{\Delta t} = \frac{+0.80 \text{ kg m/s}}{0.050 \text{ s}} = +16 \text{ N}$$

Key ideas for impulse and momentum: Analyzing situations from an impulse-momentum perspective can be very useful, as it allows us to directly connect force, velocity, and time. It is absolutely critical to account for the fact that momentum, impulse, force, and velocity are all vectors when carrying out such an analysis. **Related End-of-Chapter Exercises: 14, 15**

Let's now summarize a general method we can use to solve a problem involving impulse and momentum. We will apply this method in Exploration 6.2.

Solving a Problem Involving Impulse and Momentum

A typical impulse-and-momentum problem relates the net force acting on an object over a time interval to the object's change in momentum. A method for solving such a problem is:

1. Draw a diagram of the situation.
2. Add a coordinate system to the diagram, showing the positive direction(s). Keeping track of direction is important because force and momentum are vector quantities.
3. Organize what you know, and what you're looking for, such as by drawing one or more free-body diagrams, or drawing a graph of the net force as a function of time.
4. Apply equation 6.3 $\left(\vec{F}_{net}\Delta t = \Delta(m\vec{v}) \right)$ to solve the problem.

Essential Question 6.1**:** At some time T after a ball is released from rest, the force of gravity has accelerated that ball to a velocity v directed straight down. Taking into account impulse and momentum, what is the ball's velocity at a time $2T$ after being released?

Answer to Essential Question 6.1: The force is constant, and Equation 6.3 tells us that the velocity increases linearly with time. Thus, at a time $2T$, the velocity will be $2v$ directed down.

6-2 Relating Momentum and Impulse

In this section, we will apply the general method from the end of Section 6-1 to solve a problem using the concepts of impulse and momentum.

EXPLORATION 6.2 – An impulsive bike ride

Suki is riding her bicycle, in a straight line, along a flat road. Suki and her bike have a combined mass of 50 kg. At $t = 0$, Suki is traveling at 8.0 m/s. Suki coasts for 10 seconds, but when she realizes she is slowing down, she pedals for the next 20 seconds. Suki pedals so that the static friction force exerted on the bike by the road increases linearly with time from 0 to 40 N, in the direction Suki is traveling, over that 20-second period. Assume there is constant 10 N resistive force, from air resistance and other factors, acting on her and the bicycle the entire time.

Step 1 - ***Sketch a diagram of the situation.*** The diagram is shown in Figure 6.2, along with the free-body diagram that applies for the first 10 s and the free-body diagram that applies for the 20-second period while Suki is pedaling.

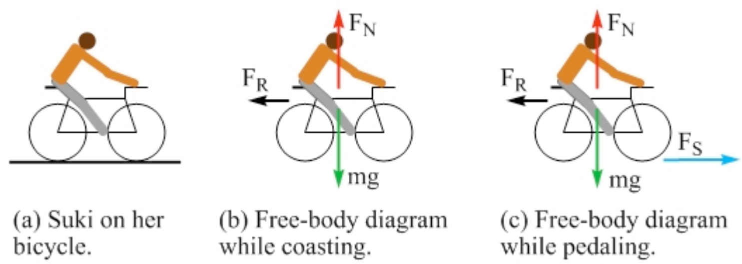

Figure 6.2: A diagram of (a) Suki on her bike, as well as free-body diagrams while she is (b) coasting and while she is (c) pedaling. Note that in free-body diagram (c), the static friction force $\vec{F}_S$ gradually increases because of the way Suki pedals.

Step 2 - ***Sketch a graph of the net force acting on Suki and her bicycle as a function of time.*** Take the positive direction to be the direction Suki is traveling. In the vertical direction, the normal force exactly balances the force of gravity, so we can focus on the horizontal forces. For the first 10 seconds, we have only the 10 N resistive force, which acts to oppose the motion and is thus in the negative direction. For the next 20 seconds, we have to account for the friction force that acts in the direction of motion and the resistive force. We can account for their combined effect by drawing a straight line that goes from –10 N at $t = 10$ s, to +30 N (40 N – 10N) at $t = 30$ s. The result is shown in Figure 6.3.

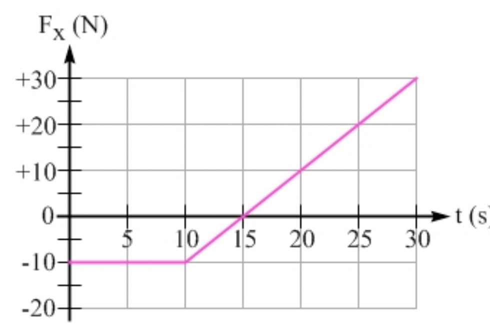

Figure 6.3: A graph of the net force acting on Suki and her bicycle as a function of time.

Step 3 - ***What is Suki's speed at t = 10 s?*** Let's apply Equation 6.3, which we can write as:

$$\vec{F}_{net}\,\Delta t = \Delta(m\,\vec{v}) = m\,\Delta\vec{v} = m\left(\vec{v}_{10s} - \vec{v}_i\right).$$

Solving for the velocity at $t = 10$ s gives:

$$\vec{v}_{10s} = \vec{v}_i + \frac{\vec{F}_{net}\,\Delta t}{m} = +8.0\text{ m/s} + \frac{(-10\text{ N})(10\text{ s})}{50\text{ kg}} = +8.0\text{ m/s} - 2.0\text{ m/s} = +6.0\text{ m/s}.$$

Thus, Suki's speed at t = 10 s is 6.0 m/s. We can also obtain this result from the force-versus-time graph, by recognizing that the impulse, $\vec{F}_{net}\,\Delta t$, represents the area under this graph over some time interval Δt. Let's find the area under the graph, over the first 10 seconds, shown in Figure 6.4. The area is negative, because the net force is negative over that time interval. The area under the graph is the impulse:

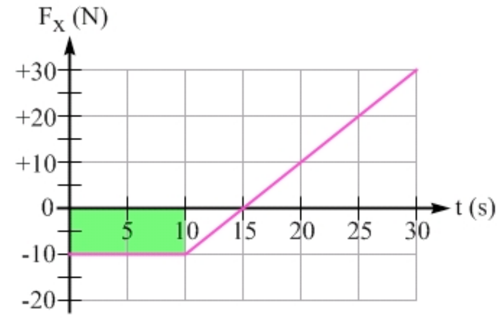

Figure 6.4: The rectangle represents the area under the graph for the first 10 s. The area is negative, because the force is negative.

$$\vec{F}_{net}\,\Delta t = -10\text{ N}\times 10\text{ s} = -100\text{ N s} = -100\text{ kg m/s}.$$

From Equation 6.3, we know the impulse is equal to the change in momentum. Suki's initial momentum is $m\vec{v}_i = 50\text{ kg}\times(+8.0\text{ m/s}) = +400\text{ kg m/s}$. Her momentum at t = 10 s is therefore $+400\text{ kg m/s} - 100\text{ kg m/s} = +300\text{ kg m/s}$. Dividing this by the mass to find the velocity at t = 10 s confirms what we found above:

$$\vec{v}_{10\text{s}} = \frac{\vec{p}_{10\text{s}}}{m} = \frac{\vec{p}_i + \Delta\vec{p}}{m} = \frac{+400\text{ kg m/s} - 100\text{ kg m/s}}{50\text{ kg}} = \frac{+300\text{ kg m/s}}{50\text{ kg}} = +6.0\text{ m/s}.$$

Step 4 - *What is Suki's speed at t = 30 s?* Let's use the area under the force-versus-time graph, between t = 10 s and t = 30 s, to find Suki's change in momentum over that 20-second period. This area is highlighted in Figure 6.5, split into a negative area for the time between t = 10 s and t = 15 s, and a positive area between t = 15 s and t = 30 s. These regions are triangles, so we can use the equation for the area of a triangle, $0.5\times base\times height$. The area under the curve, between 10 s and 15 s, is $0.5\times(5.0\text{ s})\times(-10\text{ N}) = -25\text{ kg m/s}$. The area between 15 s and 30 s is $0.5\times(15\text{ s})\times(30\text{ N}) = +225\text{ kg m/s}$. The total area (total change in momentum) is +200 kg m/s.

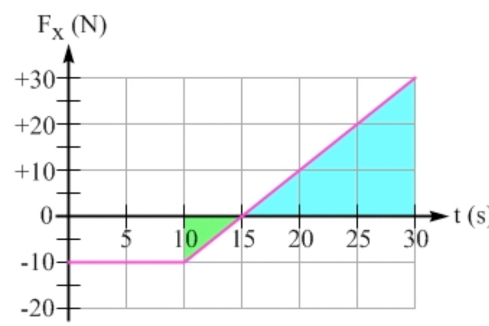

Figure 6.5: The shaded regions correspond to the area under the curve for the time interval from t = 10 s to t = 30 s.

Note that another approach is to multiply the average net force acting on Suki and the bicycle (+10 N) over this interval, by the time interval (20 s), for a +200 kg m/s change in momentum.

In step 3, we determined that Suki's momentum at t = 10 s is +300 kg m/s. With the additional 200 kg m/s, the net momentum at t = 10 s is +500 kg m/s. Dividing by the 50 kg mass gives a velocity at t = 30 s of +10 m/s.

Key idea for the graphical interpretation of impulse: The area under the net force versus time graph for a particular time interval is equal to the change in momentum during that time interval. **Related End-of-Chapter Exercises: 24, 27 – 30.**

Essential Question 6.2: Return to the 30-second interval covered in Exploration 6.2. At what time during this period does Suki reach her minimum speed?

***Answer to Essential Question 6.2*:** At $t = 15$ s. The graph in Figure 6.4 is helpful in determining when Suki reaches her minimum speed. As long as the net force is negative, Suki slows down (unless her velocity becomes negative, which never happens in this case). Suki continues to slow down until $t = 15$ s. After that time, the net force is positive, so Suki speeds up after $t = 15$ s.

6-3 Implication of Newton's Third Law: Momentum is Conserved

EXPLORATION 6.3A – Two carts collide

Let's do an experiment in which two carts, cart 1 and cart 2, collide with one another on a horizontal track, as shown in Figure 6.6. How does the momentum of each cart change? What happens to the momentum of the two-cart system? The upward normal force applied by the track on each cart is balanced by the downward force of gravity, so the net force experienced by each cart during the collision is that applied by the other cart.

Figure 6.6: Two carts colliding.

Let's use the subscripts *i* for the initial situation (before the collision), and *f* for the final situation (after the collision).

The collision changes the momentum of cart 1 from $\vec{p}_{1i}$ to $\vec{p}_{1f} = \vec{p}_{1i} + \Delta\vec{p}_1$.

Similarly, the collision changes the momentum of cart 2 from $\vec{p}_{2i}$ to $\vec{p}_{2f} = \vec{p}_{2i} + \Delta\vec{p}_2$.

The total momentum of the system beforehand is $\vec{p}_{1i} + \vec{p}_{2i}$.

The total momentum of the system afterwards is $\vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1i} + \Delta\vec{p}_1 + \vec{p}_{2i} + \Delta\vec{p}_2$.

Consider $\Delta\vec{p}_1$, the change in momentum experienced by cart 1 in the collision. This change in momentum comes from the force applied to cart 1 by cart 2 during the collision. Similarly, $\Delta\vec{p}_2$, cart 2's change in momentum, comes from the force applied to cart 2 by cart 1 during the collision. Newton's third law tells us that, no matter what, the force applied to cart 1 by cart 2 is equal and opposite to that applied to cart 2 by cart 1. Keeping in mind that the change in momentum is directly proportional to the net force, and that we're talking about vectors, this means:

$$\Delta\vec{p}_2 = -\Delta\vec{p}_1 .$$

Substituting this result into our expression for the total momentum of the system after the collision shows that momentum is conserved (momentum remains constant):

$$\vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1i} + \vec{p}_{2i} .$$

Key idea: The total momentum of the system after the collision equals the total momentum of the system before the collision. This **Law of Conservation of Momentum** applies to any system where there is no net external force. **Related End-of-Chapter Exercise: 4.**

We'll spend more time on the law of conservation of momentum in Chapter 7 but, for now, consider the following exploration.

EXPLORATION 6.3B – An explosive situation

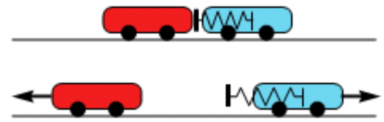

Figure 6.7: A diagram showing the initial situation (top), and the situation after the carts have moved apart (bottom).

Two carts are placed back-to-back on a horizontal track. One cart contains a spring-loaded piston. When the spring is released, the piston pushes against the other cart and the two carts move in opposite directions along the track, as shown in Figure 6.7. Assume the carts are initially at rest in the center of the track and that friction is negligible.

Step 1 - ***If the two carts have equal masses, is momentum conserved in this process?*** A good answer to this question is "it depends." The momentum of each cart individually is not conserved, because each cart starts with no momentum and ends up with a non-zero momentum. This is because each cart experiences a net force (applied by the other cart), so its momentum changes according to the impulse equation (Equation 6.3).

On the other hand, the law of conservation of momentum tells us that ***the momentum of the two-cart system is conserved because no net external force acts on this system***. The upward normal force, exerted by the track on this system, balances the downward force of gravity. Cart 1 acquires some momentum because of the force applied by cart 2, but cart 2 acquires an equal-and-opposite momentum because of the equal-and-opposite force applied to it by cart 1. The net momentum of the two-cart system is zero, even when the carts are in motion. Momentum is a vector, so the momentum of one cart is cancelled by the momentum of the other cart.

Step 2 - ***If we double the mass of one of the carts and repeat the experiment, is momentum conserved?*** Yes, the momentum of this system is conserved because no net external force acts on the system. Changing the mass of one cart will change the magnitude of the momentum acquired by each cart, but the momentum of the two-cart system is always zero, both before and after the spring is released. To conserve momentum, the force applied on cart 1 by cart 2 must be equal and opposite to the force applied on cart 2 by cart 1. Newton's third law tells us that these forces are equal and opposite, no matter how the masses compare.

Step 3 - ***If we make the experiment more interesting, by balancing the track on a brick before releasing the spring, will the track tip over after the spring is released?*** If we tried this experiment when the masses are equal, what would happen? The track would remain balanced, even when the carts are in motion, because of the symmetry. The tendency of cart 1 to tip the track one way is balanced by the tendency of cart 2 to tip it the opposite way. We don't have the same symmetry in step 2, but the track still remains balanced. The cart with half the mass of the other cart is always twice as far from the balance point. That maintains the balance, as shown in Figure 6.8.

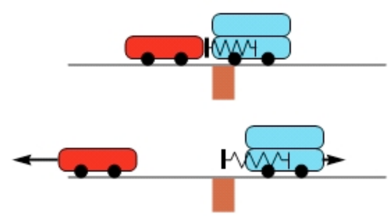

Figure 6.8: As the carts move apart, the track remains balanced on the brick even if the carts have different masses.

Key idea for momentum conservation: Even if the momenta of individual parts of a system are not conserved, the momentum of the entire system is conserved (constant), as long as no net external force acts on the system. Conservation of momentum is a consequence of Newton's third law. **Related End-of-Chapter Exercises: 44, 45.**

Essential Question 6.3: In Exploration 6.3B, the momentum of the system is always zero. Is there anything about the two-cart system that remains at rest and that shows clearly why the track doesn't tip over when balanced on the brick?

Answer to Essential Question 6.3: No matter how the masses of the two carts in Exploration 6.3B compare, the center of mass of the system remains at rest at the balance point of the track.

6-4 Center of Mass

In the previous chapters, we treated everything as a particle, or, equivalently, as a ball. A ball, or particle, thrown through the air follows a parabolic path. What if you take a non-spherical object (a pen, for instance) and throw it so it spins? Most points on the object follow complicated paths, but the center of mass still follows a parabolic curve, as shown in Figure 6.9.

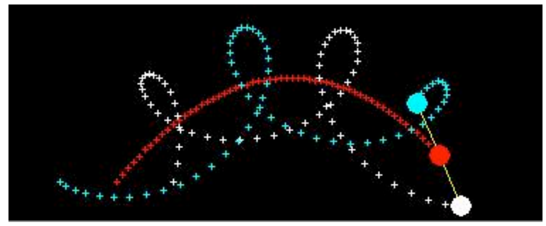

Figure 6.9: The motion of three balls on a stick. Only the red ball, located at the center of mass of the system, follows the parabolic path characteristic of free fall.

For a uniform object, the center of mass is located at the geometric center of the object. In general, the center of mass of an object, or a collection of objects, is given by Equation 6.4.

The **center of mass** is the point on an object that moves as though all the mass of the object is concentrated there. The x-coordinate of the center of mass is given by:

$$X_{CM} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} \qquad \text{(Equation 6.4: \textbf{Position of the center of mass})}$$

where the m's represent the masses of different objects in the system (or of various pieces of a single object) and the x's represent the x-coordinates of those objects or pieces. Similar equations give us the y and z-coordinates of the center of mass.

EXAMPLE 6.4A – Three balls on a stick

Three balls are placed on a meter stick. Ball 1, at the 0-cm mark, has a mass of 1.0 kg. Ball 2, at the 80-cm mark, has a mass of 3.0 kg. Ball 3, at the 90-cm mark, has a mass of 2.0 kg.

(a) If the meter stick has negligible mass, where is the system's center of mass?
(b) If the meter stick has a mass of 2.0 kg, where is the system's center of mass?

SOLUTION

(a) As usual, let's begin with a diagram of the situation (see Figure 6.10).

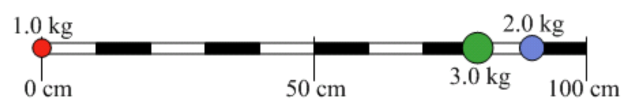

Figure 6.10: A diagram showing the position of the three balls on the meter stick.

To find the center of mass, we can substitute the given values into Equation 6.4:

$$X_{CM} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3} = \frac{0 \times 1.0 \text{ kg} + (80 \text{ cm})(3.0 \text{ kg}) + (90 \text{ cm})(2.0 \text{ kg})}{1.0 \text{ kg} + 2.0 \text{ kg} + 3.0 \text{ kg}} = \frac{420 \text{ kg cm}}{6.0 \text{ kg}} = 70 \text{ cm}.$$

(b) If the stick's mass is uniformly distributed, we can treat the stick as a fourth ball, with a mass of 2.0 kg, located at the 50-cm mark. Making use of the result from part (a), which says that the first three balls are equivalent to a single 6.0-kg ball located at the 70-cm mark, we get:

$$X'_{CM} = \frac{(70 \text{ cm})(6.0 \text{ kg}) + (50 \text{ cm})(2.0 \text{ kg})}{6.0 \text{ kg} + 2.0 \text{ kg}} = \frac{520 \text{ kg cm}}{8.0 \text{ kg}} = 65 \text{ cm}.$$

Related End-of-Chapter Exercises: 31, 32.

The center of mass is particularly useful in systems experiencing no net external force. In such systems, the motion of the system's center of mass is unchanged, even if the motion of different parts of the system changes. This is a consequence of Newton's Second Law. Without a net external force acting, the acceleration of the center of mass of the system is zero. Different parts of the system can accelerate, but the forces associated with these accelerations cancel because the net force on the system is zero. Let's now consider an example of such a system.

EXAMPLE 6.4B – Canoe move the center of mass?

A man, with a mass of 90 kg, stands 2.0 from the center of a 30 kg canoe that is floating on the calm water of a lake. Both the man and the canoe are initially at rest.

(a) If the man then moves to the point 2.0 m on the opposite side of the center of the canoe from where he starts, how far does the canoe move?

(b) How far does the man actually move relative to a fixed point on the shore?

SOLUTION

(a) We could solve this problem formally, but let's solve it conceptually by looking at Before and After pictures in Figure 6.11. First, let's determine the position of the center of mass of the system in the Before picture, before the man changes position.

Define the man's initial position as the origin (you can pick a different origin if you want), and assume the canoe's center of mass to be the middle of the canoe. We get:

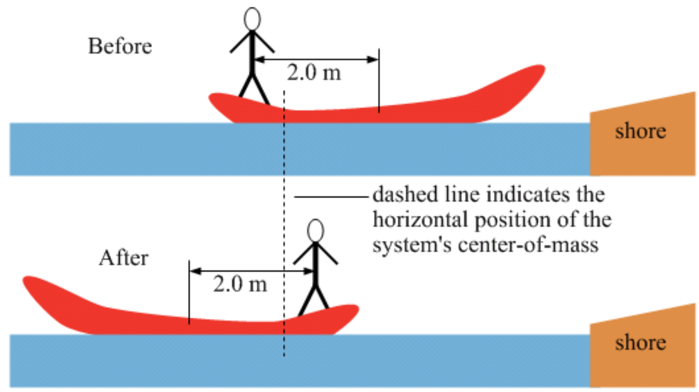

Figure 6.11: The position of the man and the canoe before and after the man moves from one end of the canoe to the other.

$$X_{CM} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} = \frac{0 \times 90 \text{ kg} + (2.0 \text{ m})(30 \text{ kg})}{90 \text{ kg} + 30 \text{ kg}} = \frac{60 \text{ kg} \cdot \text{m}}{120 \text{ kg}} = 0.50 \text{ m}$$

In the Before picture, the man is 50 cm to the left, and the canoe's center of mass is 1.5 m to the right, of the system's center of mass. The canoe's center of mass is three times farther from the system's center of mass than the man is because the canoe's mass is 1/3 of the man's mass. Because no net external force acts on the canoe-man system, when the man moves to the right, the canoe moves left in such a way that the system's center of mass remains at rest. The man moves to a position that is a mirror image of his initial position, so the After picture is a mirror image of the Before picture (placing the mirror at the system's center of mass). The canoe's center of mass moves from 1.5 m to the right of the system's center of mass to 1.5 m to the left of the system's center of mass, for a net displacement of 3.0 m to the left.

(b) Applying a similar analysis to the man, the man moves from 0.5 m to the left of the system's center of mass to 0.5 m to the right, a net displacement of 1.0 m right. Equivalently, the man moves 4.0 m to the right relative to the canoe while the canoe moves 3.0 m to the left with respect to the shore, so the man ends up moving just 1.0 m right relative to the shore.

Related End-of-Chapter Exercise: 33.

Essential Question 6.4: In Example 6.4B, what force makes the canoe move when the man starts to move? What force stops the canoe when the man stops?

Answer to Essential Question 6.4: In each case, the force of friction (static friction if there is no slipping) between the man's shoes and the canoe causes the changes in the canoe's motion.

6-5 Playing with a Constant Acceleration Equation

Once again, let's start with a familiar relationship and look at in a new way to come up with a powerful idea. Return to one of our constant acceleration equations: $v_x^2 = v_{ix}^2 + 2\vec{a}_x \Delta\vec{x}$. If we re-arrange this equation to solve for the acceleration, we get: $\vec{a}_x = \dfrac{v_x^2 - v_{ix}^2}{2\Delta\vec{x}}$.

Substituting this into Newton's second law, $\vec{F}_{net,x} = m\vec{a}_x$, gives, after some re-arranging:

$$\frac{1}{2}mv_x^2 - \frac{1}{2}mv_{ix}^2 = \vec{F}_{net,x}\,\Delta\vec{x}\ .$$

We can do the same thing in the y-direction. Adding the x and y equations gives:

$$\left(\frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2\right) - \left(\frac{1}{2}mv_{ix}^2 + \frac{1}{2}mv_{iy}^2\right) = \vec{F}_{net,x}\Delta\vec{x} + \vec{F}_{net,y}\Delta\vec{y}\ .$$

Recognizing that $v_x^2 + v_y^2 = v^2$, the left side of the equation can be simplified:

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_i^2 = \vec{F}_{net,x}\Delta\vec{x} + \vec{F}_{net,y}\Delta\vec{y}\ .$$

The right side can also be simplified, because its form matches a dot product:

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_i^2 = \vec{F}_{net} \bullet \Delta\vec{r} = F_{net}\Delta r\cos\theta\ , \qquad \text{(Equation 6.5)}$$

where θ is the angle between the net force $\vec{F}_{net}$ and the displacement $\Delta\vec{r}$.

So, we've now come up with two more useful concepts, which we name and define here.

Kinetic energy is energy associated with motion:

$$K = \frac{1}{2}mv^2 \quad ; \qquad \text{(Equation 6.6: \textbf{Kinetic energy})}$$

Work relates force and displacement $W = \vec{F}_{net} \bullet \Delta\vec{r} = F_{net}\Delta r\cos\theta$. (Eq. 6.7: **Work**)

Both work and kinetic energy have units of joules (J), and they are both scalars.

Equation 6.5, when written in the form below, is known as the **work-kinetic energy theorem**. In this case, the work is the work done by the net force.

$$\Delta K = W_{net} = F_{net}\ \Delta r\ \cos\theta\ , \qquad \text{(Eq. 6.8: \textbf{The work-kinetic energy theorem})}$$

where θ is the angle between the net force $\vec{F}_{net}$ and the displacement $\Delta\vec{r}$.

In general, when a force is perpendicular to the displacement, the force does no work. If the force has a component parallel to the displacement, the force does positive work. If the force has a component in the direction opposite to the displacement, the force does negative work.

Compare Exploration 6.5 to Exploration 6.2, in which Suki was riding her bike.

EXPLORATION 6.5 – A hard-working cyclist

Peter is riding his bicycle in a straight line on a flat road. Peter and his bike have a total mass of 60 kg and, at $t = 0$, he is traveling at 8.0 m/s. For the first 70 meters, he coasts. When Peter realizes he is slowing down, he pedals so that the static friction force exerted on the bike by the road increases linearly with distance from 0 to 40 N, in the direction Peter is traveling, over the next 140 meters. A constant 10 N resistive force acts on Peter and the bicycle the entire time.

Figure 6.12: A graph of the net force acting on Peter and his bicycle, as a function of position.

Step 1 - *Sketch a graph of the net force acting on Peter and his bicycle as a function of position.* Take the positive direction to be the direction Peter is traveling. In the vertical direction, the normal force balances the force of gravity, so we can focus on the horizontal forces. For the first 70 m, we have only the 10 N resistive force, which opposes the motion and is thus in the negative direction. For the next 140 m, we have to account for the friction force, which acts in the direction of motion, and the resistive force. We can account for their combined effect by drawing a straight line, as in Figure 6.12, that goes from –10 N at x = 70 m to +30 N (40 N – 10N) at x = 210 m.

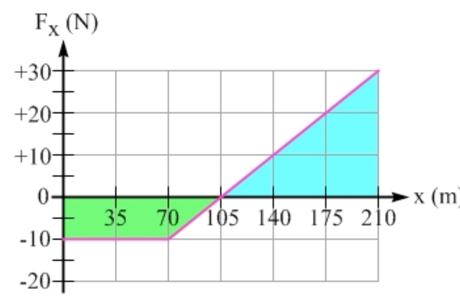

Figure 6.13: The area within the shaded regions represents the area under the curve for the region from x = 0 to x = 210 m.

Step 2 - *What is Peter's speed at x = 210 m?* Let's use the area under the F_{net} versus position graph, between x = 0 and x = 210 m, to find the net work over that distance. This area is shown in Figure 6.13, split into a negative area for the region x = 0 to x = 105 m, and a positive area between x = 105 m and x = 210 m. Each box on the graph has an area of $10 \text{ N} \times 35 \text{ m} = 350 \text{ N m}$. The negative area covers two-and-a-half boxes on the graph, while the positive area covers four-and-a-half boxes, for a net positive area of 2 boxes, or 700 N m.

The net area under the curve in Figure 6.13 is the net work done on Peter and the bicycle, which is the change in kinetic energy ($W_{net} = \Delta K = K_f - K_i$). Thus, the final kinetic energy is:

$$K_f = K_i + W_{net} = \frac{1}{2}mv_i^2 + W_{net} = \frac{1}{2}(60 \text{ kg})(8.0 \text{ m/s})^2 + 700 \text{ N m} = 1920 \text{ J} + 700 \text{ J} = 2620 \text{ J}.$$

Solving for the final speed from $K_f = (1/2)mv_f^2$ gives: $v_f = \sqrt{\dfrac{2K_f}{m}} = \sqrt{\dfrac{2 \times 2620 \text{ J}}{60 \text{ kg}}} = 9.3 \text{ m/s}$.

Key idea: The area under the net force-versus-position graph for a particular region is the work, and the change in kinetic energy, over that region. **Related End-of-Chapter Exercises: 48, 49.**

Essential Question 6.5: Initially, objects A and B are at rest. B's mass is four times larger than A's mass. Identical net forces are applied to the objects, as shown in Figure 6.14. Each force is removed once the object it is applied to has accelerated through a distance d. After the forces are removed, which object has more (a) kinetic energy? (b) momentum?

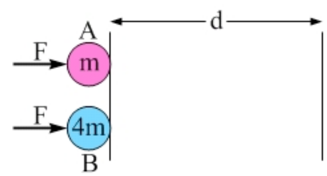

Figure 6.14: An overhead view of two objects, A and B, experiencing the same net force F as they move from rest through a distance d.

Answer to Essential Question 6.5: The two objects experience equal forces over equal displacements, so the work done is the same. Thus, the change in kinetic energy is the same for each and, because they start with no kinetic energy, **their final kinetic energies are equal.**

The change in momentum is the force multiplied by the time over which the force acts. Both objects experience the same force, but, because *B* has more mass, *B* takes more time to move through the distance *d* than *A* does. The force acts on *B* for a longer time, so ***B*'s final momentum is larger than *A*'s**. **Related End-of-Chapter Exercises: 9, 10, 52 – 54.**

6-6 Conservative Forces and Potential Energy

Let's first write down a method for solving a problem involving work and kinetic energy, similar to the method we use for solving an impulse-and-momentum problem.

A General Method for Solving a Problem Involving Work and Kinetic Energy

1. Draw a diagram of the situation.
2. Add a coordinate system to the diagram, showing the positive direction(s). Doing so helps remind us that force and displacement are vector quantities.
3. Organize what you know, perhaps by drawing a free-body diagram of the object, or drawing a graph of the net force as a function of position.
4. Apply Equation 6.8 $\left(F_{net}\,\Delta r\cos\theta = \Delta K\right)$ to solve the problem.

We now have the tools needed to investigate some intriguing ideas about energy.

EXPLORATION 6.6A – Making gravity work

Step 1 - ***Take a ball of weight mg = 10 N and move it through a distance of 2 m. How much work does gravity do on the ball during the motion?*** It is tempting to multiply 10 N by 2 m to get 20 J and say that's the work, but the work depends on the angle between the force and the displacement (see Equation 6.7, $W = F_{net}\Delta r\cos\theta$). The direction of the displacement was not given, so we can't say how much work is done.

Let's consider the extreme cases. If we move the ball up 2 m, the force of gravity and the displacement are in opposite directions, so the work done is –20 J. If we move it down 2 m, the force and displacement are in the same direction, so the work done is +20 J. So, the work done is somewhere between –20 J and + 20 J. Work can even be zero, if the displacement is horizontal.

Step 2 – ***What is the work done by gravity, if we give our 10 N ball a displacement of 2 m down at the same time we displace it 4 m horizontally?*** Gravity still does +20 J of work. All we have to worry about is the vertical motion. There is no work done by gravity for the horizontal motion.

Step 3 - ***Does the path followed make any difference?*** In Figure 6.15, point B is 2 m below, and 4 m horizontally, from A. For any path starting at A and ending at B, the work done by gravity in moving a 10-N ball is +20 J. The horizontal motion does not matter. What matters is that every path involves the same net 2 m vertical downward displacement.

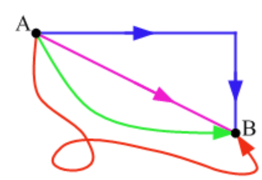

Figure 6.15: The work done by gravity, when an object is moved from point A to point B, is the same no matter what path the object is moved along.

Key idea: The work done by gravity on an object is ***path-independent***. All that matters is the position of the initial point and the position of the final point. It doesn't matter how the object gets from the initial point to the final point. **Related End-of-Chapter Exercise: 43.**

When the work done by a force is path-independent, we say the force is **conservative**. Gravity is a conservative force, and we will discuss other examples later in the book. Other conservative forces include the spring force (chapter 12) and the electrostatic force (chapter 16).

Instead of talking about the work done by a conservative force, we usually do something equivalent and talk about the change in **potential energy** associated with the force. Potential energy can, in general, be thought of as energy something has because of its position.

At the surface of the Earth, where we take the force of gravity to be constant, the work done by gravity is $W_g = -mg\Delta y$. The change in gravitational potential energy, ΔU_g, has the opposite sign: $\Delta U_g = -W_g = mg\Delta y$. (Eq. 6.9: **Change in gravitational potential energy**)

> When the force of gravity is constant, we define **gravitational potential energy** as
>
> $U_g = mgh$, (Equation 6.10: **Gravitational potential energy**)
>
> where h is the height that the object is above some reference level. We can choose any convenient level to be the reference level.

EXPLORATION 6.6B – Talking about potential energy

A 10-N ball is moved by some path from A to B, where B is 2 m lower than A. What is the ball's initial gravitational potential energy? What is its final gravitational potential energy? What is the change in the gravitational potential energy? Analyze the following conversation.

Bob: "We can use Equation 6.9 to find the change in gravitational potential energy. Because B is 2 meters lower than A, the Δy in the equation is –2 meters. Multiplying this by an mg of 10 newtons gives a change in gravitational potential energy of –20 joules."

Andrea: "If I define *B* as the level where the potential energy equals zero, then the ball's potential energy at *A* is +20 joules. The ball's potential energy changes from +20 joules to zero for a change of –20 joules."

Bob: "I agree with what you get for the change but we have to define the zero for potential energy at A. That gives the object a potential energy of –20 joules at B."

Christy: "We can each pick our own zero. It doesn't make any difference. No matter where you put the zero you get –20 joules for the change in potential energy."

Which student is correct?

Bob's first statement is correct. Andrea is correct, and so is Christy. Christy makes an important point – everyone agrees on the value of the change in potential energy, no matter which level they choose as the zero. In his second statement, Bob is incorrect about having to set the potential energy to be zero at A. You can do that, but, as Christy points out, you don't have to.

> **Key idea**: The change in potential energy, which everyone agrees on, is far more important than the actual value of the potential energy. **Related End-of-Chapter Exercise: 12.**

Essential Question 6.6: We often use terminology like "the ball's gravitational potential energy." Does the ball really have gravitational potential energy all by itself?

Answer to Essential Question 6.6: No, gravitational potential energy really does not belong to an object. Rather, it is associated with the interaction between two objects, such as the interaction between the ball and the Earth in Exploration 6.6A. We will explore this idea further when we discuss gravity in more detail in Chapter 8.

6-7 Power

Let's say you are buying a new vehicle. While you are searching the Internet to compare the latest models, an advertisement for a fancy sports car catches your eye. You read that the car can go from rest to 100 km/h in under five seconds, considerably less time than it takes a base-model Honda Civic, for instance, to do the same. Then, when you tell your friend about what you're planning, he encourages you to buy a pickup truck. The truck and the Civic have similar accelerations, but the truck can achieve that acceleration while loaded down with bikes and kayaks. What is the difference between these vehicles? Their engines can all do work, but an important difference between them is the rate at which they do work.

The ability to do work quickly is something that we celebrate. For instance, in many Olympic events, the gold medal goes to the individual who can do more work, and/or do work in less time, than the other athletes. Once again, we should name this important concept.

Power is the rate at which work is done. The unit of power is the watt, and 1 W = 1 J/s.

$$P = \frac{Work}{\Delta t} = \frac{F\Delta r\cos\theta}{\Delta t} = Fv\cos\theta \ , \qquad \text{(Equation 6.11: \textbf{Power})}$$

where θ is the angle between the force and the velocity.

EXAMPLE 6.7 – Climbing the hill

A car with a weight of mg = 16000 N is climbing a hill that is 120 m long and rises 30 m vertically. The car is traveling at a constant velocity of 72 km/h. In addition to having to contend with the component of the force of gravity that acts down the slope, the car also has to deal with a constant 1000 N in resistive forces as it climbs.

(a) What is the power provided to the drive wheels by the car's engine?

(b) The power unit the horsepower was first used by James Watt in 1782 to compare steam engines and horses. What is the car's power in units of horsepower, where 1 hp = 746 W?

SOLUTION

Let's begin by sketching a diagram of the situation (see Figure 6.16), along with a free-body diagram. If we use a coordinate system aligned with the slope, with the positive x-direction up the slope, we can re-draw the free-body diagram with all the forces parallel to the coordinate axes. Doing so involves breaking the force of gravity into components.

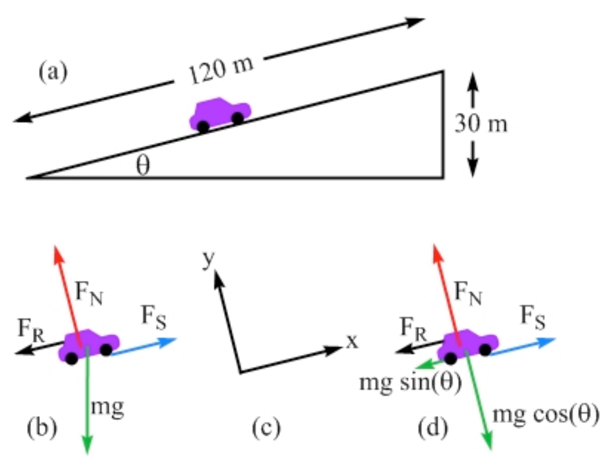

Figure 6.16: (a) A diagram for the car climbing the hill, along with (b) a free-body diagram, (c) an appropriate coordinate system, aligned with the slope, and (d) a revised free-body diagram, with all forces aligned with the coordinate system.

(a) Let's assume that this case is typical and the tires do not slip on the road surface as the car climbs the hill. If so, the force propelling the car up the slope is a static force of friction, much like the force propelling you forward when you walk is a static force of friction. This force of static friction, directed up the hill, must balance the sum of the 1000 N resistive force and the component of the force of gravity acting down the hill, which is $mg\sin\theta$. The value of $\sin\theta$ can be found from the geometry of the hill:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{30 \text{ m}}{120 \text{ m}} = \frac{1}{4}.$$

The net force directed up the hill is:

$$F_{up} = 1000 \text{ N} + mg\sin\theta = 1000 \text{ N} + 16000 \times \frac{1}{4} = 1000 \text{ N} + 4000 \text{ N} = 5000 \text{ N}.$$

The car's velocity is also directed up, so, if we multiply the force by the speed, we get the power. The speed has to be expressed in units of m/s, however. So, we perform the conversion:

$$72 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 20 \text{ m/s}.$$

The power associated with the drive wheels is:

$$P = F\,v\cos\theta = F\,v = 5000 \text{ N} \times 20 \text{ m/s} = 100000 \text{ W}.$$

(b) Converting watts to horsepower gives:

$$100000 \text{ W} \times \frac{1 \text{ hp}}{746 \text{ W}} = 134 \text{ hp}.$$

Not every car is capable of putting out that much power, but many cars are, so that's a reasonable value. The car is probably working at close to its maximum power output, however.

Related End-of-Chapter Exercise: 57, 58.

Question: A typical adult takes in about 2500 nutritional Calories of food energy in a day. Using the fact that 1 Calorie is equivalent to 1000 calories, and that 1 calorie is equivalent to 4.186 J, show that a typical adult takes in about 1 x 10^7 J worth of food energy in a day.

Answer: Much like converting from watts to horsepower, this is an exercise in unit conversion.

$$2500 \text{ Cal} \times \frac{1000 \text{ cal}}{1 \text{ Cal}} \times \frac{4.186 \text{ J}}{1 \text{ cal}} = 1.05 \times 10^7 \text{ J}.$$

Essential Question 6.7: As we have just shown, a typical adult takes in about 1 x 10^7 J of food energy in a day. Assuming this energy equals the work done by the person in a day, what average power output does this correspond to? Compare this power to the power output of a world-class cyclist, who can sustain a power output of 500 W for several hours.

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At the surface of the Earth, where we take the force of gravity to be constant, the work done by gravity is $W_g = -mg\Delta y$. The change in gravitational potential energy, ΔU_g, has the opposite sign: $\Delta U_g = -W_g = mg\Delta y$. (Eq. 6.9: **Change in gravitational potential energy**)

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EXPLORATION 6.6B – Talking about potential energy

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> **Key idea**: The change in potential energy, which everyone agrees on, is far more important than the actual value of the potential energy. **Related End-of-Chapter Exercise: 12.**

Essential Question 6.6: We often use terminology like "the ball's gravitational potential energy." Does the ball really have gravitational potential energy all by itself?

End-of-Chapter Exercises

Exercises 1 – 12 are conceptual questions designed to see whether you understand the main concepts of the chapter.

1. Three identical objects are traveling north with identical speeds *v*. Each object experiences a collision, after which the states of motion are: Object A is at rest; object B is traveling south with a speed *v*; and object C is traveling east with a speed *v*. Rank these objects, from largest to smallest, based on the magnitude of the impulse they experienced during their collision.

2. Case 1: you run with speed *v* toward a wall and then stick to it because you and the wall are covered with Velcro. Case 2: you run with speed *v* toward a wall and bounce straight back at speed *v* because the wall is covering with an elastic material. If you assume that the time during which you experience an acceleration, because of the force applied by the wall, is the same in both cases, in which case do you experience a larger force?

3. You are driving down the road at high speed. All of a sudden, you see your evil twin, driving an identical car with an equal-and-opposite velocity to you. You both apply the brakes, but it is too late and a collision is imminent. At the last instant, you see a large immovable (completely rigid) object on the side of the road. Considering only the likelihood that you will survive the crash, is it better for you to hit your evil twin or to hit the immovable object? Briefly explain your answer.

4. As you are driving along the road, you hit a mosquito, squashing it on the windshield of your car. During the collision, which object (a) exerts a force of a larger magnitude on the other? (b) experiences a change in momentum of larger magnitude? (c) experiences a change in velocity of larger magnitude?

5. Must the center of mass of an object always be located at a point where the object has some mass? If it must, explain why. If not, give an example (or two) of objects where the center of mass is located at a point where none of the mass of the object is located.

6. (a) Give an example of an object (or system of objects) such that when you make a single straight cut through the center of mass the object (or system) is split into two parts with the same mass. (b) If possible, give an example of an object or system such that when you make a single straight cut through the center of mass the two parts have different masses.

7. Consider the following four cases, in which a net force is applied to an object initially moving in the $+x$ direction with a velocity of 5 m/s. Case 1: the object's mass is 1 kg, and a force of 10 N in the $+x$ direction is applied for 1 s. Case 2: the same as case 1 except the object's mass is 2 kg. Case 3: the same as case 1 except the force is in the $-x$ direction. Case 4: the same as case 1 except the magnitude of the force is 5 N. Rank the four cases from largest to smallest based on (a) the magnitude of the change in momentum the object experiences; (b) the magnitude of the object's final momentum; (c) the object's final speed; (d) the final kinetic energy of the object.

8. Cars have crumple zones that are designed to crumple and compress when your car is in a collision. In many cases after a collision, this crumpling means that the car is ruined and you have to buy a new one (preferably with the aid of a payment from your insurance company). Is the crumple zone a huge conspiracy on the part of the auto industry, or is it

an important safety feature? Briefly explain your answer, using concepts of impulse and momentum, or work and kinetic energy.

9. Two objects, *A* and *B*, are initially at rest. The mass of object *B* is two times larger than that of object *A*. Identical net forces are then applied to the two objects, making them accelerate. Each net force is removed once the object that it is applied to has moved through a distance *d*. After both net forces are removed, how do: (a) the kinetic energies compare? (b) the speeds compare? (c) the momenta compare?

10. Repeat Exercise 9, assuming that both net forces are removed after the same amount of time instead.

11. (a) Is it possible to apply a force to an object so that the object's momentum changes but its kinetic energy remains the same? If so, give an example. (b) Is it possible to apply a force to an object so that the object's kinetic energy changes but its momentum remains the same? If so, give an example.

12. Consider Exploration 6.6B, in which Andrea and Bob chose different points as the zero point of the ball's gravitational potential energy. Do Andrea and Bob agree or disagree about the following? The value of (a) the ball's initial gravitational potential energy? (b) the ball's final gravitational potential energy? (c) the ball's change in gravitational potential energy? (d) the work done by gravity on the ball?

Exercises 13 – 18 deal with momentum and impulse.

13. Three identical objects are traveling north with identical speeds v. Each object experiences a collision, after which the states of motion are: Object A is at rest; object B is traveling south with a speed v; and object C is traveling east with a speed v. If the mass of each object is 40 kg and v = 12 m/s, find the magnitude and direction of the impulse experienced by (a) object A, (b) object B, and (c) object C.

14. Just before hitting the boards of a hockey rink, a puck is sliding along the ice at a constant velocity. As shown in Figure 6.17, the components of this velocity are 3 m/s in the direction perpendicular to the boards and 4 m/s parallel to the boards. Immediately after bouncing off the boards, the puck's velocity component parallel to the boards is unchanged at 4 m/s, and its velocity component perpendicular to the boards is 1 m/s in case A, 2 m/s in case B, and 3 m/s in case C. Without doing any calculations, rank the three cases based on the impulse the puck experienced because of its collision with the boards.

15. Return to the situation described in Exercise 14, and shown in Figure 6.17. If the puck's mass is 160 g, find the impulse applied by the boards in (a) case C; (b) case A.

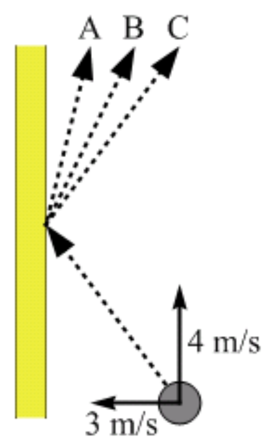

Figure 6.17: Three situations involving a hockey puck colliding with the boards, for Exercises 14 and 15.

16. An object with a mass of 5.00 kg is traveling east at 4.00 m/s. It is then subjected to a constant net force for a period of 2.00 s. In which direction should the force be applied if you want the object (a) to be moving fastest once the force is removed? (b) to experience the largest-magnitude change in momentum over the time period during which the force is applied?

17. Return to the situation described in Exercise 16. What are the magnitude and direction of the applied force if the object's velocity after the force is removed is (a) 12.0 m/s east? (b) zero? (c) 12.0 m/s west?

18. Return to the situation described in Exercise 16. What are the magnitude and direction of the applied force if the object's velocity after the force is removed is (a) 4.00 m/s north? (b) $4\sqrt{2}$ m/s northeast? (c) 8.00 m/s south?

Exercises 19 – 23 are designed to give you some practice in applying the general method for solving a problem involving impulse and momentum For each exercise, begin with the following parts: (a) Sketch a diagram of the situation. (b) Choose a coordinate system, and show it on the diagram. (c) Organize what you know, such as by drawing a free-body diagram or a graph of the net force as a function of time.

19. You throw a 200-gram ball straight up into the air, releasing it with a speed of 20 m/s. The goal here is to use impulse and momentum concepts to determine the time it takes the ball to reach its maximum height, assuming g = 10 m/s^2 down. Parts (a) – (c) as described above, where you should draw a free-body diagram of the ball after it leaves your hand for part (c). (d) What is the ball's momentum at the instant you let go of it? (e) What is the ball's momentum at the maximum-height point? (f) What is the change in the ball's momentum between the time you release it and the time it reaches its maximum height? (g) What is the force acting on the ball over this time interval? (h) Using equation 6.3, determine the time the ball takes to reach its maximum height.

20. You launch a 200-gram ball horizontally, with a speed of 30 m/s, from the top of a tall building, 80 m above the ground. The goal in this exercise is to use impulse and momentum concepts to determine the ball's momentum just before it hits the ground, assuming g = 10 m/s^2 down and air resistance is negligible. Parts (a) – (c) as described above, where you should draw a free-body diagram of the ball after it leaves your hand for part (c). (d) Using one or more constant-acceleration equations, determine the time it takes the ball to reach the ground. (e) What is the ball's momentum at the instant you let go of it? (f) Using equation 6.3, what is the change in the ball's momentum during the time it is in flight? (g) Use your answers to parts (e) and (f), noting that they are vectors, to find the ball's momentum just before it reaches the ground.

21. The Williams sisters are playing one another in the semi-finals at Wimbledon. At the instant Venus' racket makes contact with one of Serena's serves, the ball is traveling horizontally at 20 m/s, and it has no vertical velocity. The racket is in contact with the ball (which has a mass of 100 g) for 0.030 s, and the ball leaves the racket traveling at 40 m/s horizontally, in a direction exactly opposite to the path it was traveling just as her racket made contact with it. Parts (a) – (c) as described above, where you should sketch the x and y components of the average force exerted on the ball by the racket for the free-body diagram of the ball in part (c). (d) What is the x-component of the average force exerted by the racket on the ball in this case? (e) Does the average force exerted by the racket on the ball also have a non-zero y-component? Briefly explain your answer.

22. A box, with a weight of 40 N, is initially at rest on a horizontal surface. The coefficients of friction between the box and the surface are $\mu_S = 0.40$ and $\mu_K = 0.20$. You then exert a horizontal force on the box that increases linearly from 0 to 40 N over a 1.0-second period. Assume $g = 10$ m/s^2. Parts (a) – (c) as described above, where you should sketch a graph of the net force acting on the box, as a function of time, in part (c). (d) When does the box start to move? (e) What is the area under the net force versus time graph over the 1.0-second period? (f) Determine the speed of the box at the end of the 1.0-second period.

23. While you are out for a run, you see a patch of smooth ice ahead of you. You decide to slide (on your running shoes) across the ice. Your initial speed is 6.0 m/s. When you reach the end of the horizontal ice patch, after sliding for 2.0 s, your speed is 4.0 m/s. Your goal here is to determine the coefficient of kinetic friction between your running shoes and the ice, assuming that $g = 10$ m/s^2. Parts (a) – (c) as described above, where you should sketch a free-body diagram for the period you are sliding, in part (c). (d) Write an expression for the net force acting on you while you are sliding. This should involve the coefficient of kinetic friction and g. (e) Write an expression representing your change in momentum while you are sliding. (f) Use equation 6.3 to relate the expressions you wrote down in parts (d) and (e). (g) Solve for the coefficient of kinetic friction.

Exercises 24 – 30 deal with working with graphs.

24. At a time $t = 0$, a wheeled cart with a mass of 2.00 kg has an initial velocity of 5.00 m/s in the $+x$-direction. For the next 8.00 seconds, the cart then experiences a net force. As shown in the graph in Figure 6.18, the x-component of the applied force is +1.00 N for 2.00 seconds, then –4.00 N for 5.00 seconds, then +2.00 N for 1.00 seconds. (a) Sketch a graph of the x-component of the cart's momentum as a function of time. (b) What is the cart's maximum speed during the 8.00-second interval when the varying force is being applied? At what time does the cart reach this maximum speed? (c) What is the cart's minimum speed during the 8.00-second interval when the varying force is being applied? At what time does the cart reach this minimum speed?

25. Return to the situation described in Exercise 24. (a) How much work is done on the cart during the 8.00-second interval over which the force acts? (b) If the cart starts at the origin at $t = 0$, where is it at $t = 8$ s?

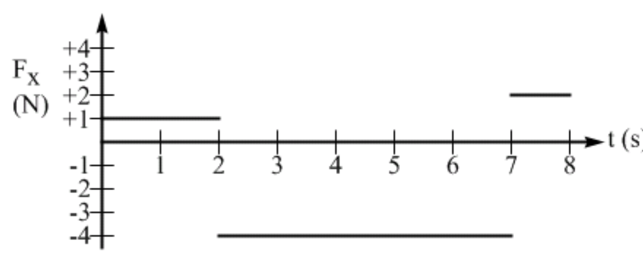

Figure 6.18: A plot of the force applied to a cart as a function of time, for Exercises 24 and 25.

26. A spaceship of mass 4000 kg is drifting at constant velocity through outer space, unaffected by any gravitational interactions. Figure 6.19 shows the trajectory followed by the spaceship in a particular *x-y* coordinate system during a 2.00 second interval. At t = 2.00 seconds, the spaceship fires its engine, producing a net force on the spaceship of 8000 N in the +y direction. The engine is turned off again after 2.00 seconds, at t = 4.00 seconds. Assume the mass of the spaceship does not change. The square boxes in the Figure 6.19 measure 1.00 m by 1.00 m. (a) Carefully plot the trajectory followed by the spaceship after t = 2.00 seconds. Note in particular where the spaceship is at t = 3.00 s, t = 4.00 s, and t = 5.00 s. (b) What is the speed of the spaceship at t = 5.00 seconds?

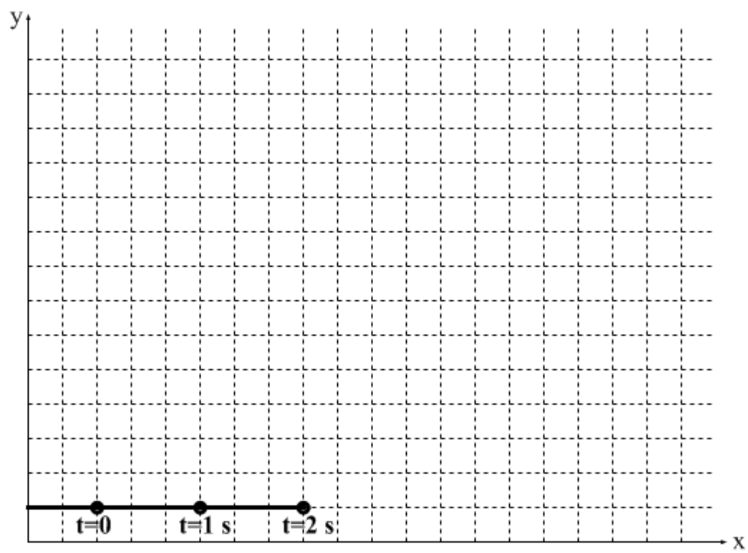

Figure 6.19: A plot of a spaceship's position as a function of time between t = 0 and t = 2 seconds, for Exercise 26.

27. An object of mass 2.0 kg is at rest at the origin, at t = 0, when it is subjected to a net force in the *x*-direction that varies in magnitude and direction as shown by the graph in Figure 6.20. (a) When does the object reach its maximum speed? (b) What is the maximum speed reached by the object? (c) What is the object's velocity at t = 8 s?

28. Repeat Exercise 27, with the only change being that the object has an initial velocity of 2.0 m/s in the negative *x* direction at t = 0.

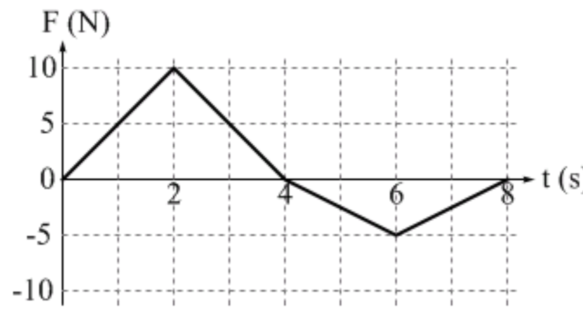

Figure 6.20: A graph of the net force in the *x*-direction that an object experiences, for Exercises 27 and 28.

29. After the time t = 0, an object of mass m = 1.0 kg is moving in the positive x direction at a constant speed of 8.0 m/s. The object is on a frictionless horizontal surface. Before t = 0, however, the object experienced a net force in the positive *x*-direction as shown in Figure 6.21. Determine the object's velocity at a time of (a) t = –1.0 s (b) t = –2.0 s (c) t = –4.0 s.

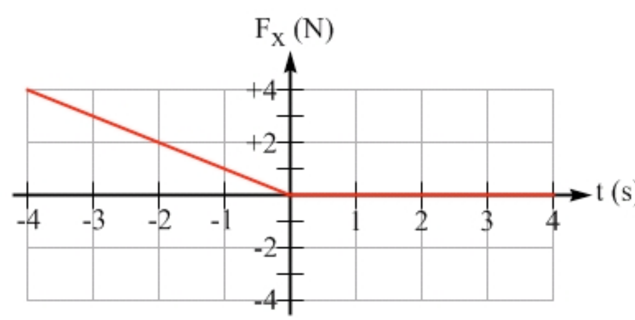

Figure 6.21: A graph of the net force applied to an object as a function of time, for Exercises 29 and 30.

30. Repeat Exercise 29, but this time use a mass of m = 0.25 kg.

Exercises 31 – 34 deal with the concept of center of mass.

31. A system consists of three balls. (a) Find the center of mass of the system, given that: Ball 1 has a mass of 2.0 kg and is located at x = +3 m, y = 0; ball 2 has a mass of 3.0 kg and is located at x = –1 m, y = –1 m; and ball 3 has a mass of 5.0 kg and is located at x = 0, y = +2 m. (b) If the mass of ball 3 is increased, the position of the center of mass shifts. In which direction does it shift?

32. A system consists of three balls at different locations on the x-axis. Ball 1 has a mass of 6.0 kg and is located at x = +3 m; ball 2 has a mass of 2.0 kg and is located at x = –1 m; ball 3 has an unknown mass and is located at x = –4 m. (a) If the center of mass of this system is located at x = –2 m, what is the mass of ball 3? (b) Let's say that you can make ball 3 as light or as heavy as you like. By adjusting the mass of ball 3, what range of positions on the x-axis can the center of mass of this system occupy?

33. A man with a mass of 120 kg is out fishing with his daughter, who has a mass of 40 kg. They are initially sitting at opposite ends of their 3.0-m boat, which has a mass of 80 kg and is at rest in the middle of a calm lake. If the man and the daughter then carefully trade places, how far does the boat move?

34. A uniform sheet of plywood measuring 4L by 4L is centered on the origin, as shown in Figure 6.22. One quarter of the sheet (the part in the first quadrant) is removed. Where is the center of mass of the remaining piece?

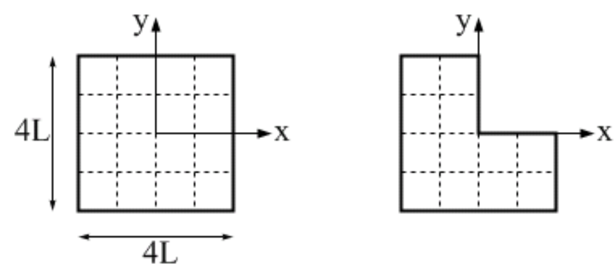

Figure 6.22: One quarter of a sheet of plywood is removed, for Exercise 34.

Exercises 35 – 39 are designed to give you some practice in applying the general method for solving a problem involving work and kinetic energy. For each exercise, begin with the following parts: (a) Sketch a diagram of the situation. (b) Choose a coordinate system, and show it on the diagram. (c) Organize what you know, such as by drawing a free-body diagram or a graph of the net force as a function of position.

35. You throw a 200-gram ball straight up into the air, releasing it with a speed of 20 m/s. The goal here is to use work and energy concepts to determine the ball's maximum height, assuming $g = 10$ m/s^2 down. Parts (a) – (c) as described above, where you should draw a free-body diagram of the ball after it leaves your hand for part (c). (d) What is the ball's kinetic energy at the instant you let go of it? (e) What is the ball's kinetic energy at the maximum-height point? (f) What is the change in the ball's kinetic energy between the point at which you release it and the point of maximum height? (g) What is the force acting on the ball over this distance? (h) Using equation 6.8, determine the distance between the point from which you released the ball and the point of maximum height.

36. You launch a 200-gram ball horizontally, with a speed of 30 m/s, from the top of a tall building, 80 m above the ground. The goal in this exercise is to use work and kinetic energy concepts to determine the ball's speed just before it hits the ground, assuming $g = 10$ m/s^2 down and air resistance is negligible. Parts (a) – (c) as described above, where you should draw a free-body diagram of the ball after it leaves your hand for part (c). (d) What is the ball's kinetic energy when you release it? (e) What is the net work done on the ball over its path from your hand to just above the ground? Hint: you can find

the net work by multiplying the net force acting on the ball by the ball's displacement in the direction of the net force. (f) Using equation 6.8, what is the ball's kinetic energy just before the ball reaches the ground? (g) What is the ball's speed just before it reaches the ground?

37. An object with a mass of 2.0 kg is following a straight path, along a line we can call the x axis. When it passes $x = 4.0$ m, it is traveling with a velocity of 8.0 m/s in the positive x direction. The object experiences a net force of 4.0 N in the positive x-direction at all locations where $x < 2.0$ m and a net force of 10 N in the positive x-direction at all locations where $x > 2.0$ m. Parts (a) – (c) as described above, where you should draw a graph of the net force acting on the ball as a function of position in part (c). The goal of this exercise is to determine, with the help of the graph, at what location the object has a velocity of 10 m/s in the positive x-direction, and at what location the object has a velocity of 2.0 m/s in the positive x-direction. (d) What is the object's kinetic energy at $x = 4.0$ m? (e) What is the object's kinetic energy when its speed is 10 m/s? (f) How much work is required to change the object's kinetic energy from what the object has at $x = 4.0$ m to what it has at the point at which the velocity is 10 m/s in the positive x-direction? Shade in the corresponding area on the force-vs.-position graph. (g) At what location will the object have a velocity of +10 m/s? (h) Follow a similar procedure to determine at what location the object will have a velocity of +2.0 m/s.

38. You are traveling in a car at 54 km/h when the car is involved in an accident. Assume that your mass is 60 kg. (a) What is your kinetic energy? (b) You are wearing your seat belt, and you come to rest after you and the car move through a distance of 2.0 m. What is the average force exerted on you by the seat belt? (c) What is your average acceleration? Express this in units of g, assuming $g = 10$ m/s^2. (d) If you are not wearing your seat belt, you may come to rest after striking the windshield and moving through a distance of 10 cm. What is your average acceleration in this case? Again, express this in units of g.

39. While you are out for a run you see a long patch of smooth ice ahead of you. You decide to slide (on your running shoes) across the ice. When you begin sliding, your speed is 6.0 m/s. When you reach the end of the horizontal ice patch, after sliding for a distance of 5.0 m, your speed is 4.0 m/s. Your goal here is to determine the coefficient of kinetic friction between your running shoes and the ice, assuming that $g = 10$ m/s^2. Parts (a) – (c) as described above, where you should sketch a free-body diagram for the period in which you are sliding, in part (c). (d) Write an expression for the net force acting on you while you are sliding. This expression should involve the coefficient of kinetic friction and g. (e) Write an expression representing your change in kinetic energy while you are sliding. (f) Use equation 6.8 to relate the expressions you wrote down in parts (d) and (e). (g) Solve for the coefficient of kinetic friction.

General Exercises and Conceptual Questions

40. A hose is used to spray water horizontally at a wall. The water has a speed of 4 m/s, and the flow rate is 5 liters per second. (a) Assuming that the water stops completely when it hits the wall, how much force does the water exert on the wall? (b) Rather than stopping completely, the water rebounds when it hits the wall. Does this change the force exerted by the water on the wall? If so, how?

41. An object has a momentum with a magnitude of 20 kg m/s and a speed of 4 m/s. It is then subjected to an impulse of 15 kg m/s in the $+x$ direction. What is the object's final velocity if the initial momentum is in the (a) $+x$ direction? (b) $-x$ direction? (c) $+y$ direction?

42. A hockey puck is sliding east at a constant velocity v over some ice. A net force F is then applied to the puck for 5 seconds. In case 1, the net force is directed west. In case 2, the net force is directed south. In case 3, the net force is directed east. The magnitude of the applied force is the same in each case. Rank the cases from largest to smallest, based on: (a) the magnitude of the change in momentum experienced by the puck, (b) the magnitude of the puck's final momentum, and (c) the work done on the puck.

43. You are shooting a free throw in basketball. If the center of the basket is 1.0 m higher, and 4.0 m horizontally, from the point at which the ball loses contact with your hands, what momentum (magnitude and direction) must the ball have when you release it, if the ball takes exactly 1.0 s to reach the center of the basket? The basketball has a mass of 0.50 kg. Use $g = 9.8$ m/s^2 for this exercise.

44. A firework of mass $10M$ is launched from the ground and follows a parabolic trajectory (assume air resistance is negligible) as shown in Figure 6.23. Its initial velocity has components $v_{ix} = 30$ m/s to the right and $v_{iy} = 20$ m/s up. It follows the parabolic trajectory shown at right. When the firework reaches its maximum height, it explodes into four pieces, A, B, C, and D (not shown on the diagram). The masses and velocities of the four pieces immediately after the explosion are:
$m_A = 1M$, $v_{Af} = 24$ m/s vertically up;
$m_B = 2M$, $v_{Bf} = 50$ m/s horizontally to the right;
$m_C = 3M$, $v_{Cf} =$ an unknown speed vertically down;
$m_D = 4M$, $v_{Df} =$ an unknown speed horizontally right or left.
(a)What is the speed of piece C after the collision? (b) What is the velocity (magnitude and direction) of piece D after the collision? (c) Before the explosion, the firework follows the typical parabolic path of an object moving under the influence of gravity alone. What path will the center of mass follow after the collision? Qualitatively, when will the center of mass divert from this path?

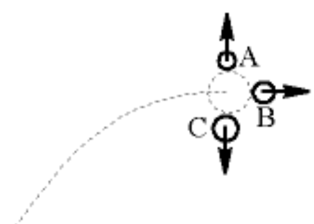

Figure 6.23: An exploding firework, for Exercises 44 and 45.

45. Repeat Exercise 44, parts (a) and (b), with the firework exploding not at the top of its trajectory, but 2.3 s after launch instead. Use $g = 10$ m/s^2, so you can do the calculations without a calculator.

46. How much work do you do on a box with a weight of 10 N in the following situations? (a) You hold the box motionless over your head for 2.0 s (b) You move the box 2.0 m horizontally at constant velocity (c) Starting and ending with the box at rest, you move the box 2.0 m straight up.

47. A box with a weight of 20.0 N is initially at rest on a horizontal surface, when a force is applied to it for 6.00 seconds. As shown in Figure 6.24, in case 1, the force is 5.00 N to the right, while in case 2, the force is 10.0 N at an angle of 60˚ above the horizontal. (a) If there is no friction between the box and the surface, in which case is more work done on the object? (b) What is the net work done in the two cases? (c) If, instead, the coefficients of friction are $\mu_s = 0.400$ and $\mu_k = 0.300$, in which case is more work done on the object? What is the net work done in the two cases now? Use $g = 10.0$ m/s^2 to simplify the calculations.

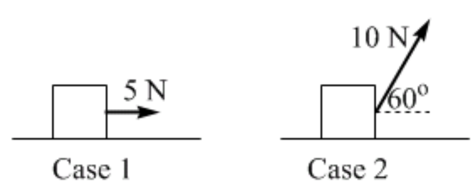

Figure 6.24: Two situations of a box subjected to a force, for Exercise 47.

48. A wheeled cart, which is free to move along the x-axis, is initially at rest at the origin. As the graph in Figure 6.25 shows, if the cart is between $x = -1$ m and $x = +2$ m, the net force is 1.00 N in the positive x-direction. If the cart is between $x = +2$ m and $x = +7$ m, the net force is 4.00 N in the negative x-direction. If the cart is between $x = +7$ m and $x = +8$ m, the net force is 2.00 N in the positive x-direction. The net force is zero at all other locations. (a) Describe, qualitatively, the resulting motion of the cart. (b) What is the maximum distance the cart gets from the origin? (c) Graph the cart's kinetic energy as a function of position as it moves. (d) If you wanted the cart to travel at least as far as $x = +8$ m, what is the minimum kinetic energy the cart needs to have at the origin?

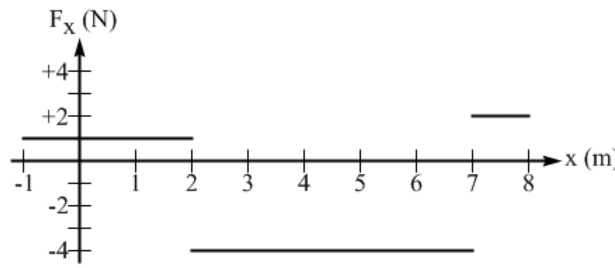

Figure 6.25: A graph showing the force applied to an object as a function of position, for Exercises 48 and 49.

49. Consider again the situation in Exercise 48. Assume the cart has a mass of 0.250 kg and is released from rest at the origin. (a) How long after the cart is released does it first pass $x = +2$ m? (b) What is the cart's maximum speed during its motion? (c) How long after it is released does the cart first return to the origin? (d) Graph the cart's velocity, as a function of time, for the first 10 seconds after its release.

50. We'll deal with springs in detail in chapter 12, but consider the situation shown in Figure 6.26. A block of mass $m = 0.25$ kg is traveling with a velocity $v = 4.0$ m/s to the left on a frictionless horizontal surface. When it reaches $x = 0$, the block encounters a spring, which exerts a force directed right on the block that depends on how much the spring is compressed. The graph shows the force the spring exerts on the block as a function of position, x. (a) How far will the block compress the spring in this case? (b) How far is the spring compressed when the block has a speed of $v = 2.0$ m/s?

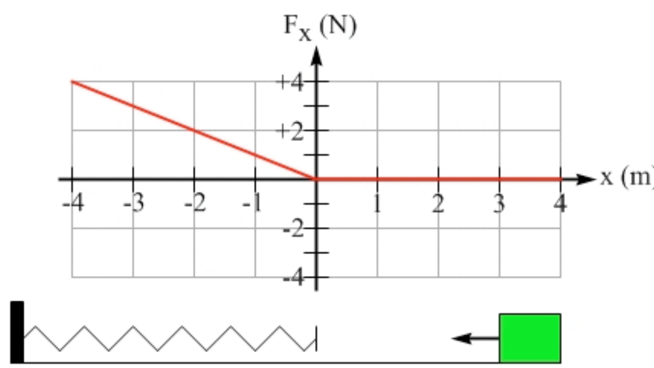

Figure 6.26: The graph shows the force a spring exerts on a block as a function of position. The diagram below the graph shows the block moving left on a frictionless surface before encountering the spring. For Exercises 50 and 51.

51. Consider the situation described in Exercise 50. How far will the block compress the spring (a) if the mass of the block is doubled? (b) if, instead, the initial velocity of the block is doubled?

52. Two identical boxes of mass m are sliding along a horizontal floor, but both eventually come to rest because of friction. Box A has an initial speed of v, while box B has an initial speed of $2v$. The coefficient of kinetic friction between each box and the floor is μ_K, and the acceleration due to gravity is g. (a) If it takes box A a time T to come to a stop, how much time does it take for box B to come to a stop? (b) Find an expression for T in terms of the variables specified in the exercise. (c) If box A travels a distance D before coming to rest, how far does box B travel before coming to rest? (d) Find an expression for D in terms of the variables specified in the exercise.

53. Return to the situation described in Exercise 52. How does T, the stopping time for box A, change if (a) m is doubled? (b) v is doubled? (c) μ_K is doubled?

54. Return to the situation described in Exercise 52. How does D, the stopping distance for box A, change if (a) m is doubled? (b) v is doubled? (c) μ_K is doubled? (d) g is doubled?

55. A car traveling 50 km/h can be brought to a stop in a distance of 40 m under controlled braking conditions. (a) Assuming the force used to bring the car to rest is the same, how much distance is required to bring the car to a stop if the car is traveling 100 km/h, twice as fast as it was originally? (b) How do the stopping times compare? (Ignore the reaction time of the driver and find the distance and time after the brakes are applied.)

56. A box, with a weight of mg = 25 N, is placed at the top of a ramp and released from rest. The ramp is in the shape of a 3-4-5 triangle, measuring 4 meters horizontally and 3 meters vertically. The box accelerates down the incline, attaining a kinetic energy at the bottom of the ramp of 55 J. There is a force of kinetic friction acting on the box as it slides down the incline. (a) Sketch a free-body diagram of the box, showing all the forces acting on it. (b) How much work does the normal force do on the box as the box slides down the incline? (c) Calculate the change in gravitational potential energy that the box experiences in this process. (d) How much work does the force of friction do on the box as the box slides down the incline? (e) What is the coefficient of kinetic friction between the box and ramp?

57. A car is accelerating from rest and takes a time T to reach speed v. (a) Assuming the force accelerating the car is constant, what is the total time (measured from the starting point) needed to reach a speed of $2v$? (b) Assuming instead that the power associated with accelerating the car is constant, what is the total time needed to reach a speed of $2v$?

58. You are cycling at a constant speed of 10 m/s. (a) If the net resistive force acting against you from things like air resistance is 35 N, what is your power output as you pedal? (a) (b) How much additional power is required to maintain this speed up a hill inclined at 8.0° with the horizontal? Assume the combined mass of you and your bicycle is 50 kg.

59. On a monthly electricity bill, the power companies charge you for the number of kilowatt-hours you consume. (a) What kind of unit is the kilowatt-hour? Is it power? Momentum? Something else? (b) Convert 1 kW-h to MKS units. (c) 1 kilowatt-hour typically costs about 20 cents. If you were somehow able to obtain your daily intake of 2500 Cal by plugging yourself into a wall socket (don't try this, of course!), how much would it cost you?

60. An energy bar contains about 200 Cal. If your brain consumes about 20 W under typical conditions, for how long does one energy bar keep the brain functioning? 1 Cal = 1000 calories, and 1 calorie is approximately 4 J.

61. Consider the Earth, with a mass of 6.0×10^{24} kg, in its orbit around the Sun, with a mass of 2.0×10^{30} kg. Assume the orbit is circular, with a radius of 1.5×10^{11} m. The Earth, traveling at 30 km/s, takes six months to travel halfway around the orbit. (a) What is the magnitude of the Earth's change in momentum over this six-month period? (b) How much work does the Sun do on the Earth over this six-month period?

62. Comment on the statements made by three students who are working together to solve the following problem, and state the answer to the problem. A cart with a mass of 2.0 kg has an initial velocity of 4.0 m/s in the positive *x*-direction. A constant net force of 8.0 N, in the positive *x*-direction, is then applied to the cart for 0.50 s. What is the cart's kinetic energy at the end of this 0.50 second interval?

Christina: I think we should use impulse here. Using impulse, we can figure out the change in velocity, and then the final velocity. Once we get that, we can use the mass and velocity to get the kinetic energy.

Sandy: Don't we need to find the acceleration? That's just 4.0 meters per second squared. Then we can use one of the constant-acceleration equations to find the final speed, and get the kinetic energy that way.

Phil: I like getting the acceleration first, but then we can find the displacement using one of the constant-acceleration equations. After that, we can get the work, which is the change in kinetic energy, and then get the final kinetic energy. They basically give us the initial kinetic energy.

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