5-1 Kinetic Friction

When two objects are in contact, the friction force, if there is one, is the component of the contact force between the objects that is parallel to the surfaces in contact. (The component of the contact force that is perpendicular to the surfaces is the normal force.) Friction tends to oppose relative motion between objects. When there is relative motion, the friction force is the kinetic force of friction. For instance, when a book slides across a table, kinetic friction slows the book.

> If there is relative motion between objects in contact, the force of friction is the kinetic force of friction (F_K). We will use a simple model of friction that assumes the force of kinetic friction is proportional to the normal force. A dimensionless parameter, called the coefficient of kinetic friction, μ_K, represents the strength of that frictional interaction.
>
> $$\mu_K = \frac{F_K}{F_N} \quad \text{so} \quad F_K = \mu_K F_N \,. \qquad \text{(Equation 5.1: \textbf{Kinetic friction})}$$
>
> Note that some typical values for the coefficient of kinetic friction, as well as for the coefficient of static friction, which we will define in Section 5-2, are given in Table 5.1. The coefficients of friction depend on the materials that the two surfaces are made of, as well as on the details of their interaction. For instance, adding a lubricant between the surfaces tends to reduce the coefficient of friction. There is also some dependence of the coefficients of friction on the temperature.

Interacting materials	**Coefficient of kinetic friction** (μ_K)	**Coefficient of static friction** (μ_S)
Rubber on dry pavement	0.7	0.9
Steel on steel (unlubricated)	0.6	0.7
Rubber on wet pavement	0.5	0.7
Wood on wood	0.3	0.4
Waxed ski on snow	0.05	0.1
Friction in human joints	0.01	0.01

Table 5.1: Approximate coefficients of kinetic friction, and static friction (see Section 5-2), for various interacting materials.

EXPLORATION 5.1 – A sliding book

You slide a book, with an initial speed v_i, across a flat table. The book travels a distance L before coming to rest. What determines the value of L in this situation?

Step 1 – ***Sketch a diagram of the situation and a free-body diagram of the book.*** These diagrams are shown in Figure 5.1.

Figure 5.1: A diagram of the sliding book and a free-body diagram showing the forces acting on the book as it slides.

The Earth applies a downward force of gravity on the book, while the table applies a contact force. Generally, we split the contact force into components and show an upward normal force and a horizontal force of kinetic friction, $\vec{F}_K$, acting on the book. $\vec{F}_K$ points to the right on the book, opposing the relative motion between the book (moving left) and the table (at rest).

Step 2 – ***Find an expression for the book's acceleration.*** We are working in two dimensions, and a coordinate system with axes horizontal and vertical is convenient. Let's choose up to be positive for the y-axis and, because the motion of the book is to the left, let's choose left to be the positive x-direction. We will apply Newton's Second Law twice, once for each direction. There is no acceleration in the y-direction, so we have: $\sum \vec{F}_y = m\vec{a}_y = 0$.

Because we're dealing with the y sub-problem, we need only the vertical forces on the free-body diagram. Adding the vertical forces as vectors (with appropriate signs) tells us that:

$$+F_N - F_G = 0 \,, \quad \text{so} \quad F_N = F_G = mg \,.$$

Repeat the process for the x sub-problem, applying Newton's Second Law, $\sum \vec{F}_x = m\vec{a}_x$. Because the acceleration is entirely in the x-direction, we can replace $\vec{a}_x$ by $\vec{a}$. Now, we focus only on the forces in the x-direction. All we have is the force of kinetic friction, which is to the right while the positive direction is to the left. Thus: $-F_K = m\vec{a}$.

We can now solve for the acceleration of the book, which is entirely in the x-direction:

$$\vec{a} = \frac{-F_K}{m} = \frac{-\mu_K F_N}{m} = \frac{-\mu_K mg}{m} = -\mu_K g \,.$$

Step 3 – ***Determine an expression for length, L, in terms of the other parameters.*** Let's use our expression for the book's acceleration and apply the method for solving a constant-acceleration problem. Table 5.2 summarizes what we know. Define the origin to be the book's starting point and the positive direction to be the direction of motion.

Initial position	$x_i = 0$
Final position	$x = +L$
Initial velocity	$+v_i$
Final velocity	$v = 0$
Acceleration	$a = -\mu_K g$

Table 5.2: A summary of what we know about the book's motion to help solve the constant-acceleration problem.

We can use equation 2.10 to relate the distance traveled to the coefficient of friction:

$$v^2 = v_i^2 + 2a\Delta x \,, \quad \text{so} \quad \Delta x = \frac{v^2 - v_i^2}{2a} \,.$$

In this case, we get $$L = \frac{0 - v_i^2}{-2\mu_K g} = \frac{v_i^2}{2\mu_K g} \,.$$

Let's think about whether the equation we just derived for L makes sense. The equation tells us that the book travels farther with a larger initial speed or with smaller values of the coefficient of friction or the acceleration due to gravity, which makes sense. It is interesting to see that there is no dependence on mass – all other parameters being equal, a heavy object travels the same distance as a light object.

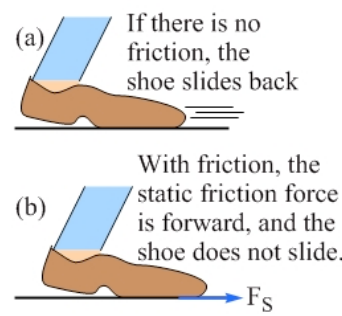
Key idea: A useful method for solving a problem with forces in two dimensions is to split the problem into two one-dimensional problems. Then, we solve the two one-dimensional problems individually. **Related End-of-Chapter Exercises 13 – 15, 31.**

Essential Question 5.1: You are standing still and you then start to walk forward. Is there a friction force involved here? If so, is it the kinetic force of friction or the static force of friction?

Answer to Essential Question 5.1: When asked this question, most people are split over whether the friction force is kinetic or static. Think about what happens when you walk. When your shoe (or foot) is in contact with the ground, the shoe does not slip on the ground. Because there is no relative motion between the shoe and the ground, the friction force is static friction.

5-2 Static Friction

If there is no relative motion between objects in contact, then the friction force (if there is one) is the static force of friction (F_S). An example is a box that remains at rest on a ramp. The force of gravity acting down the ramp is opposed by a static force of friction acting up the ramp. A more challenging example is when the box is placed on the floor of a truck. When the truck accelerates and the box moves with the truck (remaining at rest relative to the truck), it is the force of static friction that acts on the box to keep it from sliding around in the truck.



Consider again the question about the friction force between the sole of your shoe and the floor, when you start to walk. In which direction is the force of static friction? Many people think this friction force is directed opposite to the way you are walking, but the force of static friction is actually directed the way you are going. To determine the direction of the force of static friction, think about the motion that would result if there were no friction. To start walking, you push back with your foot on the floor. Without friction, your foot would slide back, moving back relative to the floor, as shown in Figure 5.2. Static friction opposes this motion, the motion that would occur if there was no friction, and thus static friction is directed forward.

Figure 5.2: On a frictionless floor, your shoe slides backward over the floor when you try to walk forward (a). Static friction opposes this motion, so the static force of friction, applied by the ground on you, is directed forward (b).

The static force of friction opposes the relative motion that would occur if there were no friction. Another interesting feature is that ***the static force of friction adjusts itself to whatever it needs to be to prevent relative motion between the surfaces in contact.*** Within limits, that is. The static force of friction has a maximum value, $F_{S,\max}$, and the coefficient of static friction is defined in terms of this maximum value:

$$\mu_S = \frac{F_{S,\max}}{F_N} \quad \text{so} \quad F_S \le \mu_S F_N . \qquad \text{(Equation 5.2: \textbf{Static friction})}$$

Let's now explore a situation that involves the adjustable nature of the force of static friction.

EXPLORATION 5.2 – A box on the floor

A box with a weight of *mg* = 40 N is at rest on a floor. The coefficient of static friction between the box and the floor is $\mu_S = 0.50$, while the coefficient of kinetic friction between the box and the floor is $\mu_K = 0.40$.

Step 1 - ***What is the force of friction acting on the box if you exert no force on the box?*** Let's draw a free-body diagram of the box (see Figure 5.3b) as it sits at rest. Because the box remains at rest, its acceleration is zero and the forces must balance. Applying Newton's Second Law tells us that $F_N = mg = 40$ N. There is no tendency for the box to move, so there is no force of friction.

Step 2 - ***What is the force of friction acting on the box if you push horizontally on the box with a force of 10 N, as in Figure 5.3a?***
Nothing has changed vertically, so we still have $F_N = mg = 40 \text{ N}$. To determine whether or not the box moves, let's use equation 5.2 to determine the maximum possible force of static friction in this case. We get:

$$F_S \leq \mu_S F_N = 0.50 \times 40 \text{ N} = 20 \text{ N}.$$

The role of static friction is to keep the box at rest. If we exert a horizontal force of 10 N on the box, the force of static friction acting on the box must be 10 N in the opposite direction, to keep the box from moving. The free-body diagram of this situation is shown in Figure 5.3c. 10 N is below the 20 N maximum value, so the box will not move.

Step 3 - ***What is the force of friction acting on the box if you increase your force to 15 N?*** This situation is similar to step 2. Now, the force of static friction adjusts itself to 15 N in the opposite direction of your 15 N force. 15 N is still less than the maximum possible force of static friction (20 N), so the box does not move.

Step 4 - ***What is the force of friction acting on the box if you increase your force to 20 N?*** If your force is 20 N, the force of static friction matches you with 20 N in the opposite direction. We are now at the maximum possible value of the force of static friction. Pushing even a tiny bit harder would make the box move.

Step 5 - ***What is the force of friction acting on the box if you increase your force to 25 N?*** Increasing your force to 25 N, which is larger in magnitude than the maximum possible force of static friction, makes the box move. Because the box moves, the friction is the kinetic force of friction, which is in the direction opposite to your force with a magnitude of

$$F_K = \mu_K F_N = 0.40 \times 40 \text{ N} = 16 \text{ N}.$$

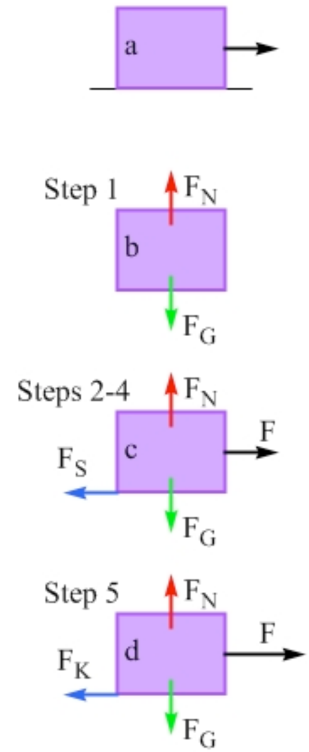


Figure 5.3: (a) The top diagram shows the box, and the force you exert on it. (b) The free-body diagram for step 1, in which you exert no force. (c) The free-body diagram that applies to steps 2 – 4, in which the force you exert is less than or equal to the maximum possible force of static friction. (d) The free-body diagram that applies to step 5, in which your force is large enough to cause the box to move.

Key ideas for static friction: The static force of friction is whatever is required to prevent relative motion between surfaces in contact. The static force of friction is adjustable only up to a point. If the required force exceeds the maximum value $F_{S,\max} = \mu_S F_N$, then relative motion will occur.
Related End-of-Chapter Exercises: 32, 34.

A microscopic model of friction

Figure 5.4 shows a magnified view of two surfaces in contact. Surface irregularities interfere with the motion of one surface left or right with respect to the other surface, giving rise to friction.

Figure 5.4: A magnified view of two surfaces in contact. The irregularities in the objects prevent smooth motion of one surface over the other, giving rise to friction.

Essential Question 5.2: What is the magnitude of the net contact force exerted by the floor on the box in step 5 of Exploration 5.2?

Answer to Essential Question 5.2: The two components of the contact force are the upward normal force and the horizontal force of kinetic friction. These two components are at right angles to one another, so we can use the Pythagorean theorem to find the magnitude of the contact force:

$$F_C = \sqrt{F_K^2 + F_N^2} = \sqrt{(16\text{ N})^2 + (40\text{ N})^2} = 43\text{ N} .$$

5-3 Measuring the Coefficient of Friction

Let's connect the force ideas to the one-dimensional motion situations from Chapter 2.

EXPLORATION 5.3 – Measuring the coefficient of static friction

Coefficients of static friction for various pairs of materials are given in Table 5.1. Here's one method for experimentally determining these coefficients for a particular pair of materials. Take an aluminum block of mass m and a board made from a particular type of wood (we could also use a block of the wood and a piece of inflexible aluminum). Place the block on the board and slowly raise one end of the board. The angle of the board when the block starts to slide gives the coefficient of static friction. How?

To answer this question, let's extend the general method for solving a problem that involves Newton's laws.

Step 1 – ***Draw a diagram of the situation.*** The diagram is shown in Figure 5.5a.

Step 2 – ***Draw a free-body diagram of the block when it is at rest on the inclined board.*** Two forces act on the block, the downward force of gravity and the upward contact force from the board. We generally split the contact force into components, the normal force perpendicular to the incline, and the force of static friction acting up the slope. This free-body diagram is shown in Figure 5.5b

Step 3 – ***Choose an appropriate coordinate system.*** In this case, if we choose a coordinate system aligned with the board (one axis parallel to the board and the other perpendicular to it), as in Figure 5.5c, we will only have to split the force of gravity into components.

Step 4 – ***Split the force of gravity into components.*** If the angle between the board and the horizontal is θ , the angle between the force of gravity and the y-axis is also θ. The component of the force of gravity acting parallel to the slope has a magnitude of $F_{Gx} = F_G \sin\theta = mg\sin\theta$. The perpendicular component is $F_{Gy} = F_G \cos\theta = mg\cos\theta$. These components are shown on the lower free-body diagram, in Figure 5.5d.

Step 5 – ***Apply Newton's second law twice, once for each direction.*** Again, we break a two-dimensional problem into two one-dimensional problems. With no acceleration in the y-direction we get: $\sum \vec{F}_y = m\vec{a}_y = 0$. Looking at the lower diagram in Figure 5.5 for the y-direction forces:

$$+F_N - mg\cos\theta = 0 ,$$ which tells us that

$$F_N = mg\cos\theta .$$

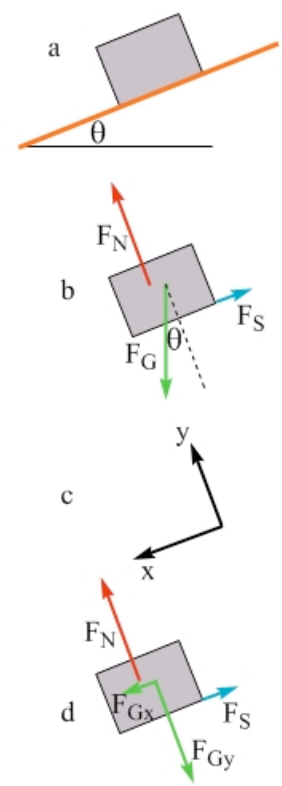


Figure 5.5: (a) A diagram showing the block on the board. (b) The initial free-body diagram of the block. (c) An appropriate coordinate system, aligned with the incline. (d) A second free-body diagram, with the forces aligned parallel to the coordinate axes.

While the box is at rest, there is no acceleration in the x-direction so: $\sum \vec{F}_x = m\vec{a}_x = 0$. Looking at the lower free-body diagram, Figure 5.5d, for the forces in the x-direction:

$mg\sin\theta - F_S = 0$, which tells us that $F_S = mg\sin\theta$.

These two equations, $F_N = mg\cos\theta$ and $F_S = mg\sin\theta$, tell us a great deal about what happens as the angle of the incline increases. When the board is horizontal, the normal force is equal to mg and the static force of friction is zero. As the angle increases, $\cos\theta$ decreases from 1 and $\sin\theta$ increases from zero. Thus, as the angle increases, the normal force (and the maximum possible force of static friction) decreases, while the force of static friction required to keep the block at rest increases. At some critical angle θ_C, the force of static friction needed to keep the block at rest equals the maximum possible force of static friction. If the angle exceeds this critical angle, the block will slide. Using the definition of the coefficient of static friction:

$$\mu_S = \frac{F_{S,\max}}{F_N} = \frac{mg\sin\theta_C}{mg\cos\theta_C} = \tan\theta_C .$$

Thus, it is easy to determine coefficients of static friction experimentally. Take two objects and place one on top of the other. Gradually tilt the objects until the top one slides off. The tangent of the angle at which sliding occurs is the coefficient of static friction.

Key idea regarding the coefficient of static friction: The coefficient of static friction between two objects is the tangent of the angle beyond which one object slides down the other.
Related End-of-Chapter Exercises: 7, 36.

The steps we used to solve the problem in Exploration 5.3 can be applied generally to most problems involving Newton's laws. Let's summarize the steps here. Then, we will get some more practice applying the method in the next section.

A General Method for Solving a Problem Involving Newton's Laws in Two Dimensions

1. Draw a diagram of the situation.

2. Draw one or more free-body diagrams, with each free-body diagram showing all the forces acting on an object.

3. For each free-body diagram, choose an appropriate coordinate system. Coordinate systems for different free-body diagrams should be consistent with one another. A good rule of thumb is to align each coordinate system with the direction of the acceleration.

4. Break forces into components that are parallel to the coordinate axes.

5. Apply Newton's second law twice to each free-body diagram, once for each coordinate axis. Put the resulting force equations together and solve.

Related End-of-Chapter Exercises 17, 19.

Essential Question 5.3: Could we modify the procedure described in Exploration 5.3 to measure the coefficient of kinetic friction? If so, how could we modify it?

Answer to Essential Question 5.3: Yes. With the top object at rest on the inclined second object, give the top object a little push to get it moving. If it slides down the second object with constant velocity, the tangent of the angle of the incline equals the coefficient of kinetic friction.

5-4 A System of Two Objects and a Pulley

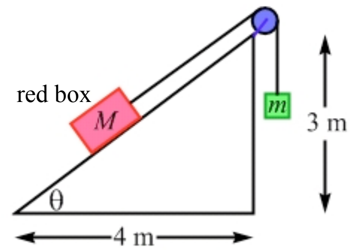


Figure 5.6: A diagram for the system of two objects and a pulley.

EXAMPLE 5.4 – Working with more than one object, and a pulley

A red box of mass M = 10 kg is placed on a ramp that is a 3-4-5 triangle, with a height of 3.0 m and a width of 4.0 m. The red box is tied to a green block of mass m = 1.0 kg by a string passing over a pulley, as shown in Figure 5.6. The coefficients of friction for the red box and the incline are $\mu_s = 0.50$ and $\mu_k = 0.25$. Use g = 10 m/s^2. When the system is released from rest, what is the acceleration?

SOLUTION

To determine if there is an acceleration, and to find the direction of any acceleration, think about what happens if there is no friction. With no friction, we have the free-body diagrams in Figure 5.7. Choose a coordinate system aligned with the slope for the red box, with the positive x-direction down the slope and a positive y-direction perpendicular to the incline. If the red box moves down the slope, the green block moves up, so a consistent coordinate system for the green block has the positive direction up.

To align all the forces with the coordinate axes, only the force of gravity acting on the red box needs to be split into components. As shown in Figure 5.7c, we get $Mg\sin\theta$ directed down the slope and $Mg\cos\theta$ perpendicular to the slope.

Applying Newton's second law, $\sum \vec{F} = m\vec{a}$, to the green block gives:

$+F_T - mg = +ma$, which tells us that $F_T = mg + ma$.

Applying Newton's second law in the x-direction for the red box gives

$+Mg\sin\theta - F_T = Ma$, which gives $Mg\sin\theta = F_T + Ma$.

Combining the equation from the green block with the equation from the red box tells us that: $Mg\sin\theta = mg + ma + Ma$. This can be re-arranged into $(M\sin\theta - m)g = (m + M)a$.

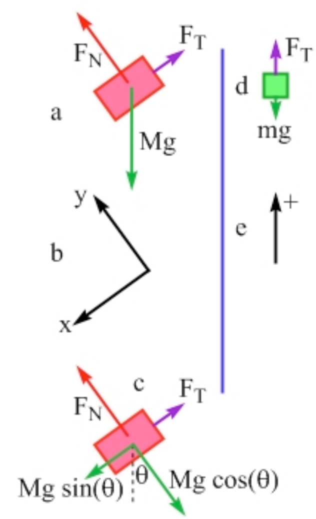


Figure 5.7: Free-body diagrams if there is no friction. (a) The free-body diagram of the red box. (b) An appropriate coordinate system for the red box. (c) The free-body diagram of the red box, with force components aligned with the coordinate system. (d) and (e), a free-body diagram and coordinate system for the green block.

The acceleration of the system, with no friction, is positive if $M\sin\theta$ exceeds m. In this case, $M\sin\theta = 10\text{ kg}\times(3/5) = 6.0\text{ kg}$ is larger than the mass m = 1.0 kg of the green block. Thus, if the system accelerates, the red box accelerates down the slope. We don't know for sure that the system accelerates, because the force of static friction could prevent any motion. The force of static friction would be directed up the ramp, as in Figure 5.8, to stop the box from sliding down the ramp. How large must the force of static friction be to prevent the system from moving?

If the system remains at rest, the acceleration is zero. Applying Newton's second law, $\sum \vec{F}_x = M\vec{a} = 0$, to the red box in the x-direction gives $+Mg\sin\theta - F_S - F_T = 0$.

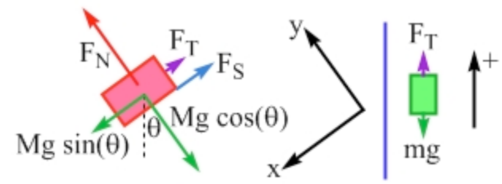


Figure 5.8: The free-body diagrams if static friction prevents motion are similar to those in Figure 5.7, except that, in this case, the forces balance and there is a force of static friction, on the red box, directed up the slope.

With no acceleration, the force equation for the green block is $F_T = mg$. Using $F_T = mg$ in the previous equation gives $+Mg\sin\theta - F_S - mg = 0$. Thus, the force of static friction needed to prevent motion is:

$$F_S = Mg\sin\theta - mg = (M\sin\theta - m)g = [10\text{ kg}\times(3/5) - 1.0\text{ kg}]g = (5.0\text{ kg})\,g.$$

Let's compare this value to the maximum possible force of static friction. Applying Newton's second law to the red box in the y-direction gives $F_N = Mg\cos\theta$, because there is no acceleration in that direction. Thus, the maximum possible force of static friction in this case is:

$$F_{S,\max} = \mu_S F_N = \mu_S Mg\cos\theta = 0.5\times(10\text{ kg})\times(4/5)\times g = (4.0\text{ kg})\,g.$$

The force of static friction required to prevent the system from accelerating is larger than its maximum possible value, which cannot happen. Thus, the system does accelerate. To find the acceleration, we again draw free-body diagrams, as in Figure 5.9. Now kinetic friction acts up the slope on the red box, and each object has a net force acting on it.

Applying Newton's second law, to the green block gives $+F_T - mg = +ma$, which tells us that $F_T = mg + ma$. A common error is to assume that $F_T = mg$, which is true only when the acceleration is zero.

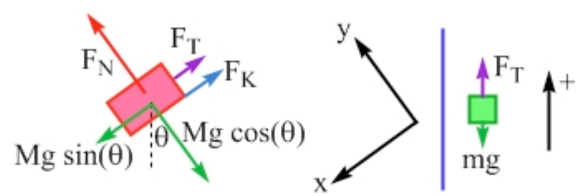


Figure 5.9: Free-body diagrams for the situation of the red box accelerating down the slope and the green block accelerating up.

Applying Newton's second law to the red box in the x-direction gives $+Mg\sin\theta - F_k - F_T = Ma$.

Substituting for the force of tension (using our result from analyzing the green block), we get $+Mg\sin\theta - F_k - mg - ma = Ma$, which we re-write as $+Mg\sin\theta - F_k - mg = (M+m)a$.

We can now substitute for the force of kinetic friction and find the acceleration:

$$a = \frac{+Mg\sin\theta - \mu_K F_N - mg}{M+m} = \frac{+Mg\sin\theta - \mu_K Mg\cos\theta - mg}{M+m}.$$

$$a = \frac{+10\text{ kg}\times(10\text{ m/s}^2)(3/5) - 0.25\times(10\text{ kg})(10\text{ m/s}^2)(4/5) - (1.0\text{ kg})\times(10\text{ m/s}^2)}{10.0\text{ kg} + 1.0\text{ kg}}.$$

$$a = \frac{+60\text{ N} - 20\text{ N} - 10\text{ N}}{11.0\text{ kg}} = \frac{+30\text{ N}}{11.0\text{ kg}} = +2.7\text{ m/s}^2.$$

Note that the role of the pulley in this situation is simply to redirect the force of tension.

Related End-of-Chapter Exercises: 18, 41, 47.

Essential Question 5.4: If an object moves in a circle at constant speed, is there an acceleration?

Answer to Essential Question 5.4: Yes. An object has an acceleration whenever its velocity changes. Although the magnitude of the velocity is constant, the direction of the velocity changes, so there must be an acceleration. We'll investigate this further in the next section.

5-5 Uniform Circular Motion

Uniform circular motion is motion in a circle with constant speed. Let's define T, the period of the uniform circular motion, to be the time it takes an object to travel around one complete circle. Because the speed is constant, we can relate the speed to the distance traveled very simply: v = distance/time = $2\pi r / T$.

As with straight-line motion, the magnitude of the acceleration is related to the speed the same way that the speed is related to the distance: $a = 2\pi v / T$.

If we combine the two equations above, we get what's called the *centripetal acceleration*.

When an object is traveling in a circular path, the object has an acceleration directed toward the center of the circle. This acceleration is known as the centripetal acceleration :

$$a_c = \frac{v^2}{r}.$$ (Equation 5.3: **Centripetal acceleration**)

The direction of the centripetal acceleration is toward the center of the circle, because the change in velocity is toward the center, as illustrated in Figure 5.10.

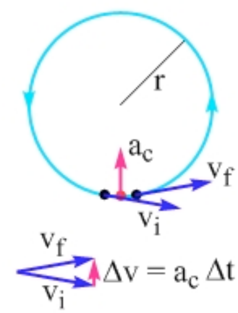


Figure 5.10: Subtracting the velocity just before the object is at the bottom of the circle ($\vec{v}_i$) from the velocity just after that point ($\vec{v}_f$) gives the change in velocity $\Delta\vec{v}$, which is directed toward the center of the circle. The centripetal acceleration is proportional to this change in velocity and thus is also directed toward the center.

Many people have heard the term "centripetal force." Is this a new force that arises because something goes in a circle? No, it is not. Let's investigate this idea. Which free-body diagram in Figure 5.11 correctly shows the force(s) acting on the Earth (E) as it orbits the Sun, when the Earth is at the position shown, to the right of the Sun? F_G is the gravitational force exerted on the Earth by the Sun, while F_C stands for centripetal force.

The correct free-body diagram is diagram 3, which shows only the force of gravity applied by the Sun on the Earth. The word "centripetal" means "directed toward the center." When an object experiences uniform circular motion, the object has a centripetal acceleration directed toward the center of the circle. The centripetal acceleration requires a net force directed toward the center, but this net force comes from one or more real forces (such as gravity, tension, or friction) or their components. There is no magical new centripetal force responsible for this motion. Thus, we will avoid the term "centripetal force" altogether, and talk about the force, or forces, responsible for the centripetal acceleration, instead.

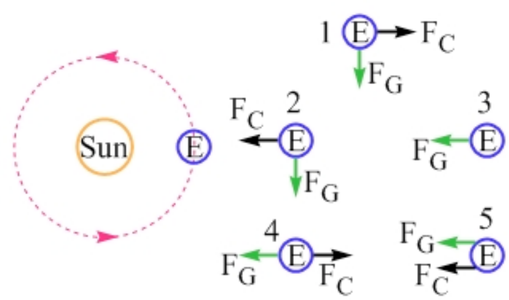


Figure 5.11: Five possible free-body diagrams for the Earth (E), as it orbits the Sun.

Figure 5.12: To avoid confusion, we will avoid the term "centripetal force" and we will not to draw a centripetal force on a free-body diagram.

EXPLORATION 5.5 – Identifying the force(s) responsible for the centripetal acceleration

Which force gives rise to the centripetal acceleration in the following situations?

Step 1 - ***While you are riding your bike, you turn a corner, following a circular arc. What is the force acting on your bike that is associated with the centripetal acceleration, keeping you going in a circle?***

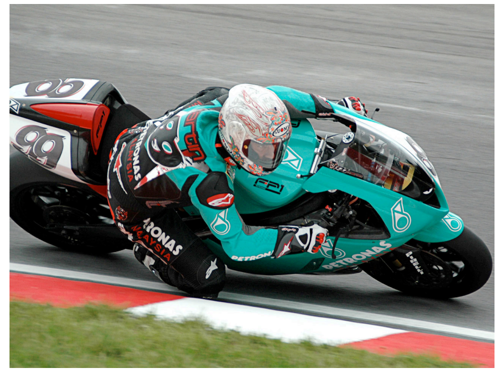
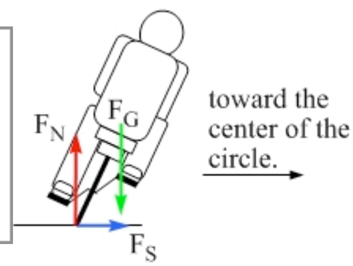


Figure 5.13: A photograph of motorcycle racer Steve Martin rounding a bend. Which force is directed in toward the center of the circular arc as the motorcycle rounds the bend? Photo credit: John Edwards, via http://www.publicdomainpictures.net.

Let's sketch a free-body diagram (see Figure 5.14). As usual, there is a downward force of gravity acting on the system consisting of you and the bike, balanced by an upward normal force applied by the road. If there is no friction acting on the bike tires, the bike would keep going in a straight line, moving away from the center of the circle. The force of friction acting on the bike tires is the force pointing toward the center of the circular arc, opposing the tendency for the bike to move out from the center of the circle. As long as the tires do not skid on the road surface, the friction force is a static force of friction.

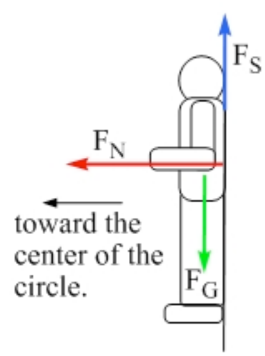
Figure 5.14: A free-body diagram for a rider-bike system while the rider is traveling around a curve.



Step 2 - ***On a carnival ride called the Rotor, shown in the opening photo of this chapter, once the ride is spinning quickly, the riders are pinned to the wall and the floor is removed from under them. Which force is directed toward the center of the circle?***

Let's sketch the free-body diagram of a person on the moving ride. As usual, the force of gravity is acting down. The person's velocity tries to carry them farther from the center of the circle, but the wall gets in the way. There is a contact force associated with the person-wall interaction. The wall exerts a normal force, directed toward the center of the circle, on the person. This is the force we're looking for. The complete free-body diagram, in Figure 5.15, also shows an upward force of friction opposing the force of gravity. This force of friction is static friction because there is no relative motion between the person and the wall.

Figure 5.15: A free-body diagram for a person on the Rotor.



Key ideas for circular motion: In uniform circular motion, there is a net force directed toward the center of the circle. We do not need to invent a magical new force to act as the net force. Instead, this net force toward the center of the circle is associated with one or more of the standard forces we already know about. **Related End-of-Chapter Exercises 11, 59.**

Essential Question 5.5: Why does the Rotor have to be spinning fast before the floor is removed?

Answer to Essential Question 5.5: The faster the ride goes, the larger the normal force that the wall exerts on the rider. The larger the normal force is, the larger the maximum possible force of static friction that the wall can exert on the rider. The upward static friction force must balance the downward force of gravity. To provide a margin of safety, the maximum possible force of static friction should exceed the rider's weight by a significant amount. If the ride is too slow, the normal force is reduced, reducing the maximum possible force of static friction. If the maximum possible force of static friction drops below the rider's weight, the rider will slide down the wall. A similar situation could occur if the coefficient of static friction, associated with the interaction between the wall and the rider's clothes, is too small.

5-6 Solving Problems Involving Uniform Circular Motion

Let's investigate a typical circular-motion situation in some detail, although first we should slightly modify our general approach to solving problems using forces. Usually, the method that we follow in a uniform circular motion situation is identical to the approach that we use for other problems involving Newton's Second Law, where we apply the equation $\sum \vec{F} = m\vec{a}$. However, for uniform circular motion, the acceleration has the special form of Equation 5.3, $a_c = v^2/r$. Thus, when we apply Newton's Second Law, it has a special form.

The special form of Newton's Second Law for uniform circular motion is:

$$\sum \vec{F} = \frac{mv^2}{r}$$ (Eq. 5.4: **Newton's Second Law for uniform circular motion**)

where the net force, and the acceleration, is directed toward the center of the circle.

EXAMPLE 5.6 – A ball on a string

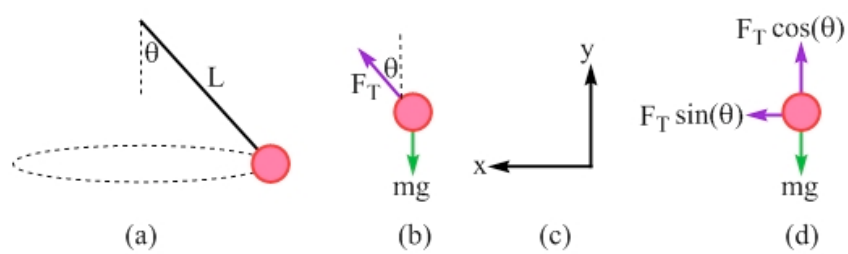
You are whirling a ball of mass m in a horizontal circle at the end of a string of length L. The ball has a constant speed v, and the string makes an angle θ with the vertical.

(a) What is the tension in the string? Express your answer in terms of m, g, and θ.
(b) What is v? Express your answer in terms of m, L, g, and/or θ.

SOLUTION

Let's apply the general method for solving problems using Newton's Laws. The first step is to draw a diagram (see Figure 5.16a) showing the ball, the string, and the circular path followed by the ball. The next step is to draw a free-body diagram showing the forces acting on the ball. Although the ball is going in a horizontal circle, the string is at an angle. As shown in Figure 5.16b, only two forces act on the ball, the downward force of gravity and the force of tension that is directed away from the ball along the string.

Figure 5.16: (a) A diagram and (b) free-body diagram for a ball being whirled in a horizontal circle at the end of a string. (c) An appropriate coordinate system. (d) A free-body diagram, with force components aligned with the coordinate system.



Now, choose an appropriate coordinate system. The key is to align the coordinate system with the acceleration. Because the ball is experiencing uniform circular motion, the acceleration is directed horizontally toward the center of the circle. We can choose a coordinate system with axes that are horizontal and vertical, as in Figure 5.16c. Finally, split the tension into components, with $F_T \cos\theta$ vertically up and $F_T \sin\theta$ toward the center of the circle, as in Figure 5.16d.

(a) To find an appropriate expression for the tension, we can apply Newton's second law in the *y*-direction. Because there is no acceleration vertically, we have:

$$\sum \vec{F}_y = m\vec{a}_y = 0 .$$

Looking at the free-body diagram to evaluate the left-hand side of this equation gives:

$$+F_T \cos\theta - mg = 0 .$$

Solving for the tension gives: $F_T = \dfrac{mg}{\cos\theta}$.

(b) To find an expression for the speed of the ball, let's apply Newton's second law in the *x*-direction. The positive *x*-direction is toward the center of the circle, in the direction of the centripetal acceleration, so we apply the special form of Newton's second law that is appropriate for use in circular motion situations. The general equation is:

$\sum \vec{F}_x = \dfrac{mv^2}{r}$, where the acceleration is directed toward the center of the circle.

Looking at the free-body diagram in Figure 5.16d, we see that there is only one force in the *x*-direction, so:

$$+F_T \sin\theta = \frac{mv^2}{r} .$$

A common error in this situation is to assume that *r*, the radius of the circular path, is equal to *L*, the length of the string. Referring to Figure 5.17, however, it can be seen that $r = L\sin\theta$.

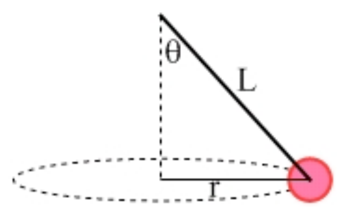


Figure 5.17: Note that *r*, the radius of the circular path, is not the same as *L*, the length of the string.

Substituting that into our equation gives:

$$F_T \sin\theta = \frac{mv^2}{L\sin\theta}, \qquad \text{so} \qquad v^2 = \frac{F_T L \sin^2\theta}{m} .$$

Using our result from part (a) to eliminate F_T gives:

$$v^2 = \frac{mgL\sin^2\theta}{m\cos\theta} = \frac{gL\sin^2\theta}{\cos\theta} .$$

Taking the square root of both sides gives: $v = \sin\theta \sqrt{\dfrac{gL}{\cos\theta}}$.

Related End-of-Chapter Exercises 21, 57.

Essential Question 5.6: If the speed of the ball in Example 5.6 is increased, what happens to *θ*, the angle between the string and the vertical?

Answer to Essential Question 5.6: As the speed increases the angle θ increases. One way to see this is to consider what happens to the tension. The vertical component of the tension remains the same, balancing the force of gravity, while the horizontal component of the tension increases (increasing the angle), because it provides the force toward the center needed for the ball to go in a circle, and that force increases as v increases. We can also convince ourselves that the angle increases, as the speed increases, by looking at the final equation in part (b) of Example 5.6.

5-7 Using Whole Vectors

The standard method of solving a problem involving Newton's laws is to break the forces into components. However, using whole vectors is an alternate approach. Let's see how whole vectors can be applied in a particular situation.

EXAMPLE 5.7 – Using whole vectors

(a) A box is placed on a frictionless ramp inclined at an angle θ with the horizontal. The box is then released from rest. Find an expression for the normal force acting on the box in this situation. What is the role of the normal force? What is the acceleration of the box?

(b) A box truck is traveling in a horizontal circle around a banked curve that is inclined at an angle θ with the horizontal. The curve is covered with ice and is effectively frictionless, so the truck can make it safely around the curve only if it travels at a particular constant speed (known as the *design speed* of the curve). Find an expression for the normal force acting on the truck in this situation. What is the role of the normal force? What is the design speed of the curve?

(c) Compare and contrast these two situations.

SOLUTION

(a) As usual, our first step is to draw a diagram, and then a free-body diagram showing the forces acting on the box, as in Figure 5.18. The two forces are the downward force of gravity, and the normal force applied by the ramp to the box. Because we are using whole vectors, we don't need to worry about splitting vectors into components. It is crucial, however, to think about the direction of the acceleration, which in this case is directed down the slope.

Let's apply Newton's second law, $\sum \vec{F} = m\vec{a}$, adding the forces as vectors. In this case, we get the right-angled triangle in Figure 5.18c. Each side of the triangle represents one vector in the equation $m\vec{g} + \vec{F}_N = m\vec{a}$. The vector $m\vec{a}$ (the net force) is parallel to the ramp, while the normal force is perpendicular to the ramp and the force of gravity is directed straight down. θ, the angle of the ramp, is the angle at the bottom of the triangle.

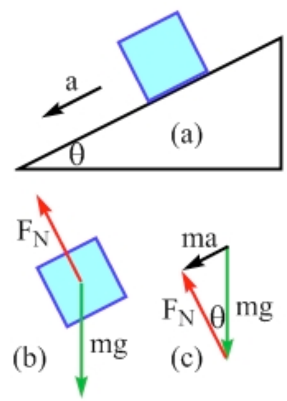


Figure 5.18: (a) A diagram, (b) free-body diagram, and (c) a right-angled triangle to show how the force of gravity and normal force combine to give the net force on the box.

Because the force of gravity is on the hypotenuse of the triangle, we get:

$$\cos\theta = \frac{F_N}{mg}, \quad \text{so} \quad F_N = mg\cos\theta .$$

The role of the normal force is simply to prevent the box from falling through the ramp.

We can find the acceleration from the geometry of the right-angled triangle:

$$\sin\theta = \frac{ma}{mg} = \frac{a}{g} .$$

This relationship gives $\vec{a} = g\sin\theta$ directed down the slope.

(b) The situation of the box truck traveling around the banked curve resembles the box on the incline. A diagram of the situation, showing the back of the truck, is illustrated in Figure 5.19. The same forces, the force of gravity and the normal force, appear on this free-body diagram as in the free-body diagram for the box. The difference lies in the direction of the acceleration, which for the circular motion situation is directed horizontally to the left, toward the center of the circle.

Again we apply Newton's second law, $\sum \vec{F} = m\vec{a}$, adding the forces as vectors, where the magnitude of the acceleration has the special form $a_c = v^2 / r$. This gives:

$m\vec{g} + \vec{F}_N = \frac{mv^2}{r}$, directed toward the center of the circle.

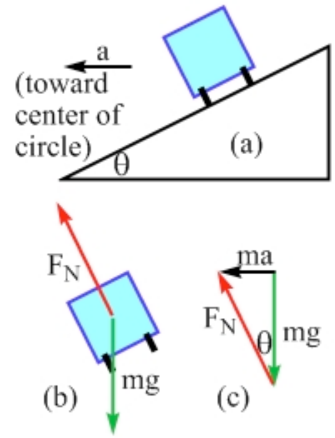


Figure 5.19: (a) A diagram, (b) free-body diagram, and (c) right-angled triangle to show how the force of gravity and normal force combine to give the net force on the truck.

Again, each force represents one side of a right-angled triangle. Now the normal force is on the hypotenuse, and clearly must be larger than it is in part (a).

Using what we know about the geometry of right-angled triangles, we get:

$\cos\theta = \frac{mg}{F_N}$, so now $F_N = \frac{mg}{\cos\theta}$.

If the angle of the ramp is larger than zero, then $\cos\theta$ is a number less than 1 and $F_N > mg$. In part (a) we had $F_N < mg$. The normal force for the box truck is larger because it has two roles. Not only does the normal force prevent the truck from falling through the incline, it must also provide the force directed toward the center to keep the truck moving around the circle.

To find the design speed of the curve, we can use the other side of the triangle:

$$\sin\theta = \frac{ma}{F_N} = \frac{mv^2}{rF_N} = \frac{mv^2 \cos\theta}{rmg} = \frac{v^2 \cos\theta}{rg}.$$

Re-arranging the preceding equation gives a design speed of $v = \sqrt{\frac{rg\sin\theta}{\cos\theta}} = \sqrt{rg\tan\theta}$.

This is an interesting result. First, there is a design speed, a safest speed to negotiate the curve. At the design speed, the vehicle needs no assistance from friction to travel around the circle. Going significantly faster is dangerous because the vehicle is only prevented from sliding toward the outside of the curve by the presence of friction - the faster you go, the larger the force of friction required. Second, the design speed does not depend on the vehicle mass, which is fortunate for the road designers. The same physics applies to a Mini Cooper as to a large truck.

(c) A key similarity is the free-body diagram: in both cases there is a downward force of gravity and a normal force perpendicular to the slope. The key difference is that the accelerations are in different directions, requiring a larger normal force in the circular motion situation.

Related End-of-Chapter Exercises: 25 – 27.

Essential Question 5.7: Consider again the situation of the truck on the banked curve. In icy conditions, is it safest to drive very slowly around the curve or to drive at the design speed?

Answer to Essential Question 5.7: Even in very low-friction conditions, it is safer to travel at the design speed than at a slower speed! If you go too slowly around a banked curve, there is a tendency for your vehicle to slide down the slope and run off the road on the inside of the curve.

5-8 Vertical Circular Motion

A common application of circular motion is an object moving in a vertical circle. Examples include roller coasters, cars on hilly roads, and a bucket of water on a string. The bucket and roller coaster turn completely upside down as they travel, so they differ a little from the situation of the car on the road, which (we hope) remains upright.

EXAMPLE 5.8A – Whirling a bucket of water

A bucket of water is being whirled in a vertical circle of constant radius *r* at a constant speed *v*. What is the minimum speed required for the water to remain in the bucket at the top of the circle?

SOLUTION

Let's apply the general method, starting with the diagram in Figure 5.20. We then draw a free-body diagram, although we have to decide whether to analyze the bucket or the water. If we consider the bucket, two forces act on it, the force of gravity and the tension in the string, both of which are directed down when the bucket is at the top of the circle. If we consider the water, there is a downward force of gravity, and a downward normal force from the bucket takes the place of the tension. The analysis is the same in both cases, so let's consider the water.

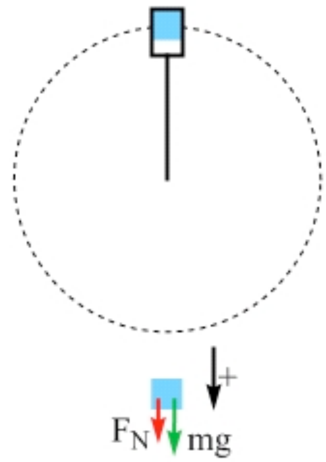


Figure 5.20: A bucket of water whirled in a vertical circle and a free-body diagram showing the forces acting on the water at the top of the loop.

Next, we choose an appropriate coordinate system. It is generally best to choose the positive direction as the direction of the acceleration, which points toward the center of the circle. When the bucket is at the top of the path, the acceleration, and the positive direction, points down. We don't need to split any forces into components. Let's apply Newton's second law, $\sum \vec{F} = m\vec{a}$.

Have a look at the free-body diagram to evaluate the left-hand side, and write the right-hand side in the usual circular-motion form. This gives:

$$+mg + F_N = +\frac{mv^2}{r} .$$

Solving for the normal force gives: $F_N = \frac{mv^2}{r} - mg$.

As long as the first term on the right exceeds the second term (in other words, as long as the normal force is positive), we're in no danger of having the water fall on us. Objects lose contact with one another (the water starts to fall out) when the normal force goes to zero. Setting the normal force to zero gives us the minimum safe speed of the bucket at the top of the circle:

$0 = \frac{mv_{\min}^2}{r} - mg$, which leads to $v_{\min} = \sqrt{gr}$.

Related End-of-Chapter Exercises: 12, 61.

EXAMPLE 5.8B – Apparent weight on a roller coaster

You are riding on a roller coaster that is going around a vertical circular loop. What is the expression for the normal force on you at the bottom of the circle?

SOLUTION

Once again, we apply the general method, starting with a diagram and a free-body diagram in Figure 5.21. We then draw a free-body diagram, which shows an upward normal force and a downward force of gravity. The system can be you or the car – the analysis is the same. Here we choose a coordinate system with a positive direction up, in the direction of the acceleration (toward the center of the circle). There is no need to split forces into components, so we can go straight to step 5 of the general method and apply Newton's Second Law:

$$\sum \vec{F} = m\vec{a} .$$

Have a look at the free-body diagram to evaluate the left-hand side, and remember that the right-hand side can be written in the usual circular-motion form. This gives:

$$+F_N - mg = +\frac{mv^2}{r} .$$

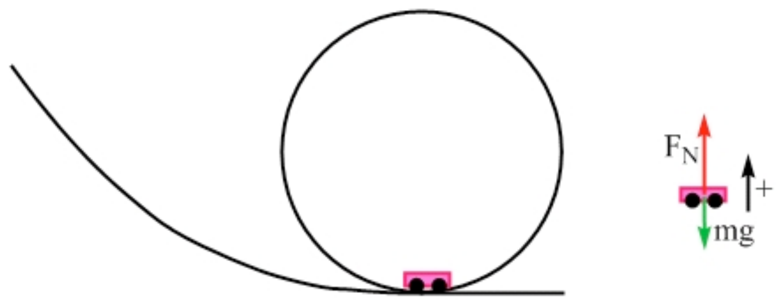


Figure 5.21: A car on a roller-coaster track (left), as well as (right) the free-body diagram when the car is at the bottom of the loop.

Solving for the normal force at the bottom of the circle gives:

$$F_{N,bottom} = \frac{mv^2}{r} + mg .$$

Let's compare our expression for the normal force on the car (or you) at the bottom of the loop to the expression for the normal force when the car (or you) is at the top. We can use the expression that we derived for the bucket at the top, in Example 5.8A, because the free-body diagram is the same in the two situations at the top of the loop.

$$F_{N,top} = \frac{mv^2}{r} - mg .$$

Note that the normal force at the bottom is larger than it is at the top. This difference is enhanced by the fact that the speed of the roller coaster at the bottom of the loop is larger than the speed at the top. Does this change in the normal force match the experience of a rider, who feels that she is lighter than usual at the top of the loop and heavier than usual at the bottom? Yes, because the normal force is the rider's apparent weight. Roller coasters are generally designed to have non-zero but fairly small normal forces at the top, so a rider feels almost weightless. At the bottom of the loop, the apparent weight can be considerably larger than *mg*, so a rider feels much heavier than usual.

Related End-of-Chapter Exercises: 20, 62.

Essential Question 5.8: You are on a roller coaster that reaches a top speed of 120 km/h at the bottom of a circular loop of radius 30 m. If you have a mass of 50 kg (and therefore a weight of 490 N), what is your apparent weight at the bottom of the loop? If the roller coaster's speed at the top of the loop has dropped to 80 km/h, what is your apparent weight at the top of the loop?

Answer to Essential Question 5.8: Let's first convert the speed from km/h to m/s, so all the units are SI units. The 120 km/h is equal to 33.3 m/s. Your apparent weight at the bottom of the loop is the normal force acting on you:

$$F_{N,bottom} = \frac{mv^2}{r} + mg = \frac{(50 \text{ kg})(33.3 \text{ m/s})^2}{30 \text{ m}} + 490 \text{ N} = +1850 \text{ N} + 490 \text{ N} = 2340 \text{ N}.$$

For the top of the loop, the speed of 60 km/h is equal to 22.2 m/s. Once again, your apparent weight is the normal force acting on you. Using the equation we found for the normal force at the top of the loop:

$$F_{N,top} = \frac{mv^2}{r} - mg = \frac{(50 \text{ kg})(22.2 \text{ m/s})^2}{30 \text{ m}} - 490 \text{ N} = +820 \text{ N} - 490 \text{ N} = 330 \text{ N}.$$

Chapter Summary

Essential Idea Regarding Applications of Newton's Laws

In this chapter we extended our understanding of what we can do with Newton's second law by applying it to situations involving friction and circular-motion situations.

Friction

The force of friction comes into play when objects are in contact. The force of friction is the component of the contact force that is parallel to the surfaces in contact (the normal force is the perpendicular component). The force of friction ***opposes relative motion*** between objects in contact. If there is relative motion between surfaces in contact (when one object is sliding over another), the friction force is the force of kinetic friction. The strength of the interaction is measured by the coefficient of kinetic friction, defined as:

$$\mu_K = \frac{F_K}{F_N} \quad \text{so} \quad F_K = \mu_K F_N. \qquad \text{(Equation 5.1: \textbf{The kinetic force of friction})}$$

If the force of friction prevents relative motion between surfaces in contact, the friction force is the force of static friction. The maximum strength of this interaction is measured by the coefficient of static friction, defined as:

$$\mu_S = \frac{F_{S,\max}}{F_N} \quad \text{so} \quad F_S \le \mu_S F_N. \qquad \text{(Equation 5.2 \textbf{The static force of friction})}$$

The coefficients of friction depend on the pair of interacting materials.

Be careful when working with static friction because, if the friction force that is required to ensure no relative motion is less than the maximum possible force of static friction, the static force of friction adjusts itself to whatever it needs to be to prevent relative motion. Two points to keep in mind about static friction are:

- we usually use Equation 5.2 with the equals sign only when we are certain we are dealing with a limiting case and the force of static friction is the maximum value;
- even the direction of the static force of friction can be counter-intuitive. A good rule of thumb is that the static force of friction opposes the relative motion that would occur if there were no friction.

A General Method for Solving a Problem Involving Newton's Laws, in Two Dimensions

The standard method of solving force problems involves splitting the forces into components, applying Newton's second law once for each direction, and combining the results. However, it should be kept in mind that some problems lend themselves to being solved with whole vectors. The steps in the standard method include:

1. Draw a diagram of the situation.

2. Draw one or more free-body diagrams, with each free-body diagram showing all the forces acting on an object.

3. For each free-body diagram, choose an appropriate coordinate system. Coordinate systems for different free-body diagrams should be consistent with one another. A good rule of thumb is to align each coordinate system with the direction of the acceleration.

4. Break forces into components that are parallel to the coordinate axes.

5. Apply Newton's second law twice to each free-body diagram, once for each coordinate axis. Put the resulting force equations together and solve.

Uniform Circular Motion – motion in a circle at constant speed

When an object travels with a speed v in a circular path of radius r, there is a centripetal acceleration directed toward the center of the circle, given by:

$$a_c = \frac{v^2}{r}.$$ (Equation 5.3:**Centripetal acceleration**)

To solve a problem involving uniform circular motion, we apply the method we use for other problems involving Newton's laws. The only change is in the form of Newton's second law:

$$\sum \vec{F} = \frac{mv^2}{r} \quad ,$$ (Eq. 5.4: **Newton's Second Law for uniform circular motion**)

where the acceleration is directed toward the center of the circle.

We avoid the term "centripetal force" because many people think of it as a new force that appears when an object travels in a circle - there is no such force! In uniform circular motion, there is a force directed toward the center of the circle but it is not a magical new force that arises just because something goes in a circle. Instead, the force is a familiar force, such as the force of gravity, the normal force, tension, or friction; or, the force is a force that we will investigate later in the book, such as the electric force between charged particles or the magnetic force.

End-of-Chapter Exercises

For all these exercises, assume that all strings are massless and all pulleys are both massless and frictionless. We will improve our model and learn how to account for the mass of a pulley in Chapter 10.

Exercises 1 – 12 are conceptual questions designed to see whether you understand the main concepts of the chapter. The first six exercises, in particular, are intended to give you more experience with free-body diagrams.

1. A box is placed in the middle of the floor of a truck. From the five free-body diagrams shown in Figure 5.22, which free-body diagram corresponds to the following situations? Assume the floor of the truck is horizontal at all times, and that for parts (a) - (d) the box does not slip on the floor of the truck. (a) The truck is moving at constant velocity to the right. (b) The truck is moving at constant velocity to the left. (c) The truck, while moving right, is speeding up. (d) The truck, while moving right, is slowing down. (e) The truck, while moving right, is stopping so quickly that the box slides over the floor of the truck. (f) The truck, while moving right, is speeding up so rapidly that the box slides over the floor of the truck.

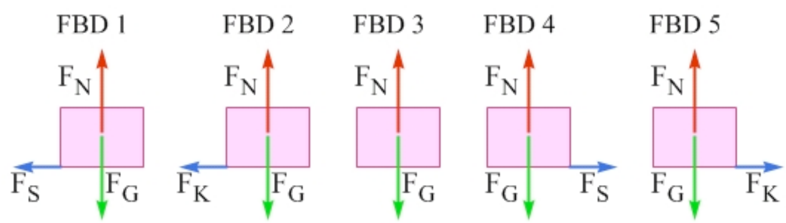


Figure 5.22: Possible free-body diagrams for a box in a truck, for Exercises 1 and 2.

2. Various possible free-body diagrams are shown in Figure 5.22 for a box on the floor of a moving truck. Imagine that you are in the back of the truck, looking forward at the box. As the truck makes a turn to the right, gently enough that the box does not slide over the floor of the truck, which free-body applies? Explain your choice.

3. Figure 5.23 shows five possible free-body diagrams (labeled FBD 1 through FBD 5) for a box on a ramp. (a) On the free-body diagrams, four different forces are shown. Briefly describe each of these forces and state what applies the force to the box. (b) Determine which free-body diagram, if any, matches each of the following situations: (i) the box remains at rest on the ramp; (ii) the box is sliding down the ramp, with some friction acting; (iii) the box is sliding up the ramp, which is frictionless.

4. Figure 5.23 shows five possible free-body diagrams (labeled FBD 1 through FBD 5) for a box on a ramp. Describe a situation in which the correct free-body diagram is FBD 3.

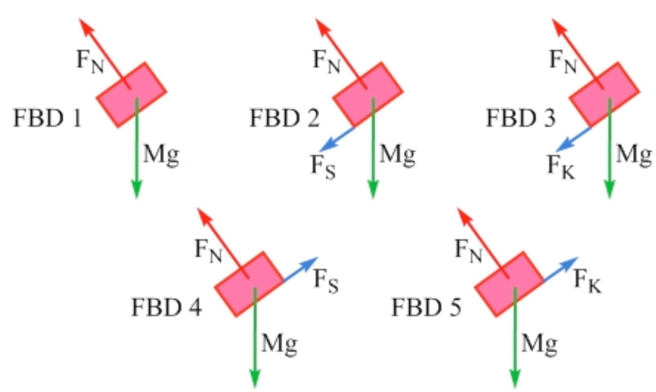


Figure 5.23: Five possible free-body diagrams for a box on a ramp, for Exercises 3 – 6.

5. Let's say that the free-body diagrams shown in Figure 5.23 are possible free-body diagrams for a box placed on the floor of a truck that is traveling on a hill. Identify the free-body diagram that could apply in each of the following situations. For parts (a) – (d),

assume the box does not slide over the floor of the truck. (a) The truck is traveling at constant velocity up the hill. (b) The truck is traveling at constant velocity down the hill. (c) The truck is at rest at a red light halfway up the hill. (d) The truck is speeding up as it climbs the hill. (e) The truck's acceleration up the hill is so large that the box slides over the floor of the truck.

6. Consider again the situation described in Exercise 5, regarding a box on the floor of a truck traveling up a hill that has a constant angle θ with respect to the horizontal. Now the truck is traveling down the hill and its speed is increasing. Which of the free-body diagrams shown in Figure 5.23 could apply in that situation, assuming the box does not slide over the floor of the truck? Select all possible answers, and explain in what situation (s) each free-body diagram applies.

7. A box with a weight of mg is initially at rest on a board that is initially horizontal. The angle between the board and the horizontal, θ, is gradually increased until the box starts to slide on the board. The coefficient of static friction between the box and the board is $\mu_S = 1.00$. (a) Plot a graph of the magnitude of the normal force applied to the box by the board, as a function of θ from $\theta = 0$ to $\theta = 90^\circ$, when θ is gradually increased from zero. (b) What else could that graph represent? (c) On the same graph, plot the magnitude of the component of the force of gravity acting on the box, as a function of θ, as θ is gradually increased. (d) What else could the graph from part (c) represent? (e) On the same graph, plot the magnitude of the maximum possible force of static friction that could act on the box if the coefficient of static friction is reduced to 0.50. (f) Briefly describe how the graphs can be used to find the angle at which the block first starts to slide on the board.

8. Three situations involving a block with a weight of 10 N and a string are shown in Figure 5.24. In cases 2 and 3, the string passes through a hook on top of the block. (a) Rank these situations based on the tension in the string, from largest to smallest (e.g., 3>1=2). (b) What is the tension in the string in case 3?

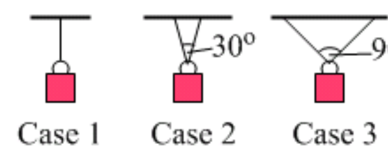


Figure 5.24: Three different situations of a block being supported by a string, for Exercise 8.

9. As shown in Figure 5.25, identical 20-N boxes are placed on identical horizontal surfaces. Each box has a string attached that passes over a pulley. In Case 1, the other end of the string is attached to a block with a weight of 8.0 N. In case 2, you simply exert an 8.0 N force straight down on the string. In both cases, enough friction acts on the box to keep the box at rest. (a) In which case is the force of friction acting on the box larger? (b) In which case is the tension in the string larger? (c) Calculate the force of friction acting on the box, and the tension in the string, in case 1. (d) What, if anything, can you say about the coefficient of static friction between the box and the surface in case 1?

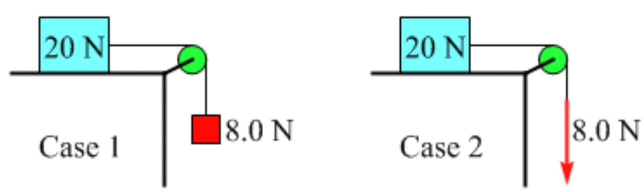


Figure 5.25: Two situations of a 20-N box on a level surface, for Exercise 9.

10. A box of mass m is placed on a frictionless ramp inclined at an angle of θ with respect to the horizontal. The ramp has a mass M and is on a frictionless horizontal surface. If the ramp remained at rest, the box would slide down the incline, but if the ramp is given a

horizontal acceleration of just the right magnitude, the box will remain at rest with respect to the ramp. (a) Using whole vectors, construct a right-angled triangle showing the normal force applied to the box, the force of gravity on the box, and the net force on the box. (b) Use the triangle to determine the acceleration of the system.

11. A disk is placed on a turntable at some distance from the center of the turntable. When the turntable rotates, the disk moves along with it without slipping. Which force, forces, or force components give rise to the disk's centripetal acceleration?

12. Take a metal coat hanger and stretch it out, as shown in Figure 5.26. It is best if a straight line through the hook of the hanger passes through the top vertex of the hanger, as shown by the dashed line in the figure. Holding the hanger vertically, balance a penny on the hook. After getting the hanger to rock back and forth on your finger, make the penny and coat hanger rotate so that at some times the penny is completely upside down. The penny should remain on the hanger at all times! (Bonus points if you can bring the system to a stop without the penny falling off.) Explain how this is possible.

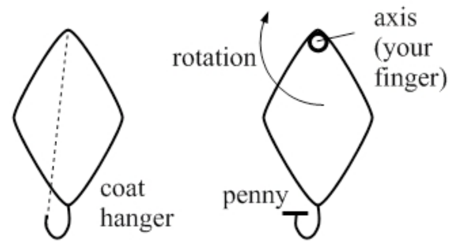


Figure 5.26: Spin a penny on the end of a stretched-out metal coat hanger, for Exercise 12.

Exercises 13 – 19 are designed to give you practice with the general method for solving a problem involving Newton's laws. For each of these exercises, start by doing the following: (a) Draw a diagram of the situation. (b) Draw one or more free-body diagram(s) showing all the forces that act on various objects or systems. (c) Choose an appropriate coordinate system for each free-body diagram. (d) If necessary, split forces into components that are parallel to the coordinate axes. (e) Apply Newton's second law enough times to obtain the necessary equations to solve the problem.

13. You pull a box, which has a weight of 20 N, across a horizontal floor at a constant velocity of 5.0 m/s. You do this by exerting a 10 N force horizontally on a string attached to the box. The goal here is to find the coefficient of kinetic friction between the box and the floor. Parts (a) – (e) as described above. (f) Find the coefficient of kinetic friction between the box and the floor.

14. Repeat Exercise 13, except that now the string between the box and your hand is at an angle of 30˚ with the horizontal, so your 10 N force has a component directed vertically up.

15. A box with a mass of 2.5 kg is initially at rest on a horizontal frictionless surface. You then pull on a string attached to the box, exerting a constant force F at an angle of 45˚ above the horizontal. This pull causes the box to travel 4.0 m horizontally in 2.0 s. Parts (a) – (e) as described above. (f) Find the box's acceleration by treating this as a one-dimensional constant-acceleration problem. (g) Find the magnitude of the force F. (h) Find the magnitude of the normal force acting on the box. Use $g = 9.8$ m/s^2.

16. In Atwood's machine (see Figure 5.27), the two blocks connected by a string passing over the pulley have masses of 1.0 kg (on the left) and 3.0 kg (on the right). The system is initially held at rest by means of a second string tied from the lighter block to the ground. Use $g = 9.8$ m/s^2. Parts (a) – (e) as described above. (f) What is the tension in the string connecting the two blocks? (g) What is the tension in the second string? (h) Without doing any calculations, describe whether the tension in the first string will increase,

decrease, or remain the same when the second string is cut. (i) Calculate the tension in the first string when the second string is cut.

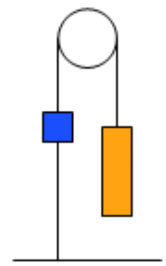


Figure 5.27: Atwood's machine, held at rest by a string connecting the smaller mass to the ground, for Exercise 16.

17. While moving out of your apartment, you push a box of textbooks, which has a mass of 20 kg, up the ramp of your moving truck. The coefficient of kinetic friction between the ramp and the box is 0.20, and the ramp is inclined at 30˚ with respect to the horizontal. Our goal here is to determine what force you push the box with, if the box travels at constant speed up the ramp and the force you exert is parallel to the ramp. Parts (a) – (e) as described above. (f) What is the magnitude of the force you exert on the box? Use $g = 9.8$ m/s^2.

18. Three blocks, A, B, and C, are connected by massless strings, as shown in Figure 5.28. The two strings pass over massless frictionless pulleys. The mass of block A is $m_A = 4.00$ kg. The mass of block B is $m_B = 8.00$ kg. Use $\mathbf{g = 10.0}$ **m/s^2**. Our first goal will be to determine the acceleration of the system if the mass of block C is $m_C = 8.00$ kg. Let's start by assuming that the table supporting block B is frictionless. Parts (a)-(e) as described above. (f) What is the acceleration of the system if $m_C = 8.00$ kg? (g) What would the mass of the block C have to be for the system to have no acceleration? (h) If the coefficient of static friction between block B and the table is $\mu_S = 0.300$, the system will remain at rest if the mass of block C is between a maximum value m_{Cmax} and a minimum value m_{Cmin}. What are m_{Cmax} and m_{Cmin}? (i) If the mass of block C is 5.00 kg, what is the force of friction exerted on block B by the table? Assume the system remains at rest.

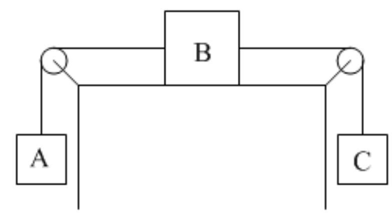


Figure 5.28: A diagram of three blocks connected by strings, for Exercise 18.

19. A box of mass $m = 3.0$ kg is placed in the center of a flatbed truck. The coefficients of friction between the box and the truck are $\mu_S = 0.40$ and $\mu_K = 0.30$. Our first goal will be to determine the force of friction acting on the box when the truck accelerates forward on a level road, starting from rest. Assume the box remains at rest relative to the truck. Parts (a) – (e) as described above. (f) If the truck's acceleration is 2.0 m/s^2, what is the magnitude and direction of the force of friction acting on the box? Is it static friction or kinetic friction? (g) What is the largest magnitude that the truck's acceleration can be for the box to remain at rest relative to the truck? (h) Which of the values stated in the problem did you not need to solve the problem?

Exercises 20 – 22 are designed to give you practice with the general method for solving a uniform circular motion problem. For each of these exercises, start by doing the following: (a) Draw a diagram of the situation. (b) Draw one or more free-body diagram(s) showing all the forces that act on various objects or systems. (c) Choose an appropriate coordinate system for each free-body diagram. (d) If necessary, split forces into components that are parallel to the coordinate axes. (e) Apply Newton's second law enough times to get the equations necessary to solve the problem, and solve the resulting equations.

20. While driving your car on a hilly road, you pass over a hill that, at the top, is a circular arc with a radius of 20 m. Parts (a) – (e) as described above. (f) How fast are you traveling if, at the top of the hill, the normal force exerted by the road on your car is 30% less than it was the previous day, when you stopped your car there to admire the view? Assume the mass of the car is the same on the two days.

21. A *whirligig conical pendulum* consists of a ball of mass M being whirled in midair in a horizontal circle at a constant speed v (see Figure 5.29). The ball is tied to a string that passes over a pulley, which rotates on frictionless bearings as the ball travels around the circle, and down through a hollow tube. A block of mass m is tied to the other end of the string. As the ball travels around the circle, the string makes an angle of θ with the horizontal, and the part of the string that goes from the ball to the pulley has a fixed length L. Parts (a) – (e) as described above. (f) In terms of the variables M, m, and/or g only, what is the magnitude of the tension in the string? (g) In terms of M, m, g, L, and/or θ only, what is v, the constant speed of the ball?

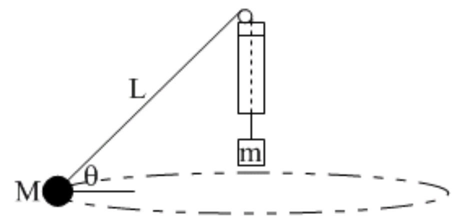


Figure 5.29: A whirligig conical pendulum, for Exercise 21.

22. Mary is whirling around on a merry-go-round at the playground that her friend Caitlin is gradually spinning faster and faster. The goal of the exercise is to determine the maximum speed Mary can travel at, if the coefficient of static friction between her shoes and the floor is μ_S. Assume that Mary is not hanging on, but is simply balancing on her feet at a distance r from the center. Parts (a) – (e) as described above. (f) Find Mary's maximum speed. (g) Which, if any, of the following would help Mary stay on the merry-go-round? (i) Moving closer to the center of the turntable. (ii) Moving closer to the edge of the turntable. (iii) Wearing a heavy backpack that increases her mass.

Exercises 23 – 27 are designed to give you practice with applying whole vectors, so take a whole-vector approach to the situations described.

23. A box with a weight of 20 N is initially at rest on a horizontal surface. The coefficients of friction between the box and the surface are $\mu_S = 0.30$ and $\mu_K = 0.20$. A force is then applied to the box at an angle of 30˚ with the horizontal so that the box has a non-zero acceleration directed entirely horizontally and no friction acts on the box. What is the magnitude of this applied force?

24. A box with a weight of 40 N remains at rest on a horizontal surface, even though a 25 N horizontal force is being applied to it. What are the magnitude and direction (the angle with respect to the horizontal) of the contact force exerted on the box by the surface?

25. A box with a weight of 40 N remains at rest on a flat surface inclined at 30˚ with respect to the horizontal, even though a 25 N horizontal force is being applied to it. (a) What are the magnitude and direction (the angle with respect to the horizontal) of the contact force exerted on the box by the surface? What are the magnitude of the (b) normal force and (c) friction force exerted on the box by the surface?

26. As shown in Figure 5.30, a disk with a weight of 2.0 N lies on a rotating turntable such that the disk does not slip on the turntable. The turntable spins at a steady rate of 1 revolution every second and the disk is 50 cm from the turntable's center. (a) Draw a right-angled triangle showing the contact force applied to the disk by the turntable, the force of gravity acting on the disk, and the disk's net force. (b) Using $g = 9.8$ m/s^2, find the angle between the horizontal and the contact force.

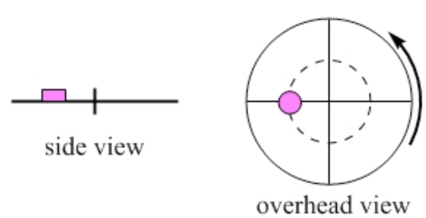


Figure 5.30: A disk on a rotating turntable, for Exercise 26.

27. A 5.0 kg box is at rest on a ramp that is at an angle of 15˚ with the horizontal. The coefficient of static friction between the box and the ramp is $\mu_S = 0.40$. Begin by sketching a free-body diagram for the box, showing the force of gravity and the contact force. (a) What are the magnitude and direction of the contact force applied by the ramp on the box? (b) What is the magnitude of the normal force applied by the ramp on the box? (c) What is the magnitude of the force of friction applied by the ramp on the box?

General problems and conceptual questions

28. Return to the situation described in Exercise 27, but this time use an approach in which you choose an appropriate coordinate system and split forces into components. A 5.0 kg box is at rest on a ramp that is at an angle of 15˚ with the horizontal. The coefficient of static friction between the box and the ramp is $\mu_S = 0.40$. Determine (a) the normal force applied by the ramp on the box, (b) the force of friction applied by the ramp on the box, and (c) the magnitude and direction of the contact force applied by the ramp on the box.

29. Return to the situation described in Exercises 27 and 28. If you would like the box to start moving, what is the minimum force you can apply if your force is directed (a) parallel to the slope? (b) perpendicular to the slope? (c) horizontally?

30. A shuffleboard disk with a mass *m* is sliding on a horizontal surface. The coefficient of kinetic friction between the disk and the surface is μ_K, and the acceleration due to gravity is *g*. In terms of the variables stated here, determine the magnitude of (a) the normal force acting on the disk, (b) the force of friction acting on the disk, and (c) the net contact force acting on the disk from the surface. (d) What is the angle between the contact force and the surface?

31. Return to the situation described in Exercise 30. If the disk has an initial speed of 4.0 m/s, $\mu_K = 0.20$, and $g = 9.8$ m/s^2, how far does the disk slide?

32. A 4.0 kg box is initially at rest on a flat surface. The coefficient of static friction between the box and the surface is $\mu_S = 0.50$. Determine the minimum force you have to apply to start the box moving if your force is directed (a) horizontally; (b) at an angle of 20˚ from the horizontal with a vertical component down; (c) at an angle of 20˚ from the horizontal with a vertical component up.

33. Return to the situation described in Exercise 32. If the angle between the force you apply and the horizontal is larger than a particular critical angle, the box will not move, no matter what magnitude your force is. (a) Is this true when your force has a downward vertical component, an upward vertical component, or in both of these cases? (b) Find an expression for the critical angle θ_C in terms of the coefficient of static friction μ_S. (c) Solve for θ_C when $\mu_S = 0.50$.

34. A box with a weight of 20 N is initially at rest on a flat surface. The coefficients of friction for the box and the surface are $\mu_S = 0.50$ and $\mu_K = 0.40$. A horizontal force with a constant direction is then applied to the box. The magnitude of the force increases with time according to the equation $F = (2.0 \text{ N/s})t$ until the box starts to move. Then, the force remains constant at the value it has when the box starts to move. (a) Sketch a graph showing the magnitude of this horizontal force, and the magnitude of the force of friction

acting on the box, as a function of time for the first 8.0 s after the horizontal force is first applied. (c) Sketch a graph of the speed of the box as a function of time over the same 8.0-second interval.

35. Figure 5.31 shows a box with a mass of 2.5 kg remaining at rest on a horizontal table. A string is tied to the box and held in a horizontal position by means of a system of two other strings, one that hangs vertically and supports a small block with a mass of 500 g, and the other that makes a 30° angle with the horizontal and is tied to a wall. Use $g = 9.8$ m/s^2. (a) What is the magnitude of the force of friction acting on the box? (b) What, if anything, can you say about the coefficient of static friction between the box and the table in this situation?

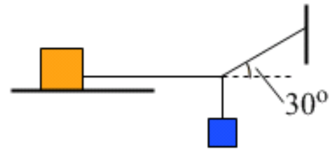


Figure 5.31: An equilibrium situation involving two blocks, for Exercise 35.

36. A box is placed at the bottom of a ramp that measures 3.00 m vertically by 4.00 m horizontally. The coefficients of friction for the box and the ramp are: $\mu_S = 0.60$ and $\mu_K = 0.50$. The box is then given an initial velocity up the ramp of 4.0 m/s. The box slows down as it slides up the ramp and eventually comes to a stop. (a) Will the box remain at rest, or will the stop just be for an instant before it slides down the ramp again? Justify your answer. (b) If you determine that the box slides down again, which takes more time, sliding up the ramp or sliding down the ramp? Justify your answer without calculating either time.

37. Return to the situation described in the previous exercise, and use $g = 10$ m/s^2. (a) How far does the box travel up the ramp? (b) How long does the box spend sliding up the ramp? (c) If you determine that the box slides down the ramp again, how long does the box spend sliding down the ramp?

38. As shown in Figure 5.32, identical 20-N boxes are placed on identical horizontal surfaces. Each box has a string attached that passes over a pulley. In Case 1, the other end of the string is attached to a block with a weight of 8.0 N. In case 2, you simply exert an 8.0 N force straight down on the string. The coefficients of friction for the box-surface interaction are the same in both cases. Although there is some friction, in both cases the box accelerates from rest to the right. Try to answer parts (a) – (c) without doing any calculations. (a) In which case is the force of friction acting on the box larger? (b) In which case is the tension in the string larger? (c) In which case is the acceleration larger? (d) What, if anything, can you say about the coefficient of static friction between the box and the surface?

39. Return to the situations shown in Figure 5.32. In each case, the coefficients of friction are $\mu_S = 0.20$ and $\mu_K = 0.10$. Use $g = 10$ m/s^2. Determine the acceleration of the 20-N box in (a) case 1 (b) case 2.

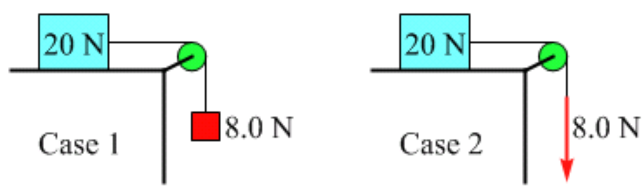


Figure 5.32: Two situations of a 20-N box on a level surface, for Exercises 38 – 40.

40. Consider the situation shown in case 1 in Figure 5.32. Assume now that the coefficients of friction between the box and the surface are $\mu_S = 0.25$ and $\mu_K = 0.20$. The 20-N box is given an initial velocity of 2.0 m/s away from the pulley. (a) How far will the box travel before coming instantaneously to rest? (b) How much time passes until the box comes instantaneously to rest? (c) Will the box remain at rest or

will it reverse direction and travel back toward the pulley? How can you tell? (d) If you decide that the box does travel back toward the pulley, determine (i) how fast it is going when it reaches its starting point; (ii) how long the return trip to the starting point takes.

41. Two identical boxes are linked by a string passing over a pulley, as in Figure 5.33. One box is on a horizontal surface, while the other is on an incline in the shape of a 3-4-5 triangle. If the surfaces are frictionless, what is the acceleration of the system?

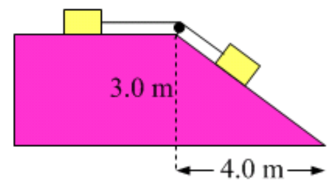


Figure 5.33: Two boxes connected by a string, for Exercises 41 – 45.

42. Return to the situation described in Exercise 41 and shown in Figure 5.33. Each of the following changes to the system is made separately – the changes are not sequential. State whether the change affects the acceleration of the system and, if so, state what happens to the acceleration. (a) Double the mass of both boxes. (b) Double the mass of the box on the incline only. (c) Double the mass of the box on the horizontal surface only. (d) Transfer the whole system to the Moon, where the acceleration due to gravity is about $1/6$ of the value here on Earth.

43. Two identical boxes are linked by a string passing over a pulley, as in Figure 5.33. One box is on a horizontal surface, while the other is on an incline in the shape of a 3-4-5 triangle. If the coefficients of friction for both box-surface interactions are $\mu_S = 0.15$ and $\mu_K = 0.10$ and the system is released from rest, what is the magnitude of the acceleration of the system?

44. Two identical boxes are linked by a string passing over a pulley, as in Figure 5.33. One box is on a horizontal surface, and the other is on an incline in the shape of a 3-4-5 triangle. Both boxes are moving such that the string between them remains taut. If the acceleration of each box has a magnitude of $g/10$, what is the coefficient of kinetic friction between the boxes and the surface? Assume the coefficient of kinetic friction is the same for both boxes. How many different solutions does this problem have? Find all the solutions.

45. Two identical boxes are linked by a string passing over a pulley, as in Figure 5.33. One box is on a horizontal surface, while the other is on an incline in the shape of a 3-4-5 triangle. Both boxes are moving with a constant non-zero velocity such that the string between them remains taut. What is the coefficient of kinetic friction between the boxes and the surface? Assume the coefficient of kinetic friction is the same for both boxes. How many different solutions does this problem have? Find all the solutions.

46. Fred pulls down on a massless rope (rope 1) with a constant force F (see Figure 5.34). This rope passes over a massless, frictionless pulley and is tied to a box with a mass $m_1 = 2.00$ kg. Rope 2 connects the first box to a second box with a mass $m_2 = 4.00$ kg. The two boxes accelerate upwards with an acceleration $a = 4.00$ m/s^2. Use $g = 10.0$ m/s^2. (a) Find the tension in rope 2. (b) Find the tension in rope 1. (c) If the boxes are initially at rest, what distance have they traveled 0.500 seconds after they start moving? (d) If the ground exerts an upward normal force on Fred of 500 N, what is Fred's mass?

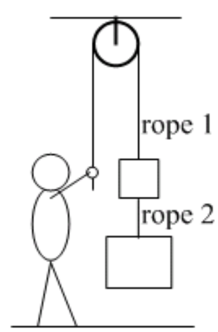


Figure 5.34: Fred accelerates a system of two boxes upward, for Exercise 46.

47. A 5.00 kg box is placed on a ramp that measures 3.00 m vertically by 4.00 m horizontally. As shown in Figure 5.33, the box is attached to a 1.00 kg ball by a massless rope that passes over a massless and frictionless pulley. A horizontal force of $F = 10.0$ N is also applied to the box. Use $g = 10.0$ m/s^2. (a) If there is no friction between the box and the ramp, what is the acceleration of the box? State clearly if the acceleration is directed up or down the ramp. (b) What value of the horizontal force F would result in zero acceleration for the box? (c) Let's introduce some friction between the box and the ramp, with a coefficient of static friction of $\mu_S = 1/3$. In this case, the box, if it starts from rest, will have no acceleration for all values of the horizontal force F between some minimum value F_{min} and some maximum value F_{max}. What are these minimum and maximum values?

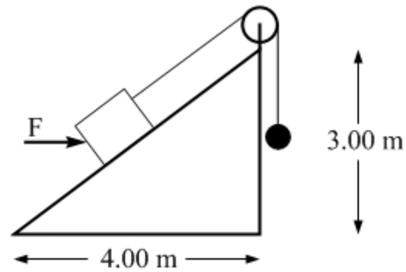


Figure 5.35: The situation described in Exercise 47.

48. A block of mass m is placed on top of a larger block of mass M and the two-block system rests on a horizontal frictionless surface. There is friction between the two blocks, with static and kinetic friction coefficients μ_S and μ_K, respectively. As shown in Figure 5.36, a horizontal force F is applied to the larger block. Express all your answers below in terms of m, M, g, F, and/or μ_S or μ_K. (a) For small values of F, the two blocks slide together on the surface. For this situation, what are: (i) the acceleration of the large block and (ii) the friction force on the small block? (b) For large values of F the small block slides with respect to the large block. What is the acceleration of the large block in this case? (c) What is the maximum value of F such that the small block does not move with respect to the large block?

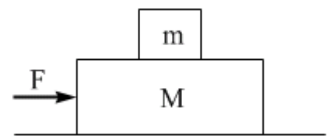


Figure 5.36: A stack of two blocks, with a force being applied to the lower block, for Exercises 48 and 49.

49. Return to the situation described in Exercise 48 and shown in Figure 5.36, but now we will use numerical values for the variables. We will also introduce some friction between the large box and the horizontal surface. Let's use $m = 5.0$ kg, $M = 10$ kg, $g = 10$ m/s^2, and say that $\mu_S = 0.40$ and $\mu_K = 0.30$ for the interaction between the blocks as well as the interaction between the large block and the horizontal surface. (a) If the blocks are initially at rest, what is the minimum value of the horizontal force F necessary to start the system moving? (b) Once the blocks are moving, what is the minimum value of the horizontal force F necessary to keep the system moving at constant velocity? (c) If the blocks are initially at rest, what is the maximum value the horizontal force F can be such that the small block does not slide on the large block?

50. As shown in Figure 5.37, Joe is placed in a large bucket that is suspended in mid-air by a rope that passes over a pulley. In Case 1, Joe holds the other end of the rope. In Case 2, Bill, on the ground, holds the other end of the rope. If Joe and the bucket are stationary, in which case is the tension in the rope larger?

51. Return to the situation described in Exercise 50. If Bill's mass is 70 kg, the combined mass of Joe and the bucket is 50 kg, and the bucket is accelerating up at the rate of 2.0 m/s^2, what is the tension in the rope in (a) Case 1? (b) Case 2?

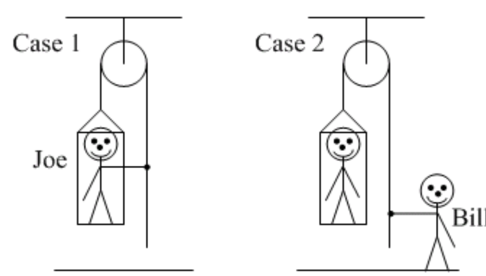


Figure 5.37: Two cases of Joe in a bucket, for Exercises 50 and 51.

52. Two boxes are side-by-side on a horizontal floor as shown in Figure 5.38. The coefficient of kinetic friction between each box and the floor is μ_K. In Case 1, a horizontal force F directed right is applied to the box of mass M and the boxes are moving to the right. In Case 2, the horizontal force F is instead directed left and applied to the box of mass m, and the boxes are moving to the left. Find an expression for the magnitude of F_{Mm}, the force the box of mass M exerts on the box of mass m, in (a) Case 1 (b) Case 2. Express your answers in terms of variables given in the problem.

Figure 5.38: Two cases involving two boxes on a horizontal floor, with an applied force, for Exercises 52 and 53.

53. Consider again the situation described in Exercise 52 and shown in Figure 5.38. Let's use m = 5.0 kg, M = 10 kg, g = 10 m/s^2, F = 20 N, and say that $\mu_K = 0.40$ for the interaction between the boxes and the floor. What are the magnitude and direction of the acceleration of the system if the boxes are initially moving to the right in (a) Case 1; (b) Case 2? What are the magnitude and direction of the acceleration of the system if the boxes are initially moving to the left in (c) Case 1; (d) Case 2?

54. Three blocks are placed side-by-side on a horizontal surface and subjected to a horizontal force F, as shown in case 1 of Figure 5.39. Doing so causes the blocks to accelerate from rest to the right. The coefficients of friction between each box and the surface are $\mu_S = 0.30$ and $\mu_K = 0.20$. Use g = 10 m/s^2, and consider case 1 only. If the horizontal force F is 40 N, what are (a) the acceleration of the system? (b) the net force acting on the 2.0 kg block? (c) the force exerted by the 5.0 kg block on the 2.0 kg block?

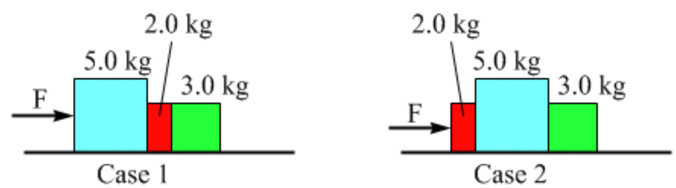


Figure 5.39: Two situations involving three blocks being pushed from the left by a horizontal force F, for Exercises 54 – 55.

55. Repeat Exercise 54, but now answer the questions for case 2, instead.

56. You get off a bus, take a plastic cube out of your pocket, and hold the cube against the front of the bus, which is vertical. When the bus starts to move, you carefully let the cube go, and step away from the bus. As the bus accelerates at 4.0 m/s^2, you notice that the cube remains in place on the front of the bus. What does this tell you about the coefficient of static friction between the cube and the bus?

57. While on a train, you are holding a string with a ball attached to it. At first, the train's velocity is constant and the string is vertical. When the train changes direction while going around a curve with a radius of 200 m, you measure the angle between the string and the vertical to be 20˚. What is the speed of the train?

58. A hockey puck, with a mass of 160 g, is placed 75 cm from the center of a horizontal turntable. The turntable begins to rotate, gradually spinning faster as its rate of rotation increases at a steady rate. The speed of the puck, as a function of time, is given by $v = (0.10 \text{ m/s}^2)t$. The coefficients of friction between the puck and the turntable are $\mu_S = 0.60$ and $\mu_K = 0.50$. (a) Assuming the force of friction is directed toward the center of the turntable, determine at what time the puck starts to slide on the turntable. Plot graphs, as a function of time, of (b) the maximum possible force of static friction, and (c) the magnitude of the actual force of friction acting on the puck.

59. A cube of mass m is placed in a funnel that is rotating around the vertical axis shown in Figure 5.40. There is no friction between the cube and the funnel, but the funnel is rotating at just the right speed needed to keep the cube rotating with the funnel. The cube travels in a circular path of radius r, and the angle between the vertical and the wall of the funnel is θ. Express your answers in terms of m, r, g, and/or θ. (a) Which force, forces, or force components give rise to the centripetal acceleration in this situation? (b) What is the normal force acting on the cube? (c) What is the speed v of the cube?

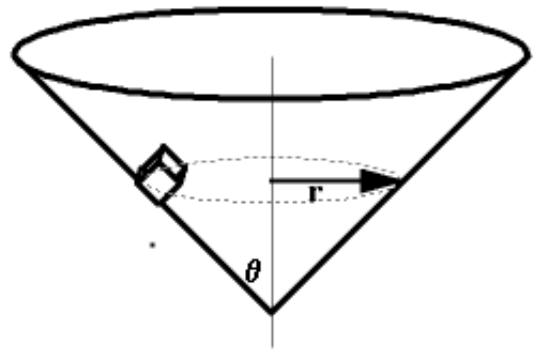


Figure 5.40: A cube in a rotating funnel, for Exercises 59 and 60.

60. Return to the situation described in Exercise 59, except now there is some friction between the cube and the funnel. Let's use the following values: $\theta = 45^\circ$, m = 500 grams, r = 20 cm, and g = 9.8 m/s^2, with coefficients of friction between the cube and the funnel of $\mu_S = 0.20$ and $\mu_K = 0.10$. (a) What is the cube's speed, if there is no force of friction acting on the cube? (b) What are the maximum and minimum values that the cube's speed can be, so that the cube will not slide up or down in the funnel?

61. Matt, with a mass of 22.0 kg, is swinging on a tire swing (a tire attached to a rope hanging from a tree). The tire itself has a mass of 5.0 kg, and the rope has a length of 4.0 m. (a) If Matt's speed is 3.0 m/s when the tire reaches the bottom of its circular arc, what is the tension in the rope at that point? (b) Matt's father, Bob, thinks that the tire swing looks like fun. If the rope will break when the tension exceeds 750 N, is it safe for Bob to jump on the swing with Matt, should Bob swing by himself, or should he not swing at all? Justify your answer. Bob's mass is 70.0 kg, and he would reach the same 3.0 m/s speed that Matt does at the bottom of the swing.

62. You are traveling on a hilly road. At a particular spot, when your car is perfectly horizontal, the road follows a circular arc of some unknown radius. Your speedometer reads 80 km/h, and your apparent weight is 30% larger than usual. (a) Are you at the bottom of a hill or the top of a hill at that instant? (b) What is the radius of the circular arc?

- Contents >
- Chapter 5: Additional Resources

Pre-session Movies on YouTube

- Friction
- A Box on a Ramp
- Uniform Circular Motion

Examples

- Sample Questions

Solutions

- Answers to Selected End of Chapter Problems
- Sample Question Solutions

Simulations

- Simulation: Uniform Circular Motion

Additional Links

- *PhET* simulation: The Ramp