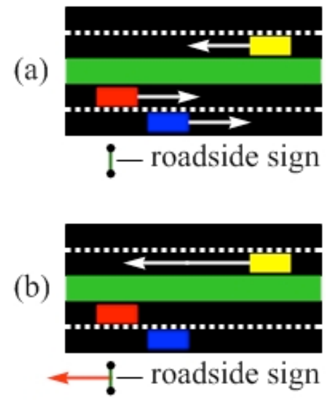
4-1 Relative Velocity in One Dimension

Before we generalize to two dimensions, let's consider a familiar situation involving relative velocity in one dimension. You are driving east along the highway at 100 km/h. The car in the next lane looks like it is barely moving relative to you, while a car traveling in the opposite direction looks like it is traveling at 200 km/h. This is your perception, even though the speedometers in all three vehicles say that each car is traveling at about 100 km/h.



How can we explain your observations? First, consider the velocity of your car relative to you. Even though your car is zooming along the highway at 100 km/h (with respect to the road), your car is at rest relative to you. To get a result of zero for the velocity of your car relative to you, we subtract 100 km/h east (your velocity with respect to the ground) from the velocity of the car with respect to the ground. This method of subtracting your velocity with respect to the ground also works to find the velocity of something else (such as an oncoming car) with respect to you. Subtracting your velocity from the velocity of other objects is equivalent to adding the opposite of your velocity to these velocities. Figure 4.1 illustrates the process.

Figure 4.1: (a) An overhead view, with arrows showing the velocities of three cars and a roadside sign, with respect to the ground. (b) The same situation, but with the velocities shown with respect to you, the driver of the bottom car.

Most relative-velocity problems can be thought of as vector-addition problems. The trick is to keep track of which vectors we're adding (or subtracting). Let's explore this idea further.

EXPLORATION 4.1 – Crossing a river

You are crossing a river on a ferry. Assume there is no current in the river (that is, the water is at rest with respect to the shore). The ferry is traveling at a constant velocity of 7.0 m/s north with respect to the shore, while you are walking at a constant velocity of 3.0 m/s south relative to the ferry.

Step 1 - ***What is the ferry's velocity relative to you?*** We can use the notation $\vec{v}_{YF}$ to denote your velocity relative to the ferry, so we have $\vec{v}_{YF} = 3.0$ m/s south. The velocity of the ferry with respect to you is exactly the opposite of your velocity with respect to the ferry, so $\vec{v}_{FY} = 3.0$ m/s north. This relation is always true: the velocity of some object A with respect to another object B is the opposite of the velocity of B with respect to A ($\vec{v}_{AB} = -\vec{v}_{BA}$).

Step 2 - ***What is your velocity relative to the shore?*** If you stood still on the ferry, your velocity relative to the shore would match the ferry's velocity with respect to the shore. In this case, however, you are moving with respect to the ferry, so we have to add the relative velocities as vectors (see Figure 4.2). Your velocity with respect to the shore is your velocity relative to the ferry plus the ferry's velocity relative to the shore:

$$\vec{v}_{YS} = \vec{v}_{YF} + \vec{v}_{FS} = 3.0 \text{ m/s south} + 7.0 \text{ m/s north} = -3.0 \text{ m/s north} + 7.0 \text{ m/s north.}$$

$$\vec{v}_{YS} = 4.0 \text{ m/s north.}$$

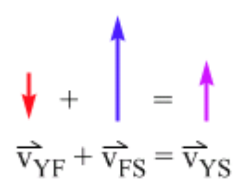


Figure 4.2: Finding your velocity relative to the shore by vector addition.

This vector-addition method is valid in general and works in more than one dimension.

A relative-velocity problem is a vector-addition problem. The velocity of an object A relative to an object C is the vector sum of the velocity of A relative to B plus the velocity of B relative to C:

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$ (Equation 4.1: **The vector addition underlying relative velocity**)

Step 3 - ***Assume now that there is a current of 2.0 m/s directed south in the river, and that the ferry's velocity of 7.0 m/s north is relative to the water. What is the ferry's velocity relative to the shore? What is your velocity relative to the shore?***

To find the ferry's velocity with respect to the shore, we can use Equation 4.1 to combine the known values of the ferry's velocity with respect to the water and the water's velocity with respect to the shore.

In this case, using the subscript *W* to refer to the water, we get:

$$\vec{v}_{FS} = \vec{v}_{FW} + \vec{v}_{WS} = +7.0 \text{ m/s north} + 2.0 \text{ m/s south} .$$

$$\vec{v}_{FS} = +7.0 \text{ m/s north} - 2.0 \text{ m/s north} = +5.0 \text{ m/s north} .$$

Going against the current, the ferry takes longer to get to its destination, because it is moving slower with respect to the shore than when there was no current.

Now that we know the ferry's velocity with respect to the shore, we can apply Equation 4.1 to find your velocity with respect to the shore.

$$\vec{v}_{YS} = \vec{v}_{YF} + \vec{v}_{FS} = +3.0 \text{ m/s south + 5.0 m/s north.}$$

$$\vec{v}_{YS} = -3.0 \text{ m/s north + 5.0 m/s north = +2.0 m/s north.}$$

An alternate approach to finding your velocity with respect to the shore is to extend the procedure of Step 2 to three vectors, as represented in Figure 4.3. In this case, we get:

$$\vec{v}_{YS} = \vec{v}_{YF} + \vec{v}_{FW} + \vec{v}_{WS} = 3.0 \text{ m/s south + 7.0 m/s north + 2.0 m/s south.}$$

$$\vec{v}_{YS} = -3.0 \text{ m/s north + 7.0 m/s north – 2.0 m/s north = +2.0 m/s north.}$$

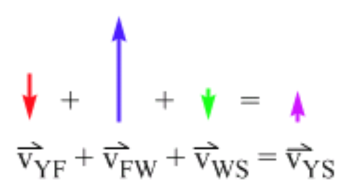


Figure 4.3: Finding your velocity relative to the shore by vector addition after adding a current in the river.

Key idea for relative velocity: Solving a relative velocity problem amounts to solving a vector-addition problem. In general, the velocity of an object A relative to an object C is the vector sum of the velocity of A relative to B plus the velocity of B relative to C: $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$.
Related End-of-Chapter Exercises: 1, 5.

Essential Question 4.1 In Step 3 of Exploration 4.1, could you adjust your speed with respect to the ferry so that you are at rest with respect to the shore? If so, how would you adjust it?

Answer to Essential Question 4.1 Yes. Adding an additional velocity of 2.0 m/s directed south will cancel the velocity of 2.0 m/s north that you are moving with respect to the shore. You were already walking at 3.0 m/s south with respect to the ferry, so you now need to be moving at 5.0 m/s south (so you need to run now) with respect to the ferry.

4-2 Combining Relative Velocity and Motion

Let's now connect relative velocity with the one-dimensional motion situations that we considered in Chapter 2.

EXPLORATION 4.2 – Who's faster?
On their way to play soccer in the World Cup, Mia and Brandi get stranded at O'Hare Airport in Chicago because of bad weather. Late at night, with nobody else around, they decide to have a race to see who is faster. They run down to a particular point, then turn around and return to their starting point. That race turns out to be a tie.

They race again, this time with Mia running on a moving sidewalk. They start at the same time, run east at top speed a distance L through the airport terminal, and then turn around and run west back to the starting point. Brandi runs at a constant speed v, while Mia runs on a moving sidewalk that travels at a speed of $v/2$. Mia runs at a constant speed v relative to the moving sidewalk. Neglect the time it takes the women to turn around at the halfway point. A diagram is shown in Figure 4.4 to help with the analysis.

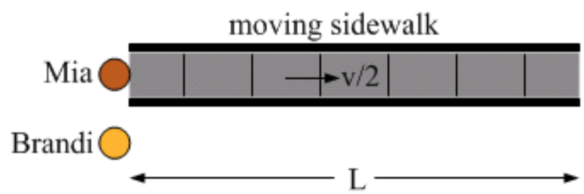


Figure 4.4: A diagram showing the positions of Mia and Brandi at the start of the race.

Step 1 - ***Make a prediction – who wins this race?*** Most people predict that this race is also a tie, because Mia is helped by the moving sidewalk for half the distance, and she has to work against the moving sidewalk for the remainder of the race.

Step 2 - ***If Brandi takes a time T to reach the turn-around point, how long does Mia take to reach the same point?*** First, let's do a relative-velocity analysis for Mia as she runs east, treating east as the positive direction. Figure 4.5 shows the vectors being added together. Using subscripts of M for Mia, S for sidewalk, and G for ground, we get:

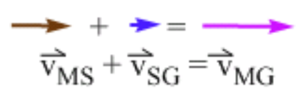


Figure 4.5: A vector diagram showing Mia's velocity relative to the ground as she runs east.

$$\vec{v}_{MG} = \vec{v}_{MS} + \vec{v}_{SG} = +v + \frac{v}{2} = +\frac{3v}{2}$$

This relative-velocity situation is really a one-dimensional motion problem in disguise. Let's analyze the motion as the two women are moving away from the start line, choosing the origin as the start line and the positive direction to the right, in the direction the women are running. Let's summarize what we know in Table 4.1.

	Mia	**Brandi**
Initial position	$x_{iM} = 0$	$x_{iB} = 0$
Final position	$x_{fM} = +L$	$x_{fB} = +L$
Initial velocity	$v_{iM} = +3v/2$	$v_{iB} = +v$
Acceleration	$a_M = 0$	$a_B = 0$
Time	$t_M = ?$	$t_B = T$

Table 4.1: Organizing the data for the outbound trips.

Let's analyze Brandi's motion to see how the time T relates to the distance L and the speed v. We can use the following constant-acceleration equation, which we used previously in Section 2-5, to relate these parameters:

$$x_{fB} = x_{iB} + v_{iB} t_B + \frac{1}{2} a_B t_B^2 .$$

Substituting appropriate values from Table 4.1 gives: $L = 0 + vT + 0$, and thus $T = L / v$.

Using a similar analysis for Mia, we start with: $x_{fM} = x_{iM} + v_{iM} t_M + \frac{1}{2} a_M t_M^2$.

Substituting appropriate values from Table 4.1, we get:

$$L = 0 + \frac{3}{2} v t_M + 0 \text{, which gives } t_M = \frac{2L}{3v} = \frac{2}{3} T.$$

So Mia has a sizable lead by the time she reaches the turn-around point.

Step 3 - ***Brandi takes an equal time T for the return trip from the turn-around point to the start/finish line. How long does Mia's return trip take?*** The vector addition in this case is shown in Figure 4.6. For Mia's return trip, her velocity relative to the ground is:

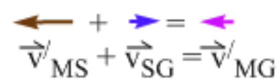


Figure 4.6: A vector diagram showing Mia's velocity relative to the ground as she runs west.

$$\vec{v}'_{MG} = \vec{v}'_{MS} + \vec{v}_{SG} = -v + \frac{v}{2} = -\frac{v}{2} .$$

The primed ($'$) values represent the values of these variables on the return trip.

The data table for the return trips is shown in Table 4.2. For Brandi, the return trip is the same as the outbound trip, so there is no need to repeat that analysis. Let's solve for Mia's time for the return trip:

$$x'_{fM} = x'_{iM} + v'_{iM} t_M + \frac{1}{2} a'_M (t'_M)^2 .$$

Substituting the values from Table 4.2 gives:

$$0 = +L - \frac{1}{2} v t'_M + 0 \text{, which gives } t'_M = \frac{2L}{v} = 2T.$$

	Mia	**Brandi**
Initial position	$x'_{iM} = +L$	$x'_{iB} = +L$
Final position	$x'_{fM} = 0$	$x'_{fB} = 0$
Initial velocity	$v'_{iM} = -v/2$	$v'_{iB} = -v$
Acceleration	$a'_M = 0$	$a'_B = 0$
Time	$t'_M = ?$	$t'_B = T$

Table 4.2: Organizing the data for the return trips.

Step 4 - ***Based on your answers to steps 2 and 3, who wins the race?*** Brandi's time for the entire trip is $2T$, the same time Mia takes just to come back along the moving sidewalk. Mia's total time is $2T + 2T/3$, so Brandi wins this race quite easily.

Key ideas: The methods we used to analyze one-dimensional motion situations in Chapter 2 can be combined with relative-velocity problems. **Related End-of-Chapter Exercises: 2, 31, 32.**

Essential Question 4.2 In Exploration 4.2, we concluded that Mia is at a disadvantage in the race when she runs on the moving sidewalk. What is a good conceptual explanation for this disadvantage?

Answer to Essential Question 4.2 Running on the moving sidewalk is a disadvantage for Mia because she spends considerably more time running against the moving sidewalk than she does running with it.

4-3 Relative Velocity in Two Dimensions

Let's modify Exploration 4.1, about your motion on a ferry, to see how things change when we switch to two dimensions. The major difference between one and two dimensions is that it is more challenging to add vectors in two dimensions than in one dimension; however, we simply need to follow what we learned about vector addition in Chapter 1.

EXPLORATION 4.3 – Crossing a river

You are crossing a river on a ferry. The ferry is pointing north, and it is traveling at a constant speed of 7.0 m/s relative to the water. The current in the river is 2.0 m/s directed southeast.

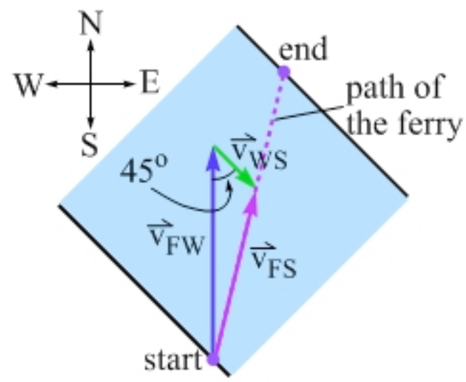


Figure 4.7: Vector diagram to find the ferry's velocity relative to the shore. We add the velocity of the ferry relative to the water to the velocity of the water relative to the shore to find the velocity of the ferry relative to the shore.

Step 1 - *What is the ferry's velocity relative to the shore?* The first step is to draw a vector diagram, as in Figure 4.7, showing how the relevant vectors combine to produce the velocity we're interested in.

We can apply the standard relative-velocity equation, using subscripts of F for the ferry, W for the water, and S for the shore, to get:

$$\vec{v}_{FS} = \vec{v}_{FW} + \vec{v}_{WS} .$$

This equation is consistent with the vector diagram in Figure 4.7. Because the direction of the current is specified as southeast, we can use a 45° angle for that vector.

Here we have a typical vector-addition problem in two dimensions. One method of solving the problem is to break the vectors into components and add the components. This method involves separating the two-dimensional problem into two different one-dimensional problems, which we do quite often in physics.

As we learned in Chapter 1, we can put together a table to help us keep the *x* and *y* components separate. Doing so makes it easy to add the vectors using the component method. Define the positive *x* direction as east, and the positive *y* direction as north.

Vector	*x* components	*y* components
$\vec{v}_{FW}$	$v_{FWx} = 0$	$v_{FWy} = +7.00$ m/s
$\vec{v}_{WS}$	$v_{WSx} = +(2.00 \text{ m/s})\cos(45^\circ) = +1.41$ m/s	$v_{WSy} = -(2.00 \text{ m/s})\sin(45^\circ) = -1.41$ m/s
$\vec{v}_{FS} = \vec{v}_{FW} + \vec{v}_{WS}$	$v_{FSx} = v_{FWx} + v_{WSx}$ $v_{FSx} = 0 + 1.41 \text{ m/s} = +1.41$ m/s	$v_{FSy} = v_{FWy} + v_{WSy}$ $v_{FSy} = +7.00 \text{ m/s} - 1.41 \text{ m/s} = +5.59$ m/s

Table 4.3: Adding the vectors using components.

We can leave the answer in terms of components, as is done at the bottom of Table 4.3, or we can specify the velocity of the ferry with respect to the shore in terms of its magnitude and direction. To specify the velocity vector in the magnitude-direction format, start by drawing the right-angled triangle corresponding to the vector and its components, as in Figure 4.8.

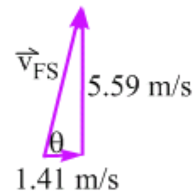


Figure 4.8: A vector diagram showing how the components add to give the net velocity vector.

The magnitude of the vector can be found using the Pythagorean theorem:

$$v_{FS} = \sqrt{1.41^2 + 5.59^2} = 5.8 \text{ m/s}.$$

The angle can be found using $\tan\theta = \frac{5.59}{1.41}$, so $\theta = 76^\circ$.

This angle is measured with respect to east. The ferry is not traveling due east, but rather somewhat north of east. Thus, we can say that the ferry's velocity relative to the shore is 5.8 m/s at an angle of 76˚ north of east (or, equivalently, 14˚ east of north).

Step 2 - ***You are standing on an observation deck at the top of the ferry, leaning against the railing on the starboard (right) side of the ferry. Suddenly someone yells from the port (left) side that there are some porpoises frolicking in the water. To get from one side of the ferry to the other in the shortest time to see the porpoises, in which direction should you run? Choose the best answer below.***

1. Run directly across the ferry, taking the shortest route to the other side (this means you will be carried along with the ferry, and the current).
2. Run partly toward the bow (front) of the ferry, so the ferry's velocity adds to your velocity and you travel at a higher speed.
3. Run partly toward the stern (rear) of the ferry, so the distance you travel relative to the shore is minimized.

If the ferry is docked and you want to cross from one side of the ferry to the other in the least time, you simply head straight across the ferry. If the ferry is moving, you do the same thing! The fact that the ferry is moving does not even cross your mind, which is good, because that is irrelevant. The reason the ferry's motion does not matter is that the ferry's motion is perpendicular to the direction you want to go, and a velocity in one direction does not affect motion in a perpendicular direction. The time for you to cross from one side of the ferry to the other is determined by how much of your velocity is directed across the ferry. If you aim yourself entirely in that direction, you minimize the time it takes to cover a particular distance in that direction.

Key ideas for relative velocity in two dimensions: The relative velocity equation, $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$, applies just as well in two or three dimensions as it does in one dimension. To solve the equation, we can apply the vector-addition methods, such as the component method, we covered in Chapter 1. Another important concept is that motion in two dimensions can be split into two independent one-dimensional motions. **Related End-of-Chapter Exercises: 7, 8.**

Essential Question 4.3 A pilot has aimed her plane north, and her airspeed indicator reads 150 km/h. The control tower reports that the wind is directed west at 50 km/h. Explain why the plane's speed relative to the ground is greater than 150 km/h, but less than 200 km/h.

Answer to Essential Question 4.3 The plane's velocity relative to the ground is the vector sum of the plane's velocity relative to the air (150 km/h north) and the air's velocity relative to the ground (50 km/h west). This sum is less than 200 km/h in a particular direction because the two vectors are not in the same direction. The plane's speed relative to the ground is 158 km/h, the length of the hypotenuse of the right-angled triangle we get by adding the vectors.

4-4 Projectile Motion

Projectile motion is, in general, two-dimensional motion that results from an object with an initial velocity in one direction experiencing a constant force in a different direction. A good example is a ball you throw to a friend. You give the ball an initial velocity when you throw it, and then the force of gravity acts on the ball as it travels to your friend. In this section, we will learn how to analyze this kind of situation.

EXPLORATION 4.4 – A race
You release one ball (ball *A*) from rest at the same time you throw another ball (ball *B*), which you release with an initial velocity that is directed entirely horizontally. You release both balls simultaneously from the same height *h* above level ground. Neglect air resistance.

Step 1 - ***Which ball travels a greater distance before hitting the ground?*** Ball *A* takes the shortest path to the ground, so ball *B* travels farther.

Step 2 - ***Which ball reaches the ground first? Why?*** We can construct a motion diagram (see Figure 4.9) by, for instance, analyzing a video of the balls as they fall. Many people think that because ball *B* travels farther it takes longer to reach the ground; however, ball *B* also has a higher speed. The reality is that both balls reach the ground at the same time. The reason is that the motion of ball *B* can be viewed as a combination of its horizontal motion and its vertical motion. The horizontal motion has no effect whatsoever on the vertical motion, so what happens vertically for ball *B* is exactly the same as what happens vertically for ball *A*.

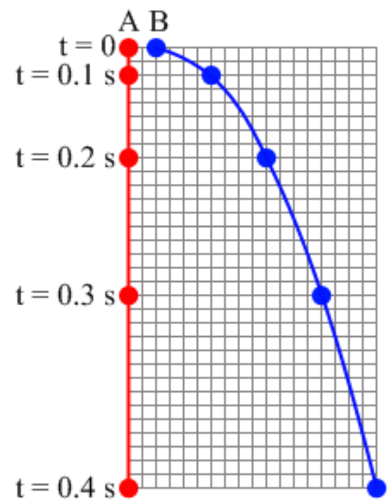


Figure 4.9: A motion diagram can be constructed from experimental evidence, such as by analyzing a video of the balls as they fall.

Key idea for projectile motion: The key idea of this chapter is the independence of x and y. The basic idea is that the motion that happens in one direction (x) is independent of the motion that happens in a perpendicular direction (y), and vice versa, as long as the force is constant.
Related End-of-Chapter Exercises: 9, 10.

The x-direction and y-direction motions are independent in the sense that each of the one-dimensional motions occurs as if the other motion is not happening. These motions are connected, though. The object's motion generally stops after a particular time, so the time is the same for the x-direction motion and the y-direction motion.

This powerful concept allows us to treat a two-dimensional projectile motion problem as two separate one-dimensional problems. We already have a good deal of experience with one-dimensional motion, so we can build on what we learned in Chapter 2. For the most part, we will deal with situations where the acceleration is constant, so all our experience with constant-acceleration situations in one dimension will be directly relevant here.

Solving a Two-Dimensional Constant-Acceleration Problem

Our general method for analyzing a typical projectile-motion problem builds on the method we used for analyzing one-dimensional constant-acceleration motion in Chapter 2. The basic idea is to split the two-dimensional problem into two one-dimensional subproblems, which we can call the *x* subproblem and the *y* subproblem.

1. Draw a diagram of the situation.
2. Draw a free-body diagram of the object showing all the forces acting on the object while it is in motion. A free-body diagram helps in determining the acceleration of the object.
3. Choose an origin.
4. Choose an *x*-*y* coordinate system, showing which way is positive for each coordinate axis.
5. Organize your data, keeping the information for the *x* subproblem separate from the information for the *y* subproblem.
6. Only then should you turn to the constant-acceleration equations. Make sure the acceleration is constant so the equations apply! We use the same three equations that we used in Chapter 2, but we customize them for the *x* and *y* subproblems, as follows:

Equation from Chapter 2	*x*-direction equations	*y*-direction equations
$v = v_i + at$ (2.7)	$v_x = v_{ix} + a_x t$ (4.2x)	$v_y = v_{iy} + a_y t$ (4.2y)
$x = x_i + v_i t + \frac{1}{2}at^2$ (2.9)	$x = x_i + v_{ix} t + \frac{1}{2}a_x t^2$ (4.3x)	$y = y_i + v_{iy} t + \frac{1}{2}a_y t^2$ (4.3y)
$v^2 = v_i^2 + 2a\Delta x$ (2.10)	$v_x^2 = v_{ix}^2 + 2a_x \Delta x$ (4.4x)	$v_y^2 = v_{iy}^2 + 2a_y \Delta y$ (4.4y)

Table 4.4: Constant acceleration equations for two-dimensional projectile motion. The equation numbers are shown in parentheses after each equation.

Let's apply the method above to analyze the race from Exploration 4.4. We begin by sketching a motion diagram and a free-body diagram for each ball, and continue the analysis in the next section. On the motion diagram for ball *B*, show the separate *x* (horizontal) and *y* (vertical) motions.

The motion diagrams are shown in Figure 4.10, while the free-body diagram of each ball is shown in Figure 4.11. Let's consider the motion from just after the balls are released until just before the balls make contact with the ground. Because the only force acting on either ball is the force of gravity, the same free-body diagram applies to both objects.

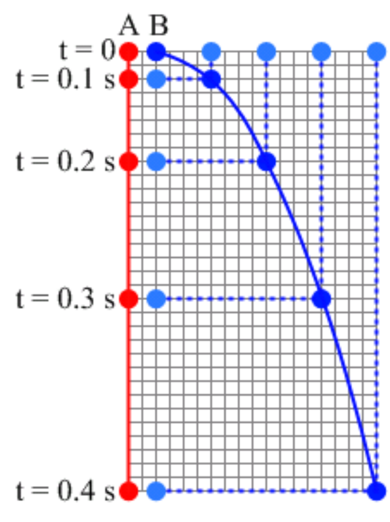


Figure 4.10: Motion diagram for balls *A* and *B*. For ball *B*, the vertical and horizontal motions are shown separately. These two independent motions combine to give the parabolic path followed by ball *B*.

Essential Question 4.4 The free-body diagrams in Figure 4.11 imply that the balls have the same mass. What would happen if the balls had different masses?

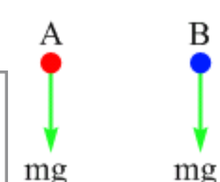


Figure 4.11: Free-body diagrams for balls *A* and *B*. From the instant just after you release the balls until the instant just before the balls hit the ground, the only force acting on either ball is the force of gravity, so the balls have identical free-body diagrams. The balls travel along different paths only because their initial velocities are different.

Answer to Essential Question 4.4 Assuming that we can neglect air resistance, the relative mass of the balls is completely irrelevant. If B's mass was double A's mass, for instance, the force of gravity on B would be twice that on A, but both balls would still have an acceleration of $\vec{g}$, and the two balls would still hit the ground simultaneously.

4-5 The Independence of x and y

A key to understanding projectile motion is the independence of x and y, the fact that the horizontal (x-direction) motion is completely independent of the vertical (y-direction) motion. Let's exploit this concept to continue our analysis of the race from Exploration 4.4.

EXPLORATION 4.5 – Analyzing the race

Step 1 – ***Find the acceleration of each ball.*** The free-body diagram in Figure 4.11, combined with Newton's second law, tells us that the acceleration of each object is simply the acceleration due to gravity, $\vec{g}$. This comes from:

$$\vec{a} = \frac{\Sigma \vec{F}}{m} = \frac{m\vec{g}}{m} = \vec{g}\,.$$

Step 2 – ***Use the general method to find the time it takes the balls to reach the ground.*** Because the motion is directed right and down, let's choose positive directions as $+x$ to the right and $+y$ down, and set the origin for each ball to be the point from which it is released. Choosing up as positive, with an origin at ground level, would also work well.

Let's say both balls fall through a vertical distance of h, and that the initial velocity of ball B is directed horizontally with a velocity of v_i. Table 4.5 shows how we organize the data. Note how the data for the x-direction (horizontal) motion for ball B are kept separate from the data for the y-direction (vertical) motion.

Component	**Ball *B*, *x* direction**	**Ball *B*, *y* direction**	**Ball *A*, *y* direction**
Initial position	$x_{iB} = 0$	$y_{iB} = 0$	$y_{iA} = 0$
Final position	$x_B = ?$	$y_B = +h$	$y_A = +h$
Initial velocity	$v_{ixB} = +v_i$	$v_{iyB} = 0$	$v_{iyA} = 0$
Final velocity	$v_{xB} = +v_i$	$v_{yB} = ?$	$v_{yA} = ?$
Acceleration	$a_{xB} = 0$	$a_{yB} = +g$	$a_{yA} = +g$

Table 4.5: Organizing the data for ball A (dropped from rest) and ball B (with an initial velocity that is horizontal). Note that everything is the same for the two balls in the y-direction, which is vertical.

One of the most common errors in analyzing a projectile-motion situation is to mix up information from the x and y directions, such as by using the acceleration due to gravity as the acceleration in the horizontal direction. Organizing the data into a table like the one above makes such errors far less likely. Including the appropriate sign on all vectors is another way to reduce errors, because it reminds us to think about which sign is correct and whether we really want a + or a –. A statement like $y_B = +h$ tells us that the final vertical position of ball B is a distance h from the origin in the positive y-direction.

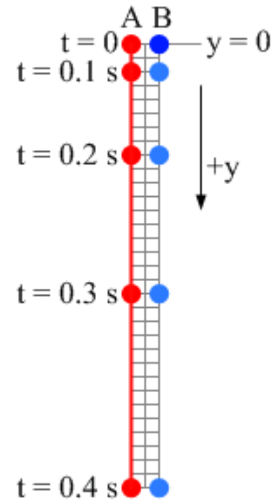


Figure 4.12: A motion diagram for the vertical components of the motion for the balls.

Can we use the data from Table 4.5 to justify the conclusion from Exploration 4.4 that the two balls reach the ground at the same time? Absolutely. The appropriate motion diagrams are shown in Figure 4.12. Analyzing the y-direction subproblem for ball B, we can use equation 4.3y to find an expression for the time to reach the ground.

$$\vec{y} = \vec{y}_i + \vec{v}_{iy}t + \frac{1}{2}\vec{a}_y t^2 ,$$

$$+h = 0 + 0 + \frac{1}{2}gt^2 .$$

This gives $t^2 = +\dfrac{2h}{g}$.

Therefore, the time for ball B to reach the ground is $t = \sqrt{\dfrac{2h}{g}}$.

Does the answer make any sense? First, it does have the right units. Second, it says that if we increase h the ball takes longer to reach the ground, which makes sense. Third, it says that the larger the acceleration due to gravity the smaller the time the ball takes to reach the ground, which also sounds right. Note that if we solve for the time ball A takes to reach the ground we get exactly the same result, because A has the same initial position, final position, initial vertical velocity, and vertical acceleration as B.

Step 3 – ***Find an expression for the horizontal distance traveled by ball B before it reaches the ground.*** Even though we're dealing with the x subproblem, we can use the time from the y subproblem – that is often key to solving projectile motion problems. The motion diagram for the x-direction motion is shown in Figure 4.13. One way to find the horizontal distance that ball B travels is to use Equation 4.3x (see Table 4.4 in Section 4.4 for the equations).

$$x = x_i + v_{ix}t + \frac{1}{2}a_x t^2 ,$$

$$x = 0 + v_i t + 0 = +v_i\sqrt{\frac{2h}{g}} .$$

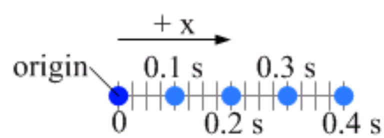


Figure 4.13: A motion diagram for the horizontal component of ball B's motion.

Again, be careful not to mix the x information with the y information. Here, for instance, we can use Table 4.5 to remind us that the acceleration in the x direction is zero. The motion diagram in Figure 4.13, showing constant-velocity motion, confirms that the acceleration in the x direction is zero.

Key idea for projectile motion: One way to solve a projectile-motion problem is to break the two-dimensional problem into two independent one-dimensional problems, ***linked by the time***, and apply the one-dimensional constant-acceleration methods from Chapter 2.
Related End-of-Chapter Exercises: 44, 45.

Essential Question 4.5 When a sailboat is at rest, a beanbag you release from the top of the mast lands in a bucket that is on the deck at the base of the mast. Will the beanbag still land in the bucket if you release the beanbag from rest when the sailboat is moving with a constant velocity?

Answer to Essential Question 4.5 Yes. From your perspective, moving with the sailboat at a constant velocity, the beanbag still drops straight down from rest. From the perspective of someone at rest on shore, the beanbag follows a parabolic motion. Its horizontal motion keeps it over the bucket at all times, though, so it still falls into the bucket because the bucket, beanbag, and boat all have the same horizontal velocity. We're neglecting air resistance here, by the way.

4-6 An Example of Projectile Motion

When a soccer goalkeeper comes off the goal line to challenge a shooter, the shooter can score by kicking the ball so that it goes directly over the goalkeeper, and then comes down in time to either bounce into, or fly into, the net. This is known as chipping the goaltender, because the shooter chips the ball to produce the desired effect. Let's analyze this situation in detail.

EXAMPLE 4.6 – Chipping the goaltender

Precisely 1.00 s after you kick it from ground level, a soccer ball passes just above the outstretched hands of a goaltender who is 5.00 m away from you, and lands on the goal line before bouncing into the net for the winning goal in a soccer match. The goaltender's fingertips are 2.50 m above the ground. Neglect air resistance, and use $g = 9.81$ m/s^2.

(a) At what angle did you launch the ball?
(b) What was the ball's initial speed?
(c) Assuming the ground is completely level, how long does the ball spend in the air?

SOLUTION

As usual, we should be methodical. A diagram of the situation is shown in Figure 4.14. The diagram includes an appropriate coordinate system, with the origin at the launch point. Table 4.6 summarizes what we know, separated into x and y components.

Take the direction of the horizontal component of the ball's motion to be the positive x direction, and take up to be the positive y direction.

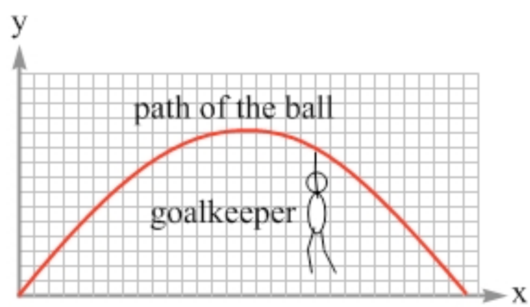


Figure 4.14: The flight of the soccer ball. The squares on the grid measure 0.25 m × 0.25 m.

Component	*x* direction	*y* direction
Initial position	$x_i = 0$	$y_i = 0$
Final position	$x = ?$	$y = 0$
Initial velocity	$v_{ix} = ?$	$v_{iy} = ?$
Final velocity	$v_x = v_{ix}$	$v_y = ?$
Acceleration	$a_x = 0$	$a_y = -g$
At t = 1.00 s	$x_{t=1} = +5.00$ m	$y_{t=1} = +2.50$ m

Table 4.6: Organizing the data for the problem.

(a) ***At what angle did you launch the ball?*** It is tempting to draw a right-angled triangle with a base of 5.00 m and a height of 2.50 m and take the angle between the base and the hypotenuse to be the launch angle, but doing so is incorrect. The ball follows a parabolic path that curves down, not a straight path to just above the goaltender's fingertips. Thus, the launch angle is larger than the angle of that particular right-angled triangle.

The launch angle is the angle between the ball's initial velocity and the horizontal, so let's work on determining the initial velocity. We can use what we know about the ball at t = 1.00 s to help us. Once again, we will make use of the equations in Table 4.4.

To find the x component of the initial velocity, use equation 4.3x: $x_{t=1} = x_i + v_{ix}t + \frac{1}{2}a_x t^2$.

$x_{t=1} = 0 + v_{ix}\,t + 0$.

This gives $v_{ix} = \frac{x_{t=1}}{t} = \frac{+5.00 \text{ m}}{1.00 \text{ s}} = +5.00 \text{ m/s}$.

To find the y component of the initial velocity, use equation 4.3y: $y_{t=1} = y_i + v_{iy}t + \frac{1}{2}a_y t^2$.

$y_{t=1} = 0 + v_{iy}t - \frac{1}{2}gt^2$.

This gives $\vec{v}_{iy} = \frac{y_{t=1} + gt^2/2}{t} = \frac{+2.50 \text{ m} + 4.90 \text{ m}}{1.00 \text{ s}} = +7.40 \text{ m/s}$.

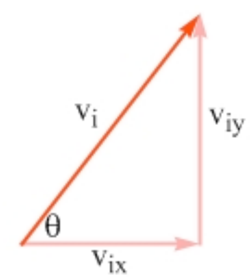


Figure 4.15: The right-angled triangle that we use to find the launch angle and launch speed of the ball.

From the two components of the initial velocity, we can determine the launch angle θ and the initial speed. The geometry of the situation is shown in Figure 4.15.

$\tan\theta = \frac{v_{iy}}{v_{ix}} = \frac{7.40 \text{ m/s}}{5.00 \text{ m/s}}$, so $\theta = 56.0^\circ$.

(b) ***What was the initial speed of the ball?*** The launch speed is the magnitude of the initial velocity. Applying the Pythagorean theorem to the triangle in Figure 4.15 gives:

$v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{(5.00 \text{ m/s})^2 + (7.40 \text{ m/s})^2} = 8.93 \text{ m/s}$.

(c) ***Assuming the ground is completely level, how long does the ball spend in the air?*** We should not assume that the ball passes over the goaltender when the ball is at its maximum height. Here is a good rule of thumb: If you do not have to assume something, don't assume it! Instead, let's make use of Equation 4.3y to determine the time for the entire flight:

$y = y_i + v_{iy}t + \frac{1}{2}a_y t^2$.

$0 = 0 + v_{iy}t - \frac{1}{2}gt^2$.

At first glance it looks as though we have to use the quadratic formula to solve this equation, but we can simply divide through by a factor of t. Doing so gives:

$0 = +v_{iy} - \frac{1}{2}gt$.

Solving for t gives $t = \frac{2v_{iy}}{g} = \frac{2 \times 7.405 \text{ m/s}}{9.81 \text{ m/s}^2} = \frac{14.81 \text{ m/s}}{9.81 \text{ m/s}^2} = 1.51 \text{ s}$.

Related End-of-Chapter Exercises: 13, 17, 18.

Essential Question 4.6 Consider again the ball from Example 4.6. How does the time it takes the ball to reach maximum height compare to the time for the entire flight? Explain.

Answer to Essential Question 4.6 The result we found in Example 4.6, that the total time for the flight is $t_{\text{impact}} = 2v_{iy} / g$, is a special case that applies only when the projectile lands at the same height from which it was launched. In this special case, the time for the entire flight is twice the time to reach maximum height (thus, $t_{\text{max height}} = v_{iy} / g$). This result comes about because of the symmetry of the motion. The second half of the motion, when the projectile is coming down, is a mirror image of the first half of the motion, when the projectile is going up. These two halves take exactly the same time (neglecting air resistance). The projectile's speed on the way up at a particular height is equal to its speed on the way down as it passes through that same height.

4-7 Graphs for Projectile Motion

Graphs can give us a lot of information about a particular motion. Let's consider how to draw a set of graphs for the motion of the soccer ball in Example 4.6.

EXAMPLE 4.7 – Graphs for the ball chipped over the goalkeeper
Draw graphs showing, as a function of time, the *x*-component of position, *x* component of velocity, and *x* component of acceleration for the soccer ball in Example 4.6. Take *x* to be horizontal, with the positive *x* direction pointing from the launch point toward the goalkeeper and the net. Draw another set of graphs for the *y* component, taking the positive *y* direction to be up.

SOLUTION
A graph can be thought of as a picture of an equation. To graph these various parameters as a function of time, it can be helpful to write down the corresponding equation. Graphs for the *x* components are shown in Figure 4.16.

To graph the *x* component of the position as a function of time, take what we know about the ball and substitute these known quantities into Equation 4.3x (see Table 4.4 in Section 4.4):

$$x = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$
$$x = 0 + (+5.00 \text{ m/s})t + 0 \, .$$
$$x = +(5.00 \text{ m/s})t$$

This is a linear function starting at the origin and having a slope of $+5.00$ m/s.

To graph the *x* component of the velocity as a function of time, take what we know about the ball and fill in equation 4.2x:

$$v_x = v_{ix} + a_x t$$
$$v_x = +5.00 \text{ m/s} + 0 \, .$$
$$v_x = +5.00 \text{ m/s}$$

The *x* component of the velocity is constant, so the graph of this velocity component is a horizontal line, as shown in the middle graph in Figure 4.16.

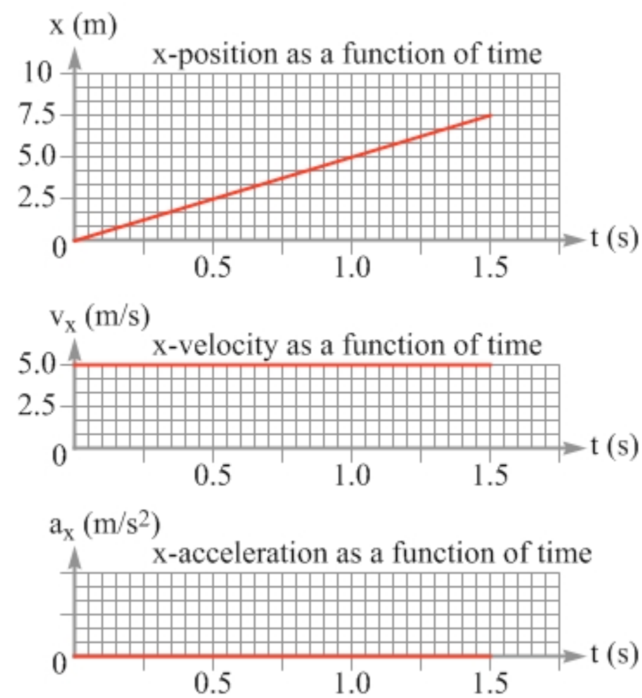


Figure 4.16: Graphs of the horizontal components of the ball's position, velocity, and acceleration.

The graph of the acceleration in the *x* direction is also a horizontal line, being drawn along the time axis, because there is no acceleration in the *x* direction.

In the *y* direction, we can do something similar. The three *y*-component graphs are drawn in Figure 4.17. To graph the *y* component of the position as a function of time, take what we know about the ball and fill in equation 4.3y:

$$y = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$

$$y = 0 + (7.40 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2.$$

The graph representing this equation is a parabola curving downward. Note the symmetry of the graph, with the second half of the motion being a mirror-image of the first half. Equation 4.4y can be used to find the maximum height the ball rises above the ground, knowing that at maximum height the *y* component of the velocity is zero. From Equation 4.4y we get:

$$v_y^2 = v_{iy}^2 + 2a_y\Delta y \,,$$

which gives $\Delta y = \dfrac{v_y^2 - v_{iy}^2}{2a_y} = \dfrac{0-(7.40 \text{ m/s})^2}{-2(9.81 \text{ m/s}^2)} = \dfrac{54.76 \text{ m}^2/\text{s}^2}{19.62 \text{ m/s}^2} = 2.79 \text{ m}$.

It is helpful to know the value of the maximum height when we graph the *y* component of the position.

To graph the *y* component of the velocity as a function of time, we take what we know about the ball and fill in Equation 4.2y:

$$v_y = v_{iy} + a_y t = +(7.40 \text{ m/s}) - (9.81 \text{ m/s}^2)t \,.$$

A graph of the *y* component of the velocity is a straight line with a *y*-intercept of +7.40 m/s and a slope of –9.81 m/s^2. Note that we can tell when the ball reaches maximum height from both the *y*-position graph and the *y*-velocity graph.

The *y* component of the acceleration is constant at –9.81 m/s^2, so that graph is a horizontal line.

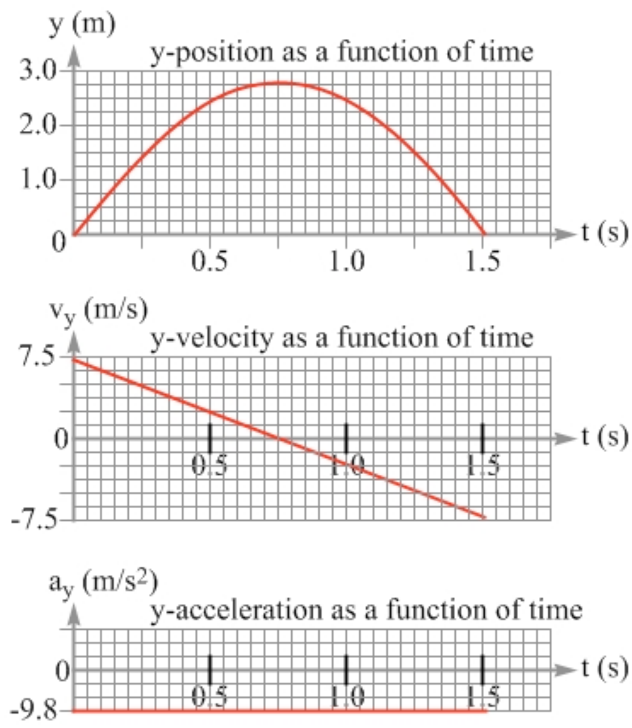


Figure 4.17: Graphs of the vertical components of the ball's position, velocity, and acceleration.

Related End-of-Chapter Exercises: 47 – 49.

Essential Question 4.7 Compare the two sets of graphs in Example 4.7 to the graphs for motion with constant velocity, and motion with constant acceleration, that we drew in Chapter 2. What similarities are there?

Answer to Essential Question 4.7 Because the ball has constant velocity in the *x* direction, the graphs for the *x* components of the ball's motion match the constant-velocity graphs from Chapter 2. Similarly, because the ball has constant acceleration in the *y* direction, the graphs for the *y* components of the ball's motion match the constant-acceleration graphs from Chapter 2.

4-8 Range, and One Final Example

The **range** of a projectile is the horizontal distance it travels before striking the ground. If a projectile lands at the same height from which it was launched, what launch angle gives the maximum range if the initial speed of the projectile is constant? Let's answer this question with the aid of the answer to Essential Question 4.6, that the time-of-flight when a projectile lands at the height from which it was launched is $t_{\text{impact}} = 2v_{iy} / g$. Substituting this time into equation 4.3x, taking the origin to be the launch point and recalling that the acceleration is zero, gives:

$$\vec{x} = \vec{x}_i + \vec{v}_{ix}t + \frac{1}{2}\vec{a}_x t^2 = 0 + \vec{v}_{ix}t + 0 = \vec{v}_{ix}t \,.$$

Therefore, $\text{range} = v_{ix}\, t_{\text{impact}} = \dfrac{2v_{ix}v_{iy}}{g}$.

If the launch angle θ is measured from the horizontal, we have $v_{ix} = v_i \cos\theta$ and $v_{iy} = v_i \sin\theta$. Substituting these expressions into the range equation above gives:

$$\text{range} = \frac{2v_{ix}v_{iy}}{g} = \frac{2(v_i\cos\theta)(v_i\sin\theta)}{g} = \frac{2v_i^2 \sin\theta\, \cos\theta}{g} \,.$$

This can be simplified with the trigonometric identity $2\sin\theta\, \cos\theta = \sin(2\theta)$:

$$\text{range} = \frac{v_i^2 \sin(2\theta)}{g} \,.$$

This equation applies only when the projectile lands at the height from which it was launched.

So, the range is proportional to the square of the initial speed, is inversely proportional to *g*; and depends on the launch angle in an interesting way. A graph of $\sin(2\theta)$ is shown in Figure 4.18. If the launch angle θ is between 0 and 90˚, then 2θ is between 0 and 180˚. Keeping v_i and *g* constant, the maximum range is achieved when $\sin(2\theta)$ is maximized. This occurs when $2\theta = 90˚$, so $\theta = 45˚$ is the launch angle that gives the maximum range when the projectile lands at the same height from which it was launched.

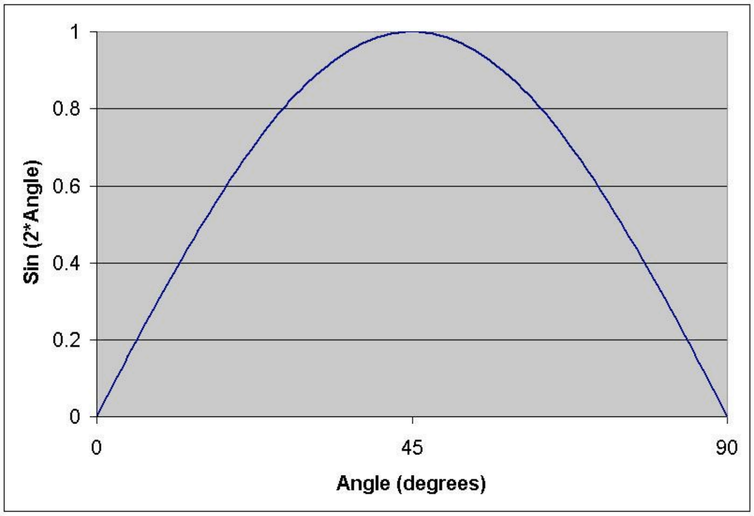


Figure 4.18: A graph of $\sin(2\theta)$ as a function of θ.

Related End-of-Chapter Exercises: 57 – 62.

EXAMPLE 4.8 – A corner kick
A corner kick, in which a player directs the ball into a crowd of players in front of the net, is one of the most exciting plays in soccer. You are taking the kick, and you want the ball, while it is on the way down, to be precisely 2.00 m above the ground when it reaches your teammate, who will attempt to head the ball into the net. Your teammate is 30.0 m away, and you kick the ball from ground level at an angle of 25.0° with respect to the horizontal. What initial speed should you give the ball? Use $g = 9.81$ m/s^2. (a) Sketch a diagram of the situation, and organize what you know in a data table, keeping the x (horizontal) information separate from the y (vertical) information. (b) Use the x information to find an expression for the time it takes the ball to reach your teammate. (c) Use the y information, and the time, to find the ball's initial speed.

SOLUTION (a) Define the origin as the launch point, and the positive directions so that toward your teammate is the positive x direction, and up is the positive y direction. Figure 4.19 shows a diagram of the situation, and the data is given in Table 4.7.

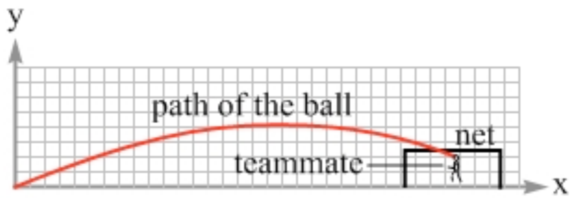


Figure 4.19: The flight of the soccer ball. The squares on the grid measure 1 m × 1 m.

Component	x-direction	y-direction
Initial position	$x_i = 0$	$y_i = 0$
Final position	$x_f = +30.0$ m	$y_f = +2.00$ m
Initial velocity	$v_{ix} = +v_i \cos(25°)$	$v_{iy} = +v_i \sin(25°)$
Acceleration	$a_x = 0$	$a_y = -9.81$ m/s^2

Table 4.7: Organizing the data for the problem.

(b) Here, we can use Equation 4.3x, $x = x_i + v_{ix}t + (0.5)a_x t^2$, which reduces to $x = v_{ix}t$. Solving for the time the ball takes to reach your teammate, we get: $t = \dfrac{x}{v_{ix}} = \dfrac{+30.0\text{ m}}{v_i \cos(25°)}$.

(c) We can now substitute our expression for time into equation 4.3y:

$$y = y_i + v_{iy}t + \frac{1}{2}a_y t^2: \qquad +2.00\text{ m} = v_i \sin(25°) \times \left(\frac{+30.0\text{ m}}{v_i \cos(25°)}\right) + \frac{1}{2}(-9.81\text{ m/s}^2) \times \left(\frac{+30.0\text{ m}}{v_i \cos(25°)}\right)^2.$$

Simplifying, we get: $+2.00\text{ m} = +13.989\text{ m} - \dfrac{5374.40\text{ m}^3/\text{s}^2}{\text{v}_\text{i}^2}$.

This gives $+\dfrac{5374.40\text{ m}^3/\text{s}^2}{\text{v}_\text{i}^2} = +11.989\text{ m}$, so $v_i = \sqrt{\dfrac{5374.40\text{ m}^3/\text{s}^2}{11.989\text{ m}}} = 21.2\text{ m/s}$.

This is fast, but not unusual for a typical soccer game.

Related End-of-Chapter Exercises: 19, 28, 63.

Essential Question 4.8 Return again to the situation described in Example 4.8. Could we have found the answer more easily by applying the range equation from the previous page?

Answer to Essential Question 4.8 No. The range equation applies only in the special case when a projectile's initial and final heights are the same. In this case, the final position is 2.00 m higher than the initial position.

Chapter Summary

Essential Idea - The Independence of x and y

This powerful concept enables us to split a two-dimensional problem into two one-dimensional problems, in a situation like projectile motion when the force applied to the projectile is constant. The motion in one direction is completely unaffected by the motion in the perpendicular direction, except that the two motions share the same time.

Relative Velocity

A relative velocity problem is a vector addition problem. The velocity of A to C is the vector sum of the velocity of A relative to B plus the velocity of B relative to C:

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC} \qquad \text{(Equation 4.1)}$$

Rules of Thumb for Projectile Motion (motion under the influence of gravity alone)

- The motion is symmetric, with the downward part of the motion being a mirror image of the upward part of the motion.
- The larger the initial vertical velocity, the higher the projectile goes.
- The higher a projectile goes, the longer the time of flight.
- Range is maximized when the launch angle is 45˚, as long as the projectile lands at the same height from which it was launched.

General Method for Solving a Two-Dimensional Motion Problem

The basic method is to split a two-dimensional motion problem into two one-dimensional subproblems, which we can call the *x* subproblem and the *y* subproblem.

1. Draw a diagram of the situation.
2. Draw a free-body diagram of the object showing all the forces acting on the object while it is in motion. A free-body diagram helps in determining the acceleration of the object.
3. Choose an origin.
4. Choose an *x*-*y* coordinate system, showing which way is positive for each direction.
5. Organize your data, keeping the information for the *x* subproblem separate from the information for the *y* subproblem.
6. As long as the acceleration is constant, apply the constant-acceleration equations. These equations are based on the equations from Chapter 2:

***x* equations**		***y* equations**	
$v_x = v_{ix} + a_x t$	(Equation 4.2x)	$v_y = v_{iy} + a_y t$	(Equation 4.2y)
$x = x_i + v_{ix} t + \frac{1}{2} a_x t^2$	(Equation 4.3x)	$y = y_i + v_{iy} t + \frac{1}{2} a_y t^2$	(Equation 4.3y)
$v_x^2 = v_{ix}^2 + 2 a_x \Delta x$	(Equation 4.4x)	$v_y^2 = v_{iy}^2 + 2 a_y \Delta y$	(Equation 4.4y)

End-of-Chapter Exercises

Exercises 1 – 12 are conceptual questions designed to see whether you understand the main concepts of the chapter.

1. You are going to paddle your kayak from one side of a river to the other side. The current in the river is directed downstream, as is usual for a river. How should you point your kayak (upstream, perpendicular to the riverbank, or downstream) so that you cross from one side of the river to (a) anywhere on the other side in the shortest possible time? (b) the point directly across the river from your starting point? Briefly justify your answers.

2. A ballistic cart is a wheeled cart that can launch a ball in a direction perpendicular to the way the cart moves and can then catch the ball again if it falls back down on the cart. Holding the cart stationary on a horizontal track, you confirm that the ball does indeed land in the cart after it is launched. You now give the cart a quick push so that, after you release it, the cart rolls along the track with a constant velocity. If the ball is launched while the cart is rolling, will it land in front of the cart, behind the cart, or in the cart? Briefly justify your answer.

3. On an assignment, you are asked to find the time it takes for a ball launched with a particular initial velocity to reach the surface of the water some distance below. When setting up the exercise, you choose the origin to be at the base of the cliff. Your friend chooses the origin to be at the top of the cliff, from where the ball was launched. Who is right, or is there no one right place to choose as the origin? What do you and your friend agree on? What do you disagree on? Explain.

4. On an assignment, you are asked to find the time it takes for a ball launched with a particular initial velocity to reach the surface of the water some distance below. When setting up the exercise, you choose an *x*-*y* coordinate system in which the positive *y* direction is up, while your friend chooses an *x*-*y* coordinate system in which the positive *y* direction is down. Who is right, or is there no one correct direction for the positive *y*-axis? What do you and your friend agree on? What do you disagree on? Explain.

5. Consider the trajectories of three objects, labeled *A*, *B*, and *C*, shown in Figure 4.20. Rank these objects from largest to smallest, based on (a) their times of flight, (b) the *y* component of their initial velocities, (c) the *x* component of their initial velocities, and (d) their launch speeds.

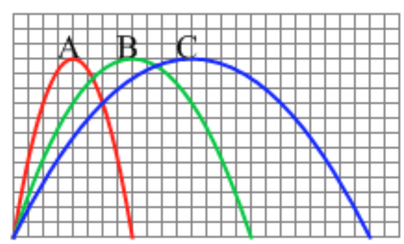


Figure 4.20: The trajectories of three projectiles, for Exercise 5.

6. The trajectories of two objects, *D* and *E*, are shown in Figure 4.21. The grid shown on the diagram is square. (a) Which object has the longer time of flight? Briefly explain. (b) If object *E* has a time of flight of *T*, what is object *D*'s time of flight? (c) If object *E* has a constant horizontal velocity of v_{ix} to the right, what is the constant horizontal velocity of object *D*?

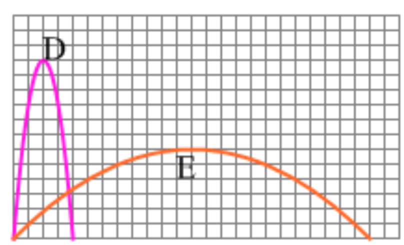


Figure 4.21: The trajectories of two projectiles, for Exercise 6.

7. The trajectories of three objects, E, F, and G, are shown in Figure 4.22. Assume that the grid shown on the diagram is square, and note that object E is launched at an angle of 45° with respect to the horizontal. Rank these three projectiles from largest to smallest, based on (a) their times of flight, (b) the y component of their initial velocities, (c) the x component of their initial velocities, and (d) their launch speeds.

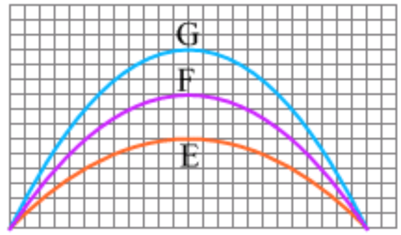


Figure 4.22: The trajectories of three projectiles, for Exercise 7.

8. The motion diagram in Figure 4.23 shows a ball's position at regular intervals as the ball flies from left to right through the air. Copy the motion diagram onto a sheet of graph paper. (a) On the same graph, sketch the motion diagram for a ball that also starts at the lower left corner, has the same initial vertical velocity as the ball in Figure 4.23, but has a horizontal velocity 50% larger than that of the ball in Figure 4.23. (b) On the same graph, sketch the motion diagram for a ball that has the same starting point, half the initial vertical velocity, and twice the horizontal velocity, of the ball in Figure 4.23.

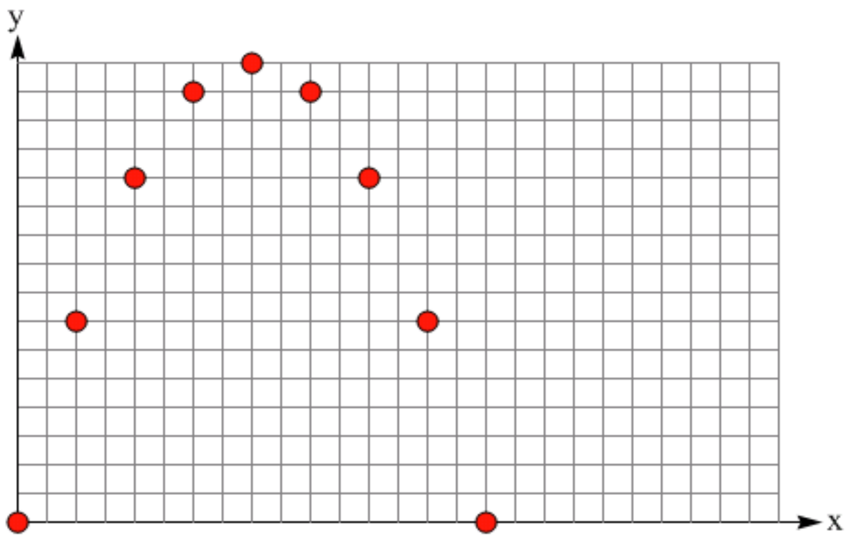


Figure 4.23: A motion diagram showing the position of a ball at regular time intervals, as the ball moves from left to right, for Exercises 8 and 9.

9. Consider the motion diagram shown in Figure 4.23. Assuming the ball's position is shown at 1-second intervals and that $g = 10$ m/s^2, sketch graphs of the ball's: (a) y velocity as a function of time, (b) y position as a function of time, (c) x velocity as a function of time, and (d) x position as a function of time.

10. To answer a typical projectile-motion problem, it is not necessary to know the mass of the projectile. Why is this?

11. You find that when you throw a ball with a particular initial velocity, the ball reaches a maximum height H, has a time of flight T, and covers a range R when landing at the same height from which it was launched. If you throw the ball again but now you double the launch speed, keeping everything else the same, what are the ball's (a) maximum height, (b) time of flight, and (c) range?

12. You find that when you throw a ball on Earth with a particular initial velocity, the ball reaches a maximum height *H*, has a time-of-flight *T*, and covers a range *R* when landing at the same height from which it was launched. You now travel to the Moon, where the acceleration due to gravity is one-sixth of what it is on Earth, and throw the ball with the same initial velocity. What are (a) the maximum height, (b) the time of flight, and (c) the range of the ball on the Moon?

Exercises 13 – 20 deal with relative-velocity situations.

13. You are driving your car on a highway, traveling at a constant velocity of 110 km/h north. Ahead of you on the road is a truck traveling at a constant velocity of 90 km/h north. On the other side of the road, coming toward you, is a motorcycle traveling at a constant velocity of 100 km/h south. (a) What is your velocity relative to the truck? (b) What is the truck's velocity relative to you? (c) What is your velocity relative to the motorcycle? (d) What is the motorcycle's velocity relative to the truck? (e) After you pass the truck and the motorcycle roars past on the other side of the road, do any of the answers above change? Explain.

14. You paddle your canoe at a constant speed of 4.0 km/h relative to the water. You are canoeing along a river that is flowing at a constant speed of 1.0 km/h. If you paddle for 60 minutes downstream (with the current) and then turn around, how long does it take you to get back to your starting point?

15. Return to the situation described in Exercise 14, and let's say that the river is 300 m wide. (a) If you paddle your canoe so you cross from one side of the river to the other in the shortest possible time, how long does it take to cross the river? (b) If instead you paddle your canoe so that you land at the point directly across the river from where you started, following a straight line, how long does it take you?

16. Return again to the situation described in Exercises 14 and 15. Compare what happens if, in case 1, you aim your canoe at an angle of 15˚ upstream to what happens if, in case 2, you aim your canoe 15˚ downstream. (a) In which case does it take you longer to cross the river? How much time does it take you to cross in that case? (b) How far upstream or downstream are you when you reach the far side in (i) case 1? (ii) case 2?

17. Four people are moving along a sidewalk. Ron's velocity relative to Susan is 2.0 m/s east; Susan's velocity relative to Tamika is 6.0 m/s west; and Tamika's velocity relative to Ulrich is 3.0 m/s east. What is Ulrich's velocity relative to (a) Susan? (b) Ron? (c) a lamppost at rest with respect to the sidewalk?

18. Return to the situation described Exercise 17, except now Ron is crossing the street and his velocity relative to Susan has components of 3.0 m/s north and 2.0 m/s east. Find Ulrich's velocity relative to Ron now, expressing it in (a) component form and (b) magnitude-and-direction form.

19. You are flying your airplane at a constant speed of 200 km/h relative to the air. As you pass over Zurich, Switzerland, you check your map and find that Frankfurt, Germany, is 300 km due north, so you point your plane due north. (a) If there is no wind blowing, how long should it take you to get to Frankfurt? (b) After that amount of time has passed you are shocked to find that, instead of being over Frankfurt, you are near the town of Tenneville, Belgium, precisely 225 km due west of Frankfurt. What is the velocity of the wind? (c) You now immediately change direction so you can fly to Frankfurt. In which direction should you point your plane, and how long does it take you to reach Frankfurt? (Let's hope you have plenty of fuel!)

20. Having learned from your experience of Exercise 19, the next time you fly from Zurich to Frankfurt you account for the wind, which is blowing west at a speed of 120 km/h. (a) If your plane travels at a constant speed of 200 km/h relative to the air, in which direction should you point your plane so that you travel due north, toward Frankfurt? (b) How long does it take you to fly from Zurich to Frankfurt this time?

Exercises 21 – 24 deal with the concept of the independence of *x* and *y*.

21. You climb 20 meters up the mast of a tall ship and release a ball from rest so it hits the deck of the ship at the base of the mast. Use $g = 10$ m/s^2 to simplify the calculations. (a) How long does it take the ball to reach the deck of the ship? (b) From the same point, 20 meters up the mast, you launch a second ball so that its initial velocity is 1.0 m/s directed horizontally and east. How long does this ball take to reach the deck of the ship? (c) How far is the second ball from the base of the mast when it hits the deck?

22. Returning to the situation described in Exercise 21, assume now that the ship is traveling with a constant velocity of 4.0 m/s east, and that the water is calm so the mast remains vertical. Once again you climb 20 meters up the mast and release one ball from rest, relative to you, and give a second ball an initial velocity of 1.0 m/s, directed east, relative to you. (a) Does the fact that the ship is moving affect the time it takes either ball to fall to the deck of the ship? Explain. (b) How far from the base of the mast does the first ball land now? (c) How far from the base of the mast does the second ball land now? (d) From the point of view of a seagull sitting motionless on a buoy watching the tall ship sail past, what is the horizontal displacement of the second ball as it falls to the deck of the ship?

23. You are standing inside a bus that is traveling at a constant velocity of 8 m/s along a straight horizontal road. You toss an apple straight up into the air and then catch it again 1.2 seconds later at the same height from which you let it go. Use $g = 10$ m/s^2 to simplify the calculations. Answer these questions based on what you see. In other words, consider the motion of the apple relative to you. (a) What is the maximum height reached by the apple? (b) What is the initial velocity of the apple? (c) How far does the apple move horizontally (relative to you)?

24. Repeat Exercise 23, but now answer the questions from the perspective of a person at rest outside the bus, looking in through the bus windows. For part (c), in particular, how far does the apple move horizontally relative to the person at rest outside the bus?

Exercises 25 – 36 are designed to help you practice applying the general method for solving projectile-motion problems. For each exercise, (a) draw a diagram. (b) Sketch a free-body diagram, showing the forces acting on the object. (c) Choose an origin, and mark it on your diagram. (d) Choose an *x*-*y* coordinate system and draw it on your diagram, showing which way is positive for each coordinate axis. (e) Organize your data in a table, keeping the information for the *x* subproblem separate from the information for the *y* subproblem.

25. Before game 1 of the 2004 World Series, Johnny Damon and Trot Nixon are playing catch in the outfield at Fenway Park. The two players are 23.0 m apart. Damon throws the ball to Nixon so that it reaches a maximum height of 15.5 m above the point he released it. Assume that Nixon catches the ball at the same level from which Damon releases it, that air resistance is negligible, and that $g = 9.81$ m/s^2. Answer parts (a) through (e), as described above. (f) At what initial velocity did Damon throw the ball?

26. Working as an accident reconstruction expert, you find that a car that was driven off a horizontal road over the edge of a 15-m-high cliff traveled 42 m horizontally before impact. Use $g = 9.8$ m/s^2. Answer parts (a) through (e) as described above. (f) At what speed was the car moving when it left the road? Express the speed in m/s, km/h, and miles/h. (g) At what speed was the car moving just before impact?

27. You toss a set of keys up to your friend, who is leaning out a window above you. You are at ground level in front of the window, and you release the keys 2.0 m horizontally from, and 6.0 m vertically below, the point where your friend catches them. The keys happen to be at their maximum height point when they are caught by your friend. Answer parts (a) through (e) as described above. (f) With what initial velocity did you launch the keys?

28. You throw a ball with an initial speed v_i at an angle of θ above the horizontal. The ball lands at a point some distance away that is a height *h* below the point from which it was launched. Answer parts (a) through (e) as described above. (f) In terms of *g*, v_i, θ, and *h*, determine an expression for the time the ball is in flight. Think about what would happen if *h* was zero, or if the landing point was some height *h* above the launch point instead.

29. You throw a ball so that it is in the air for 4.56 s and travels a horizontal distance of 50.0 m. Use $g = 9.81$ m/s^2, and assume the ball lands at the same height from which it was launched. Answer parts (a) through (e) as described above. (f) What was the ball's initial velocity?

30. Repeat Exercise 29, but now assume the ball lands at a height 20.0 m below the point from which it was launched.

31. Taking a corner kick in a soccer game, you kick the ball from ground level so that, after reaching a maximum height of 3.5 m above the ground, the ball reaches your teammate 22 meters away. Your teammate makes contact with the ball when the ball is 2.0 m off the ground and heads the ball into the net. Set the origin to be the point from which you kicked the ball, take up to be the positive *y* direction, and point the positive *x* direction from you toward your teammate. Answer parts (a) through (e) as described above. (f) Determine the *x* and *y* components of the ball's initial velocity. (g) Plot graphs of the ball's *x* position, *y* position, *x* velocity, and *y* velocity, all as a function of time. (h) Plot a graph of the ball's *y* position as a function of its *x* position.

32. You throw a ball from the edge of a vertical cliff overlooking the ocean. The ball is launched with an initial velocity of 12.0 m/s at an angle of 34.0˚ above the horizontal, from a height of 55.0 m above the water. Neglect air resistance, and use g = 9.81 m/s^2. Answer parts (a) through (e), as described above. (f) How long does it take the ball to reach the water? (g) Assuming you throw the ball so the horizontal component of its velocity is perpendicular to the cliff face, how far is it from the base of the cliff when it hits the water?

33. Repeat parts (a) through (e) of Exercise 32, but, this time, choose a different origin. Show that you still get the same answers for parts (f) and (g).

34. Repeat parts (a) through (e) of Exercise 32, but, this time, reverse the direction you take to be positive for the vertical coordinate axis. Show that you still get the same answers for parts (f) and (g).

35. A spacecraft with a mass of 8000 kg is drifting through deep space with a constant velocity of 5.0 m/s in the positive y direction. The thrusters are then turned on so the craft experiences a constant acceleration of 4.0 m/s^2 in the positive x direction for a period of 5.0 s (assume the mass lost in this process is negligible). The thrusters are then turned off. Take the origin to be the position of the spacecraft when the thrusters are first turned on. Answer parts (a) through (e) as described above. (f) How far is the spacecraft from the origin when the thrusters are turned off? (g) How far is the spacecraft from the origin after another 5.0 s has passed?

36. Repeat Exercise 35, but this time assume that, when the thrusters are turned on, the spacecraft experiences a constant acceleration of 4.0 m/s^2 in the positive x direction as well as a constant acceleration of 3.0 m/s^2 in the positive y direction. Once again, the thrusters are turned off after 5.0 s.

Exercises 37 – 40 deal with common applications of projectile motion.

37. For a typical kickoff in a football game, the “hang time” (the time the ball spends in the air) is 4.0 s, and the ball travels 50 m. Neglecting air resistance, determine (a) the vertical component of the initial velocity, (b) the horizontal component of the initial velocity, (c) the magnitude and direction, relative to the horizontal, of the initial velocity.

38. A basketball hoop is 3.05 m above the floor, and the horizontal distance from the free-throw line to a point directly below the center of the hoop is 4.60 m. Assuming that a basketball player releases the ball at an angle of 60.0° with respect to the horizontal, from a point that is located exactly 1.70 m above the free-throw line, what should the launch speed be so the ball passes through the center of the hoop?

39. In 1983, Chris Bromham set a world record by jumping a motorcycle over 18 double-decker buses. If Mr. Bromham launched the motorcycle off a ramp with a launch speed of 42 m/s and traveled a horizontal distance of 63 m, landing at the same height from which he left the ramp, at what angle was the launch ramp inclined? Note that such ramps are inclined at angles considerably less than 45°.

40. The Russian shot-putter Natalya Lisovskaya set a world record, in 1987, for the shot put that was still unbroken at the time this book went to press. Neglect air resistance. Assume the shot was launched from a height of 1.800 m above the ground, at an angle of 43.00° above the horizontal, and with a speed of 14.32 m/s. Use g = 9.810 m/s, and determine the distance of the record throw.

General Problems and Conceptual Questions

41. In Exploration 4.2, let's say $L = 36.0$ m and $v = 8.0$ m/s. By what distance does Brandi win the race?

42. In Exploration 4.2, let's say $L = 36.0$ m and $v = 8.0$ m/s. (a) Aside from the very start of the race, at how many different instants are the two women the same distance from the start line at the same time, between the time the race starts and the time Brandi arrives at the finish line? (b) When are these instants, and how far from the start line are the two women when these instants occur?

43. You are driving your car at a constant velocity of 110 km/h north. As you pass under a bridge, a train is passing over the bridge, traveling at a constant speed of 60 km/h. What is your velocity relative to the train if (a) the train is traveling due east? (b) the train is traveling at an angle of 15˚ south of east?

44. A ballistic cart is a wheeled cart that can launch a ball in a direction perpendicular to the way the cart moves and can then catch the ball again if it falls back down on the cart. Holding the cart stationary on a horizontal track, you confirm that the ball does indeed land in the cart after it is launched. Let's say that the cart launches the ball with an initial velocity of 4.00 m/s up relative to the cart while the cart is rolling with a constant velocity of 3.00 m/s to the right. Using $g = 9.81$ m/s^2, determine (a) the time it takes the ball to return to the height from which it was launched, (b) the displacement of the cart during this time, and (c) the displacement of the ball during this time.

45. In 1991, Mike Powell of the United States set a world record of 8.95 m in the long jump. With this jump, Powell broke Bob Beamon's record of 8.90 m, which was set at the Mexico City Olympic Games in 1968. Estimate how fast Powell was going when he left the ground, knowing that he was trying to jump as far as he could. How does your value compare to the top speed of a world-class sprinter?

46. While playing catch with your friend, who is located due north of you, you throw a ball such that its initial velocity components are 20 m/s up and 7.0 m/s north. Some time later, your friend catches the ball at the same height from which you released it. Considering the motion from the instant just after you released it until just before your friend caught it, what is (a) the ball's average velocity over this interval? (b) the ball's average acceleration over this interval?

47. Consider the trajectory of object B in Figure 4.24. At what angle from the horizontal was it launched? Assume the grid shown on the diagram is square.

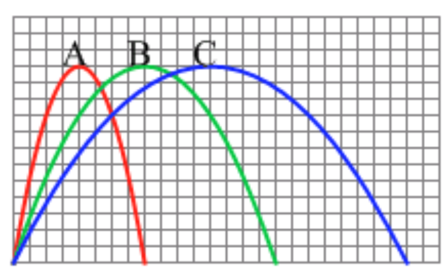


Figure 4.24: The trajectories of three projectiles. For Exercise 47, just focus on object B.

48. The motion diagram in Figure 4.25 shows a ball's position at regular intervals as the ball flies from left to right through the air. (a) On the x-axis, draw the motion diagram corresponding to the x subproblem. What does this tell you about the ball's velocity and acceleration in the x direction? (b) On the y-axis, draw the motion diagram corresponding to the y subproblem. What does this tell you about the ball's velocity and acceleration in the y direction? If the ball's position is shown at 1-second intervals and $g = 10$ m/s^2, (c) what is the maximum height reached by the ball? (d) How far does the ball travel horizontally?

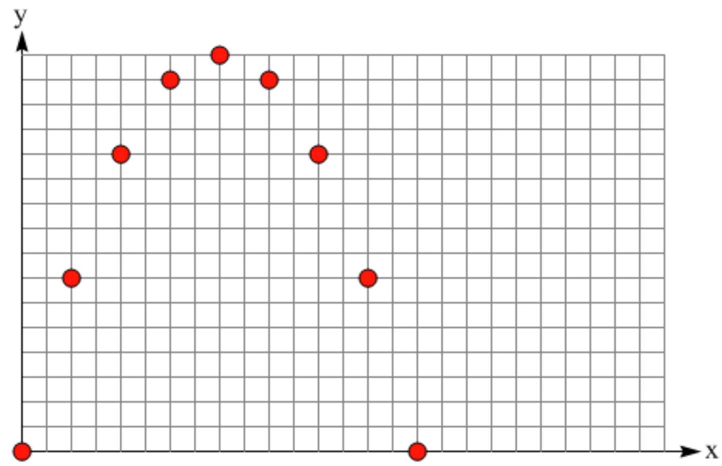


Figure 4.25: A motion diagram showing the position of a ball at regular time intervals, as the ball moves from left to right, for Exercise 48.

49. A ball is launched and travels for exactly 3 seconds before being caught. Graphs of the ball's x velocity and y velocity are shown in Figure 4.26, where the acceleration due to gravity is assumed to be $g = 10$ m/s^2. (a) Is the ball caught at a level that is higher, lower, or the same as the level from which it was launched? How do you know? (b) How far does the ball travel horizontally during the 3-second period? (c) At what instant does the ball reach its maximum height? Justify your answer.

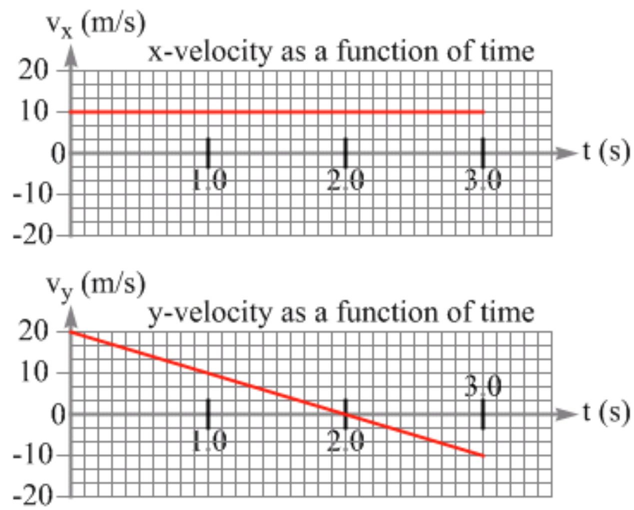


Figure 4.26: Graphs of the x-velocity and y-velocity for a ball thrown into the air. For Exercises 49 – 51.

50. Consider again the situation described in Exercise 49, and the graphs of the ball's x and y velocity shown in Figure 4.26. Sketch corresponding graphs of the ball's (a) x acceleration, (b) y acceleration, (c) x position, and (d) y position. Draw each graph as a function of time.

51. On a piece of graph paper, sketch the motion diagram corresponding to the velocity graphs in Figure 4.26, showing the position of the ball at 0.50-second intervals between $t = 0$ and $t = 3$ seconds.

52. A ball is launched with an initial velocity of 12.0 m/s at an angle of 30.0˚ above the horizontal. It lands some time later at the same height from which it was launched. Using the standard component method of analysis, and using $g = 9.81$ m/s^2, determine the time the ball is in flight.

53. You kick a soccer ball from ground level with an initial velocity of 7.50 m/s at an angle of 30.0˚ above the horizontal. The ball hits a wall 8.00 m away. How far up the wall is the point of impact?

54. You kick a soccer ball from ground level with an initial velocity of 7.50 m/s at an angle of 30.0˚ above the horizontal. When the ball strikes a wall that is some distance from you, the ball is 50.0 cm off the ground. How far away is the wall? Find all possible answers, assuming that the ball does not bounce before hitting the wall. Use $g = 9.80$ m/s^2.

55. Standing in the middle of a perfectly flat field, you kick a soccer ball from ground level, launching it with an initial speed of 20 m/s. Using $g = 10$ m/s^2 to simplify the calculations, and assuming you can use any launch angle between 0 and 90˚, measured from the horizontal, determine the following: (a) the shortest and longest times the ball can spend in the air before hitting the ground, (b) the maximum height the ball can reach in its flight, and (c) the maximum range of the ball.

56. Returning to the situation of Exercise 55, what launch angle(s) give a range that is 75% of the maximum range?

57. You are playing catch with your friend, who is standing 20.0 m from you. When you first throw the ball, you launch it at an angle of 65.0˚ and the ball falls 5.00 m short of reaching your friend. (a) At what speed did you launch the ball? (b) If you kept the launch angle the same, at what initial speed should you launch the ball on the next throw so the ball reaches your friend? (c) If instead you kept the launch speed the same as in part (a), at what launch angle should you launch the ball so it reaches your friend? Assume the ball lands at the same height from which it was launched for all parts of this exercise.

58. You are having a snowball fight with your friend, who is 7.0 m away from you. Knowing some physics, you throw one snowball at an angle of 70˚ above the horizontal, launching it so that the snowball will hit your friend at the same height from which you let it go. You wait for a short time interval, and then launch a second snowball from the same point, but at an angle of 20˚ above the horizontal. If you want both snowballs to hit your friend in the same place simultaneously, how long should the time interval be between throwing the two snowballs?

59. You repeat the two-snowball situation described in Exercise 58, but this time with different launch angles. You observe that the maximum height (measured from the launch point) of one snowball is exactly 5 times higher than the maximum height reached by the other snowball. At what angles were the two snowballs launched, assuming they were launched with the same speed?

60. You repeat the two-snowball situation described in Exercise 58, but this time with different launch angles. You observe that the time of flight for one snowball is exactly 5 times longer than the time of flight of the other snowball. At what angles were the two snowballs launched, assuming they were launched with the same speed?

61. You flick a coin from a tabletop that is 1.30 m above the floor, giving the coin an initial speed of 2.40 m/s when it leaves the table. Calculate the horizontal distance between the launch point and the point of impact if the launch angle, relative to the horizontal, is (a) 0˚, (b) 30˚, and (c) 45˚.

Figure 4.27: This decorative fountain shows the water following parabolic trajectories. After being sprayed from a pipe in the fountain, each stream of water is acted upon by gravity and follows a two-dimensional path as the stream deflects toward the Earth. (Photo credit: Edward Hor / iStockphoto.)

62. The photograph in Figure 4.27 shows the parabolic paths followed by water in a decorative fountain. If the water emerging from one of the pipes has an initial speed of 4.0 m/s, and is projected at an angle of 60° above the horizontal, determine the maximum height reached by the water above the level of the top end of the pipe.

63. Three students are trying to solve a problem that involves a ball being launched, at a 30° angle above the horizontal, from the top of a cliff, and landing on the flat ground some distance below. The students know the launch speed, the acceleration due to gravity, and the height of the cliff. They are looking for the time the ball spends in the air. Comment on the part of their conversation that is recorded below.

Avi : I think we have to do the problem in two steps. First, we find the point where the ball reaches its maximum height, and then we go from that point down to the ground.

T.J. : I think we can do it all in one step. The equations can handle it, just going all the way from the initial point to the ground.

Kristin: I think we need two steps, too, but I would do it differently than Avi. What if we first find the point where the ball comes down to the same height from where it was launched, and then we go from that point down to the ground?

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Chapter 4: Additional Resources

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