2-1 Position, Displacement, and Distance

In describing an object's motion, we should first talk about position – where is the object? A position is a vector because it has both a magnitude and a direction: it is some distance from a zero point (the point we call **the origin**) in a particular direction. With one-dimensional motion, we can define a straight line along which the object moves. Let's call this the *x*-axis, and represent different locations on the *x*-axis using variables such as $\vec{x}_0$ and $\vec{x}_1$, as in Figure 2.1.

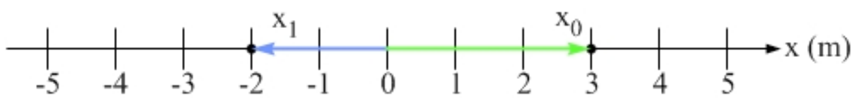


Figure 2.1: Positions $\vec{x}_0$ = +3 m and $\vec{x}_1$ = –2 m, where the + and – signs indicate the direction.

If an object moves from one position to another we say it experiences a **displacement**.

Displacement: a vector representing a change in position. A displacement is measured in length units, so the MKS unit for displacement is the meter (m).

We generally use the Greek letter capital delta (Δ) to represent a change. If the initial position is $\vec{x}_i$ and the final position is $\vec{x}_f$ we can express the displacement as:

$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$. (Equation 2.1: **Displacement in one dimension**)

In Figure 2.1, we defined the positions $\vec{x}_0$ = +3 m and $\vec{x}_1$ = –2 m. What is the displacement in moving from position $\vec{x}_0$ to position $\vec{x}_1$? Applying Equation 2.1 gives $\Delta\vec{x} = \vec{x}_1 - \vec{x}_0 = -2\text{ m} - (+3\text{ m}) = -5\text{ m}$. This method of adding vectors to obtain the displacement is shown in Figure 2.2. Note that the negative sign comes from the fact that the displacement is directed left, and we have defined the positive *x*-direction as pointing to the right.

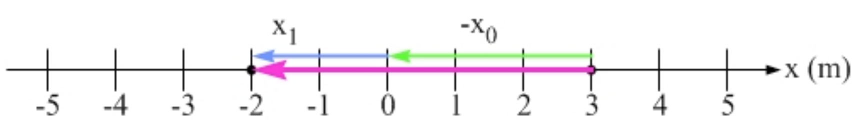


Figure 2.2: The displacement is –5 m when moving from position $\vec{x}_0$ to position $\vec{x}_1$. Equation 2.1, the displacement equation, tells us that the displacement is $\Delta\vec{x} = \vec{x}_1 - \vec{x}_0$, as in the figure. The bold arrow on the axis is the displacement, the vector sum of the vector $\vec{x}_1$ and the vector $-\vec{x}_0$.

To determine the displacement of an object, you only have to consider the change in position between the starting point and the ending point. The path followed from one point to the other does not matter. For instance, let's say you start at $\vec{x}_0$ and you then have a displacement of 8 meters to the left followed by a second displacement of 3 meters right. You again end up at $\vec{x}_1$, as shown in Figure 2.4. The total distance traveled is the sum of the magnitudes of the individual displacements, 8 m + 3 m = 11 m. The net displacement (the vector sum of the individual displacements), however, is still 5 meters to the left: $\Delta\vec{x} = -8\text{ m} + (+3\text{ m}) = -5\text{ m} = \vec{x}_1 - \vec{x}_0$.

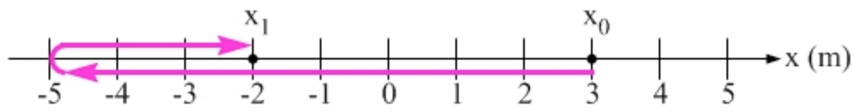


Figure 2.3: The net displacement is still –5 m, even though the path taken from $\vec{x}_0$ to $\vec{x}_1$ is different from the direct path taken in Figure 2.2.

EXAMPLE 2.1 – Interpreting graphs

Another way to represent positions and displacements is to graph the position as a function of time, as in Figure 2.4. This graph could represent your motion along a sidewalk.

(a) What happens at a time of $t = 40$ s?

(b) Draw a diagram similar to that in Figure 2.3, to show your motion along the sidewalk. Add circles to your diagram to show your location at 10-second intervals, starting at $t = 0$.

Using the graph in Figure 2.4, find (c) your net displacement and (d) the total distance you covered during the 50-second period.

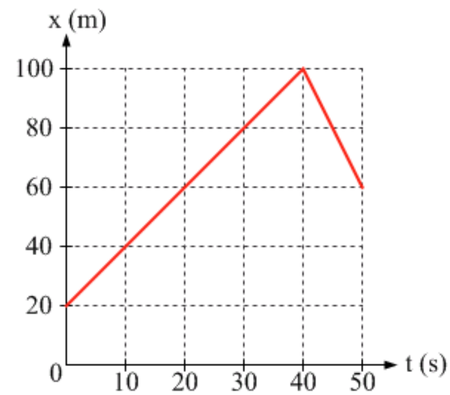


Figure 2.4: A graph of the position of an object versus time over a 50-second period. The graph represents your motion in a straight line as you travel along a sidewalk.

SOLUTION

(a) At a time of $t = 40$ s, the graph shows that your motion changes from travel in the positive *x*-direction to travel in the negative *x*-direction. In other words, at $t = 40$ s you reverse direction.

(b) Figure 2.5 shows one way to turn the graph in Figure 2.4 into a vector diagram to show how a series of individual displacements adds together to a net displacement. Figure 2.5 shows five separate displacements, which break your motion down into 10-second intervals.

(c) The displacement can be found by subtracting the initial position, +20 m, from the final position, +60 m. This gives a net displacement of $\Delta\vec{x}_{net} = \vec{x}_f - \vec{x}_i = +60 \text{ m} - (+20 \text{ m}) = +40 \text{ m}$.

A second way to find the net displacement is to recognize that the motion consists of two displacements, one of +80 m (from +20 m to +100 m) and one of –40 m (from +100 m to +60 m). Adding these individual displacements gives $\Delta\vec{x}_{net} = \Delta\vec{x}_1 + \Delta\vec{x}_2 = +80 \text{ m} + (-40 \text{ m}) = +40 \text{ m}$.

(d) The total distance covered is the sum of the magnitudes of the individual displacements. Total distance = 80 m + 40 m = 120 m.

Related End-of-Chapter Exercises: 7 and 9

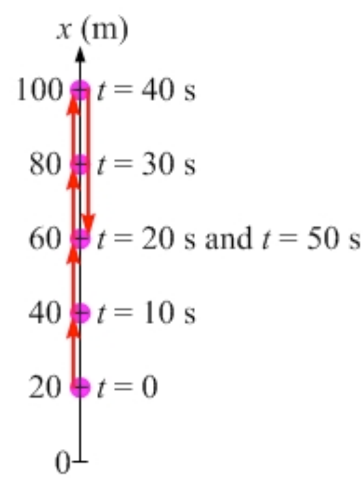


Figure 2.5: A vector diagram to show your displacement, as a sequence of five 10-second displacements over a 50-second period. The circles show your position at 10-second intervals.

Essential Question 2.1: In the previous example, the magnitude of the displacement is less than the total distance covered. Could the magnitude of the displacement ever be larger than the total distance covered? Could they be equal? Explain. *(The answer is at the top of the next page.)*

Answer to Essential Question 2.1: The magnitude of the net displacement is always less than or equal to the total distance. The two quantities are equal when the motion occurs without any change in direction. In that case, the individual displacements point in the same direction, so the magnitude of the net displacement is equal to the sum of the magnitudes of the individual displacements (the total distance). If there is a change of direction, however, the magnitude of the net displacement is less than the total distance, as in Example 2.1.

2-2 Velocity and Speed

In describing motion, we are not only interested in where an object is and where it is going, but we are also generally interested in how fast the object is moving and in what direction it is traveling. This is measured by the object's velocity.

Average velocity: a vector representing the average rate of change of position with respect to time. The SI unit for velocity is m/s (meters per second).

Because the change in position is the displacement, we can express the average velocity as:

$$\bar{\vec{v}} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\text{net displacement}}{\text{time interval}} .$$ (Equation 2.2: **Average velocity**)

The bar symbol ($\bar{\ }$) above a quantity means the average of that quantity. The direction of the average velocity is the direction of the displacement.

"Velocity" and "speed" are often used interchangeably in everyday speech, but in physics we distinguish between the two. Velocity is a vector, so it has both a magnitude and a direction, while speed is a scalar. Speed is the magnitude of the instantaneous velocity (see the next page). Let's define average speed.

Average Speed = $\bar{v} = \dfrac{\text{total distance covered}}{\text{time interval}}$ (Equation 2.3: **Average speed**)

In Section 2-1, we discussed how the magnitude of the displacement can be different from the total distance traveled. This is why the magnitude of the average velocity can be different from the average speed.

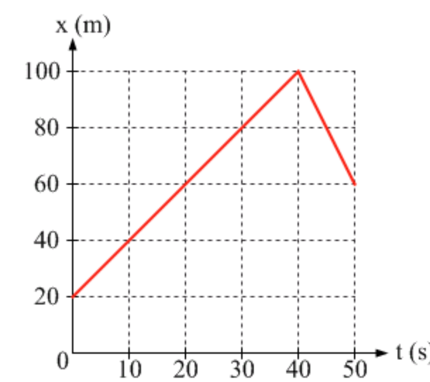


Figure 2.6: A graph of your position versus time over a 50-second period as you move along a sidewalk.

EXAMPLE 2.2A – Average velocity and average speed

Consider Figure 2.6, the graph of position-versus-time we looked at in the previous section. Over the 50-second interval, find: (a) the average velocity, and (b) the average speed.

SOLUTION

(a) Applying Equation 2.2, we find that the average velocity is:

$$\bar{\vec{v}} = \frac{\Delta \vec{x}}{\Delta t} = \frac{+40\,\text{m}}{50\,\text{s}} = +0.80\ \text{m/s} .$$

The net displacement is shown in Figure 2.7. We can also find the net displacement by adding, as vectors, the displacement of +80 meters, in the first 40 seconds, to the displacement of –40 meters, which occurs in the last 10 seconds.

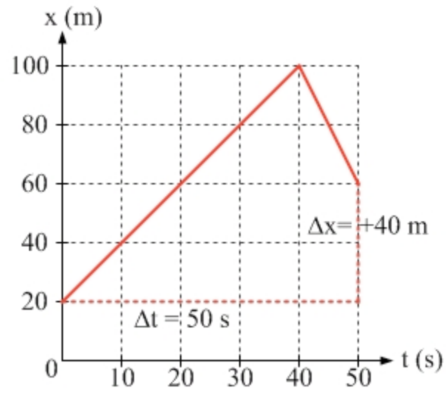


Figure 2.7: The net displacement of +40 m is shown in the graph.

(b) Applying Equation 2.3 to find the average speed,

$$\bar{v} = \frac{\text{total distance covered}}{\text{time interval}} = \frac{80\,\text{m} + 40\,\text{m}}{50\,\text{s}} = \frac{120\,\text{m}}{50\,\text{s}} = 2.4\,\text{m/s}.$$

The average speed and average velocity differ because the motion involves a change of direction. Let's now turn to finding instantaneous values of velocity and speed.

Instantaneous velocity: a vector representing the rate of change of position with respect to time at a particular instant in time. A practical definition is that the instantaneous velocity is the slope of the position-versus-time graph at a particular instant. Expressing this as an equation:

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t}. \qquad \text{(Equation 2.4: \textbf{Instantaneous velocity})}$$

Δt is sufficiently small that the velocity can be considered to be constant over that time interval.

Instantaneous speed: the magnitude of the instantaneous velocity.

EXAMPLE 2.2B – Instantaneous velocity

Once again, consider the motion represented by the graph in Figure 2.6. What is the instantaneous velocity at (a) $t = 25$ s? (b) $t = 45$ s?

SOLUTION

(a) Focus on the slope of the graph, as in Figure 2.8, which represents the velocity. The position-versus-time graph is a straight line for the first 40 seconds, so the slope, and the velocity, is constant over that time interval. Because of this, we can use the entire 40-second interval to find the value of the constant velocity at any instant between $t = 0$ and $t = 40$ s.
Thus, the velocity at t = 25 s is

$$\vec{v}_1 = \frac{\text{rise}}{\text{run}} = \frac{\text{displacement}}{\text{time}} = \frac{+80\text{ m}}{40\text{ s}} = +2.0\text{ m/s}.$$

(b) We use a similar method to find the constant velocity between $t = 40$ s and $t = 50$ s:
At $t = 45$ s, the velocity is

$$\vec{v}_2 = \frac{\text{displacement}}{\text{time}} = \frac{-40\text{ m}}{10\text{ s}} = -4.0\text{ m/s}.$$

Related End-of-Chapter Exercises: 2, 3, 8, 10, and 11

Essential Question 2.2: For the motion represented by the graph in Figure 2.6, is the average velocity over the entire 50-second interval equal to the average of the velocities we found in Example 2.2B for the two different parts of the motion? Explain.

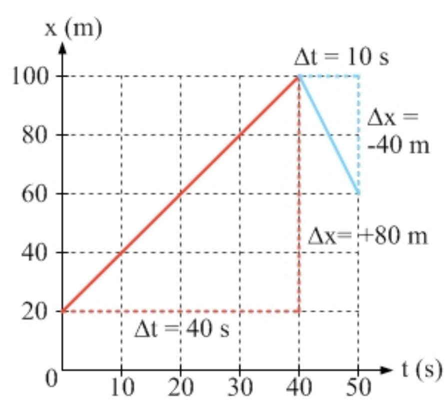


Figure 2.8: The velocity at any instant in time is determined by the slope of the position-versus-time graph at that instant.

Answer to Essential Question 2.2: If we take the average of the two velocities we found in Example 2.2B, $\vec{v}_1 = +2.0\,\text{m/s}$ and $\vec{v}_2 = -4.0\,\text{m/s}$, we get $-1.0\,\text{m/s}$. This is clearly not the average velocity, because we found the average velocity to be +0.80 m/s in Example 2.2A . The reason the average velocity differs from the average of the velocities of the two parts of the motion is that one part of the motion takes place over a longer time interval than the other (4 times longer, in this case). If we wanted to find the average velocity by averaging the velocity of the different parts, we could do a weighted average, weighting the velocity of the first part of the motion four times more heavily because it takes four times as long, as follows:

$$\bar{\vec{v}} = \frac{4\times(+2.0\,\text{m/s}) + 1\times(-4.0\,\text{m/s})}{4+1} = \frac{+4.0\,\text{m/s}}{5} = +0.8\,\text{m/s}\,.$$

2-3 Different Representations of Motion

There are several ways to describe the motion of an object, such as explaining it in words, or using equations to describe the motion mathematically. Different representations give us different perspectives on how an object moves. In this section, we'll focus on two other ways of representing motion, drawing motion diagrams and drawing graphs. We'll do this for motion with constant velocity - motion in a constant direction at a constant speed.

EXPLORATION 2.3A – Learning about motion diagrams

A motion diagram is a diagram in which the position of an object is shown at regular time intervals as the object moves. It's like taking a video and over-laying the frames of the video.

Step 1 - *Sketch a motion diagram for an object that is moving at a constant velocity.* An object with constant velocity travels the same distance in the same direction in each time interval. The motion diagram in Figure 2.9 shows equally spaced images along a straight line. The numbers correspond to times, so this object is moving to the right with a constant velocity.

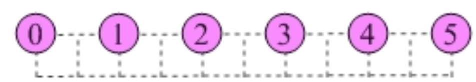


Figure 2.9: Motion diagram for an object that has a constant velocity to the right.

Step 2 - *Draw a second motion diagram next to the first, this time for an object that is moving parallel to the first object but with a larger velocity.* To be consistent, we should record the positions of the two objects at the same times. Because the second object is moving at constant velocity, the various images of the second object on the motion diagram will also be equally spaced. Because the second object is moving faster than the first, however, there will be more space between the images of the second object on the motion diagram – the second object covers a greater distance in the same time interval. The two motion diagrams are shown in Figure 2.10.

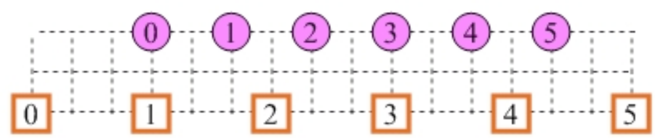


Figure 2.10: Two motion diagrams side by side. These two motion diagrams show objects with a constant velocity to the right but the lower object (marked by the square) has a higher speed, and it passes the one marked by the circles at time-step 3.

Key ideas: A motion diagram can tell us whether or not an object is moving at constant velocity. The farther apart the images, the higher the speed. Comparing two motion diagrams can tell us which object is moving fastest and when one object passes another.

Related End-of-Chapter Exercises: 23 and 24

EXPLORATION 2.3B – Connecting velocity and displacement using graphs
As we have investigated already with position-versus-time graphs, another way to represent motion is to use graphs, which can give us a great deal of information. Let's now explore a velocity-versus-time graph, for the case of a car traveling at a constant velocity of +25 m/s.

Step 1 - ***How far does the car travel in 2.0 seconds***? The car is traveling at a constant speed of 25 m/s, so it travels 25 m every second. In 2.0 seconds the car goes 25 m/s × 2.0 s, which is 50 m.

Step 2 – ***Sketch a velocity-versus-time graph for the motion. What on the velocity-versus-time graph tells us how far the car travels in 2.0 seconds?*** Because the velocity is constant, the velocity-versus-time graph is a horizontal line, as shown in Figure 2.11.

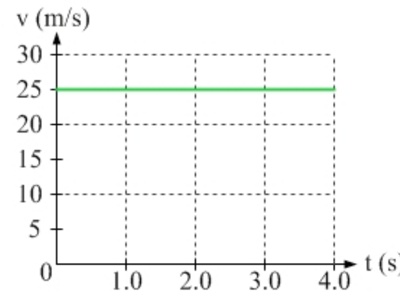


Figure 2.11: The velocity-versus-time graph for a car traveling at a constant velocity of +25 m/s.

To answer the second question, let's re-arrange Equation 2.2, $\bar{\vec{v}} = \frac{\Delta\vec{x}}{\Delta t}$, to solve for the displacement from the average velocity.

$\Delta\vec{x} = \bar{\vec{v}}\,\Delta t$. (Equation 2.5: **Finding displacement from average velocity**)

When the velocity is constant, the average velocity is the value of the constant velocity. This method of finding the displacement can be visualized from the velocity-versus-time graph. The displacement in a particular time interval is the area under the velocity-versus-time graph for that time interval. "The area under a graph" means the area of the region between the line or curve on the graph and the *x*-axis. As shown in Figure 2.12, this area is particularly easy to find in a constant-velocity situation because the region we need to find the area of is rectangular, so we can simply multiply the height of the rectangle (the velocity) by the width of the rectangle (the time interval) to find the area (the displacement).

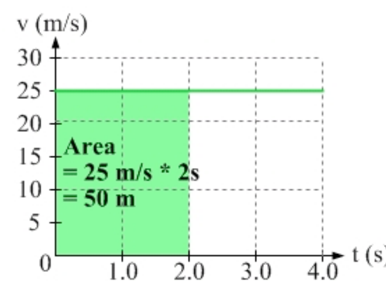


Figure 2.12: The area under the velocity-versus-time graph in a particular time interval equals the displacement in that time interval.

Key idea: The displacement is the area under the velocity-versus-time graph. This is true in general, not just for constant-velocity motion.

Deriving an equation for position when the velocity is constant

Substitute Equation 2.1, $\Delta\vec{x} = \vec{x}_f - \vec{x}_i$, into Equation 2.5, $\Delta\vec{x} = \vec{v}\,\Delta t$.

This gives: $\vec{x}_f - \vec{x}_i = \vec{v}\,\Delta t = \vec{v}\left(t_f - t_i\right)$.

Generally, we define the initial time t_i to be zero: $\vec{x}_f - \vec{x}_i = \vec{v}\,t_f$.

Remove the "f" subscripts to make the equation as general as possible: $\vec{x} - \vec{x}_i = \vec{v}\,t$.

$\vec{x} = \vec{x}_i + \vec{v}\,t$. (Equation 2.6: **Position for constant-velocity motion**)

Such a position-as-a-function-of-time equation is known as an **equation of motion**.

Related End-of-Chapter Exercises: 3, 17, and 48.

Essential Question 2.3: What are some examples of real-life objects experiencing constant-velocity motion? *(The answer is at the top of the next page.)*

Answer to Essential Question 2.3: Some examples of constant velocity (or at least almost-constant velocity) motion include (among many others):

- A car traveling at constant speed without changing direction.
- A hockey puck sliding across ice.
- A space probe that is drifting through interstellar space.

2-4 Constant-Velocity Motion

Let's summarize what we know about constant-velocity motion. We will also explore a special case of constant-velocity motion - that of an object at rest.

EXPLORATION 2.4 – Positive, negative, and zero velocities

Three cars are on a straight road. A blue car is traveling west at a constant speed of 20 m/s; a green car remains at rest as its driver waits for a chance to turn; and a red car has a constant velocity of 10 m/s east. At the time $t = 0$, the blue and green cars are side-by-side at a position 20 m east of the red car. Take east to be positive.

Step 1 – ***Picture the scene: sketch a diagram showing this situation.*** In addition to showing the initial position of the cars, the sketch at the middle left of Figure 2.13 shows the origin and positive direction. The origin was chosen to be the initial position of the red car.

Step 2 - *Sketch a set of motion diagrams for this situation*. The motion diagrams are shown at the top of Figure 2.13, from the perspective of someone in a stationary helicopter looking down on the road from above. Because the blue car's speed is twice as large as the red car's speed, successive images of the blue car are twice as far apart as those of the red car. The cars' positions are shown at 1-second intervals for four seconds.

Step 3 - *Write an equation of motion (an equation giving position as a function of time) for each car.* Writing equations of motion means substituting appropriate values for the initial position $\vec{x}_i$ and the constant velocity $\vec{v}$ into Equation 2.5, $\vec{x} = \vec{x}_i + \vec{v}\,t$. The equations are shown above the graphs in Figure 2.13, using the values from Table 2.1.

	Blue car	**Green car**	**Red car**
Initial position, $\vec{x}_i$	+20 m	+20 m	0
Velocity, $\vec{v}$	-20 m/s	0	+10 m/s

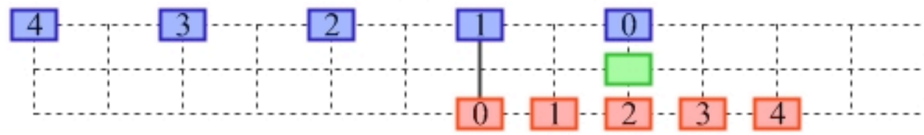
Table 2.1: Organizing the data for the three cars.

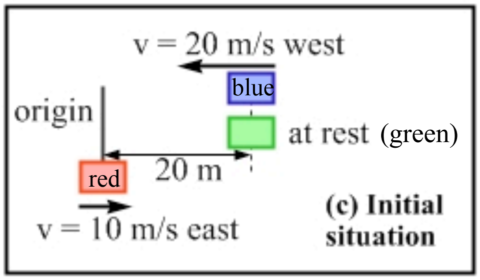
Step 4 - *For each car sketch a graph of its position as a function of time and its velocity as a function of time for 4.0 seconds.* The graphs are shown at the bottom of Figure 2.13. Note that the position-versus-time graph for the green car, which is at rest, is a horizontal line because the car maintains a constant position. An object at rest is a special case of constant-velocity motion: the velocity is both constant and equal to zero.

Key ideas: The at-rest situation is a special case of constant-velocity motion. In addition, all we have learned about constant-velocity motion applies whether the constant velocity is positive, negative, or zero. This includes the fact that an object's displacement is given by $\vec{x} = \vec{x}_i + \vec{v}\,t$; the displacement is the area under the velocity-versus-time graph; and the velocity is the slope of the position-versus-time graph. **Related End-of-Chapter Exercise: 43**

(a) Description of the motion in words: Three cars are on a straight road. A blue car has a constant velocity of 20 m/s west; a green car remains at rest; and a red car has a constant velocity of 10 m/s east. At t = 0 the blue and green cars are side-by-side, 20 m east of the red car.

(b) Motion diagram



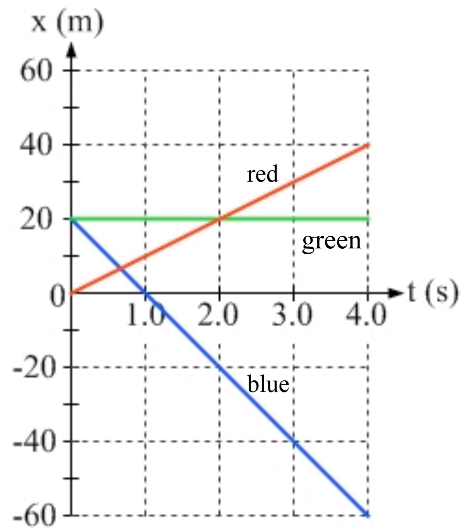


(d) Equations of motion

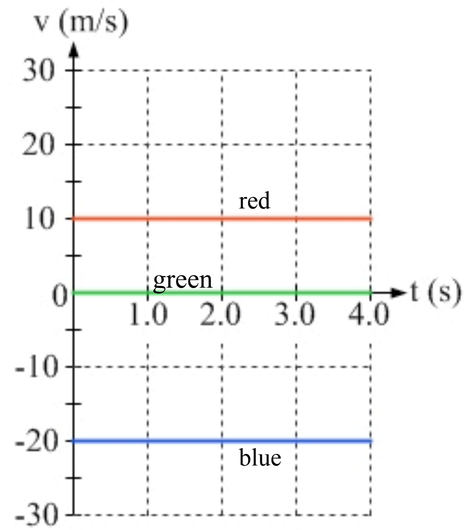
Blue car: $\vec{x}_B = +20\text{ m} - (20\text{ m/s})t$

Green car: $\vec{x}_G = +20\text{ m}$

Red car: $\vec{x}_R = +(10\text{ m/s})t$



(e) Position versus time graphs



(f) Velocity versus time graphs

Figure 2.13: Multiple representations of the constant-velocity motions of three cars. These include (a) a description of the motion in words; (b) a motion diagram; (c) a diagram of the initial situation, at t = 0 (this is shown in a box); (d) equations of motion for each car; (e) graphs of the position of each car as a function of time; and (f) graphs of the velocity of each car as a function of time. Each representation gives us a different perspective on the motion.

Essential Question 2.4: Consider the graph of position-versus-time that is part of Figure 2.13. What is the significance of the points where the different lines cross? *(The answer is at the top of the next page.)*

Answer to Essential Question 2.4: The crossing points tell us the time and location at which one car passes another. For instance, the red car passes the green car at t = 2 s, +20 m from the origin.

2-5 Acceleration

Let's turn now to motion that is not at constant velocity. An example is the motion of an object you release from rest from some distance above the floor.

EXPLORATION 2.5A – Exploring the motion diagram of a dropped object

Step 1 - ***Sketch a motion diagram for a ball that you release from rest from some distance above the floor, showing its position at regular time intervals as it falls.*** The motion diagram in Figure 2.14 shows images of the ball that are close together near the top, where the ball moves more slowly. As the ball speeds up, these images get farther apart because the ball covers progressively larger distances in the equal time intervals. How do we know how much space to include between each image? One thing we can do is to consult experimental evidence, such as strobe photos of dropped objects. These photos show that the displacement from one time interval to the next increases linearly, as in Figure 2.14.

Step 2 - ***At each point on the motion diagram, add an arrow representing the ball's velocity at that point. Neglect air resistance.*** The arrows in Figure 2.14 represent the velocity of the ball at the various times indicated on the motion diagram. Because the displacement increases linearly from one time interval to the next, the velocity also increases linearly with time.

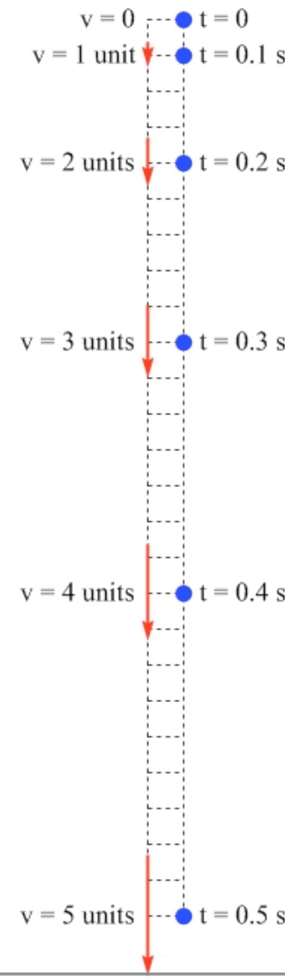


Figure 2.14: A motion diagram, and velocity vectors, for a ball released from rest at t = 0. The ball's position and velocity are shown at 0.1-second intervals.

Key idea: For an object dropped from rest, the velocity changes linearly with time.
Related End-of-Chapter Exercises: 57 - 60

Another way to say that the ball's velocity increases linearly with time is to say that the rate of change of the ball's velocity, with respect to time, is constant:

$$\frac{\Delta \vec{v}}{\Delta t} = \text{constant} .$$

This quantity, the rate of change of velocity with respect to time, is referred to as the **acceleration**. Acceleration is related to velocity in the same way velocity is related to position.

Average acceleration: a vector representing the average rate of change of velocity with respect to time. The SI unit for acceleration is m/s^2.

$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\text{change in velocity}}{\text{time interval}} . \qquad \text{(Equation 2.7: \textbf{Average acceleration})}$$

The direction of the average acceleration is the direction of the change in velocity.

Instantaneous acceleration: a vector representing the rate of change of velocity with respect to time at a particular instant in time. The SI unit for acceleration is m/s^2.

A practical definition of instantaneous acceleration at a particular instant is that it is the slope of the velocity-versus-time graph at that instant. Expressing this as an equation:

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} . \qquad \text{(Equation 2.8: \textbf{Instantaneous acceleration})}$$

Δt is small enough that the acceleration can be considered to be constant over that time interval.

Note that, on Earth, objects dropped from rest are observed to have accelerations of 9.8 m/s^2 straight down. This is known as $\vec{g}$, the acceleration due to gravity.

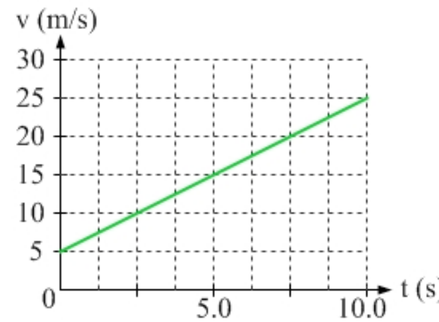


Figure 2.15: A graph of the velocity of a bus as a function of time.

EXPLORATION 2.5B – Graphs in a constant-acceleration situation

The graph in Figure 2.15 shows the velocity, as a function of time, of a bus moving in the positive direction along a straight road.

Step 1 – ***What is the acceleration of the bus? Sketch a graph of the acceleration as a function of time.*** The graph in Figure 2.15 is a straight line with a constant slope. This tells us that the acceleration is constant because the acceleration is the slope of the velocity-versus-time graph. Using the entire 10-second interval, applying equation 2.8 gives:

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{+25\text{ m/s} - (+5\text{ m/s})}{10\text{ s}} = \frac{+20\text{ m/s}}{10\text{ s}} = +2.0\text{ m/s}^2 .$$

The acceleration graph in Figure 2.16 is a horizontal line, because the acceleration is constant. Compare Figures 2.15 and 2.16 to the graphs for the red car in Figure 2.13. Note the similarity between the acceleration and velocity graphs in a constant-acceleration situation and the velocity and position graphs in a constant-velocity situation.

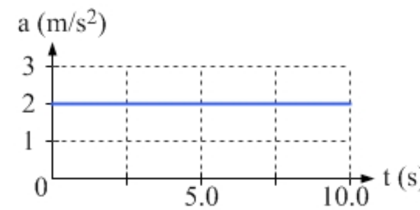


Figure 2.16: A graph of the acceleration of the bus as a function of time.

Step 2 - ***What on the acceleration-versus-time graph is connected to the velocity?*** The connection between velocity and acceleration is similar to that between position and velocity - the area under the curve of the acceleration graph is equal to the change in velocity, as shown in Figure 2.17. This follows from Equation 2.8, which in a constant acceleration situation can be written as $\Delta \vec{v} = \vec{a}\,\Delta t$.

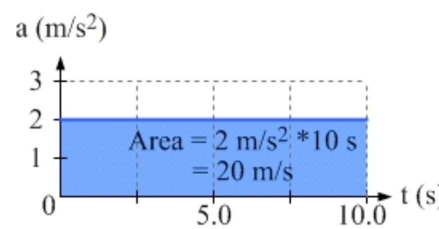


Figure 2.17: In 10 seconds, the velocity changes by +20 m/s. This is the area under the acceleration-versus-time graph over that interval.

Key ideas: The acceleration is the slope of the velocity-versus-time graph, while the area under the acceleration-versus-time graph for a particular time interval represents the change in velocity during that time interval.

Related End-of-Chapter Exercises: 16 and 32.

Essential Question 2.5: Consider the bus in Exploration 2.5B. Would the graph of the bus' position as a function of time be a straight line? Why or why not?

Answer to Essential Question 2.5: The position-versus-time graph is not a straight line, because the slope of such a graph is the velocity. The fact that the bus' velocity increases linearly with time means the slope of the position-versus-time graph also increases linearly with time. This actually describes a parabola, which we will investigate further in the next section.

2-6 Equations for Motion with Constant Acceleration

In many situations, we will analyze motion using a model in which we assume the acceleration to be constant. Let's derive some equations that we can apply in such situations. In general, at some initial time $t_i = 0$, the object has an initial position $\vec{x}_i$ and an initial velocity of $\vec{v}_i$, while at some (usually later) time *t,* the object's position is $\vec{x}$ and its velocity is $\vec{v}$.

Acceleration is related to velocity the same way velocity is related to position, so we can follow a procedure similar to that at the end of Section 2-3 to derive an equation for velocity.

Substitute $\vec{v}_f - \vec{v}_i$ for $\Delta\vec{v}$ in the rearrangement of Equation 2.8, $\Delta\vec{v} = \vec{a}\,\Delta t$.

This gives: $\vec{v}_f - \vec{v}_i = \vec{a}\,\Delta t = \vec{a}\left(t_f - t_i\right)$.

Generally, we define the initial time t_i to be zero: $\vec{v}_f - \vec{v}_i = \vec{a}\,t_f$.

Remove the "f" subscripts to make the equation as general as possible: $\vec{v} - \vec{v}_i = \vec{a}\,t$. We also generally remove the vector symbols, although we must be careful to include signs.

$v = v_i + a\,t$. (Equation 2.9: **Velocity for constant-acceleration motion**)

A second equation comes from the definition of average velocity (Equation 2.2):

Average velocity = $\bar{\vec{v}} = \dfrac{\Delta\vec{x}}{\Delta t} = \dfrac{\vec{x} - \vec{x}_i}{t - t_i} = \dfrac{\vec{x} - \vec{x}_i}{t}$.

If the acceleration is constant the average velocity is simply the average of the initial and final velocities. This gives, after again dropping the vector symbols:

$\dfrac{v_i + v}{2} = \dfrac{x - x_i}{t}$. (Equation 2.10: **Connecting average velocity and displacement**)

Equation 2.10 is sometimes awkward to work with. If we substitute $v_i + at$ in for v (see Equation 2.9) in Equation 2.10, re-arranging produces an equation describing a parabola:

$x = x_i + v_i t + \dfrac{1}{2}at^2$. (Equation 2.11: **Position for constant-acceleration motion**)

We can derive another useful equation by combining equations 2.9 and 2.10 in a different way. Solving equation 2.9 for time, to get $t = \dfrac{v - v_i}{a}$, and substituting the right-hand side of that expression in for *t* in equation 2.10, gives, after some re-arrangement:

$v^2 = v_i^2 + 2a\,\Delta x$. (Eq. 2.12: **Connecting velocity, acceleration, and displacement**)

An important note about positive and negative signs.

When we make use of the equations on the previous page, we must be careful to include the appropriate positive or negative signs that are built into each of the variables. The first step is to choose a positive direction. If the initial velocity is in that direction, it goes into the equations with a positive sign. If the initial velocity is in the opposite direction (the negative direction), it goes into the equations with a negative sign. Apply a similar rule for the final velocity, the acceleration, the displacement, the initial position, and the final position. For all of those quantities, the sign is associated with the direction of the corresponding vector.

Motion with constant acceleration is an important concept. Let's summarize a general, systematic approach we can apply to situations involving motion with constant acceleration.

A General Method for Solving a One-Dimensional Constant-Acceleration Problem

1. **Picture the scene.** Draw a diagram of the situation. Choose an origin to measure positions from, and a positive direction, and show these on the diagram.
2. **Organize what you know, and what you're looking for.** Making a table of data can be helpful. Record values of the variables used in the equations below.
3. **Solve the problem.** Think about which of the constant-acceleration equations to apply, and then set up and solve the problem. The three main equations are:

$v = v_i + at$. (Equation 2.9: **Velocity for constant-acceleration motion**)

$x = x_i + v_i t + \frac{1}{2}at^2$. (Equation 2.11: **Position for constant-acceleration motion**)

$v^2 = v_i^2 + 2a\,\Delta x$. (Equation 2.12: **Connecting velocity, acceleration, and displacement**)

4. **Think about the answer(s)**. Check your answers to see if they make sense.

EXAMPLE 2.6 – Working with variables

In physics, being able to work with variables as well as numbers is an important skill. This can also produce insights that working with numbers does not. Let's say an object is dropped from rest from the top of a building of height *H*, while another object is dropped from rest from the top of a building of height 4*H*. Assuming both objects fall under the influence of gravity alone (that is, they have the same acceleration), compare the times it takes them to reach the ground.

SOLUTION

$v_i = 0$, because the objects are dropped from rest. Take the initial position to be the top of the building in each case, so $x_i = 0$. This reduces Equation 2.11 to $x = at^2/2$. Because the acceleration is the same in each case, the equation tells us the position is proportional to the square of the time. To quadruple the final position, as we are doing, we need to increase the time by a factor of two. The fall from the building that is four times as high takes twice as long. This is illustrated by the motion diagram in Figure 2.18.

Related End-of-Chapter Exercises: 52 and 55

Essential Question 2.6: Return to the situation described in Example 2.6. Compare the velocities of the objects just before they hit the ground.

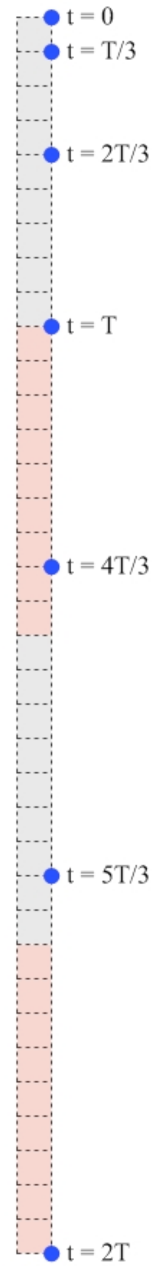


Figure 2.18: Falling from rest for double the time quadruples the distance traveled. The height of the diagram is 4*H*, four times the height of the smaller building in Example 2.6.

Answer to Essential Question 2.6: Using $v_i = 0$ reduces Equation 2.9 to $v = at$. Doubling the time doubles the final velocity, so the object dropped from the building that is four times higher has a final velocity twice as large as that of the other object. Equation 2.12 gives the same result.

2-7 Example Problem

Let's look at the various representations of motion with constant acceleration, considering the example of a ball tossed straight up in the air.

EXPLORATION 2.7 – A ball tossed straight up

You toss a ball straight up into the air. The ball takes 2.0 s to reach its maximum height, and an additional 2.0 s to return to your hand. You catch the ball at the same height from which you let it go, and the ball has a constant acceleration because it is acted on only by gravity. Consider the motion from the instant just after you release the ball until just before you catch it.

Step 1 – *Picture the scene – draw a diagram of the situation.* The diagram in Figure 2.19 shows the initial conditions, the origin, and the positive direction. We are free to choose either up or down as the positive direction, and to choose any reference point as the origin. In this case let's choose the origin to be the point from which the ball was released, and choose up to be positive.

Step 2 – *Organize the data.* Table 2.2 summarizes what we know, including values for the acceleration and the initial position. We need these values for the constant-acceleration equations. Because the ball moves under the influence of gravity alone, and we can assume the ball is on the Earth, the acceleration is the acceleration due to gravity, 9.8 m/s^2 directed down. Because down is the negative direction, we include a negative sign: $a = -9.8\,\text{m/s}^2$.

Parameter	Value
Origin	Launch point
Positive direction	up
Initial position	$x_i = 0$
Initial velocity	$v_i = +\_\_\_\_\_\ \text{m/s}$
Acceleration	$a = -9.8\ \text{m/s}^2$
Position at t = 4.0 s	$x_{t=4\,\text{s}} = x_i = 0$

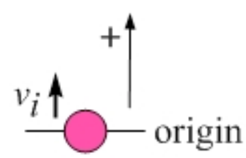


Figure 2.19: A diagram of the initial situation.

Table 2.2: Summarizing the information that was given about the ball.

Step 3 – *Solve the problem.* In this case, we want to draw graphs of the acceleration, velocity, and position of the ball, all as a function of time. To do this we should first write equations for the acceleration, velocity, and position. The acceleration is constant at $a = -9.8\,\text{m/s}^2$. Knowing the acceleration allows us to solve for the initial velocity. One way to do this is to re-arrange Equation 2.9 to give $v_i = v - at$. Because $v = 0$ at t = 2.0 s (the ball is at rest for an instant when it reaches its maximum height) we get $v_i = v - at = 0 - (-9.8\ \text{m/s}^2)(2.0\ \text{s}) = +19.6\ \text{m/s}$.

Knowing the initial velocity enables us to write equations for the ball's velocity (using Equation 2.9) and position (using Equation 2.11) as a function of time. The equations, and corresponding graphs, are part of the multiple representations of the motion shown in Figure 2.20. Note that the position versus time graph is parabolic, but the motion is confined to a line.

Step 4 - *Sketch a motion diagram for the ball.* The motion diagram is shown on the right in Figure 2.15. Note the symmetry of the up and down motions (this is also apparent from the graphs). The motion of the ball on the way down is a mirror image of its motion on the way up.

(a) Description of the motion: A ball you toss straight up into the air reaches its maximum height 2.0 s after being released, taking an addition 2.0 s to return to your hand. It experiences a constant acceleration from the moment you release it until just before you catch it.

(b) Equations (up is positive): Acceleration-versus-time: $a = -9.8\text{ m/s}^2$

Velocity-versus-time: $v = +19.6\text{ m/s} - (9.8\text{ m/s}^2)t$

Position-versus-time: $x = +\left(19.6\text{ m/s}\right)t - \left(4.9\text{ m/s}^2\right)t^2$

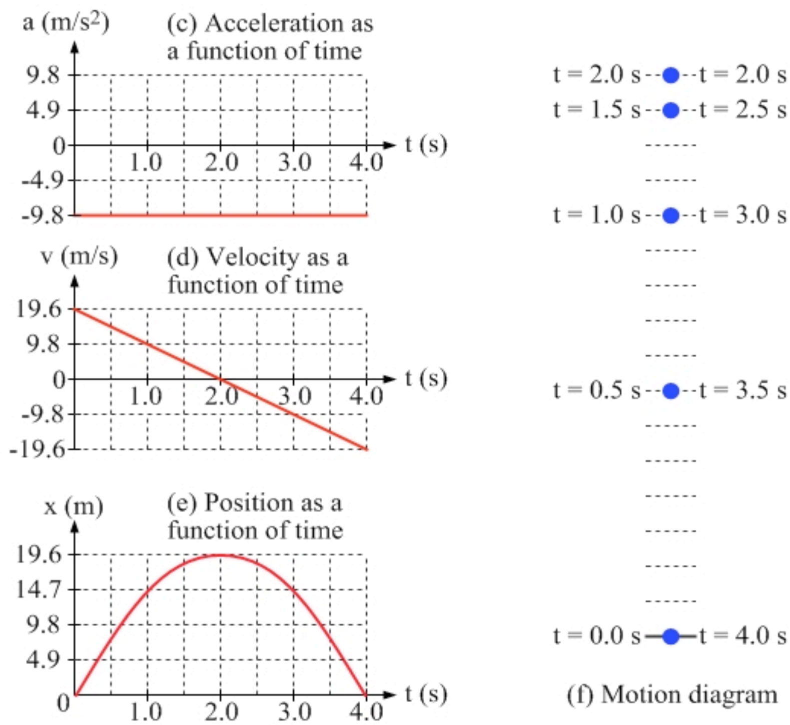


Figure 2.20: Multiple representations of a ball thrown straight up, including (a) a description in words; (b) equations for the ball's acceleration, velocity, and position; graphs giving the ball's (c) acceleration, (d) velocity, and (e) position as a function of time; and (f) a motion diagram. These different perspectives show how various aspects of the motion evolve with time.

Key ideas: At the Earth's surface the acceleration due to gravity has a constant value of $g = 9.8\text{ m/s}^2$ directed down. We can thus apply constant-acceleration methods to situations involving objects dropped or thrown into the air. For an object that is thrown straight up the downward part of the trip is a mirror image of the upward part of the trip.
Related End-of-Chapter Exercises: 33, 61, and 62.

Essential Question 2.7: Consider again the ball in Exploration 2.7. The ball comes to rest for an instant at its maximum-height point. What is the ball's acceleration at that point?

Answer to Essential Question 2.7: A common misconception is that the ball's acceleration is zero at the maximum-height point. In fact, the acceleration is $\vec{g}$, 9.8 m/s^2 down, during the entire trip.

This is what is shown on the acceleration graph, for one thing – the graph confirms that nothing special happens to the acceleration at t = 2.0 s, even though the ball is momentarily at rest. One reason for this is that the ball is under the influence of gravity the entire time.

2-8 Solving Constant-Acceleration Problems

Consider one more example of applying the general method for solving a constant-acceleration problem.

EXAMPLE 2.8 – Combining constant-acceleration motion and constant-velocity motion

A car and a bus are traveling along the same straight road in neighboring lanes. The car has a constant velocity of +25.0 m/s, and at t = 0 it is located 21 meters ahead of the bus. At time t = 0 , the bus has a velocity of +5.0 m/s and an acceleration of +2.0 m/s^2.

When does the bus pass the car?

SOLUTION

1. **Picture the scene – draw a diagram.** The diagram in Figure 2.21 shows the initial situation, the positive direction, and the origin. Let's choose the positive direction to be the direction of travel, and the origin to be the initial position of the bus.

2. **Organize the data.** Data for the car and the bus is organized separately in Table 2.3, using subscripts C and B to represent the car and bus, respectively.

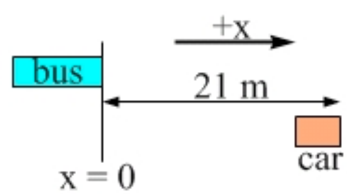


Figure 2.21: A diagram showing the initial positions of the car and the bus, the position of the origin, and the positive direction.

	Car	**Bus**
Initial position	$x_{iC} = +21\,\text{m}$	$x_{iB} = 0$
Initial velocity	$v_{iC} = +25\,\text{m/s}$	$v_{iB} = +5.0\,\text{m/s}$
Acceleration	$a_C = 0$	$a_B = +2.0\,\text{m/s}^2$

Table 2.3: Summarizing the information that was given about the car and the bus.

3. **Solve the problem.** Let's use Equation 2.8 to write expressions for the position of each vehicle as a function of time. Because we summarized all the data in Table 2.3 we can easily find the values of the variables in the equations.

For the car: $x_C = x_{iC} + v_{iC}\,t + \frac{1}{2}a_C\,t^2 = +21\,\text{m} + (25\,\text{m/s})t$.

For the bus: $x_B = x_{iB} + v_{iB}\,t + \frac{1}{2}a_B\,t^2 = 0 + (5.0\,\text{m/s})t + (1.0\,\text{m/s}^2)t^2$.

The bus passes the car when the vehicles have the same position. At what time does $x_C = x_B$? Set the two equations equal to one another and solve for this time (let's call it t_1).

$$+21\,\text{m} + (25\,\text{m/s})t_1 = (5.0\,\text{m/s})t_1 + (1.0\,\text{m/s}^2)t_1^2$$

Bringing everything to the left side gives: $(1.0\,\text{m/s}^2)t_1^2 - (20\,\text{m/s})t_1 - 21\,\text{m} = 0$.

We can solve this with the quadratic equation, where $a = 1.0 \text{ m/s}^2$, $b = -20 \text{ m/s}$, and $c = -21 \text{ m}$.

$$t_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+(20 \text{ m/s}) \pm \sqrt{(400 \text{ m}^2/\text{s}^2) + (84 \text{ m}^2/\text{s}^2)}}{2.0 \text{ m/s}^2} = \frac{+(20 \text{ m/s}) \pm (22 \text{ m/s})}{2.0 \text{ m/s}^2}.$$

The equation gives us two solutions for t_1: Either $t_1 = +21$ s or $t_1 = -1.0$ s. Clearly the answer we want is +21 s. The other number does have a physical significance, however, so let's try to make some sense of it. The negative answer represents the time at which the car would have passed the bus if the motion conditions after $t = 0$ also applied to the period before $t = 0$.

4. **Think about the answer.** A nice way to check the answer is to plug $t_1 = +21$ s into the two position-versus-time equations from the previous step. If the time is correct the position of the car should equal the position of the bus. Both equations give positions of 546 m from the origin, giving us confidence that $t_1 = +21$ s is correct.

Note: This example is continued on the accompanying web site, solving for the time at which the car and the bus have the same velocity.

Related End-of-Chapter Exercises: 54 and 56.

Chapter Summary

Essential Idea: Describing motion in one dimension.

The motion of many objects (such as you, cars, and objects that are dropped) can be approximated very well using a constant-velocity or a constant-acceleration model.

Parameters used to Describe Motion

Displacement is a vector representing a change in position. If the initial position is $\vec{x}_i$ and the final position is $\vec{x}_f$ we can express the displacement as:

$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$. (Equation 2.1: **Displacement in one dimension**)

Average Velocity: a vector representing the average rate of change of position with respect to time. The MKS unit for velocity is m/s (meters per second).

$\overline{\vec{v}} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\text{net displacement}}{\text{time interval}}$. (Equation 2.2: **Average velocity**)

While velocity is a vector, and thus has a direction, speed is a scalar.

Average Speed = $\overline{v} = \frac{\text{total distance covered}}{\text{time interval}}$ (Equation 2.3: **Average speed**)

Instantaneous Velocity: a vector representing the rate of change of position with respect to time at a particular instant in time. The MKS unit for velocity is m/s (meters per second).

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t} \text{,} \qquad \text{(Equation 2.4: \textbf{Instantaneous velocity})}$$

where Δt is small enough that the velocity can be considered to be constant over that interval.

Instantaneous Speed: the magnitude of the instantaneous velocity.

Average Acceleration: a vector representing the average rate of change of velocity with respect to time. The MKS unit for acceleration is m/s^2.

$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\text{change in velocity}}{\text{time interval}} \text{.} \qquad \text{(Equation 2.7: \textbf{Average acceleration})}$$

Instantaneous Acceleration: a vector representing the rate of change of velocity with respect to time at a particular instant in time.

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \text{,} \qquad \text{(Equation 2.8: \textbf{Instantaneous acceleration})}$$

where Δt is small enough that the acceleration can be considered constant over that interval.

The ***acceleration due to gravity,*** $\vec{g}$, is 9.8 m/s^2, directed down, at the surface of the Earth.

A General Method for Solving a 1-Dimensional Constant-Acceleration Problem

1. **Picture the scene.** Draw a diagram of the situation. Choose an origin to measure positions from, and a positive direction, and show these on the diagram.
2. **Organize what you know, and what you're looking for.** Making a table of data can be helpful. Record values of the variables used in the equations below.
3. **Solve the problem.** Think about which of the constant-acceleration equations to apply, and then set up and solve the problem. The three main equations are:

$$v = v_i + at \text{.} \qquad \text{(Equation 2.9: \textbf{Velocity for constant-acceleration motion})}$$

$$x = x_i + v_i t + \frac{1}{2}at^2 \text{.} \qquad \text{(Equation 2.11: \textbf{Position for constant-acceleration motion})}$$

$$v^2 = v_i^2 + 2a\,\Delta x \text{.} \qquad \text{(Equation 2.12: \textbf{Connecting velocity, acceleration, and displacement})}$$

4. **Think about the answer(s)**. Check your answers, and/or see if they make sense.

Graphs

The velocity is the slope of the position-versus-time graph; the displacement is the area under the velocity-versus-time graph. The acceleration is the slope of the velocity-versus-time graph; the change in velocity is the area under the acceleration-versus-time graph.

Constant Velocity and Constant Acceleration

The motion of an object at rest is a special case of constant-velocity motion. In constant-velocity motion the position-versus-time graph is a straight line with a slope equal to the velocity.

Constant-velocity motion is a special case of constant-acceleration motion. In one dimension, if the acceleration is constant and non-zero the position-versus-time graph is quadratic while the velocity-versus-time graph is a straight line with a slope equal to the acceleration.

End-of-Chapter Exercises

Exercises 1 – 12 are conceptual questions that are designed to see if you have understood the main concepts of the chapter.

1. Refer to the position-versus-time graph in Figure 2.22 for your motion along a straight sidewalk. Consider the following four time intervals:
   Interval 1: between t = 0 and t = 10 s
   Interval 2: between t = 5 s and t = 10 s
   Interval 3: between t = 10 s and t = 15 s
   Interval 4: between t = 15 s and t = 35 s.
   Rank these intervals, from largest to smallest, based on the (a) distance traveled during the interval; (b) magnitude of the displacement during the interval. Express your rankings in a form like 2>1=3>4.

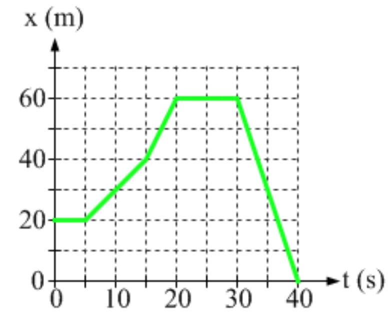


Figure 2.22: A position-versus-time graph, for Exercises 1 – 3.

2. Refer again to the position-versus-time graph in Figure 2.22 for your motion along a straight sidewalk. Consider the following four time intervals:
   Interval 1: between t = 0 and t = 10 s
   Interval 2: between t = 5 s and t = 10 s
   Interval 3: between t = 10 s and t = 15 s
   Interval 4: between t = 15 s and t = 35 s.
   Rank these intervals, from largest to smallest, based on the: (a) average speed over the interval; (b) magnitude of the average velocity over the interval. Express your rankings in a form like 2>1=3>4.

3. Compare the graph in Figure 2.22, showing your position-versus-time as you move along a straight sidewalk, to the graph in Figure 2.23, showing your velocity-versus-time for a different motion along the same straight sidewalk. (a) In a few sentences, make up a story to accompany the motion depicted in the position-versus-time graph. (b) In a few sentences, make up a story to accompany the motion depicted in the velocity-versus-time graph.

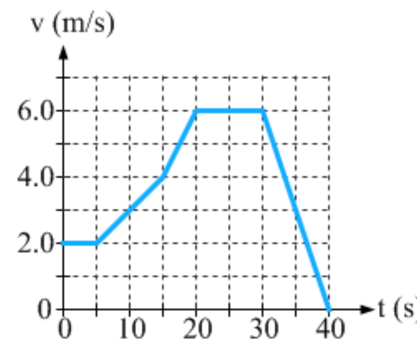


Figure 2.23: A velocity-versus-time graph, for Exercises 3 – 4.

4. Three of your friends look at the velocity-versus-time graph in Figure 2.23, depicting your motion along a straight sidewalk. They make the following comments:
   Andy: "The graph shows that you were at rest between 0 and 5 seconds, and at rest again between 20 and 30 seconds."
   Jennifer: "The graph shows that you were moving in one direction the entire time, until the end when you come to rest."
   Sue: "The graph shows that you were traveling one way for a while, and then later you were traveling in the opposite direction."

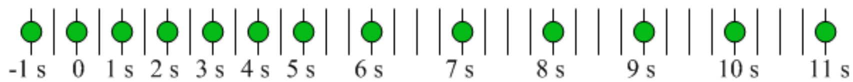
   What do you agree or disagree with about each of these comments?

5. Consider the motion diagram in Figure 2.24, showing the position of an object at regular time intervals as it moves in one dimension. Is this a complete motion diagram, or is there anything missing that would help us to better determine what the object's motion is like?

Figure 2.24: Motion diagram, for Exercise 5.

6. Consider the motion diagram in Figure 2.25, showing the position of an object at 1-second time intervals as it moves in one dimension. (a) Over the time interval from $t = 0$ to $t = 10$ s, describe the object's motion. (b) When does the object experience a non-zero acceleration? (c) Describe a real-life situation that could match this motion diagram.

Figure 2.25: Motion diagram, for Exercise 6.



7. Consider the positions in Figure 2.26. What is the total distance traveled in moving from (a) $\vec{x}_0$ directly to $\vec{x}_1$? (b) $\vec{x}_1$ directly to $\vec{x}_0$? (c) $\vec{x}_0$ to $\vec{x}_2$ and then to $\vec{x}_1$? What is the displacement in moving from
(d) $\vec{x}_0$ to $\vec{x}_1$? (e) $\vec{x}_1$ to $\vec{x}_0$?
(f) $\vec{x}_0$ to $\vec{x}_2$ and then to $\vec{x}_1$?

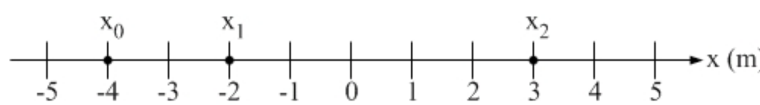


Figure 2.26: Three different positions ($\vec{x}_0$, $\vec{x}_1$, and $\vec{x}_2$) along an *x*-axis, for Exercise 7.

8. Describe a situation that matches each of the following, or state that it is impossible. (a) A person is traveling vertically down but with an upward acceleration. (b) A car has a velocity directed east and an acceleration directed east. (c) A baseball is at rest but has a non-zero acceleration.

9. Describe a situation that matches each of the following, or state that it is impossible. (a) An object has no acceleration and yet it is moving. (b) An object has a non-zero acceleration but it remains at rest.

10. Come up with an example that matches the following description of a motion, or state that it is impossible, assuming that the motion is in 1 dimension. (a) An object's average velocity is zero, but it did cover some distance over the time interval in question. (b) An object's velocity is positive for more than half the time, but its net displacement is negative. (c) An object's instantaneous velocity is, for some fraction of a particular time interval, equal to four times its average velocity over that time interval.

11. Consider the motion diagrams in Figure 2.27, showing the positions of two objects at 1-second intervals starting at $t = 0$ as both objects move to the right. Assume that whenever either object accelerates, its acceleration is constant and it accelerates for exactly 1 second. During the time interval depicted, which object has (a) the largest instantaneous speed? (b) the smallest instantaneous speed? (c) the average velocity with the largest magnitude over the interval $t = 0$ to $t = 9$ s?

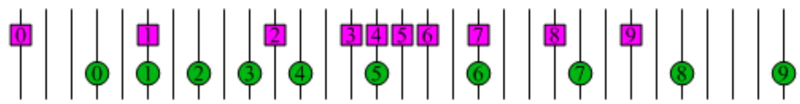


Figure 2.27: A pair of motion diagrams for Exercises 11 – 12. The numbers shown correspond to the time in seconds (e.g., the circle with the 4 in it shows the location of the bottom object at $t = 4$ s).

12. Consider the motion diagrams in Figure 2.27, showing the positions of two different objects at 1-second intervals starting at $t = 0$ as both objects move to the right. You can assume that whenever either object accelerates its acceleration is constant and it accelerates for exactly 1 second. Note that your answers to this exercise should be of the form "At $t = 4$ s", or "At some time during the 1-s interval between $t = 3$ s and $t = 4$ s." During the time interval depicted are there any times when the objects have
(a) the same horizontal position? If so, state when this occurs.
(b) the same velocity? If so, state when this occurs.
(c) the same acceleration? If so, state when this occurs.

Exercises 13 – 18 deal with calculating averages.

13. In 1973, the horse Secretariat set the record for the 1.25-mile Kentucky Derby with a time of 1:59 2/5 (one minute, 59.4 seconds). What was Secretariat's average speed in m/s?

14. The gold medalists in four events at the 2004 Athens Olympics were as follows: In the women's cycling road race, consisting of 9 laps around a 13.2 km/lap course, Sara Carrigan of Australia won in a time of 3:24:24 (3 h, 24 min, 24 s); in the 50-meter freestyle swim, Gary Hall of the USA swam one length of the pool in 21.93 s; in men's single sculls rowing, Olaf Tufte of Norway rowed the 2000 m course in 6:49.30 (6 min, 49.30 s); and in the women's 100-meter race on the track Yuliya Nesterenko of Belarus won in a time of 10.93 s. (a) Rank these athletes based on their average speed over their respective races, from largest to smallest. (b) Rank them instead based on the magnitude of their average velocity.

15. In the women's 200-meter backstroke event in swimming at the 2004 Athens Olympics, Kirsty Coventry of Zimbabwe swam the four lengths of the pool in a time of 2:09.19 (2 min., 9.19 s). At the instant Coventry touched the wall to win the race, the eighth-place swimmer in the race, Aya Terakawa of Japan, still had a few meters left to swim. (a) Which of these two swimmers had the largest average speed over the 2:09.19 time interval from the start of the race to when Kirsty Coventry touched the wall? (b) Over the same time interval, which swimmer had an average velocity with a larger magnitude? Briefly justify your answers.

16. The following times are given for the Porsche 911 Turbo Cabriolet to achieve a particular speed when accelerating from rest: 0 to 60 mph (miles per hour) in 3.8 s; 0 to 100 mph in 9.2 s; and 0 to 130 mph in 16.0 s. (a) In each of the three cases, what is the magnitude of the car's average acceleration? (b) Does the Porsche exhibit constant acceleration, or not? Briefly comment.

17. You are competing in a duathlon, an event that involves running and cycling. This duathlon involves running once around a particular loop, cycling twice around the same loop, and then finishing the race by again running once around the loop. If your average speed when running is 4.0 m/s and your average speed when cycling is 6.0 m/s, what was your average speed for the race? Assume that the time spent during the run-bike and bike-run transitions is negligible.

18. You take a trip, covering a total distance of 20 km. For the first 10 km you travel on horseback at an average speed of 20 km/h. You then switch to a different mode of transportation. What speed should you average over the second 10 km of the trip if you want your average speed for the entire trip to be (a) 10 km/h? (b) 30 km/h? (c) 40 km/h?

Exercises 19 - 26 deal with interpreting graphs.

19. The graph in Figure 2.28 shows your motion as you move along a straight sidewalk. Over the 40-second interval what is (a) your net displacement? (b) the total distance traveled?

20. The graph in Figure 2.28 shows your motion as you move along a straight sidewalk. Over the 40-second interval what is (a) your average velocity? (b) your average speed?

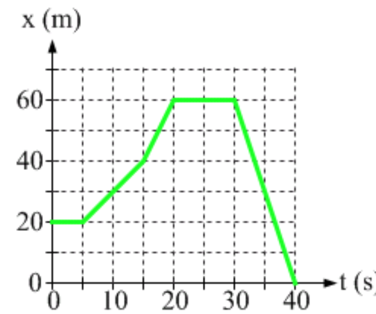


Figure 2.28: A position-versus-time graph, for Exercises 19 – 21.

21. Refer again to the position-versus-time graph in Figure 2.28 for your motion along a straight sidewalk. (a) Sketch the corresponding velocity-versus-time graph for the motion. (b) What is your instantaneous velocity at t = 25 s? (c) What is your average velocity over the interval between t = 0 and t = 25 s? (d) What is your instantaneous velocity at t = 35 s? (e) What is your average velocity over the interval between t = 0 and t = 35 s?

22. If all you were given was the graph of velocity-versus-time in Figure 2.29, and you knew the motion depicted was for your motion along a straight sidewalk, could you answer the following questions? Simply write "Yes" if you can answer a particular question, and if you cannot explain why not. (a) What is your velocity at t = 25 s? (b) What is your acceleration at t = 25 s? (c) What is your position at t = 25 s? (d) What is your displacement between t = 0 and t = 25 s? (e) Are you walking east or west? (f) Are you walking forward or backward?

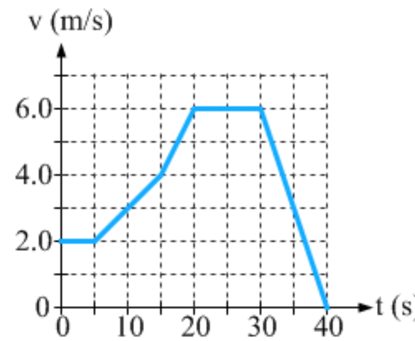


Figure 2.29: A velocity-versus-time graph, for Exercises 22 – 26.

23. Refer again to the velocity-versus-time graph in Figure 2.29 for your motion along a straight sidewalk. If your position at t =0 is +20 m from some convenient origin, determine your position, velocity, and acceleration at the following times: (a) t = 10 s, (b) t = 25 s, (c) t = 35 s.

24. Refer again to the velocity-versus-time graph in Figure 2.29 for your motion along a straight sidewalk. If your position at t = 0 is +20 m from some convenient origin, sketch corresponding graphs of your position as a function of time as well as your acceleration as a function of time.

25. Refer again to the velocity-versus-time graph in Figure 2.29 for your motion along a straight sidewalk. Over the 40-second interval, what is your (a) total distance traveled? (b) net displacement? (c) average speed? (d) average velocity?

26. Refer again to the velocity-versus-time graph in Figure 2.29 for your motion along a straight sidewalk. Sketch the corresponding motion diagram showing your position, which you can represent by circles or X's, at 5-second intervals starting from t = 0.

Exercises 27 – 34 deal with understanding and interpreting motion diagrams.

Figure 2.30: Motion diagram for Exercises 27 – 29.

27. Consider the motion diagram in Figure 2.30, showing the position of an object at regular time intervals as it moves in one dimension. Assume that the object's acceleration is constant throughout the time interval covered by the motion diagram and that the object does not reverse direction during the motion. (a) In what direction is the object moving? (b) In what direction is the acceleration? (c) In one or two sentences, describe a real-life situation that could match this motion diagram.

28. Consider the motion diagram in Figure 2.30, showing the position of an object at regular time intervals as it moves in one dimension. The object starts from rest from the left-most point and then has a constant acceleration directed right for the entire time interval covered by the motion diagram (and continues to have this acceleration afterwards). If the vertical lines in the picture are 0.20 m apart and the time it takes the object to go from the initial position to the final position is 6.0 s, determine (a) the object's speed as it passes through the last position shown in the diagram, and (b) its acceleration.

29. Consider the motion diagram in Figure 2.30, showing the position of an object at regular time intervals as it moves in one dimension. Let's say that the object is moving from right to left, and experiencing a constant acceleration that brings it instantaneously to rest at the left-most point. The successive positions of the object are shown at 0.20 s intervals, starting from the right-most point at $t = 0$. The vertical lines in the picture are 0.20 m apart. (a) Make a copy of the diagram, labeling the positions shown on the diagram with the times. (b) Assuming the object still experiences the same constant acceleration after $t = 1.0$ s, add to the diagram the positions of the object at 0.20 s intervals between $t = 1.0$ s and $t = 2.0$ s. (c) What is the object's acceleration? (d) What is the object's velocity at $t = 0$? (e) What is the object's average velocity over the interval from $t = 0$ to $t = 1.0$ s?

30. Sketch a motion diagram for an object on the x-axis that (a) has a constant position; (b) has a constant acceleration of 1.0 m/s^2 in the negative x direction.

31. (a) Sketch a motion diagram for an object on the x-axis that is moving with a constant non-zero velocity in the positive x direction. (b) Add a motion diagram for a second object moving on a path that is parallel to the first object, but with a velocity 1.5 times larger than that of the first object.

32. Consider the motion diagram in Figure 2.31, showing the position of an object at 1-second time intervals as it moves in one dimension. Assume that the time is known precisely. If the object's velocity at $t = 4$ s is 4.0 m/s to the right, determine the object's average velocity over the time intervals (a) $t = 0$ to $t = 4$ s; (b) $t = 4$ s to $t = 7$ s; (c) $t = 7$ s to $t = 10$ s; (d) $t = 0$ to $t = 10$ s.

Figure 2.31: Motion diagram for Exercises 32 – 35.

33. Repeat Exercise 32, but now determine the object's average acceleration over the given time intervals instead of the average velocity.

34. Consider the motion diagram in Figure 2.31, showing the position of an object at 1-second time intervals as it moves in one dimension. Assume the time is known precisely. If the object's velocity at t = 0 is 2.0 m/s to the right, determine the object's (a) displacement, (b) average velocity, and (c) average acceleration over the time interval t = 0 to t = 8 s. (d) Would your answers to parts (a) – (c) change if you considered the time interval t = 0 to t = 7 s instead? State whether the magnitude of each of these vector quantities would increase, decrease, or stay the same.

Exercises 35 – 38 deal with transforming between various representations of motion.

35. Consider the motion diagram in Figure 2.31, showing the position of an object at 1-second time intervals as it moves along the x-axis. If, at t = 0, the object's velocity is 2.0 m/s to the right, and its position is x = 0 m, plot a graph of the object's (a) position, (b) velocity, and (c) acceleration over the interval t = 0 to t = 10 s. Assume the object has a constant non-zero acceleration lasting 1 second during this interval.

36. A plot of an object's position as a function of time is shown in Figure 2.32. (a) Briefly describe a real-life situation that would match the motion described by the graph. (b) Plot the corresponding velocity graph. (c) Draw the corresponding motion diagram.

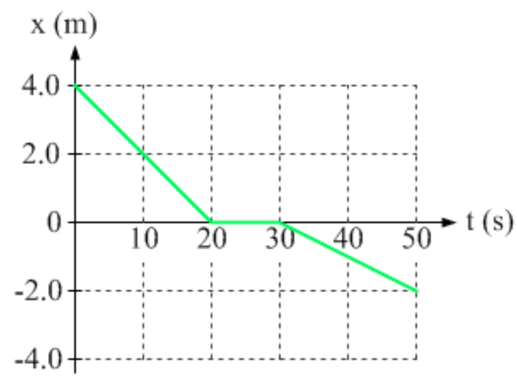


Figure 2.32: A graph of position-versus-time, for Exercise 36.

37. You are running along a straight path at constant velocity. After a few seconds, you see your friend, so you stop to chat. After a few seconds, she asks you to walk with her, so you start walking in the same direction you were originally running. (a) Sketch a graph showing your velocity as a function of time. (b) Sketch a graph showing your position as a function of time. (c) Draw a motion diagram for this motion.

38. A plot of an object's acceleration as a function of time is shown in Figure 2.33. (a) Sketch a velocity graph that would match this acceleration graph. (b) Comment on whether there is only one correct answer to part (a). (c) Briefly describe a real-life situation that would match this motion.

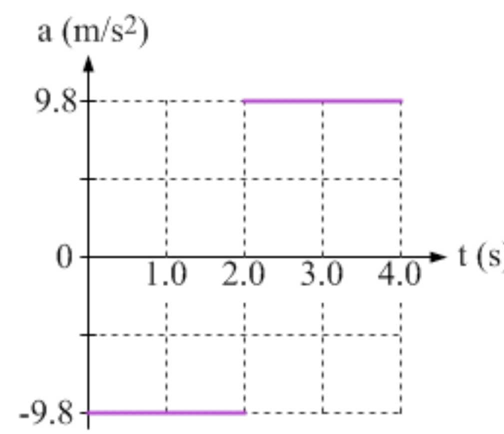


Figure 2.33: A graph of acceleration as a function of time, for Exercise 38.

Exercises 39 – 48 are designed to give you practice in solving a typical one-dimensional motion problem. Try to solve all exercises using a systematic approach. For each of these exercises start by doing the following parts. (a) **Picture the scene -** draw a diagram of the situation. Choose an origin to measure displacements from and mark that on the diagram. Choose a positive direction and indicate that with an arrow on the diagram. (b) **Organize the data -** create a table summarizing everything you know, as well as the unknowns you want to solve for.

39. You release a ball from rest, and it drops exactly 1.8 m to the floor below. Your goals in this exercise are to determine the ball's velocity just before impact and the time it takes the ball to reach the ground. Use g = 10 m/s^2. Carry out parts (a) – (b) as described above. (c) Which equation(s) will you use to determine the ball's velocity just before impact? (d) Solve for that velocity. (e) Which equation(s) will you use to determine the time it takes the ball to reach the ground? (f) Solve for that time.

40. When a traffic light turns green, you accelerate from rest along the road with a constant acceleration of 3.0 m/s^2. The goal here is to determine how long it takes you to reach the speed limit of 80 km/h. Carry out parts (a) and (b) as described above. (c) Which equation(s) will you use to find the time it takes to reach 80 km/h? (d) Find that time.

41. You throw a baseball straight up into the air, giving it an initial speed of 12 m/s. The baseball hits the ground 2.6 seconds later. To make the calculations easy, use g = 10 m/s^2. Your goals in this exercise are to solve for the initial height above the ground from which you launched the ball, and to find the maximum height above the ground reached by the ball in its motion. Carry out parts (a) and (b) as described above. (c) Which equation(s) will you use to find the initial height above the ground from which the ball was launched? (d) Solve for that initial height. (e) Which equation(s) will you use to find the maximum height above the ground reached by the ball? (f) Solve for that maximum height.

42. In this exercise you will analyze the method you used in the previous exercise, so all these questions pertain to what you did to solve the previous exercise. (a) Is there only one correct choice for the origin? Why did you make the choice you made? (b) Is there only one correct choice for the positive direction? Would your answers to (d) and (f) above change if you chose the opposite direction to be positive? (c) Find an alternative method to determine the initial height above the ground from which the ball was launched, and show that it gives the same answer as the method you used in the previous exercise. (d) Find an alternative method to determine the maximum height above the ground reached by the ball in its motion, and show that it gives the same answer as the method you used in the previous exercise.

43. A toy car is rolling down a ramp. When it passes a particular point you determine that it is traveling at a speed of 20 cm/s, and in the next 1.0 seconds it travels 40 cm. The goal of this exercise is to determine the car's acceleration, which we will assume to be constant. Carry out parts (a) and (b) as described above. (c) Which equation(s) will you use to determine the acceleration? (d) What is that acceleration?

44. Repeat parts (c) and (d) of Exercise 43, but calculate the answers another way, without using the equation(s) you used in Exercise 43.

45. You give a toy car an initial velocity of 1.00 m/s directed up a ramp. The car takes a total of 4.00 s to roll up the ramp and then roll back down again into your hand. Assuming you catch the car at the same point from which you released it, and that the acceleration is constant through the entire motion, the goal of the exercise is to determine the maximum distance the car was from your hand during the motion. Carry out parts (a) and (b) as

described above. (c) Which equation(s) will you use to determine the maximum distance the car was from your hand during the motion? (d) What is that maximum distance?

46. One of the events in the X Games is the Street Luge, in which competitors race on wheeled carts down an incline. Let's say that in one of the races a competitor starts from rest and then covers 100 m in 5.0 seconds. The goal of this exercise is to determine what the acceleration is, assuming it to be constant. Carry out parts (a) and (b) as described above. (c) What is the acceleration?

47. A professional baseball pitcher can throw a baseball with a speed of 150 km/h. Assuming the pitcher accelerates the ball from rest through a distance of 2.0 m, the goal of this exercise is to determine the ball's acceleration and the time over which this acceleration occurs. Carry out parts (a) and (b) as described above. (c) What is the ball's acceleration? (d) What is the time over which the acceleration occurs?

48. You're driving along a straight road at a speed of 48.0 km/h when you see a deer in the road 35.0 m ahead of you. After applying the brakes it takes you 2.00 s to bring your car to rest, but there is a reaction time period (the time between when you first see the deer and when you first apply the brakes) during which the car continues to travel at 48.0 km/h. The goal of this exercise is to answer the question: Assuming the acceleration is constant during the braking phase of the motion, what is the longest your reaction time can be if you are to stop the car before reaching the deer? Carry out parts (a) and (b) as described above. Note that there are two phases to the motion, a constant velocity phase and a constant-acceleration phase, so you should clearly separate the information for the two phases in your table. (c) Briefly describe the method you will use to solve the exercise. (d) Solve for your maximum possible reaction time.

General problems and conceptual questions

49. In Figure 2.13, we sketched graphs of the position and velocity of three cars as a function of time as they moved with constant velocity. Sketch a graph of the acceleration of each car as a function of time.

50. Consider the three positions shown in Figure 2.34. Starting at $\vec{x}_0$, it takes you 9.0 s to walk to $\vec{x}_2$ and then an additional 3.0 s to walk to $\vec{x}_1$. For this motion, find your (a) average speed and (b) average velocity.

Figure 2.34: Three different positions ($\vec{x}_0$, $\vec{x}_1$, and $\vec{x}_2$) along an *x*-axis, for Exercise 50.

51. In the men's 100-meter track race at the 2004 Athens Olympics, Justin Gatlin of the USA won the gold medal with a time of 9.85 s. Shawn Crawford (USA) ended up fourth in a time of 9.89 s. Estimate the distance separating these two runners at the finish line.

52. On August 16, 1960, Joe Kittinger of the United States Air Force jumped from a helium balloon from a height of 102,800 feet. After being in free fall for 4 minutes and 36 seconds, and falling for 85,300 feet, he opened his parachute and eventually landed safely on the ground. Assume that the acceleration was constant to answer parts (a) and (b). (a) What was the acceleration during the free fall? (b) What was Kittinger's speed just before he opened his parachute? (c) Comment on whether you think Kittinger's acceleration was constant during the fall. Depending on what source you read, Kittinger either broke, or came close to breaking, the sound barrier during this event.

53. Who was Colonel John Paul Stapp and why is he relevant to the material in this chapter? In particular, what was the magnitude of the maximum acceleration he experienced?

54. You throw a ball straight up into the air and catch it again at the same height from which you let it go. Considering the motion from the instant you release the ball until the instant you catch it again, the motion takes a time T and the ball reaches a maximum height H above the point where you release it. You now repeat the process, but this time the ball has twice the initial velocity of the previous motion. (a) How high above the release point does the ball go this time? (b) How long does this motion take?

55. You throw a ball straight up into the air, releasing it with a speed of 20 m/s. Assuming g = 10 m/s^2, you catch the ball 4.0 seconds later at the same point from which you let it go. Consider the motion from just after you release the ball until just before it returns to your hand. For the round trip, determine the ball's: (a) average velocity; (b) average speed; (c) average acceleration.

56. While a commuter train is stopped to pick up passengers, a freight train goes by at a constant speed of 36 km/h. Exactly 1 minute after the front of the freight train passes the front of the commuter train, the commuter train starts to move. After another 1 minute of constant acceleration, the commuter train reaches a speed of 72 km/h, and then moves at constant speed. Both trains are going the same way on parallel tracks. How much more time passes until the front of the commuter train passes the front of the freight train?

57. Two balls are launched at the same time. Ball A is released from rest from the top of a tall building of height H. Ball B is fired straight up from the ground with an initial velocity such that it just reaches the top of the same building. Neglect air resistance. (a) Which ball has the largest magnitude acceleration at the point they pass one another? (b) If ball A takes a time T to reach the ground, and ball B takes the same time T to reach the top of the building, which ball has the highest speed at time $T/2$? (c) How far from the ground are the two balls when they pass one another? Express your answer in terms of H. (d) Sketch a graph showing the velocity of ball A, and the velocity of ball B, as a function of time. Start from when the balls are launched, and end when ball A reaches the ground.

58. A cat and a dog are having a 100 m race. When the starting gun goes off, the dog lies down for a nap. The cat moves forward with a constant acceleration, reaching a speed of 2.0 m/s when she is 20 m from the starting line. After reaching that point, the cat travels at a constant velocity of 2.0 m/s until crossing the finish line. After 45 seconds, the dog wakes up from his nap and covers the 100 m with a constant acceleration of 2.0 m/s^2. (a) Who wins the race? Clearly justify your answer. (b) What is the distance between the animals when the winner crosses the finish line? (c) What is the distance between the animals at the only time (other than at the start of the race) they have the same velocity?

59. Consider the motion diagrams in Figure 2.35, showing the positions of two different objects at 1-second intervals starting at $t = 0$. Assume the top object, which has its position at regular intervals denoted by squares, is at $x = 0$ at $t = 0$, and that the vertical lines in the diagram are 1.0 m apart. Plot a graph of this object's (a) position, (b) velocity, and (c) acceleration over the interval between $t = 0$ and $t = 9$ s. You can assume that whenever the object accelerates its acceleration is constant and it accelerates for exactly 1 second.

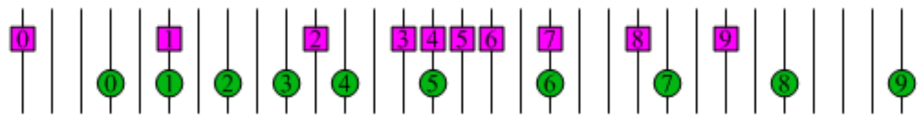


Figure 2.35: A pair of motion diagrams for Exercises 59 – 60. The numbers shown correspond to the time in seconds (e.g., the circle with the 4 in it shows the location of the bottom object at $t = 4$ s).

60. Consider the motion diagrams in Figure 2.35, showing the positions of two different objects at 1-second intervals starting at $t = 0$ as both objects move to the right. Which of the objects has the average velocity with the largest magnitude over the time interval (a) $t = 0$ to $t = 4$ s, (b) $t = 0$ to $t = 6$ s, (c) $t = 2$ s to $t = 7$ s?

61. You overhear two of your classmates discussing the issue of the acceleration of a ball that is tossed straight up into the air. Comment on each of their statements.

Jim: As the ball goes up, it slows down, so the acceleration is negative, while as the ball comes down again it speeds up, so the acceleration is positive. In other words, the acceleration changes sign when the ball changes direction.

Karen: What about the magnitude of the acceleration? I think it decreases to zero as the ball travels up, and then increases again as the ball comes down.

Jim: That sounds like the velocity. Remember that the acceleration is just the acceleration due to gravity, so that has a constant magnitude the whole time.

62. Referring back to the previous question, the conversation continues with two more classmates joining the discussion. Again, comment on each statement as they discuss the ball's acceleration at the instant it reaches the highest point.

Maria: Karen, isn't the ball's acceleration constant the whole time, even at the highest point? Are you saying that the acceleration is zero at that point?

Jose: I think Karen's right about that. Acceleration is velocity over time, so when the velocity goes to zero the acceleration must be zero, too.

Maria: Actually, isn't acceleration the change in velocity over some time interval? So, don't we have to consider how the velocity changes rather than worrying about what the value of the velocity is?

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Chapter 2: Additional Resources

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- Answers to Selected End of Chapter Problems
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- Simulation: Matching Graphs in One Dimension
- Simulation: Car Race (Constant Velocity vs. Constant Acceleration)

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- *PhET* simulation: The Moving Man