1-1 Physics, Models, and Units

You will most likely be devoting several months to learn physics. Physics is the study of something, but of what? Take a minute and write one sentence describing what you think physics is.

Physics encompasses many different topics, but a nice one-sentence description is that physics is the study of how the world works. Physics could just as easily be described as the study of how the universe works, or the study of how things work. Physics can be very practical, explaining how a toaster works, for instance. It can also be mind-blowing stuff, as we will see when we talk about the quantum nature of the atom and Einstein's theory of relativity. It is also important to keep in mind that physics is a science. Physics can, in some sense, also be thought of as a logical, systematic approach to analyzing physical situations.

Another important question to ask is, what is this book, *Essential Physics*? This is your guide to specific areas of physics. In some sense it is a history book as well as a science book, covering much of the same ground that was covered by natural philosophers, scientists, and physicists over the past 2500 years or so. This book is certainly not a comprehensive look at all of physics – you can always dig deeper to find more information – but it should at least give you a reasonable basis for understanding the world around you.

Models in Physics

In this book, we will see plenty of questions about spaceships in outer space, balls, cars, people, etc. When we come to analyze situations involving such objects, however, we will often use simplifying assumptions (such as assuming that no air resistance acts on a ball in a particular case) and we will often use models in which we replace the object by something simpler.

There are several reasons for using simplifying assumptions, including:

- The assumptions can allow us to solve a particular problem in a straightforward way.
- Anything we neglect should only have a minor impact on the result if we were to account for it.
- We may not know enough about the situation to include whatever it is we're neglecting. In some cases we will address that as we go through the book, such as neglecting friction initially and then learning how to account for friction.
- The mathematical methods that would be required to solve the problem more generally are above the level of this book.

When we use a model that neglects something like air resistance, the answer we come up may not match what really happens. If the model is good, though, the answer should be a reasonable approximation of what happens. It's important to think about how an answer would change if other factors were included. For instance, if we neglect air resistance and calculate that a particular baseball hit by Josh Hamilton just clears the outfield fence for a home run, that ball would, in reality, probably be caught by an outfielder, because air resistance tends to slow an object down and reduce how far it travels.

If you start to think that we neglect too much in this book and you're interested in doing more, that's terrific. This book is merely an introduction to physics, and there is plenty of exciting physics involved with going above and beyond what we'll cover here.

Applying a model often means treating complicated objects, such as the car shown in Figure 1.1, as a simpler object, such as a particle. A modern car is rather complicated. Its shape is designed to reduce air resistance; it generally has anti-lock brakes and air bags to increase passenger safety; its engine operates under the laws of thermodynamics; and its onboard computer and accompanying electric circuits are the analog of a human's brain and central nervous system. To understand the motion of such a car along a road, however, we can generally ignore many of these complicated systems. In the early chapters of this book, for instance, we will treat cars, people, etc., as particles. We will use this **particle model** a great deal, mainly so we can focus on the big picture rather than on subtle details that depend on the precise object. As we continue through the book, our models will become more sophisticated, but when we use models we're keeping a saying of Albert Einstein's in mind: make the problem as simple as possible, but no simpler.

Figure 1.1: A photograph of a 2006 Volkswagen Jetta. A modern car is an incredibly complex object, but to understand the motion of a car like this along a straight road, we can treat the car as a particle.
Photo credit: Wikimedia Commons.

Units

Physics is an experimental science. Each time we measure something, we need to be aware of the units of the measurement. Take a minute and measure the length of this page. What do you get? It would be meaningless to say 8.5, or 21.2, or 212, but if you said 8.5 inches, or 21.2 centimeters, or 212 millimeters, that would be fine. Those three measurements are equivalent, and you could measure the length of the page in other length units, too, such as feet, miles, furlongs, kilometers, or light-years. Just remember that a measurement requires both a number and a unit.

In this book we will primarily use SI (système international) units. SI has several base units, including the meter (m) as the unit of length, the kilogram (kg) as the unit of mass, and the second (s) as the unit of time. Being based on powers of ten, SI is easy to use, unlike the English system of units in which you have to remember conversion factors such as how many cups are in a gallon. In the metric system the prefix tells you which power of 10 to use. Table 1.1 gives some examples of conversion factors, with a more complete table inside the front cover of the book.

Name	Prefix	Power of ten	Example
mega	M	x 10^6	92.9 MHz – frequency of an FM radio station
kilo	k	x 10^3	110 km/h – speed limit on some Canadian highways
centi	c	x 10^{-2}	30 cm – approximately equal to 1 foot
milli	m	x 10^{-3}	500 mg – mass of Vitamin C in a Vitamin C capsule
micro	µ	x 10^{-6}	150 µm – diameter of a human hair
nano	n	x 10^{-9}	400 to 700 nm – wavelength range of visible light

Table 1.1: Common prefixes in the metric system.

Essential Question1.1: What is a good definition of a physical model?

Note that each section in the book ends with an Essential Question. The answer to each Essential Question is given at the top of the following page, but you should resist the temptation to immediately turn the page to look at the answer. Spending some time yourself thinking about the answer will really help you to learn the material.

Answer to Essential Question 1.1: A model is simplified version of physical situation. Using a model enables us to focus on the key elements of a particular situation, and is one way of getting a good idea of what is going on without having to consider every fine detail.

1-2 Unit Conversions, and Significant Figures

It is often necessary to convert a value from one set of units to another. To do this, we need to know the appropriate conversion factors. For instance, in Example 1.2 we will make use of these conversion factors:

- 1 hour = 3600 seconds
- 1 km = 1000 m
- 1 mile = 1.609344 km

EXAMPLE 1.2 – Unit conversions

At the 2009 World Championships in Athletics, held in Berlin, the Jamaican sprinter Usain Bolt set a world record for the 200-meter dash by running that distance in a time of 19.19 s. Assuming he ran exactly 200 m in this time, what was Usain Bolt's average speed during the race in (a) m/s; (b) km/h; (c) miles per hour?

SOLUTION

(a) The first thing we need to do is to understand what an average speed is. Average speed is the total distance covered divided by the time in which it was covered. If we divide the given distance by the given time we'll get the answer we're looking for:

$$\text{Usain Bolt's average speed was: } \frac{200 \text{ m}}{19.19 \text{ s}} = 10.422094841063 \text{ m/s}$$

This brings up the idea of **significant figures**, because you certainly do not want to quote an answer with 14 significant figures, as is shown above. Instead, round off the answer to four significant figures, because there are four in the time of 19.19 s. The rule is, ***when you multiply or divide numbers you look at the number of significant figures in the values going into the calculation and round off to the smallest number of significant figures***. Here, we're saying that the distance of 200 m is exact (see the assumption stated in the example), so that number has an infinite number of significant figures, while the time has four significant figures.

It would be more realistic to make the following argument. Lengths on a track, particularly at a major international competition such as the World Championships, are measured very accurately. For argument's sake, let's say the 200 meter distance is accurate to within 1 centimeter. Thus, the distance Usain Bolt ran was 200.00 m, seeing as 1 cm = 0.01 m. There are five significant figures in 200.00, so when dividing a number with five significant figures by one with four, we should round off our final answer to four significant figures.

Thus, Usain Bolt's average speed was 10.42 m/s.

(b) To convert from m/s to km/h, we need to know that there are 1000 m in 1 km, and that there are 3600 s in 1 hour. Then, we simply set these conversion factors up as ratios so that the units cancel properly, as follows:

$$10.4221 \frac{\text{m}}{\text{s}} \times \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{3600 \text{ s}}{1 \text{ h}} = 37.52 \text{ km/h} .$$

We treat conversion factors as having an infinite number of significant figures and we remember that the minimum number of significant figures in the factors going into the average speed in m/s was four. Thus, our final answer in this case should also have four significant figures. In carrying out the calculation, however, six digits are shown for the average speed in m/s, even though we know the last two are not significant (this is why the final answer is rounded off to four significant figures in part (a)). We could even keep the 14 digits we had originally – the reason for keeping at least a couple of extra digits, and ***only rounding off at the end of the calculation when you state the final answer***, is to state your answer as accurately as possible.

37.52 km/h does not differ by much from the 37.51 km/h we would get if we had started the conversion process with 10.42 m/s, but the 37.52 km/h value is more accurate.

(c) To state the average speed in miles per hour, we could start with the average speed in m/s and convert; however, it requires less work to start from km/h, so let's do that. Again, let's add an extra couple of digits for the intermediate values and round off to four significant figures at the end.

$$37.51956 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ mile}}{1.609344 \text{ km}} = 23.31 \text{ miles/h}.$$

So, we have now stated Usain Bolt's average speed in three equivalent ways, all with different units. Don't forget to be amazed by how fast that is!

Significant figures

If we add or subtract numbers, the rules are a little different from what we do when we multiply or divide. Let's add the following three distances: 341.2 m, 25 cm, and 0.3367 m. First we need to convert everything to the same units. We could convert everything to meters, for instance. Then, do the addition:

341.2 m + 0.25 m + 0.3367 m = 341.7867 m.

At this point, we need to round off correctly. Here, we look at decimal places, not significant figures. The first number goes to 1 decimal place, the second number to 2 decimal places, and the third number goes to 4 decimal places. Round off the final answer to 1 decimal place, because that's the smallest number of decimal places in any of the numbers going into the sum. ***When adding or subtracting, round off to the smallest number of decimal places.*** In this case, our final answer would be 341.8 m.

Many people get confused by zeroes, and whether to count them as significant figures. Leading zeroes do not count, but trailing zeroes do count as significant figures. If you forget, just convert a value to scientific notation and count the significant digits.

Related End-of-Chapter Exercises: 1, 2, 3, 11, 17.

Essential Question 1.2: How many significant figures are there in the value 0.0035 m? How many are in the value 35.00 m?

Answer to Essential Question 1.2: There are only two significant figures in the value 0.0035 m, because it can be written as 3.5×10^{-3} m, which has only two significant figures. There are four significant figures in the value 35.00 m, because it can be written as 3.500×10^{1} m, which has four significant figures. Trailing zeroes are very important! 3.5×10^{1} m and 3.500×10^{1} m represent the same length, but in the second case we know the length with greater precision than we do in the first case.

1-3 Trigonometry, Algebra, and Dimensional Analysis

Solving a physics problem often involves the geometry of right-angled triangles. Such a triangle is shown in Figure 1.2. In a right-angled triangle there are several relationships between the angle shown in the diagram and the different sides of the triangle, including:

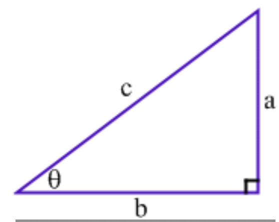


Figure 1.2: A right-angled triangle.

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{a}{c}; \quad \cos\theta = \frac{adjacent}{hypotenuse} = \frac{b}{c}; \quad \tan\theta = \frac{opposite}{adjacent} = \frac{a}{b}.$$

In a right-angled triangle, the Pythagorean Theorem relates the three sides:

$$c^2 = a^2 + b^2. \qquad \text{(Equation 1.1: \textbf{The Pythagorean theorem})}.$$

A few special right-angled triangles include:

- the 3-4-5 triangle in which the sides are in a 3:4:5 ratio.
- The 5-12-13 triangle in which the sides are in a 5:12:13 ratio.
- The 30˚-60˚-90˚ triangle in which the sides are in a 1: $\sqrt{3}$:2 ratio.

Many triangles do not have a 90˚ angle. For a general triangle, such as that in Figure 1.3, if we know the length of two sides and one angle, or the length of one side and two angles, we can use the Sine Law and the Cosine Law to find the other sides and angles.

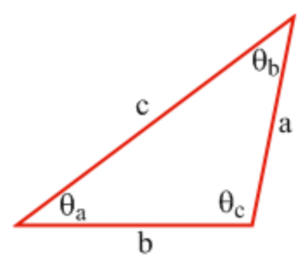


Figure 1.3: A general triangle.

$$\frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}. \qquad \text{(Equation 1.2: \textbf{Sine Law})}$$

$$c^2 = a^2 + b^2 - 2ab\cos\theta_c. \qquad \text{(Equation 1.3: \textbf{Cosine Law})}$$

Algebra

In addition to understanding what concepts to apply in solving a particular physics problem, you will need to know how to manipulate equations to solve for a particular unknown. In other words, you'll need to do algebra.

EXAMPLE 1.3A – Solving an equation using algebra

Solve for *v* in the following equation: $4v^2 - 7 = 3 - v^2$. Take a minute to solve the equation on your own before looking at the solution.

SOLUTION

To solve for a particular variable, you generally isolate that variable on one side and place everything else on the other side. Taking a step-by-step approach gives:

1. Bring all v terms to the left by adding v^2 to both sides: $5v^2 - 7 = 3$
2. Isolate the v term on the left by adding 7 to both sides: $5v^2 = 10$
3. Divide by 5: $v^2 = 2$
4. Solve for v: $v = \pm\sqrt{2}$

It is tempting to say that $v = \sqrt{2}$, but it is important to remember that the negative square root is also a possibility.

We did not concern ourselves with units above, but whenever you come up with an equation it is a good idea to do some dimensional analysis (that is, check your units). If the units check out, that does not necessarily mean your equation is correct. If your units do not check out, however, you know for sure there is something wrong with the equation.

EXAMPLE 1.3B – Using dimensional analysis

You're trying to solve for the velocity of a ball, 3 seconds after you throw it straight up in the air. You know that the velocity v has units of m/s, and you know the following parameters (defining the positive direction to be up): the initial velocity is $v_i = 20\text{ m/s}$; the acceleration is $a = -10\text{ m/s}^2$; and the time is $t = 3\text{ s}$. Your friend Sara says the equation connecting these variables is: $v = v_i + at/2$. Your friend Bob claims the equation is: $v = v_i + at^2$. Can dimensional analysis (checking the units) help you to rule out one or both of these equations as incorrect?

SOLUTION

Let's try both equations, keeping careful track of the units as we go.

Sara's method: $v = v_i + \frac{1}{2}at = 20\frac{\text{m}}{\text{s}} + \frac{1}{2}\left(-10\frac{\text{m}}{\text{s}^2}\right)(3\text{ s}) = 20\frac{\text{m}}{\text{s}} - 15\frac{\text{m}}{\text{s}}$.

For Sara's equation, the left-hand side (v) has units of m/s, and both terms on the right-hand side also have units of m/s. This is good. ***Quantities that are added or subtracted must have the same units***, and ***the units on one side of an equation must match the units on the other***.

Bob's method: $v = v_i + at^2 = 20\frac{\text{m}}{\text{s}} + \left(-10\frac{\text{m}}{\text{s}^2}\right)(3\text{ s})^2 = 20\frac{\text{m}}{\text{s}} - 90\text{ m}$.

Bob's equation is incorrect, because the two terms on the right do not have the same units, and the units of the last term do not match the units of the left side of the equation.

In fact, *neither Sara nor Bob has the correct equation*. As we will see in chapter 2, the correct equation relating the velocity to the initial velocity, acceleration, and time is $v = v_i + at$. Dimensional analysis let us know that Bob's equation was incorrect, but it could not tell us that Sara's equation had an extra factor of ½ in one term, because that extra factor had no units associated with it. Dimensional analysis can be helpful, but it is just one tool in our problem-solving toolkit, and it needs to be used appropriately.

Related End-of-Chapter Exercises: 38, 45.

Essential Question 1.3: What is the connection between the Pythagorean theorem and the Cosine law?

Answer to Essential Question 1.3: The Pythagorean theorem is a special case of the Cosine Law that applies to right-angled triangles. With an angle of 90˚ opposite the hypotenuse, the last term in the Cosine Law disappears because cos(90˚) = 0, leaving $c^2 = a^2 + b^2$.

1-4 Vectors

It is always important to distinguish between a quantity that has only a magnitude, which we call a **scalar**, and a quantity that has both a magnitude and a direction, which we call a **vector**. When we work with scalars and vectors we handle minus signs quite differently. For instance, temperature is a scalar, and a temperature of +30˚C feels quite different to you than a temperature of –30˚C. On the other hand, velocity is a vector quantity. Driving at +30 m/s north feels much the same as driving at –30 m/s north (or, equivalently, +30 m/s south), assuming you're going forward in both cases, at least! In the two cases, the speed at which you're traveling is the same, it's just the direction that changes. So, ***a minus sign for a vector tells us something about the direction of the vector; it does not affect the magnitude (the size) of the vector***.

When we write out a vector we draw an arrow on top to represent the fact that it is a vector, for example $\vec{A}$. *A*, drawn without the arrow, represents the magnitude of the vector.

EXPLORATION 1.4 – Vector components

Consider the vectors $\vec{A}$ and $\vec{B}$ represented by the arrows in Figure 1.4 below. The vector $\vec{A}$ lines up exactly with one of the points on the grid. The vector $\vec{B}$ has a magnitude of 4.00 m and is directed at an angle of 63.8˚ below the positive *x*-axis. It is often useful (if we're adding the vectors together, for instance) to find the **components** of the vectors. In this Exploration, we'll use a two-dimensional coordinate system with the positive *x*-direction to the right and the positive *y*-direction up. Finding the *x* and *y* components of a vector involves determining how much of the vector is directed right or left, and how much is directed up or down, respectively.

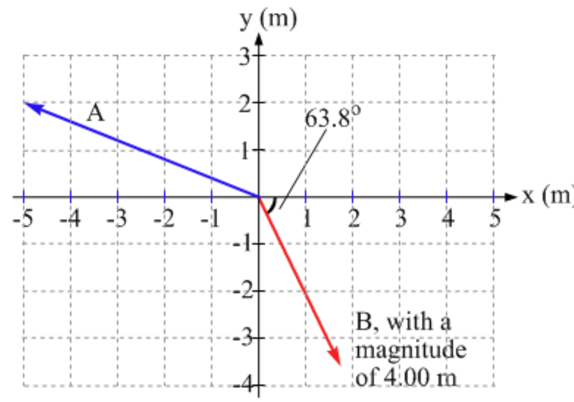


Figure 1.4: The vectors $\vec{A}$ and $\vec{B}$.

Step 1 - *Find the components of the vector* $\vec{A}$**.** The *x* and *y* components of $\vec{A}$ ($\vec{A}_x$ and $\vec{A}_y$, respectively) can be determined directly from Figure 1.4. Conveniently, the tip of $\vec{A}$ is located at an intersection of grid lines. In this case, we go exactly 5 m to the left and exactly 2 m up, so we can express the *x* and *y* components as:

$\vec{A}_x = +5\text{ m}$ to the left, or $\vec{A}_x = -5\text{ m}$ to the right.

$\vec{A}_y = +2\text{ m}$ up.

This makes it look like we know the components of $\vec{A}$ to an accuracy of only one significant figure. The components are known far more precisely than that, because $\vec{A}$ lines up exactly with the grid lines. The components of $\vec{A}$ are shown in Figure 1.5.

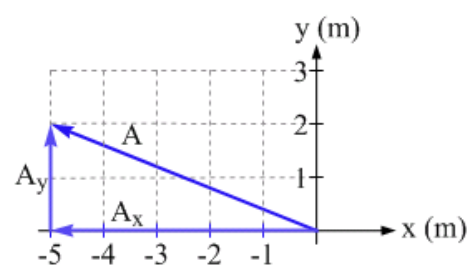


Figure 1.5: Components of the vector $\vec{A}$.

Step 2 – ***Express the vector $\vec{A}$ in unit-vector notation.*** Any vector is the vector sum of its components. For example, $\vec{A} = \vec{A}_x + \vec{A}_y$. This is shown graphically in Figure 1.5. It is rather long-winded to say $\vec{A} = -5$ m to the right + 2 m up. We can express the vector in a more compact form by using **unit vectors**. A unit vector is a vector with a magnitude of 1 unit. We will draw a unit vector with a carat (^) on top, rather than an arrow, such as $\hat{x}$. This notation looks a bit like a hat, so we say $\hat{x}$ as "x hat". Here we make use of the following unit vectors:

$\hat{x} =$ a vector with a magnitude of 1 unit pointing in the positive *x*-direction

$\hat{y} =$ a vector with a magnitude of 1 unit pointing in the positive *y*-direction

We can now express the vector $\vec{A}$ in the compact notation: $\vec{A} = (-5\text{ m})\ \hat{x} + (2\text{ m})\ \hat{y}$.

Step 3 - ***Find the components of the vector $\vec{B}$.*** We will handle the components of $\vec{B}$ differently from the method we used for $\vec{A}$, because $\vec{B}$ does not conveniently line up with the grid lines like $\vec{A}$ does. Although we could measure the components of $\vec{B}$ carefully off the diagram, we will instead use the trigonometry associated with right-angled triangles to calculate these components because we know the magnitude and direction of the vector.

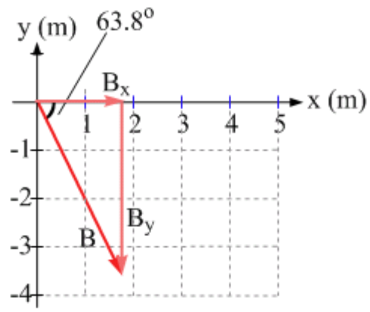


Figure 1.6: Components of the vector $\vec{B}$.

As shown in Figure 1.6, we draw a right-angled triangle with the vector as the hypotenuse, and with the other two sides parallel to the coordinate axes (horizontal and vertical, in this case). The *x*-component can be found from the relationship:

$$\cos\theta = \frac{B_x}{B}. \qquad \text{So } B_x = B\cos\theta = (4.00\text{ m})\cos(63.8^\circ) = 1.77\text{ m}.$$

We can use trigonometry to determine the magnitude of the component and then check the diagram to get the appropriate sign. From Figure 1.6, we see that the *x*-component of $\vec{B}$ points to the right, so it is in the positive *x*-direction. We can then express the *x*-component of $\vec{B}$ as:

$$\vec{B}_x = (+1.77\text{ m})\,\hat{x}.$$

The *y*-component can be found in a similar way:

$$\sin\theta = \frac{B_y}{B}. \qquad \text{So, } B_y = B\sin\theta = 4.00\sin(63.8^\circ) = 3.59\text{ m}.$$

The *y*-component of $\vec{B}$ points down, so it is in the negative *y*-direction. Thus:

$$\vec{B}_y = -(3.59\text{ m})\,\hat{y}.$$

The vector $\vec{B}$ can now be expressed in unit-vector notation as:

$$\vec{B} = \vec{B}_x + \vec{B}_y = (1.77\text{ m})\,\hat{x} - (3.59\text{ m})\,\hat{y}.$$

Key ideas for vectors: It can be useful to express a vector in terms of its components. One convenient way to do this is to make use of unit vectors; a unit vector is a vector with a magnitude of 1 unit. **Related End-of-Chapter Exercises: 6, 18.**

Essential Question 1.4: Temperature is a good example of a scalar, while velocity is a good example of a vector. List two more examples of scalars, and two more examples of vectors.

Answer to Essential Question 1.4: Other examples of scalars include mass, distance, and speed. Examples of vectors, which have directions associated with them, include displacement, force, and acceleration.

1-5 Adding Vectors

EXAMPLE 1.5 – Adding vectors

Let's define a vector $\vec{C}$ as being the sum of the two vectors $\vec{A}$ and $\vec{B}$ from Exploration 1.4. A vector that results from the addition of two or more vectors is called a **resultant vector**.

(a) Draw the vectors $\vec{A}$ and $\vec{B}$ tip-to-tail to show geometrically the resultant vector $\vec{C}$.

(b) Use the components of vectors $\vec{A}$ and $\vec{B}$ to find the components of $\vec{C}$.

(c) Express $\vec{C}$ in unit-vector notation.

(d) Express $\vec{C}$ in terms of its magnitude and direction.

SOLUTION

(a) To add the vectors geometrically we can move the tail of $\vec{B}$ to the tip of $\vec{A}$, or the tail of $\vec{A}$ to the tip of $\vec{B}$. The order makes no difference. If we had more vectors, we could continue the process, drawing them tip-to-tail in sequence. The resultant vector always goes from the tail of the first vector to the tip of the last vector, as is shown in Figure 1.7.

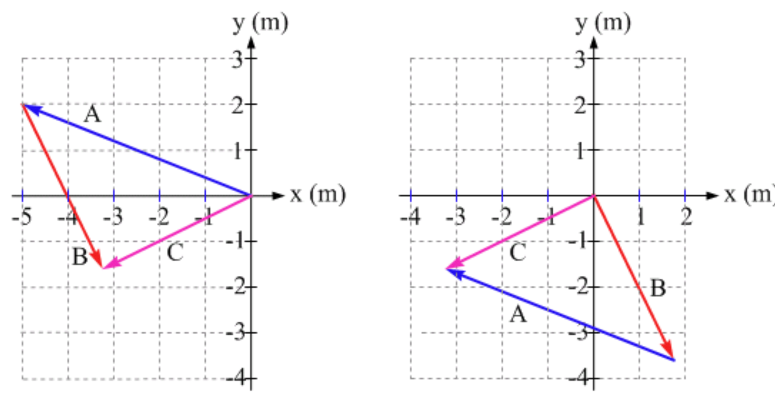


Figure 1.7: Adding vectors geometrically, tip-to-tail. In (a), the tail of vector $\vec{B}$ is placed at the tip of $\vec{A}$; in (b), the tail of vector $\vec{A}$ is placed at the tip of $\vec{B}$. The same resultant vector $\vec{C}$ is produced - the order does not matter.

(b) Now let's add the vectors using their components. We already know the x and y components of $\vec{A}$ and $\vec{B}$ (see Exploration 1.4), so we can use those to find the components of the resultant vector $\vec{C}$. Table 1.2 demonstrates the process. Note that the components of $\vec{A}$ are shown here to two decimal places, even though we know them with more precision. Because we'll be adding the components of $\vec{A}$ to the components of $\vec{B}$, which we know to two decimal places, our final answers should also be expressed with two decimal places.

Vector	*x*-component	*y*-component
$\vec{A}$	$\vec{A}_x = -(5.00 \text{ m})\,\hat{x}$	$\vec{A}_y = +(2.00 \text{ m})\,\hat{y}$
$\vec{B}$	$\vec{B}_x = (1.77 \text{ m})\,\hat{x}$	$\vec{B}_y = -(3.59 \text{ m})\,\hat{y}$
$\vec{C} = \vec{A} + \vec{B}$	$\vec{C}_x = \vec{A}_x + \vec{B}_x$ $\vec{C}_x = -(5.00 \text{ m})\,\hat{x} + (1.77 \text{ m})\,\hat{x}$ $\vec{C}_x = -(3.23 \text{ m})\,\hat{x}$	$\vec{C}_y = \vec{A}_y + \vec{B}_y$ $\vec{C}_y = +(2.00 \text{ m})\,\hat{y} - (3.59 \text{ m})\hat{y}$ $\vec{C}_y = -(1.59 \text{ m})\hat{y}$

Table 1.2: Adding the vectors $\vec{A}$ and $\vec{B}$ using components. The process is shown pictorially in Figure 1.8.

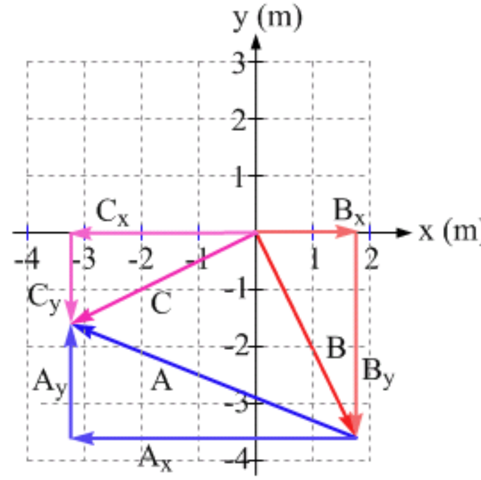


Figure 1.8: This figure illustrates the process of splitting the vectors into components when adding. Each component of the resultant vector, $\vec{C}$, is the vector sum of the corresponding components of the vectors $\vec{A}$ and $\vec{B}$.

Note that we are solving this two-dimensional vector-addition problem by using a technique that is very common in physics – splitting a two-dimensional problem into two separate one-dimensional problems. It is very easy to add vectors in one dimension, because the vectors can be added like scalars with signs. To find $\vec{C}_x$, for instance, we simply add the *x*-components of $\vec{A}$ and $\vec{B}$ together. To find $\vec{C}_y$, we carry out a similar process, adding the *y*-components of $\vec{A}$ and $\vec{B}$. After finding the individual components of $\vec{C}$, we then combine them, as in parts (c) and (d) below, to specify the vector $\vec{C}$.

(c) Using the bottom line in Table 1.2, the vector $\vec{C}$ can be expressed in unit-vector notation as:
$\vec{C} = \vec{C}_x + \vec{C}_y = -(3.23 \text{ m})\,\hat{x} - (1.59 \text{ m})\,\hat{y}$.

(d) If we know the components of a vector we can draw a right-angled triangle (see Figure 1.9) in which we know the lengths of two sides. Applying the Pythagorean theorem gives the length of the hypotenuse, which is the magnitude of the vector $\vec{C}$.

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{3.23^2 + 1.59^2} = \sqrt{12.961} = 3.60 \text{ m}$$

To find the angle between $\vec{C}$ and $\vec{C}_x$ we can use the relationship:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{C_y}{C_x}.$$

This gives $\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{1.59}{3.23}\right) = 26.2^\circ$.

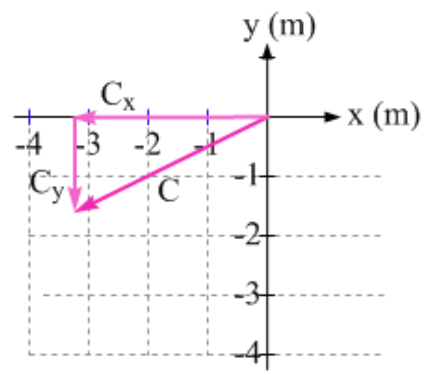


Figure 1.9: The components of the vector $\vec{C}$.

We have dropped the signs from the components, but, in stating the vector $\vec{C}$ correctly in magnitude-direction form, we can check the diagram to make sure we're accounting for which way $\vec{C}$ points: $\vec{C}$ = 3.60 m at an angle of 26.2° below the negative *x*-axis. The phrase "below the negative *x*-axis" accounts for the fact that the vector $\vec{C}$ has negative *x* and *y* components.

Related End-of-Chapter Exercises: 24 – 30.

Essential Question 1.5: Consider again the vectors $\vec{A}$ and $\vec{B}$ from Exploration 1.4 and Example 1.5. If the vector $\vec{D}$ is equal to $\vec{A} - \vec{B}$, express $\vec{D}$ in terms of its components.

Answer to Essential Question 1.5: $\vec{D} = -(6.77\text{ m})\,\hat{x} + (5.59\text{ m})\,\hat{y}$.

1-6 Coordinate Systems

Now that we have looked at an example of the component method of vector addition, in Example 1.5, we can summarize the steps to follow.

A General Method for Adding Vectors Using Components

1. Draw a diagram of the situation, placing the vectors tip-to-tail to show how they add geometrically.
2. Show the coordinate system on the diagram, in particular showing the positive direction(s).
3. Make a table showing the *x* and *y* components of each vector you are adding together.
4. In the last line of this table, find the components of the resultant vector by adding up the components of the individual vectors.

Coordinate systems

A coordinate system typically consists of an *x*-axis and a *y*-axis that, when combined, show an origin and the positive directions, as in Figure 1.10. A coordinate system can have just one axis, which would be appropriate for handling a situation involving motion along one line, and it can also have more than two axes if that is appropriate. An important part of dealing with vectors is to think about the coordinate system or systems that is/are appropriate for dealing with a particular situation. Let's explore this idea further.

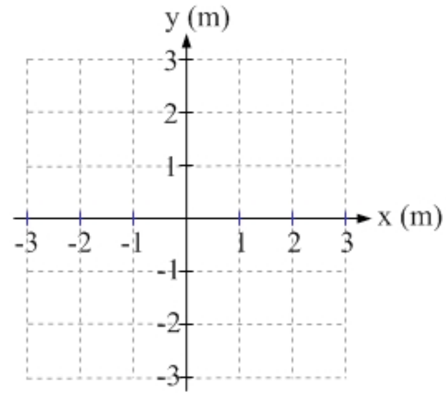


Figure 1.10: A typical *x*-*y* coordinate system.

EXPLORATION 1.6 – Buried treasure

While stranded on a desert island you find a note sealed inside a bottle that is half-buried near a big tree. Unfolding the note, you read: "Start 1 pace north of the big tree. Walk 10 paces northeast, 5 paces southeast, 6 paces southwest, 7 paces northwest, 4 paces southwest, and 2 paces southeast. Then dig." Realizing that your paces might differ in length from the paces of whoever left the note, rather than actually pacing out the distances you begin by drawing an *x*-*y* coordinate system in the sand, with positive *x* directed east and positive *y* directed north. After struggling to split the six vectors into components, however, you wonder whether there is a better way to solve the problem.

Step 1 - ***Is there only one correct coordinate system, or can you choose from a number of different coordinate systems to calculate a single resultant vector that represents the vector sum of the six vectors specified in the note?*** Any coordinate system will work, but there may be one coordinate system that makes the problem relatively easy, while others involve significantly more work to arrive at the answer. It's always a good idea to spend some time thinking about which coordinate system would make the problem easiest.

In fact, you should also think about whether the component method is even the easiest method to use to solve the problem. Adding vectors geometrically would also be a relatively easy way of solving this problem. Thinking about adding them geometrically (it might help to look at the six displacements, as sketched in Figure 1.11), in fact, leads us straight to the most appropriate coordinate system.

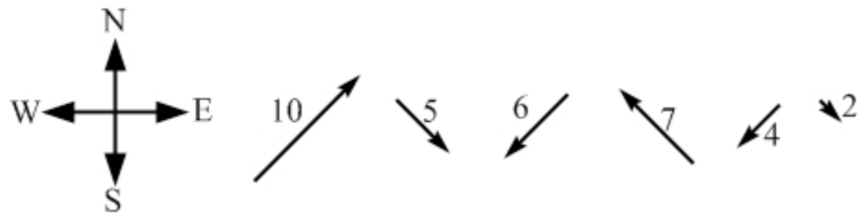


Figure 1.11: A sketch of the six displacements specified on the treasure map.

Step 2 - ***What would be the simplest coordinate system to use to find the resultant vector?*** One thing to notice is that the directions given are northeast, southeast, southwest, or northwest. An appropriate coordinate system is one that is aligned with these directions. For instance, we could point the positive x-direction northeast, and the positive y-direction northwest. In that case, out of the six different displacements, three are entirely in the x-direction and the other three are entirely in the y-direction. This makes the problem straightforward to solve. Figure 1.12 shows the vectors grouped by whether they are parallel to the x-axis or parallel to the y-axis.

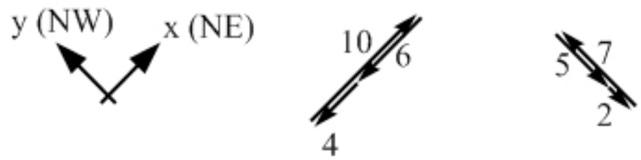


Figure 1.12: Choosing a coordinate system that fits the problem can make the problem easier to solve. In this case we have three vectors aligned with the x-axis and three vectors aligned with the y-axis.

Step 3 - ***Where should you dig?*** To determine where to dig, focus first on the displacements that are either in the $+x$ direction (10 paces northeast) or the $-x$ direction (6 paces southwest, and 4 paces southwest). Since the total of 10 paces southwest exactly cancels the 10 paces northeast, there is no net displacement along the x-axis.

Now turn to the y-axis, where we have 7 paces northwest (the $+y$ direction) and a total of $5 + 2 = 7$ paces southeast (the $-y$ direction). Once again these exactly cancel. Because the two components are zero, the resultant displacement vector has a magnitude of zero. You should dig at the starting point, 1 pace north of the tree (assuming you can figure out which way north is!). Digging at that spot, you find a box with a few car batteries, a 12-volt lantern, a solar cell, several wires, and a physics textbook. Reading through the book you figure out how to wire the solar cell to the batteries so the batteries are charged up while the sun shines, and you then figure out how to wire the batteries to the lantern to create a bright light you can use to signal passing planes. Using this system, you are rescued just a few days later, although you make sure to bury everything again carefully near the tree, and place the map back in the bottle, to help the next person who gets stranded there.

Key ideas for coordinate systems: Thinking carefully about the coordinate system to use can save a lot of work. Any coordinate system will work, but, in some cases, choosing the most appropriate coordinate system can make a problem considerably easier to solve.
Related End-of-Chapter Exercises: 5, 31, 41, 42.

Essential Question 1.6: In Exploration 1.6, the six displacements of 10 paces, 5 paces, 6 paces, 7 paces, 4 paces, and 2 paces happen to completely cancel one another because of their particular directions. If you could adjust the directions of each of the six vectors to whatever direction you wanted, what is the maximum distance they could take you away from the starting point?

Answer to Essential Question 1.6: If you lined up all six vectors in the same direction, you would end up 34 paces away from the starting point. When the vectors point in the same direction (and only in this case) you can add their magnitudes. 10 + 5 + 6 + 7 + 4 + 2 = 34 paces.

1-7 The Quadratic Formula

EXAMPLE 1.7 – Solving a quadratic equation

Sometime, such as in some projectile-motion situations, we will have to solve a quadratic equation, such as $2.0x^2 = 7.0 + 5.0x$. Try solving this yourself before looking at the solution.

SOLUTION

The usual first step is to write this in the form $ax^2 + bx + c = 0$, with all the terms on the left side. In our case we get: $2.0x^2 - 5.0x - 7.0 = 0$.

We could graph this on a computer or a calculator to find the values of *x* (if there are any) that satisfy the equation; we could try factoring it out to find solutions; or we can use the quadratic formula to find the solution(s). Let's try the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$ (Equation 1.4: **The quadratic formula**)

In our example, with $a = 2.0$, $b = -5.0$, and $c = -7.0$, the two solutions work out to:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{+5.0 + \sqrt{25 + 56}}{4.0} = \frac{+5.0 + 9.0}{4.0} = +3.5$$, with appropriate units.

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{+5.0 - \sqrt{25 + 56}}{4.0} = \frac{+5.0 - 9.0}{4.0} = -1.0$$, with appropriate units.

These values agree with the graph of the function shown in Figure 1.13. The graph crosses the *x*-axis at two points, at $x = -1$ and also at $x = +3.5$.

Related End-of-Chapter Exercises: 36, 46.

Essential Question 1.7: Could you have a quadratic equation in the form $ax^2 + bx + c = 0$ that had no solutions for *x* (at least, no real solutions)? If so, what would happen when you tried to solve for *x* using the quadratic formula? What would the graph look like?

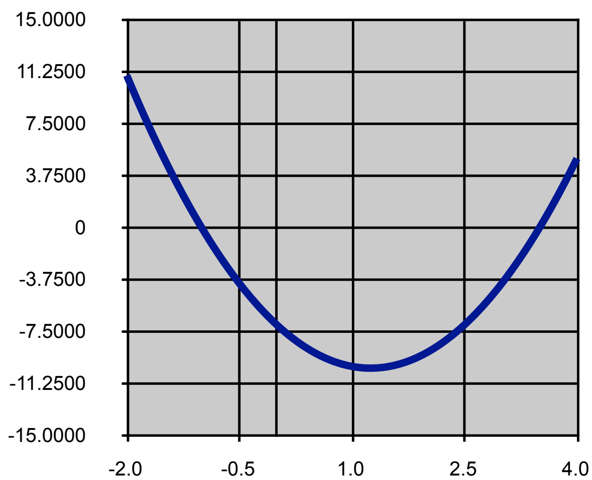


Figure 1.13: A graph of the quadratic equation $2.0x^2 - 5.0x - 7.0 = 0$, for Example 1.7.

Answer to Essential Question 1.7: Yes, you could have an equation with no real solutions. In that case when you applied the quadratic formula you would get a negative under the square root, while the graph would still be parabolic but would not cross (or touch) the *x*-axis.

Chapter Summary

Essential Idea

Physics is the study of how things work, and in analyzing physical situations we will try to apply a logical, systematic approach. Some of the basic tools we will use include:

Units

Our primary set of units is the système international (SI), based on meters, kilograms, and seconds, and four other base units. SI is widely accepted in science worldwide, and convenient because conversions are based on powers of ten. Converting between units is straightforward if you know the appropriate conversion factor(s).

Significant Figures

Three useful guidelines to follow when rounding off include:

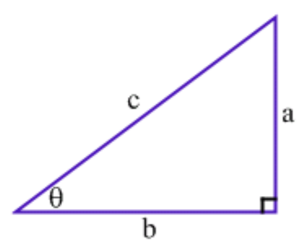
1. Round off only at the end of a calculation when you state the final answer.

2. When you multiply or divide, round your final answer to the smallest number of significant figures in the values going into the calculation.

3. When adding or subtracting, round your final answer to the smallest number of decimal places in the values going into the calculation.

Trigonometry

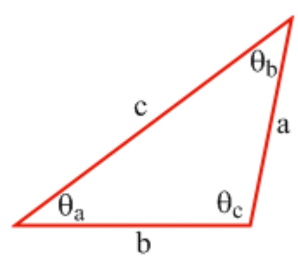
In a right-angled triangle we use the following relationships:



$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{a}{c}; \qquad \cos\theta = \frac{adjacent}{hypotenuse} = \frac{b}{c}; \qquad \tan\theta = \frac{opposite}{adjacent} = \frac{a}{b}.$$

We relate the three sides using: $c^2 = a^2 + b^2$. (Eq. 1.1: **The Pythagorean Theorem**)

Many triangles do not have a 90˚ angle. For a general triangle, such as that in Figure 1.3, if we know the length of two sides and one angle, or the length of one side and two angles, we can use the Sine Law and the Cosine Law to find the other sides and angles.



$$\frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}.$$ (Equation 1.2: **Sine Law**)

$$c^2 = a^2 + b^2 - 2ab\cos\theta_c.$$ (Equation 1.3: **Cosine Law**)

Vectors

A vector is a quantity with both a magnitude and a direction. Vectors can be added geometrically (drawn tip-to-tail), or by using components.

A unit vector is a vector with a length of one unit. A unit vector is denoted by having a carat on top, which looks like a hat, like $\hat{x}$ (pronounced "x hat").

A vector can be stated in unit-vector notation or in magnitude-direction notation.

A Method for Adding Vectors Using Components

1. Draw a diagram of the situation, placing the vectors tip-to-tail to show how they add geometrically.
2. Show the coordinate system on the diagram, in particular showing the positive direction(s).
3. Make a table showing the x and y components of each vector you are adding together.
4. In the last line of this table, find the components of the resultant vector by adding up the components of the individual vectors.

Algebra and Dimensional Analysis

Dimensional analysis can help check the validity of an equation. Units must be the same for values that are added or subtracted, as well as the same on both sides of an equation.

A quadratic equation in the form $ax^2 + bx + c = 0$ can be solved by using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

(Equation 1.4: **The quadratic formula**)

End-of-Chapter Exercises

Exercises 1 – 10 are conceptual questions that are designed to see if you have understood the main concepts of the chapter.

1. You can convert back and forth between miles and kilometers using the approximation that 1 mile is approximately 1.6 km. (a) Which is a greater distance, 1 mile or 1 km? (b) How many miles are in 32 km? (c) How many kilometers are in 50 miles?

2. (a) How many significant figures are in the number 0.040 kg? (b) How many grams are in 0.040 kg?

3. You have two numbers, 248.0 cm and 8 cm. Rounding off correctly, according to the rules of significant figures, what is the (a) sum, and (b) product of these two numbers?

4. Figure 1.14 shows an 8-15-17 right-angled triangle. For the angle labeled θ in the triangle, express (as a ratio of integers) the angle's (a) sine, (b) cosine, and (c) tangent.

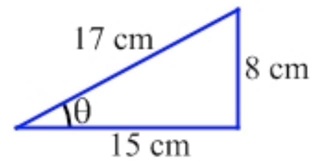


Figure 1.14: An 8-15-17 triangle, for Exercise 4.

5. You are adding two vectors by breaking them up into components. Your friend is adding the same two vectors, but is using a coordinate system that is rotated by 40° with respect to yours. Assuming you both follow the component method correctly, which of the following do the two of you agree on and which do you disagree about? (a) The magnitude of the resultant vector. (b) The direction of the resultant vector. (c) The *x*-component of the resultant vector. (d) The *y*-component of the resultant vector. (e) The angle between the *x*-axis and the resultant vector.

6. Three vectors are shown in Figure 1.15, along with an *x*-*y* coordinate system. Find the *x* and *y* components of (a) $\vec{A}$, (b) $\vec{B}$, and (c) $\vec{C}$.

7. The vectors $\vec{A}$ and $\vec{B}$ are specified in Figure 1.15. What is the magnitude and direction of the vector $\vec{A} + \vec{B}$?

8. You have two vectors, one with a length of length 4 m and the other with a length of 7 m. Each can be oriented in any direction you wish. If you add these two vectors together what is the (a) largest-magnitude resultant vector you can obtain? (b) smallest-magnitude resultant vector you can obtain?

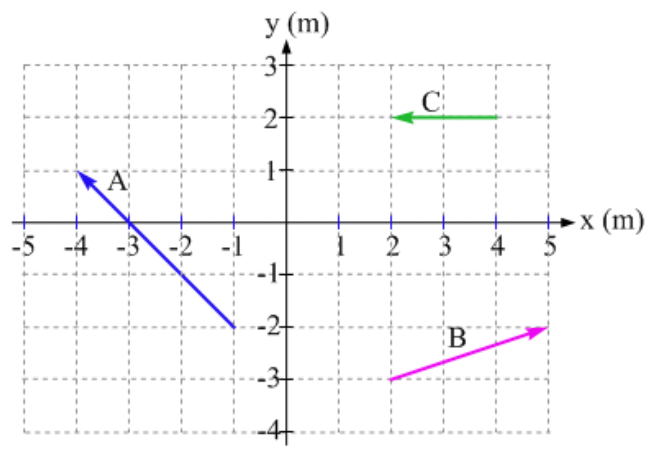


Figure 1.15: The vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$, for Exercises 6 and 7.

9. You have three vectors, with lengths of 4 m, 7 m, and 9 m, respectively. If you add these three vectors together, what is the (a) largest-magnitude resultant vector you can obtain? (b) smallest-magnitude resultant vector you can obtain?

10. You have three vectors, with lengths of 4 m, 7 m, and 15 m, respectively. If you add these three vectors together what is the (a) largest-magnitude resultant vector you can obtain? (b) smallest-magnitude resultant vector you can obtain?

Exercises 11 – 17 deal with unit conversions.

11. Using the conversion factors you find in some reference (such as on the Internet), convert the following to SI units. In other words, express the following in terms of meters, kilograms, and/or seconds. (a) 1.00 years (b) 1.00 light-years (c) 8.0 furlongs (d) 12 slugs (e) 26 miles, 385 yards (the length of a marathon).

English Unit	Metric Unit
1 inch	2.54 cm
1 foot	______ cm
1 foot	______ m
______ feet	1 m
1 mile	______ km
______miles	1 km

Table 1.3: A table of conversion factors for length units.

12. Using the fact that 1 inch is precisely 2.54 cm, fill in Table 1.3 to create your own table of conversion factors for various length units.

13. If someone were to give you a 50-carat diamond, what would be its mass in grams?

14. Fill in Table 1.4 to create your own table of conversion factors for various mass units.

English Unit	Metric Unit
1 ounce	28.35 g
1 lb.	______ g
1 lb.	______ kg
______ lbs.	1 kg
1 stone	______ kg
______stones	1 kg

Table 1.4: A table of conversion factors for mass units.

15. (a) Which is larger, 1 acre or 1 hectare? (b) If you own a plot of land that has an area of exactly 1 hectare and it is square, what is the length of one of its sides, in meters? (c) If your 1-hectare lot is rectangular with a width of 20 m, how long is it?

16. What is your height in (a) inches? (b) cm? What is your mass in (c) pounds? (d) kg?

17. Firefighters using a fire hose can spray about 1.0 x 10^2 gallons of water per minute on a fire. What is this in liters per second? Assume the firefighters are in the USA. (Why is this assumption necessary?)

Exercises 18 – 26 deal with various aspects of vectors and vector components.

18. In Exploration 1.4, we expressed the vector $\vec{A}$ in terms of its components. Assuming the magnitude of each component is known to three significant figures, express $\vec{A}$ in terms of its magnitude and direction.

19. Using the result of Exercise 18 to help you, and aided by Figure 1.7, use the Sine Law and/or the Cosine Law to determine the magnitude and direction of the vector $\vec{C}$ shown in Figure 1.7. The vectors $\vec{A}$ and $\vec{B}$ are defined in Exploration 1.4. Hint: you may find it helpful to use geometry to first determine the angle between the vectors $\vec{A}$ and $\vec{B}$. Show your work.

20. Two vectors $\vec{Q}$ and $\vec{R}$ can be expressed in unit-vector notation as follows: $\vec{Q} = (3.0\text{ m})\hat{x} + (4.0\text{ m})\hat{y}$ and $\vec{R} = (5.0\text{ m})\hat{x} - (7.0\text{ m})\hat{y}$. Express the following in unit-vector notation: (a) $6\vec{Q}$ (b) $6\vec{Q} - 4\vec{R}$ (c) $4\vec{R} - 6\vec{Q}$.

21. Repeat Exercise 20, but express your answers in magnitude-direction format instead.

22. See Exercise 20 for the definitions of the vectors $\vec{Q}$ and $\vec{R}$. (a) Is it possible to solve for the number a in the equation $\vec{Q}+a\vec{R}=0$? If it is possible, then solve for a; if not, explain why not. (b) How many different values of b are there such that the sum $\vec{Q}+b\vec{R}$ has only an x-component? Find all such values of b.

23. Three vectors are shown in Figure 1.16, along with an x-y coordinate system. Use magnitude-direction format to specify vector (a) $\vec{A}$ (b) $\vec{B}$ (c) $\vec{C}$.

24. The vectors $\vec{A}$ and $\vec{C}$ are shown in Figure 1.16. Consider the following vectors: 1. $\vec{A}$; 2. $\vec{C}$; 3. $\vec{A}+\vec{C}$; 4. $\vec{A}-\vec{C}$. Rank those four vectors by their magnitude, from largest to smallest. Use notation such as 3 > 2 = 4 > 1.

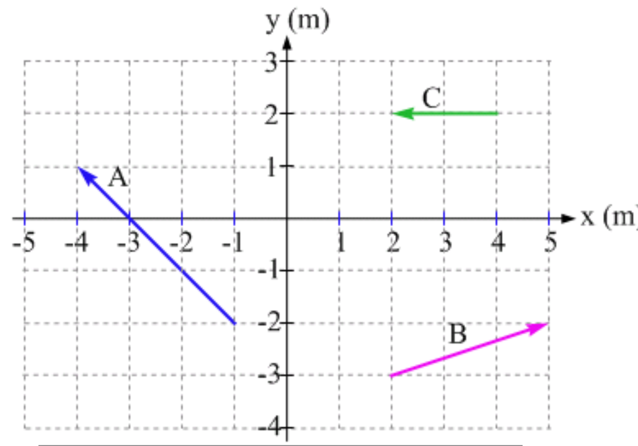


Figure 1.16: The vectors $\vec{A}, \vec{B},$ and $\vec{C}$, for Exercises 23 – 31.

25. Three vectors are shown in Figure 1.16. (a) Use the geometric method of vector addition (add vectors tip-to-tail) to draw the vector representing $\vec{B}+\vec{C}$. (b) Specify that resultant vector in unit-vector notation.

26. Three vectors are shown in Figure 1.16. (a) Use the geometric method of vector addition (add vectors tip-to-tail) to draw the vector representing $\vec{A}+\vec{B}+\vec{C}$. (b) Specify that resultant vector in unit-vector notation. (c) Specify that resultant vector in magnitude-direction notation.

Exercises 27 – 31 are designed to give you practice in applying the component method of vector addition. For each exercise, start with the following: (a) Draw a diagram showing how the vectors add geometrically (the tip-to-tail method). (b) Show the coordinate system (we'll use the standard coordinate system shown in Figure 1.16 above). (c) Make a table showing the x and y components of each vector you're adding together. (d) In the last row of the table, find the components of the resultant vector.

In addition to the vectors $\vec{A}, \vec{B},$ and $\vec{C}$ shown in Figure 1.16, let's make use of these two vectors: the vector $\vec{D}=(-6.00 \text{ m})\hat{x}+(-2.00 \text{ m})\hat{y}$, and the vector $\vec{E}$ with a magnitude of 5.00 m at an angle of 30° above the positive x-axis.

27. Find the resultant vector representing $\vec{A}+\vec{B}+\vec{C}$. Answer parts (a) – (d) as specified above. (e) State the resultant vector in magnitude-direction notation.

28. Find the resultant vector representing $\vec{A}+\vec{D}$. Answer parts (a) – (d) as specified above. (e) State the resultant vector in unit-vector notation.

29. Find the resultant vector representing $\vec{C} - \vec{E}$. Answer parts (a) – (d) as specified above. (e) State the resultant vector in magnitude-direction format.

30. Find the resultant vector representing $\vec{B} + \vec{D}$. Answer parts (a) – (d) as specified above. (e) State the resultant vector in unit-vector notation. (f) State the resultant vector in magnitude-direction format.

31. Repeat Exercise 30, but now use an *x-y* coordinate system that is rotated so the positive *x*-direction is the direction of the vector $\vec{B}$.

Exercises 32 – 36 involve applications of the physics concepts addressed in this chapter.

32. In 1999, NASA had a high-profile failure when it lost contact with the Mars Climate Orbiter as it was trying to put the spacecraft into orbit around Mars. Do some research and write a paragraph or two about what was responsible for this failure, and how much the project cost.

33. In 1983, an Air Canada Boeing 767 airplane was nicknamed "The Gimli Glider." Discuss the events that led to the plane getting this nickname, and how they relate to the topics in this chapter.

34. One way to travel from Salt Lake City, Utah, to Billings, Montana, is to first drive 660 km north on interstate 15 to Butte, Montana, and then drive 370 km east to Billings on interstate 90. If you did this trip by plane, instead, traveling in a straight line between Salt Lake City and Billings, how far would you travel?

35. In the sport of orienteering, participants must plan carefully to get from one checkpoint to another in the shortest possible time. In one case, starting at a particular checkpoint, Sam decides to take a path that goes west for 600 meters, and then go northeast for 400 meters on another path to reach the next checkpoint. Between the same two checkpoints, Mary decides to take the shortest distance between the two checkpoints, traveling off the paths through the woods instead. What is the distance Mary travels along her route, and in what direction does she travel between the checkpoints?

36. You throw a ball almost straight up, with an initial speed of 10 m/s, from the top of a 20-meter-high cliff. The approximate time it takes the ball to reach the base of the cliff can be found by solving the quadratic equation $-20 \text{ m} = (10 \text{ m/s})t - (5.0 \text{ m/s}^2)t^2$. Solve the equation to find the approximate time the ball takes to reach the base of the cliff.

General problems and conceptual questions

37. Prior to the 2004 Boston Marathon, the Boston Globe newspaper carried a story about a running shoe called the Nike Mayfly. According to the newspaper, the shoes were designed to last for 62.5 miles. Does anything strike you as odd about this distance? If so, what?

38. Do the following calculations make any sense? Why or why not? If any make sense, what could they represent?

(a) $a = 17\frac{\text{m}}{\text{s}} \times 3.2\text{ s}$ (b) $b = 17\frac{\text{m}}{\text{s}} + 3.2\text{ s}$ (c) $c = \frac{751\text{ m}}{\cos\left(23^\circ\right)}$ (d) $d = (751\text{ m}) - \cos\left(23^\circ\right)$.

39. The distance from Dar es Salaam, Tanzania to Nairobi, Kenya, is 677 km, while it is 1091 km from Dar es Salaam to Kampala, Uganda. (a) Using these numbers alone, can you determine the distance between Nairobi and Kampala? Briefly justify your answer. (b) Using only these numbers, what is the minimum possible distance between Nairobi and Kampala? (c) What is the maximum possible distance?

40. Use the information given in Exercise 39, combined with the fact that it is 503 km from Nairobi to Kampala, to construct a triangle with the three cities at the vertices. What is the angle between the two lines that meet at (a) Dar es Salaam? (b) Nairobi? (c) Kampala?

41. Figure 1.17 shows overhead views of similar sections of two different cities where the blocks are marked out in a square grid pattern. In City A, the streets run north-south and east-west, while in City B, the streets are at some angle with respect to those in City A. In City A, Anya intends to walk from the lower left corner (marked by a blue dot) to the upper right corner (marked by a yellow dot). In City B, Boris will walk a similar route between the colored dots, ending up due north of his starting point. For both cities, use a coordinate system where positive *x* is east and positive *y* is north. (a) Assuming Anya goes along the streets, marked in black, choose a route for Anya to follow that involves her changing direction only once. Express her route in unit-vector notation. (b) How many blocks does she travel? (c) Assuming Boris goes along the streets, choose a route for Boris to follow that involves him changing direction only once. Break Boris' trip into two parts, the first ending at the corner where he changes direction and the second starting there, and express each part as a vector in unit-vector notation. (d) What do you get when you add the two vectors from part (c)?

42. Return to the situation described in Exercise 39, where for City B we used a coordinate system where positive *x* is east and positive *y* is north. Comment on the relative advantages and disadvantages of that coordinate system over one in which the coordinate axes are aligned parallel to the streets.

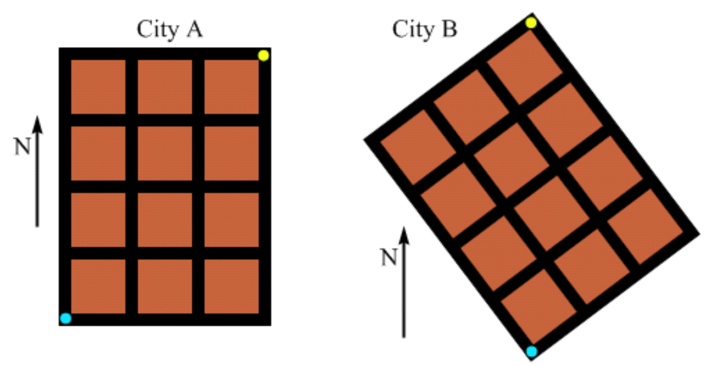


Figure 1.17: Overhead views of a 3 block by 4 block region in two different cities, for Exercises 41 and 42.

43. The top of a mountain is 2100 m north, 3300 m west, and 2300 m vertically above the initial location of a mountain climber. (a) What is the straight-line distance between the top of the mountain and the climber? (b) Later in the climb, the climber finds that she is 1200 m south, 900 m east, and 1100 m vertically below the top of the mountain. What is the minimum distance she has traveled from her starting point? (c) Checking her handheld GPS (global positioning system) receiver, she finds she has actually traveled a distance 2.5 times larger than the answer to part (b). How is this possible?

44. In Figure 1.18, four successive moves are shown near the end of a chess match. First, Black moves his Pawn (P); then, White moves her Queen (Q); then Black moves his Knight (K); and White moves her Rook (R). Using a traditional *x*-*y* coordinate system with positive *x* to the right and positive *y* up, we can express the movement of the Queen as +4 units in the *x*-direction and -4 units in the *y*-direction. Using similar notation express the movement of the (a) Pawn; (b) Knight; and (c) Rook.

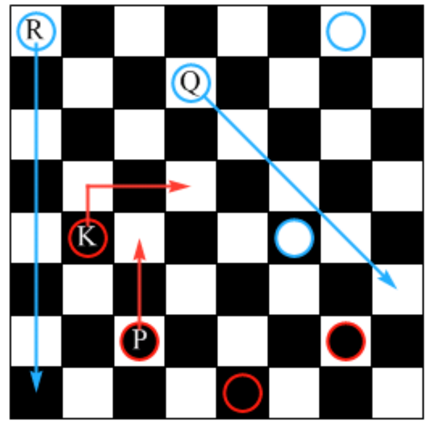


Figure 1.18: Four successive moves in a chess match.

45. Solve for *x* in the following expressions:

    (a) $5x - 7 = 2x + 5$. (b) $3 = \frac{4}{x} + \frac{4}{2x}$.

46. Often, when we solve a one-dimensional motion problem, which we will spend some time doing in the next chapter, we need to solve a quadratic equation to find the time when some event happens. For instance, solving for the time it takes a police officer to catch a speeding motorist could involve solving an equation of the form:

    $$\left(2\ \text{m/s}^2\right)t^2 - \left(10\ \text{m/s}\right)t - 100\ \text{m} = 0,$$

    where the *m* stands for meters and *s* stands for seconds. (a) What are the two possible solutions for *t*, the time when the police officer catches up to the motorist? (b) Which solution is the one we want to keep as the solution to the problem?

47. In the optics section of this book, we will use an equation to relate the position of an image formed by a lens (known as the image distance, d_i) to the position of the object with respect to the lens (the object distance, d_o) and the focal length, *f*, of the lens. (a) Use dimensional analysis to determine which of the following three equations could correctly relate those three lengths.

    Equation 1: $d_i = \frac{d_o - f}{d_o f}$; Equation 2: $d_i = \frac{d_o f}{d_o - f}$; Equation 3: $d_i = \frac{d_o f - f}{d_o}$.

    If you think any of the above are dimensionally incorrect, explain why. (b) If you found one or more of the three equations to be dimensionally correct, does this guarantee that the equation is the correct way to relate these three lengths? Explain.

48. Three students are having a conversation. Comment on each of their statements.

Ruben: The question says, what's the magnitude of the resultant vector obtained by adding a vector of length 3 units to a vector of length 4 units. That's just 7 units, right?

Marta: I think it is 5 units, because you can make a 3-4-5 triangle.

Kaitlyn: It depends on what direction the vectors are in. If the 3 and the 4 are in the same direction, you get 7 when you add them. If you have the 3 and the 4 perpendicular to one another, you get the 3-4-5 triangle. I think you can get a resultant of anything between 5 units and 7 units, depending on the angle between the 3 and the 4.

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