Chapter 7

Oozing and Growing: Percolation

Imagine a forest in which the trees were planted by scattering seeds randomly all over the ground. The trees are now full-grown, tall and green. Some trees stand alone; others are in clusters. The forest is peaceful on a quiet, windless day.

Now imagine that all the trees along one edge of the forest are suddenly set on fire. As each tree burns, the fire spreads rapidly to the trees next to it. Every tree in the same cluster burns, as the fire spreads from each tree to its closest neighbors. But fire does not spread from one cluster to another, because it cannot jump across the space that separates clusters; there is no wind.

If the trees are planted sparsely, with just a few trees scattered over a large area, chances are the fire will burn only a small number of trees and then die out; most of the forest will be safe. That's because sparsely planted trees, scattered randomly, tend to stand alone or in small clusters separated from one another.

In contrast, suppose that the trees are planted very densely, with lots of trees crowded into a small area. Then the fire is likely to spread through nearly the entire forest. That's because densely planted trees, scattered randomly, tend to form large clusters. There may even be one large cluster that reaches from one side of the forest to the other. The trees in such a large cluster form a connected path that allows the fire to spread from tree to tree across the entire forest.

A cluster that reaches from one side of the forest to the other is said to *percolate*, and the model we have just described for a burning forest is called *percolation*. We say a cluster percolates when it forms a connected path from one side of the forest to the other. The cluster that spans the forest is called the "percolating cluster." (A "percolator" is a device for making coffee. In a coffee percolator, water percolates—*finds a connected path*—through the narrow spaces in a thick layer of ground coffee.)

Think again about the burning forest. When tree density is *low*, trees *do not* form a percolating cluster, and the fire does not spread very far. In contrast, when the density is *high*, trees *do* form a percolating cluster, and the fire spreads across the forest. But what happens in between? And what does a percolating cluster look like, anyway? Can a percolating cluster be a fractal?

To answer these questions, we study the method of growing a "forest" by hand. We call this method a **model**. Then we study the same model using the computer.

HandsOn 30: Growing a Scattered-Seed Forest

Work in pairs. You need:

- a pencil,
- a ruler,
- a sheet of plain paper or graph paper,
- a green marker (any color will do if green is not available), and
- a red marker (a pencil or pen will do).

First create a random forest on a square grid, with 25 cells in the grid.

1. Draw a 5×5 square grid on a piece of paper, with each square approximately 2 centimeters on a side. Save time by using graph paper if it is available.

2. One student places a finger on the uppermost left square while the other student flips a coin.

3. If the coin is heads, use the green marker to make a green dot (representing a tree) on the square. If the coin is tails, leave that square blank.

4. Move the finger to the next square to the right.

5. Flip the coin. If it is heads, draw a green dot on the square; if it is tails, leave it blank.

6. Repeat this process until the coin has been flipped once for each square in the grid.

> Q7.1: Now the model forest is grown. Does it look to you like a real forest? How is it different?

We are now ready to start a fire in our model forest, and see how far it spreads. The fire starts along the left edge of the forest and spreads from one tree to another if they are neighbors (up, down, right, left) on the grid. Here's how it works:

7. Place a red circle around each of the green dots (trees) that lie along the left edge of the grid (forest). A tree with a red circle is on fire.

8. Look at each tree that is on fire. Is there an unburned tree in the next square to the right? If so, draw a red circle around it, indicating that it has caught fire.

9. Now look at each burning tree. Is there an unburned tree in any neighboring square (up, down, right, left) ? If so, draw a red circle around it. This is how the fire spreads. (We are assuming that fire does *not* spread directly between two trees that are *diagonally* next to one another.)

10. Continue to "spread" the fire from each burning tree to any unburned tree in a neighboring square (up, down, right, left) until it can go no further, so that there is no unburned tree left next to a burning tree.

> Q7.2: Now you have burned your forest. Did the fire percolate? That is, did it spread all the way from the left edge to at least one tree on the right edge? Or did it stop part way across the grid?

11. Talk to the other teams in your class. In what percentage of the forests did the fire percolate?

> Q7.3: Now you have seen what happens in a model forest where about 50% of the squares are occupied by trees. What would happen if there were more trees? Would the fire be more or less likely to spread all the way across?

Grow a new forest on the same size grid, but this time roll a die for each square, placing a tree if the die comes up 1, 2, 3 or 4 and leaving the square blank if the die shows 5 or 6. Now *predict* what you expect to happen when you start a fire along the left edge. Will more or fewer fires burn across to the right side of the forest than before? Try it!

END ACTIVITY

SimuLab 16: Modeling a Scattered-Seed Forest

Now play **BlaZe**, the computer game of forest management and fire fighting. Here is how the game works. You are responsible for growing and harvesting trees on a plot of land. The more trees you harvest, the higher your income. Unfortunately, your forest is located in a high-risk fire region; when the forest is grown, a blaze begins along the left edge. The more trees survive the fire, the more your profit.

The **BlaZe** program allows you to select the density of tree growth, the probability, labeled p, that a tree will grow on each site. In the preceding activity, you flipped a coin, so the probability was one half, or $p = 0.5$. In **BlaZe** you can select the value of p between 0 (no trees) and 1 (a tree on every site). You can also select the dryness of the forest, which determines how fast the fire spreads.

What value of p, the tree probability, should you use? If you plant trees at low probability, trees will usually be separated from one another (low density), so the fire will not spread. As a result, not many trees will burn, but you won't harvest many trees either, since there are not many trees altogether. On the other hand, if you use a high tree probability, the trees will typically be next to one another (high density), fire will spread across the forest, and again you won't have much of a harvest. The trick is to find a particular tree density (a particular probability p) that gives you the chance for a large harvest but allows you to control the spread of fire.

In addition to controlling tree density, you can fight the fire by dumping water on individual trees. Your job is to maneuver your helicopter and drop water to stop the spread of the fire. The helicopter moves to the position where you place the cursor and drops water when you click the mouse. Any tree that you dump water on does not burn. The idea is to wet trees ahead of the fire to stop it from spreading. You have a limited amount of water to dump on the forest.

When the fire has burned out, your score is shown in the lower right of the screen. The score depends on the number of unburned trees and the dryness of the forest.

Q7.4: If you are using these materials in a classroom setting, as many class members as possible should try the **BlaZe** program, selecting different tree probabilities and dryness settings in order to win the highest score. Get the whole class together and discuss your results. Who got the highest score? Did this person have a strategy, or did it just happen? Was there more than one winning strategy? Go back to the game and test the strategies. Does the strategy work for other players? If there is more than one strategy, which one is best?

Q7.5: What is the largest tree probability p at which you can keep the fire from reaching the opposite side (right side) of the forest, no matter how effective your strategy? Do you "run up against a wall" at some value of p, finding it impossible to keep the fire from crossing the forest for higher values of p?

END ACTIVITY

SimuLab 17: Gathering Statistics in BlaZe

What if no one is around to fight the fire after the forest is grown? (You may own so many forest plots in different locations that you cannot fly the helicopter over each one in person!) When there is no active fire fighting, we call the resulting forest *unmanaged*.

Q7.6: What is your best strategy for maximizing the number of trees that survive fire in an unmanaged forest?

Investigate this question by using the **Gather Statistics** command under the **Mode** menu. Now there will no longer be a water supply or helicopter to deliver the water. The forest just burns from the left without hindrance except for the barriers provided by empty spaces between trees. The number of spaces depends on the setting of the probability p that determines the density of trees in the forest.

Try various probability settings. You can use the **Continuous** button to set up a number of automatic runs at a given probability. The results are summarized for you in the **Table** window under the **Windows** menu. The table automatically records data from every run made in Gather Statistics mode. You can take averages of any set of runs, as well as delete runs from the table. You can also print the table using the **Print Table** command menu, or export the table into a commercial spreadsheet using **Export Table**. Both of these commands are under the **File** menu.

> Q7.7: Which value of the probability p gives you the greatest number of surviving trees, on the average, in an unmanaged forest?

One column in the table records whether or not the fire burns all the way from one side of the forest to the other side. A graph of these cases is titled **Fraction Burned Across** in the **Windows** menu.

> Q7.8: Is there any relation between the probability that gives you the most surviving trees and the lowest probability at which the fire burns across the forest most of the time?

END ACTIVITY

HandsOn 31: Growing a Forest from a Single Tree

An old, mighty oak tree stands alone in a field. Much of the ground is covered with randomly-scattered rocks. Autumn arrives, and the tree drops acorns on the ground nearby.

One acorn lands on a rock and dies. Another acorn lands on dirt, takes root, and a new tree grows.

The next autumn, every tree (both the original tree and all its "children") drops acorns on the ground next to it. Again, an acorn that lands on a rock dies. An acorn that lands on an existing tree dies also. But an acorn that lands on dirt takes root and grows.

As the years go by, generation after generation of oaks spread away from the original mighty oak. If the area is rocky, with only small patches of exposed dirt, the trees probably won't spread very far. That's because the spreading cluster of oaks has to stop when it is surrounded on all sides by rocks. But if the area is not rocky, has lots of exposed dirt, the cluster of trees spreads and spreads. It does not stop until it reaches the edge of the field.

We start by growing a "forest" by hand. Then we will go on to grow the forest using a computer simulation.

For this exercise, you might want to work with a partner. Each pair will need:

- a checkerboard,
- a set of red and black checkers, and
- a handful of pennies, 20 or so.

> Q7.9: *Predict!* What do you expect will happen? What will your pattern of trees look like when it is finished? Will your pattern of trees look exactly like everyone else's pattern? As your forest grows, do you think it will reach the edge? What might keep it from reaching the edge?

Now carry out the following steps:

1. Place a red checker on a square near the center of the checkerboard. This stands for the old, mighty oak whose acorns start the forest growth.

   Now imagine that this tree drops acorns on the neighboring squares, up, down, right, and left. For each acorn, we must ask: does it grow into a new tree? Or is that square occupied by a big rock where no trees can grow?

Let's assume that for each neighboring square there is a 50% chance that a tree grows, and a 50% chance that the square contains a big rock. That is, the "tree probability" is 50%, and the "rock probability" is 50%.

2. Shake up four pennies between your cupped hands and, without looking at them, put one penny in each of the four neighboring squares right, left, up, and down (see Figure 7.1).

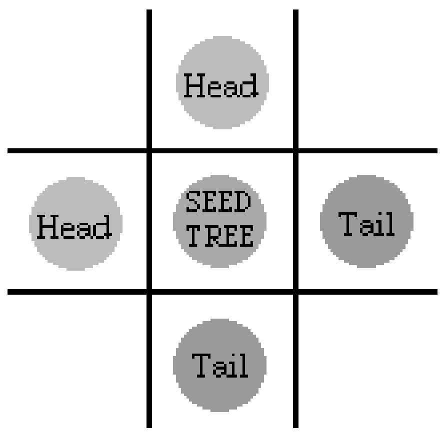

Figure 7.1: Checkerboard with 4 pennies and 1 checker.

3. Look at the four pennies. A head means a new tree; a tail means a rock. Replace each head with a red checker (tree), each tail with a black checker (rock).

4. Shake some pennies up again and place one in every empty square next to a red checker (tree) using the squares up, down, right, and left of the tree.

5. Again replace each head with a red checker (tree), and each tail with a black checker (rock).

6. Repeat steps 4 and 5 until the forest reaches the edge or cannot grow any further (all trees are surrounded by rocks).

On the board in front of the class, draw a table with two columns. Label the first column “Reach edge” and the second column “Not reach edge.” As each pair of students completes growing the forest, they report on whether or not their forest reached the edge. Enter their result in the table. Then each pair starts growing a new forest from the beginning. When each group has completed growing three forests, stop the activity and look at the table on the board. Have a discussion about the following questions:

Q7.10: Why don’t you get the same pattern every time?

Q7.11: Why do some of the patterns reach the edge and others do not? Why don’t they all behave the same?

Q7.12: If we grew 1000 forests, what fraction of these forests would reach the edge? Do you have a definite percentage in mind?

Q7.13: Would you expect the patterns to be different if we used a single die instead of a coin, and placed a tree on the site for numbers 1, 2, or 3 and a rock for numbers 4, 5, or 6? Would a greater or smaller fraction of the forests reach the edge than for the original game? Explain the reasoning behind your answer.

Q7.14: Suppose we throw a single die and place a tree for number 1 but a rock for numbers 2, 3, 4, 5, or 6? In this case would a greater or smaller fraction of patterns reach the edge?

Q7.15: Suppose we throw a die and place a tree on the site for numbers 1, 2, 3, 4, or 5 and a rock for number 6 only. How will this change the pattern of the forest? Will it increase or decrease the chance of the forest reaching the edge?

Q7.16: When the tree probability drops from 0.5 to 0.1, is the forest more likely or less likely to reach the edge? What about when the tree probability rises from 0.5 to 0.9. Is growth to the edge more likely or less likely in this case? Suppose we grow many forests, each with a different tree probability, starting at 0.1 and increasing to 0.9. Will the fraction reaching the edge increase or decrease as tree probability increases? Will this increase or decrease take place gradually as probabilities get bigger or suddenly at a particular probability?

END ACTIVITY

HandsOn 32: Jello Experiment

We have found percolation to be useful for understanding how forest fires spread and how clusters of trees grow, but it can also be used to understand many other problems. One example is gelation, the process by which a liquid solution of gelatin (made of long-chain molecules—polymers) turns into a gel. Jello brand gelatin, a popular dessert, is a gel.

To make Jello, one starts with a solution of long-chain polymers and water. During the process of gelation, the polymers link together into larger chains. The chains become interconnected and form a connected path across the sample, they can act as “bags” to contain the water, creating a gel instead of a liquid. A liquid pours; a gel wiggles.

In the same way, if clusters of trees form a connected path across a forest, then there is a good chance that a fire will burn across the forest. Thus, connectivity is the key idea for both forest fires and gels.

What gives Jello its strange properties? It is similar in some ways to a liquid and in other ways to a solid. The ideas you have explored earlier in this chapter will help you to understand the characteristics of Jello, and gels in general. In particular, there is a critical concentration of gelatin molecules that makes Jello into a "wiggly solid," which should be compared with the critical probability in the case of percolation. The critical concentration is needed to create the bag compartments that contain the water, thus making jello into a wiggly solid. Below the critical concentration the bags do not form a connected path across the structure, the water is not contained, and the substance flows as a liquid.

In the laboratory, experiment with different concentrations of gelatin to see if you can find the lowest concentration that turns the liquid to a wiggly solid.

Q7.17: How does raising the temperature affect your results?

END ACTIVITY

SimuLab 18: Percolating Forests

1. Start the application called **BlaZe**.

2. Choose a tree density, here called p, with a value of 0.500 by using the mouse to click on the numbers. This means that the probability is one-half that a given square will have a tree on it; the same as the probability that a coin flip will be heads.

3. Click on **Go** and see what happens. A picture of the forest appears on your screen, with trees shown in green. A fire starts burning on the left side of the forest and spreads toward the right. As each tree burns it turns red, then the ashes left behind turn blue.

4. There is a helicopter in this game that we will use later on to try to stop the fire from spreading. For now, though, just watch the forest burn. Don't move the mouse or click it until the fire burns out.

5. How far does the fire spread across the forest? Does it percolate—that is, does it reach the opposite side of the grid?

6. Grow and burn more forests using a tree probability of 0.500. Do any of these forests have fires that percolate? Carry out at least 10 trials, and note in your lab book what percent of your trials produce fires that percolate.

7. Repeat the experiment using tree probability $p = 0.600$. Burn 10 or more forests. What fraction of these $p = 0.600$ forests have a fire that percolates?

8. Repeat the experiment using tree probability $p = 0.700$. Burn 10 or more forests. What fraction of these $p = 0.700$ forests have a fire that percolates?

9. Stop and think. Do you notice a significant change in the number of fires that percolate between probabilities $p = 0.5$ and 0.7? Guess: Will the number of fires that percolate be radically different for $p = 0.3$ than for $p = 0.5$? Will the number of fires that percolate be radically different for $p = 0.9$ than for $p = 0.7$? Try a few at these lower and higher values of p to check out your guesses.

The tree probability p at which approximately half of the fires percolate is called the *critical probability.* Below the critical probability, the fire is unlikely to percolate, spread across the forest. Above the critical probably the fire almost always percolates.

Q7.18: What do you think is the critical probability for the forest modeled in **BlaZe**?

Q7.19: What's so special about the critical probability? If you were a forest ranger, you might want to keep the forest density below the critical probability, so that if a fire starts the whole forest won't burn down.

Q7.20: Can you describe the relationship between the different test tubes of Jello at various concentrations and the forest generated by the BlaZe simulation?

Now let's go back to the **BlaZe** program and test your skills as a forest ranger!

1. Start the **BlaZe** program again. Set the tree probability at $p = 0.600$.
2. Your job is to maneuver your helicopter and drop water to stop the spread of the fire. The helicopter moves toward the position where you place the cursor and drops water when you click the mouse button. Any tree that you dump water on does not burn. The idea is to wet trees ahead of the fire and to stop its spread. Are there certain places—so-called tree bridges—where you can wet just one or two trees and cut off the fire easily?
3. When the fire is finished burning, your score flashes on the screen. The score depends on the number of trees you saved, and the quantity of water used; your score decreases the more water you use to stop the fire. If your score is 0, you were not able to stop the fire from spreading. Try again!

Q7.21: What happens if you play with a tree probability of $p = 0.5$, or $p = 0.7$? Which of these cases leads to a higher score?

END ACTIVITY

7.1 What Do You Think?

Q7.22: If you were the owner of a tree farm, how would you maximize your profit by growing the largest number of trees, while taking account of the danger of fire? The answer depends on the number of *tree bridges*, strings of trees that connect one cluster to another. There are few tree bridges when there are few trees; this occurs when tree probability p is low. For low p you may not have to use any water at all: the fire just burns itself out in isolated clusters of trees. But few trees means low profit. So try the alternative: grow many trees by choosing a high tree probability p. But then there are many tree bridges, so many in fact that you may not have time to dump water on all of them before the fire arrives. So most of the trees burn and your profit is low again.

What is the trick to making a high profit? What tree density p should you use and why? (To encourage you to actually save trees, your score in the game is based on the number of *threatened* trees that you manage to save, along with a penalty for the amount of water used.)

Q7.23: Write an essay on the possible connection between the microscopic molecular interactions of various concentrations of Jello and the forest program **BlaZe** at various tree probabilities.

Q7.24: We noticed in the **BlaZe** program that the forest fire follows the "path of least resistance" by spreading through a connected network of trees. In an essay talk about systems in nature that may follow this same principle and explain the mechanisms involved. For example, electrons in electrical circuits follow the path of least resistance: if a circuit breaks off into two branches A and B where the equivalent resistance of A is 100 times the equivalent resistance of B, the flow of electrons through branch A will be 100 times less than the flow through B. Hint: Thinking of how fluids flow through sewer systems or how erosion occurs on a beach may help get you started.

Research Projects

Try the suggestion below, design your own, or write an essay using any of the questions throughout this chapter as inspiration.

BlaZe has some advanced features that permit further investigations on your own:

1. The burned trees display can be saved as a picture. It can also be exported as a file for the separate **Fractal Dimension** program. This allows you to measure the fractal dimension of the burned portions of different forests grown with various probabilities. This collection of save commands are available under the **File** menu.

2. Instead of starting the fire from the left, the fire can be started by random lightning strokes, using the command **Random Lightning** under the **Mode** menu. Lightning falls on a randomly-chosen unburned tree. The fire then burns all connected trees grouped together in that cluster. When that cluster is entirely burned, lightning falls on another randomly-chosen unburned tree. This continues until all trees are burned. A command in the **Mode** menu gives different colors to different clusters, making it easier to distinguish them from one another. A graph called **Cluster Sizes** available under the **Windows** menu plots the number of clusters of various sizes.