Option Basics

Robert Singarella

American vs European Style

- American Style Options:
  - May be exercised at any time before the option expires
  - All stocks and exchange traded funds (ETFs) have American-style options
  - Has absolutely nothing to do with geographic location
- European Style Options:
  - May only be exercised on the day they expire
  - Major indices (S&P 500, DJIA, FTSE 100, DAX, NASDAQ,…) have European-style options
  - You cannot buy an index, so index options are cash settled
  - Still has nothing to do with geography

In/At/Out-of-the-money ("Moneyness")

- An option is said to be "In the money" (ITM) if it currently has some intrinsic value
  - A call is ITM when the stock price is greater than the strike price
  - A put is ITM when the stock price is below the strike price
- An option is "Out of the money" (OTM) if it currently has no intrinsic value
  - A call is OTM when the stock price is below the strike price
  - A put is OTM when the stock price is above the strike price
- Sometimes, when the stock price is close to the strike price, you say the option is "At the money" (ATM)
- If the stock price is very far from the strike price you will sometimes hear it referred to as "deep in the money" or "deep out of the money"

Strike Price

- Usually denoted by "K"
- Generally listed in 0.5, 1, 2.5, or 10 point increments depending on price level
  - Example, AAPL trades at around $500 and so the strike prices are listed in $10 increments
  - AMD trades at around $4.00 and strike prices are listed in $0.50 increments
  - Details at (CBOE Option Specifications)
- Adjustments to a contract's size, deliverable and/or strike price may be made to account for stock splits or mergers

Other common symbols

- S_t – usually denotes the stock price at time t
  - Be careful because this S_t is sometimes used as a constant, a variable, and a stochastic process
- K – strike price
- r – risk free rate of return (annualized, continuously compounded)
  - More advanced models sometimes describe the risk free rate as a stochastic process
- A good rule of thumb is that a subscript 't' usually is present for a stochastic process rather than a deterministic function of time or a constant, but the stock price is an exception to this because the 't' is usually present regardless

More symbols (volatility)

- σ – **volatility**, generally the square root of the variance of the log returns over (a,t) is used, choice of 'a' is up to you
  - It makes sense that you should select 'a' so that $t-a > T-t$
- σ or σ_{IV} – **implied volatility**, this is what value of σ is required to make the model give you the correct answer
  - Always double check what σ refers to, the implied volatility is NOT a statistical measure based on past data
  - Implied volatility is a very good proxy for how large the risk premium for an option is. It gives you an indication of how much risk the market believes the investment carries.
  - Implied volatility is usually not equal to the measured volatility, in fact, you will most likely not find them to be equal or even close to equal

Symbols (cont.)

- t – current time (in years)
- T – usually used as the point in time when the option expires
- $\tau \equiv T - t$
- $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-z^2/2}\, dz$ (Standard Normal Cumulative Distribution Function)
- μ - the drift rate of the stock price in the Black-Scholes model
- Π - the value of a portfolio
- $V(t, S_t)$ – the price of a financial derivative as a function of time and the *stochastic process* S_t
- $C(t, S_t)$ – the price of a European call option, the context will determine if S_t is a stochastic process or a simple variable
- $P(t, S_t)$ – the price of a European put option, S_t depends on context

Assumptions for Black-Scholes

- Black-Scholes prices European style options, not American style
- There exists a riskless asset and it has a constant rate of return, r.
- Assume that the instantaneous log returns of the stock price is given by geometric Brownian motion with a constant drift rate, μ, and a constant volatility, σ
- Assume that there is no arbitrage opportunity in the market (this can be thought of as saying that there is no way to game the market and make a riskless profit on the stock…sort of, look up arbitrage if you want to get a better idea of what it means; this is a required assumption)
- You can buy and sell any amount, including non-integer values, of the stock including short-selling (i.e. you can buy e^{π} shares of the stock if you want, for example)
- The market is frictionless (assume there are no transaction costs or fees)

The Black-Scholes Equation

- $$\frac{\partial V}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$
- Boundary conditions for a call: C(t,S) and S is a number
- $C(t,0) = 0$ for all t
- $C(t,S) \to S \; as \; S \to \infty$
- $C(t,T) = max\{0, S - K\}$
- Note that you will replace V in the partial differential equation with C or P when you deal with a call or put respectively
- That means the differential equation holds for both calls and puts
- I'll show how to get the Black-Scholes equation on the board, but here is a link to a more advanced extension of the model for anyone interested

Black-Scholes Formula

- Black-Scholes Solutions (note: I am suppressing the subscript 't' in the stock price symbol in order to make it clear that in this context it is NOT a stochastic process)
- $d_1 \equiv \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S}{K}\right) + (T-t)(r + \frac{\sigma^2}{2})\right]$
- $d_2 \equiv \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S}{K}\right) + (T-t)\left(r - \frac{\sigma^2}{2}\right)\right] = d_1 - \sigma\sqrt{T-t}$
- $C(t, S \mid r, \sigma, K, T) = S\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$
- $P(t, S \mid r, \sigma, K, T) = C(t, S|r, \sigma, K, T) - S + Ke^{-r(T-t)}$
- $P(t, S \mid r, \sigma, K, T) = Ke^{-r(T-t)}\Phi(-d_2) - S\Phi(-d_1)$
- Note: the variables after the ' | ' in the call and put formulas above are the parameters that you need to price the option, but they are treated as constants when solving the Black-Scholes equation