

# **INTERDEPENDENCIES AND INTERCONNECTEDNESS IN THE GLOBAL FINANCIAL VILLAGE**

**Dror Y. Kenett**

**Department of Physics, Boston University**

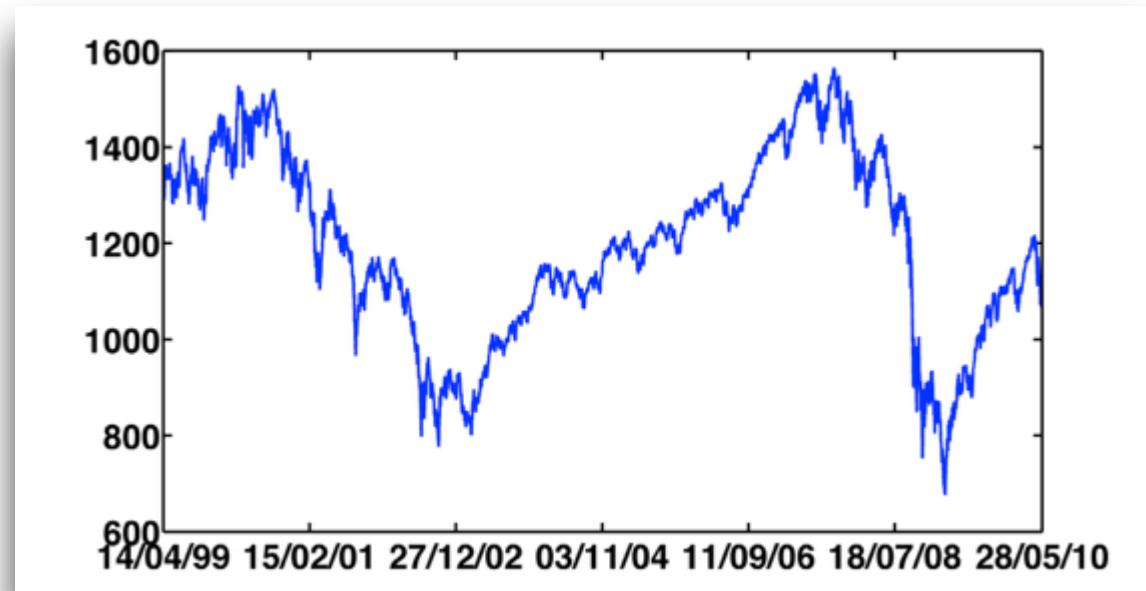
# **Outline**

- (1) Introduction**
  - Financial time series
  - Stock correlations
  - Dynamics of stock correlations
- (2) Global financial village**
  - Market intra and meta correlation
  - Financial Seismograph
- (3) Dependency and Influence**
- (4) Examples of network projects**
  - I. Cascading failures in industry networks
  - II. Overlapping communities in networks
  - III. Failure and recovery in networks
  - IV. Evolution of networks
  - V. Cascading failures in the financial system
  - VI. Interdependent networks
- (5) Discussion**

# Outline

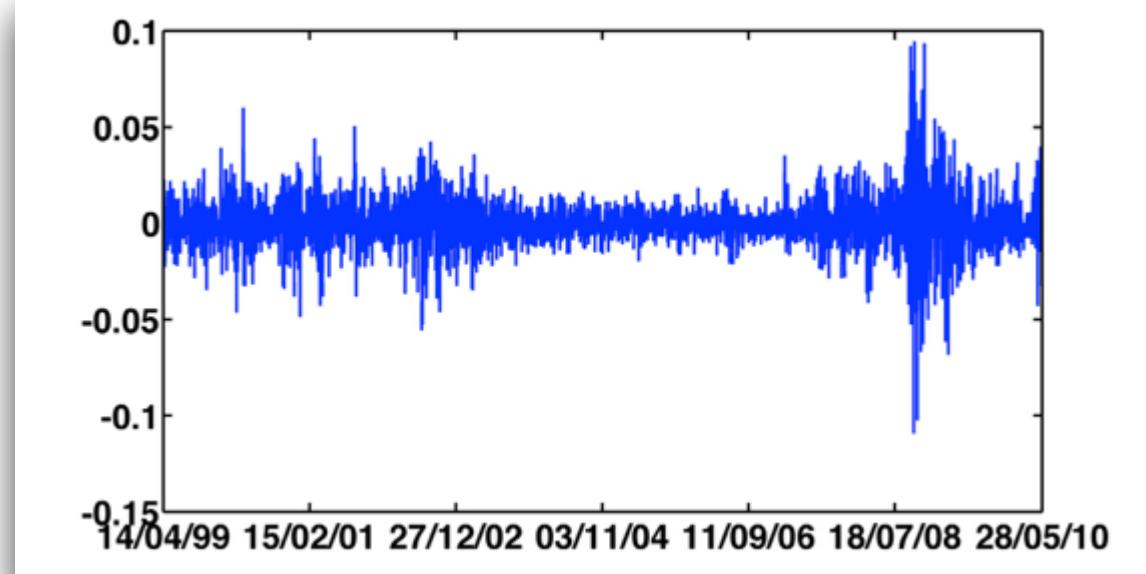
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# S&P500 Price



$$r_i(t) = \log[P_i(t)] - \log[P_i(t-1)]$$

# S&P500 Return

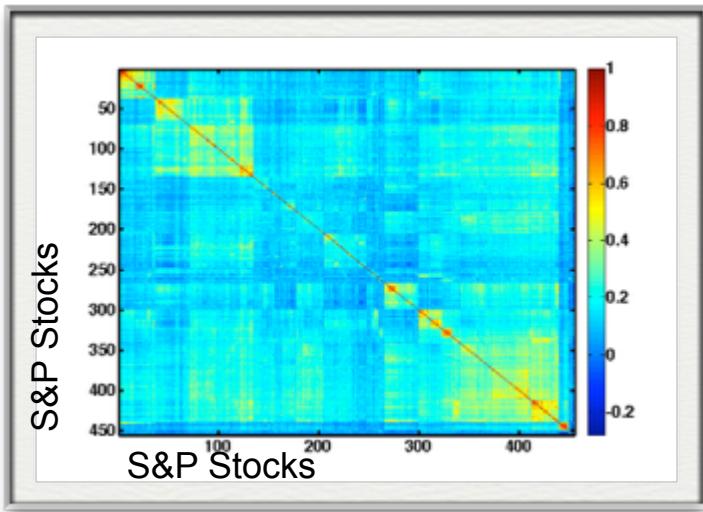


# Stock correlations

$$C(i,j) = \frac{\langle (r_i - \langle r_i \rangle) \cdot (r_j - \langle r_j \rangle) \rangle}{\sigma_i \cdot \sigma_j}$$

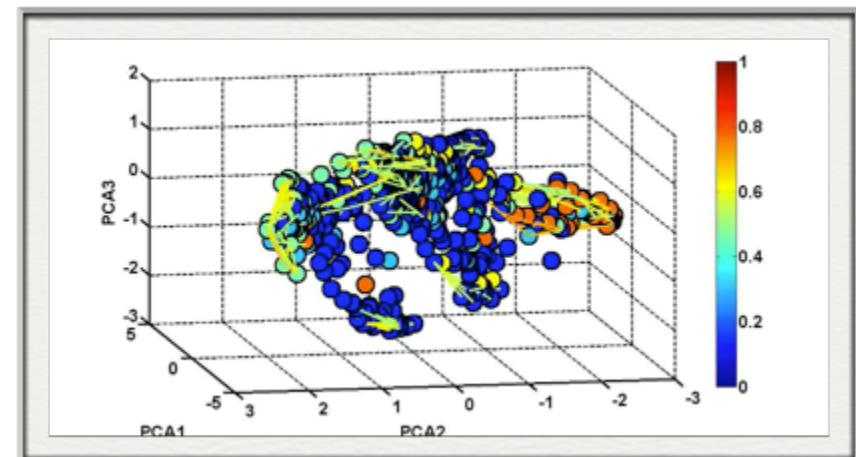
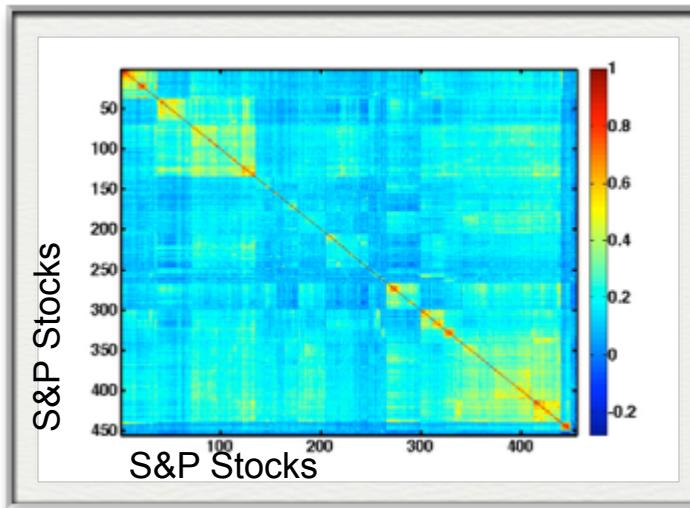
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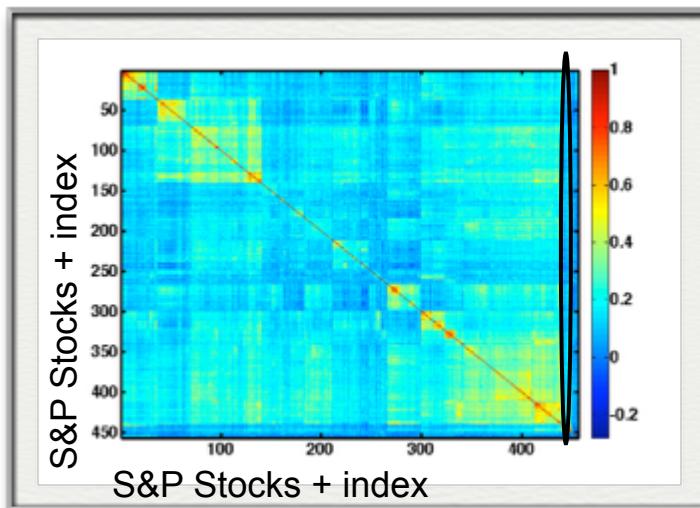
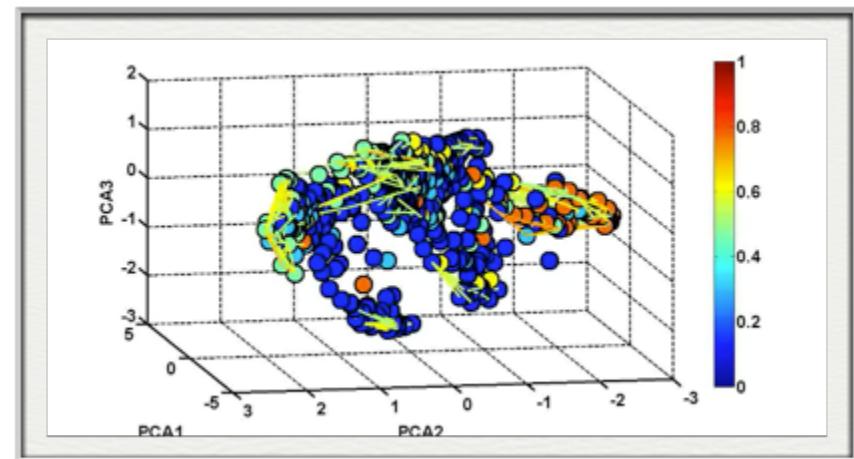
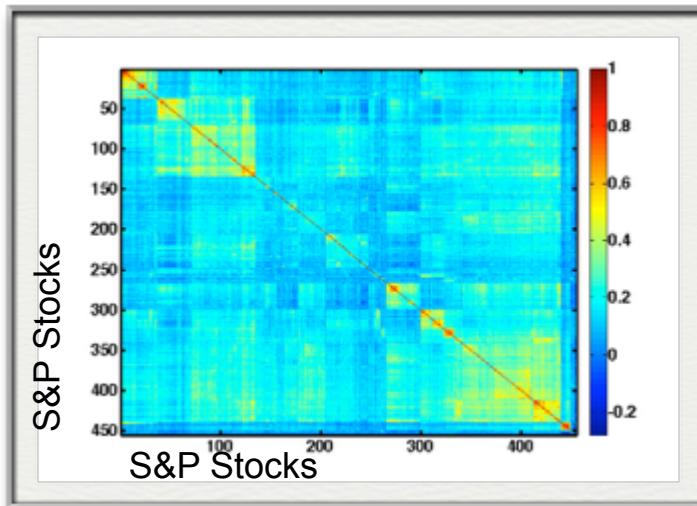
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# Stock correlations

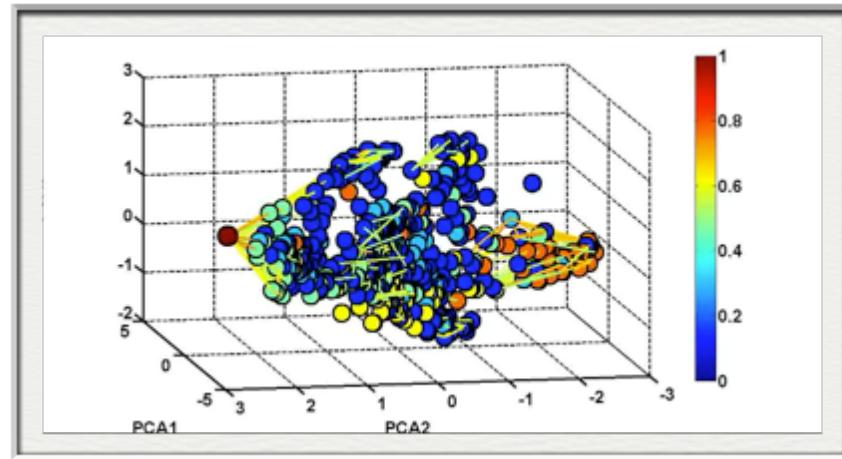
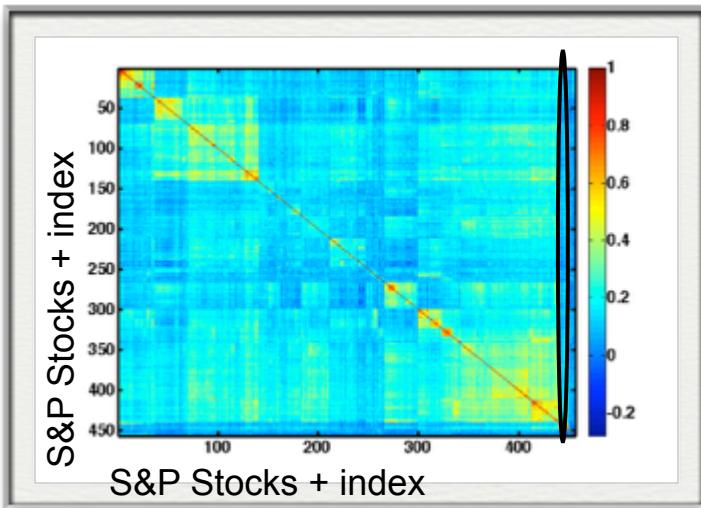
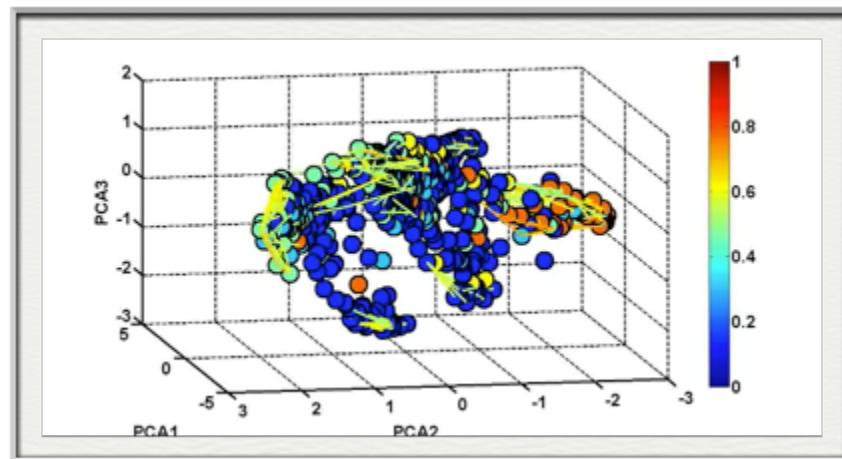
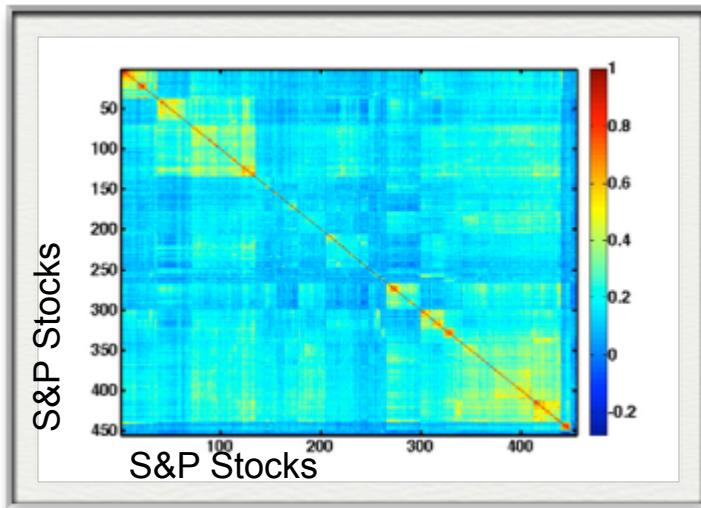
$$C(i,j) = \frac{\langle (r_i - \langle r_i \rangle) \cdot (r_j - \langle r_j \rangle) \rangle}{\sigma_i \cdot \sigma_j}$$



Scatter plot of S&P Stocks + index on the first two principal components (PCA1 and PCA2).

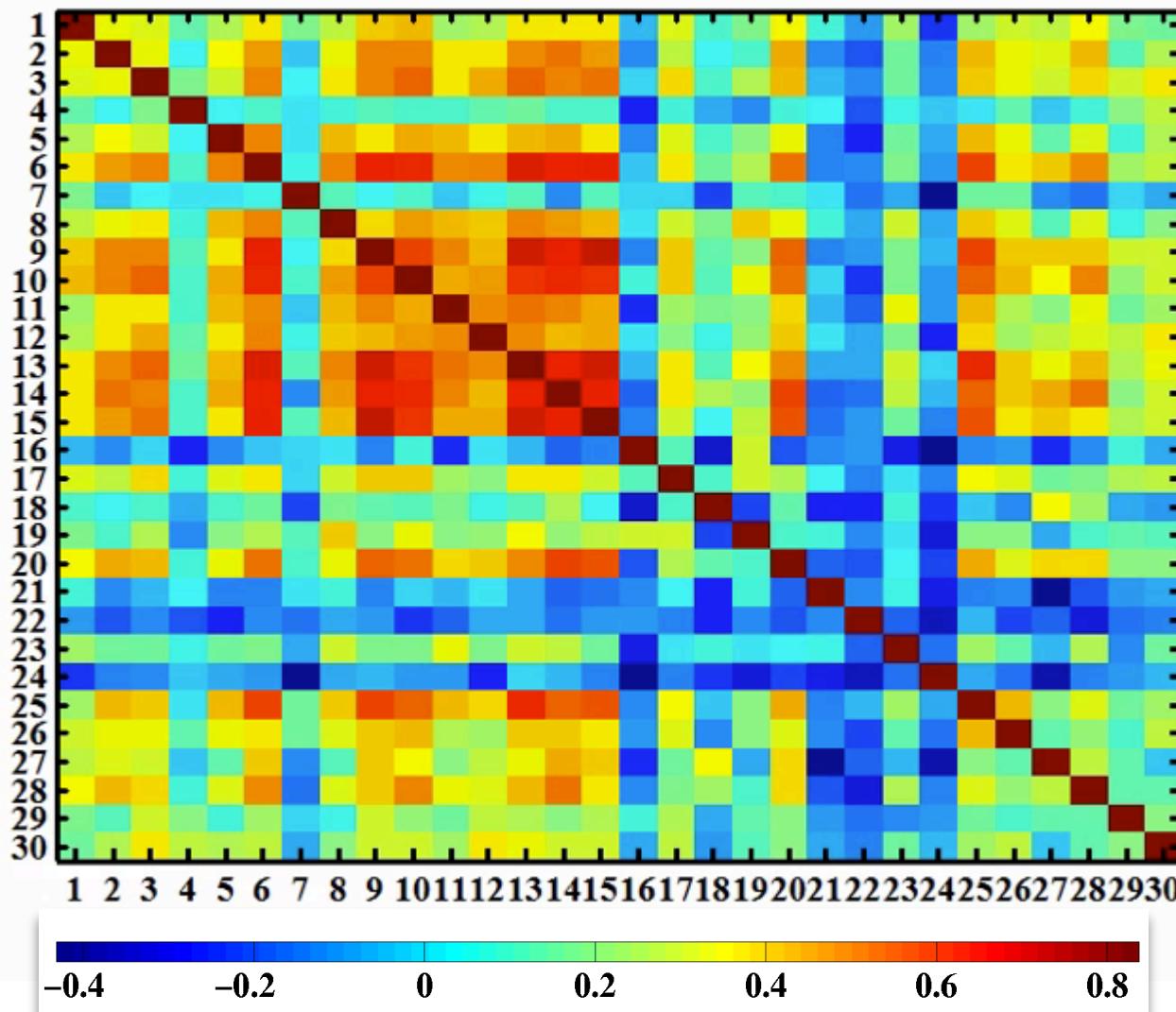
# Stock correlations

$$C(i,j) = \frac{\langle (r_i - \langle r_i \rangle) \cdot (r_j - \langle r_j \rangle) \rangle}{\sigma_i \cdot \sigma_j}$$



# Dynamics of stock correlations

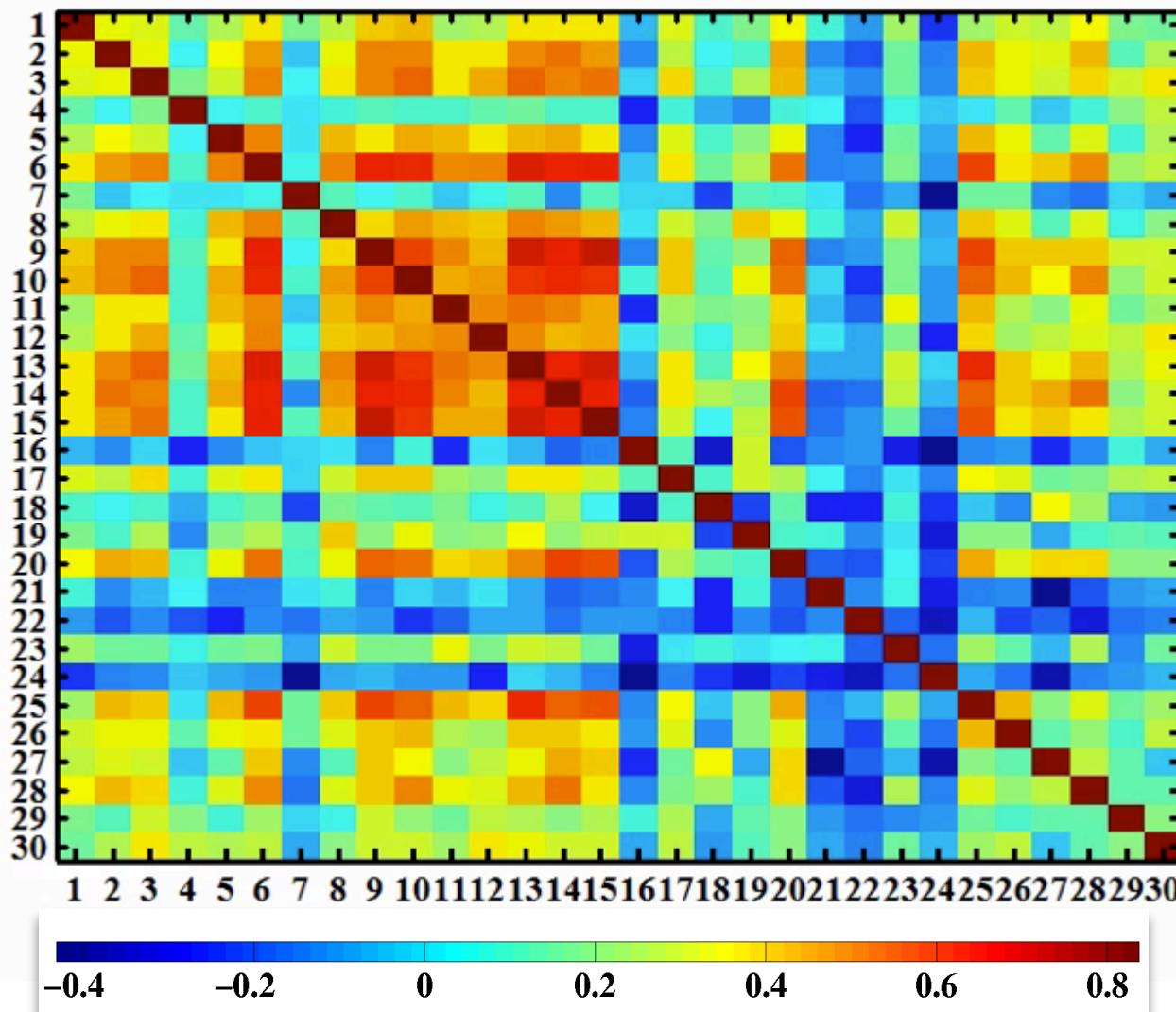
days = 1-101



Dow Jones  
Stocks

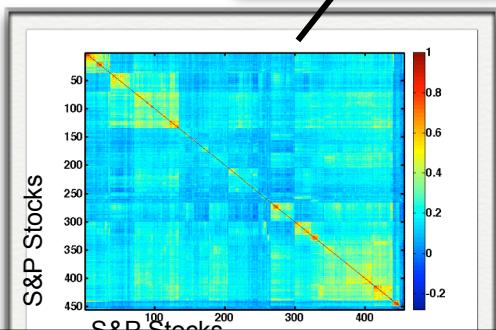
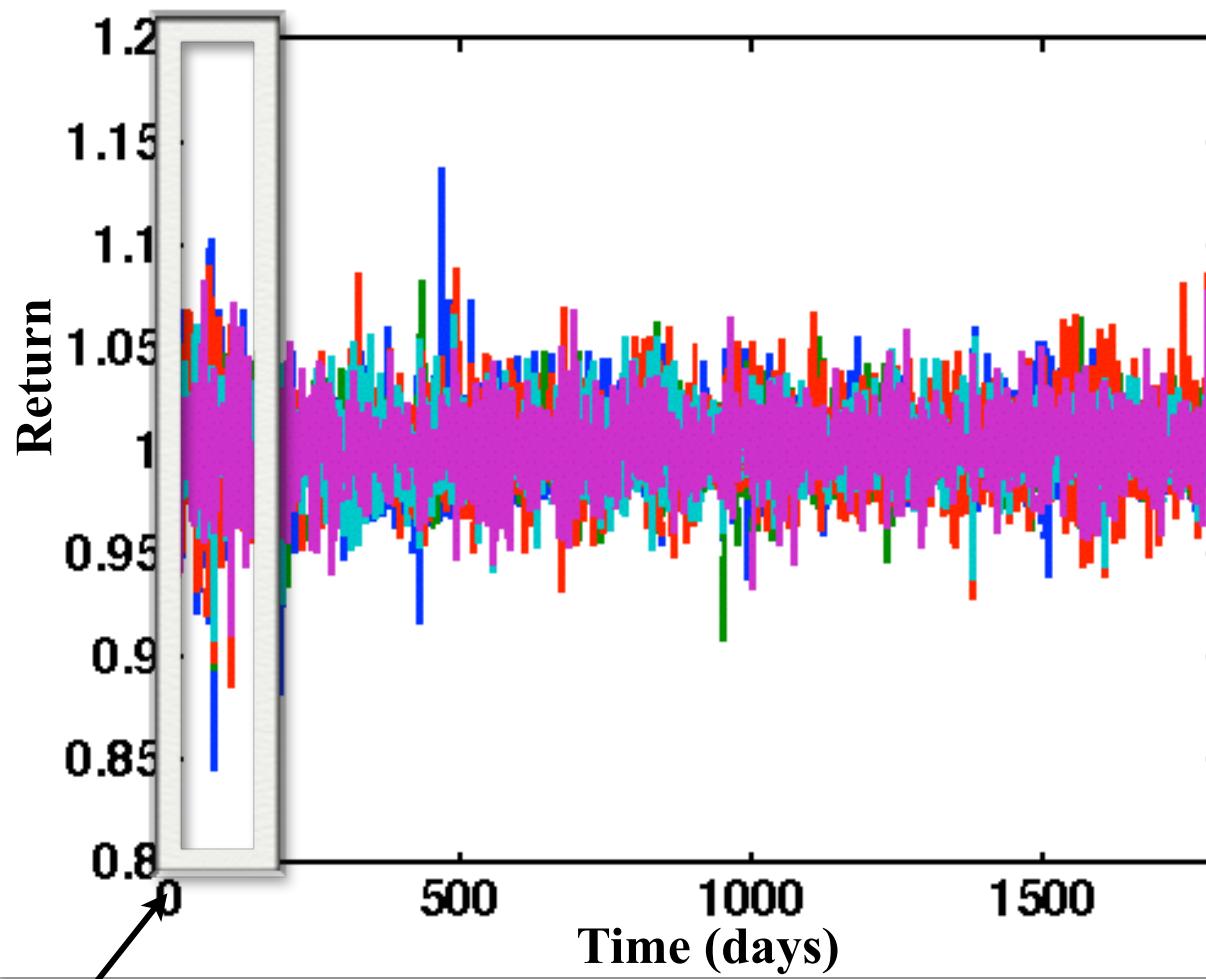
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Dow Jones  
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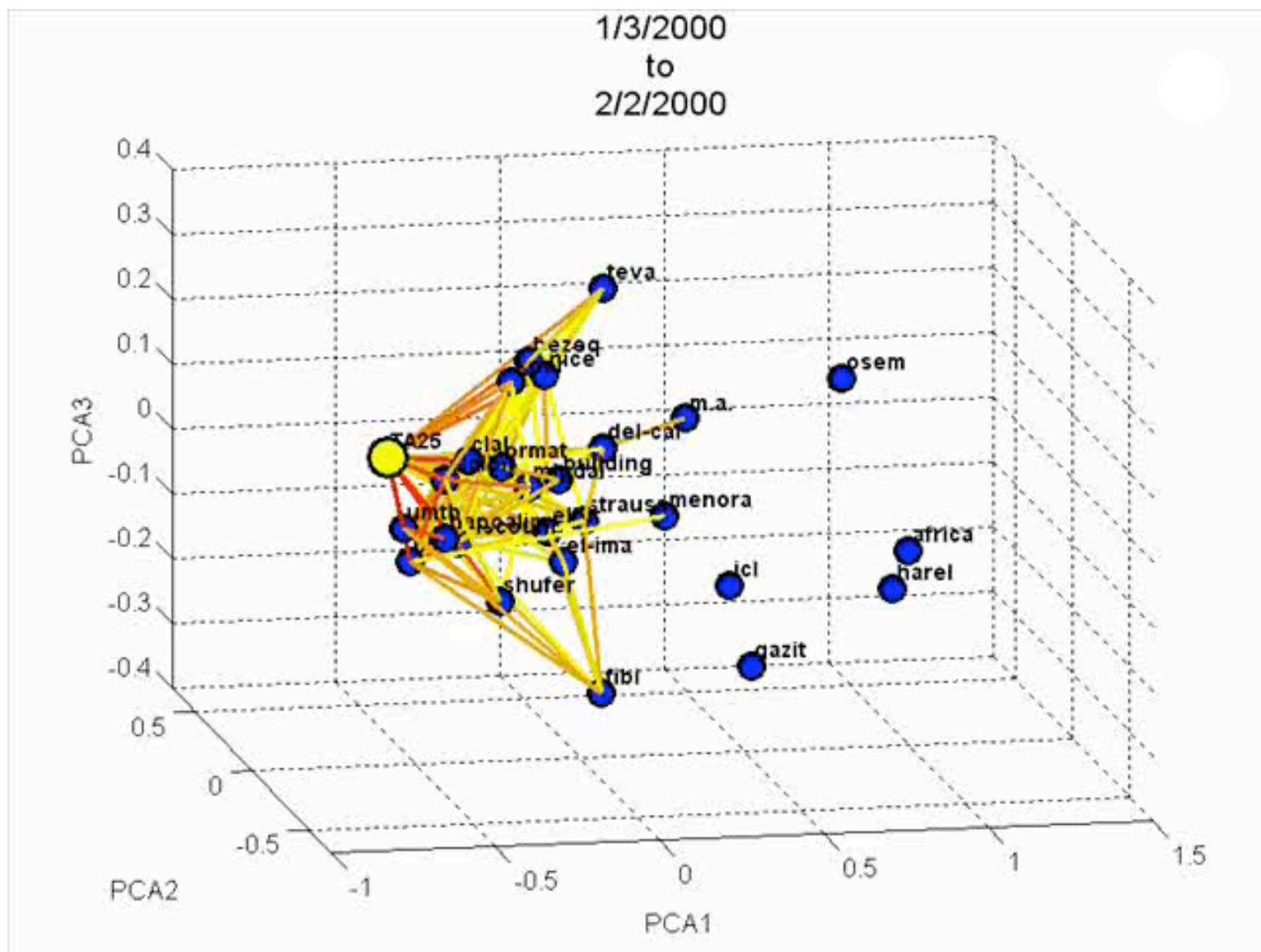
# Dynamics of stock correlations



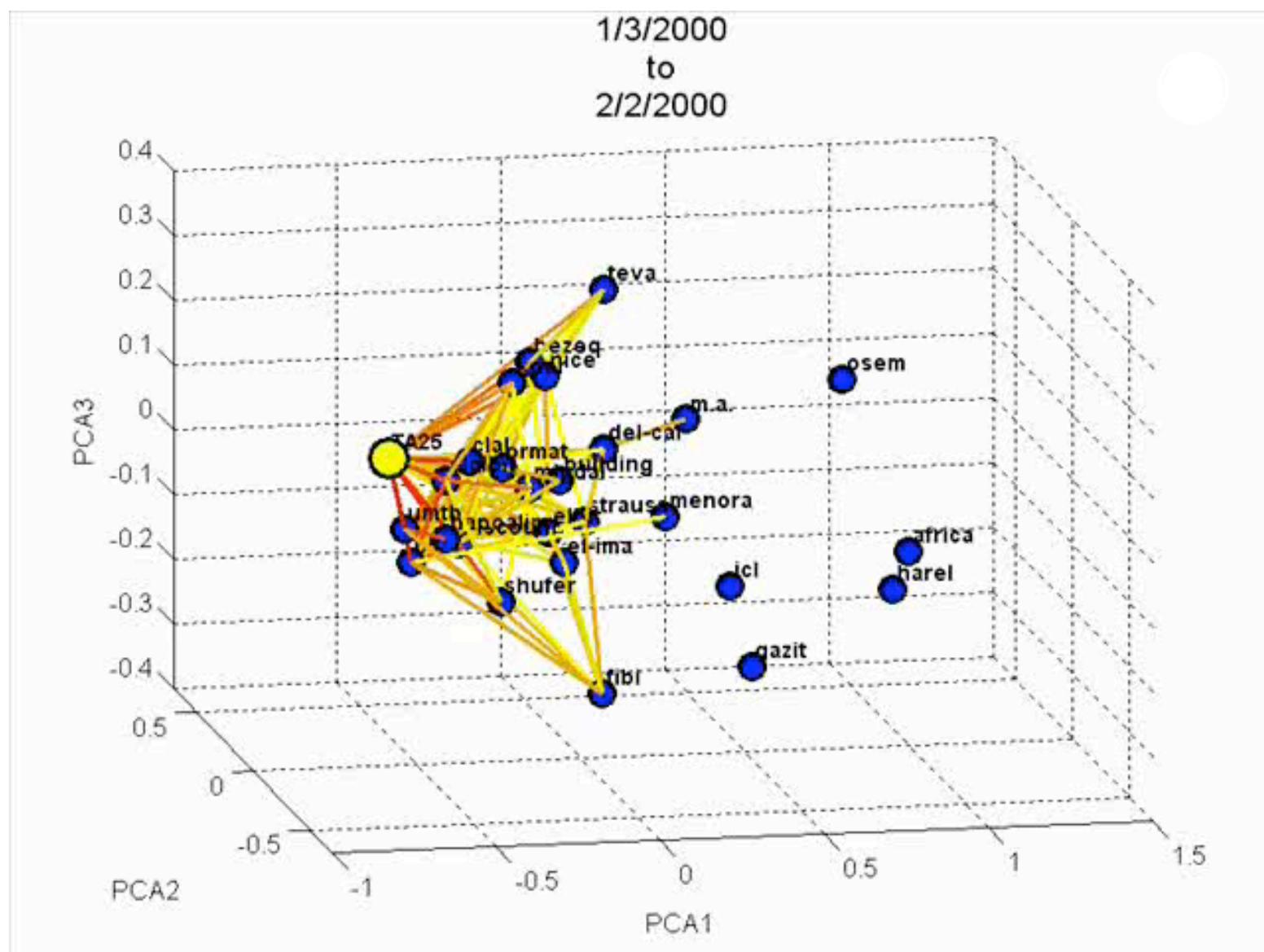
$\rightarrow \rightarrow \rightarrow \langle \overline{C(i,j)} \rangle$

The diagram shows a vertical arrow pointing to the right, which then points to a mathematical expression:  $\langle \overline{C(i,j)} \rangle$ . This expression represents the average value of the correlation coefficient  $C(i,j)$  across all pairs of stocks in the S&P index.

# Example: Tel-Aviv market

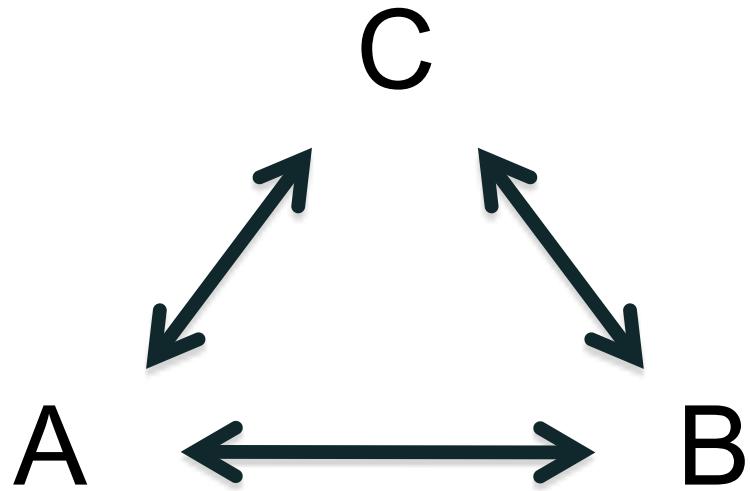


# Example: Tel-Aviv market



# Quantifying functional relationships

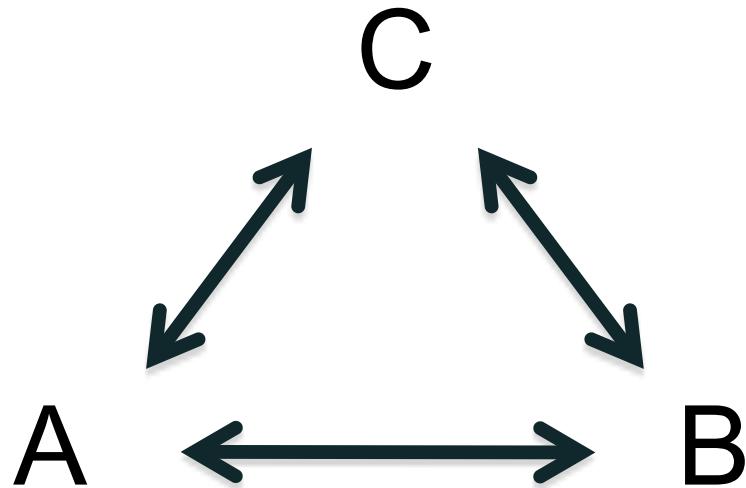
## Correlation



$$C(i,j) = \frac{\langle (r_i - \langle r_i \rangle) \cdot (r_j - \langle r_j \rangle) \rangle}{\sigma_i \cdot \sigma_j}$$

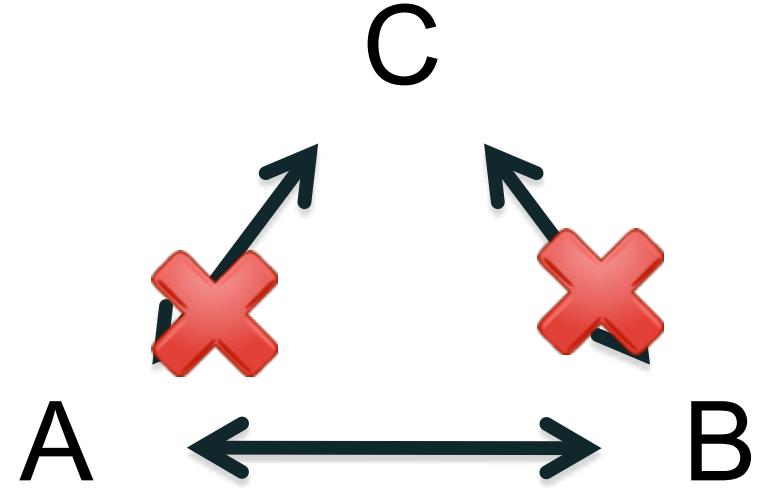
# Quantifying functional relationships

## Correlation



$$C(i,j) = \frac{\langle (r_i - \langle r_i \rangle) \cdot (r_j - \langle r_j \rangle) \rangle}{\sigma_i \cdot \sigma_j}$$

## Partial Correlation



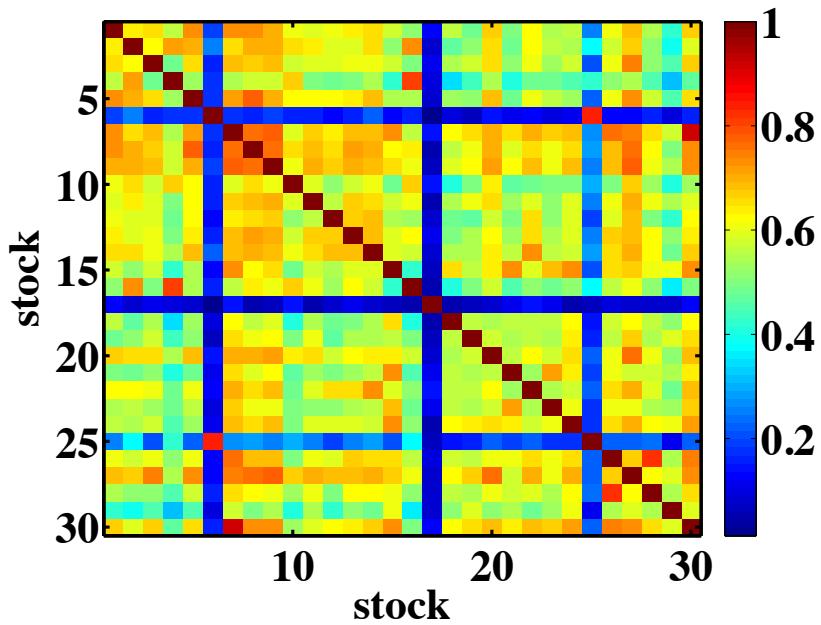
$$PC(i,j | m) = \frac{C(i,j) - C(i,m) \cdot C(j,m)}{\sqrt{(1 - C^2(i,m)) \cdot (1 - C^2(j,m))}}$$

## PARTIAL CORRELATION:

The partial correlation (residual correlation) between  $i$  and  $j$  given  $m$ , is the correlation between  $i$  and  $j$  after removing their dependency on  $m$ ; thus, it is a measure of the correlation between  $i$  and  $j$  after removing the affect of  $m$  on their correlation

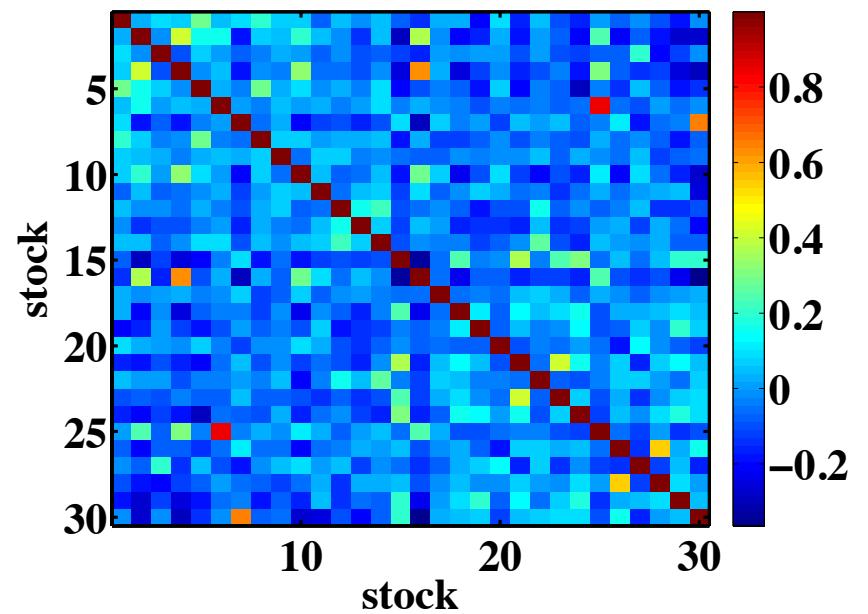
# Quantifying functional relationships

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$$C(i,j) = \frac{\langle (r_i - \langle r_i \rangle) \cdot (r_j - \langle r_j \rangle) \rangle}{\sigma_i \cdot \sigma_j}$$

## Partial Correlation



$$PC(i,j \mid m) = \frac{C(i,j) - C(i,m) \cdot C(j,m)}{\sqrt{(1 - C^2(i,m)) \cdot (1 - C^2(j,m))}}$$

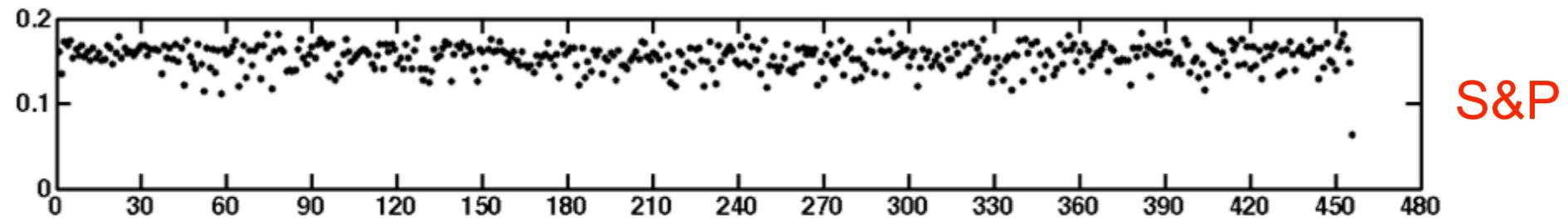
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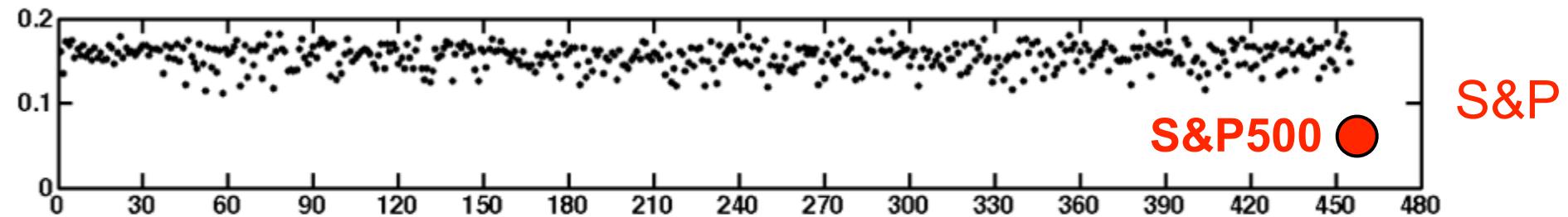
# **Partial Correlations Example**

Yoash Shapira, Dror Y. Kenett, and Eshel Ben-Jacob,  
Physical Journal B. vol. 72, no. 4, pp. 657-669 (2009)

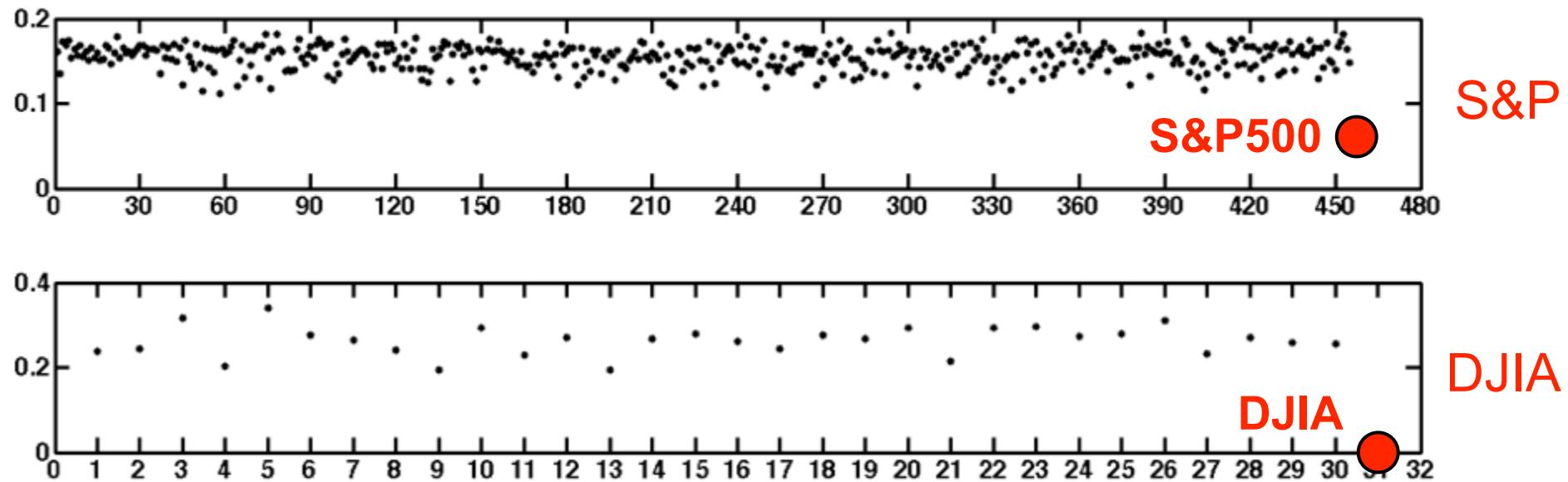
# Partial Correlations Example



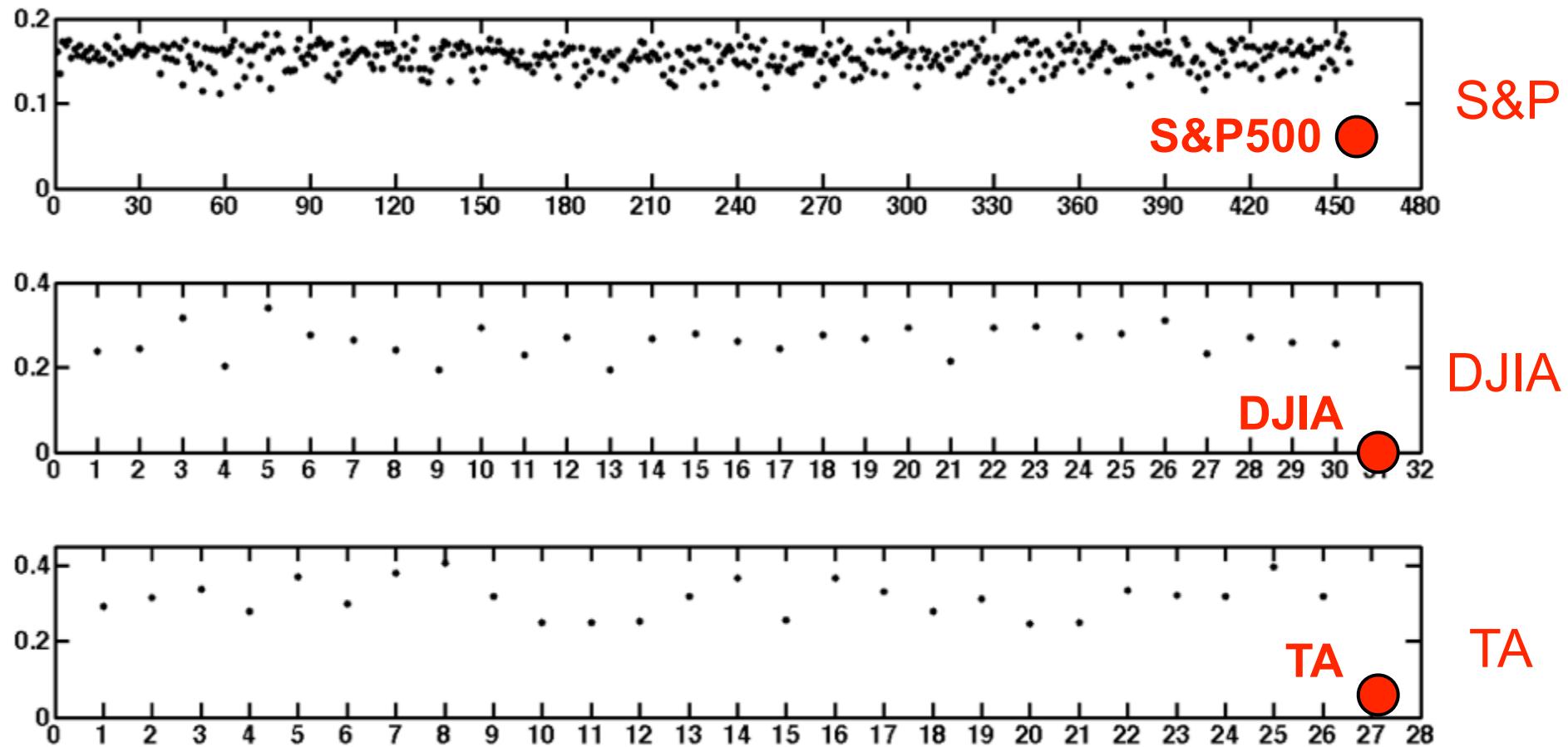
# Partial Correlations Example



# Partial Correlations Example



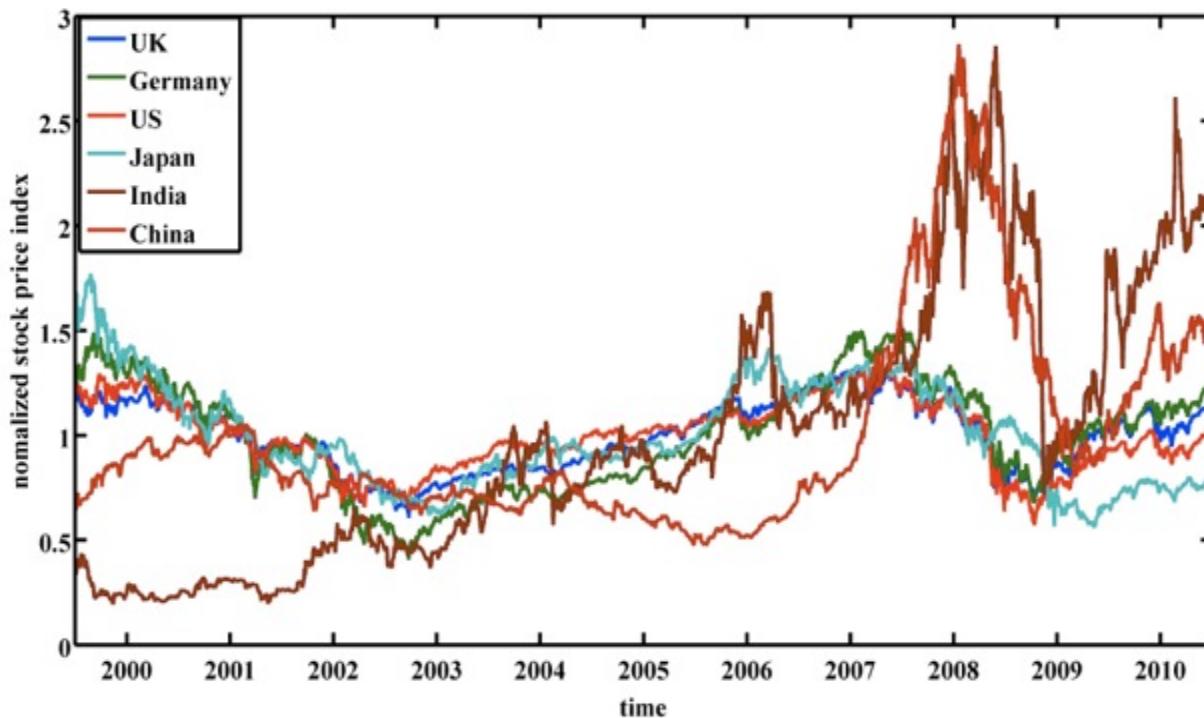
# Partial Correlations Example



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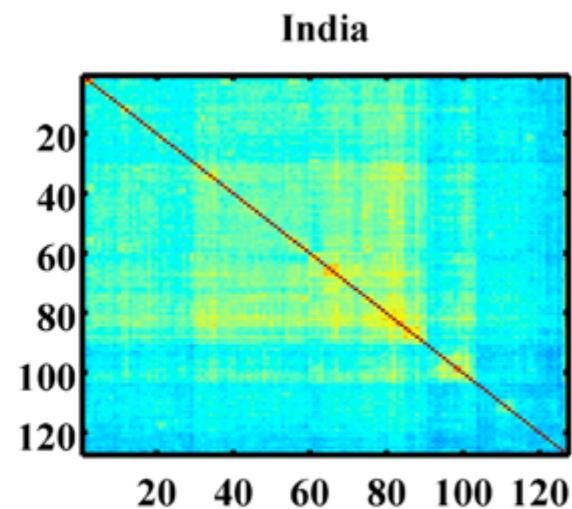
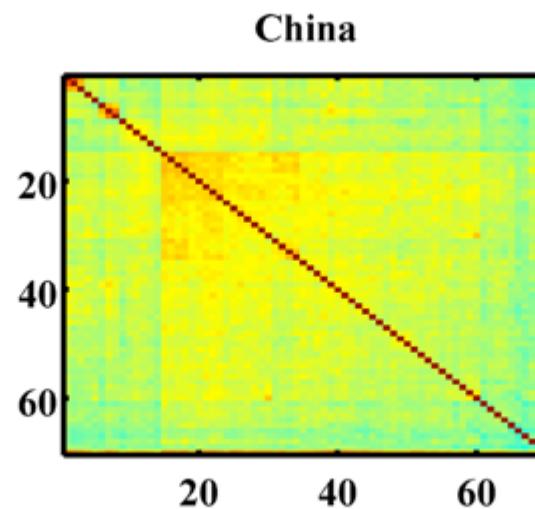
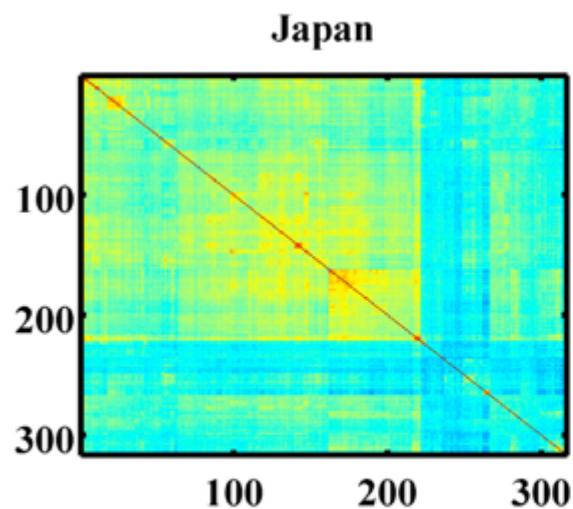
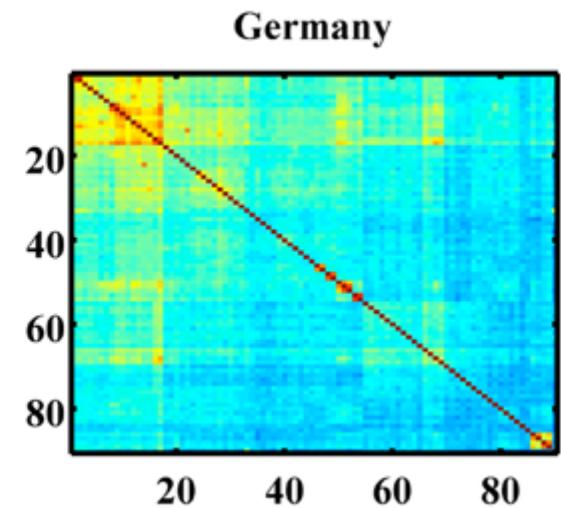
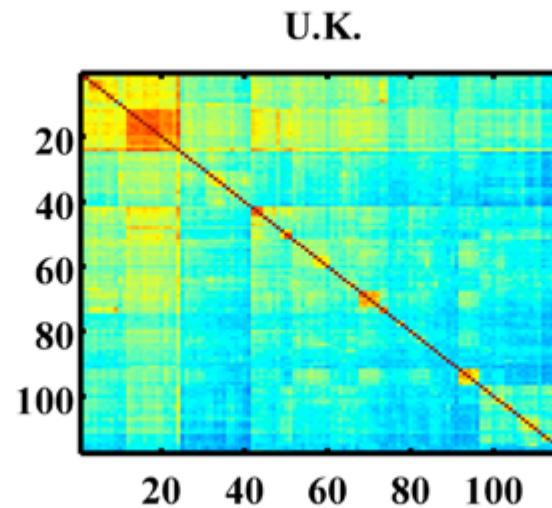
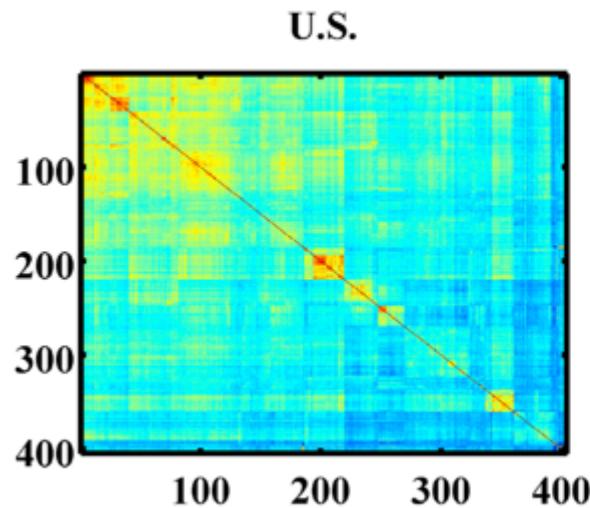
# Financial Global village



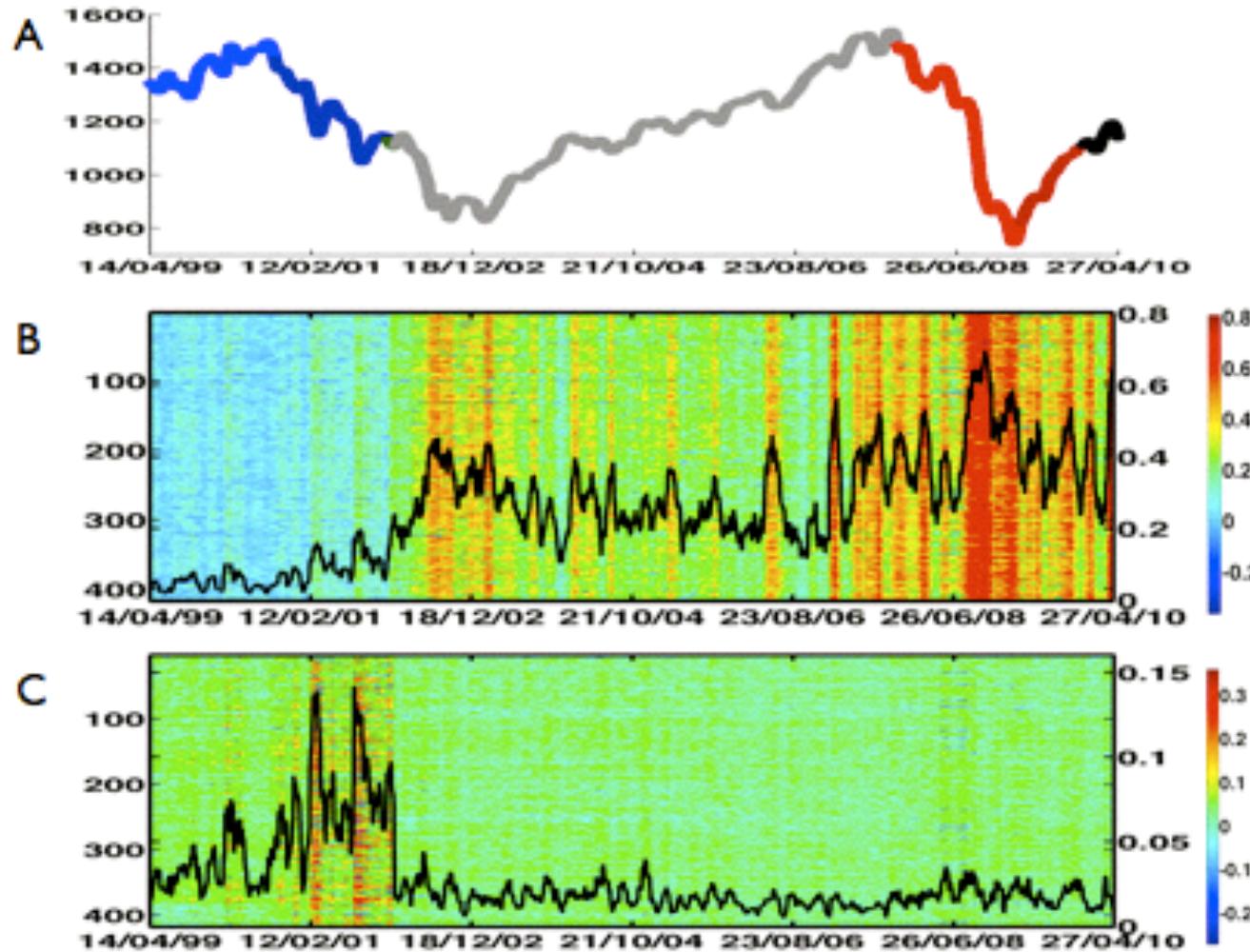
Market	Stocks used	Index used	# before	# filtered
US	S&P 500	S&P 500	500	403
UK	FTSE 350	FTSE 350	356	116
Germany	DAX Composite	DAX 30 Performance	605	89
Japan	Nikkei 500	Nikkei 500	500	315
India	BSE 200	BSE 100	193	126
China	SSE Composite	SSE Composite	1204	69

# Stock correlations

$$C(i,j) = \frac{\langle (r_i - \langle r_i \rangle) \cdot (r_j - \langle r_j \rangle) \rangle}{\sigma_i \cdot \sigma_j}$$



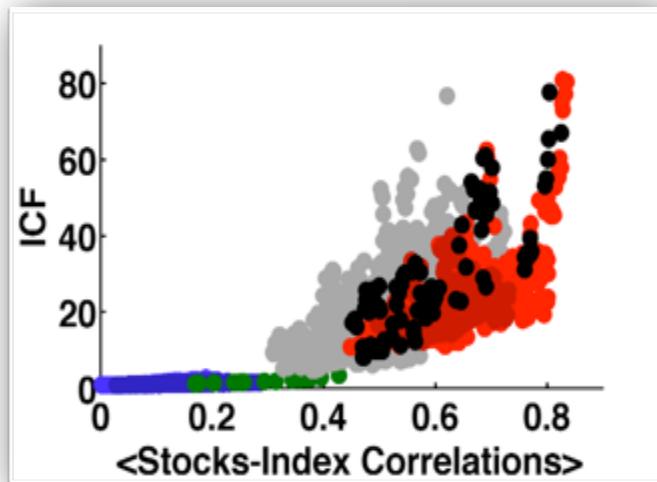
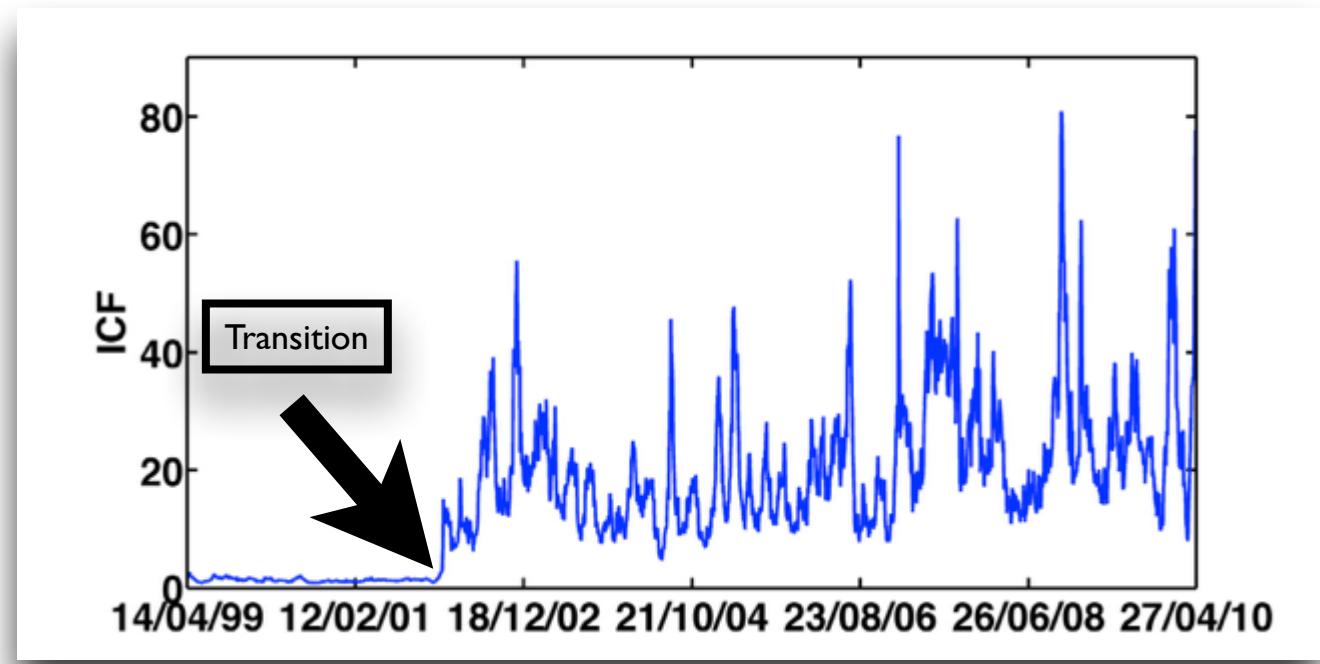
# Example: Dynamics of correlations of S&P500 stocks, in the US market



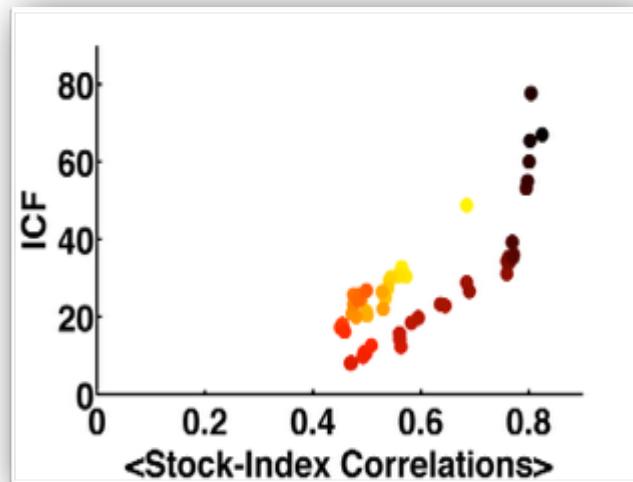
Dror Y. Kenett, Yoash Shapira, Asaf Madi, Sharron Bransburg-Zabary, Gitit Gur-Gershgoren, and Eshel Ben-Jacob (2011), Index cohesive force analysis reveals that the US market became prone to systemic collapses since 2002, PLoS ONE 6(4): e19378

# Correlations of S&P500 stocks, in the US market: Phase Transition?

$$ICF(\tau) \equiv \frac{\langle C(i,j) \rangle_\tau}{\langle PC(i,j | m) \rangle_\tau}$$

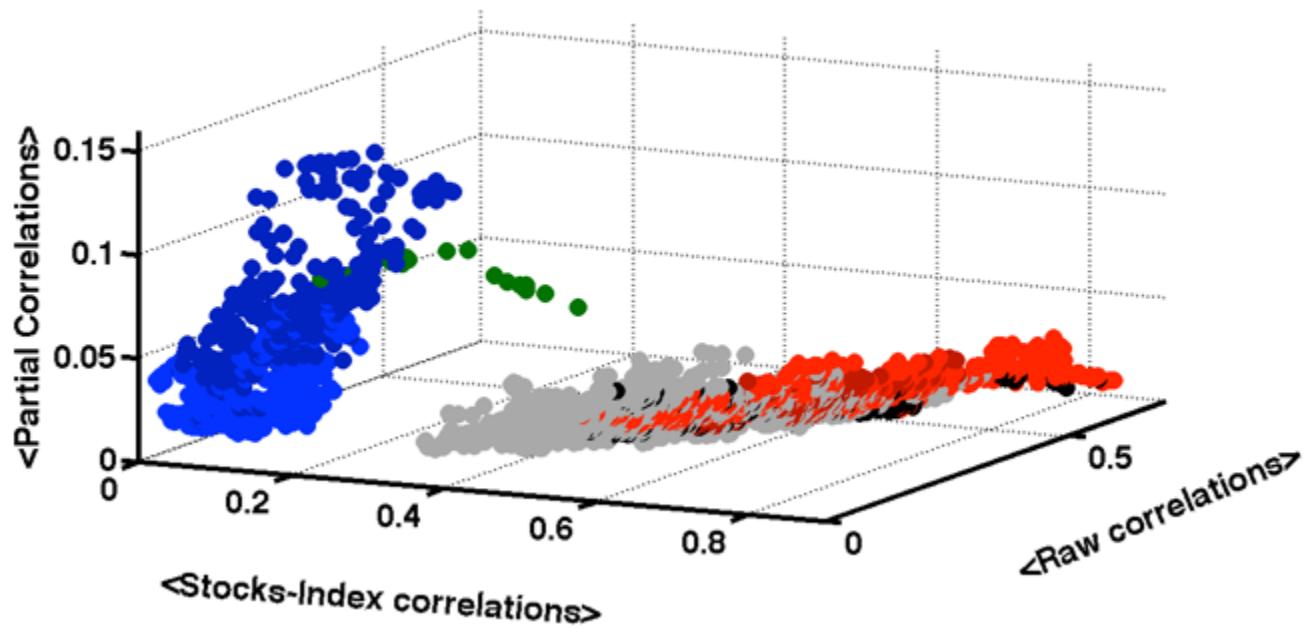


1999 - 2010

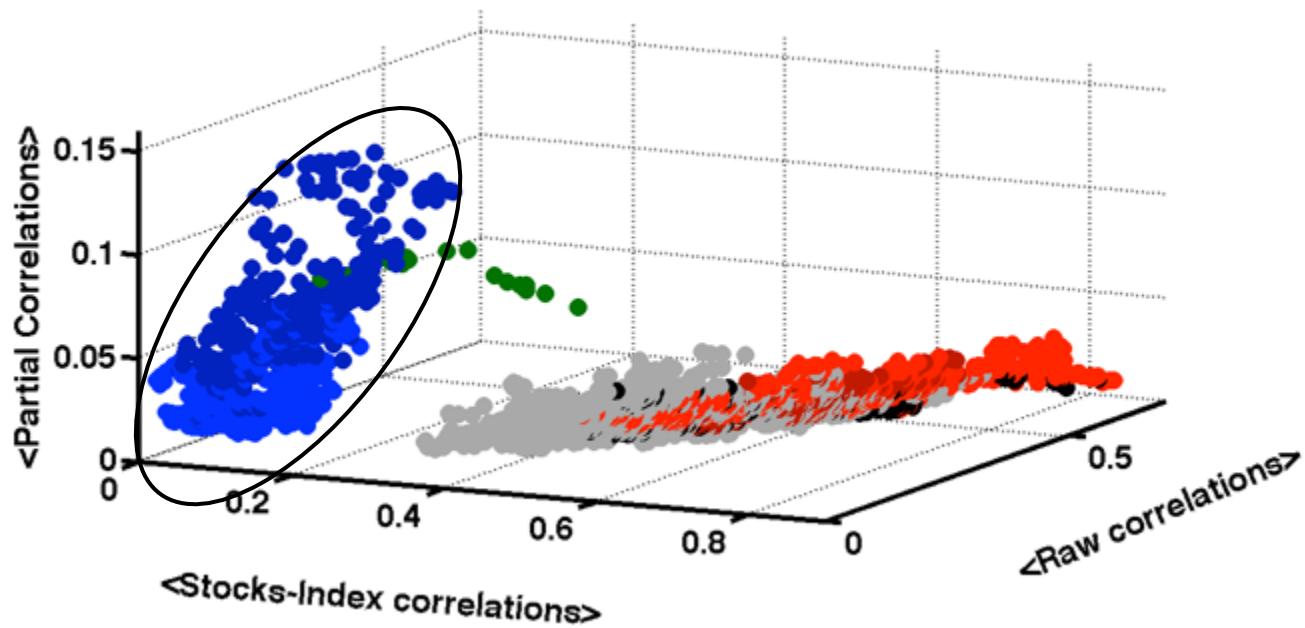


2010

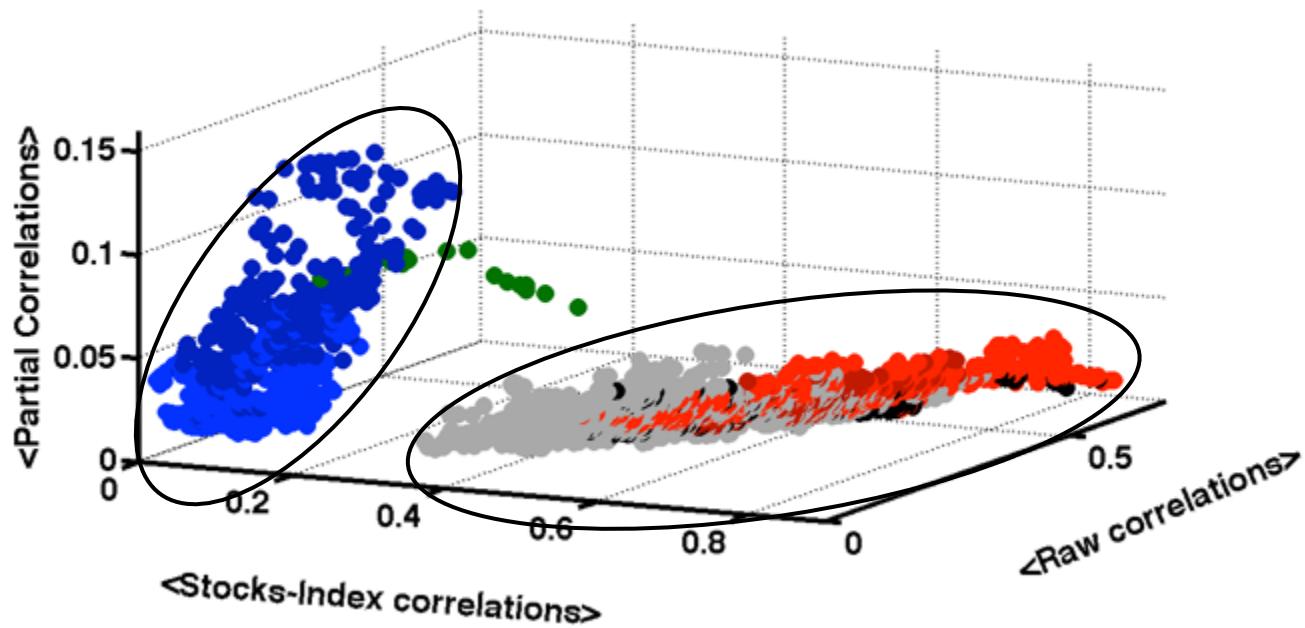
# Financial States and Transitions



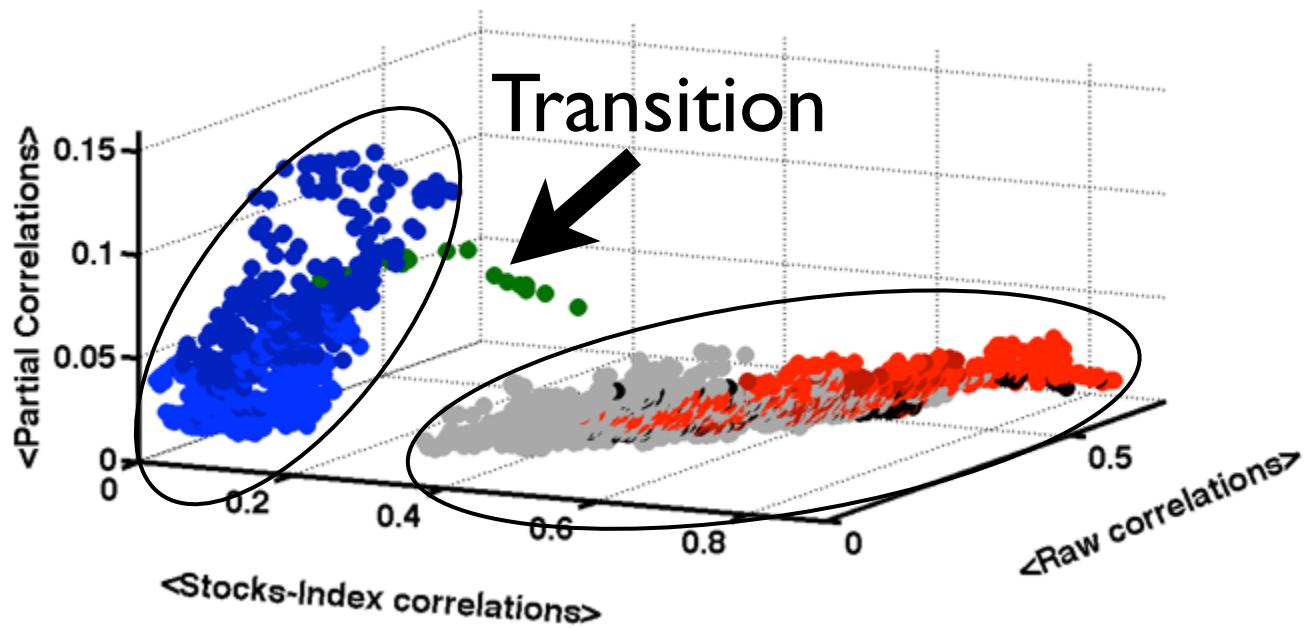
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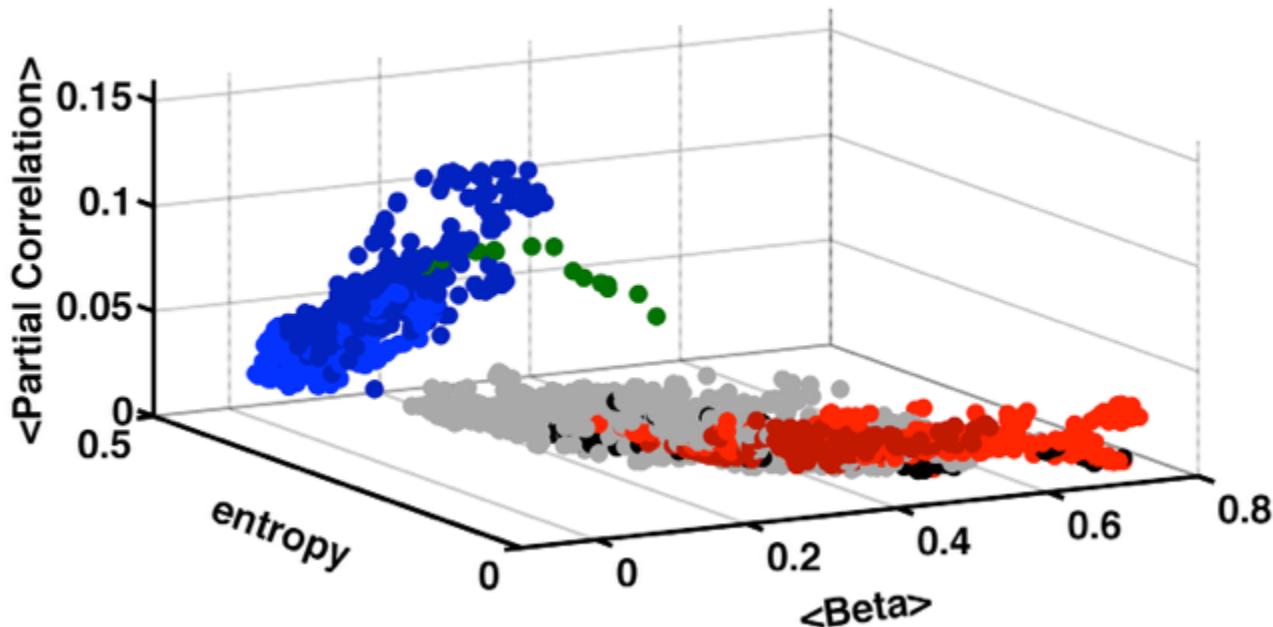
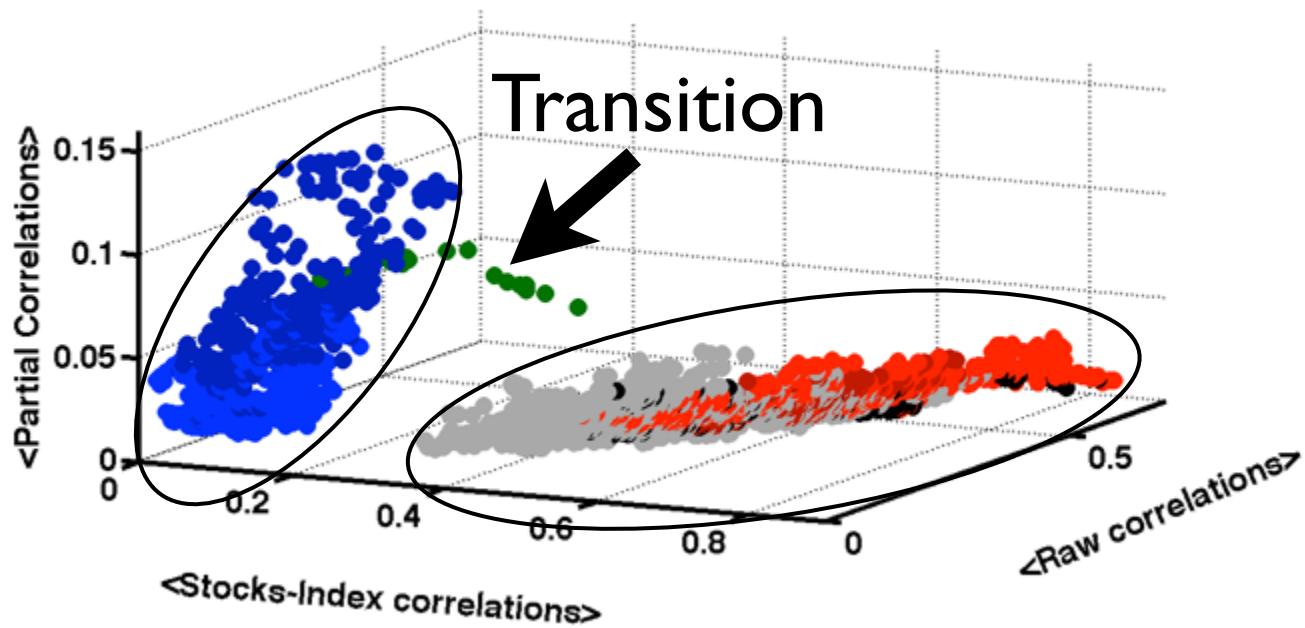
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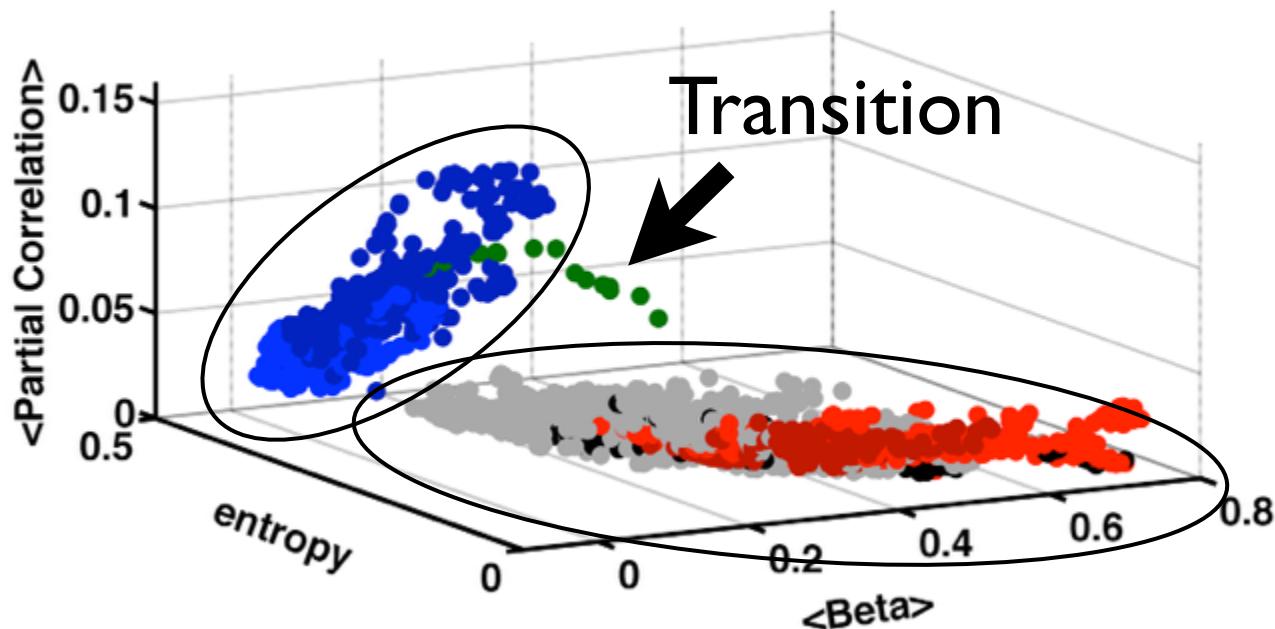
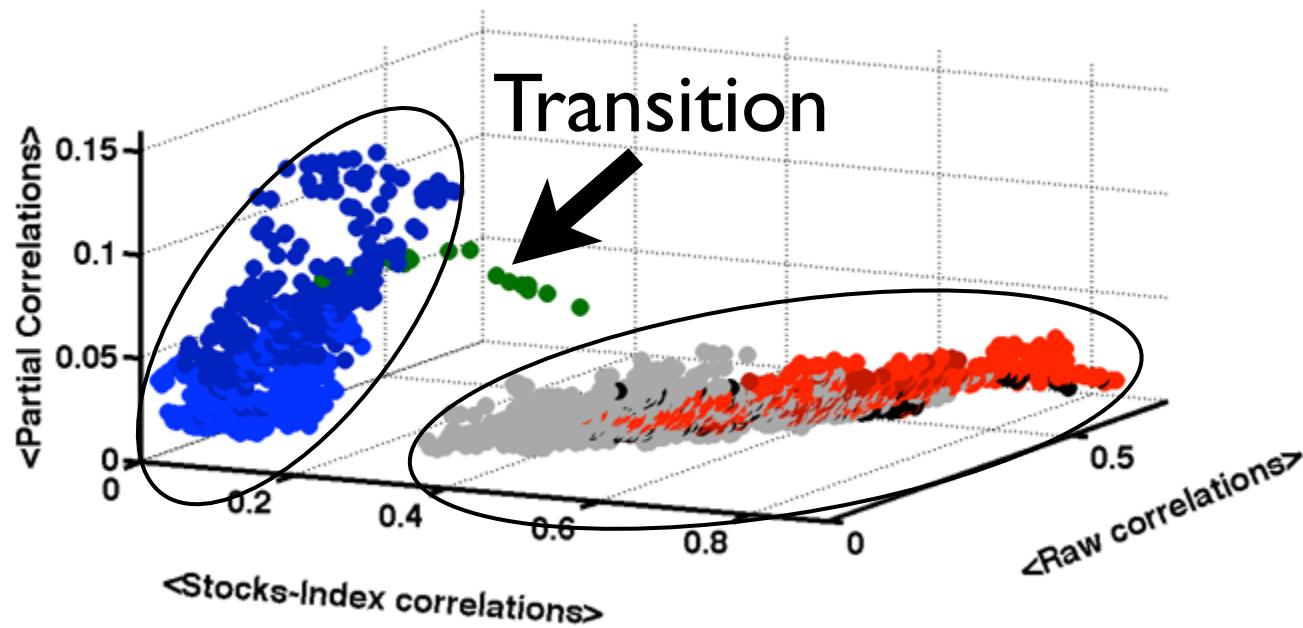
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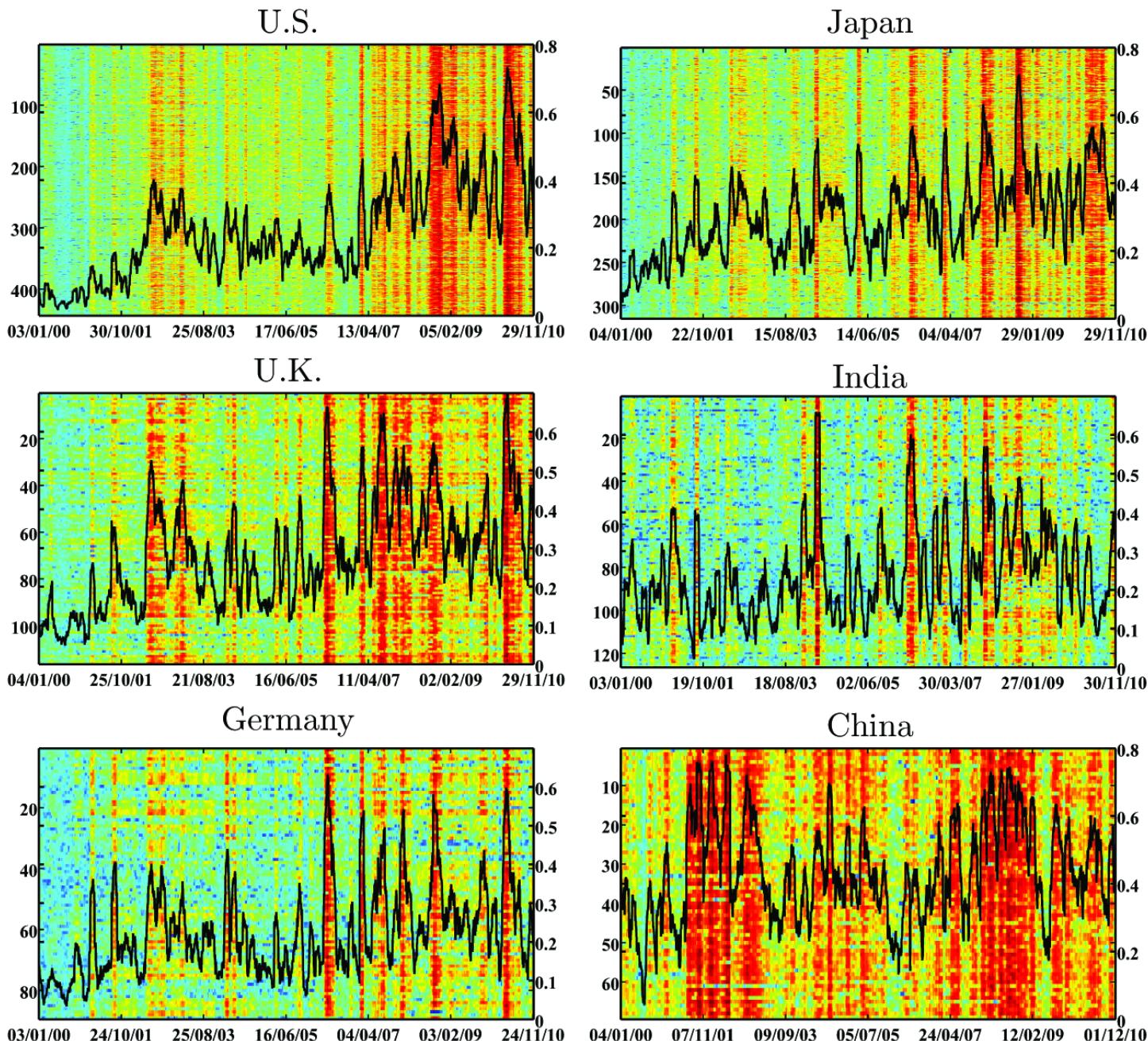
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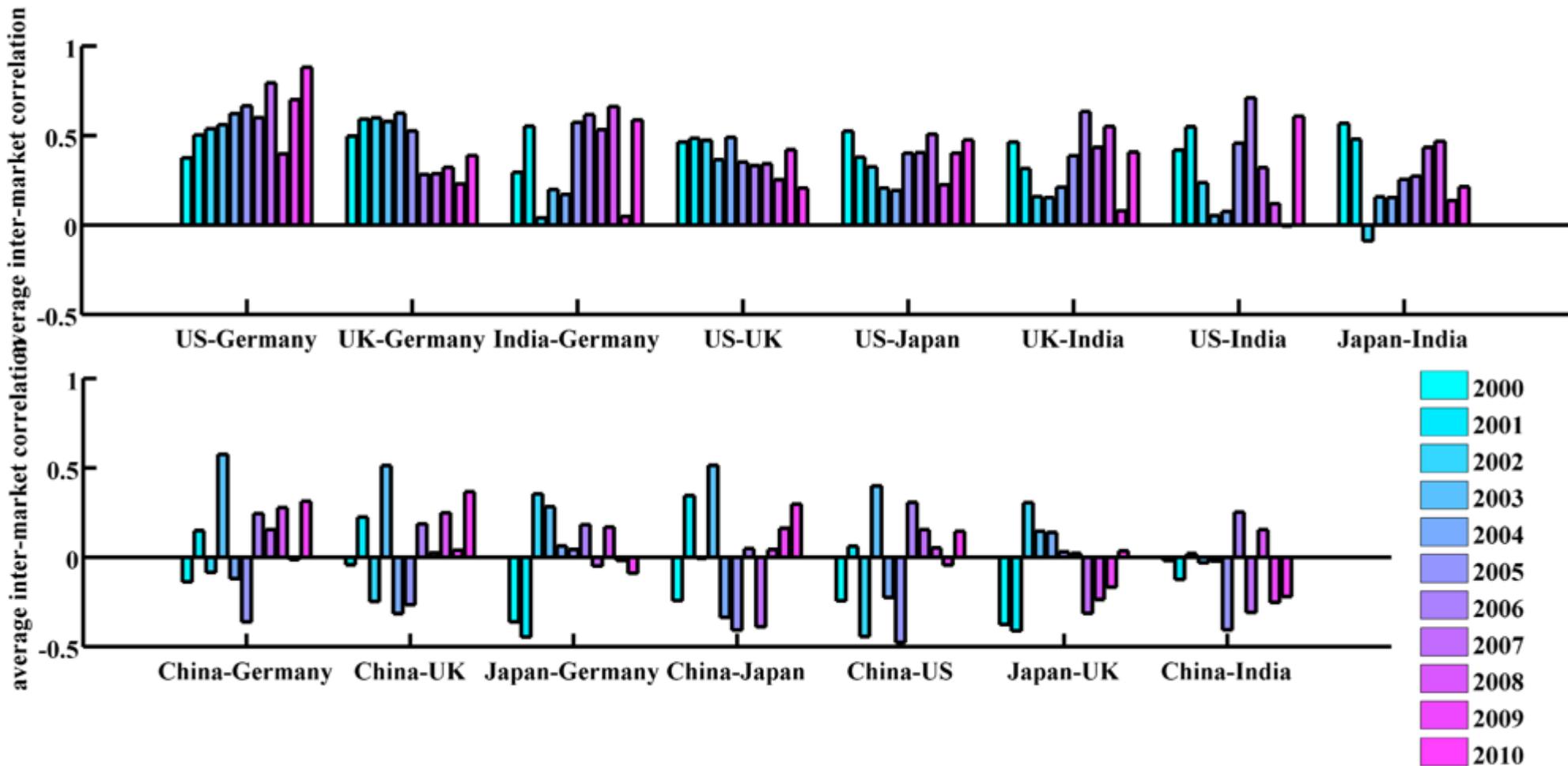
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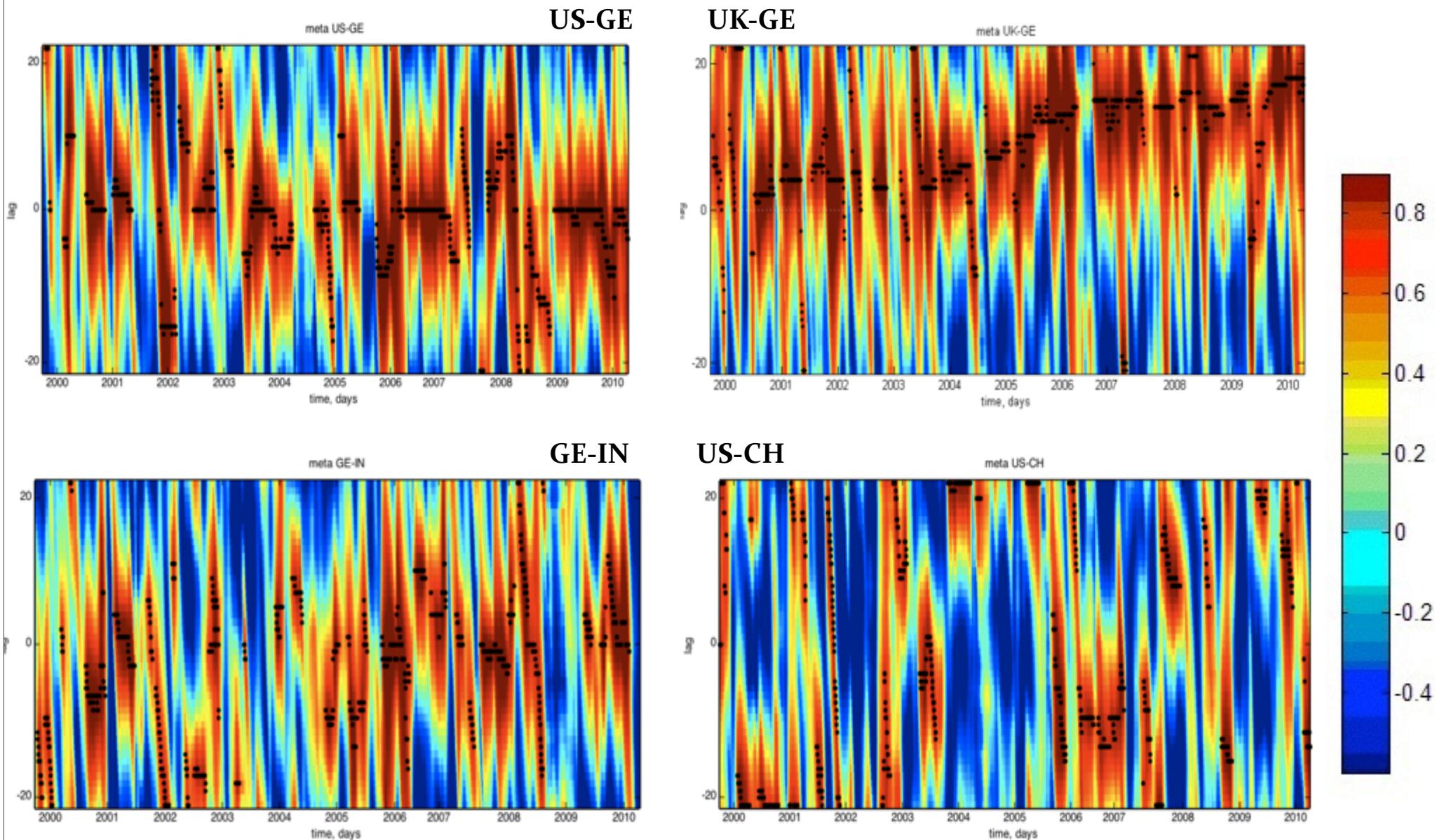
# Stock market Correlations



# Market Meta-Correlation

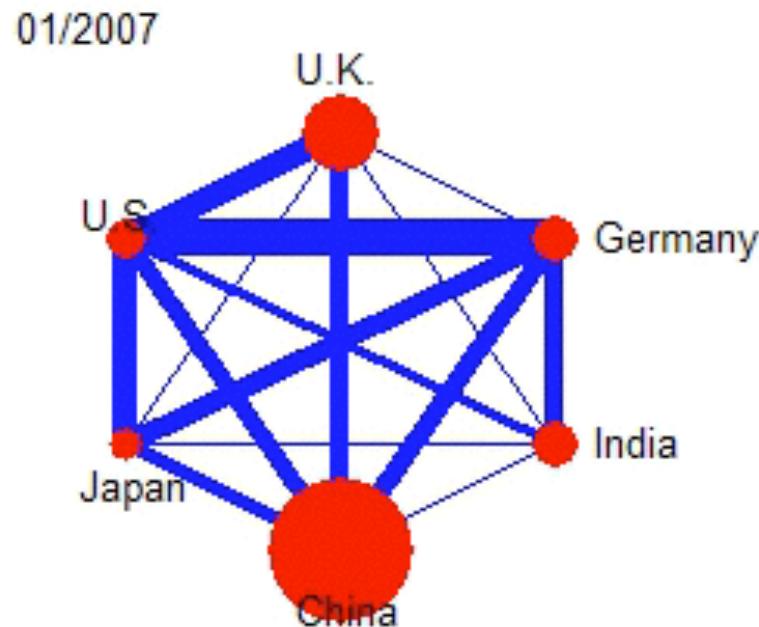


# Question: Can correlations in one market predict correlations in a second market?

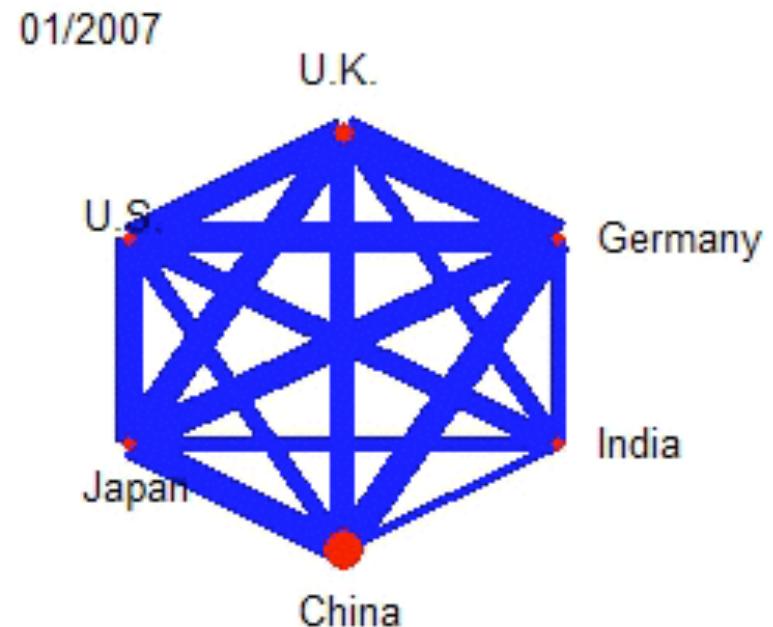


# Goal: Financial Seismograph

Intra correlation & Metacorrelation



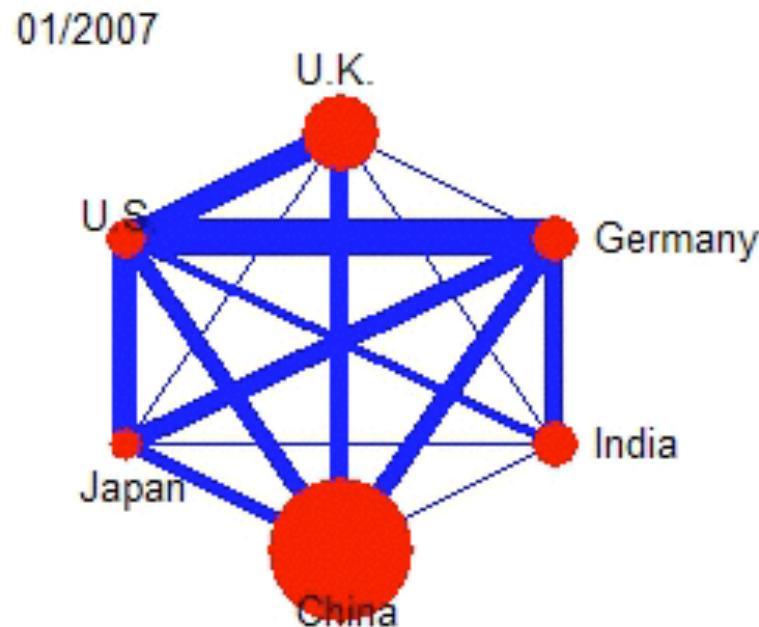
Index volatility & Index correlation



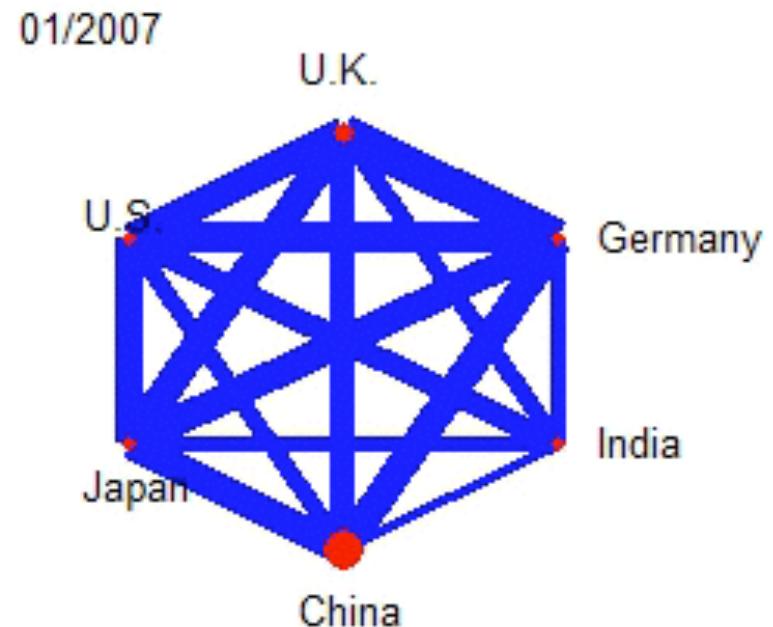
**Financial Seismograph:** Analysis and visualization of how correlations in one market can propagate and influence correlations in a different market

# Goal: Financial Seismograph

Intra correlation & Metacorrelation



Index volatility & Index correlation



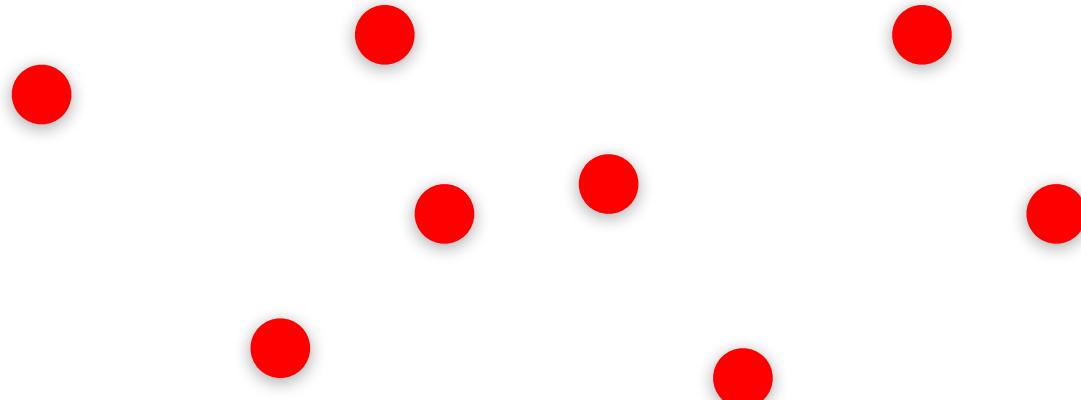
**Financial Seismograph: Analysis and visualization of how correlations in one market can propagate and influence correlations in a different market**

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# **What is a network?**

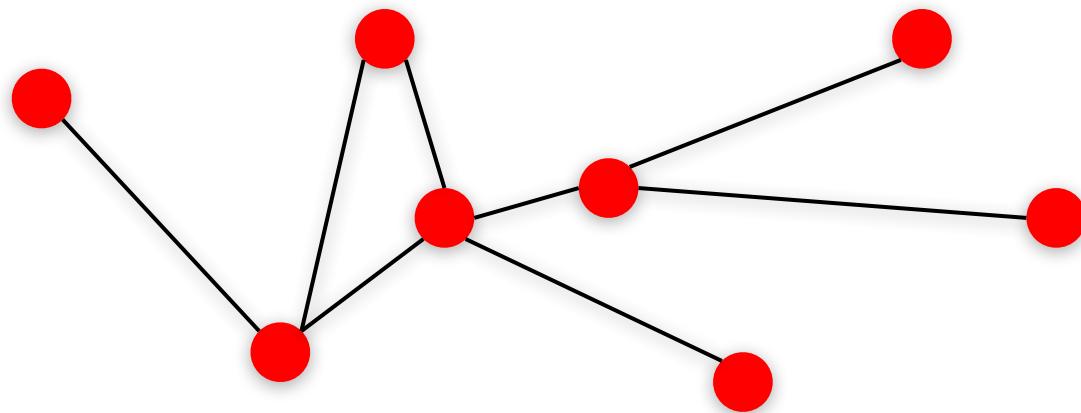
# What is a network?



- **components:** nodes, vertices

N

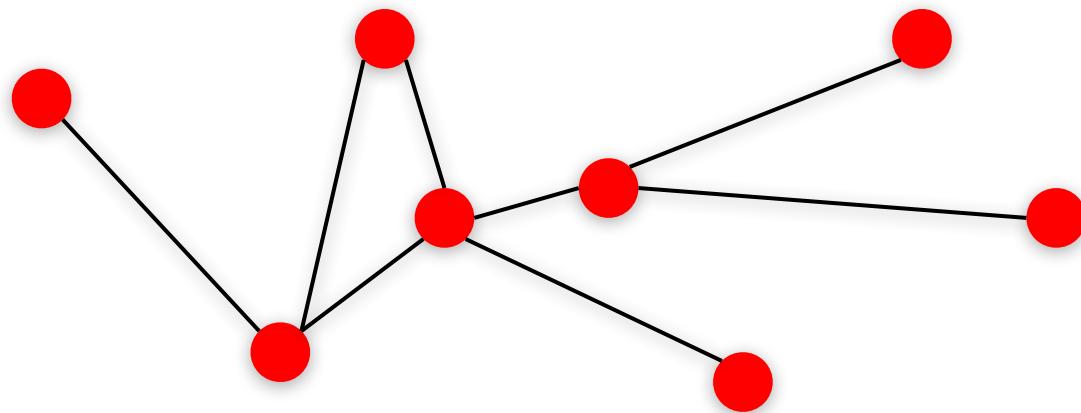
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- **components:** nodes, vertices      N

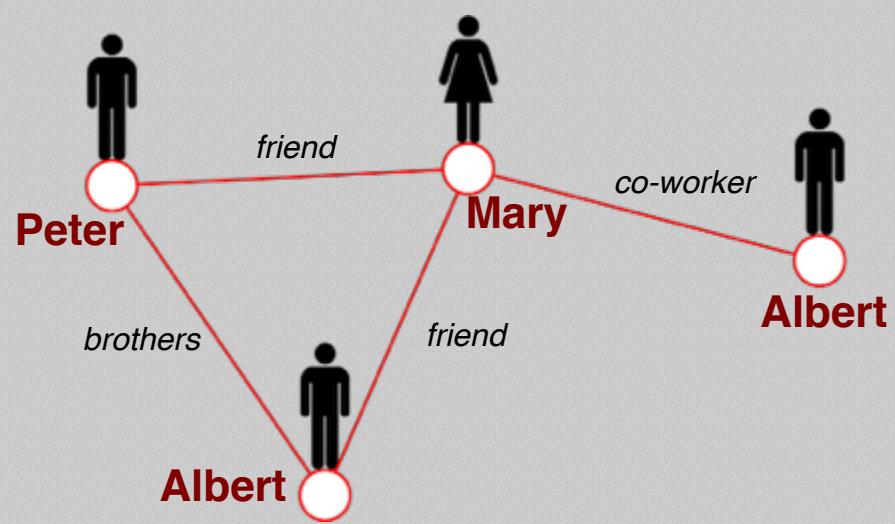
- **interactions:** links, edges      L

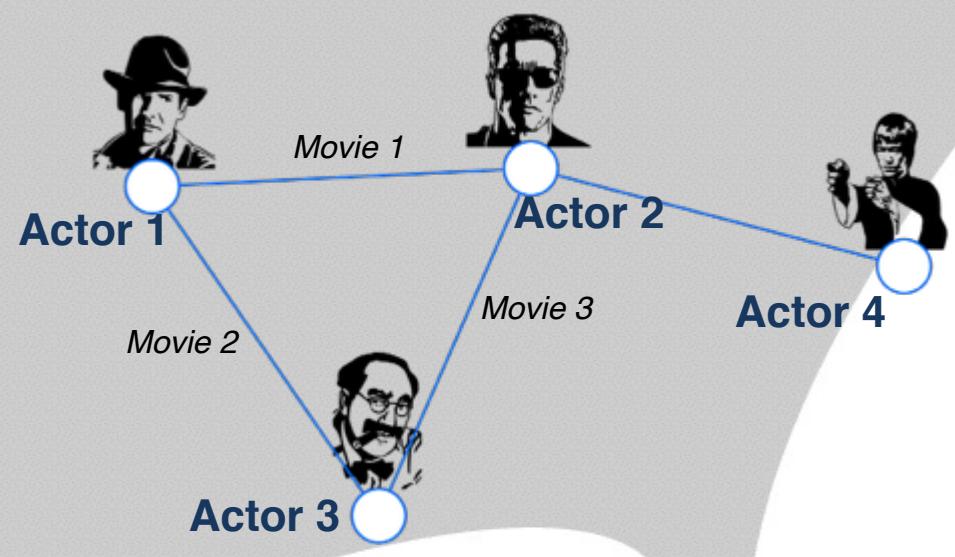
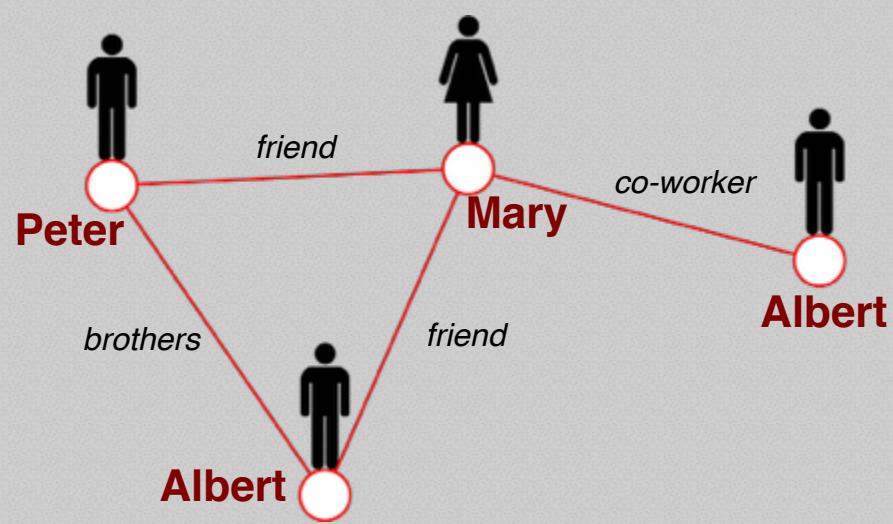
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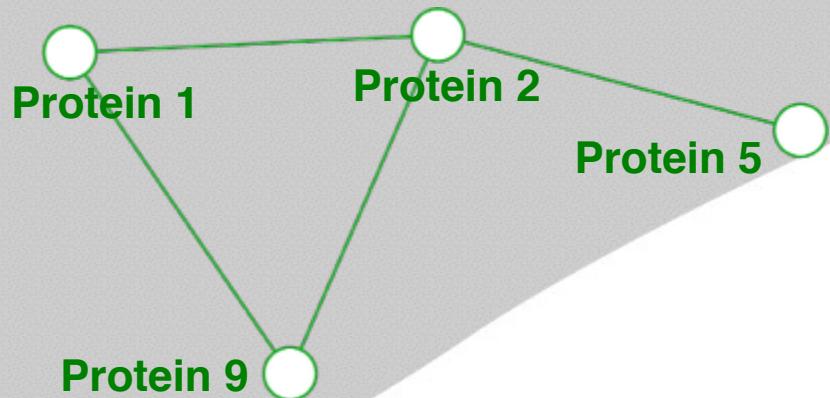
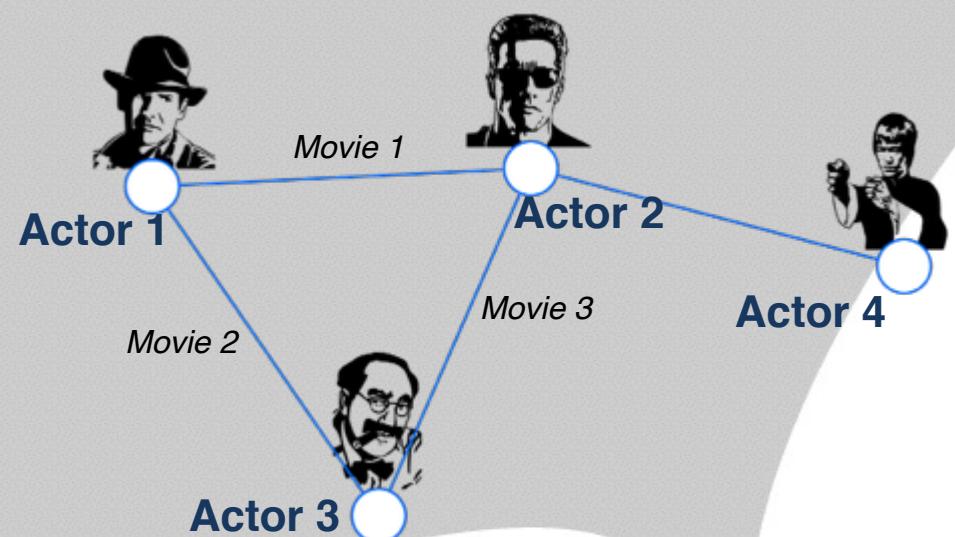
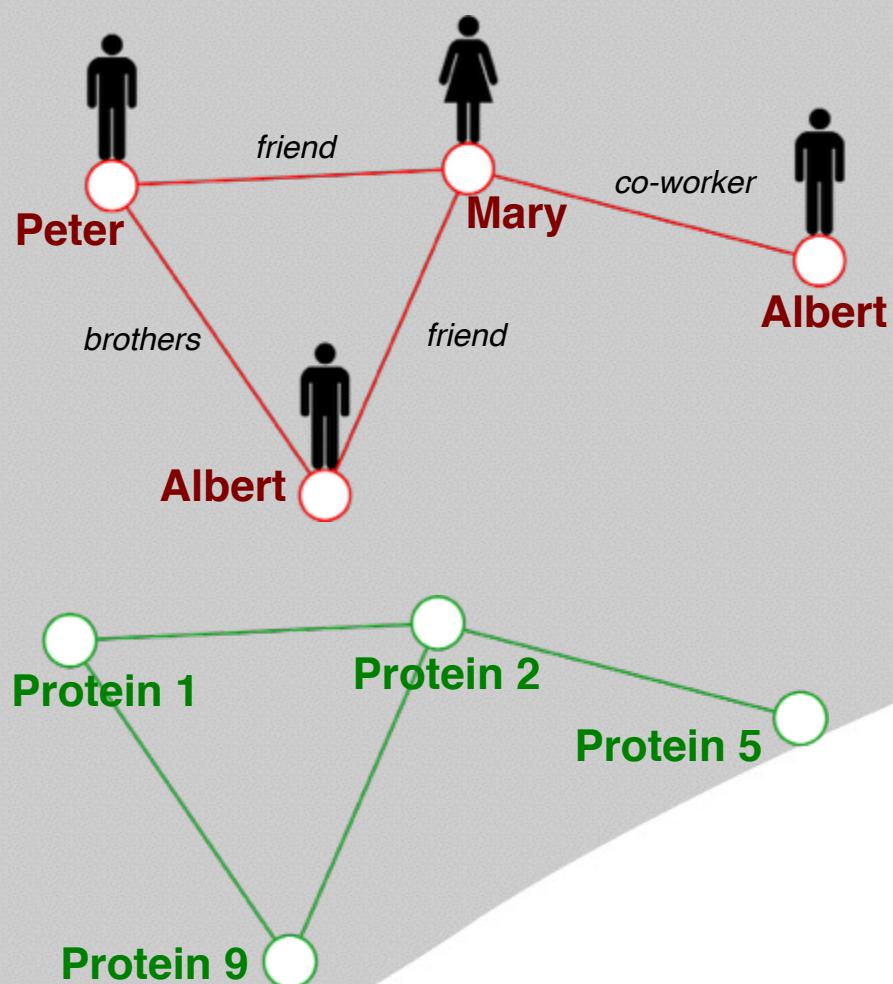


- **components:** nodes, vertices      N
- **interactions:** links, edges      L
- **system:** network, graph      (N,L)

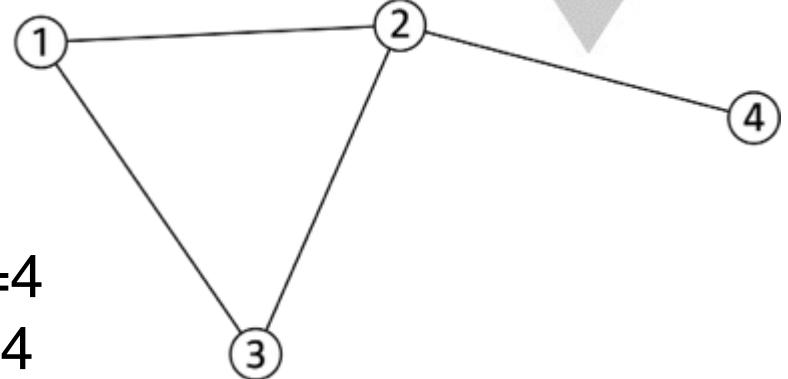






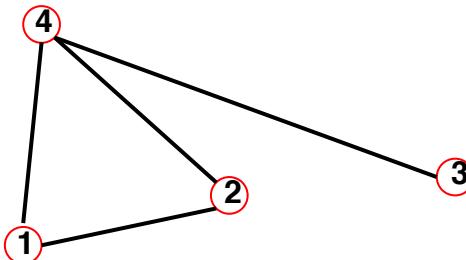


$N=4$   
 $L=4$

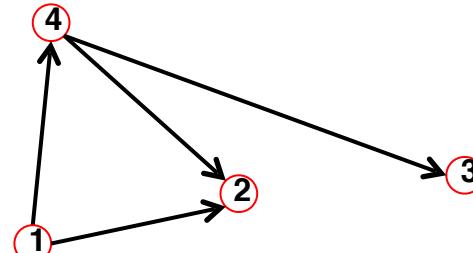


# The Adjacency Matrix

Undirected



Directed



$A_{ij}=1$  if there is a link between node  $i$  and  $j$

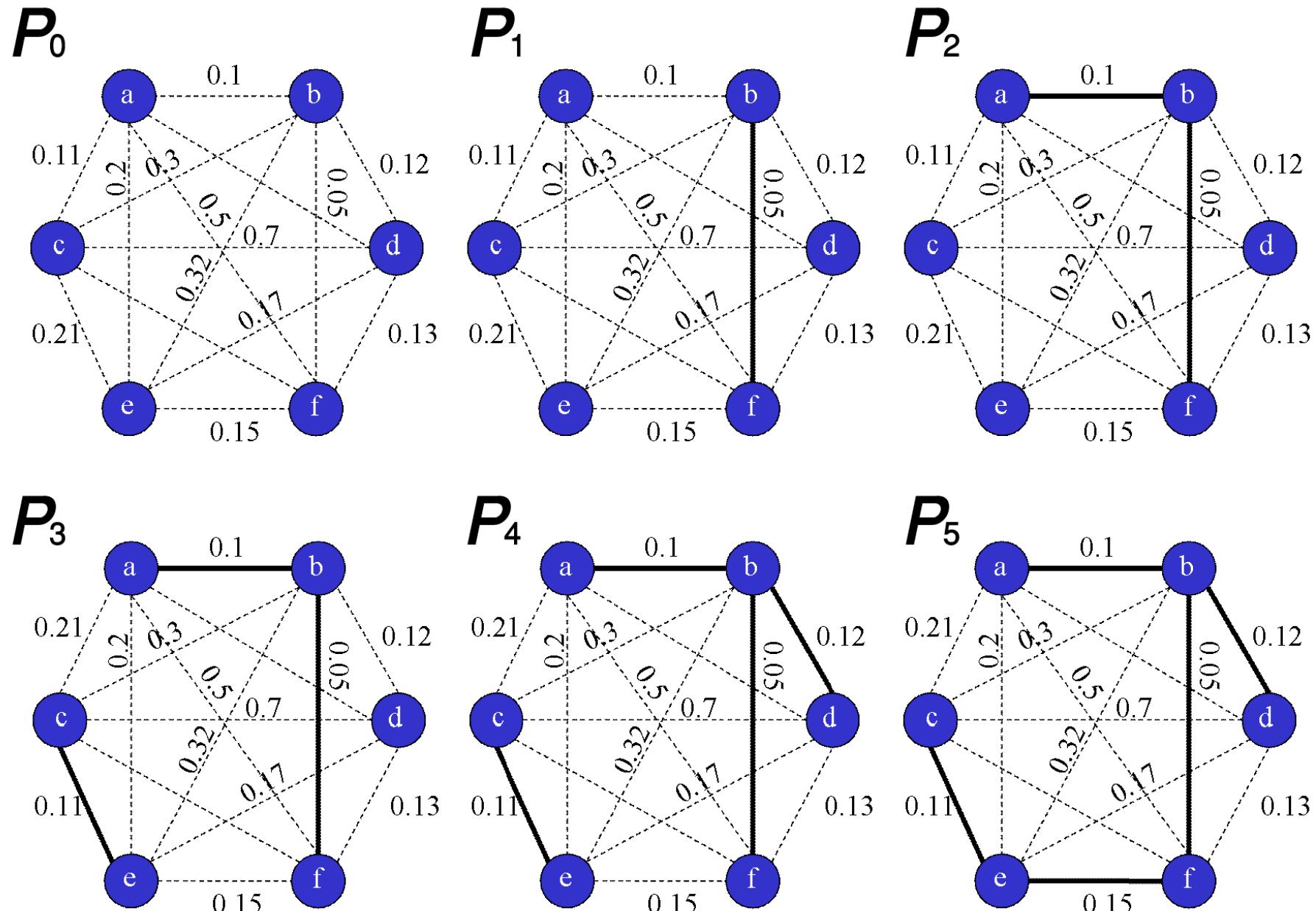
$A_{ij}=0$  if nodes  $i$  and  $j$  are not connected to each other.

$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$$A_{ij} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Note that for a directed graph (right) the matrix is not symmetric.

	a	b	c	d	e	f
a	0	0.1	0.11	0.4	0.2	0.5
b	0.1	0	0.3	0.12	0.32	0.05
c	0.11	0.3	0	0.7	0.21	0.5
d	0.4	0.12	0.7	0	0.17	0.13
e	0.2	0.32	0.21	0.17	0	0.15
f	0.5	0.05	0.5	0.13	0.15	0



# Stock Dependency Networks

1. Calculate partial correlation  $PC(i,k | j) \quad j = 1,2,\dots,N$

2. Correlation Influence

$$D(i,k | j) \equiv C(i,k) - PC(i,k | j)$$

3. Stock Dependency  $d(i | j) = \frac{1}{N-1} \sum_{k \neq j,i}^{N-1} D(i,k | j)$

4. Construct Planar Graph (PMFG, Tumminello *et al.*, PNAS 2005)

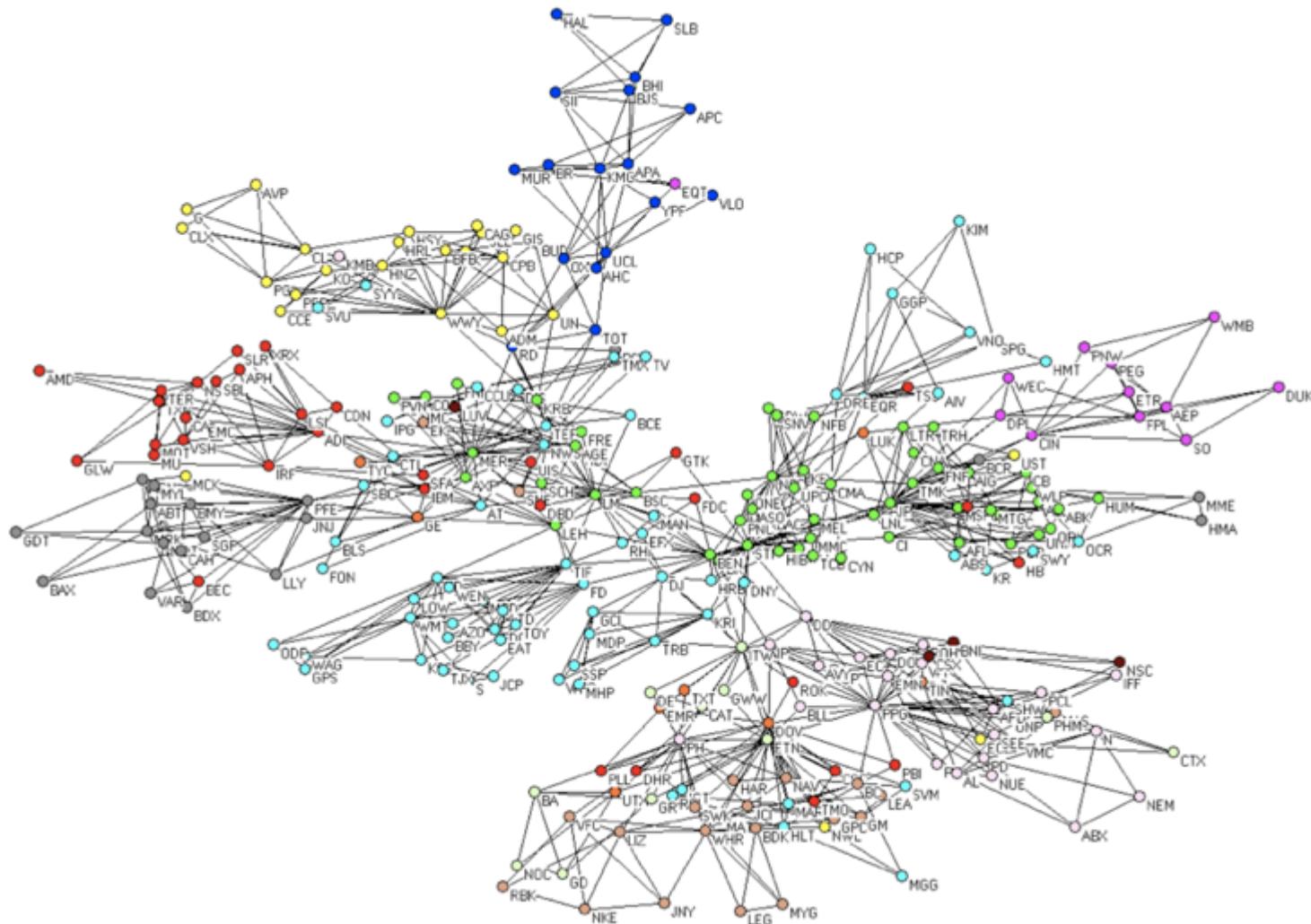
Dror Y. Kenett, Michele Tumminello, Asaf Madi, Girit Gur-Gershoren, Rosario N. Mantegna, and Eshel Ben-Jacob (2010), Dominating clasp of the financial sector revealed by partial correlation analysis of the stock market, PLoS ONE 5(12), e15032

# Data

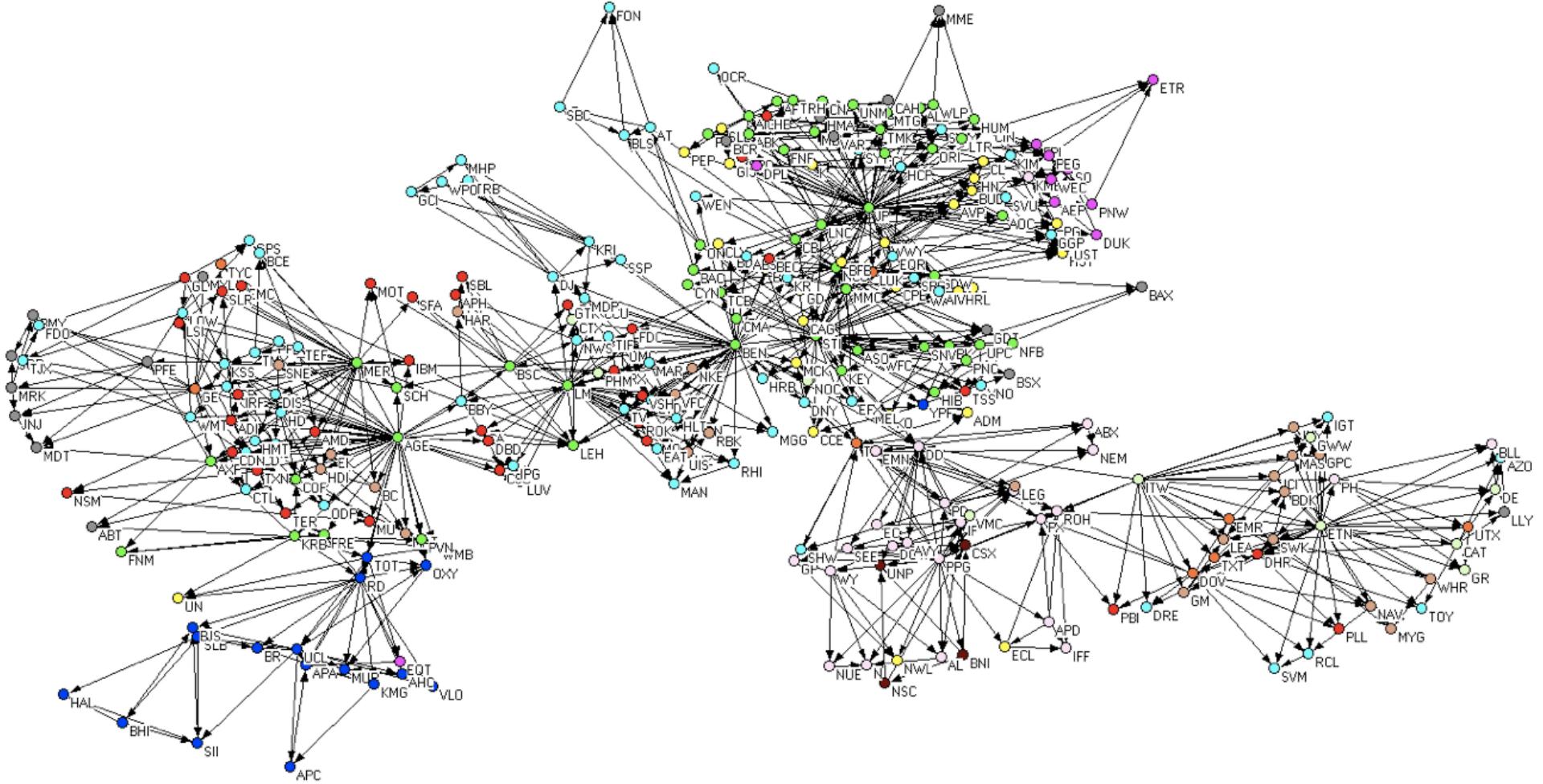
N = 300      T = 748

Index	Sector	# stocks
1	Basic Materials	24
2	Consumer Cyclical	22
3	Consumer Non Cyclical	25
4	Capital Goods	12
5	Conglomerates	8
6	Energy	17
7	Financial	53
8	Healthcare	19
9	Services	69
10	Technology	34
11	Transportation	5
12	Utilities	12

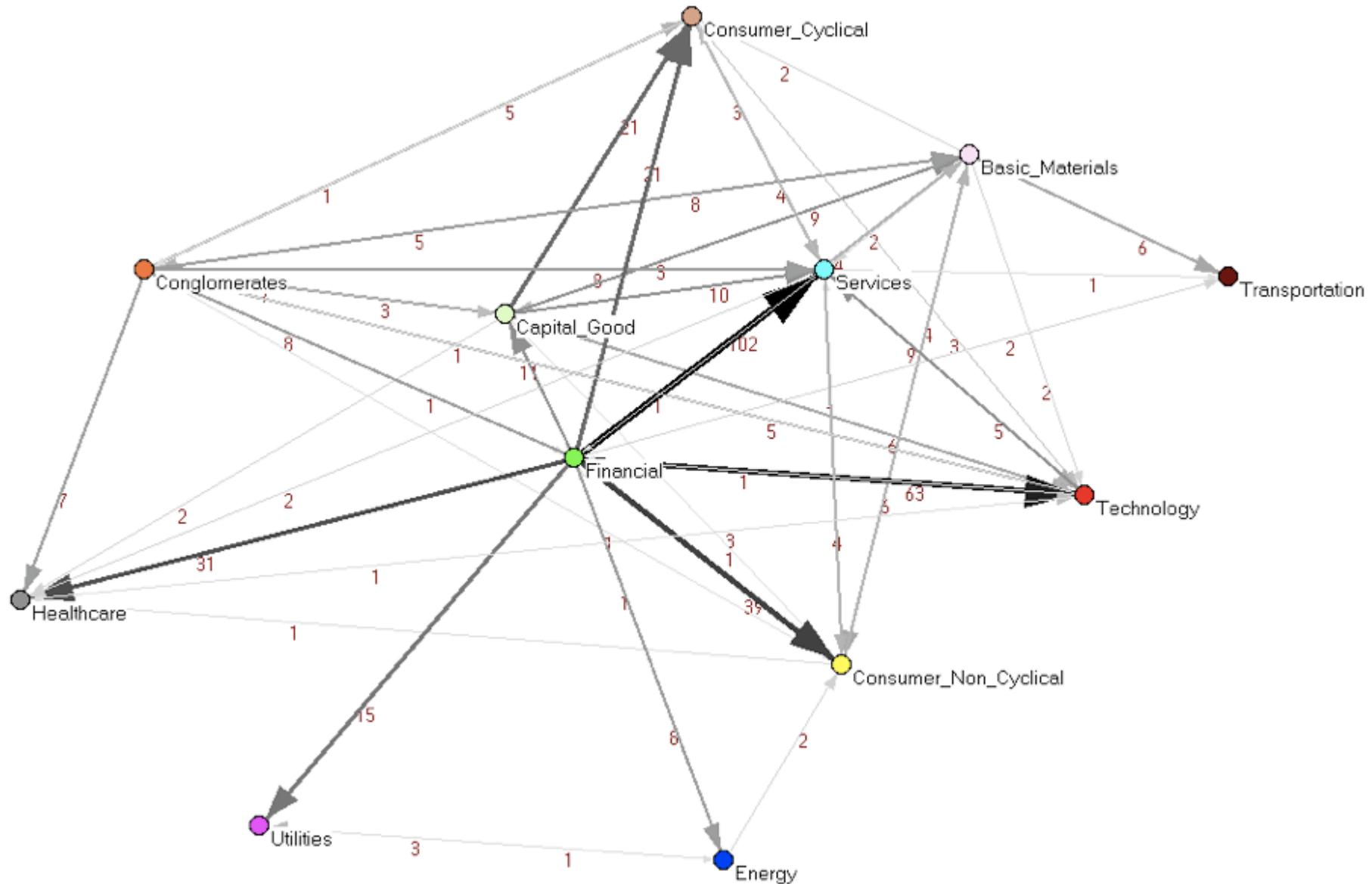
# Stock Dependency Network: S&P Stocks



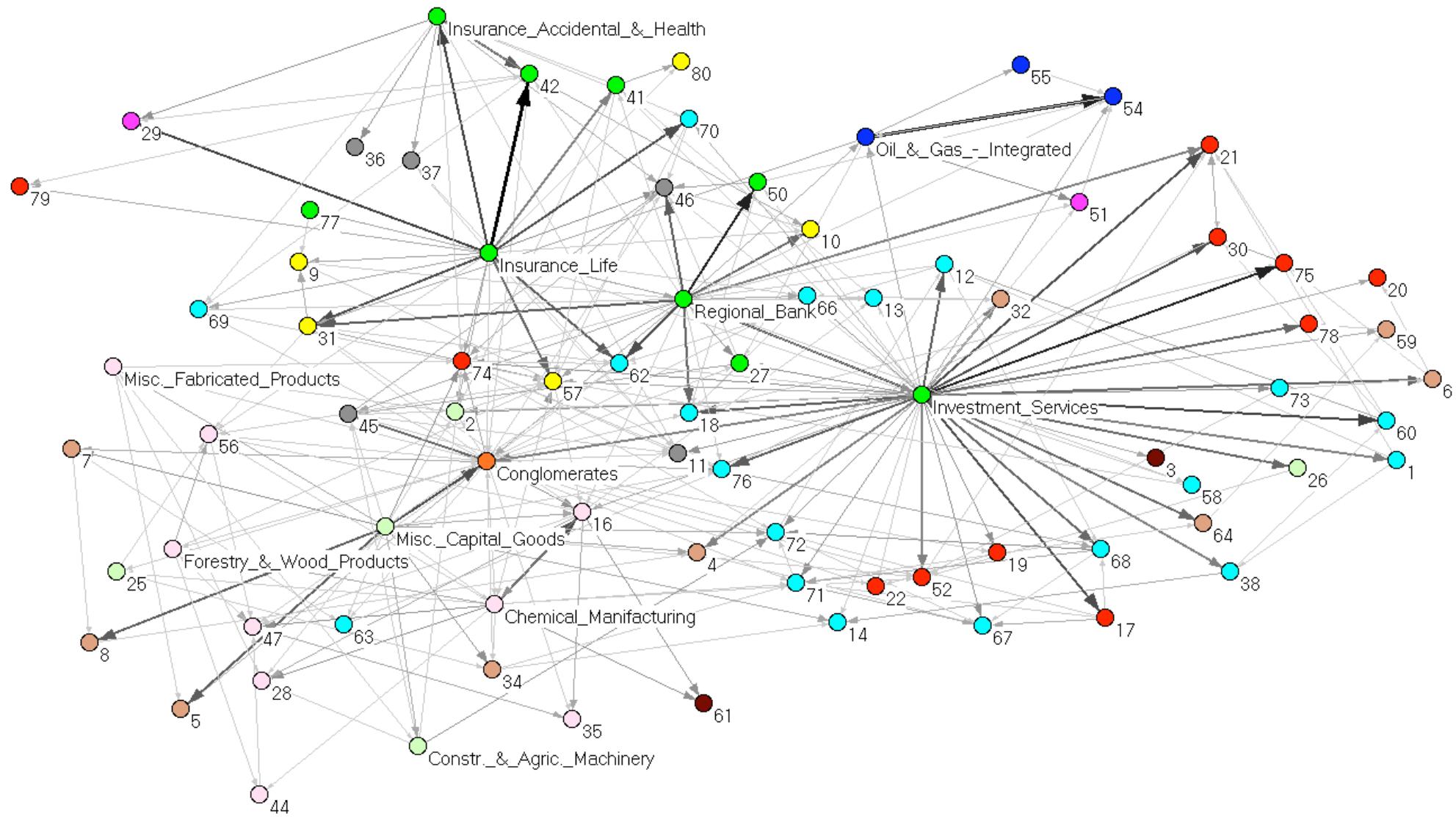
# Stock Dependency Network: S&P Stocks



# Sector Dependency Network



# Sector Dependency Network



# Theoretical Models

## Simple Index

$$r_i = \gamma_i f + \sqrt{1 - \gamma_i^2} \varepsilon_i, \quad i = 1, \dots, N,$$

$$\langle r_i f \rangle = \gamma_i \langle f^2 \rangle = \gamma_i,$$

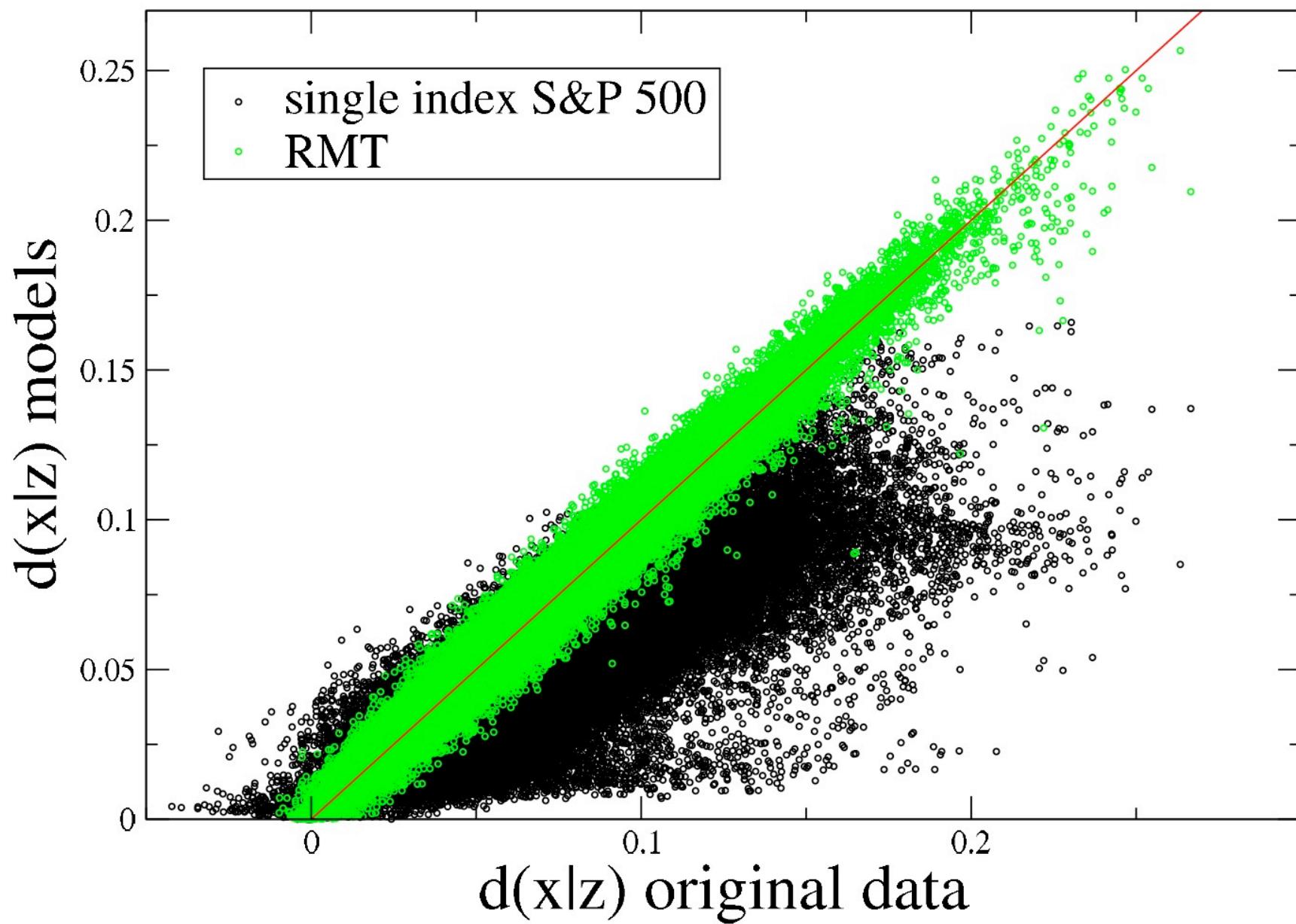
$$\rho_{i,j}(SI) = \langle r_i r_j \rangle = \gamma_i \gamma_j$$

## RMT

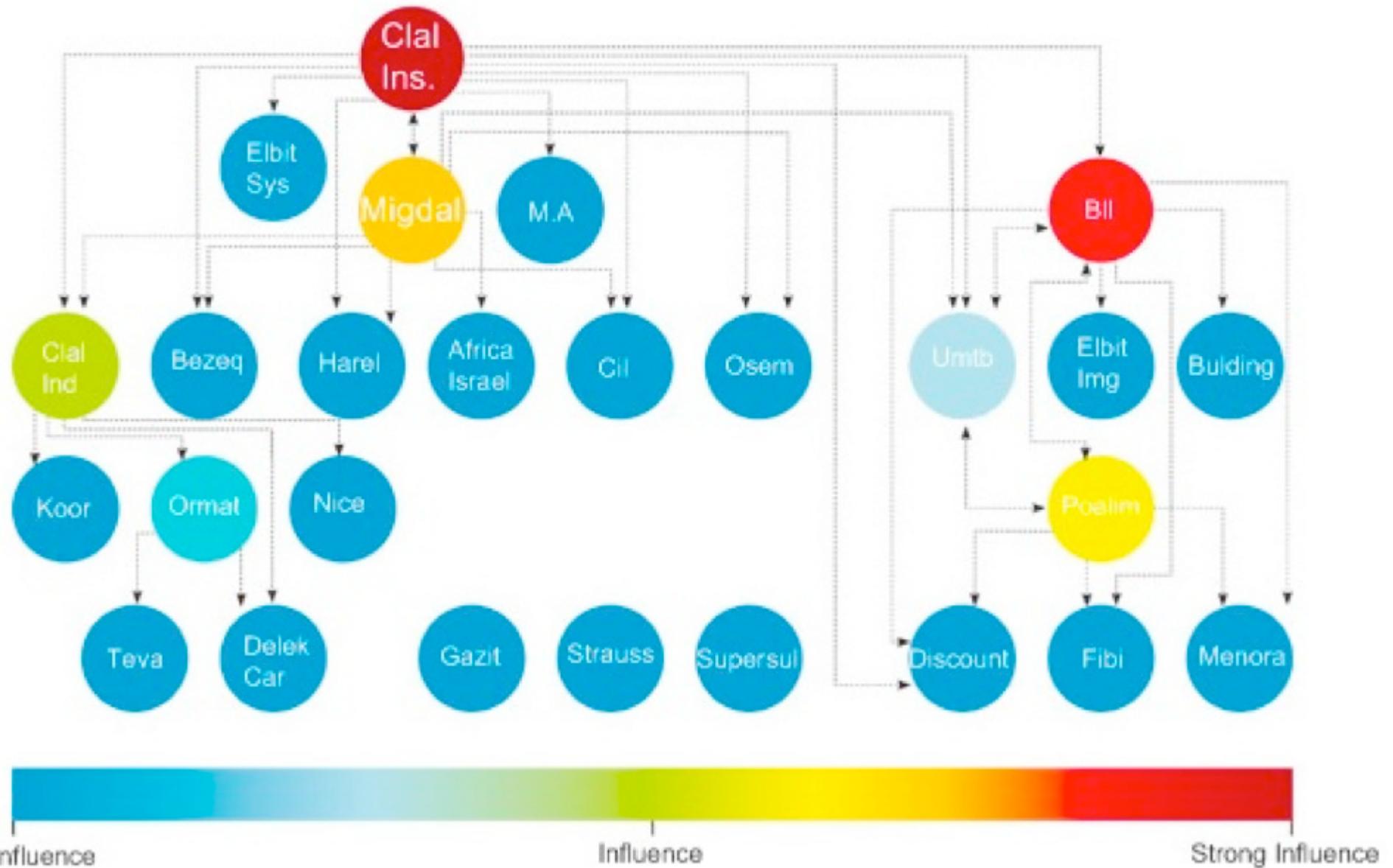
$$r_i = \sum_{h=1}^K \gamma_{i,h} \sqrt{\lambda_h} f_h + \sqrt{1 - \sum_{h=1}^k \gamma_{i,h}^2 \lambda_h} \varepsilon_i \quad i = 1, \dots, N,$$

$$\lambda_{\max} = \left( 1 - \frac{\lambda_1}{N} \right) \left( 1 + \frac{N}{T} + 2 \sqrt{\frac{N}{T}} \right)$$

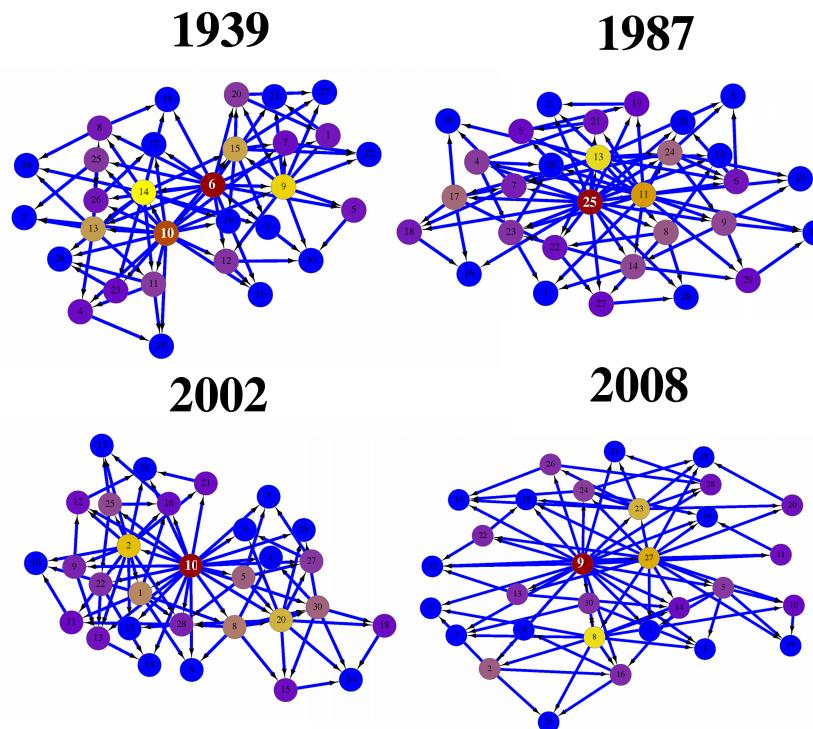
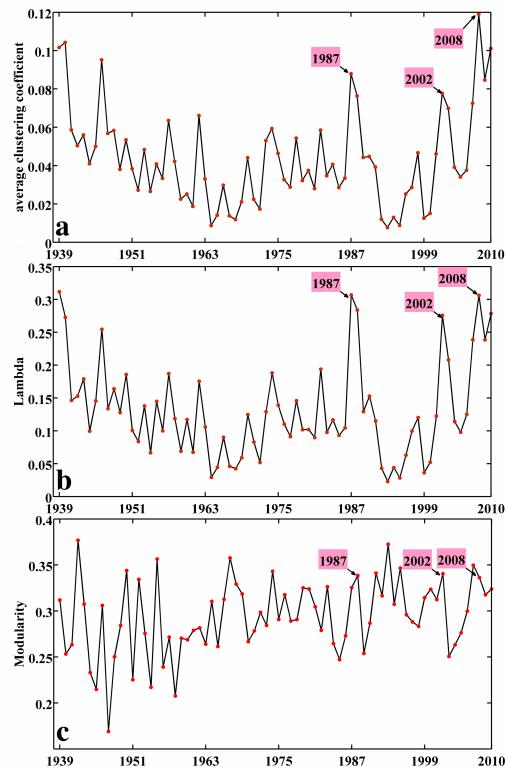
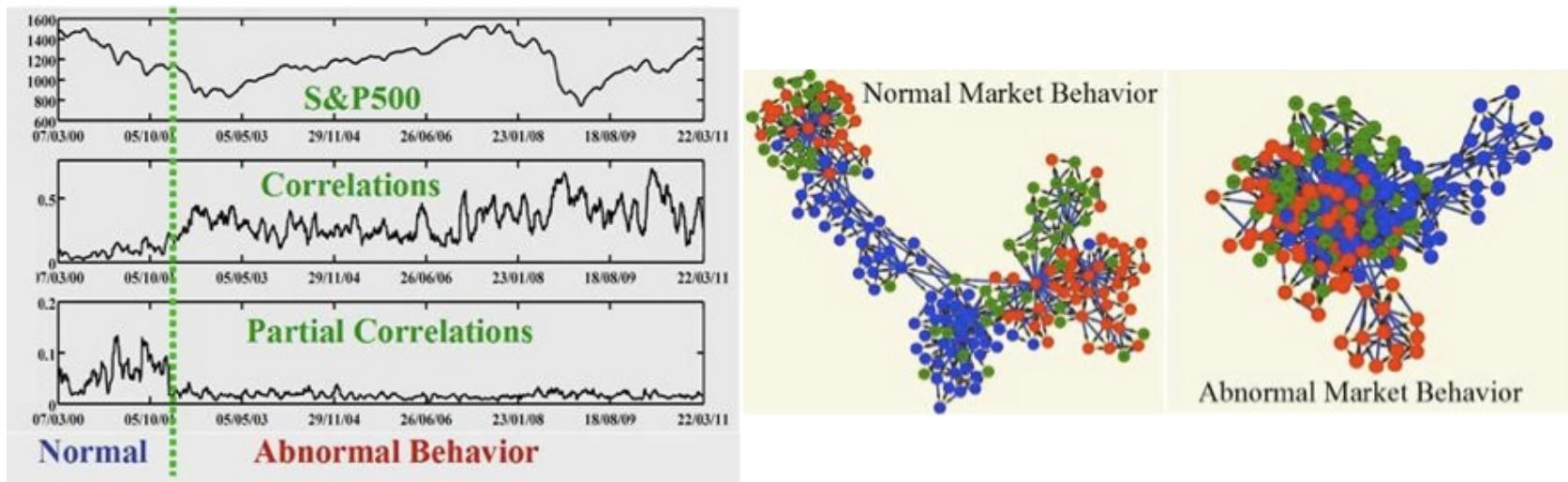
$$\rho_{i,j}(RMT) = \langle r_i r_j \rangle = \sum_{h=1}^K \gamma_{i,h} \gamma_{j,h} \lambda_h$$



# Case study - Tel-Aviv market



# Market states

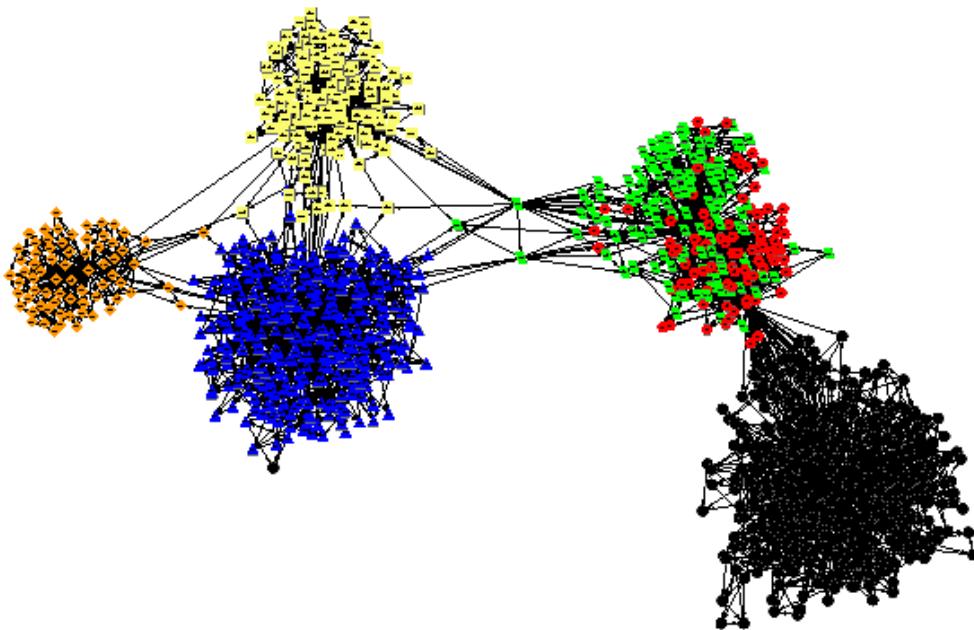


# Market dynamics

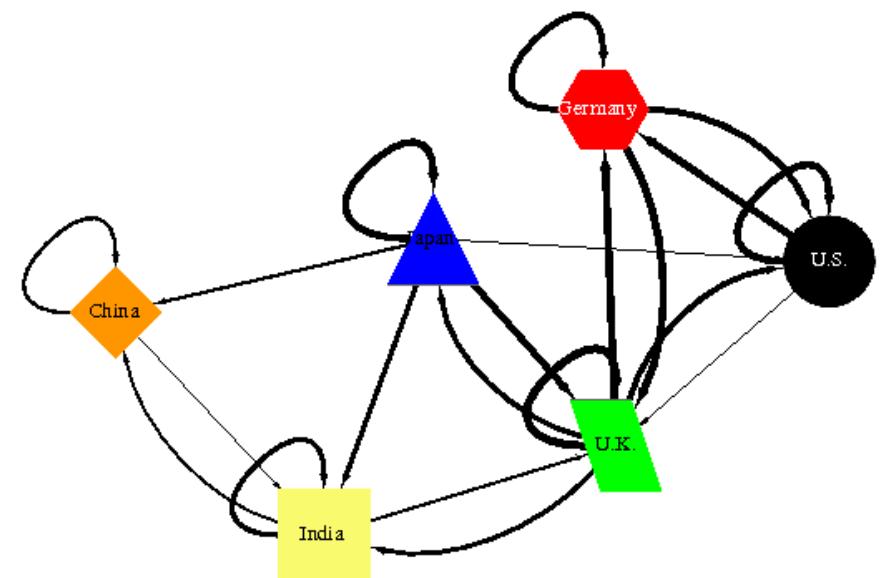
# **Interdependencies in the global financial village**

**Network analysis of influence and dependencies between Companies/Countries**

**Stock dependency network**

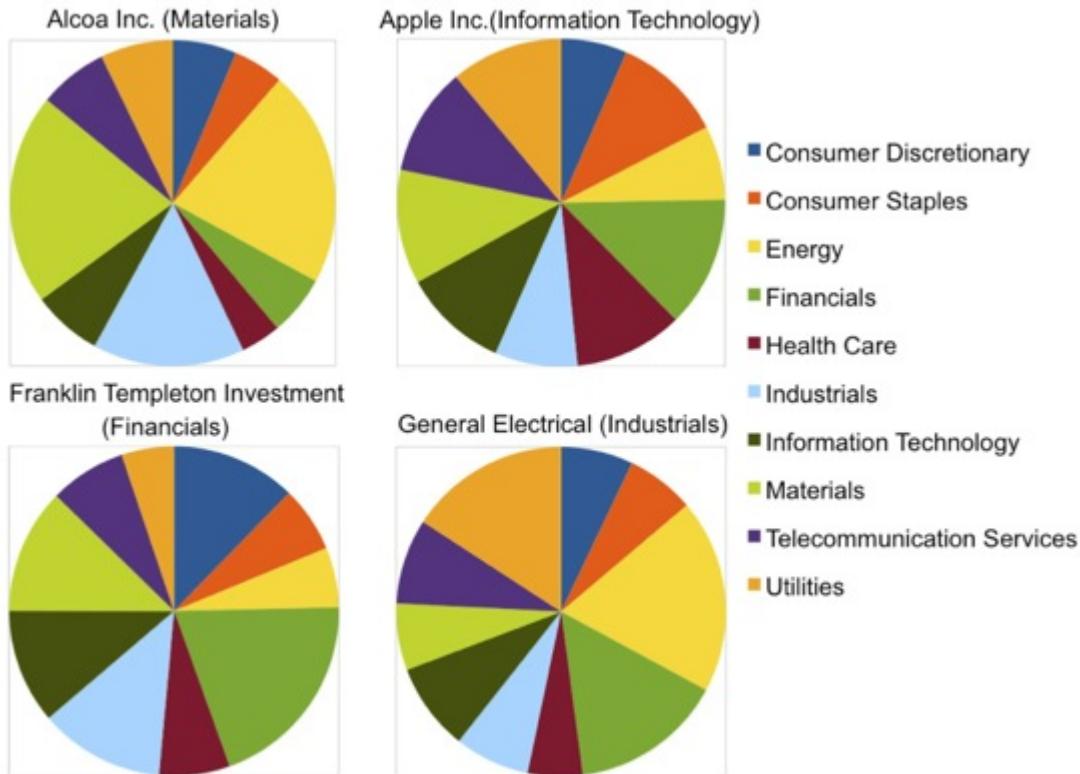


**Country dependency network**

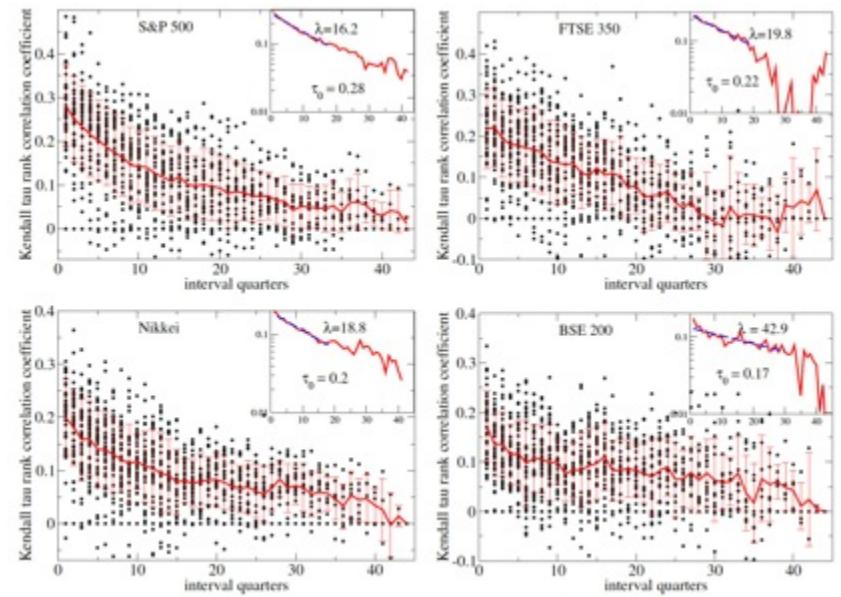
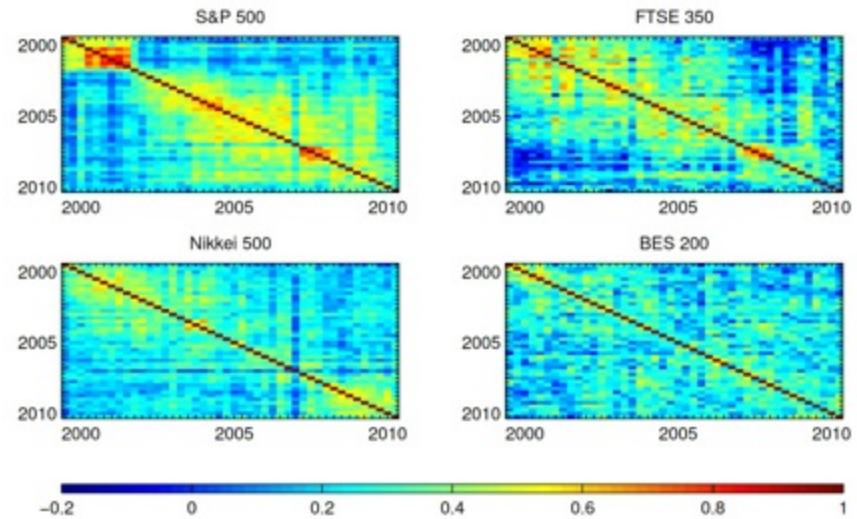


# Investigating market structure

1)



2)

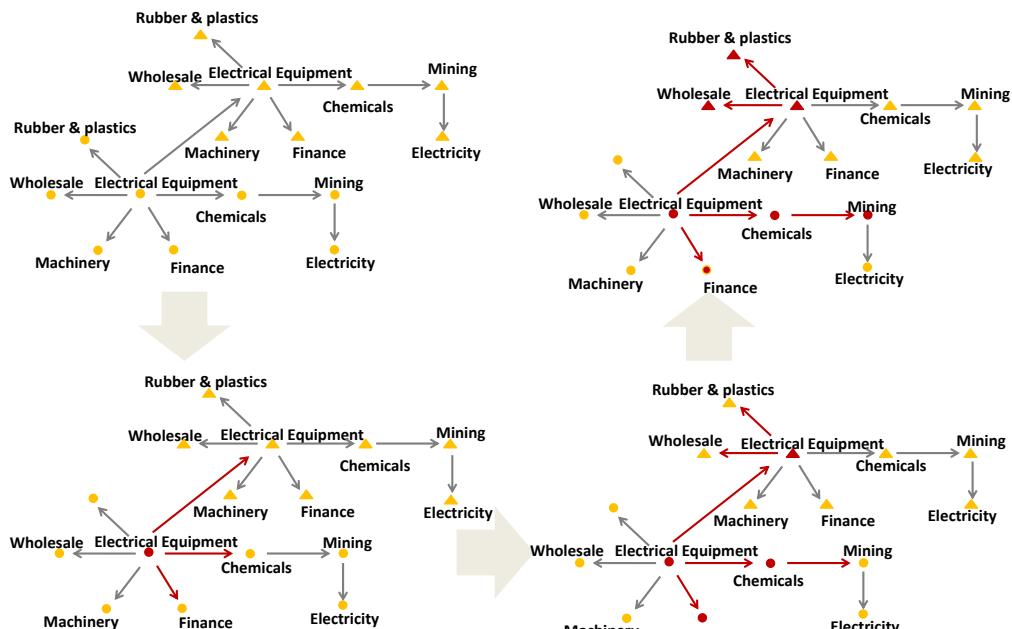


# Outline

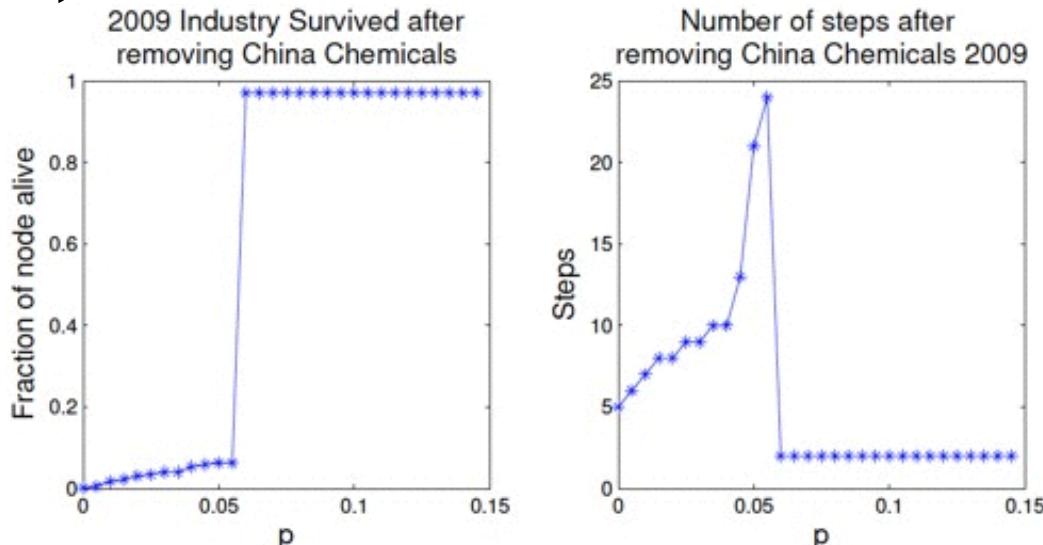
- (1) **Introduction**
  - Financial time series
  - Stock correlations
  - Dynamics of stock correlations
- (2) **Global financial village**
  - Market intra and meta correlation
  - Financial Seismograph
- (3) **Dependency and Influence**
- (4) **Examples of network projects**
  - I. Cascading failures in industry networks
  - II. Overlapping communities in networks
  - III. Failure and recovery in networks
  - IV. Evolution of networks
  - V. Cascading failures in the financial system
  - VI. Interdependent networks
- (5) **Discussion**

# I. Cascading failures in industry networks

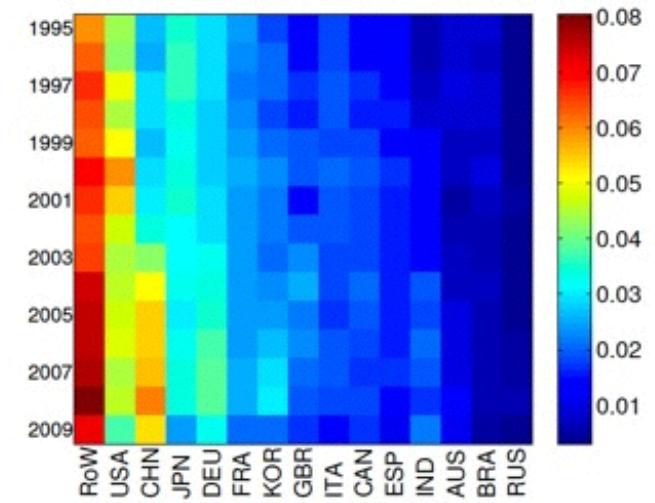
1)



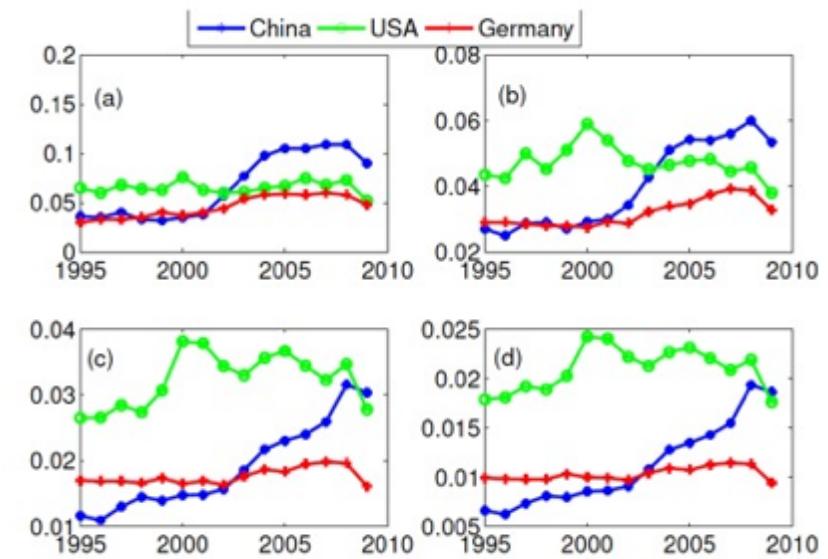
2)



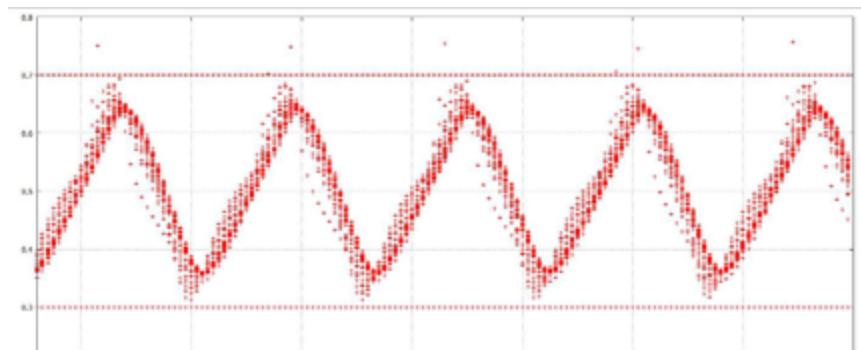
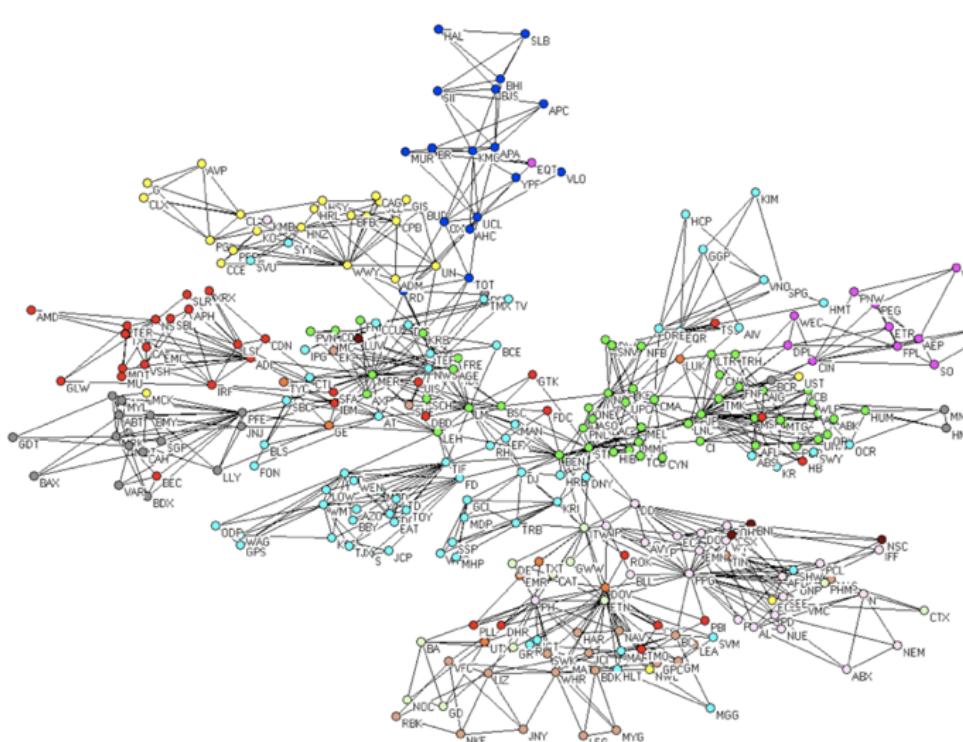
3)



4)

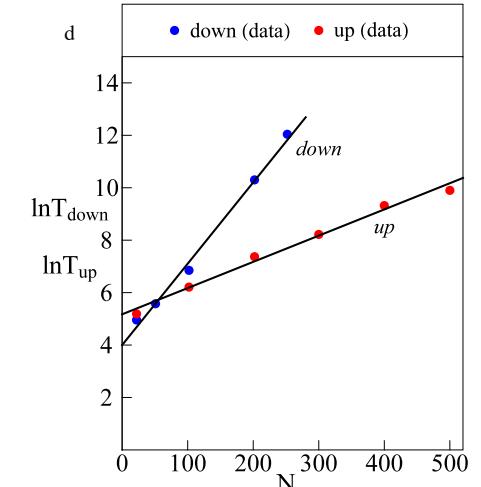
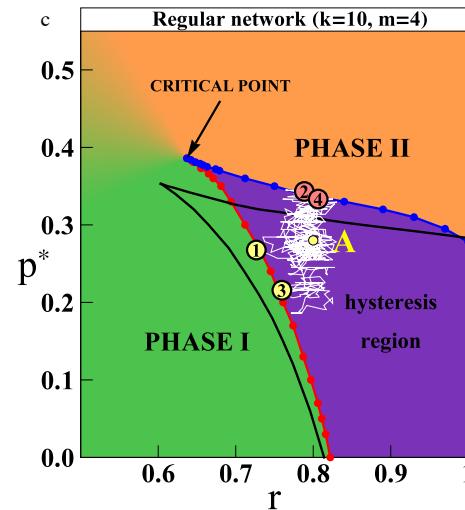
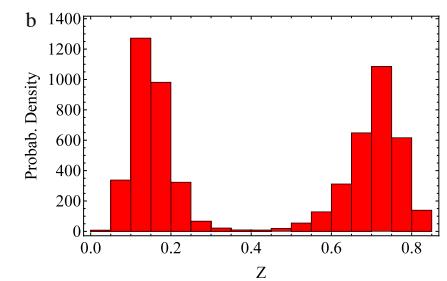
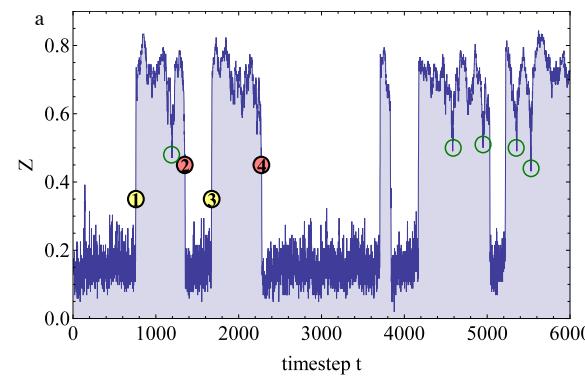


## II. Overlapping communities



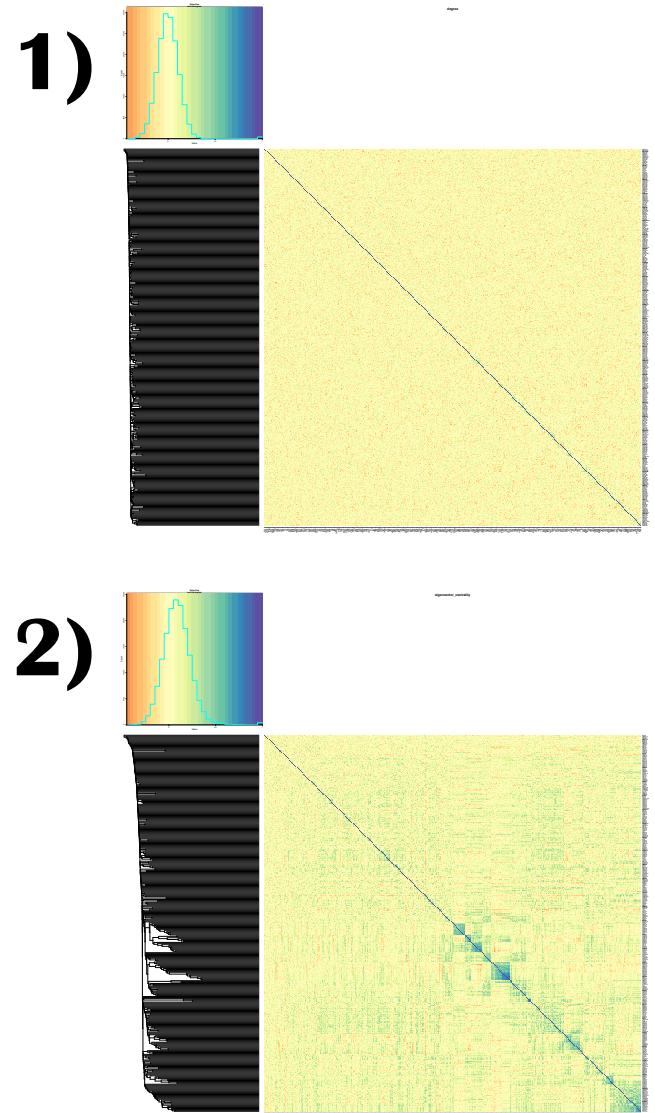
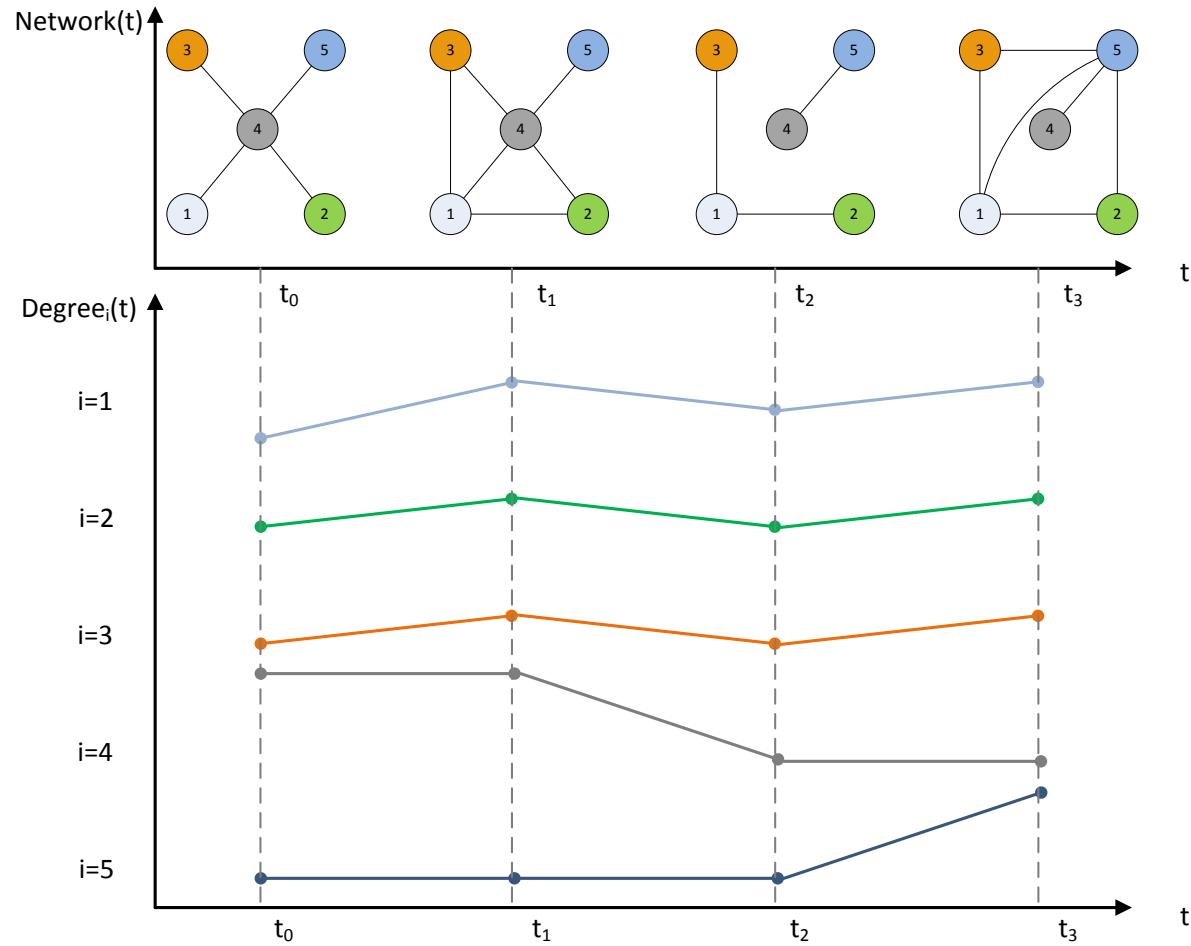
$$\dot{\phi}_i = \omega_i + \frac{d}{k_i + k_{p_i}} \sum_{j=1}^N \sin(\phi_j - \phi_i) + \frac{d_p k_{p,i}}{k_i + k_{p_i}} \sin(\phi_{p_i} + \phi_i) \quad i = 1, \dots, N$$

# III. Failure and recovery in networks

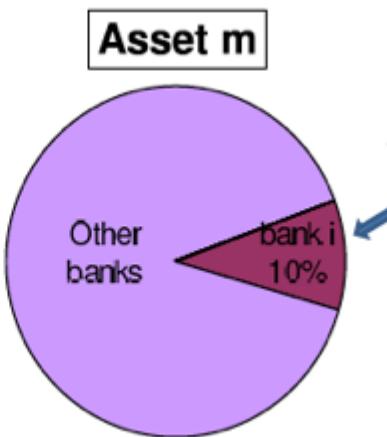
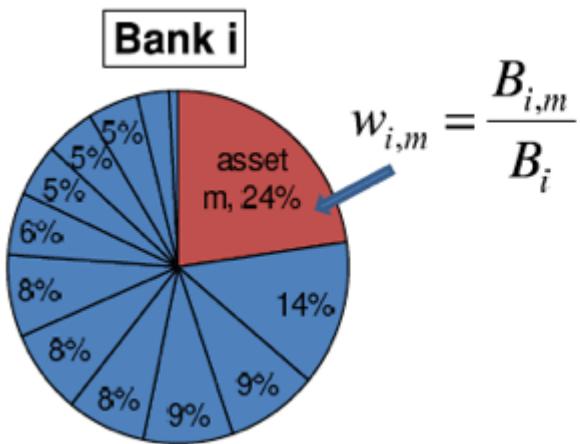


Antonio Majdandzic, Boris Podobnik, Sergey Buldyrev, Dror Y. Kenett, Shlomo Havlin, and H. Eugene Stanley (in press),  
Macroscopic phase-flipping in Dynamical Networks, Nature Physics

# IV. Evolution of networks

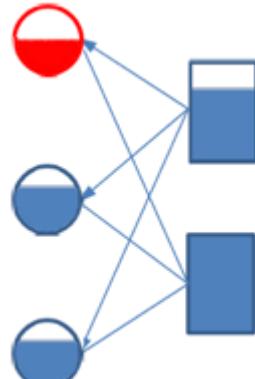
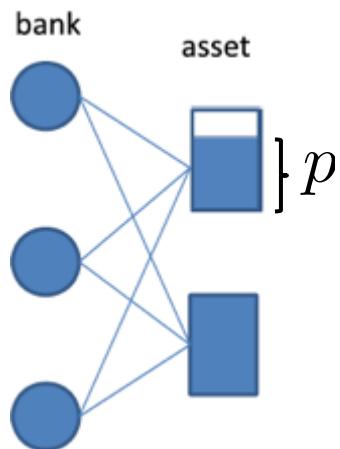


# V. Cascading failures in the financial system

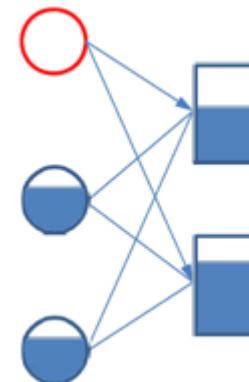


Bipartite Model

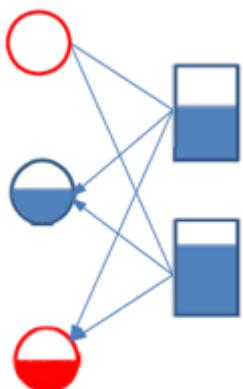
$B_i$ : Total asset of bank  $i$ .  
 $B_{i,m}$ : The amount of asset  $m$  that bank  $i$  owns.  
 $A_m$ : Total market value of asset  $m$ .



fail when  
asset < liability

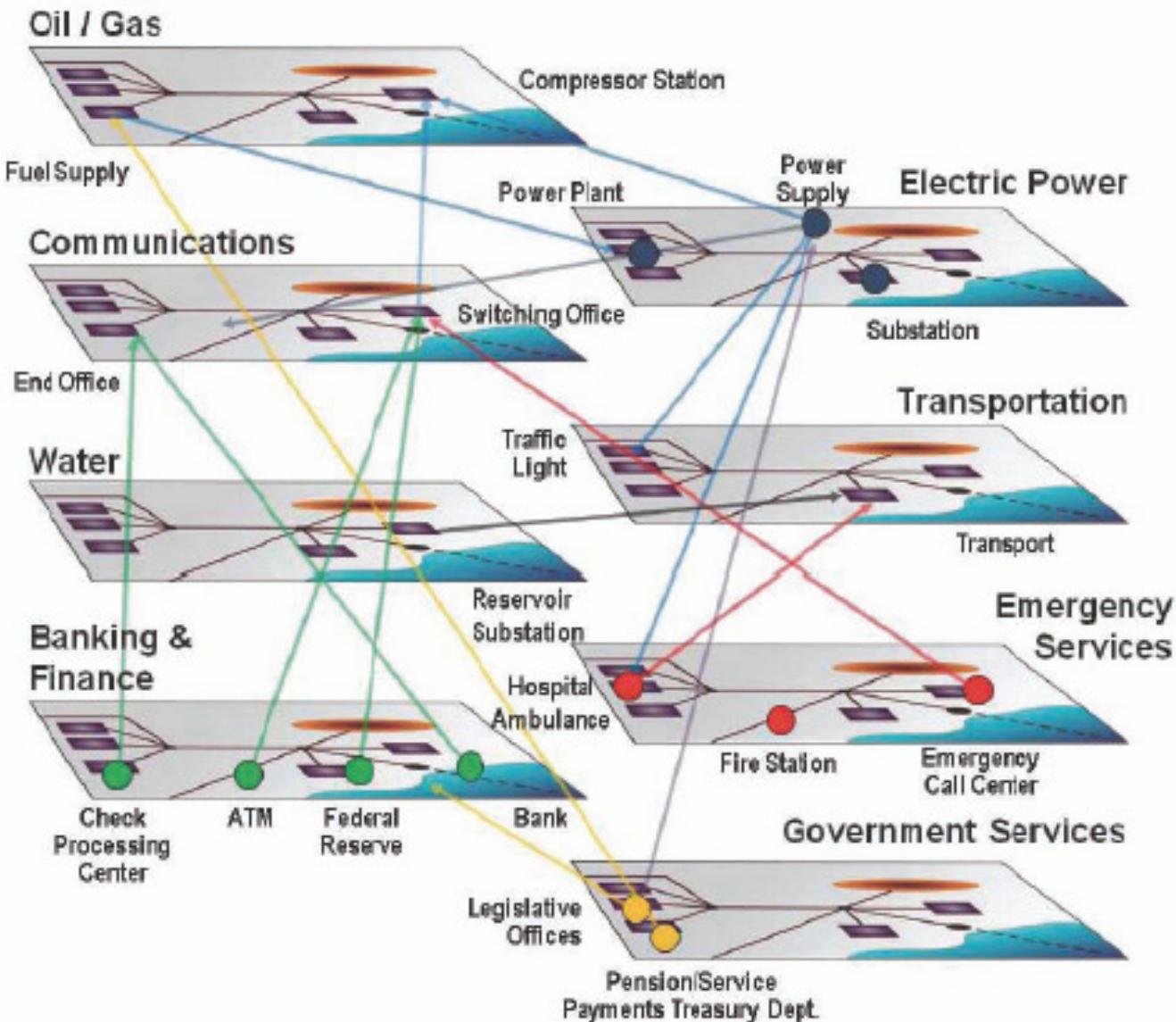


assets depreciate  
 $\alpha B_{i,m}$



1-p: initial shock to an asset  
 $\alpha$  : liquidity parameter  
describes market's reaction to bank failure

# VI. Interdependent networks



Buldyrev, S. V., Parshani, R., Paul, G., Stanley, H. E., & Havlin, S. (2010). Catastrophic cascade of failures in interdependent networks. *Nature*, 464(7291), 1025-1028.

# Summary

- Correlations in Financial Markets
- Market meta-correlation
- Uniformity and Multiformity of markets
- Bottom-up and Top-down effects
- State dependent correlations

# Applications

- Prediction
- Risk management
- Portfolio construction and optimization
- Stability of financial markets
- Financial contagion
- Financial seismograph

# **Thank You.**

# **Questions?**

**Email: drorkenett@gmail.com**