A Markov-Switching Stochastic Volatility Model with Jumps

Econophysics

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Outline

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1. Introduction
2. The model and its estimation
   - The model
   - Estimation Method
3. Empirical Results
   - The data
   - Estimation of the SVJMS model
   - Estimation of the SVJMS model

The Model

- Thanks to Merton's seminal paper of 1973, continuous-time modeling has become very popular for asset pricing
- Jump-Diffusions and more generally continuous-time Markov processes are specified in economics and finance by:

$$dX_t = \mu(X_t; \theta)dt + \sigma(X_t; \theta)dB_t + dJ_t$$

- $\mu(X_t; \theta)$ is the drift term
- $\sigma(X_t; \theta)$ is the diffusion term
- B_t is a Brownian motion
- J_t is a jump process with intensity and size which are state dependent

Stochastic Volatility(Heston - 1993)

Let introduce $Y_t = \log S_t$ (S_t is the stock price) and let the volatility be stochastic: we get the famous Heston(1993) square-root stochastic volatility model:

$$\begin{pmatrix} dY_t \\ dV_t \end{pmatrix} = \begin{pmatrix} \mu + \beta V_t \\ \kappa(\theta - V_t) \end{pmatrix} dt + \sqrt{V_t} \begin{pmatrix} 1 & 0 \\ \rho\sigma_v & \sqrt{(1-\rho^2)}\sigma_v \end{pmatrix} dW_t \quad (1)$$

- Bates(1996) added jumps to the returns only, and solved the model with GMM (and option data)
- Eraker et al.(2003) added jumps to returns and volatility, and solved the model with MCMC (using prices data)

$$\begin{pmatrix} dY_t \\ dV_t \end{pmatrix} = \begin{pmatrix} \mu + \beta V_{t-} \\ \kappa(\theta - V_{t-}) \end{pmatrix} dt + \sqrt{V_{t-}} \begin{pmatrix} 1 & 0 \\ \rho\sigma_v & \sqrt{(1-\rho^2)}\sigma_v \end{pmatrix} dW_t + \begin{pmatrix} dZ_t^y \\ dZ_t^v \end{pmatrix} \quad (2)$$

Jump Estimation

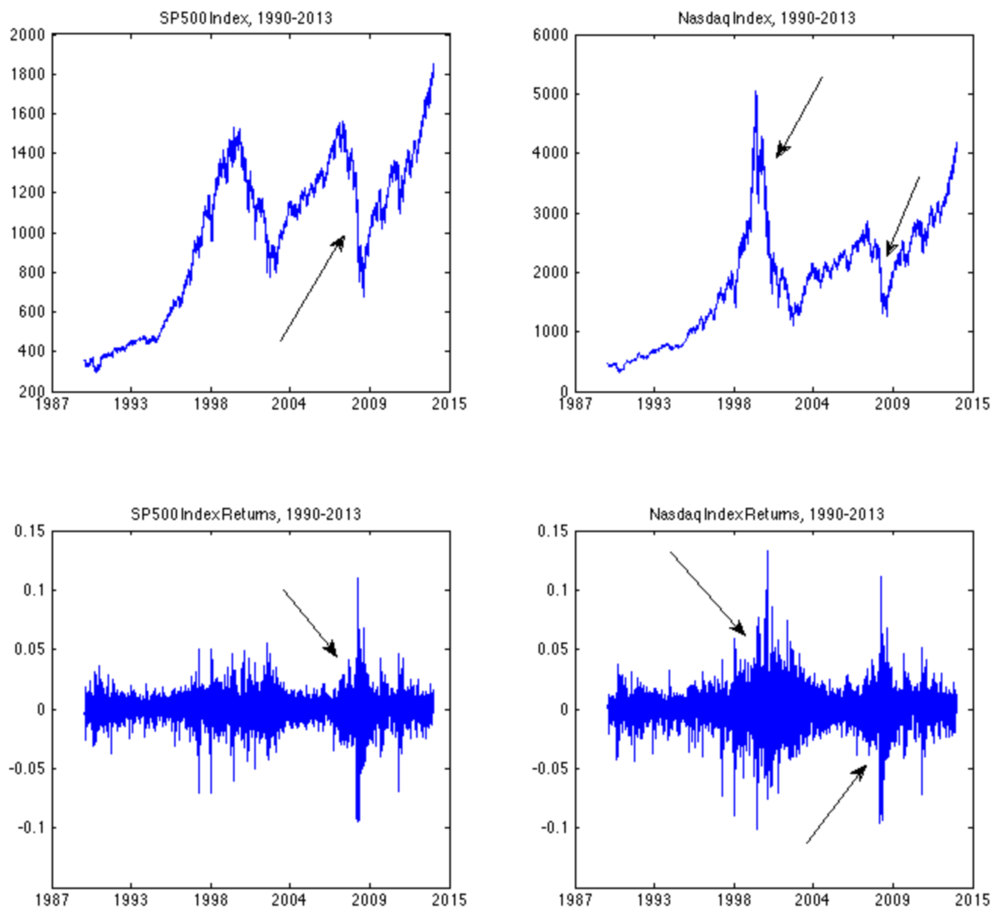
- Todorov(2010) emphasizes the importance of jumps in explaining the dynamics and pricing of the variance premium
- Bollerslev and Todorov (2011) offered a new nonparametric estimation of jump tails
- They found a strong temporal variation in the jump intensity and sizes
- Christoffersen et al. (2012) also provide evidence for time-varying jump intensities

Markov-Switching models

- Hamilton(1989) was the first to introduce this notion of regime switching. He defined the parameters of an autoregression as the outcome of a discrete-state Markov process.
- This triggered a multitude of papers modeling regime-switching
- Regarding stochastic volatility, Casarin(2003 and 2013) attempted to model the volatility parameters as Markov-switching, for a log stochastic volatility model.

This motivates our choice to model the jump intensity and sizes to follow Markov process. We would expect:

- In stress period: a cluster of jumps, and of possibly big size
- In calm period: not many jumps, of medium size


SP500 Index, 1990-2013
Nasdaq Index, 1990-2013
SP500 Index Returns, 1990-2013
Nasdaq Index Returns, 1990-2013
1987 1993 1998 2004 2009 2015

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Let $Y_t = \log S_t$, where S_t is the stock price. Then:

$$dY_t = (\mu + \beta V_{t-})dt + \sqrt{V_{t-}}dW_t^y + \xi_{s_t}^y dN_{s_t}^y$$
$$dV_t = \kappa(\theta - V_{t-})dt + \sigma_v \sqrt{V_{t-}}dW_t^v$$

where $V_{t-} = \lim_{s \to t} V_s$, $\rho = corr(W_t^y, W_t^v)$, $W_t^i (i = y, v)$ is a standard Brownian motion in $\mathbb{R}^2$, $N_{s_t}^y$ is a Poisson process with Markov-switching intensity $\lambda_{s_t}^y$, and $\xi_{s_t}^y \sim N(\mu_{s_t}^y, \sigma^y)$ is the Markov-switching jump size in returns. The Markov chain s_t have the following transition probability:

$$\mathbb{P}(s_t = 2 | s_t = 1) = \epsilon_1$$
$$\mathbb{P}(s_t = 1 | s_t = 2) = \epsilon_2$$

MCMC Methods

- We use here Markov Chain Monte Carlo methods.
- In the bayesian setup, we want to draw our vector of parameters Θ from $p(\Theta|Y)$ (Y is the vector of observations)
- problem: we don't know $p(\Theta|Y)$
- We use the Gibbs algorithm

Data Augmentation

We also have one problem: we have several latent variables (V_t, J_t, ξ_t^y, s_t). Jacquier et al. (1994) were the first to implement a Gibbs cycle to estimate a stochastic volatility model. For this, they used the concept of data augmentation of Tanner and Wong (1987).

- We cannot draw from $p(\Theta|Y)$
- We can then add some parameters, for instance λ
- Consider then $p(\Theta, \lambda|Y)$ via $p(\Theta|\lambda, Y)$ and $p(\lambda|\Theta, Y)$.

To apply the Gibbs algorithm here, we also need to discretize our continuous-time jump-diffusion:

$$\begin{aligned} Y_{(t+1)\Delta} - Y_{t\Delta} = & \quad \mu\Delta + \sqrt{V_{t\Delta}\Delta}\epsilon^{y}_{(t+1)\Delta} + \xi^{y}_{s_{(t+1)\Delta}} J^{y}_{s_{(t+1)\Delta}} \\ V_{(t+1)\Delta} - V_{t\Delta} = & \quad \kappa(\theta - V_{t\Delta})\Delta + \epsilon_v \sqrt{V_{t\Delta}}\epsilon^{v}_{(t+1)\Delta} \end{aligned} \tag{3}$$

where $J^{y}_{s_{(t+1)\Delta}}$ equals 1 if a jump occurs, $\epsilon^{i}_{(t+1)\Delta}$ $(i = y, v)$ are standard normal variables with correlation ρ, and Δ is the time-discretization interval (one-day here). Once discretized, the jump times are Bernoulli random variables with intensity $\lambda^{y}_{s_t}\Delta$.

We need to be able to compute the posterior distribution, which summarizes all the information regarding the latent variables (volatility V, the jump times J and size ξ^y, the Markov chain S) and the parameters $\Theta = (\mu, \kappa, \theta, \sigma_v, \rho, \mu_1^y, \mu_2^y, \sigma^y, \lambda_1, \lambda_2, \epsilon_1, \epsilon_2)$. The posterior distribution combines the likelihood and the prior:

$$p(\Theta, J, \xi^y, V, S|Y) \propto p(Y|\Theta, J, \xi^y, V, S)p(\Theta, J, \xi^y, V, S) \tag{4}$$

Gibbs Cycle

volatility: $p(V_{t\Delta}|V_{(t-1)\Delta},\ V_{(t+1)\Delta},\ \Theta,\ J,\ \xi^y,\ S,\ Y),\ t = 1, \ldots, T$

jump times: $p(J_{t\Delta} = 1|\Theta,\ \xi^y,\ V,\ S,\ Y),\ t = 1, \ldots, T$

jump sizes: $p(\xi^y_{s_{t\Delta}}|\Theta,\ J_{t\Delta} = 1,\ V,\ S,\ Y),\ t = 1, \ldots, T$

states: $p(s_{t\Delta}|s_{(t-1)\Delta},\ s_{(t+1)\Delta},\ \Theta,\ J,\ \xi^y,\ V,\ Y),\ t = 1, \ldots, T$

parameters: $p(\Theta_i|\Theta_{-i},\ \mathrm{J},\ \xi^y,\ V,\ Y),\ i = 1, \ldots, k$

where Θ_{-i} denotes the parameter vector except the i^{th} one, and k is the number of parameters. Drawing from these distributions is not always straightforward. We need to add a Metropolis Step to draw the volatility.

The algorithm will provide a set of draws $\{V^{(m)}, J^{(m)}, \xi^{y(m)}, \xi^{v(m)}, S^{(m)}, \Theta^{(m)}\}_{m=1}^{M}$, M being the number of Monte Carlo simulations. These draws are samples from the posterior distribution.
Because the spot volatilities, the jump times and sizes and the states are drawn from the posterior distribution, the Monte Carlo estimates of these processes is given by:

$$\mathbb{E}[X_{t\Delta}|Y] \approx \frac{1}{M}\sum_{m=1}^{M} X_{t\Delta}^{(m)} \tag{5}$$

where X can be V, ξ^y, J, S. Eraker et al. (2003) also notes a desirable feature of the MCMC methods: $\mathbb{E}[X_{t\Delta}|Y]$ is estimated and not $\mathbb{E}[X_{t\Delta}|Y, \hat{\Theta}]$, which means that we integrate out all the parameter uncertainty.

Outline

- Estimate the models using the $S\&P500$(SPX) and the Nasdaq(NSX) returns
- We use the January 2, 1990 to December 31, 2013 period
- We have 6,049 daily observations - all from the CRSP

Table : Summary Statistics

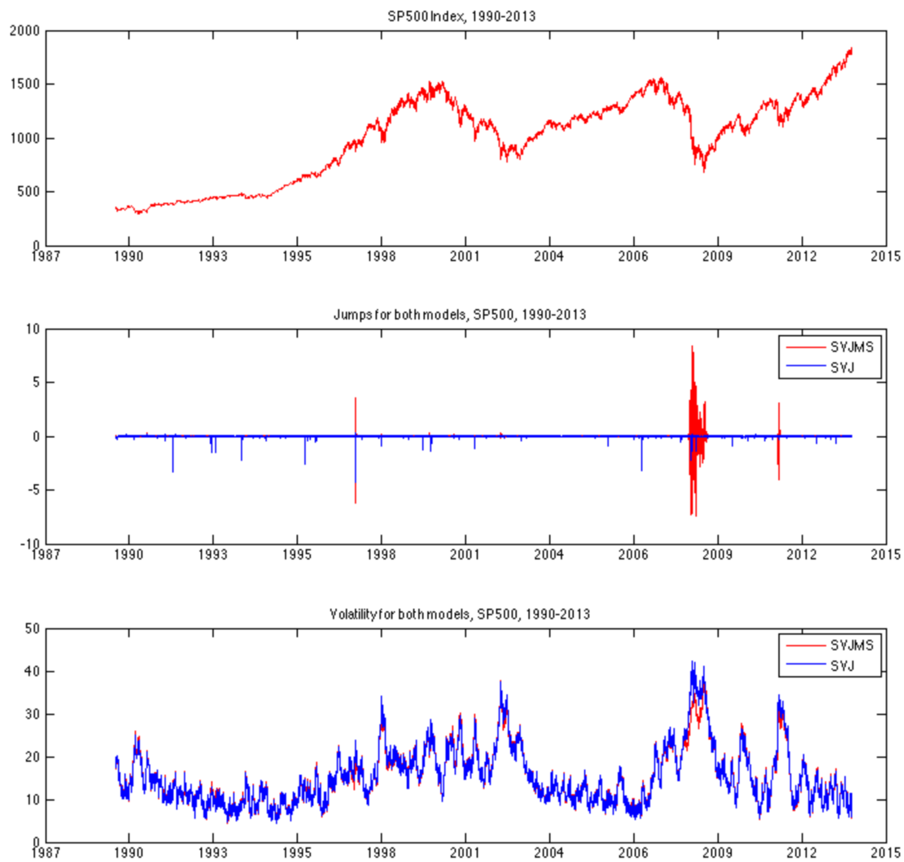
	Mean	Volatility	Skewness	Kurtosis	Min	Max
SP500 90-13	6.82	18.35	-0.24	11.62	-9.47	10.96
Nasdaq 90-13	9.20	24.05	-0.08	9.04	-10.17	13.25

$S\&P500$, 1990-2013

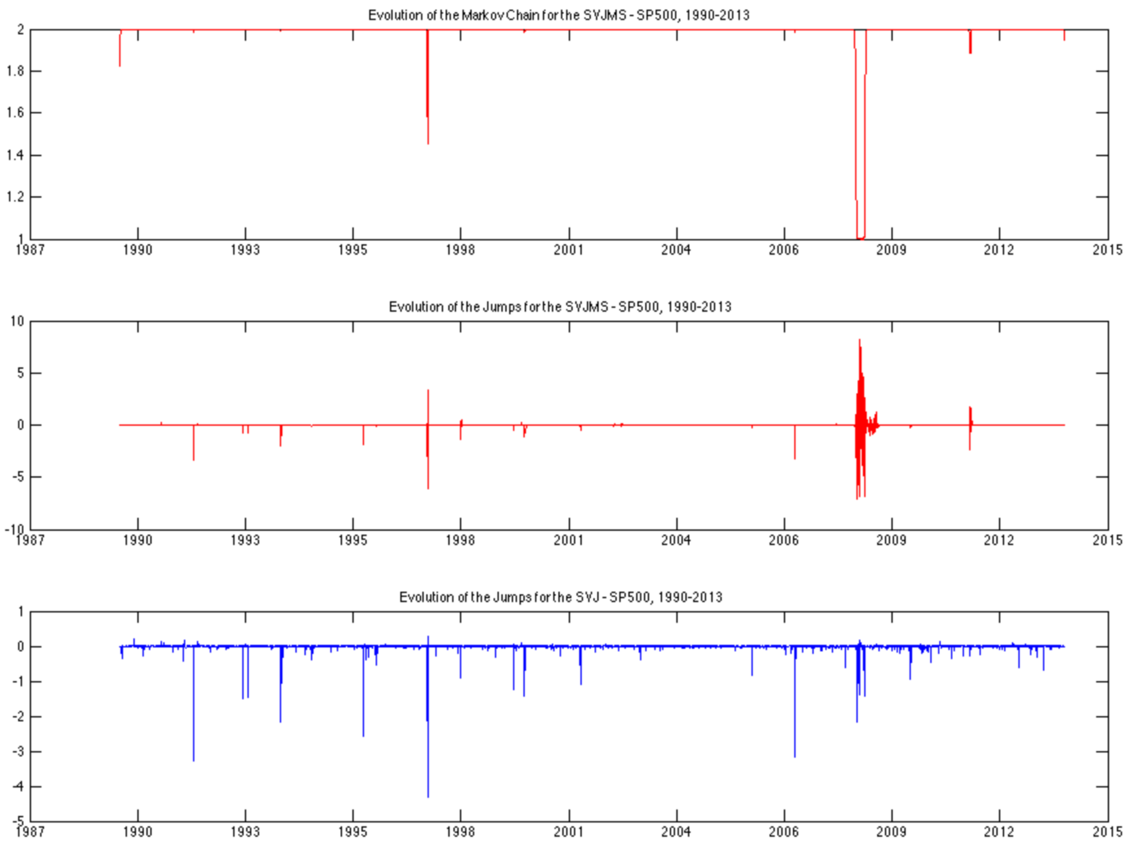
The intensity of the jumps is Markov-switching (λ_{s_t}).

0	SVJ	SVJMS
μ	0.00289(0.0098)	0.0243(0.0096)
θ	1.1921(0.3290)	1.2058(0.3137)
κ	0.0166(0.0026)	0.0180(0.0020)
σ_v	0.1496(0.0097)	0.1559(0.0089)
ρ	-0.7168(0.0281)	-0.7124(0.0303)
μ^y	-1.3900(0.6998)	-0.4717(0.5037)
σ^y	1.7214(0.3475)	3.3840(0.4940)
λ_1	0.0094(0.0050)	0.8278(0.1011)
λ_2	0	0.0025(0.0014)
ϵ_1	1	0.0426(0.0165)
ϵ_2	0	0.0005(0.0003)

S&P500, 1990-2013



S&P500, 1990-2013

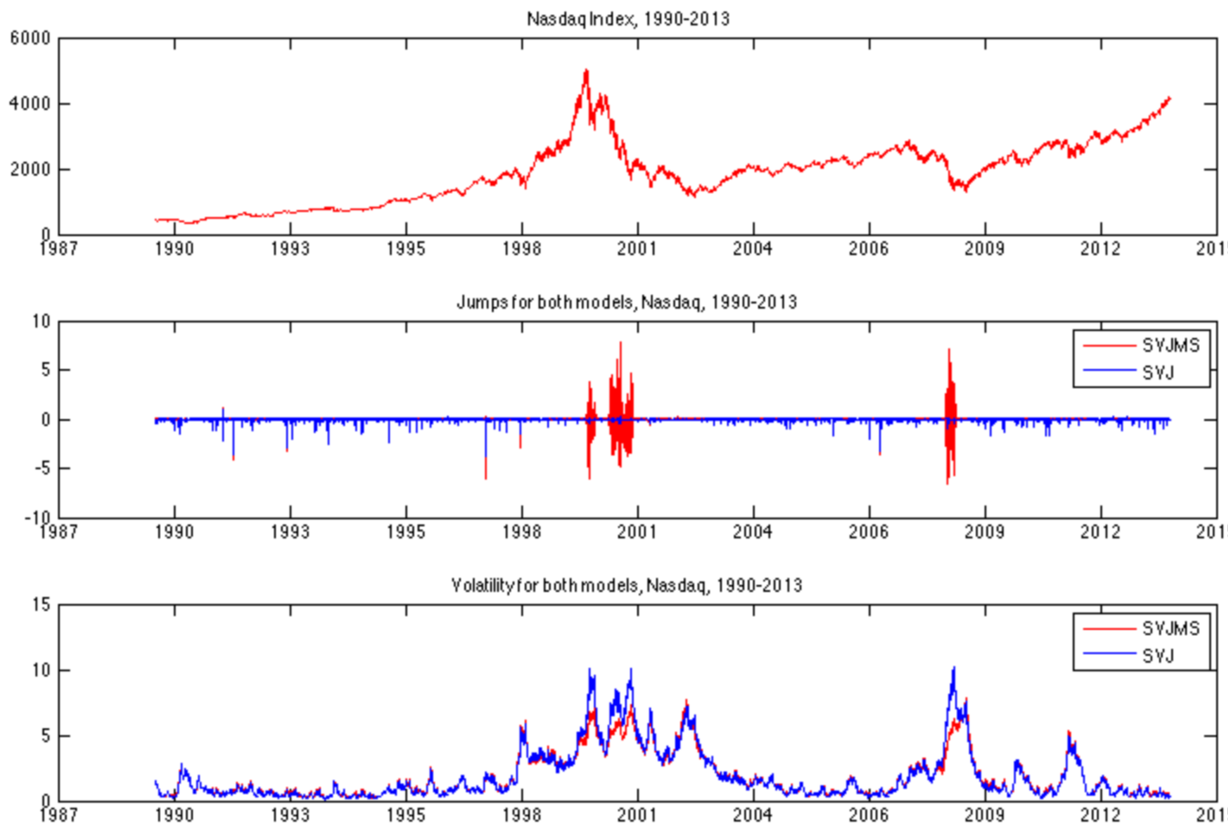


We consider jumps in returns only, and the jump size and the jump intensity to be Markov-switching $(\lambda_{s_t}, \mu^y_{s_t})$.

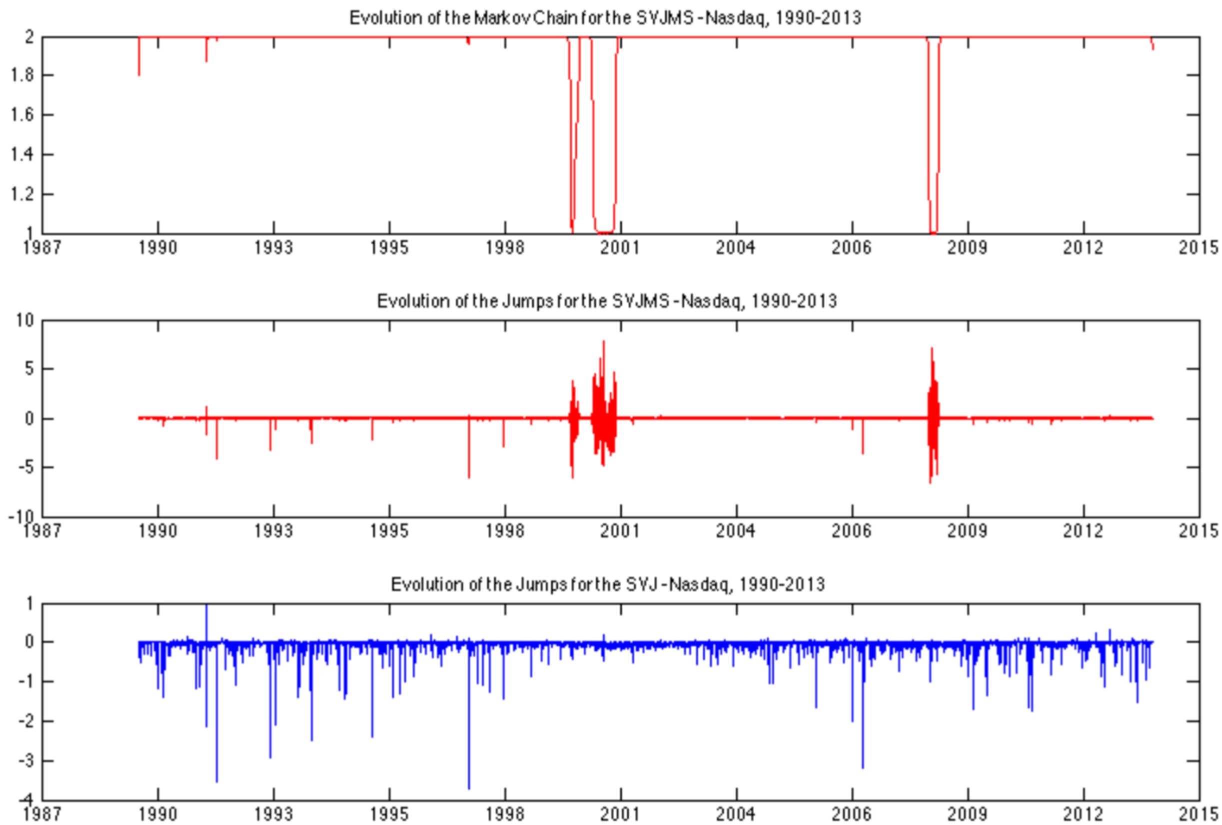
Table : Parameter Comparison for SVJMS model with the SVJ model - NDX

	Nasdaq, 1990-2013	
	SVJ	SVJMS
μ	0.10 (0.0160)	0.0721(0.0125)
θ	1.3874(0.4459)	1.7000(0.4981)
κ	0.013(0.0020)	0.011(0.0025)
σ_v	0.1459(0.0124)	0.1405(0.0104)
ρ	-0.5596(0.0407)	-0.5224(0.0432)
μ^y_1	- 0.979(0.2500)	-0.4432(0.3032)
μ^y_2	- 0.979(0.2500)	-2.1321 (0.9219)
σ^y	1.2226(0.1610)	3.0431(0.3005)
λ^y_1	0.0495(0.0203)	0.8696(0.0616)
λ^y_2	0.0495(0.0203)	0.0053(0.0019)
ϵ_1		0.0268(0.0104)
ϵ_2		0.0011(0.0004)

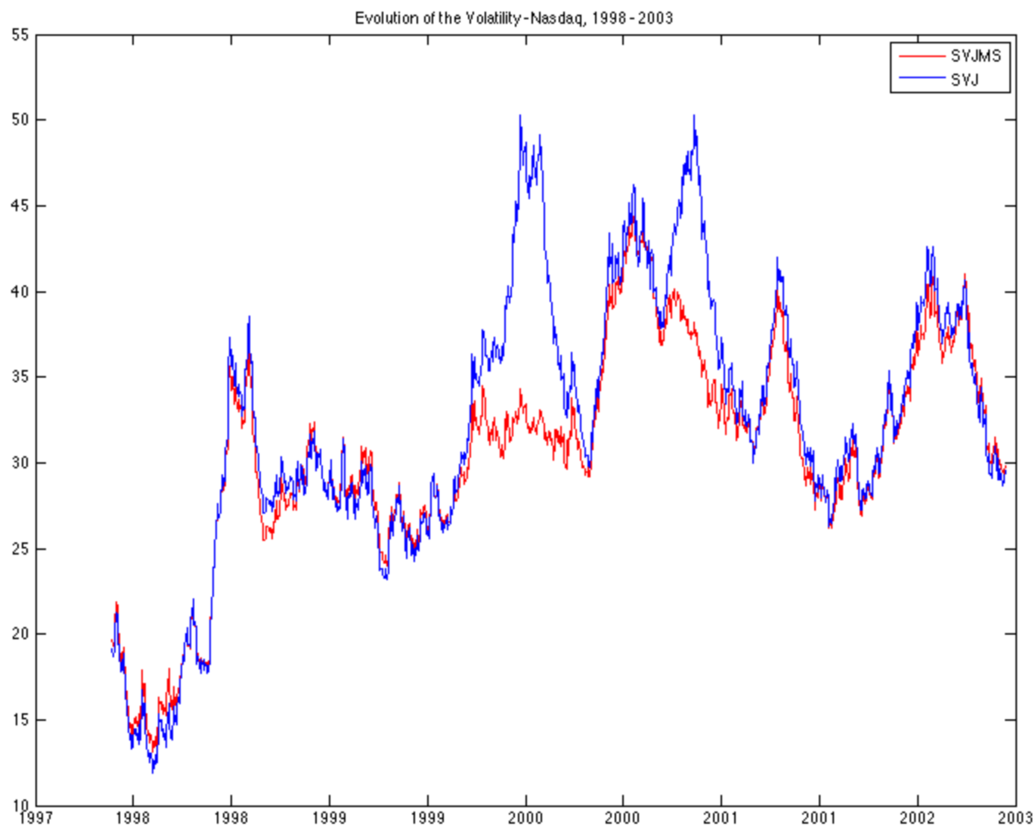
Nasdaq, 1990-2013



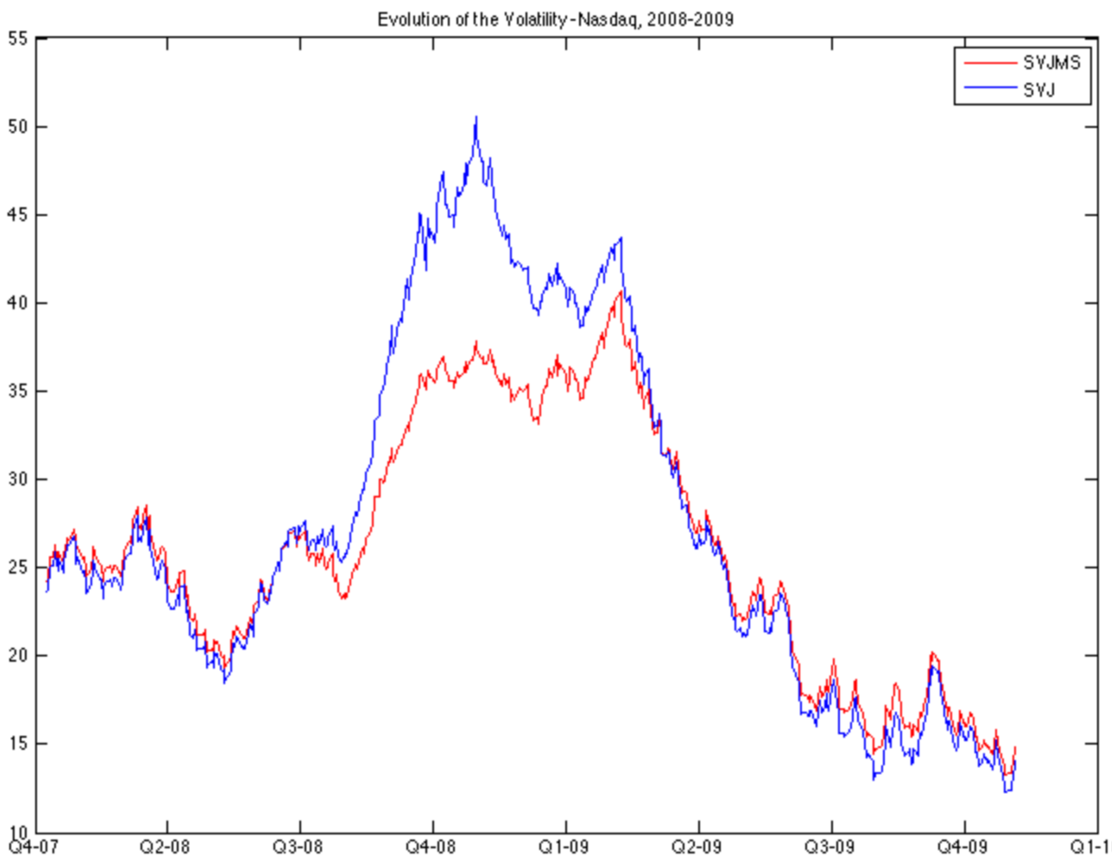
Nasdaq, 1990-2013



Nasdaq, 1998-2003, Tech Bubble



Nasdaq, 2008-2009, Financial Crisis



Total variance decomposition

$$\frac{\mathbb{E}[(\xi_t^y)^2]\lambda_y}{\bar{V} + \mathbb{E}[(\xi_t^y)^2]\lambda_y}$$

Table : Variance Decompositions

	Spot	Volatility	Total	Volatility	Jump Variance	(% total)
	SPX	NDX	SPX	NDX	SPX	NDX
SVJ	17.33	18.7	17.66	21.15	3.72	8.05
SVJMS	17.43	21.15	18.42	23.78	10.47	20.87

Conclusion and future work for that project

- SVJMS model seems promising, we find evidence of two regimes and the volatility dynamics are changed in crisis times
- The convergence of the model can be sensitive to the parameters
- Explore option pricing implications of having different volatility dynamics, especially during stressed periods
- Compute the odds ratios to evaluate which model is more likely

ANY QUESTIONS ?