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Self-similarity of banking network

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Abstract

We analyzed a network structure formed by monetary transactions between financial institutions. We present a procedure to extract a network structure from a set of records of transactions. The extracted network has self-similarity described by a power-law degree distribution. We also introduce a propagation function to describe the self-similarity. A model of network formation based on a mean-field type interaction is proposed to reproduce the self-similarity of the network.

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Keywords: Banking; Network; Mean-field

1. Introduction

Statistical physics is a field that concerns behaviors of systems composed of many elements interacting each other. The economic system is one of such systems. But physicists started to study economic phenomena only very recently in terms of the statistical physics. Though the history of "econophysics" is shorter than those of other traditional fields, the study of economic phenomena has been already one of the central issues of the statistical physics [1,2].

The economic system is composed of individuals and organizations, and the interaction between them is transactions of properties. Since transactions in the modern

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economic society is based on transactions of money in many cases, the role of financial institutions is especially important, for financial institutions provide means of monetary transactions for individuals and other organizations. The financial institutions such as banks themselves perform monetary transactions each other forming a network of monetary transactions, which we call a banking network.

Recently, structure of networks formed in the natural world is one of hot issues in the field of the statistical physics. To make the definition clear, a network is a structure formed by nodes connected by links. It has been recognized that many network structures formed in nature, such as the structures of the Internet [3], the world-wide web [4], and metabolic networks [5], present a power-law degree distributions. That is, the number k of the links connected to a node, which is called a degree, follows a power-law distribution

$$N(\geqslant k) \propto k^{-\gamma}, \tag{1}$$

where $N(\geqslant k)$ is the number of nodes with k or more links. The value of the exponent γ ranges from 1 to 2 in many cases. Note that, in our paper, the distribution is presented by the cumulative form, thus the values of the exponent γ differ by 1 from those of the probability density function. A power-law distribution is often related to the concept of fractals [6] and describes geometrical or statistical self-similarity.

The structure of the banking network is also an intriguing problem of the statistical physics. Also, it is useful to know about the structure and the nature of the network because the knowledge may be applicable to a financial policy to keep the efficiency and the stability of the economic systems.

In this paper, we investigate the structure of the banking network. The banking network is extracted from records of real monetary transactions between financial institutions. The procedure for the extraction is discussed in the following section. We show that the network has self-similarity described by a power-law degree distribution. In Section 3, we propose a model of network formation based on a mean-field type interaction. The model proves that the scale-free network by the model is at a critical point of a phase transition. In Section 4, we discuss the structure of the banking network comparing it with those of the networks by the network formation models. In the discussion, we introduce a propagation function to describe the global self-similarity of the banking network. A summary is attached as the last section.

2. The self-similarity of the banking network

2.1. Observation of monetary transactions

The observation of the network structure of monetary transactions between financial institutions has not been performed so far, presumably, because there has been no means to record individual transactions in detail. The recent development of the computer technology has made the observation possible.

In the following analysis, we use the records of monetary transactions provided by the Bank of Japan. The Bank of Japan is the Japanese central bank and provides several means of monetary transactions for the financial institutions in the country. One of them is the transaction through current accounts. Each financial institution has its own current account at the Bank of Japan, and performs its transactions by transferring money from the account to the others. The Bank of Japan has established an online system to maintain the efficiency of the transactions, and now all the transactions between financial institutions through the current accounts are performed on the online system. This means that the electrical records of all the transactions between financial institutions through the current accounts are left on the computer system of the Bank of Japan. This is the data we analyze in this paper. [1]

The observation period of the analyzed records is 1 month period of June 2001. About 150 thousand transactions were performed within the period, and the total amount of the transacted money was about 730 trillion yen. The bank of Japan provides the online system not only for banks but also for other kinds of financial institutions such as securities firms and call loan corporations. In this paper, however, we call all kinds of financial institutions collectively "banks" for simplicity. A record of a transaction in the analyzed data provides information of the origin and the destination of the transaction, the amount of the transferred money, the time the transaction performed, and so on. The origin and the destination of a transaction in the data are described by bank codes and branch codes. But, in the following analysis, we neglect branches of banks. That is, for example, when a branch a of a bank A performs a transaction to a branch b of a bank B, it is simply interpreted as a transaction from the bank A to the bank B. The transactions between the bank A and the bank B are collected regardless of the directions of the transactions, and the total amount of the transferred money, Q_{AB}, and the number of the transactions, W_{AB}, are calculated, respectively. We extract a network structure of monetary transactions from the data of Q and W.

2.2. The structure of the banking network

First of all, we have to define nodes and links of the banking network. In the case of the structure of computer networks such as the Internet, the definition of the networks is obvious. Each computer or hub forms a node, and a cable connecting any pair of them is interpreted as a link. In the case of the banking network, though individual banks naturally define the nodes, there is no explicit definition of the links. In a sense, all pairs of banks in the economic system are connected to each other because each bank can perform a transaction with any other bank through the online system. However, an intimate pair of banks may perform more intensive transactions than other pairs. Here, we consider such a network structure formed by inhomogeneity of transactions. Thus, we have to somehow define the links between banks from the intensity of transactions. The easiest way to define links in this direction may be to

[1] Though the Bank of Japan records all the transactions through the current accounts, a part of transactions, such as payment for a purchase of national bonds, is not included in the analyzed data.

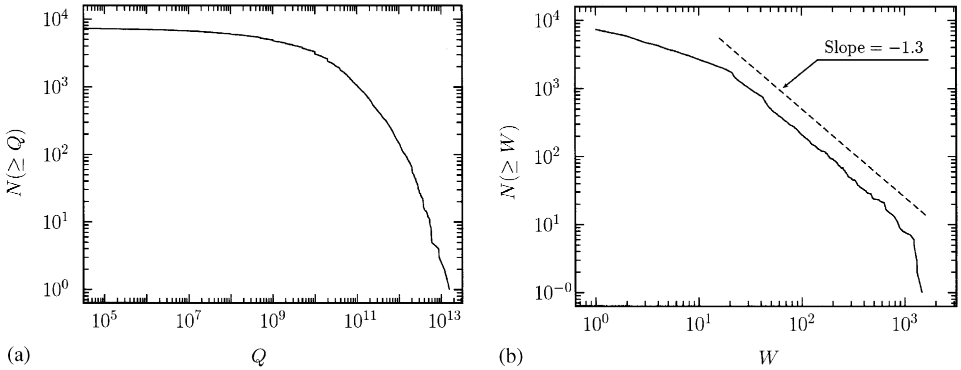

Fig. 1. The cumulative distributions of (a) the transacted money Q and (b) the transaction number W between pairs of banks, respectively. The graph of W shows an apparent kink at about $W = 20$, while the graph of Q is smooth in all the range. The dashed line shows a power-law relation $N(\geqslant W) \propto W^{-1.3}$.

introduce a threshold for the intensity of transactions and to define a link when the intensity exceeds the threshold. For this purpose, we check the statistics of Q and W.

Fig. 1 shows the cumulative distributions of Q and W. As in Fig. 1(a), the distribution of Q has a smooth curve in all the observational rage without a specific kink. Though we can set a threshold for Q at, for example, the average value of Q, it is rather artificial and its physical meaning is ambiguous. On the other hand, the graph of W in Fig. 1(b) has a clear kink at about $W = 20$. Since the number of the weekdays in the observation period is 21, the transaction number $W = 20$ roughly means the frequency of one transaction a weekday. Moreover, the graph above the kink fits well by a power-law function

$$N(\geqslant W) \propto W^{-1.3}, \tag{2}$$

which implies statistical self-similarity of the transactions.

Because of the clear physical meaning of the kink and the self-similarity implied by the power law, we set the threshold for W at $W_t = 20$ and define a link between a pair of banks when W between them is larger or equal to W_t regardless of the value of Q.

Fig. 2 shows the network structure extracted by the above definition. In the figure a bank with the larger number of links is placed the closer to the center aside from several exceptions. As in the figure, a bank placed close to the edge has only a couple of links directed toward the center. In fact, of 63,903 pairs formed by 358 nodes in the network, only 1785 pairs are connected by links. This means that most part of the banking network is composed of pairs without links.

Fig. 3 is the cumulative degree distribution of the network in log–log scale. The straight part of the graph shows a power-law distribution Eq. (1) with the exponent $\gamma = 1.1$. This result proves that the banking network, in fact, has a self-similarity described by Eq. (1).

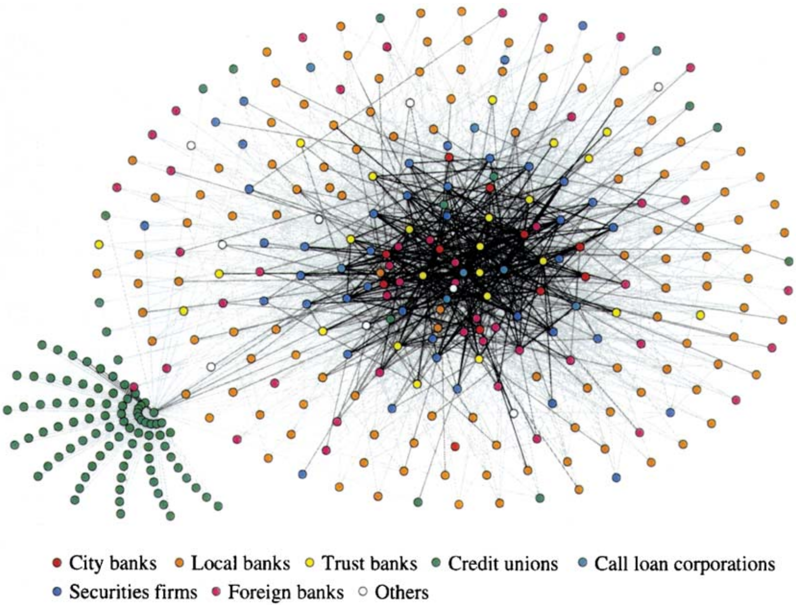

Fig. 2. The figure of the banking network. The banks are arranged so that a bank with the larger number of links is placed the closer to the center of the figure with a few exceptions. The colors of the nodes show the kinds of the financial institutions. The darkness of the lines shows the frequency of transactions.

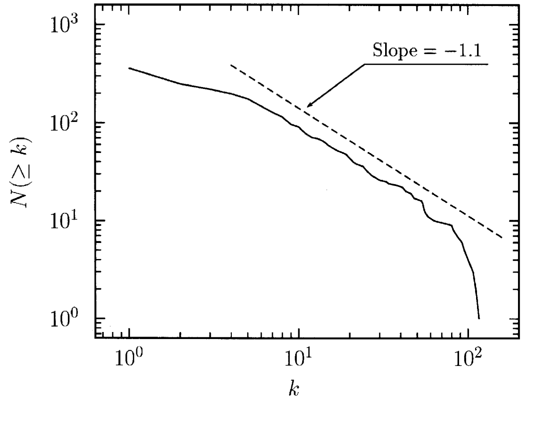

Fig. 3. The cumulative degree distribution of the banking network. The dashed line shows a power-law relation $N(\geqslant k) \propto k^{-1.1}$.

3. A mean-field model of scale-free networks

Barabási and Albert proposed a stochastic model of network formation which produces a scale-free network [7]. However, when we compare the network by the model with the banking network, we notice that there are a few significant differences between them.

An important point of the differences is that the banking network is not based on static links like the case of the Internet. In the case of the Internet, the network is composed of cables as links which connect individual computers or hubs as nodes. When a new computer is introduced to the network, it is connected to the existing network by a cable. Once a cable is connected to the network, disconnection or reconnection rarely occurs unless it is really necessary. The model by Barabási and Albert, which we call a growth model hereafter, simulates this situation, and is based on the idea of the growth of network, in which new nodes are connected one by one by links to the existing network. And once a node is connected to the network, the links are not rearranged. In the case of the banking network, however, the idea of static links is unsuitable. Each bank in the network can easily perform a transaction to another at any moment. When some deformation of the network, such as establishment or collapse of a bank, occurs, the network can reform its structure and adapt to the deformation.

Another point of the differences between the banking network and the growth model is that the links in the banking network have weight. The links of the banking network represent the heavy transactions between pairs of banks, and involve the information of the frequency of transactions. In the case of the growth model, however, links are binary and have only two state between a pair of nodes, connected and unconnected.

The most remarkable fact of the banking network is that it organizes itself into a scale-free network only by the dynamics of links without addition of nodes, for establishment of a new bank occurs rarely. In other words, the banking network has dynamic link and nearly constant number of nodes, while the growth model is composed of static links and growing nodes. Though some models are proposed to include dynamics or weight of links [8], their dynamics of links is not similar to that of the banking network.

We propose a model of network formation based on a mean-field interaction between nodes as a model of the banking network.

3.1. The description of the model

The model is composed of N nodes, and the number N of the nodes are unchanged during the time evolution of the system. Each node is assigned weight m. The nodes are sorted by the order of the weight m and numbered so that the node with the largest m is numbered one and that the node with the smallest m is numbered N. At time t, the ith and the jth nodes in the system with weight $m_i(t)$ and $m_j(t)$, respectively, interact each other with weight $w = w_{ij}$. We assume a mean-field type interaction and the weight w_{ij} of the interaction is defined by the weight of the nodes as

$$w_{ij}(t) = m_i(t) \times m_j(t) . \tag{3}$$

Tentative weight of the nodes at time $t+1$, $m_i'(t+1)$, is determined by the weight of the interaction as

$$m_i'(t+1) = \sum_{j=i+1}^{N} w_{ij}(t) + \frac{m_0}{N} , \tag{4}$$

where m_0 is a positive constant. This means that the new weight of the ith node is determined by the sum of the constant m_0/N and the weight of the interactions to the ith node from the nodes with smaller weight than $m_i(t)$. Finally, the tentative node weight $m_i'(t+1)$ is normalized and the new node weight $m_i(t+1)$ is calculated by

$$m_i(t+1) = \frac{m_i'(t+1)}{\sum_{j=1}^{N} m_j'(t+1)} . \tag{5}$$

The time evolution of the system is performed by repeating the procedures Eqs. (3)–(5). The order of the nodes is unchanged during the time evolution. Thus, the model is deterministic. The randomness in the model comes only from the initial state and the model expands the initial randomness.

The model includes several concepts that are not obvious. Especially, the procedure Eq. (4) may look odd because it sums up only the weight of the interactions from the nodes with smaller weight, not from the nodes with larger weight. We interpret above procedures in terms of the banking network as follows.

The weight of a node is interpreted as a credit of a bank. A bank performs transactions with another with the credit of the origin and the destination. The procedure Eq. (3) describes the situation, and the weight of a link w represents the frequency of the transactions.

The credit of a bank is determined by the transactions it has performed. However, not all the transactions of the bank contribute to the credit of the bank. We generally think that a large bank is more reliable than a small bank. When a bank performs a transaction to a bank larger than itself, the transaction is performed because the bank credits the larger bank. Thus, the transaction contributes to the credit of the larger bank. On the other hand, when a bank performs a transaction to a bank smaller than itself, it is performed only because of necessity. Thus, the transaction does not contribute to the credit of the smaller bank. Summarizing the above situation, new credit of a bank is determined by the sum of transactions from the banks smaller than itself. A bank also performs transactions with its customers such as depositors in addition to other banks. This also contributes to the credit of the bank. Background transactions of this kind are represented by m_0 in the model. The procedure Eq. (4) describes these in a simplified way.

The procedure Eq. (5) describes time decay of the credit. That is, credit earned by a bank does not have an eternal effect, and the effect decreases with time exponentially. We introduce the normalization of the credit instead of the time decay of the credit. Since the total credit in the system is prone to increase by the injection of the background transaction m_0, the normalization has the same effect to that of the time decay.

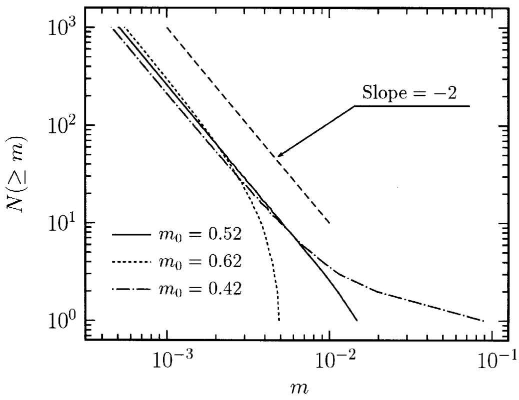

Fig. 4. The cumulative distributions of the node weight obtained by the mean-field model for various values of m_0. The distribution follows a power law $N(\geqslant m) \propto m^{-\tau}$ with the exponent $\tau = 2$ when m_0 is tuned to the critical value $m_0 = 0.52$. When m_0 is smaller than the critical value, the system is composed of one node with large weight and others with small weight, while, when m_0 is larger than the critical value, all the nodes in the system have similar weight each other.

3.2. Results

The numerical simulation of the model was performed on a system with $N = 1000$ nodes. At the initial stage $t=0$ of the system, the weight of the nodes $m_i(0)$ is given by a uniform random number ranging $[0, 1)$ and normalized by the procedure Eq. (5). The iteration of the above procedure is performed up to $t = 100$. We observe the behavior of the model with various values of the parameter m_0.

After sufficient number of the iteration, the system reaches to a steady state, where the distribution of the node weight m_i is unchanged by the iteration any more. The iteration number $t = 100$ is sufficient to get the steady state. We present the cumulative distributions of m_i for 3 values of m_0 at the steady states in Fig. 4. When the value of the parameter m_0 is somewhat small, there appears one node with large weight in the system. On the other hand, when the value of the parameter m_0 is large, the system reaches a state where all the nodes has similar values of m_i. And between these two states, when the parameter is tuned to the proper value $m_0 = 0.52$, the system reaches to a state where the cumulative distribution of m_i follows a power law

$$N(\geqslant m) \propto m^{-\tau} \tag{6}$$

with the value of τ close to 2. This situation is similar to that occurs in second-order phase transition [9], and indicates that the power-law distribution Eq. (6) is a result of a critical phenomenon.

The self-similarity of the banking network was observed by a power-law degree distribution. We observe the self-similarity of the model in the same way. As in the case of the banking network, the nodes in the model are not explicitly connected by links. Thus, we set a threshold w_t for the weight of the interaction w, and define a link between a pair of nodes with weight larger or equal to w_t.

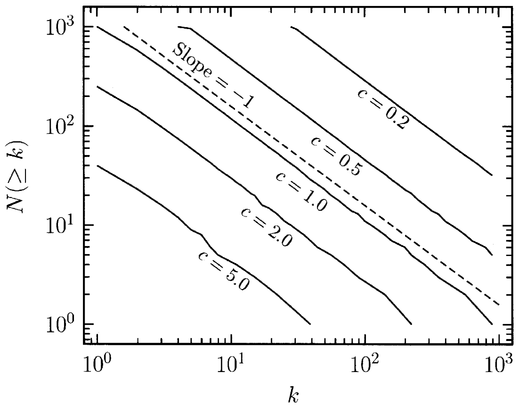

Fig. 5. The cumulative degree distributions of the network by the mean-field model for various values of the threshold w_t. The threshold $w_t = c \times m_{\max} \times m_{\min}$ is presented by the values of c in the figure. The dashed line shows a power-law relation $N(\geqslant k) \propto k^{-1}$.

Fig. 5 shows cumulative degree distributions for various values of the threshold. The values of the threshold are set by

$$w_t = c \times m_{\max} \times m_{\min} , \tag{7}$$

where c is a positive parameter and $m_{\max}$ and $m_{\min}$ are the maximum and the minimum values of m_i in the system, respectively. The values of the threshold are presented by the values of c in the figure. As in the figure, the degree distributions follow power-laws. The exponents of the power-laws are always close to $\gamma = -1$, which is also close to that of banking network $\gamma = -1.1$, regardless of the value of the threshold. This proves that our model successfully reproduces the self-similarity of the banking network.

A power-law distribution with exponent $\gamma = 1$ is often confused to be unphysical because the average of the distribution diverges. A power-law distribution observed in a system with a finite size inevitably has a cutoff. The position of the cutoff increases with the system size, and the average of the distribution is comparable to the value of the cutoff. Namely, the divergence of the average simply means that the average of the distribution depends on the system size. Thus, a power-law distribution with exponent $\gamma = 1$ is not unphysical.

When the distribution of the node weight m follows a power-law Eq. (6) and the mean-field interaction Eq. (3) is assumed, it is easy to show by a scaling relation that the link-number distribution of the model always follows a power-law Eq. (1) with an exponent $\gamma = 1$.

The distribution of the weight of the interactions w on a node with weight m_c is

$$N(\geqslant w) = N(\geqslant m_c \times m) \propto m^{-\tau} = \left(\frac{m_c}{w}\right)^{\tau} \tag{8}$$

by Eqs. (3) and (6). When the threshold of w is set at w_t and links are defined by the interactions larger or equal to the threshold, the number k of the links connected to the node is calculated by Eq. (8) as

$$k = N(\geqslant w_t) \propto \left(\frac{m_c}{w_t}\right)^{\tau} . \tag{9}$$

This means that the number of the links connected to the node is an increasing function of the weight m_c. The number of the nodes with weight larger or equal to m_c in the system is, of course, by Eq. (6),

$$N(\geqslant m_c) \propto m_c^{-\tau} . \tag{10}$$

Since k is an increasing function of m_c, a relation $N(\geqslant m_c) = N(\geqslant k)$ holds. Substituting this relation and Eq. (9) into Eq. (10), we get

$$N(\geqslant k) \propto k^{-1} . \tag{11}$$

This means that the exponent γ of the power-law degree distribution Eq. (1) of the network by the mean-field model is $\gamma = 1$.

4. Discussion

In this section, we discuss the structure of the banking network by comparing it to those of the networks obtained by the mean-field model and the growth model. The structure of the banking network is directly related to the stability and the efficiency of the network. Albert, Jeong, and Barabási studied tolerance of networks against removal of nodes [10]. They reported that a scale-free network is vulnerable to attacks on hub-like nodes. However, this does not necessarily mean that a hub in a network is a harm for the network. A hub in a network generally makes accesses between nodes in the network efficient. Thus, the efficiency and the safety of a network are in a relation of a trade-off. Here, we discuss the structure of scale-free networks in views of the efficiency and the safety by means of a power-law degree distribution and a propagation function of the network.

4.1. The comparison by the degree distribution

The local structure of the banking network, which concerns connectivity of links around a single node, is described by the degree distribution Eq. (1). The distribution follows a power-law with the exponent $\gamma = 1.1$. The mean-field model reproduces the power-law degree distribution with the exponent $\gamma = 1$, which shows good agreement with that of the banking network. The growth model also reproduces a power-law degree distribution. However, the exponent $\gamma = 2$ is a little bit larger than that of the banking network.

The smaller power exponent of the degree distribution of the banking network compared to that of the growth model is interpreted that the banking network is more efficient than the network by the growth model.

Consider two networks which have the same number of nodes and the same number of links but different values of the exponent γ. A small value of γ means that the degree distribution has a long tail in a graph. This again means that the network has large number of hub-like nodes with relatively large number of links.

Generally speaking, when a network has many hubs, it is more efficient than a network with small number of hubs. When a node tries to access to another node in a network, if there is a hub connecting both the origin and the destination, it takes only two steps to access to the destination: from the origin to the hub and from the hub to the destination. If there is no hub connecting the origin and the destination in the network, the access has to take a detour.

Thus, the banking network has more efficient structure than that of the network formed by the growth model.

4.2. The comparison by a propagation function

Though the relative number of hubs in a network is described by the exponent γ of the degree distribution Eq. (1), it does not describe the connectivity of hubs. When a node which is connected to a hub try to access to another node that is connected to another hub, if the two hubs are connected directly affects the global efficiency of the network. To describe the global connectivity of a network, we introduce a propagation function of a network, which is similar to a number-radius relation [11].

We consider a network composed of nodes and weighted links like the banking network. Length of a link with weight w is defined by the reciprocal of w. In the case of the banking network, the length of a link is proportional to the average time interval of transactions on the link since the weight of links in this case is the frequency of transactions. Based on the length of links, the "distance" r_{ij} between the ith node and the jth node is measured along the path between the two nodes. More precisely, the distance r_{ij} is the sum of the length of the links along the path connecting the ith node and the jth node. When there are more than one paths between the two nodes, the shortest path is chosen and the distance r_{ij} is defined to be the length of the shortest path. Note that the "distance" defined here does not always mean a geometrical distance, because it does not necessarily fulfill a triangle inequality. However, the distance between a pair of nodes on the banking network has a clear physical meaning, for it is roughly proportional to the time for the effect of a transaction to propagate between the pair.

The propagation function for the ith node, $n_i(r)$, is defined by the number of nodes placed within distance r from the ith node. And the propagation function for the network, $n(r)$, is defined by the sum of $n_i(r)$ for all nodes, that is,

$$n(r) = \sum_i n_i(r) . \tag{12}$$

The propagation function for the banking network is presented in Fig. 6 in log–log scale. The graph shows a power-law relation

$$n(r) \propto r^D , \tag{13}$$

where the value of the exponent D is close to 2.

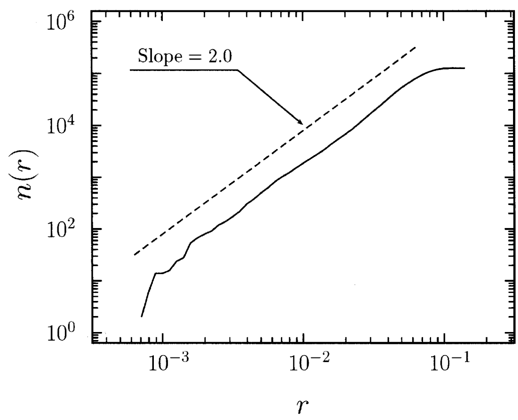

Fig. 6. The propagation function of the banking network. The dashed line shows a power-law relation $n(r) \propto r^{2.0}$.

As we mentioned before, the concept of the propagation function is similar to that of the number-radius relation. The number-radius relation is related to the concept of self-similarity when it follows a power law [11]. Though the "distance" between two banks on the banking network cannot be strictly interpreted as a geometrical distance, the propagation function describes how the effect of a transaction propagates on the network. The propagation function is a representation of the structure of the network. The relation Eq. (13) indicates that the banking network has a self-similarity and it is described by the exponent D. Though there has been an attempt to describe self-similarity of a network by a spectral dimension [12], our method is far easier.

The self-similarity of the banking network may be interpreted in terms of the connectivity of hubs. That is, nodes are connected to a node forming a hub, and hubs are again connected to a node forming a superhub.

We also plot the propagation function of the networks by the growth model and the mean-field model in Fig. 7. To calculate the length of the links properly, the links of the network by the growth model is weighted by uniform random numbers. The graphs show steep slopes in log–log scale. The exponents of the best-fit power-law relations are $D = 4.0$ for the mean-field model and $D = 5.1$ for the growth model, respectively, though the graphs are so steep that both of them may describe exponential relations.

The smaller exponent of the banking network compared to that of the simulated networks may be related to the safety of the network. Suppose a bank is collapsed in the banking network. Since the propagation function of the banking network describes the average number of banks within a radius from a bank, and the radius is roughly proportional to the time for a transaction from the bank to propagate, the exponent $D=2$ of the propagation function means that the effect of the collapse reaches increasing number of banks only with square of time. In the cases of the simulated networks, however, the effect reaches banks with the fourth or the fifth power of time. Thus, the banking network has a safer structure against collapse of a node compared to those of the networks by the models.

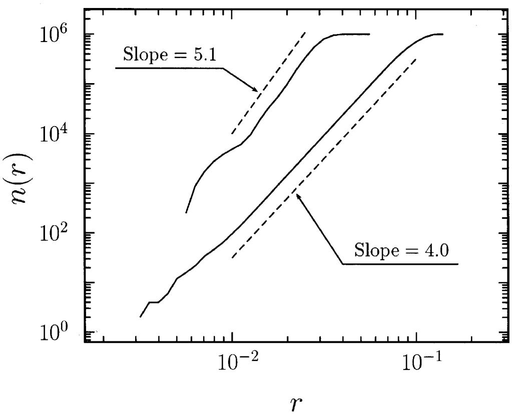

Fig. 7. The propagation functions of the networks by the growth model (above) and the mean-field model (below), respectively. The dashed lines show power-law relations $n(r) \propto r^{5.1}$ and $n(r) \propto r^{4.0}$, respectively.

5. Summary

In this paper, we studied the structure of the banking network.

The most significant feature of the banking network is its dynamic, weighted links. Our procedure discussed in Section 2 is a way to extract a "binary network" from the network with dynamic, weighted links. The extracted binary network, in fact, shows self-similarity consistent with that of the binary networks studied so far.

Though we analyzed the banking network in terms of a degree distribution according to the tradition in this field, it may not be suitable to see the banking system as a binary network. Rather, it may be more natural to see it as an ensemble of nodes interacting each other with various intensity. In this case, of course, the nodes are banks and the interaction is monetary transactions. The binary banking network is only a way to observe the complex interactions of the banking system.

In the procedure to extract the banking network in Section 2, we defined links by setting a threshold for the frequency of transactions between pairs of banks. On the other hand, it may seem more natural to define links by setting a threshold for the total amounts of transacted money between pairs of banks. In our scheme, the frequency of transactions plays more important role than the amount of transacted money. The reason we think is that a transaction of money directly affects credit of a bank. When a bank trades money, the transaction should follow the conditions imposed by the contract such as the deadline for the payment. However small the payment is, the bank loses its credit when it commits the default. Thus, banks tend to perform necessary number of transactions regardless of the amounts of money.

The credit of a bank may play more important role than we recognized so far in the banking system. In Section 4 we discussed the structure of the banking network by means of the degree distribution and the propagation function. In the discussion, we implicitly assumed that a pair of banks unconnected by a direct link had to perform the transactions through a common hub. However, a direct transaction between the pair is always possible in the case of the banking network. The existence of hub-like banks in the system suggests that a bank prefers to transact with a familiar, fixed partner rather than to transact directly with an unfamiliar partner. In this situation, it is obvious that the credit of the partners affects the behavior of the bank.

The concept of the credit of a bank also played an important role in the mean-field model of a scale-free network. In the model, the weight of a bank is determined by the sum of the credit to the bank, not by the sum of the weight of all the interactions to the bank.

The mean-field model proved that a scale-free network can be formed by a simple mean-field interaction between nodes without addition of nodes. It also suggests that a cause of a scale-free network may be a self-organized criticality [13].

It has been unraveled that a system of self-organized criticality is somehow related to a second-order phase transition with its control parameter fixed at the critical condition. In the case of the sand-pile model, for example, only when the number of sand grains is conserved, the system shows the critical behavior [14]. Generalizing the model so that the increasing rate of sand grains is the control parameter of the system, the system shows non-critical behavior when the number of sand grains is unconserved. In other words, the critical point of the model is at the point where the increasing rate is exactly zero. The sand-pile model is fixed at the critical point of the generalized model because the conservation of sand grains is naturally realized in the model.

The mean-field model proposed in this paper shows a phase transition, and, only with the fine tuning of the control parameter to the critical point, the model produces a scale-free network. In the case of banking network, however, the network seems to show the criticality without apparent tuning of parameters. This implies that the mean-field model is a generalized model of the banking network and that the banking network has a condition to fix the control parameter at the critical point. The critical condition is still to be unraveled.

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