(*Problem 1.*)

(*1*)

$$p_k = k^{-\gamma} \Big/ \int_{k_{min}}^{k_{max}} k^{-\gamma} \, dk = \left(\frac{k_{max}^{-\gamma+1} - k_{min}^{-\gamma+1}}{\gamma - 1} \right)^{-1} k^{-\gamma}.$$

(*2*)

$$<k> = \int_{k_{min}}^{k_{max}} k^{-\gamma+1} \, dk \Big/ \int_{k_{min}}^{k_{max}} k^{-\gamma} \, dk = \frac{\gamma - 1}{\gamma - 2} \frac{k_{max}^{-\gamma+2} - k_{min}^{-\gamma+2}}{k_{max}^{-\gamma+1} - k_{min}^{-\gamma+1}}.$$

$$L = \frac{N <k>}{2}.$$

(*3*)

$$N \, k \, p_k$$

(*4*)

$$\frac{N \, k \, p_k}{L}$$

(*5*)

$$<k>_{neighbor} = \int_{k_{min}}^{k_{max}} k \left(\frac{N \, k \, p_k}{L} \right) dk \Big/ \int_{k_{min}}^{k_{max}} \left(\frac{N \, k \, p_k}{L} \right) dk$$

$$= \int_{k_{min}}^{k_{max}} k^{-\gamma+2} \, dk \Big/ \int_{k_{min}}^{k_{max}} k^{-\gamma+1} \, dk = \frac{\gamma - 2}{\gamma - 3} \frac{k_{max}^{-\gamma+3} - k_{min}^{-\gamma+3}}{k_{max}^{-\gamma+2} - k_{min}^{-\gamma+2}}.$$

$<k>_{neighbor}$ is always larger than $<k>$.

(*6*)

```
N@124 / Mean@{305, 573, 118, 400, 173, 358, 217, 886, 103, 162}

NSolve[ (γ - 1) (γ - 3) / (γ - 2) ^2 == %, γ]
```

0.376328

$\{\{\gamma \to 3.26626\}, \{\gamma \to 0.733743\}\}$

Facebook friend network: $\gamma \approx 3.27$
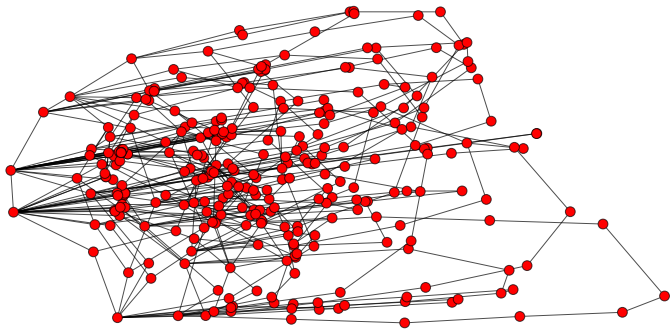
(*Problem 2.*)

```
n = 300; γ = 2.5;
```

```
vertexList = RandomVariate[ZipfDistribution[n - 1, γ - 1], n] + 0;
RandomGraph[DegreeGraphDistribution[vertexList],
 VertexStyle → ▮, EdgeStyle → ▮, GraphLayout → "SpiralEmbedding"]
```
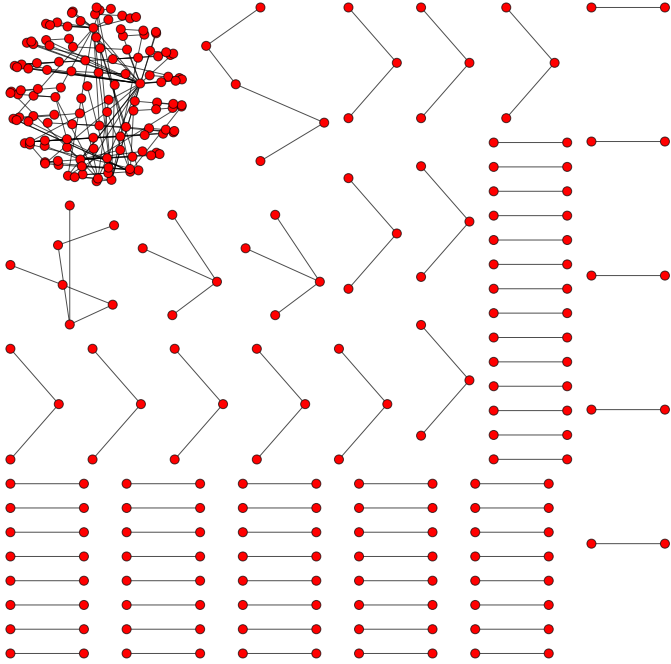


```
vertexList = RandomVariate[ZipfDistribution[n - 2, γ - 1], n] + 1;
RandomGraph[DegreeGraphDistribution[vertexList], VertexStyle → ▮,
 EdgeStyle → ▮, GraphLayout -> "HighDimensionalEmbedding"]
```

```
vertexList = RandomVariate[ZipfDistribution[n - 3, γ - 1], n] + 2;
RandomGraph[DegreeGraphDistribution[vertexList],
 VertexStyle → ■, EdgeStyle → ■, GraphLayout -> "BalloonEmbedding"]
```