HOMEWORK 2

Problem 1: Erdős-Rényi network

Given an Erdős-Rényi random graph which has $N = 3000$ nodes and linking probability $p = 10^{-3}$, answer the following questions:

1. Find $\langle L \rangle$, the expectation of how many links the graph should have.
2. Find the average degree $\langle k \rangle$.
3. Which regime should the graph be in? Subcritical regime? Supercritical regime?
4. Find the critical linking probability $p_c$.
5. The average path length between two arbitrary nodes of an Erdős-Rényi graph can be proved to be $d = \ln N / \ln \langle k \rangle$. Find it for this graph. Does it make sense to you?
6. How many nodes would this graph at least have if the graph had an average path length of $d = 100$? You can see that the ”length” dimension of networks is usually very small compared to their size.
7. Find the degree distribution $P(k)$ (approximated by a Poisson distribution).

Problem 2: Cayley tree

A Cayley tree is a symmetric regular tree. It is constructed by starting from a central node of degree $k$; the $k$ neighbors of the central node also have degree $k$ each and become the first layer; the tree expands until the outermost layer (which is away from the central node by a distance of $r$) and the outermost nodes only have degree one and are called ”leaves”.

1. How many nodes are reachable in $n$ steps from the central node (in terms of $k$ and $r$)?
2. What is the average path length between two arbitrary leaves?
3. Show that the ”length” dimension of Cayley tree is also much smaller than its size.

-Sean.