PY895, Network Science, Fall 2015

Dr. Jianxi Gao, Mr. Antonio Majdandzic; Prof. H. E. Stanley

Homework 15-03, due 6 Oct, 9:00am (before the class)

Please write your name at the top of your homework, and staple all pages together.

1. Cayley tree

A Cayley tree is a symmetric regular tree, constructed starting from a central node of degree k. Every node in the network at distance d from the central node has degree k, until we reach the nodes at distance P that have degree one and are called the leaves of the network. Assume here k = 4.

- a) Show that the number of nodes reachable in d steps from the central node is $4 \times 3^{d-1}$ for $d \in [1, P]$;
- b) Show that the diameter D is given by D = 2P;
- c) Find an expression for the diameter D of the network in terms of the total number of nodes N;
- d) Does the network display the small-world distance property?

2. Snob-ism in social networks

Consider a network where links are not drawn completely at random, but with a tendency to connect nodes which share some common traits. As a simplistic model we assume that nodes have different colors, such that there are N red nodes and N blue nodes. The probability for an edge between nodes of identical color is p and the probability for an edge between nodes of different color is q. The network is snobbish if p > q. For q = 0 the result is a network of two disjoint clusters each consisting of unicolored nodes.

a) Evaluate the minimal values of p and q required in order to reach global connectivity (with a high probability). b) Show that for large networks consisting of many nodes, even very snobbish networks will still become small worlds.

In another variant of this model we assume that q = 0, however a small fraction of the community are purple nodes, which have an equal probability p of connecting with both red and blue nodes (Assume that the average degree is k).

- c) The red and blue communities are said to be interactive if a typical red node is just two steps away from some blue node and vice versa. Evaluate the fraction of purple nodes required for the communities to become interactive.
- d) Show that if $k \gg 1$ a very small purple community is sufficient.
- e) Briefly describe the implications of this model on the structure of social (or other) networks.