

# Multifractal analysis of managed and independent float exchange rates

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## Abstract

We investigate multifractal properties of daily price changes in currency rates using the multifractal detrended fluctuation analysis (MF-DFA). We analyze managed and independent floating currency rates in eight countries, and determine the changes in multifractal spectrum when transitioning between the two regimes. We find that after the transition from managed to independent float regime the changes in multifractal spectrum (position of maximum and width) indicate an increase in market efficiency. The observed changes are more pronounced for developed countries that have a well established trading market. After shuffling the series, we find that the multifractality is due to both probability density function and long term correlations for managed float regime, while for independent float regime multifractality is in most cases caused by broad probability density function.

*Key words:* currency rate, managed float, independent float, market efficiency, multifractal detrended fluctuation analysis,

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## 1 Introduction

The foreign exchange market (FX) is the worlds largest and most liquid financial market. Its huge trading volume, high degree of liquidity, diversification of traders, geographical dispersion, amongst other factors make it uniquely challenging for empirical analysis, forecasting, and model development. The exchange rate regimes followed by governments across the world are crucial determinants of the foreign-exchange market. After World War II, governments adopted the Bretton Woods system where currencies were pegged against the U.S. dollar, which was in turn pegged to gold. Bretton Woods system helped countries avoid inflation and establish credibility of their currencies, but also removed their ability to conduct an independent monetary policy. Consequently, in 1971 the U.S. dollar switched to a floating currency, a move many major governments followed. Floating currencies are made up of two exchange-rate regimes: managed float and independent float. Exchange rates under the independent float regime fluctuate according to the foreign-exchange market, whereas rates under the managed float regime, (also known as dirty float), fluctuate on a daily basis and are influenced by government intervention. Transitions from managed to independent float regimes depend on various economic, political, and market factors. This brings the question how rate transitions affect market efficiency and economic welfare. As an extremely complex system, the FX market represents an ideal polygon for testing the usefulness of various methods including fractals, multifractals, and chaos theory, as tools to quantify market dynamics [1–7]. Multifractal properties as a measure of efficiency of financial markets were extensively studied [8–11], however less is known about efficiency of different exchange rate regimes [7,12]. In this work we apply the Multifractal Detrended Fluctuation Analysis (MF-DFA) [13] to compare the properties of the Australian Dollar (AUD), Brazilian Real (BRL), Malaysian Ringgit (MYR), New Zealand Dollar (NZD), South Korean Won (KRW), Sweden Krona (SEK), Taiwanese New Dollar (TWD), and Thai Baht (THB) per US Dollar (USD) exchange rate before and after the transition from managed to independent float regimes. We analyze logarithmic returns of daily closing exchange rates and find parameters that describe multifractal spectrum: position of maximum  $\alpha_0$ , width of the spectrum  $W$ , and skew parameter  $r$ . We also apply the MF-DFA analysis on the shuffled series to identify the effects of long term correlations and probability density function. This paper is organized as follows: We first describe the data and present the methodology, then present the results of our analysis, and finally we draw conclusions.

## 2 Methodology

Multifractal time series are characterized by a hierarchy of scaling exponents corresponding to different scaling behavior of many interwoven subsets of a series [13]. For non-stationary processes several methods have been proposed, such as the wavelet transform modulus maxima (WTMM) method [14], multifractal detrended fluctuation analysis (MF-DFA) [13], and multifractal moving average analysis [15]. In this work we use the MF-DFA method which has been successfully applied in various phenomena such as physiological signals [16], hydrological processes [17], geophysical data [18], forest fires records [19] and financial time series [8,9,11].

The MF-DFA method proceeds as follows [13]: (i) Integrate the original temporal series  $x(i)$ ,  $i = 1, \dots, N$  to produce  $y(k) = \sum_{i=1}^k [x(i) - \langle x \rangle]$ , where  $\langle x \rangle$  is the mean value of  $x(i)$ ,  $k = 1, \dots, N$ . (ii) Divide the integrated series  $y(k)$  into  $N_n = \text{int}(N/n)$  non-overlapping segments of length  $n$ . Calculate the local trend  $y_i(k)$  from a  $m$ th order polynomial regression in each segment and subtract it from  $y(k)$  to detrend the integrated series. (iii) Calculate the detrended variance of each segment (by subtracting the local trend) and average over all segments to obtain the  $q$ th order fluctuation function:

$$F_q(n) = \left\{ \frac{1}{N_n} \sum_{i=1}^{N_n} \left[ \frac{1}{n} \sum_{k=(i-1)n+1}^{in} [y(k) - y_i(k)]^2 \right]^{q/2} \right\}^{1/q} \quad (1)$$

where  $q$  can take any real value except zero. (iv) Repeat this calculation to find the fluctuation function  $F_q(n)$  for many different box sizes  $n$ . If long-term correlations are present,  $F_q(n)$  should increase with  $n$  as a power law  $F_q(n) \sim n^{h(q)}$ , where the scaling exponent  $h(q)$  (also called generalized Hurst exponent) is calculated as the slope of the linear regression of  $\log F_q(n)$  versus  $\log n$ .

The generalized Hurst exponent is a decreasing function of  $q$  for multifractal time series and constant for monofractal processes. For positive (negative) values of  $q$ , exponent  $h(q)$  describes the scaling of large (small) fluctuations [13]. The exponent relates to the classical multifractal exponent defined by the standard partition multifractal formalism as  $\tau(q) = qh(q) - 1$ , where  $\tau(q)$  is a linear function for monofractal signals and a nonlinear one for multifractal signals [13]. Multifractal series are also described by the singularity spectrum  $f(\alpha)$  through the Legendre transform

$$\alpha(q) = d\tau(q)/dq, f(\alpha) = q\alpha - \tau(q) \quad (2)$$

where  $f(\alpha)$  denotes the fractal dimension of the series subset characterized

by the Holder exponent  $\alpha$ . For monofractal signals, the singularity spectrum produces a single point in the  $f(\alpha)$  plane, whereas multifractal processes yield a humped function [13].

Multifractality in a time series may be caused by: i) a broad probability density function for the values of the time series; and ii) different long-term correlations for small and large fluctuations. To determine the type of multifractality one should analyze the corresponding randomly shuffled series. The shuffled series from multifractals of type ii) exhibit simple random behavior with  $h(q) = 0.5$  and  $f(\alpha)$  reduced to a single point, while for multifractals of type i) the original  $h(q)$  dependence (and the width of multifractal spectrum) is not changed. If the shuffled series demonstrates weaker multifractality than the original one, both kinds of multifractality are present [13]. In order to measure the complexity of the series, we fit the singularity spectra to a fourth degree polynomial

$$f(\alpha) = A + B(\alpha - \alpha_0) + C(\alpha - \alpha_0)^2 + D(\alpha - \alpha_0)^3 + E(\alpha - \alpha_0)^4 \quad (3)$$

and calculate the multifractal spectrum parameters: position of maximum  $\alpha_0$ ; width of the spectrum  $W = \alpha_{max} - \alpha_{min}$ , obtained from extrapolating the fitted curve to zero; and skew parameter  $r = (\alpha_{max} - \alpha_0) / (\alpha_0 - \alpha_{min})$  where  $r = 1$  for symmetric shapes,  $r > 1$  for right-skewed shapes, and  $r < 1$  for left-skewed shapes. Roughly speaking, a small value of  $\alpha_0$  suggests the underlying process is more regular in appearance. The width of the spectrum  $W$  measures the degree of multifractality in the series (the wider the range of fractal exponents, the richer the structure of the series). The skew parameter  $r$  determines which fractal exponents are dominant: fractal exponents that describe the scaling of small fluctuations for right-skewed spectrum, or fractal exponents that describe the scaling of large fluctuations for left-skewed spectrum. These parameters lead to a method of measuring the complexity of the series: a signal with a high value of  $\alpha_0$ , a wide range  $W$  of fractal exponents, and a right-skewed shape  $r > 1$  may be considered more complex than one with opposite characteristics [20].

### 3 Data and analysis

We analyze the currency rates during the managed float and independent float regimes of 8 different countries: Australia, Brazil, Malaysia, New Zealand, South Korea, Sweden, Taiwan, and Thailand (<http://finance.yahoo.com/>). For each currency regime we calculate the logarithmic returns  $R_i(t) = \ln P_i(t + \Delta t) - \ln P_i(t)$ , where  $P_i(t)$  is the daily closing currency rate at time  $t$  and  $i$  represents the index of the time series. The time series for the currency rate returns of the countries listed above are shown in Fig. 1.

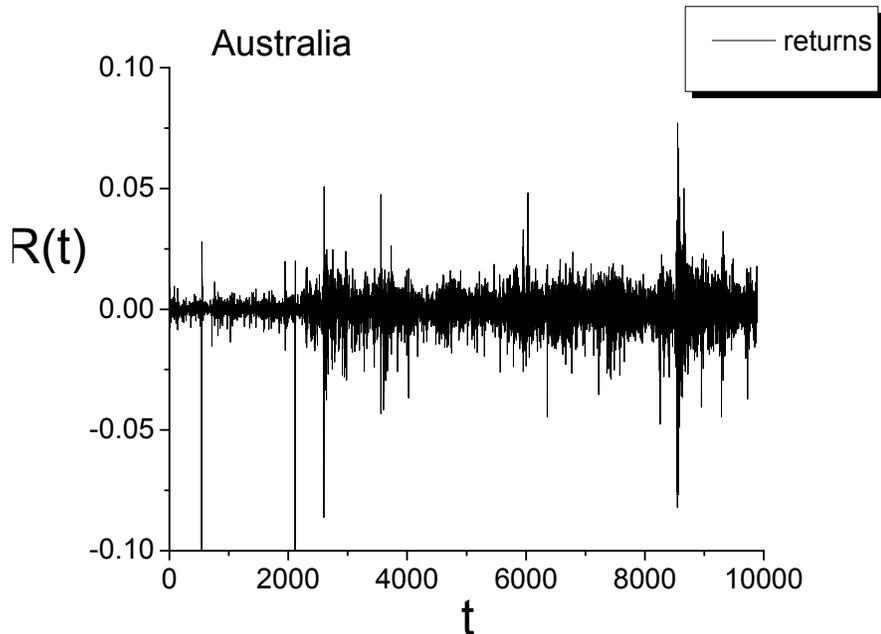


Fig. 1. Currency returns  $R$  for managed and independent float regimes of Australia for the period.

We apply the MF-DFA method to the logarithmic returns of several countries, fitting the local trends with a second degree polynomial  $m = 2$ . We also perform a fourth order polynomial regression on the singularity spectra  $f(\alpha)$  to determine the position of maximum  $\alpha_0$  and the zeroes of the polynomial  $\alpha_{max}$  and  $\alpha_{min}$ . Then we calculate the complexity parameters  $\alpha_0$ ,  $W$ ,  $r$  and use them to determine the multifractal properties of the time series. The multifractal spectra of all currency rates are shown in Fig. 2.

The measures of complexity  $(\alpha_0, W, r)$  are shown in Tab. 1. It is seen from Figure 2 and Table 1 that after the transitions from managed to independent float regime: (i) The position of maximum of  $f(\alpha)$  spectrum approaches an uncorrelated regime  $\alpha_0 \rightarrow 0.5$ , an indicator of increased market efficiency where  $\alpha_0$  approximates the overall Hurst exponent [20]; the exception is Thailand currency for which the value of  $\alpha_0$  shifts slightly away from 0.5. For all countries, managed floating regime is characterized by week persistence correlations, except for Brazil where anti persistent correlations are observed. (ii) The width  $W$  of the multifractal spectra shortens, which suggests lower complexity for independent float periods (and lower market risk) with the exception of Malaysia for which  $W$  increases, and South Korea which does not show significant change in  $W$ . (iii) The values of asymmetry parameter  $r$  reveal that for Australia, Brazil, New Zealand and Thailand multifractality is more influenced by the scaling of large fluctuations (left skewed spectrum) for both

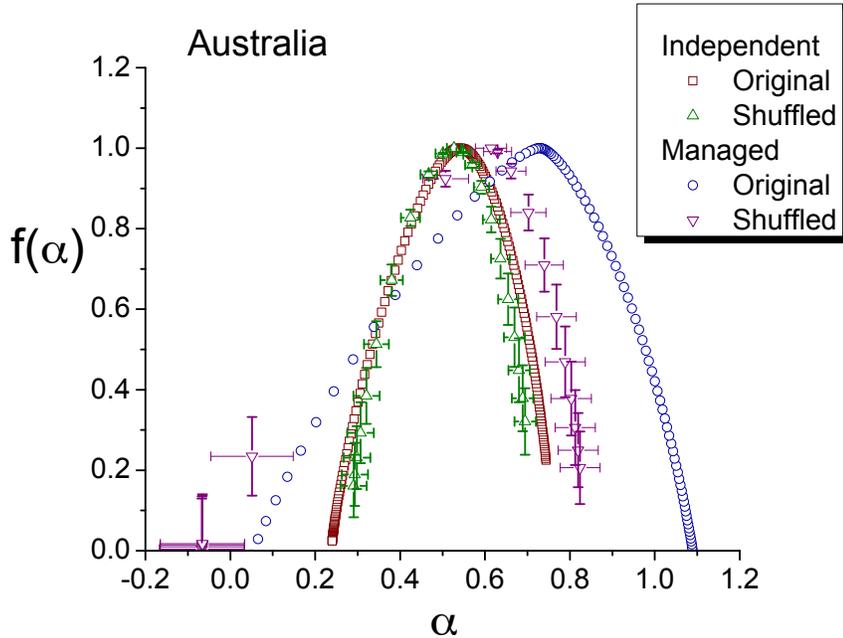


Fig. 2. Multifractal spectrum  $f(\alpha)$  for currency returns  $R$  of the original and shuffled series. The dotted lines represent regression curves to the fourth order polynomial form.

exchange rate regimes while for Taiwan small fluctuations contribute more to multifractality (right skewed spectrum) in both periods. For Malaysia (South Korea) the contribution from large (small) fluctuations in managed floating is followed by switching to the contribution of small (large) fluctuations in independent floating. For Sweden managed floating regime is characterized by the contribution of small fluctuations (right skewed spectrum), followed by equal contribution of both large and small fluctuations (symmetric spectrum) after the transition of currency regime. (iv) It seems that developed countries (Australia, New Zealand, Sweden) experience larger shifts in  $\alpha_0$  and larger decrease in  $W$  than other countries considered having the emerging markets likely due to larger trading markets and hence greater economic benefits.

We also shuffle the time series of currency rate changes and then apply the MF-DFA analysis for all countries, to determine the type of multifractality of the series. The shuffling procedure performed  $1000 \times N$  transpositions on each series and was repeated 100 times with different random number generator seeds. The multifractal spectra of original and shuffled series are shown in Figure 2. We find that for managed float regime the width of  $f(\alpha)$  spectrum decreases after shuffling, indicating that the multifractality stems from both broad probability density function and long term correlations. The exception is Brazil, for which the multifractal spectrum becomes wider after shuffling.

As recently discussed by Barunik et al. [21], the increase of multifractality after shuffling is found in various financial time series and may be caused by short-memory time correlations in the data. For independent float regime the multifractal spectrum remains unchanged for developed countries Australia, New Zealand and Sweden, as well as for Brazil and South Korea, indicating the broad probability density function as the source of multifractality. For other emerging countries Malaysia, Taiwan and Thailand, both probability density function and long term correlations contribute to multifractality, as indicated by the decrease of the width of the spectrum. In general, after the transition from managed to independent float regime, developed countries experience larger shifts in  $\alpha_0$  toward the uncorrelated regime, larger decrease in the width of the spectrum  $W$  and broad probability density function as the source of multifractality.

Table 1  
Multifractal parameters  $\alpha_0$ ,  $W$  and  $r$  for currency rate returns  $R$

country	Managed Float			Independent Float		
	$\alpha_0$	$W$	$r$	$\alpha_0$	$W$	$r$
Australia	0.6966	1.028	0.613	0.540	0.535	0.748
Brazil	0.3952	0.922	0.583	0.598	0.652	0.823
Malaysia	0.646	0.776	0.737	0.607	1.122	2.517
New Zealand	0.665	1.153	0.719	0.531	0.503	0.905
South Korea	0.634	0.842	1.354	0.589	0.823	0.722
Sweden	0.653	1.122	1.162	0.526	0.417	1.013
Taiwan	0.937	1.185	1.386	0.688	0.962	1.180
Thailand	0.526	1.040	0.470	0.599	0.829	0.942

#### 4 Conclusion

In this work we apply MF-DFA method to compare multifractal behavior of various exchange rates during the periods of managed and independent float regimes. We calculate the multifractal spectra and estimate the complexity parameters from four degree polynomial fit. We find that in most analyzed cases after the transition from a managed to independent float regime the position of maximum of multifractal spectrum shifts towards the uncorrelated regime  $\alpha_0 = 0.5$ , and the width of the spectrum decreases. This indicates that the transition from a managed float to independent float exchange rate regime is followed by an increase in market efficiency. By comparing the multifractal spectra of original and shuffled data we find that in most cases for managed

float regime the multifractality is due to both broad probability density function and long term correlations, whereas it is due only to broad probability density function during the independent float regime. For developed countries the transition from managed to independent float regime, leads to larger shifts in  $\alpha_0$  toward the uncorrelated regime, larger decrease in the width of the spectrum  $W$  than for emergent countries and broad probability density function as the source of multifractality. To confirm our findings, exchange rates from other countries should be systematically analyzed.

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To be determined.

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