Thursday, 19 March 2015
Econophysics

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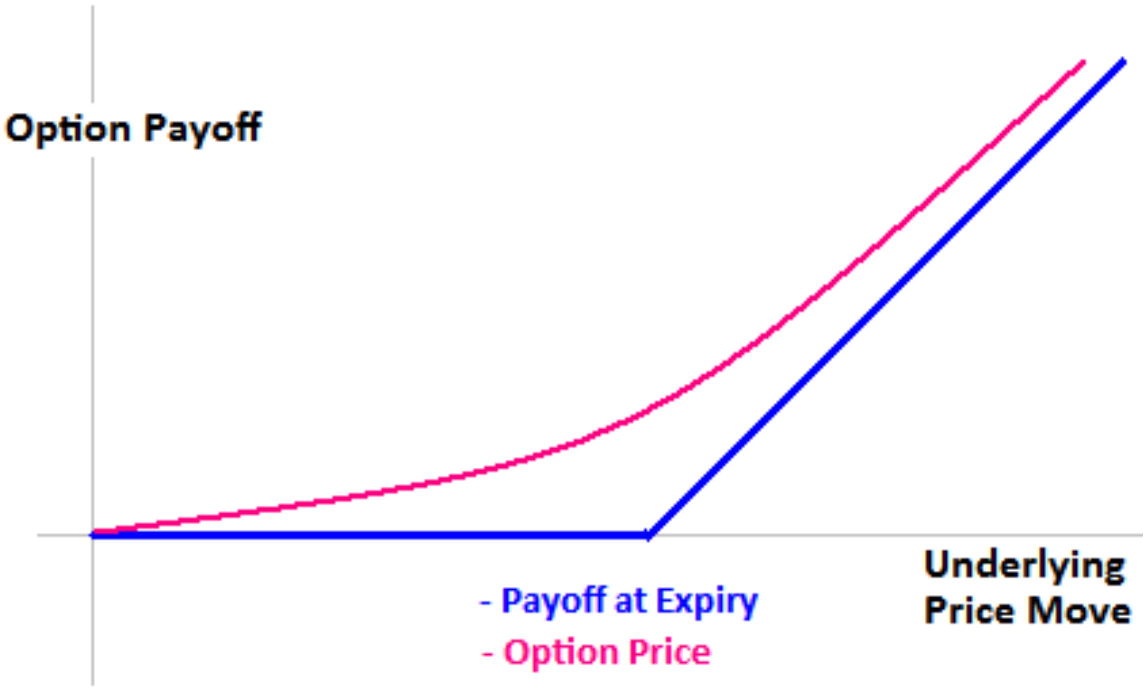
Current (spot) price of European call (if the stock does not pay dividends) is higher then its intrinsic value.



Options

- **Put option (European):**

-Tied to a specific asset, for example a stock.

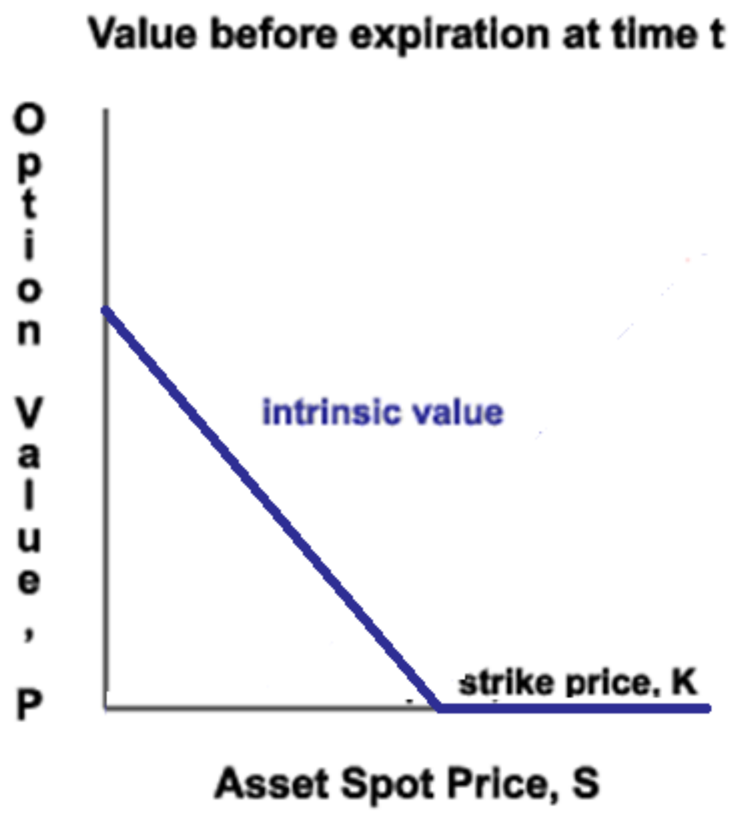
K- “strike price”

S- stock value

T-time to maturity

The option gives the owner the ***right*** to SELL the stock for price K at some specified time T.

The owner does not need to execute this right.



American options

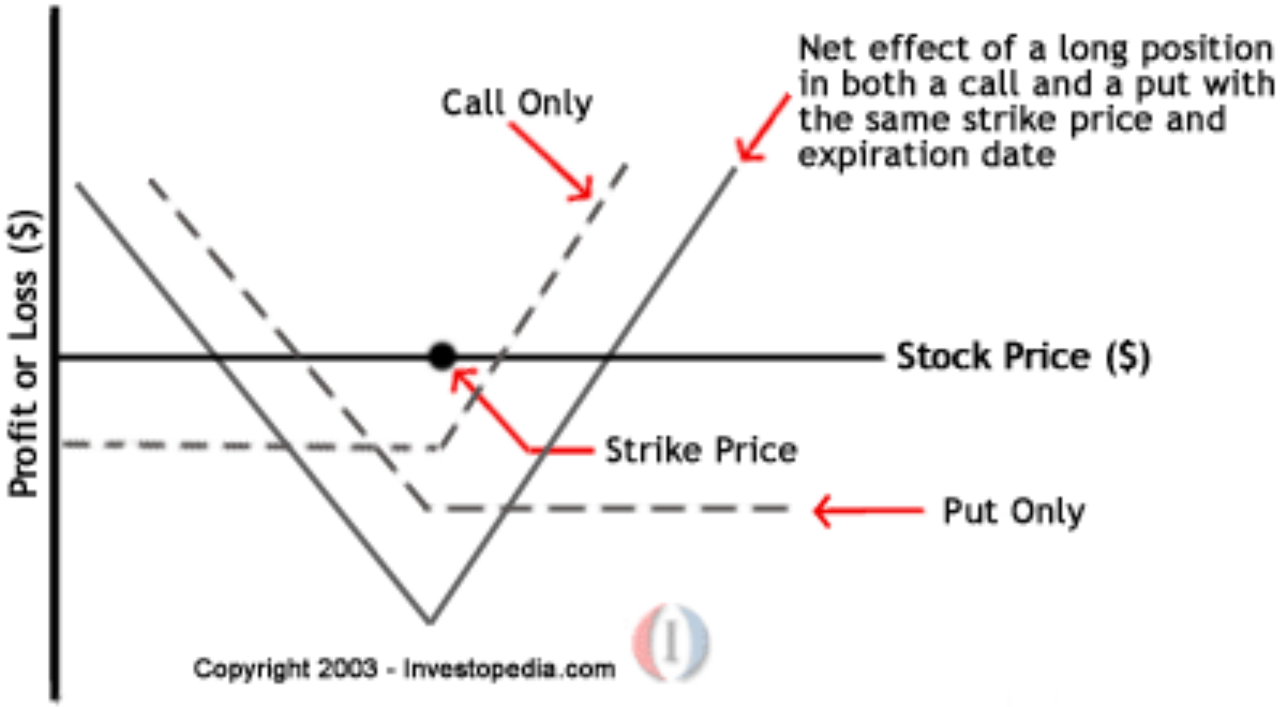
- Same as European, but can be executed at any moment prior the maturity

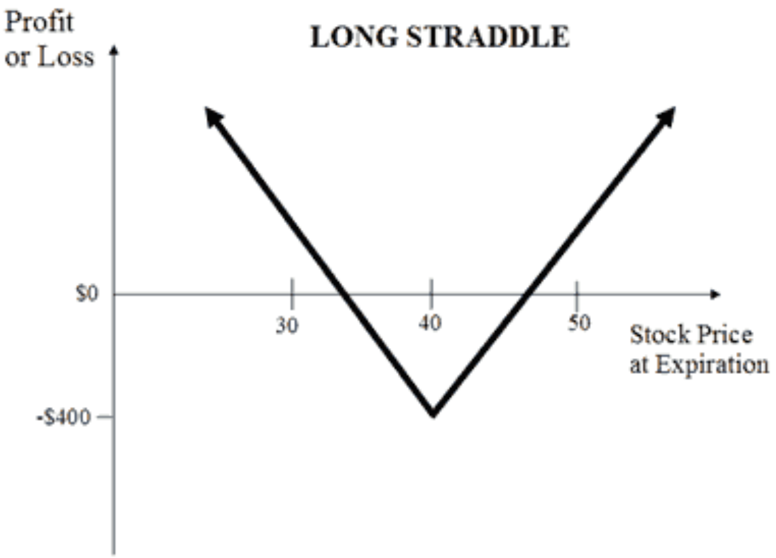
- American call

- American put

Example of a **derivative**

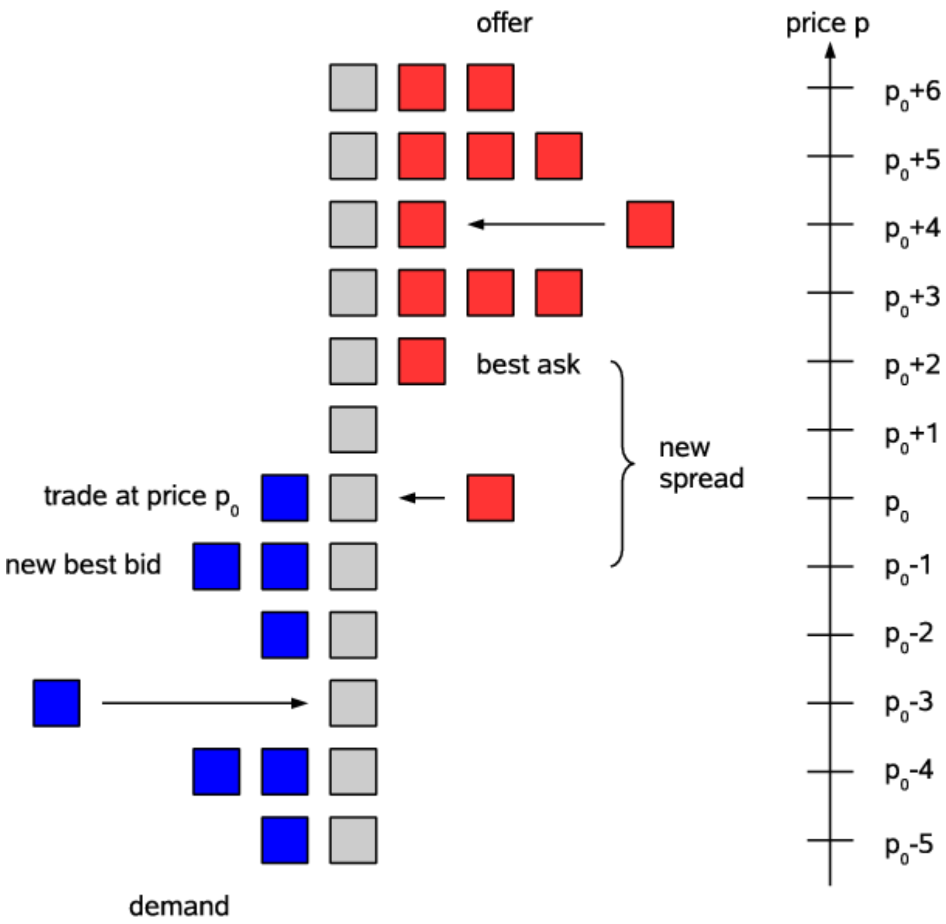
- Does it make sense to buy both call and a put?

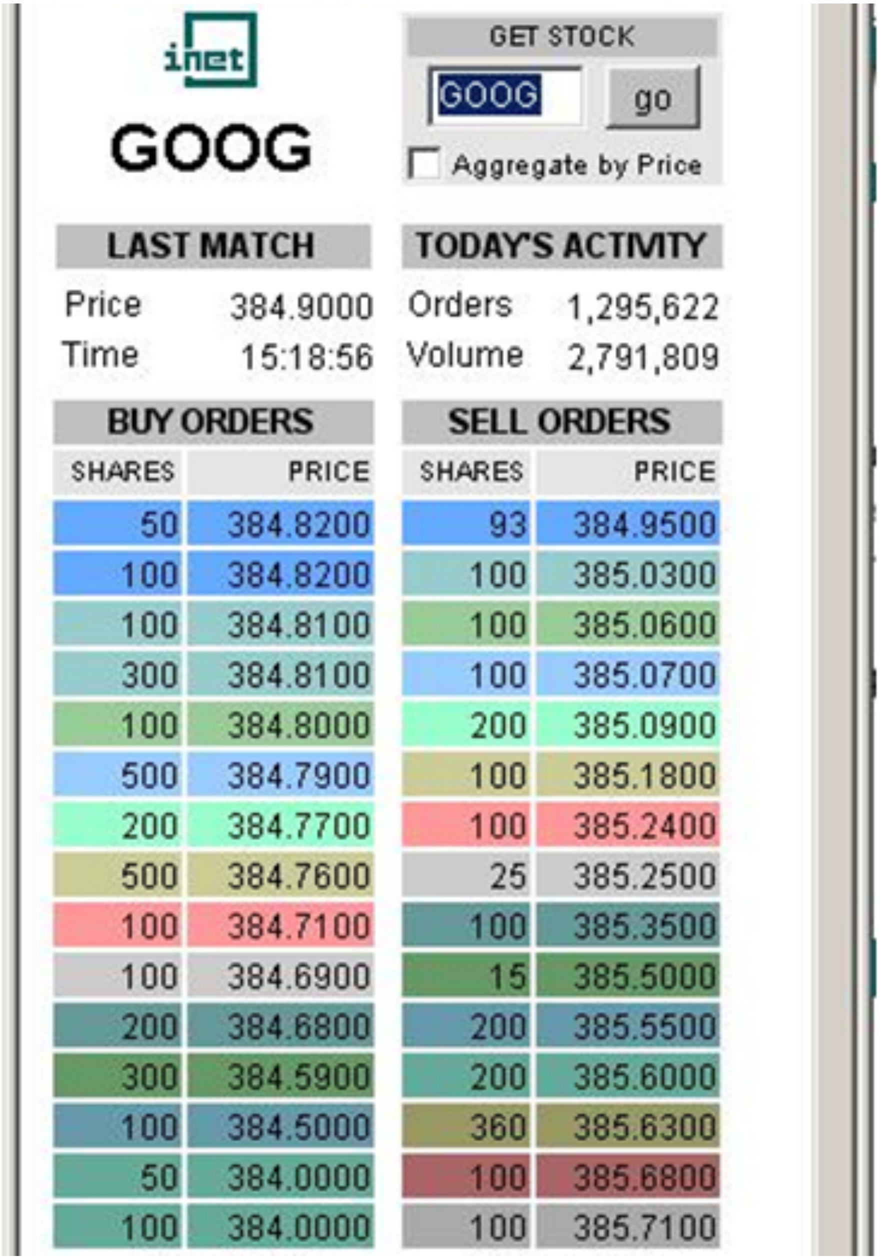




Its price is positive! This derivative is called STRADDLE

Limit order book





inet

GOOG

GET STOCK

GOOG go

Aggregate by Price

LAST MATCH	
Price	384.9000
Time	15:18:56

TODAY'S ACTIVITY	
Orders	1,295,622
Volume	2,791,809

BUY ORDERS		SELL ORDERS	
SHARES	PRICE	SHARES	PRICE
50	384.8200	93	384.9500
100	384.8200	100	385.0300
100	384.8100	100	385.0600
300	384.8100	100	385.0700
100	384.8000	200	385.0900
500	384.7900	100	385.1800
200	384.7700	100	385.2400
500	384.7600	25	385.2500
100	384.7100	100	385.3500
100	384.6900	15	385.5000
200	384.6800	200	385.5500
300	384.5900	200	385.6000
100	384.5000	360	385.6300
50	384.0000	100	385.6800
100	384.0000	100	385.7100

TRADING GAME

Each player initially has: **100 “dollars”**

Player 1: $100
Player 2: $100
Player 3: $100
Player 4: $100
Player 5: $100
Player 6: $100
Player 7: $100
Player 8: $100
…

Round 1

Asset: The result of rolling 6 dice.

Value= $\mathbf{X_1 + X_2 + X_3 + X_4 + X_5 + X_6}$

Rules:
1. Each player bids.
2. **I accept 3 best bids.**
3. The Value is revealed, the 3 players receive it and sell it on the market.

4. They earn the following profit: Profit=**Value-Bid**

Analysis of round 1:
This situation was an aggressive "market order".

Expectation[Value]=6*3.5=21

When you buy something, you want to bid below the expected value.

If there are many players, bid-ask spread narrows (in real markets the bid-ask spread is very narrow)

Round 2

- Same game one more time.

Asset: The result of rolling 6 dice.

Value= $\mathbf{X_1 + X_2 + X_3 + X_4 + X_5 + X_6}$

Profit=**Value-Bid**

However, this time I accept an unknown number of bids.

Round 3

- A big player comes in: an investor who has a large amount of Dice Technologies LTD stock. He wants to get rid of this stock quickly so he can raise money for other investments.

- He/she and announces that **he/she is selling a huge amount of shares** of Dice Technologies LTD.
- He is willing to take **<u>7 best bids</u>**!!!!
- [you must not communicate between each others]

- Again, the game is the same.

Value= $\mathbf{X_1 + X_2 + X_3 + X_4 + X5 + X_6}$

Profit=**Value-Bid**

- **Analysis of the *limit order book* for this round:**

- **Large offer decreases the price of an asset.**

- **Large demand increases the price.**

- **When you are on the market, you should not disclose (tell) what are your intentions.**

- **The information about how badly you want something is valuable to other players, and they will use it against you. When you want to buy or sell a huge amount of stocks, you should do it little by little.**
- **Still, there are computer programs on the market trying to detect your true intentions!**
- **If you are a broker, *front running* (using information against your client) is illegal.**

Time to sell some derivatives!!!

Auction:

Derivative 1:

Receiving 3 points every round until someone reaches the score of 150.

Time to sell some derivatives!!!

Auction:

Derivative 2:

Profit= $10 – t +(number of people above 130)*4

Can be executed at any moment. (American style)

→ Initial value: $7 + variable part

→ One component slowly depreciating in time, but the third term has a potential for a significant profit.

Round 4

- **The result of rolling 2 dice.**
- Value= $\mathbf{X_1 * X_2}$
- 4 best bids are accepted

Round 5 – one more roll?

- **The result of rolling 2 dice.**
- Value= $\mathbf{X_1 * X_2}$
- 4 best bids are accepted

Round 6

- **Asset: $10 + (rolling 3 dice) – (rolling 3 dice one more time)**
- Value= **$10 + X1+X2+X3-(X4+X5+X6)**
- 5 highest bids accepted

Round 7

- Value= [Population of Nigeria in millions]/10

- One best bid for the Value

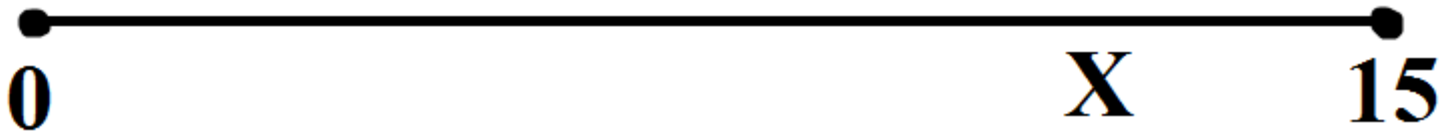
Round 8

(NA / secret)

Game theory

- On the market, many players are smart and they try to predict other people's actions
- what other player will do if you do something?
- What they think   that you think   that they think?
- See on Wikipedia: Nash Equilibrium

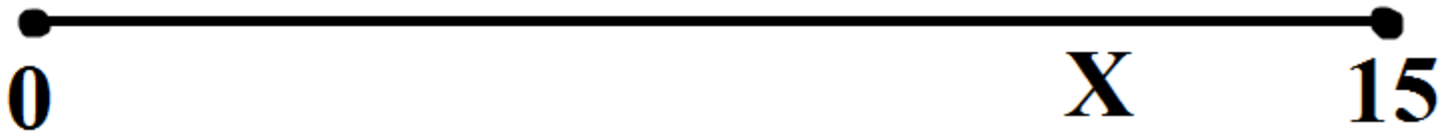
Round 9



- Choose a secret integer X between 0 and 15.
- Write it on the paper and do not show to anybody.
- If there are N players, there are N points on this line (some possibly overlapping).

- Prize= X + 8*|distance to your closest neighbor|

Round 10



- Choose a secret integer X between 0 and 15.

Prize= - \$20 + 8*|distance to your closest neighbor|

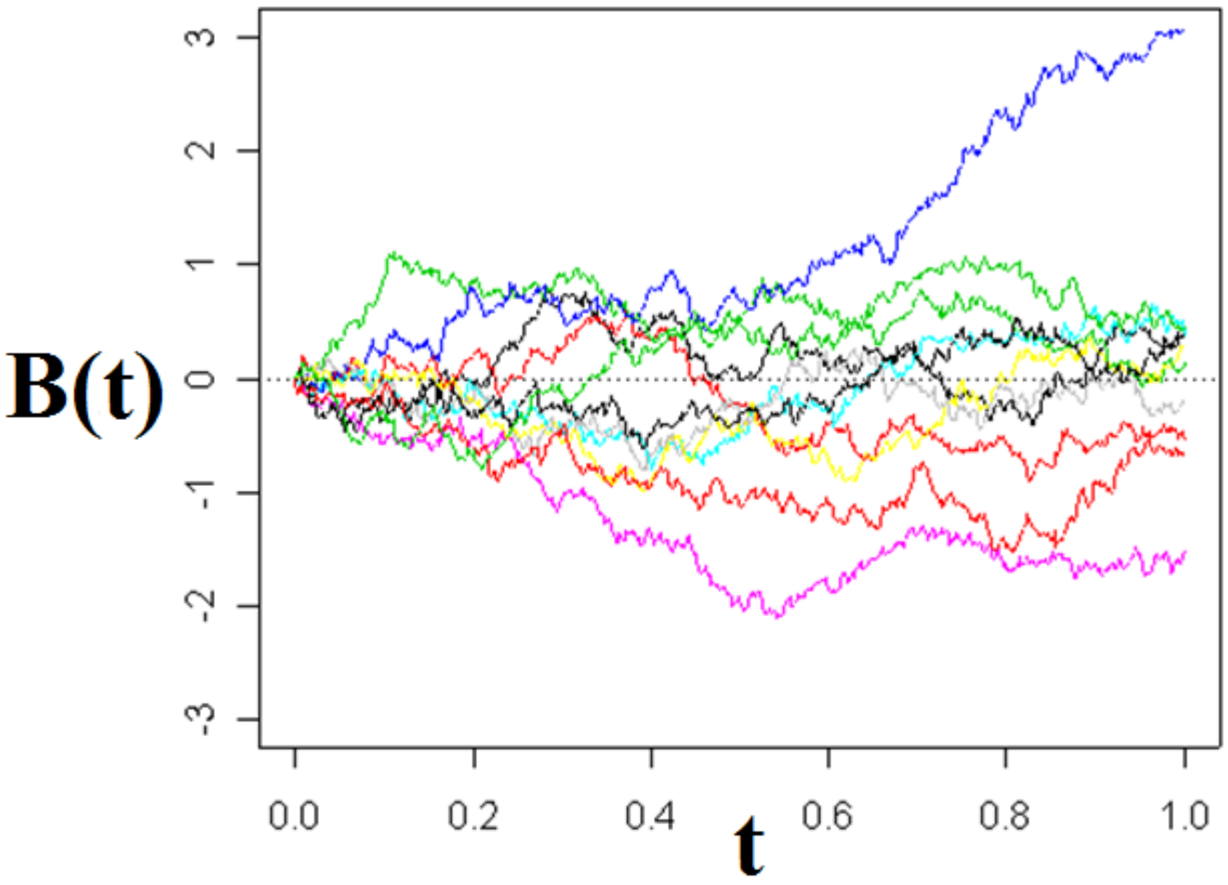
- You can choose to **fold** (not play this game).
- In that case, write "FOLD" on your paper.

How do we price options?

- We need a special math for that

Introduction to STOCHASTIC CALCULUS

- Brownian process



- We already know that if B(0)=0, then

$$E[B(t)^2] = \sigma t$$

Or for the unit standard deviation we have:

$$E[B(t)^2] = t$$

Lets observe an infinitesimal interval of time:

$$E[dB(t)^2] = dt$$

$$E[dB(t)^2] = dt$$

- Now, we claim: $$dB(t)^2 = dt$$

Sketch of a proof:

Observe the following stochastic “variable”: $dB(t)^2 - dt$

Its expectation value is: 0

Using $E(X^4) = 3\sigma^4$ for normal stochastic variables, it can be shown that its variance is equal to $2(dt)^2$. When dt is small, squared dt is even smaller, so we can informally write $dB(t)^2 - dt = 0$, or

$$dB(t)^2 = dt$$

- This is a very simple but powerful equation, and it produces a totally different calculus- **Ito calculus**

- Ito calculus may give strange results

- For example, if y=B(t), where B(t) is a Brownian process:

$$\int y\, dy \neq \frac{1}{2} y^2 + C$$

Here C cannot be *any numerical constant*, like for standard integrals.
You need to forget most of the things that you learnt in the ordinary calculus.

- In Ito calculus, second order differentials cannot always be neglected. For example, if $f = f(x, y)$
  then sometimes

$$df \neq \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

- We need to keep higher order terms, if x or y are stochastic processes

- Model for stock prices: geometric Brownian motion

$$dS = \mu S\, dt + \sigma S\, dB$$

- Now we finally come to options (or, general derivatives).

Assume a variable (derivative price f) is some function of S and t:

$$f = f(S, t)$$

Then

$$df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial S} dS + \frac{1}{2}\frac{\partial^2 f}{\partial S^2} (dS)^2$$

$$dS = \mu S\, dt + \sigma S\, dB$$

$$df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial S} dS + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\,(dS)^2$$

If we plug in the process for the stock into the total differential for f:

$$df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial S}[\mu S\, dt + \sigma S\, dB] + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\,[\mu S\, dt + \sigma S\, dB]^2$$

Using $dB(t)^2 = dt$ and neglecting (dt)^2, we get:

$$df = \left[\frac{\partial f}{\partial t} + \mu S \frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2}\right] dt + \frac{\partial f}{\partial S}\,\sigma S\, dB$$