PY 538 Homework #2

In this problem set you will make use of S&P 500 index data, found here: `http://finance.yahoo.com/q/hp?s=\%5EGSPC+Historical+Prices`

(Search "s&p 500 yahoo finance" on Google, click the top link, then click "Historical Prices" on the left of the page.)

Click "Download to Spreadsheet" to download the data.

Load the "Close" prices into your favorite software. Note that the data is formatted so that the most recent price is at the top; you may need to reverse the prices to put them in chronological order.

Finally, for the following problems it may be useful to write a function to compute the autocorrelation of any time series at some lag l. To do this, you might write two functions: one to lag a series, and another that will compute the correlation between any two series. Then you can just feed a "lag 0" version and a "lag l" version of your time series to the correlation function. Be careful in handling the lengths of the time series. Most software for scientific or statistical computing has an autocorrelation function readily available, however, and you're free to use any method you like. Please ask if you need help with this step.

Problems:

1. Make a scatter plot, where the vertical axis gives the S&P 500 Close price on day t, and the horizontal axis gives the S&P 500 Close price on day $t-1$. Does this mean that you can predict the Close price of the S&P 500 tomorrow?

2. Transform the close prices, P_t, into logarithmic returns:

$$r_t = \log(P_t) - \log(P_{t-1}).$$

   Using the returns, make the same scatter plot as you did for the Close prices. Next, plot the autocorrelation function of the returns r_t, out to some sensible number of lags (perhaps 10). What do you see?

3. Compute the time series of volatility:

$$v_t = r_t^2.$$

Plot the autocorrelation function of v_t. Compare with the autocorrelation for the returns.

4. Finally, we will explore how modeling a time series can allow one to "remove" its autocorrelation. Fit the following linear model:

$$v_t = \beta_0 + \beta_1 v_{t-1} + \beta_2 v_{t-2} + \cdots + \beta_q v_{t-q} + \epsilon_t$$

out to some lag q. Feel free to start with $q = 1$. Obtain a time series of the residuals ϵ_t, and plot the autocorrelation function of these residuals. Compare with the autocorrelation function of the volatility v_t, and examine the effect of increasing q. How might this model be used when quantifying the "risk" of investing in the S&P 500? What advantages does it have over simply quoting the historical variance of the returns r_t?