Asset-Exchange Model and wealth distribution

Shan Huang
Boston University, Physics Department

(Dated: May 5, 2017)

Asset-Exchange Model (AEM) is commonly used in analysis of wealth flow and wealth distribution within a closed system. I discussed AEM and its extension, Yard Sell Model(YSM), based on study of Boghosian et al. They demonstrated the effectiveness of YSM with Analytic and Numeric method and predicted of existence of wealth oligarch in human society.

I. INTRODUCTION

The scientific study of economic inequality has drawn a major interest due to the ever growing uneven distribution of wealth within the whole society. According to Oxfam International, in 2010, 388 individuals holds as much wealth as half of the world population. And this number has, in 2016, decreased to 62 individuals (Hardoon and Ayele, 2016). It is a very important but extremely hard problem to analyze, explain and give prediction to the wealth distribution in human society because in reality this involves all different kinds of economic activities and has also a deep relationship with government policies on wealth redistribution, which varies from nation to nation. Finding a model simple enough but still keep track of the main characteristic of human wealth distribution which allows people to not only explain the inequality but also investigate the dynamic of wealth has been a long desire.

A class of models that has been used to study such problem are called Asset-Exchange Models (AEMs) (Angle, 1986; Hayes, 2002). AEMs have simplified the complicated specific macroeconomic phenomenon using the most basic but sufficiently conclusive human economic activity: wealth exchanging between individuals. The idea of this model is to introduce a fixed number of individuals (agents) in one system, they exchange their wealth randomly with other agents with some rules which can differ from each specific model. This way, one can easily see the dynamics of wealth distribution by simply looking at the population of agents with given wealth. Based on AEMs, different exchange rules have been implemented to the models. The total number of agents and wealth in the system could be changed in extended versions.

Among different versions of AEMs, Boghosian et al. analyzed and made modification to Yard Sell Model (YSM) (Chakraborti, 2002; Hayes, 2002), derived a Boltzmann equation for basic YSM and showed that this Boltzmann equation can be reduced to a Focker-Plank equation (FP), which describes Markovian Processes. Using this, he predicted the wealth condensation, a phenomenon that a finite portion of wealth being hold by a negligible amount of agent, which is exactly what happened in real-life situation.

II. MODEL AND NOTATION

A. Yard Sell Model

The Yard Sell Model is based on AEM idea that a fixed amount of agents holding a fixed amount of total wealth exchanging their wealth in pairwise way,

$$N_P = \int_0^\infty dw P(w,t) \tag{1}$$

$$W_P = \int_0^\infty dw P(w,t) w \tag{2}$$

where N_P is the total number of agent, W_p is the total amount of wealth.

To describe the dynamics of wealth distributions, they use a random walk based on the YSM, in which two agents are randomly chosen to transact, and the magnitude of wealth exchanged is a fraction of the minimum wealth of the two agents. The winner is determined by a random variable $\eta \in \{-1, +1\}$. This trading rule implemented is the most important part of YSM, which is an analogy to real-life scenario, and is the reason of wealth condensation to occur. If we focus on a single agent whose wealth transitions from z to w as a result of the transaction, this gives rise to the random walk:

$$w - z = \eta \min(z, x) \tag{3}$$

where x is wealth of the other agent during the trade.

B. Modifications to Yard Sell Model

In order to make the model closer to real-life scenario, Boghosian et al. added two important parameters to the system, tax and wealth-attained advantage (WAA)

Taxation is the most important way of wealth redistribution used by every government to reduce the inequality in individual incomes. In YSM, tax is implemented by introducing two additional parameters τ and T_P:

$$T_p = \int_0^\infty \tau(z) P(z,t) dz \tag{4}$$

Where $\tau(z)$ denotes the fraction of wealth that will be taken from the agent with amount wealth z. $T_P(t)$ is the total amount of wealth being taxed from all the agents at time t. This part of wealth will be distributed evenly among the agent, making the random walk in eq.(**??**) becomes:

$$w - z = \eta \min(z, x) - \tau(z)z + T_p/N_p \tag{5}$$

WAA is another important factor that one would consider when it comes to real-life wealth dynamics. In the basic YSM, the games played between each pair of the agents are assumed to be fair so that both agents have same chance of 50% to win the game. But it is not often the case. One can argue that in real world, the richer one is, the more likely one is able to get resources to help him/her gain wealth. It is easier for rich people to make money than the poor. Thus Boghosian introduced WAA factor . Now the game between agents are not played fairly. Instead, the game will be in favor of the wealthier agent depends on the difference of wealth between two agents:

$$E(z) = \zeta \frac{N_p}{T_p}(z - x) \tag{6}$$

$E(z)$ denotes the difference in the probability that agent with wealth z will win compared to the fair number 50%.

III. RESULT

A. Analytic Result

Lorentz curve is a useful tool in analyzing Markovian Processes.

Define:

$$L(w,t) = \frac{1}{W_p}\int_0^w dx P(x,t)x \tag{7}$$

$$F(w,t) = \frac{1}{N_p}\int_0^w dx P(x,t) \tag{8}$$

Where $L(w)$ is Fraction of total wealth held by agents have less wealth than w, $F(w)$ is the fraction of these agents with respect to the total number of agents. Lorentz curve plots $L(w) - F(w)$.

Boghosian showed that the random walk discussed above can be describe by Fokker-Plank equation:

$$\begin{gathered} \frac{d}{dw}[(B + \frac{w^2}{2}A)P] = \{\tau(\frac{W_P}{N_P} - w) \\ -\zeta[\frac{N_P}{W_P}(B - \frac{w^2}{2}A) + (1 - 2L)w]\}P \end{gathered} \tag{9}$$

where A and B are partial moments:

$$A_P(w,t) = \frac{1}{N_p}\int_0^w dx P(x,t) \tag{10}$$

$$B_P(w,t) = \frac{1}{N_p}\int_0^w dx P(x,t)\frac{x^2}{2} \tag{11}$$

Boghosian showed that when time goes to infinity, there exists a steady state:

$$\lim_{t\to\infty} P(w,t) = P(w) \tag{12}$$

Instead of plotting $P(w)$, Lorentz curve was plotted. Boghosian showed that when $\tau < \zeta$, Lorentz curve passes point $(1, 1-c)$, where $c = \tau/\zeta$ (See FIG. **??**). This means there is a finite amount of fraction of total wealth c is held by a negligible amount of people. They are called Oligarch.

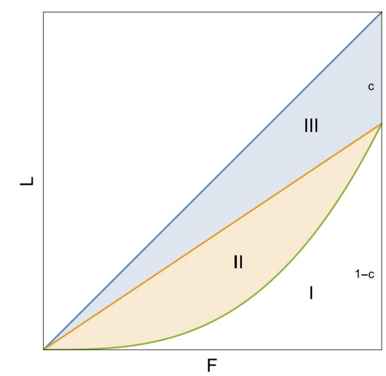

FIG. 1. Lorenz plot with partial wealth condensation. A fraction c of the wealth has condensed, and $1c$ is distributed classically.

B. Numerical Result

Numerically solving the Fokker-Plank equation, regarding and as two parameters, one can do a least square fit to real-life data to test the efficiency of the model. Boghosian used data from 2013 Survey of Customer Finances as the fitting material. By varying the two parameters one can get the overall fitting error as function in parameter space, as shown in FIG. **??**. Then one will pick up the point with lowest error value as the parameters to use.

The resulting fitting curve from numerically solving the Fokker-Plank equation is shown in FIG. **??**, the corresponding parameters chosen are $\tau = 0.024$ and $\zeta = 0.028$

One can also calculate the Gini coefficient, which is defined as twice the area under the perfect equality curve (the diagonal line) in Lorentz curve diagram. It can also be expressed through FIG. **??**:

$$G = \frac{II + III}{I + II + III} \tag{13}$$

The calculated Gini Coefficient for the fitted curve is $G_{fit} = 0.834$, compare to Gini Coefficient for the data $G_{data} = 0.8423$, its extremely close. Also the numerical

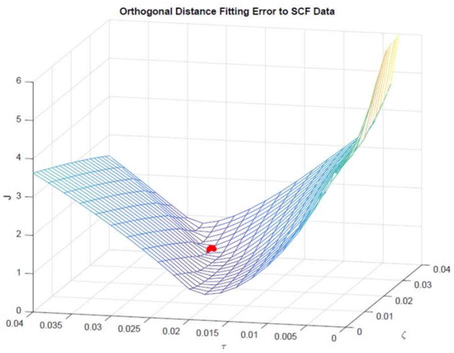

FIG. 2. Discrepancy $J(\tau, \zeta)$ plotted on (τ, ζ) plane, with minimum at $\tau = 0.024$ and $\zeta = 0.028$. Because $\zeta > \tau$, this indicating partial wealth condensation in U.S. in 2013.

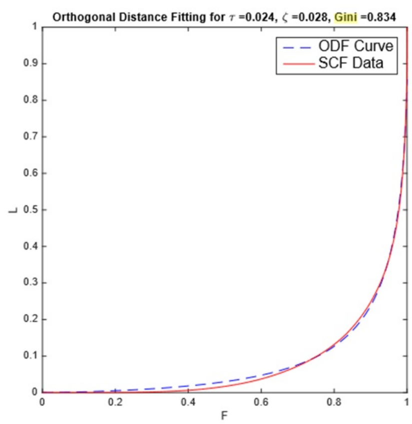

FIG. 3. Solution to Eq. (**??**) for $\tau = 0.024$ and $\zeta = 0.028$ (blue, dashed curve), and the actual Lorenz curve from the U.S. in 2013 (red curve).

result proved that $c = 0.12$, which means the oligarch of the system would hold 12% percent of total wealth in America. And the statistics shows that in 2013, the top 0.01% population in the U.S. holds 11.1% of total wealth, which matches with prediction quite well.

IV. CONCLUSION

The Asset-Exchange Models and its extension, Yard Sell Model is very useful in studying dynamics of wealth flow and redistribution due to its highly compressed but still efficient way of expressing human economic activities. It is essentially a Markovian random walk process which can be described by Fokker-Plank equation. Boghosians work demonstrate a way of solving the problem analytically and predicted wealth condensation phenomenon.

Following his method, one can add different parameters or changing the setups. One important topic studied by W. Klein et al. is that when the impact of economic growth to the equality. Here instead of a fixed amount of total wealth, they assumed that the total wealth W_P is growing with time while amount of individuals N_P stays constant. They found that the phrase A rising tide lifts all the boats is not necessarily true, but that the effect of growth on all levels of agents depends on the nature of growth. As the benets of growth are weighted more towards the wealthy inequality increases but the wealth of all agents in the system grows. However, when the benets of growth are skewed too much toward the wealthy the poor and middle class no longer benet from the growth and the richest agents eventually accrue all the wealth.

[1] B. M. Boghosian, A. Devitt-Lee, J. Li, J. A. Marcq and H. Wang. *Describing Realistic Wealth Distributions with the Extended Yard-Sale Model of Asset Exchange*, arXiv:1604.02370 [q-fin.GN],2016

[2] B. M. Boghosian, A. Devitt-Lee, H. Wang. *The Growth of Oligarchy in a Yard-Sale Model of Asset Exchange - A Logistic Equation for Wealth Condensation*, DOI: 10.5220/0005956501870193, 2016

[3] J. Angle. *Thesurplus theory of social stratification and the size distribution of personal wealth.*, Social Forces, 65:293-326, 1986

[4] B. Hayes. *Follow the money*, American Scientists, 90:400-405, 2002

[5] D. Hardoon and S. Ayele. *An economy for the 1%*, Technical report, Oxfam International. 210 Oxfam Briefing Paper (Oxfam Davos Report), 2016

[6] K. Liu, N. Lubbers, W. Klein, J. Tobochnik, B.Boghosian, H. Gould. *The Effect of Growth on Equality in Model s of the Economy*, 2013