A Study on the Foreign Exchange Market

Chon Kit Pun, Tianchi Chen

May 8, 2015

Abstract

In this report we investigated the foreign exchange (FX) market. First we looked at the correlation of return of currency pairs in four of the most traded currencies in the world: US Dollar (USD), Euro (EUR), Japanese Yen (JPY), British Pound (GBP) and two other currencies in Asia: Chinese Yuan (CNY) and Hong Kong Dollar (HKD). Next we studied the 'arbitrage of return' and the bid-ask spread time series of different combination of currencies. Interesting results were seen in the data and were discussed.

1 Introduction

The Foreign exchange market, or the FX market for short, is a decentralized, 24 hours a day 5 days a week market. Unlike a centralized market like the New York Stock Exchange (NYSE), there is no centralized physical location for trading in the FX market. Instead FX is an over-the counter (OTC) market where trading occurs in a peer-to-peer fashion without the supervision of an exchange. Another feature of the FX is high liquidity due to its large trading volume.

The FX market is divided into two categories: the spot market and the future/forward market. The spot market is where currencies are traded at the current price. To buy a currency using another currency one look at the ask price of the currency pair. To sell, one look at the bid price. The difference between the bid and the ask price is called the bid-ask spread. This spread is an indication of liquidity of the market. Low trading volume usually makes the spread larger and lowers the liquidity of the market. The future market is where contracts are traded. The contract called future is an agreement of one party to buy and the other party to sell a currency at a pre-determined fixed exchange rate at some specified future date. Futures are usually used for hedging and for mitigating risk associated with the changing of rate.

Factor affecting the exchange rate Here we are going to outline several most important factors affecting the FX rate. The first one is interest rate. There is a higher demand for a currency which country has a higher interest rate than the demand for a currency with lower interest rate. This is due to the less in the cost in borrowing from a low interest rate country and a higher return for investing in a higher interest rate country. Consequently the currency in high demand will increase in value with respect to the other currency with low demand.

The second one is inflation rate. An increase in inflation rate due to an increase in money supply will lower the value of the money simply because there is more supply of that currency in the market. When supply exceeds demand, the value of a currency drops.

The third one is terms of trade. When a (domestic) country imported more than it exported to another (foreign) country, there will be a net increase in demand of the foreign currency. This will in turn weaken the domestic currency value with respect to the foreign currency.

2 Data and methodology

We looked at six currencies in this report: USD, EUR, JPY, GBP, CNY and HKD from the period January 3 2000 to March 13 2015. The raw data we used [from OANDA] was the daily average exchange rate within that period of time. There were total number of 5549 trading days in this period.

The (logarithmic) return of the exchange rate of a currency pair is calculated as

$$r(t) = ln(S(t+1)/S(t)) \tag{1}$$

where S(t) is the daily average exchange rate at day t

3 Data Analysis

Correlation of return

We looked at the correlation of return of two currency pairs with a common base currency. Now lets have a look at figure 11 and 12, the return of JPY/CNY and JPY/HKD. By just look at those returns the time series look very similar. To verify this, figure 13 is the plot of the volatility for JC(short for JPY/CNY) and JH(short for JPY/HKD). The JH plot in black almost covered the JC plot in red. This volatility plot shows that JC and JH are extremely similar and their time series almost move at the same pace. What does it mean? This means that the currency CNY and HKD are highly correlated via currency JPY.

Here is another example of JE and JU. As shown in figure 14. The black series represents JE, the red one represents JU. This is figure is the plot of volatility of JE and JU. By looking at the plot. One can see that around certain values JE and JU are very concentrated. However, the one-dimensional series plot can not tell us around which particular value does the two currency correlated much. Therefore we plan to do a scattered plot of all six currencies. which turned out to give us a much intuitive picture about how different currencies correlates.

Let's first look at figure 15. This figure is the scatter plot for all other 5 currencies via CNY. Such 2-d scatter plot can directly tell us how two different currencies correlates. How strong is their correlation. In this figure which 5 by 5 matrix, the subplot in the first row, fourth column shows that there is a strong positive correlation be USD and HKD via CNY. Similarly, the other subplots shows more fuzziness. which means others pairs are not strongly correlated. Similar plots are seen in the figure 16 which is the scatter plot for GBP. It turns out that USD and CNY, USD and HKD, CNY and HKD are strongly correlated. other pairs are not.

Figure 17 which is the scatter plots for EUR shows some interesting features. the only strongly correlated currencies via EUR is USD and CNY. USD and HKD is partially positive strong correlated. By speculation, we think that maybe at some certain time due to certain policy the UDS and HKD are less positive correlated. Other pair currencies in this figure show more fuzziness which means less correlations.

The most interesting plot is figure 18. which turns out to have some discreteness feature between different pair currencies. The discreteness basically arise from the fact that the density plot of some currency turns out only have certain peak and it goes to zero at certain value. That is why the scattered plot shows discreteness. Although they are discrete. different pairs of currencies still show positively correlated. The CNY and HKD are the most strongly correlated one and there shows no discreteness between them. However, the discreteness feature can actually tell us more about two currencies via JPY. For example. the subplot in the first column, the fourth row. which shows that CNY and USD are positively correlated but only at discrete values. the middle line data points turns out to be less than the side lines. If we carry out a cluster analysis of this data. we could probably see that if the return of JC is at negative value, it is more likely that JU also take negative value around -0.005.

Arbitrage of return

Arbitrage means an opportunity for an investor to make a riskless profit whenever there is inefficiency in pricing in the market. A type of arbitrage in FX is the so-called triangle arbitrage. It basically involves exchange rates of three pairs of currency. For example we look at USD/EUR, EUR/GBP and GBP/USD. An arbitrage occurs if:

$$(USD/EUR) * (EUR/GBP) * (GBP/USD) \neq 1 \tag{2}$$

where each term on the left hand side are the ask price of the base currency. Obviously if equation (2) ever happened, investors will take advantage of this opportunity to make a riskless profit and subsequently change the demand for currencies in such a way that the product on the left hand side of equation (2) becoming 1. Such arbitrage opportunities do occur, but only exist of the order of seconds (Danial Fenn) before it vanishes. Therefore the resolution of our data did not allow us to investigate the behavior of this kind of arbitrage.

Given such constraint on our data, we instead looked at the 'arbitrage of return', which is defined as

$$R(USD/EUR)(t) * R(EUR/GBP)(t)–R(USD/GBP)(t) \tag{3}$$

where the R(*)(t) operator is the log return defined in equation (1). The motivation to look at the arbitrage of return is that it is natural to expect a daily change in exchange rate between A and C should be proportional to the change in exchange rate between A and B and that of B and C. For example, if USD/EUR and EUR/GBP both increases, say (USD/EUR)(t+1) = 1.05(USD/EUR)(t) and (EUR/GBP)(t+1) = 1.03(EUR/GBP)(t), then we should expect USD/GBP to also increases by (USD/GBP)(t+1) = (1.051.03)(USD/GBP)(t). Equation (3) will not be zero if there is any deviation from this expectation. Obviously the arbitrage of return is nota classical arbitrage in the sense that there is not necessarily an opportunity to make risk-less profit if equation (3) is non-zero. Instead, it looks at the trend of how the changes of exchange rates are related to each other. From now on we would use the word arbitrage and the phrase arbitrage of return interchangeably.

4 Time series of the arbitrage of return

Figure 1,2: arbitrage of return of UEG, UGE and EGU Figure 3: autocorrelation of the arbitrage of return

Three features could be seen from figure 1 and 2. First, the arbitrage of returns were very close to zero and fluctuated around zero. This matched with our prediction that equation (3) should not be hugely violated. Second, the time series were periodic with a period of seven days. This should not be surprise since returns are zero in weekends as trading are small or close to zero. Lastly, we observed occasional spikes which did not belong to the periodic structure of the time series. Those spikes were of order 5 to 10 times larger than the normal fluctuation.

Studying the autocorrelation of the arbitrage time series offered some additional insight. In figure 3 we looked at the arbitrage of return between the three pair of currencies: USD/EUR, EUR/GBP and USD/GBP. We observed that the autocorrelation was negative at day 1. An intuitive explanation for this behavior is that it is unlikely that the arbitrage will deviate further away from zero if it was already away from zero today. This seems to be reasonable to expect arbitrage to converge back to equilibrium (zero) quickly. Another feature observed in the autocorrelation is periodicity of the time series: the autocorrelation spikes at every seven days.

In summary, three features were observed in the arbitrage time series: periodicity, unexpected spikes and negative lag 1 autocorrelation.

5 Modeling

As discussed in previous section, arbitrage deviated from zero is likely to converge back to equilibrium the next day. Motivated by this intuition we propose the following model:

$$R(t+1) - R(t) = -\alpha R(t) + \beta \epsilon(t) \tag{4}$$

where R(t) is the same log return defined in equation (3);$\epsilon(t)$ is the Gaussian noise term with mean = 1 and standard deviation = 1 and α and β are positive coefficients. controls the sensitivity of future change in arbitrage to today's deviation, and β controls the randomness of the time series. What equation (4) means is that return of arbitrage is likely to change in the opposite direction of the deviation of arbitrage today. Rearranging equation (4) will give the following model for the arbitrage time series:

$$R(t+1) = (1-\alpha)R(t) + \beta \epsilon(t) \tag{5}$$

Which is also called the autoregressive AR(1) model.

To see how well the AR(1) model explain our data, we compared the autocorrelation of the AR(1) time series with the data. Figure 4 showed that both the model and the data have negative lag 1 autocorrelation. However, what the AR(1) could not describe is the periodicity of the data, which should not be surprised as we did not factor that feature into our model. Another feature the AR(1) could not capture is of course the spikes, which were mostly likely due to the injection of new information into the market that change the behavior of the investors dramatically.

Figure 4: autocorrelation of arbitrage and AR(1)

5.1 Bid-Ask spread

Figure 5,6: Arbitrage of return and bid-ask spread

Next we looked at the bid-ask spread time series of the three currencies pair used in the arbitrage analysis. Similar to the arbitrage time series, the bid-ask spread series also showed a periodicity of seven days and unexpected huge spikes. Figure 5 compared the arbitrage time series with the spread series in a three months window from mid May to mid August in 2008.

From figure 5, we noticed that spread are the highest in weekends. This could be explained by the fact that there are less trading, and thus less liquidity during weekends. Since it is less probable to match up sellers who are willing to sell at a lower price with buyers who are willing to buy at a higher price, spread increases. Another observation is that the spread for EUR/GBP is larger than the spread for USD/EUR and USD/GBP. This could be due to the fact that US dollar is traded much more often than Euro and Pound; and since dollar has a higher trading volume in comparison to the other two currencies, spread becomes smaller.

Figure 6 showed a surprising similarity of the location of spikes in the arbitrage time series and the spread time series. One difference, of course, is that the spread can only be negative (bid is always higher than ask) while the arbitrage of return can either be positive or negative. Given these observations, we instead looked at the 'return' of bid-ask spread and compared it with the arbitrage series.

Figure 7,8: Arbitrage of return and return of bid-ask spread

The return of bid-ask spread reproduces the periodic behavior of the arbitrage series and both share a similar profile. Figure 7 compared both time series in a three months window and showed how similar the return of spread and the arbitrage series looks. Next we looked at how well the return of spread described the spikes in the arbitrage. Figure 8 compared both time series in a longer (8 years) period. We noticed that the spike at around 2010 shows up in both the arbitrage and the return of spread series for EUR/GBP. Other smaller but noticeable spikes in the arbitrage, like the one in February 2007 and December 2009 were subtly seen in the return of spread in EUR/GBP. Both the return of spread of USD/GBP and USD/EUR, however, do not display a corresponding spike structure.

Next we repeated the analysis and looked at the arbitrage and the spread for USD/CNY, CNY/GBP and USD/GBP. Figure 9 and 10 compared both series and we saw that the return of spread of CNY/GBP and USD/CNY closely resembled the profile of the arbitrage series. The return of spread of USD/GBP, however, was different from that of the other two currency pairs and did not share a similar profile with the arbitrage series. This seemed to suggest a relationship between the return of spread and the arbitrage and in the next section we are going to discuss the possible explanations for these observations.

6 Discussion

Figure 5 and 7 showed that arbitrage is zero during weekends (Saturday to Sunday) but deviates from zero before (Friday to Saturday) and after (Sunday to Monday) the weekend. In contrast, bid-ask spread is largest during weekend (Saturday and Sunday). Therefore the arbitrage of return seemed not to be related to the bid-ask spread but was instead related to the change in spread. When we compared the arbitrage to the change of spread, we noticed that an increase in spread corresponded to a positive arbitrage while a decrease in spread corresponded to a negative arbitrage.

The sign of change in spread should not be random as we knew that spread increased when approaching weekends and decreased when approaching weekdays. When we looked at the return of spread we noticed that spread changes were usually positive before weekends and negative after weekends. On the other hand, there should not be a preference in the sign in the arbitrage: arbitrage fluctuated around zero and deviation from zero in either direction was equally likely. Therefore we expected that the sign of arbitrage before and after weekends should be random. However we consistently noticed that arbitrages were positive before weekend and negative after weekend. This is an interesting feature in the arbitrage time series that contradicted with our intuition.

The reason for the similarity between the change in spread and the arbitrage of return was speculated to be due to the fact that a higher spread makes the market less efficient and thus results in a higher arbitrage. In fact, we speculated that the two series reacted similarly with respect to a common factor: trading volume. As implied in previous section, a high spread is due to a low trading volume. We also speculated that a low trading volume will result in a higher arbitrage, applying the same argument of how the efficiency of the market affects the arbitrage. Also, the spread and the arbitrage might react with different magnitude with respect to the trading volume, which might explain why the spikes in the two series are not scaled properly.

Lastly, exchange rates are very much affected by monetary policies. As described in the introduction, interest rate affects the exchange rate directly. In this report we have omitted all other factors (e.g. interest rate, terms of trade etc) that actually had a significant effect on the exchange rate. In such sense there are more that we could incorporate in our analysis. Also, the resolution of our data introduced a significant error to the analysis. In particular, our daily average data covered up all the detail happening within one day and any drastic intraday changes in the exchange rate will yield a daily averages that deviate from our expectation (i.e. equation 3).

7 Conclusion

By doing two dimensional scattered plot. We see that it can gives us more information especially the correlation between different pair currencies via certain currency. Also some currency exchange return shows some discrete features. We noticed that the return of bid-ask spread and the arbitrage of return time series shared many similarities: both exhibit periodicity and spikes. We speculated that the similarity between these two series is due to a common underlying factor, such as trading volume, which both the spread and arbitrage are in theory closely related to. Also, the resolution of the data limited our scope of investigation and also introduced error to our analysis. Finally, acknowledging that exchange rate actually depends on many other factors, we believed that by incorporating other data such as interest rate would give a more complete analysis to our study.

Figure 1:

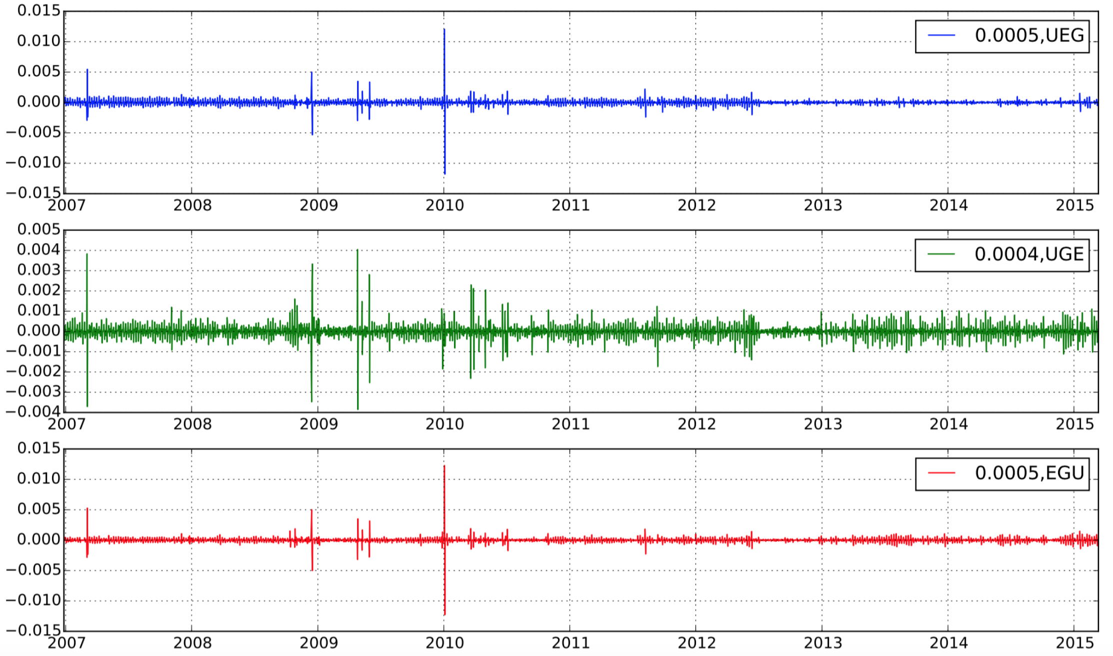

Figure 2:

Figure 3:

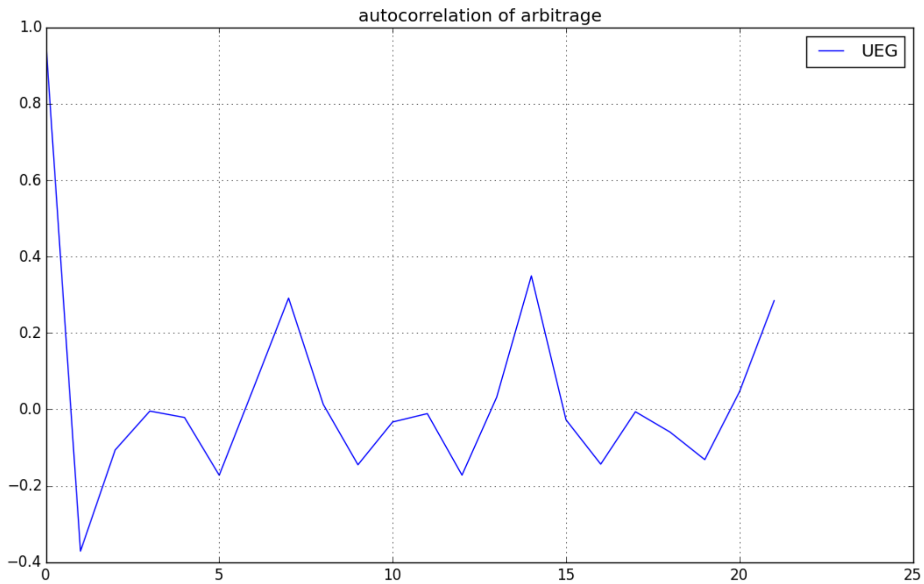

Figure 4:

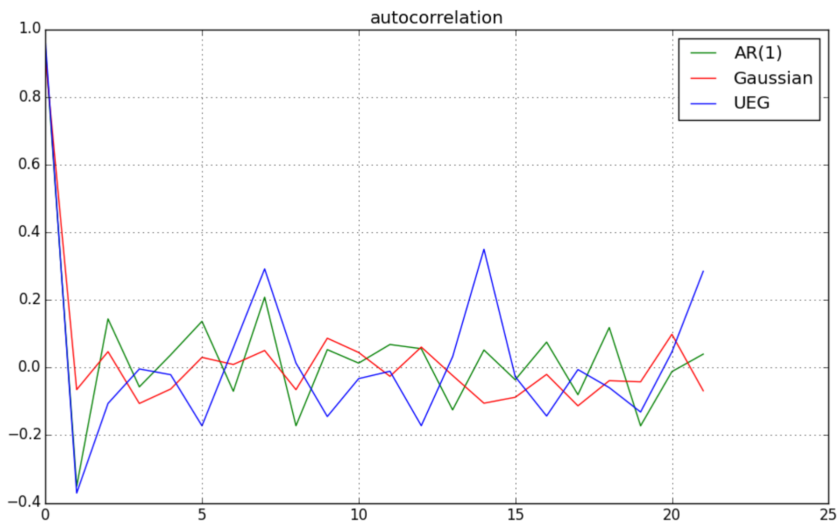

Figure 5:

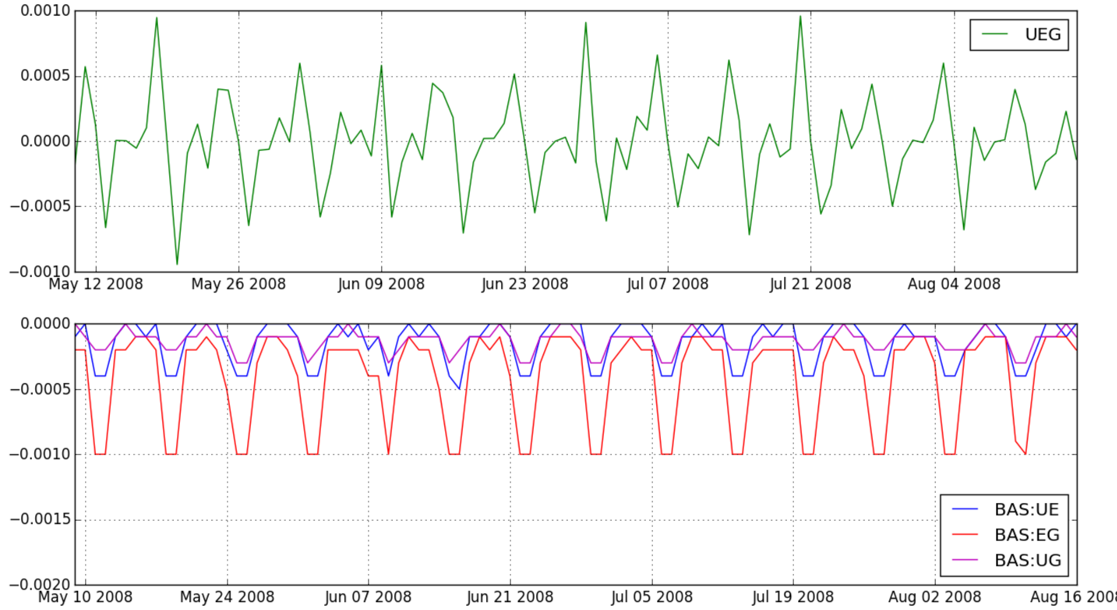

Figure 6:

Figure 7:

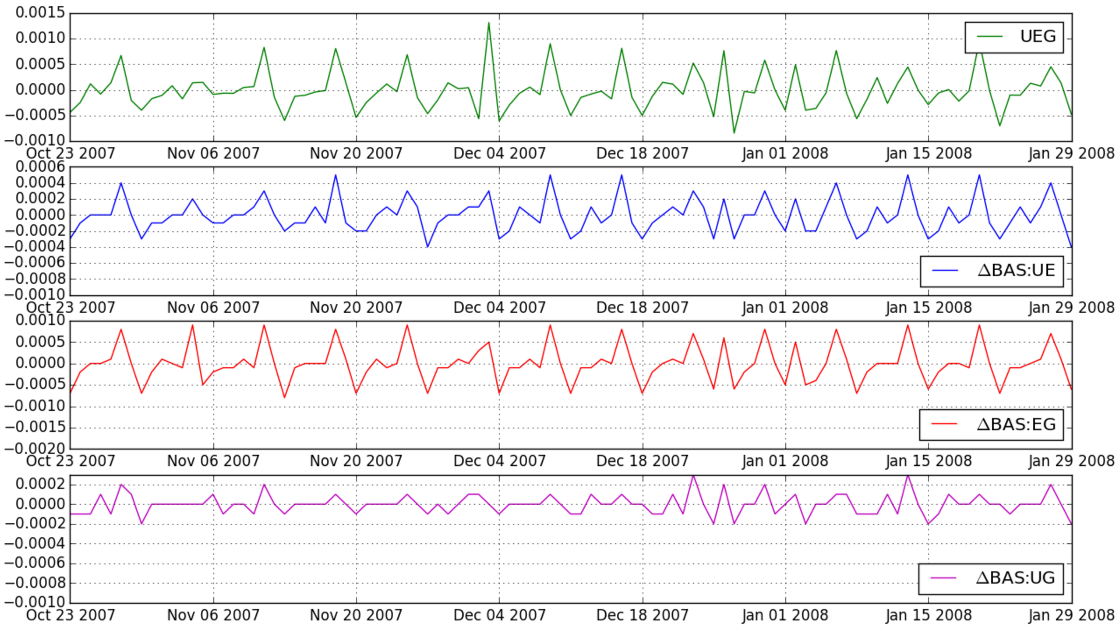

Figure 8:

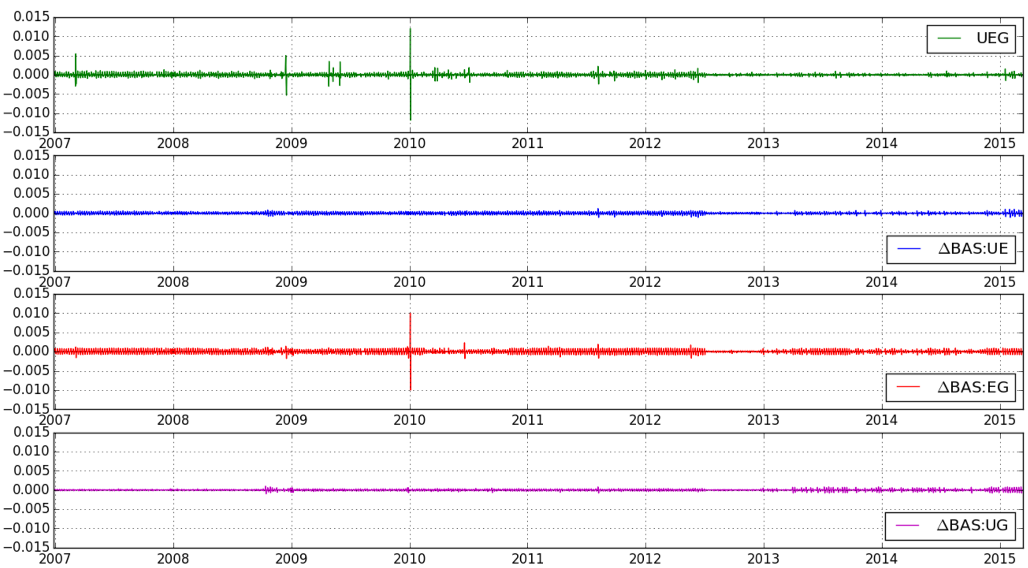

Figure 9:

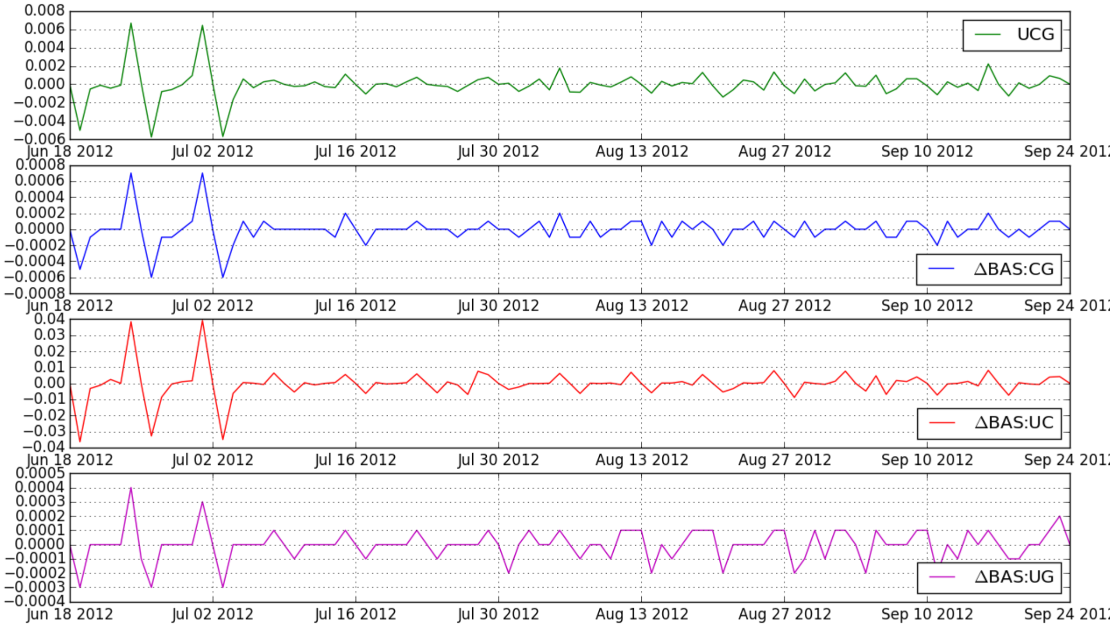

Figure 10:

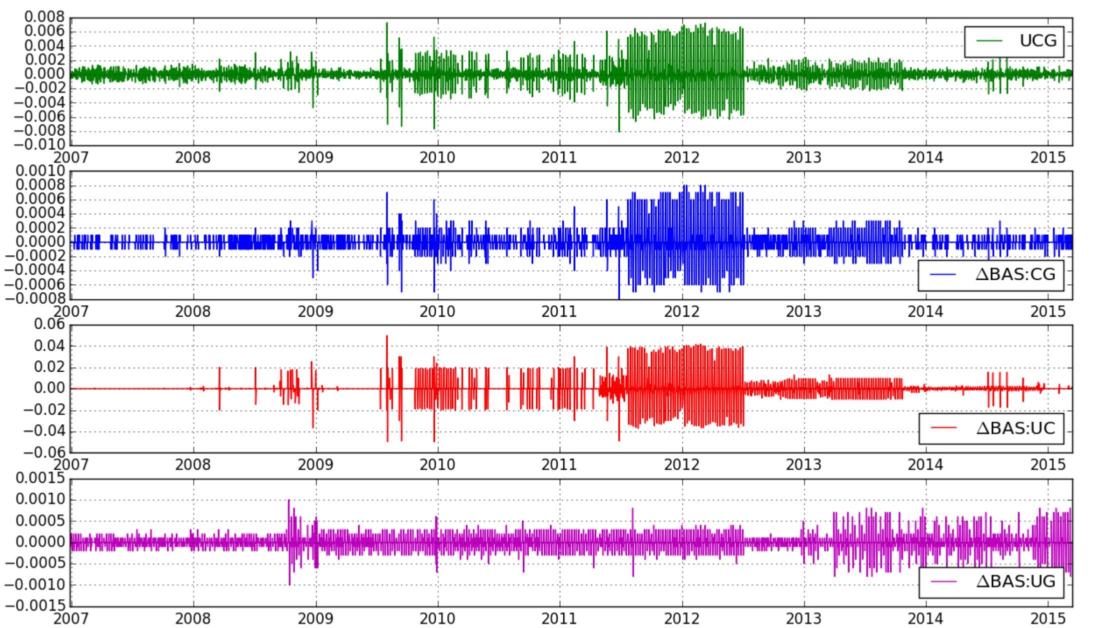

Figure 11: JC

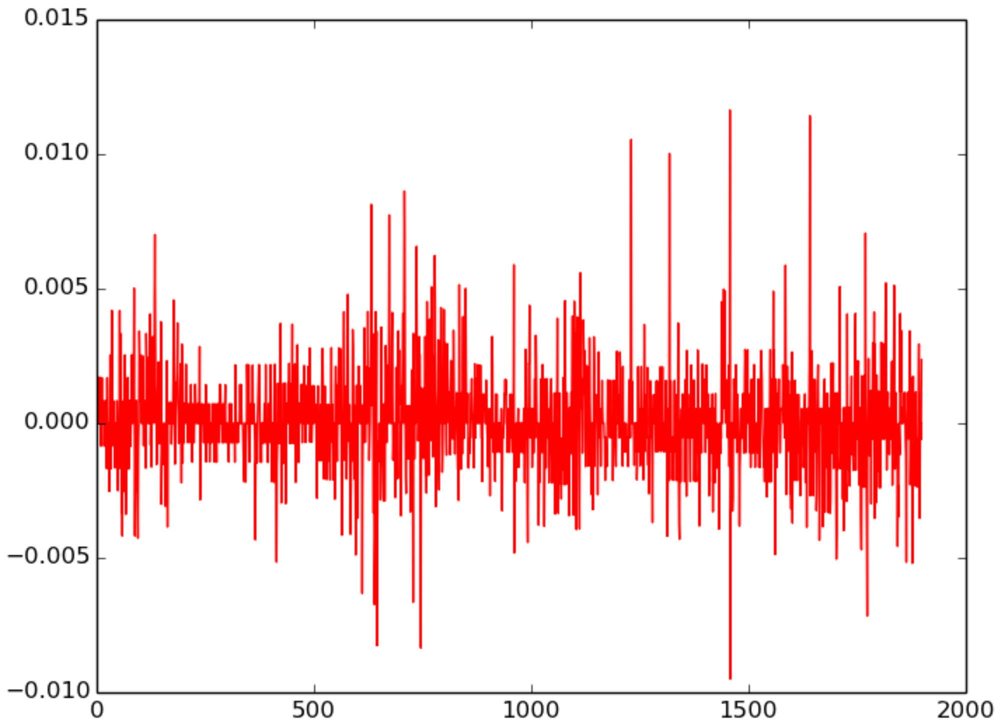

Figure 12: JH

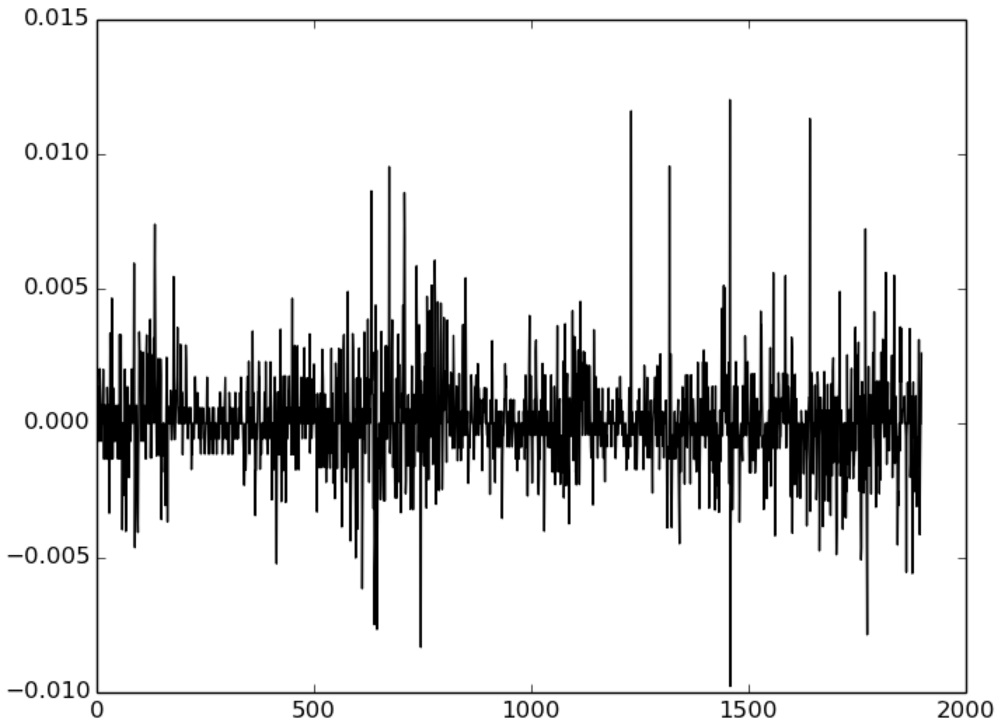

Figure 13: JC JH volatility: Black is JH, Red is JC

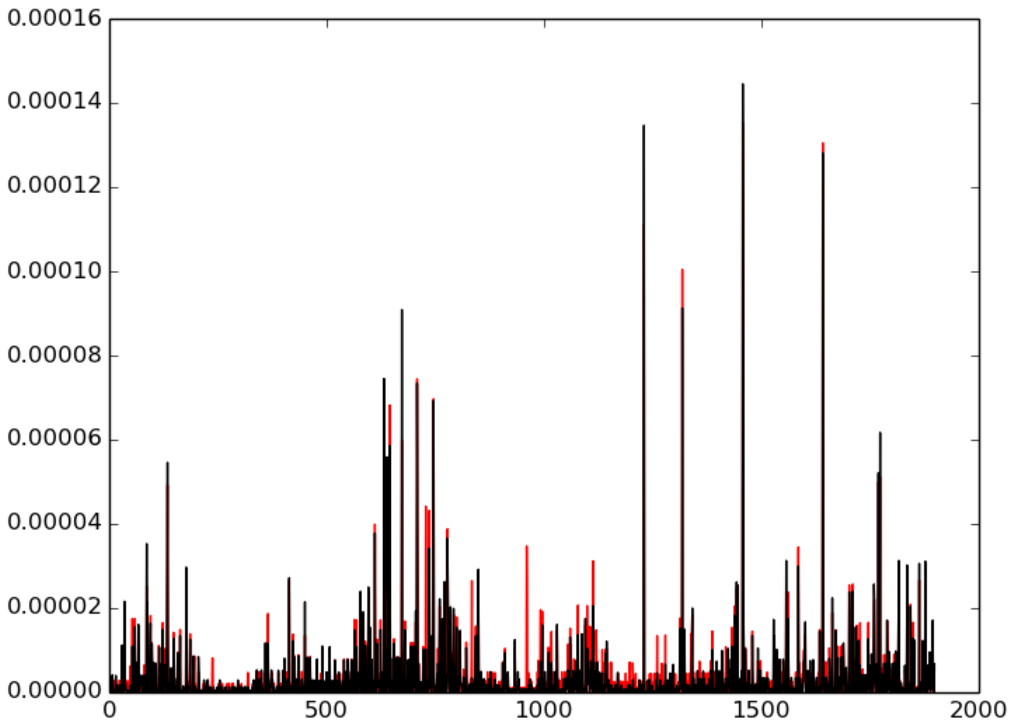

Figure 14: JE JU volatility: Black is JE, Red is JU

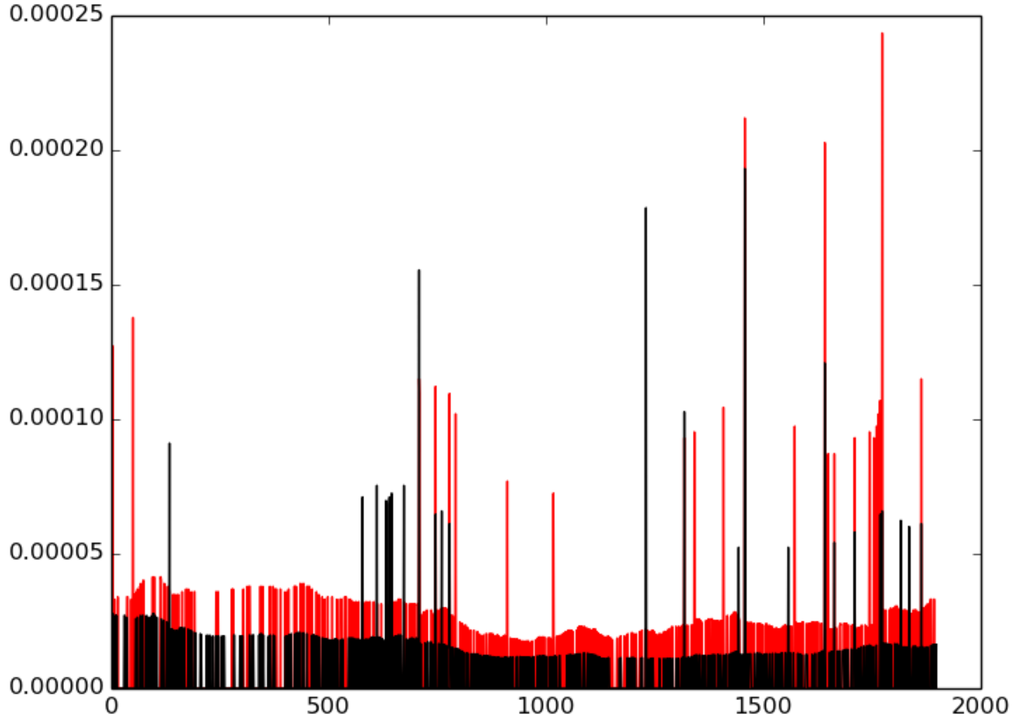

Figure 15: scatter plot for CNY

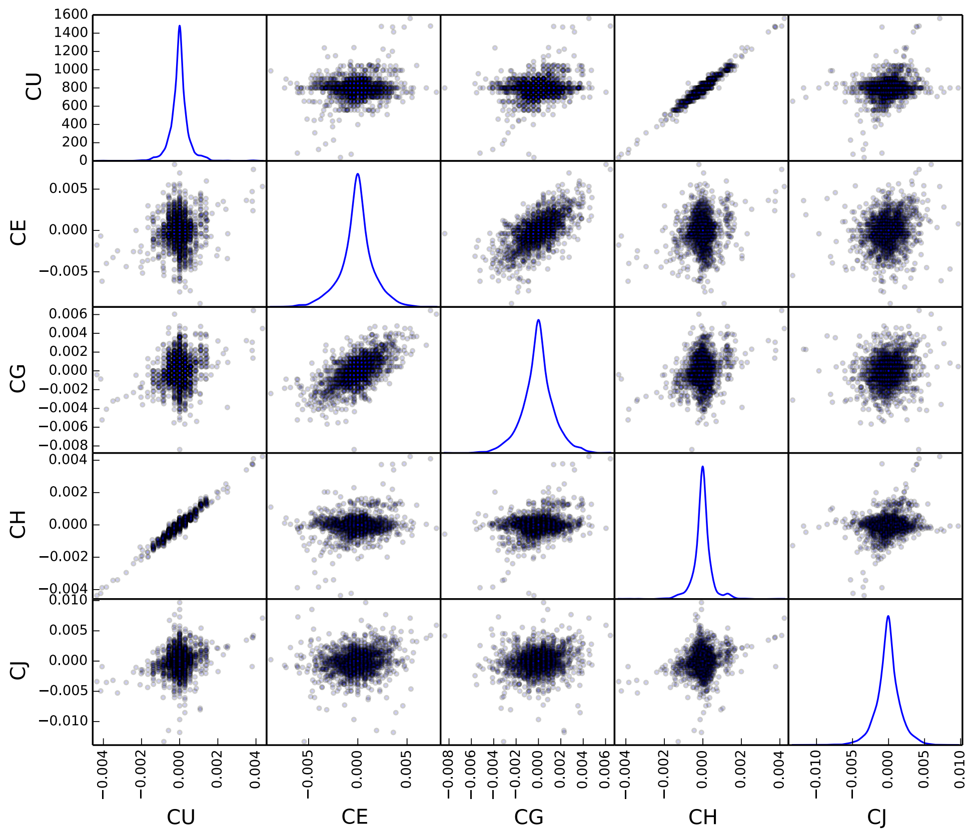

Figure 16: scatter plot for GBP

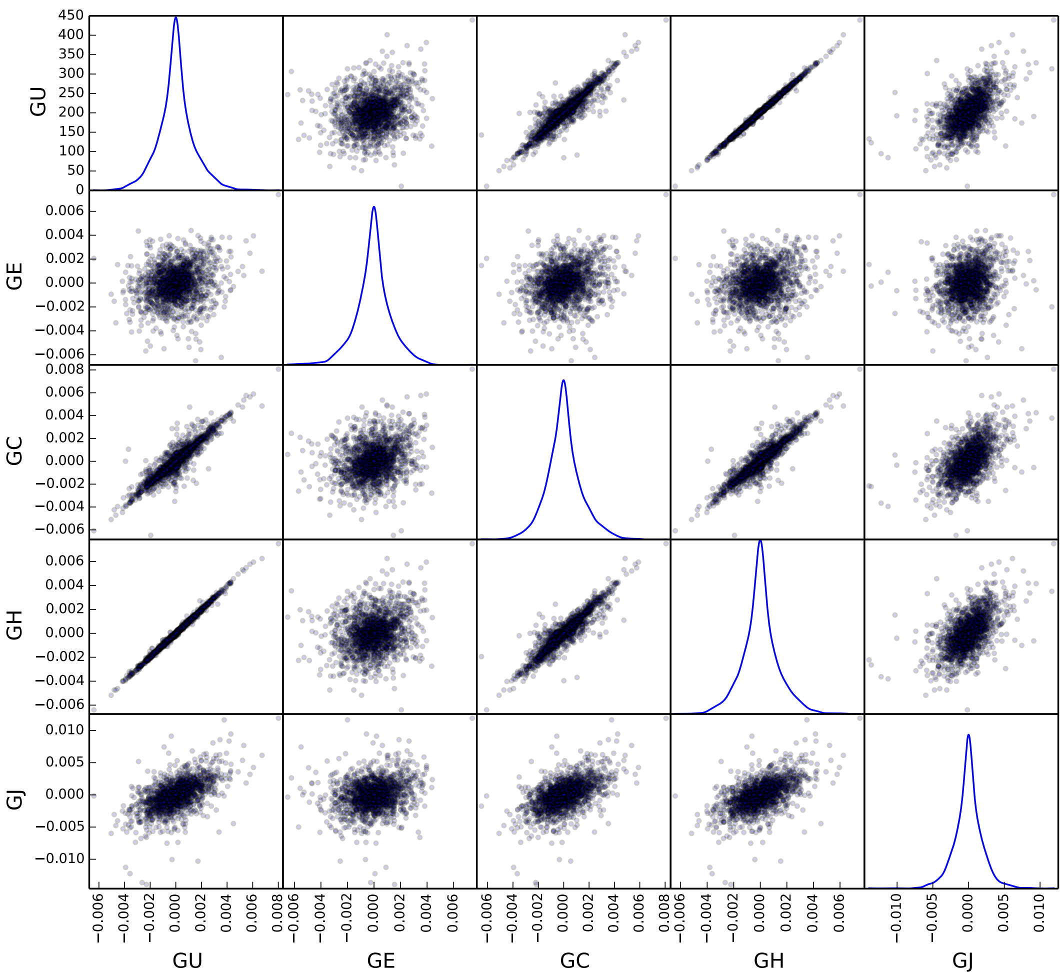

Figure 17: scatter plot for EUR

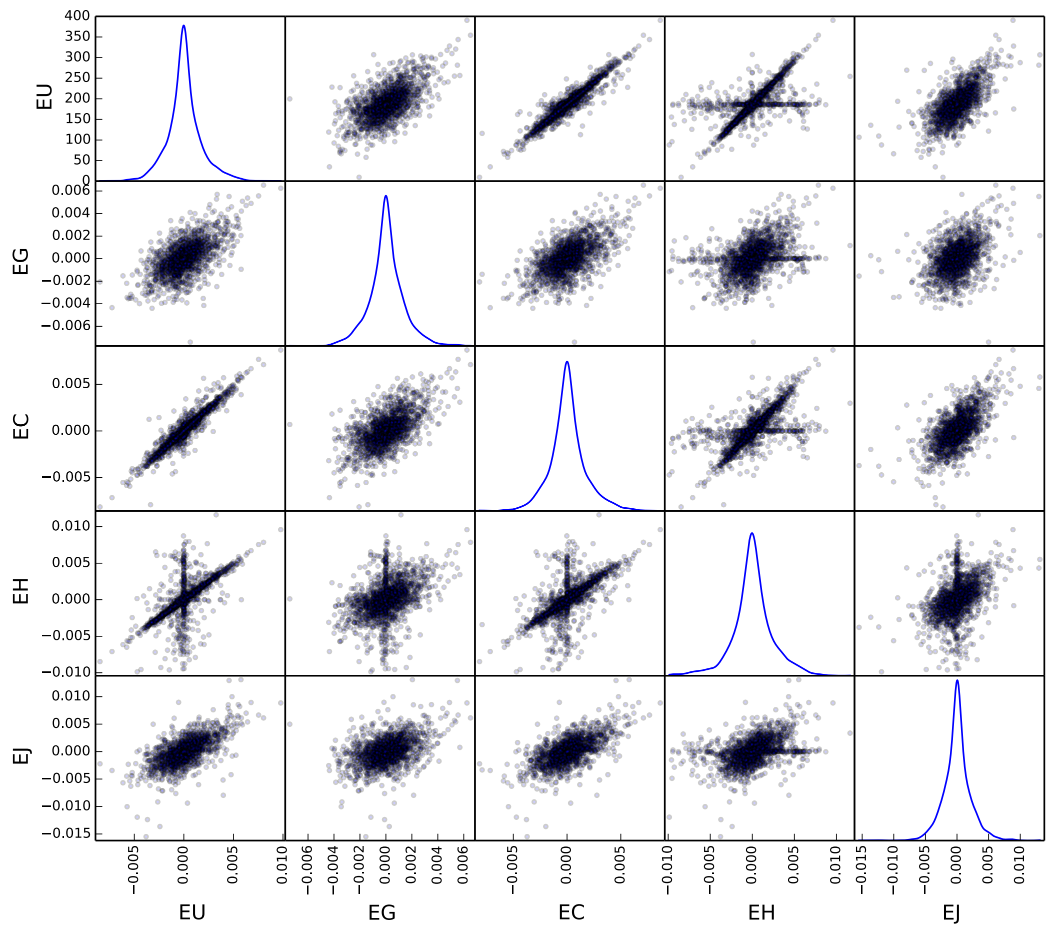

Figure 18: scatter plot for JPY